ITP-UU-14/02, SPIN-14/02

A photon propagator on de Sitter in covariant gauges

Silvije Domazet[a*] and Tomislav Prokopec[b1, 2, *]

[1a] *Theoretical Physics Division, Rudjer Bošković Institute, P.O.Box 180, HR-10002 Zagreb, Croatia*

[2b] *Institute for Theoretical Physics (ITP) & Spinoza Institute, Utrecht University, Postbus 80195, 3508 TD Utrecht, The Netherlands*

Abstract

We construct a de Sitter invariant photon propagator in general covariant gauges. Our result is a natural generalization of the Allen-Jacobson photon propagator in Feynman gauge. Our propagator reproduces the correct response to a point static charge and the one-loop electromagnetic stress-energy tensor, strongly suggesting that it is suitable for perturbative calculations on de Sitter.

PACS numbers: 98.80.-k, 04.62.+v

I. INTRODUCTION

In their classic paper Allen and Jacobson [1] have obtained the photon propagator in de Sitter invariant gauges. They have considered both massless and massive photons, in which case the propagator must be transverse on both legs (a minor error was subsequently corrected by Tsamis and Woodard in [2]). In this paper we generalize the result of Allen and Jacobson and derive a photon propagator in general covariant gauges, in which the photon propagator shows explicit dependence on the gauge parameter ξ $(-\infty < \xi < \infty)$. One can use this propagator to perform loop calculations of various quantities and investigate gauge independence of physical observables by studying whether they depend on ξ.

While the transverse Allen-Jacobson propagator was shown to give physically reasonable answers (see *e.g.* the two-loop stess-energy calculations [3–5] performed with the (corrected) transverse propagator from Ref. [2]), it was argued in [6] that the Allen-Jacobson massless photon propagator in Feynman gauge does not give an acceptable one-loop self-mass for a charged scalar field on de Sitter. The authors Kahya and Woodard attribute this unphysical behavior to the insistence of Allen and Jacobson to respect de Sitter symmetry. When a simple non-invariant propagator from Ref. [6] was used, one gets physically acceptable answers for the one-loop scalar self-mass. Yet, the two propagators were calculated in different gauges, and it remained unclear how the choice of gauge affects the self-mass, and the resulting (in principle measurable) change in the field amplitude. Of course, a photon propagator in general covariant gauges presented here allows to explicitly investigate how gauge dependence enters into quantities such as self-energies.

Vector propagators on de Sitter are useful for studying various (perturbative) quantum properties of theories on de Sitter space. Up to now, quantum loop effects on de Sitter have been investigated in scalar electrodynamics in Refs. [7–16] and in [3–6], where photon and scalar field mass generation has been studied, as well as quantum backreaction from created inflationary photons and charged scalars [3–5]. Recently, a study the one-loop quantum gravitational effects on the photon vacuum polarization on de Sitter has appeared in [14, 15, 17, 18], albeit gauge dependence of the results has not yet been properly addressed. Since de Sitter is the model space for inflation, understanding the physics of de Sitter is of crucial importance for understanding inflationary models (in which the Hubble parameter is an adiabatic function of time). An important problem of de Sitter space is known as linearization instability [19–22]. Even though de Sitter space it non-compact, spatial sections of de Sitter in global coordinates (with positively curved spatial sections) are compact, and hence for these sections the usual considerations apply, according to which no net charge can be placed on a compact space. This follows immediately from the Gauss's law

*sdomazet@irb.hr; T.Prokopec@uu.nl

$\nabla_\mu F^{\mu\nu} = (-g)^{-1/2}\partial_\mu[(-g)^{1/2}F^{\mu\nu}] = J^\nu$ which, when written in the integral form, implies

$$\int_{\Sigma_t} d^3x\sqrt{\gamma}\nabla_i F^{i0} = \int_{\Sigma_t} d^3x\partial_i[\sqrt{\gamma}F^{i0}] = \int_{\Sigma_t} d^3x\sqrt{\gamma}J^0 \equiv Q(t)\,, \tag{1}$$

where $Q(t)$ denotes the total charge on the spatial section Σ_t at time t and γ is the determinant of the spatial part of the metric (here it is assumed that one works in coordinates in which $g_{0i} = 0$ and g_{00} is independent on spatial coordinates). Eq. (1) can be also written as,

$$Q(t) = \int_{\partial\Sigma_t} dS\sqrt{\gamma_S}\hat{n}_i F^{i0}\,, \tag{2}$$

where n_i is the unit vector orthogonal to the boundary surface $S = \partial\Sigma_t$ and γ_S is the determinant of the induced two dimensional metric on $\partial\Sigma_t$. Since spatial sections of a compact space have no boundary, the integral in (2) must vanish, and therefore no net charge can be placed on a compact surface. De Sitter space is non-compact however, and hence strictly speaking this consideration applies only to global sections of de Sitter space, which are compact. In this paper we consider the response to a point charge on flat sections of de Sitter space, which are non-compact, and hence the above restriction does not apply.

Before embarking on calculations in de Sitter space-time we shall first lay out a calculation of the response to a static point charge on Minkowski space. The (Keldysh) photon propagator on Minkowski background in covariant gauges and in $D = 4$ space-time dimensions is given by,

$$\imath[_\mu\Delta_\nu^{ab}](x;x') = \eta_{\mu\nu}\imath\Delta_0^{ab}(x;x') - \imath(1-\xi)\partial_\mu\partial_\nu\int d^4x''\Delta_0^{ac}(x;x'')(\sigma^3)^{cd}\Delta_0^{db}(x'';x')\,, \tag{3}$$

where a, b, c, d (a summation over the repeated indices is assumed), $\sigma^3 = \mathrm{diag}(1,-1)$ and

$$\imath\Delta_0^{ab}(x;x') = \frac{1}{4\pi^2}\frac{1}{[\Delta x^{ab}(x;x')]^2}\,, \tag{4}$$

is the massless scalar propagator on Minkowski in four space-time dimensions and

$$[\Delta x^{++}(x;x')]^2 \;=\; -(|t-t'|-\imath\epsilon)^2 + \|\vec{x}-\vec{x}\,'\|^2\,, \qquad [\Delta x^{+-}(x;x')]^2 = -(t-t'+\imath\epsilon)^2 + \|\vec{x}-\vec{x}\,'\|^2 \tag{5}$$

$$[\Delta x^{--}(x;x')]^2 \;=\; -(|t-t'|+\imath\epsilon)^2 + \|\vec{x}-\vec{x}\,'\|^2\,, \qquad [\Delta x^{-+}(x;x')]^2 = -(t-t'-\imath\epsilon)^2 + \|\vec{x}-\vec{x}\,'\|^2\,. \tag{6}$$

The $(++)$ and $(--)$ components of the Keldysh propagator correspond to the Feynman and anti-Feynman propagator, respectively, and the $(+-)$ and $(-+)$ components are the negative and positive frequency Wightman functions, respectively, which are useful *e.g.* for non-equilibrium problems of statistical physics.

The electromagnetic potential for a static point charge can be obtained from the formula,

$$A_\mu(x) = \int d^4x'[_\mu\Delta_\nu^{\mathrm{ret}}](x;x')j^\nu(x')\,, \tag{7}$$

where the current density is given by

$$j^\nu = \delta^\nu_{\ 0}e\delta^3(\vec{x})\,, \tag{8}$$

and $[_\mu\Delta_\nu^{\mathrm{ret}}]$ is the retarded photon propagator given by

$$\begin{aligned}
[_\mu\Delta_\nu^{\mathrm{ret}}](x;x') \;&=\; [_\mu\Delta_\nu^{++}](x;x') - [_\mu\Delta_\nu^{+-}](x;x') \\
&=\; \eta_{\mu\nu}\left(\Delta_0^{++}(x;x') - \Delta_0^{+-}(x;x')\right) \\
&\quad -(1-\xi)\partial_\mu\partial_\nu\int d^4x''\left(\Delta_0^{++}(x;x'') - \Delta_0^{+-}(x;x'')\right)\left(\Delta_0^{++}(x'';x') - \Delta_0^{+-}(x'';x')\right)\,,
\end{aligned} \tag{9}$$

where

$$\begin{aligned}
\Delta_0^{++}(x;x') - \Delta_0^{+-}(x;x') \;&=\; -\frac{1}{2\pi}\theta(\eta-\eta')\delta\left((\eta-\eta')^2 - \|\vec{x}-\vec{x}\,'\|^2\right) \\
&=\; -\frac{1}{4\pi}\theta(\eta-\eta')\frac{\delta\left(\eta-\eta' - \|\vec{x}-\vec{x}\,'\|\right)}{\|\vec{x}-\vec{x}\,'\|}\,.
\end{aligned} \tag{10}$$

Inserting this into (9) yields,

$$[{}_\mu\Delta_\nu^{\rm ret}](x;x') \;=\; -\frac{\eta_{\mu\nu}}{4\pi}\frac{\delta\left(\eta-\eta'-\|\vec{x}-\vec{x}\,'\|\right)}{\|\vec{x}-\vec{x}\,'\|} - \frac{1-\xi}{16\pi^2}\partial_\mu\partial_\nu\int d^4x''\frac{\delta\left(\eta-\eta''-\|\vec{x}-\vec{x}\,''\|\right)}{\|\vec{x}-\vec{x}\,''\|}\frac{\delta\left(\eta''-\eta'-\|\vec{x}''-\vec{x}\,'\|\right)}{\|\vec{x}''-\vec{x}\,'\|}\,. \tag{11}$$

We can now insert this into the potential equation (7) and make use of the source (8). This gives

$$A_\mu(x) = \frac{e\delta^0_\mu}{4\pi r}\int_{\eta_0}^{\eta} d\eta'\,\delta\left(\eta-\eta'-r\right) - (1-\xi)\frac{e}{16\pi^2}\partial_\mu\partial_0\int d^3x''\int_{\eta_0}^{\eta}d\eta''\int_{\eta_0}^{\eta''}d\eta'\,\frac{\delta\left(\eta-\eta''-\|\vec{x}-\vec{x}\,''\|\right)}{\|\vec{x}-\vec{x}\,''\|}\frac{\delta\left(\eta''-\eta'-r''\right)}{r''}\,, \tag{12}$$

where $r=\|\vec{x}\|$, $r'=\|\vec{x}\,'\|$ and $r''=\|\vec{x}\,''\|$. Performing the necessary integrations (see Appendix A) results in,

$$A_\mu(x) \;=\; \frac{e\delta^0_\mu}{4\pi r}\theta(\Delta\eta_0-r) - (1-\xi)\frac{e}{4\pi}\partial_\mu\frac{1}{r}\int_0^{\Delta\eta_0}d\Delta\eta\Big\{\theta(\Delta\eta-r)-\theta(r)\theta(r-2\Delta\eta+\Delta\eta_0)+2\theta(r-\Delta\eta)-\theta(r-\Delta\eta_0)\Big\}\,, \tag{13}$$

where $\Delta\eta'=\eta-\eta'$ and $\Delta\eta_0=\eta-\eta_0$. This result can be written as

$$A_\mu(x)=\delta^0_\mu\mathcal{A}_0(x)+\partial_\mu\Lambda(x)\,, \tag{14}$$

where

$$\mathcal{A}_0(x)=\frac{e}{4\pi r}\theta(\Delta\eta_0-r)\,,\qquad \Lambda(x)=-\,(1-\xi)\frac{e}{8\pi}\theta(\Delta\eta_0-r)\Big(1+\frac{\Delta\eta_0}{r}\Big)\,. \tag{15}$$

From Eq. (14) it is clear that Λ in (15) is a pure gauge contribution, and does not affect physics. On the other hand, $\mathcal{A}_0$ contributes to the physical electric field as

$$\vec{E}=-\nabla A_0+\partial_0\vec{A}=-\nabla\mathcal{A}_0=\frac{e}{4\pi}\frac{\vec{r}}{r^3}\left\{\theta(\Delta\eta_0-r)+r\delta(\Delta\eta_0-r)\right\}\,,\qquad \vec{B}=\nabla\times\vec{A}=0\,. \tag{16}$$

The time dependence (on $\Delta\eta_0$) in the above expressions arises from unphysical initial conditions. Namely, the state at $\eta=\eta_0$ is chosen such as if there was no charge at $\eta<\eta_0$ which generates a light-cone starting at η_0 and propagating along $\Delta\eta_0=r$. Of course, one cannot create or destroy a charge, hence these initial conditions are unphysical, and one gets the physical answer by sending $\eta_0\to-\infty$, in which case all time dependence (in the physical part of the electric field) disappears and we get

$$\mathcal{A}_0=\frac{e}{4\pi r}\,,\qquad \vec{E}=\frac{e}{4\pi}\frac{\vec{r}}{r^3}\,,\qquad \vec{B}=0\,, \tag{17}$$

which is (obviously) the correct answer.

II. CALCULATING THE PROPAGATOR

After giving a short account of the calculation in Minkowski space we now turn our attention to solving the equivalent problem in de Sitter space. The relevant curved space action is given by

$$S_{\rm EM}=\int d^4x\sqrt{-g}\Big[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{1}{2\xi}(\nabla_\mu A^\mu)^2\Big]\,, \tag{18}$$

where $F_{\mu\nu}=\nabla_\mu A_\nu-\nabla_\nu A_\mu=\partial_\mu A_\nu-\partial_\nu A_\mu$ is the field strength and the last term is added as a covariant gauge fixing term ($\xi\in(-\infty,+\infty)$). By varying the action (18) one gets that the photon field satisfies the equation,

$$L^{\mu\nu}A_\nu=0\,. \tag{19}$$

where

$$L^{\mu\nu}=g^{\mu\nu}\Box-\nabla^\nu\nabla^\mu+\frac{1}{\xi}\nabla^\mu\nabla^\nu=g^{\mu\nu}\Box-\Big(1-\frac{1}{\xi}\Big)\nabla^\mu\nabla^\nu-R^{\mu\nu}\,. \tag{20}$$

Here $R^{\mu\nu}$ denotes the Ricci tensor which in de Sitter space equals to $H^2(D-1)g^{\mu\nu}$. The equation satisfied by the photon propagator is then

$$L^{\mu\nu}(x)\imath[_\nu\Delta_\alpha](x;x') = \left[g^{\mu\nu}\Box - \nabla^\nu\nabla^\mu + \frac{1}{\xi}\nabla^\mu\nabla^\nu\right]\imath[_\nu\Delta_\alpha](x;x') = (\partial^\mu\partial'_\alpha y)\frac{\imath\delta^D(x-x')}{\sqrt{-g}(-2H^2)}\,, \tag{21}$$

plus the equation $L^{\beta\alpha}(x')\imath[_\nu\Delta_\alpha](x;x') = (\partial'^\beta\partial_\nu y)[\imath\delta^D(x-x')/(-2H^2\sqrt{-g})]$, which is automatically satisfied when the following exchange symmetry is imposed,

$$\imath[_\nu\Delta_\alpha](x;x') = \imath[_\alpha\Delta_\nu](x';x)\,. \tag{22}$$

The factor $-2H^2$ in the denominator on the right hand side of (21) is chosen to ensure the correct normalization of the propagator. Indeed, because of the delta function, $(\partial^\mu\partial'_\alpha y)$ on the right hand side of Eq. (21) can be replaced by $-2H^2\delta^\mu_\alpha$, such that in the limit when $H\to 0$ the propagator (21) reduces to its Minkowski counterpart (3), as it should. Our *Ansatz* for the propagator consists of two de Sitter invariant tensor structures, each multiplying a de Sitter invariant scalar structure function, so that

$$\begin{aligned}\imath[_\nu\Delta_\alpha](x;x') &= (\partial_\nu\partial'_\alpha y)\times f_1(y) + (\partial_\nu y)\times(\partial'_\alpha y)\times f_2(y)\\ &= (\partial_\nu\partial'_\alpha y)\times A_1(y) + \partial_\nu\partial'_\alpha A_2(y)\,,\end{aligned} \tag{23}$$

where

$$y(x;x') \equiv y^{++}(x;x')\,,\qquad y^{ab}(x;x') = aa'H^2[\Delta x^{ab}(x;x')]^2\qquad (a,b=+,-)\,, \tag{24}$$

where $[\Delta x^{ab}(x;x')]^2$ are given in Eqs. (5–6). The latter form in (23) is motivated by the tensor structure of the photon propagator on Minkowski space, and as we will see below it can be used to significantly simplify our equations for the scalar structure functions.

A. Solving the photon equation on de Sitter

In Appendix B we have shown how the photon operator (20) acts on the propagator in the equation of motion (21), when the propagator is represented in terms of de Sitter invariant tensor structures (more precisely bi-vectors) and scalar structure functions $f_1(y)$ and $f_2(y)$ as in Eq. (23). Since the two tensor structures in appendix (B5) are mutually independent, Eq. (B5) implies the following two scalar equations,

$$(4y-y^2)f_1'' + \Big(D-1+\frac{1}{\xi}\Big)(2-y)f_1' - \frac{D}{\xi}f_1 - \Big(1-\frac{1}{\xi}\Big)(4y-y^2)f_2' - \Big(D-1-\frac{D+1}{\xi}\Big)(2-y)f_2 = \frac{\imath\delta^D(x-x')}{H^2\sqrt{-g}(-2H^2)} \tag{25}$$

$$-\Big(1-\frac{1}{\xi}\Big)(2-y)f_1'' + \Big(D-1-\frac{D+1}{\xi}\Big)f_1' + \frac{4y-y^2}{\xi}f_2'' + \Big(1+\frac{D+3}{\xi}\Big)(2-y)f_2' - \Big((D-1)+\frac{D+1}{\xi}\Big)f_2 = 0\,. \tag{26}$$

It turns out that it is more convenient to represent these equations in the $A_1 - A_2$ basis, defined in the second line of Eq. (23), which implies,

$$f_1 = A_1 + A_2'\,;\qquad f_2 = A_2''\,. \tag{27}$$

The rationale for this choice will soon become apparent. Namely, in this basis Eqs. (25–26) become,

$$(4y-y^2)A_1'' + \Big((D-1)+\frac{1}{\xi}\Big)(2-y)A_1' - \frac{D}{\xi}A_1 + \frac{1}{\xi}(4y-y^2)A_2''' + \frac{D+2}{\xi}\Big)(2-y)A_2'' - \frac{D}{\xi}A_2' = \frac{\imath\delta^D(x-x')}{(-2H^4)\sqrt{-g}} \tag{28}$$

$$-\Big(1-\frac{1}{\xi}\Big)(2-y)A_1'' + \Big((D-1)-\frac{D+1}{\xi}\Big)A_1' + \frac{1}{\xi}(4y-y^2)A_2'''' + \frac{D+4}{\xi}(2-y)A_2''' - \frac{2(D+1)}{\xi}A_2'' = 0\,. \tag{29}$$

The latter equation (29) can be easily integrated once to give,

$$-\Big(1-\frac{1}{\xi}\Big)(2-y)A_1' + \Big(D-2-\frac{D}{\xi}\Big)A_1 + \frac{1}{\xi}(4y-y^2)A_2''' + \frac{D+2}{\xi}(2-y)A_2'' - \frac{D}{\xi}A_2' = 0\,, \tag{30}$$

where we set the integration (y-independent) constant to zero. By inserting Eq. (30) into Eq. (28) we see that the equation for A_1 decouples from that for A_2. The result is the following inhomogeneous Gauss's hypergeometric equation for A_1, with the usual delta function source on the light-cone:

$$\frac{1}{H^2}\Big(\Box - (D-2)H^2\Big)A_1(y) \equiv (4y-y^2)A_1'' + D(2-y)A_1' - (D-2)A_1 = \frac{\imath\delta^D(x-x')}{(-2H^4)\sqrt{-g}}\,, \tag{31}$$

such that the de Sitter invariant photon equation on de Sitter space for the gauge invariant scalar structure function reduces to that of a scalar field with a (photon) mass term given by

$$m_A^2 = (D-2)H^2\,. \tag{32}$$

Furthermore, Eq. (31) is gauge independent (any dependence on ξ has dropped out). This is a very welcome feature since it tells us that the $A_1 - A_2$ basis (27) separates the photon propagator into a gauge independent part and a gauge dependent part.

Requiring that at light-cone the propagator reduces to a Hadamard form yields a unique solution to the Feynman (time ordered) propagator in Eq. (31) (*cf. e.g.* the Appendix in Ref. [23]),

$$\imath\Delta(x;x') \equiv A_1(y(x;x')) = -\frac{H^{D-4}}{2(4\pi)^{D/2}}\frac{\Gamma(\frac{D-1}{2}+\nu_D)\Gamma(\frac{D-1}{2}-\nu_D)}{\Gamma(\frac{D}{2})} \times {}_2F_1\Big(\frac{D-1}{2}+\nu_D,\frac{D-1}{2}-\nu_D;\frac{D}{2};1-\frac{y}{4}\Big), \tag{33}$$

where

$$\nu_D^2 = \Big(\frac{D-1}{2}\Big)^2 - \frac{m^2}{H^2}\,; \qquad \frac{m^2}{H^2} = D-2 \longrightarrow \nu_D = \frac{D-3}{2} \tag{34}$$

and

$$y(x;x') \equiv y^{++}(x;x') = a(\eta)a(\eta')H^2\big[-(|\eta-\eta'|-\imath\epsilon)^2 + \|\vec{x}-\vec{x}\,'\|^2\big]\,.$$

To get the other three elements of the 2×2 Keldysh propagator, one needs to replace $y = y^{++}$ by y^{--} and $y^{\pm\mp}$ in (33), respectively, see Eq. (24). Since $\nu_D = (D-3)/2$, the general solution (33) can be simplified to,

$$A_1(y) = -\frac{H^{D-4}}{2(4\pi)^{D/2}}\frac{\Gamma(D-2)}{\Gamma(\frac{D}{2})} \times {}_2F_1\Big(D-2,1;\frac{D}{2};1-\frac{y}{4}\Big)\,. \tag{35}$$

The gauge independent part of the Keldysh propagator is then simply,

$$\imath\Delta^{ab}(x;x') = -\frac{H^{D-4}}{2(4\pi)^{D/2}}\frac{\Gamma(D-2)}{\Gamma(\frac{D}{2})} \times {}_2F_1\Big(D-2,1;\frac{D}{2};1-\frac{y^{ab}(x;x')}{4}\Big)\,; \qquad (a,b,=\pm)\,. \tag{36}$$

If we are interested in the behavior of A_1 near the light cone ($y\sim 0$), then Eq. (9.131.2) of Gradshteyn and Ryzhik [25] can be used to transform (35) into

$$A_1(y) = -\frac{H^{D-4}}{2(4\pi)^{D/2}}\Big[\Gamma\Big(\frac{D-2}{2}\Big)\Big(\frac{y}{4}\Big)^{-(D-2)/2} \times {}_1F_0\Big(\frac{D-2}{2};\frac{y}{4}\Big) - \frac{\Gamma(D-2)}{\Gamma(\frac{D}{2})} \times {}_2F_1\Big(D-2,1;\frac{D}{2};\frac{y}{4}\Big)\Big]\,. \tag{37}$$

Obviously, it is the first part of the propagator (37) that yields the Hadamard behavior near the light-cone, $A_1(y) \propto y^{-(D-2)/2} \propto (\Delta x^2)^{-(D-2)/2}$, where $\Delta x^2 \simeq 0$.

What remains to be done is to solve for A_2, which can be done by solving the following inhomogeneous equation (30),

$$(4y-y^2)A_2''' + (D+2)(2-y)A_2'' - DA_2' = -\big(1-\xi\big)(2-y)A_1' + \big(D-(D-2)\xi\big)A_1\,, \tag{38}$$

Since we have previously established that A_1 is independent of the gauge parameter ξ, from Eq. (38) we see that A_2 depends on ξ. Next, we can integrate Eq. (38) once to get

$$\begin{aligned}(4y-y^2)A_2'' + D(2-y)A_2' &= \frac{1}{(4y-y^2)^{(D-2)/2}}\frac{d}{dy}\Big[(4y-y^2)^{\frac{D}{2}}A_2'\Big] \\ &= -(1-\xi)(2-y)A_1 + \Big((D-1)-(D-3)\xi\Big)I[A_1] \equiv s_\xi(y)\,. \end{aligned} \tag{39}$$

What this tells us is that the Green's function for A_2 is that of the scalar d'Alembertian, $\Box G(x;x') = H^2\delta^D(x-x')/\sqrt{-g}$, which is known to have no de Sitter invariant solution of a Hadamard form. This can be easily seen from the expression after that first equality in (39), which can be easily integrated, to yield (up to a constant) for the Green's function of A_2,

$$\imath G_{A_2}(x;x') \propto \int\frac{dy}{(4y-y^2)^{D/2}} \propto (4y-y^2)^{1-\frac{D}{2}} \times {}_2F_1\Big(1,2-D;2-\frac{D}{2};\frac{y}{4}\Big)\,. \tag{40}$$

Acting with the scalar d'Alembertian on this solution, and taking a careful account of the $\imath\epsilon$ prescription (for the time ordered propagator) reveals that this solution is a response to a source $\propto \delta^D(x-x')$ located at the light-cone where $y=0$, as it should, but also to an additional source at the antipodal point, where $y=4$ (because of the $\propto (4-y)^{1-D/2}$ behavior in (40) close to $y=4$). Since there is no source at the antipodal point, this behavior is unphysical. This simply means that there exists no de Sitter invariant Green's function of the Hadamard form that solves Eq. (39). One can proceed in two ways: (a) ignore that problem and write down a de Sitter invariant form for the solution (even though that means that a fictitious source at the antipodal point will also contribute), or (b) construct a proper Green's function for the problem that respects the Hadamard form but breaks de Sitter symmetry. In the light of this discussion, it is unclear to us how Allen and Jacobson in [1] could have obtained a photon propagator that is both de Sitter invariant and of the Hadamard form.

To be more concrete, choosing the de Sitter invariant option leads to

$$\imath G_{A_2} = \frac{2^{D-5}H^{D-2}\Gamma\left(\frac{D-2}{2}\right)}{\pi^{D/2}}(4y-y^2)^{1-D/2} \times \left\{{}_2F_1\Big(1,2-D;2-\frac{D}{2};\frac{y}{4}\Big) - {}_2F_1\Big(1,2-D;2-\frac{D}{2};1-\frac{y}{4}\Big)\right\}, \tag{41}$$

where in order to cancel the annoying constant term $\propto 1/(D-4)$ that arises when expanding the hypergeometric function in (40) around $D=4$, we have added a second hypergeometric function which also solves the same equation. This is legitimate, since the indefinite integral in (40) is defined up to a constant. The expression (41) leads to the following $D\to 4$ limit

$$\imath G_{A_2} = \frac{H^2}{4\pi^2}\left\{\frac{1}{y} - \frac{1}{2}\ln y - \frac{1}{4-y} + \frac{1}{2}\ln(4-y)\right\}, \tag{42}$$

which again clearly exhibits the presence of an unphysical source at the antipodal point at $y=4$. As we have already mentioned, avoiding this problem by resorting to the second option means one could take a propagator that respects spatial translations but breaks the more general de Sitter symmetry [24]

$$\begin{aligned}\imath\Delta_{\rm new}(x;x') \;=\; & \frac{H^{D-2}}{(4\pi)^{D/2}}\left\{-\sum_{n=0}^{\infty}\frac{1}{n-\frac{D}{2}+1}\frac{\Gamma\big(n+\frac{D}{2}\big)}{\Gamma\big(n+1\big)}\Big(\frac{y}{4}\Big)^{n-(D/2)+1} - \frac{\Gamma\big(D-1\big)}{\Gamma\big(\frac{D}{2}\big)}\pi\cot\Big(\frac{\pi D}{2}\Big)\right.\\ & \left. + \sum_{n=1}^{\infty}\frac{1}{n}\frac{\Gamma\big(n+D-1\big)}{\Gamma\big(n+\frac{D}{2}\big)}\Big(\frac{y}{4}\Big)^{n} + \frac{\Gamma\big(D-1\big)}{\Gamma\big(\frac{D}{2}\big)}\ln(aa')\right\},\end{aligned} \tag{43}$$

for which the $D\to 4$ limit is given by

$$\imath\Delta_{\rm new}(x;x') \;\overset{D\to 4}{\longrightarrow}\; \frac{H^2}{(4\pi)^2}\left\{\frac{4}{aa'H^2(\Delta x)^2} - 2\ln\left(\frac{H^2(\Delta x)^2}{4}\right) - 1\right\} \tag{44}$$

For both choices the Minkowski limit ($H\to 0$, $a,a'\to 1$) gives the massless scalar propagator on Minkowski space, as it should, namely

$$\imath\Delta_{\rm new}(x;x') \;\overset{\rm Mink}{\longrightarrow}\; \frac{\Gamma\big(\frac{D-2}{2}\big)}{4\pi^{D/2}}\frac{1}{(\Delta x^2)^{\frac{D-2}{2}}}, \tag{45}$$

which for $D\to 4$ reduces to

$$\imath\Delta_{\rm new}(x;x') \;\overset{D\to 4,{\rm Mink}}{\longrightarrow}\; \frac{1}{4\pi^2}\frac{1}{(\Delta x)^2}. \tag{46}$$

Once equipped with the Green's function for A_2 we can write the solution for A_2 as

$$A_2^{ab}(x;x') = H^2\int d^Dx'' G^{ac}_{A_2}(x;x'')(\sigma^3)^{cd}s_\xi^{db}(x'';x'), \tag{47}$$

where the source is given by,

$$s_\xi(x'';x') = \{-(1-\xi)(2-y)A_1(y) + \big((D-1)-(D-3)\xi\big)I[A_1](y)\}(x'';x'). \tag{48}$$

Having in mind the result (35) we see that finding the contribution of $I[A_1]$ to the above expression amounts to evaluating the following integral

$$\int dy\; {}_2F_1\Big(D-2,1;\frac{D}{2};1-\frac{y}{4}\Big). \tag{49}$$

After making the substitution, $z = 1 - y/4$, the relevant integral is

$$\int dz\ {}_2F_1\Big(a, b; c; z\Big) = \frac{c-1}{a-1} \times \frac{{}_2F_1\Big(a-1, b-1; c-1; z\Big) - 1}{b-1}\,, \tag{50}$$

where for convenience we have added an integration constant. This gives

$$\begin{aligned} I[A_1^{db}(y)] &= \frac{H^{D-4}}{(4\pi)^{D/2}}\frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \times \frac{1}{D-3}\lim_{b\to 1}\left[\frac{{}_2F_1\Big(D-3, b-1; \frac{D-2}{2}; 1-\frac{y^{db}}{4}\Big) - 1}{b-1}\right] \\ &= \frac{2H^{D-4}}{(4\pi)^{D/2}}\sum_{n=1}^{\infty}\frac{\Gamma(D-3+n)}{\Gamma(\frac{D}{2}-1+n)}\frac{(1-y^{db}/4)^n}{n}\,, \end{aligned} \tag{51}$$

and the $D \to 4$ limit of this result is

$$I[A_1^{db}] = -\frac{1}{8\pi^2}\ln\left(\frac{y^{db}}{4}\right). \tag{52}$$

Since in the same limit

$$A_1^{ab}(y) = -\frac{1}{8\pi^2}\frac{1}{y^{ab}}\,, \tag{53}$$

the corresponding expression for s_ξ (48) in $D = 4$ is

$$s_\xi^{db}(x''; x') = \frac{1}{8\pi^2}\left\{(1-\xi)\left(\frac{2}{y^{db}} - 1\right) - (3-\xi)\ln\left(\frac{y^{db}}{4}\right)\right\}(x''; x')\,. \tag{54}$$

Together with the choice for the Green's function for A_2, which we take to be the de Sitter invariant one (41–42), expressions (47), (48) and (54) constitute the necessary ingredients for the sought for de Sitter invariant photon propagator (23) in covariant gauges. In this work we have chosen to construct the photon propagator as a convolution in position space (47), such that – in Minkowski space – it yields a standard algebraic form in momentum space. This is to be contrasted with the work of Allen and Jacobson [1] and Kahya and Woodard [6], which represent A_2 as a function of $y(x; x')$. From Eq. (39) one sees that $A_2(y)$ can be easily written as a double indefinite integral over y. A discussion of the difficulties one faces when attempting to implement such a procedure (in covariant gauges and in the case of $D = 4$) is presented in Appendix C.

III. RESPONSE TO A STATIC POINT CHARGE

Here we shall use the photon propagator (23), (41), (47), (51) and (48) to calculate the response to a static point charge on de Sitter space in $D = 4$. The relevant equation is the suitable generalization of Eq. (7) to curved space-times,

$$A_\mu(x) = \int d^4x'\sqrt{-g(x')}\big[{}_\mu\Delta_\nu^{\rm ret}\big](x; x')J^\nu(x')\,, \tag{55}$$

where the charge current density is given by

$$J^\nu(x') = \frac{e\delta_0^\nu}{a'}\frac{\delta^3(\vec{x}\,')}{\sqrt{\gamma(x')}}\,. \tag{56}$$

Here $\gamma(x') = (a')^6$ is the determinant of the spatial part of the metric. The retarded propagator needed for this is given by

$$\imath[{}_\nu\Delta_\alpha^{\rm ret}](x; x') = \imath[{}_\nu\Delta_\alpha^{++}](x; x') - \imath[{}_\nu\Delta_\alpha^{+-}](x; x')\,, \tag{57}$$

where

$$\imath[{}_\nu\Delta_\alpha^{ab}](x; x') = (\partial_\nu\partial'_\alpha y) \times A_1^{ab}\big(y(x; x')\big) + \partial_\nu\partial'_\alpha A_2^{ab}\big(y(x; x')\big)\,, \tag{58}$$

and $A_2^{ab}(x;x')$ is given in (47). Making use of

$$G^{\rm ret}_{A_2} = G^{++}_{A_2} - G^{+-}_{A_2} = G^{-+}_{A_2} - G^{--}_{A_2} \,; \qquad s^{\rm ret}_{\xi} = s^{++}_{\xi} - s^{+-}_{\xi} = s^{-+}_{\xi} - s^{--}_{\xi} \,, \tag{59}$$

one easily finds

$$\imath[{}_\nu\Delta^{\rm ret}_\alpha](x;x') = (\partial_\nu\partial'_\alpha y)\times A_1^{\rm ret}\big(y(x;x')\big) + \partial_\nu\partial'_\alpha A_2^{\rm ret}\big(y(x;x')\big)\,, \tag{60}$$

where

$$A_1^{\rm ret}(x;x') = A_1^{++}(x;x') - A_1^{+-}(x;x')\,; \qquad A_2^{\rm ret}(x;x') = H^2\int d^4x'' G^{\rm ret}_{A_2}(x;x'')s^{\rm ret}_\xi(x'';x')\,. \tag{61}$$

Now from

$$\begin{aligned}
\frac{1}{y^{++}(x;x')} - \frac{1}{y^{+-}(x;x')} &= \frac{1}{y^{-+}(x;x')} - \frac{1}{y^{--}(x;x')} = -\frac{2\pi\imath\theta(\eta-\eta')}{H^2aa'}\delta(\|\vec{x}-\vec{x}'\|^2 - (\eta-\eta')^2)\\
\frac{1}{4-y^{++}(x;x')} - \frac{1}{4-y^{+-}(x;x')} &= \frac{2\pi\imath\theta(\eta-\eta')}{H^2aa'}\delta\Big(\frac{4}{H^2aa'} - \|\vec{x}-\vec{x}'\|^2 + (\eta-\eta')^2\Big)\\
&= \frac{\pi\imath\theta(\eta-\eta')}{H^2aa'\|\vec{x}-\vec{x}'\|}\delta\Big(\eta+\eta' + \|\vec{x}-\vec{x}'\|\Big)
\end{aligned} \tag{62}$$

and

$$\begin{aligned}
\ln\bigg(\frac{y^{++}(x;x')}{y^{+-}(x;x')}\bigg) &= \ln\bigg(\frac{y^{-+}(x;x')}{y^{--}(x;x')}\bigg) = 2\pi\imath\theta(\eta-\eta')\theta\big(\eta-\eta' - \|\vec{x}-\vec{x}'\|\big) \qquad (63)\\
\ln\frac{4-y^{++}(x;x')}{4-y^{+-}(x;x')} &= -2\pi\imath\theta(\eta-\eta')\theta\Big(\|\vec{x}-\vec{x}'\|^2 - \frac{4}{H^2aa'} - (\eta-\eta')^2\Big) = -2\pi\imath\theta(\eta-\eta')\theta\big(\eta+\eta' + \|\vec{x}-\vec{x}'\|\big)
\end{aligned}$$

and Eqs. (42), (53) and (54) we get

$$A_1^{\rm ret}(x;x') = \frac{\imath}{8\pi}\frac{\theta(\eta-\eta')}{H^2aa'\|\vec{x}-\vec{x}'\|}\delta(\eta-\eta' - \|\vec{x}-\vec{x}'\|) \tag{64}$$

$$\begin{aligned}
G^{\rm ret}_{A_2}(x;x'') = -\frac{\theta(\eta-\eta'')}{4\pi}\Bigg(&\frac{1}{aa''\|\vec{x}-\vec{x}''\|}\Big(\delta(\eta-\eta'' - \|\vec{x}-\vec{x}''\|) + \delta\big(\eta+\eta'' + \|\vec{x}-\vec{x}''\|\big)\Big)\\
&+ H^2\theta\big(\eta-\eta'' - \|\vec{x}-\vec{x}''\|\big) + H^2\theta\big(\eta+\eta'' + \|\vec{x}-\vec{x}''\|\big)\Bigg)
\end{aligned} \tag{65}$$

$$s^{\rm ret}_\xi(x'';x') = -\imath\frac{\theta(\eta''-\eta')}{4\pi H^2}\bigg\{\frac{1-\xi}{aa''\|\vec{x}''-\vec{x}'\|}\delta(\eta''-\eta' - \|\vec{x}''-\vec{x}'\|) + \big(3-\xi\big)H^2\theta\big(\eta''-\eta' - \|\vec{x}''-\vec{x}'\|\big)\bigg\}. \tag{66}$$

Upon inserting these expressions into Eq. (58) and (47) one gets,

$$\begin{aligned}
[{}_\nu\Delta^{\rm ret}_\alpha](x;x') &= \frac{\partial_\nu\partial'_\alpha y(x;x')}{2H^2aa'}\frac{\theta\big(\eta-\eta'\big)}{4\pi\|\vec{x}-\vec{x}'\|}\delta\big(\eta-\eta' - \|\vec{x}-\vec{x}'\|\big) \qquad (67)\\
&+ \frac{1}{(4\pi)^2}\partial_\nu\partial'_\alpha\int d^4x''\bigg\{\theta\big(\eta-\eta''\big)\bigg[\frac{1}{aa''\|\vec{x}-\vec{x}''\|}\Big(\delta\big(\eta-\eta''-\|\vec{x}-\vec{x}''\|\big) + \delta\big(\eta+\eta''+\|\vec{x}-\vec{x}''\|\big)\Big)\\
&\qquad + H^2\theta\big(\eta-\eta'' - \|\vec{x}-\vec{x}''\|\big) + H^2\theta\big(\eta+\eta'' + \|\vec{x}-\vec{x}''\|\big)\bigg]\\
&\qquad \times\theta\big(\eta''-\eta'\big)\bigg[\frac{1-\xi}{a'a''\|\vec{x}''-\vec{x}'\|}\delta\big(\eta''-\eta' - \|\vec{x}''-\vec{x}'\|\big) + (3-\xi)H^2\theta\big(\eta''-\eta'-\|\vec{x}''-\vec{x}'\|\big)\bigg]\bigg\}\\
&\equiv \aleph_{\nu\alpha}(x;x') + \partial_\nu\partial'_\alpha\lambda(x;x')\,.
\end{aligned}$$

From the form of this expression we see that, just as in the Minkowski case (11), only the (gauge independent) term in the first line can contribute to physical quantities, while the other terms which depend on ξ cannot contribute.

To illustrate how this works in some detail we shall now calculate the electromagnetic response to a static point charge (56) on four dimensional de Sitter space. From (55) and (67) we see that the gauge independent contribution to the electromagnetic potential

$$\begin{aligned}\tilde{\mathcal{A}}_\mu(x) &= \int d^4x' \sqrt{-g(x')} \aleph_{\nu\alpha}(x;x') J^\alpha(x') \qquad (68)\\ &= \frac{e}{8\pi\|\vec{x}\|}\int_{\eta_o}^{\eta} d\eta' \Big\{ \Big[y(x;x')\delta^0_\mu + 2\delta^0_\mu a(\eta)H(\eta-\eta') + 2a(\eta')H\eta_{\mu\alpha}(x^\alpha - x'^\alpha) - 2\eta_{\mu 0}\Big]\theta(\eta-\eta')\delta\big(\eta-\eta'-\|\vec{x}\|\big)\Big\}.\end{aligned}$$

where we integrated over $\vec{x}\,'$. If we introduce $r = \|\vec{x}\|$ and $\Delta\eta = \eta - \eta'$ and change the variable of integration to $\Delta\eta$, Eq. (68) becomes,

$$\tilde{\mathcal{A}}_\mu = \frac{e}{8\pi r}\int_0^{\Delta\eta_0} d(\Delta\eta) \Big\{ \Big[y(x;x')\delta^0_\mu + 2\delta^0_\mu a(\eta)H\Delta\eta + 2a(\eta-\Delta\eta)H\eta_{\mu\alpha}(x^\alpha - x'^\alpha) - 2\eta_{\mu 0}\Big]\delta\big(\Delta\eta - r\big)\Big\} \quad , \qquad (69)$$

where $\Delta\eta_0 = \eta - \eta_0$. Since $\Delta\eta_0 \geq \Delta\eta \geq 0$, Eq. (69) is easily integrated to yield,

$$\tilde{\mathcal{A}}_\mu = \frac{e\theta(\Delta\eta_0 - r)}{8\pi r}\Big[2\delta^0_\mu a(\eta)H\Delta\eta + 2a(\eta-\Delta\eta)H\eta_{\mu\alpha}(x^\alpha - x'^\alpha) - 2\eta_{\mu 0}\Big]_{\Delta\eta = r} \qquad (70)$$

For the time-like component we get

$$\tilde{\mathcal{A}}_0 = \frac{e}{4\pi}\left(\frac{1}{r} - \frac{1}{\eta} + \frac{1}{\eta - r}\right)\theta(\Delta\eta_0 - r)\,, \qquad (71)$$

and for the space-like components we obtain

$$\tilde{\mathcal{A}}_i = -\frac{e}{4\pi}\frac{x^i}{r(\eta - r)}\theta(\Delta\eta_0 - r)\,. \qquad (72)$$

As in the Minkowski case, the physical result is recovered when $\eta_0 \to -\infty$, in which case (71–72) reduce to,

$$\tilde{\mathcal{A}}_0 = \frac{e}{4\pi}\left(\frac{1}{r} - \frac{1}{\eta} + \frac{1}{\eta - r}\right), \qquad \tilde{\mathcal{A}}_i = -\frac{e}{4\pi}\frac{x^i}{r(\eta - r)}\,. \qquad (73)$$

A part of this potential is pure gauge. Indeed, Eqs. (73) can be recast as

$$\tilde{\mathcal{A}}_0 = \mathcal{A}_0 + \partial_0\Lambda_1(x)\,, \qquad \tilde{\mathcal{A}}_i = \partial_i\Lambda_1(x)\,, \qquad \mathcal{A}_0 = \frac{e}{4\pi r}\,, \qquad \Lambda_1(x) = \frac{e}{4\pi}\ln\left(1 - \frac{r}{\eta}\right). \qquad (74)$$

such that the gauge independent part of the potential on de Sitter is simply,

$$\mathcal{A}_\mu(x) = \frac{e}{4\pi r}\delta_\mu^{\;0}\,. \qquad (75)$$

The second part of the response potential arises from the gauge dependent part of the propagator (67), and it is of the form,

$$\delta A_\mu(x) = e\partial_\mu\int d\eta'\partial_0'\lambda(x;x')|_{\vec{x}\,'\to 0} \equiv \partial_\mu\Lambda_2(x)\,, \qquad (76)$$

and hence it is pure gauge. This conclusion is legitimate, provided the η' and x'' integrals (the latter appearing in (67)) are all finite, which we have checked to be the case.

To summarize, we have found that the response to a point charge on de Sitter (present from $\eta_0 \to -\infty$) yields an electric field that is conformal to the Minkowski response, and a vanishing magnetic field,

$$A_\mu(x) = \frac{e}{4\pi r}\delta_\mu^{\;0} + \partial_\mu\Lambda(x)\,, \qquad E_i = F_{0i} = \partial_0 A_i - \partial_i A_0 = \frac{e}{4\pi r^2}\frac{r^i}{r}\,, \qquad B_i = \epsilon_{ijl}\partial_j A_l = 0\,, \qquad (77)$$

where $\Lambda = \Lambda_1 + \Lambda_2$. From these results we see that, because electromagnetism is conformal on cosmological spaces in $D = 4$, (apart from the conformal rescaling of the fields by a power of the scale factor) the expansion of the Universe plays no role in the electromagnetic fields generated by a static point electric charge.

IV. THE ONE-LOOP STRESS ENERGY TENSOR

In this section we calculate the one-loop stress energy tensor from our propagator. This is in principle important for the calculation of the quantum backreaction on de Sitter space, albeit we do not expect a large backreaction from photons, since they couple conformally in four space-time dimensions.

We start with the well known formula for the photon stress energy tensor,

$$T_{\mu\nu} = \Big(\delta_\mu^{\ \alpha}\delta_\nu^{\ \gamma}g^{\beta\delta} - \frac{1}{4}g_{\mu\nu}g^{\alpha\gamma}g^{\beta\delta}\Big)F_{\alpha\beta}F_{\gamma\delta}\,, \tag{78}$$

which relates $T_{\mu\nu}$ to $F_{\alpha\beta}F_{\gamma\delta}$. The one-loop contribution to the expectation value for this quadratic operator can be obtained from,

$$\langle\Omega|F^a_{\alpha\beta}(x)F^b_{\gamma\delta}(x)|\Omega\rangle_{1\text{ loop}} = \Big\{\partial'_\gamma\Big(\partial_\alpha[{}_\beta\Delta^{ab}_\delta](x;x')-\partial_\beta[{}_\alpha\Delta^{ab}_\delta](x;x')\Big)-\partial'_\delta\Big(\partial_\alpha[{}_\beta\Delta^{ab}_\gamma](x;x')-\partial_\beta[{}_\alpha\Delta^{ab}_\gamma](x;x')\Big)\Big\}_{x'\to x}. \tag{79}$$

Taking account of Eqs. (58), (35), (36) and (37), we see from

$$\imath[{}_\beta\Delta^{ab}_\delta](x;x') = (\partial_\beta\partial'_\delta y)A_1(y^{ab}) + \partial_\beta\partial'_\delta A_2(y^{ab}) \tag{80}$$

that, because of the antisymmetrization of the indices α and β, the A_2-term does not contribute to the expectation value in (79). Furthermore, since $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = \nabla_\mu A_\nu - \nabla_\nu A_\mu$, all of the ordinary derivatives in (79) can be replaced by covariant derivatives. With these remarks, Eq. (79) becomes

$$\begin{aligned}\langle\Omega|F^a_{\alpha\beta}(x)F^b_{\gamma\delta}(x)|\Omega\rangle_{1\text{ loop}} \;=\; &\Big\{\nabla'_\gamma\Big(\nabla_\alpha[(\partial_\beta\partial'_\delta y)A_1(y^{ab})] - \nabla_\beta[(\partial_\alpha\partial'_\delta y)A_1(y^{ab})]\Big)\\ &-\nabla'_\delta\Big(\nabla_\alpha[(\partial_\beta\partial'_\gamma y)A_1(y^{ab})] - \nabla_\beta[(\partial_\alpha\partial'_\gamma y)A_1(y^{ab})]\Big)\Big\}(x;x')_{x'\to x}\,.\end{aligned} \tag{81}$$

Any dependence on the gauge parameter ξ (which is in the structure function A_2) has dropped out, such that (81) is manifestly gauge independent. Now, since $\nabla_\alpha\partial_\beta\partial'_\delta y = -H^2 g_{\alpha\beta}(x)\partial'_\delta y$ is symmetric in $\{\alpha,\beta\}$, the first covariant derivatives in (81) act only on A_1, resulting in,

$$\begin{aligned}\langle\Omega|F^a_{\alpha\beta}(x)F^b_{\gamma\delta}(x)|\Omega\rangle_{1\text{ loop}} \;=\; &\Big\{\nabla'_\gamma\Big((\partial_\beta\partial'_\delta y)(\partial_\alpha y)A'_1(y^{ab}) - (\partial_\alpha\partial'_\delta y)(\partial_\beta y)A'_1(y^{ab})\Big)\\ &-\nabla'_\delta\Big((\partial_\beta\partial'_\gamma y)(\partial_\alpha y)A'_1(y^{ab}) - (\partial_\alpha\partial'_\gamma y)(\partial_\beta y)A'_1(y^{ab})\Big)\Big\}_{x'\to x}\,.\end{aligned} \tag{82}$$

Analogously, because of the symmetry in the primed indices, the primed covariant derivatives in (82) commute through the first terms and one gets,

$$\langle\Omega|F^a_{\alpha\beta}(x)F^b_{\gamma\delta}(x)|\Omega\rangle_{1\text{ loop}} \;=\; \Big\{4(\partial_\alpha\partial'_{[\gamma}y)(\partial'_{\delta]}\partial_\beta y)A'_1(y^{ab}) - 4(\partial_{[\alpha}y)(\partial_{\beta]}\partial'_{[\gamma}y)(\partial'_{\delta]}y)A''_1(y^{ab})\Big\}_{x'\to x} \tag{83}$$

Because of the equalities,

$$(\partial'_\delta\partial_\beta y)_{x'\to x} = -2H^2 g_{\delta\beta}(x)\,, \qquad (\partial_\beta y)_{x'\to x} = 0\,, \tag{84}$$

the second term in (83) vanishes and we find,

$$\langle\Omega|F^a_{\alpha\beta}(x)F^b_{\gamma\delta}(x)|\Omega\rangle_{1\text{ loop}} = 16H^4(g_{\alpha[\gamma}g_{\delta]\beta})[A'_1(y^{ab})]_{x'\to x}\,. \tag{85}$$

From Eq. (37) we see that the coincident limit of $A'_1(y)$ equals,

$$[A'_1(y^{ab})]_{x'\to x} = \frac{1}{8}\frac{H^{D-4}}{(4\pi)^{D/2}}\frac{\Gamma(D-1)}{\Gamma\big(\frac{D}{2}+1\big)} \;\stackrel{D\to4}{\longrightarrow}\; \frac{1}{128\pi^2}\,, \tag{86}$$

where, to obtain this result in the spirit of dimensional regularization we have assumed that $\Re[D] < 0$ (recall that this procedure of analytic continuation subtracts automatically all power law divergences). When Eq. (86) is inserted into (85), one obtains

$$\langle\Omega|F^a_{\alpha\beta}(x)F^b_{\gamma\delta}(x)|\Omega\rangle_{1\text{ loop}} = \frac{2H^D}{(4\pi)^{D/2}}\frac{\Gamma(D-1)}{\Gamma\big(\frac{D}{2}+1\big)}\times g_{\alpha[\gamma}g_{\delta]\beta}(x) \;\stackrel{D\to4}{\longrightarrow}\; \frac{H^4}{8\pi^2}\times g_{\alpha[\gamma}g_{\delta]\beta}(x)\,, \tag{87}$$

independent on the polarities $a, b = \pm$, as was to be expected for a one-loop coincident correlator. Finally, upon inserting (87) into (78), one gets for the renormalized one-loop stress energy tensor,

$$\langle\Omega|T_{\mu\nu}(x)|\Omega\rangle_{1\ \rm loop} = -\frac{H^D}{(4\pi)^{D/2}}\frac{\Gamma(D)}{\Gamma\big(\frac{D}{2}+1\big)}\frac{D-4}{4}g_{\mu\nu}(x) \;\; \overset{D\to 4}{\longrightarrow} \;\; 0\,. \tag{88}$$

This means that, at the one-loop order and in $D = 4$, the electromagnetic stress-energy tensor on de Sitter space does not break conformal symmetry. Namely, when taken together, the de Sitter and conformal symmetry require that the expectation value of $T_{\mu\nu}$ taken in a de Sitter invariant state must vanish. That it must be so can be argued as follows. The de Sitter symmetry alone requires $\langle\Omega|T_{\mu\nu}(x)|\Omega\rangle = T(x)g_{\mu\nu}$, where T is some scalar function of x. But the only scalar function of x consistent with the de Sitter symmetry is a constant, $T(x) = T_0$. Next, the conformality of $\langle\Omega|T_{\mu\nu}(x)|\Omega\rangle$ requires that its trace must vanish, $g^{\mu\nu}\langle\Omega|T_{\mu\nu}(x)|\Omega\rangle = 4T_0 = 0$, from which it immediately follows that $T_0 = 0$, in agreement with (88). Of course, the results (87–88) are not new; compare, for example, Eqs. (87–88) with Eqs. (40) and (42) in Ref. [3]. The new ingredient here is the photon propagator in general covariant gauges, which we used to show that the results (87–88) are fully gauge independent.

V. CONCLUSIONS AND DISCUSSION

In this paper we have found a de Sitter invariant propagator in general covariant gauges. We have used it to calculate the response to a static point charge, and the one loop stress energy tensor. Also the gauge independence of the results is demonstrated. In searching for the de Sitter invariant solution we needed to introduce an unphysical source at the antipodal point in one of the steps. As it turned out this had no bearing on physical results since the contribution of those terms appears only in the pure gauge part of the propagator. Our propagator sheds some light on the question whether it is possible to find de Sitter invariant propagators in gauge theories and allows for perturbative calculations on de Sitter, which is the model space for inflation. Although the results presented here are interesting in their own right, what we really hope for is that an analogous treatment can be applied to construct a graviton propagator on de Sitter in general covariant gauges. Indeed, constructing a photon propagator in covariant gauges is a first step towards constructing a graviton propagator in general covariant gauges. Apart from the difference in the number of vector indices and de Sitter invariant tensors, the photon and graviton propagators differ in one more important aspect. Namely, while in de Sitter space and near four space-time dimensions electromagnetism couples to de Sitter in a nearly conformal manner, such that the effect of Universe's expansion on the creation of photons is nearly minimal, gravitons strongly break conformal invariance, implying abundant production of gravitons on de Sitter background, strongly suggesting that de Sitter symmetry is broken in (perturbative) quantum gravity. Therefore, understanding the interplay between de Sitter breaking, gauging, and graviton production is of a crucial importance to understanding the stability of de Sitter space. Recently graviton propagators were constructed in (generalized) de Donder gauges [26, 27] as well as in a non-covariant gauge [28]. These propagators can be used to study loop quantum effects on de Sitter space [29], as well as the stability of de Sitter space. The problem that still remains elusive is a general proof that de Sitter symmetry must be broken by gravitons. If proved, this theorem would immediately imply a (perturbative) instability of de Sitter space. Already a long time ago Allen and Folacci [30] have shown that a minimally coupled massless scalar field necessarily breaks de Sitter symmetry, thus perturbatively destabilizing de Sitter space. While the existence of minimally coupled scalar fields can be questioned, it is very hard to argue against the existence of gravitons, which underpins the importance of understanding perturbative stability of quantum gravity on de Sitter space.

Appendix A: The photon propagator on Minkowski background

In this appendix we perform explicit integrations of section I. Notice first that Eq. (12) can be written in the following form,

$$A_\mu(x) \;=\; \frac{e\delta^0_\mu}{4\pi r}\theta(\Delta\eta_0 - r) - (1-\xi)\frac{e}{8\pi}\partial_\mu\partial_0\int_{\eta_0}^{\eta}d\eta''\int_{\eta_0}^{\eta''}d\eta'\int_{-1}^{1}dz\frac{\delta\left((\eta-\eta'') - \|\vec{x}-\vec{x}''\|\right)}{\|\vec{x}-\vec{x}''\|}(\eta''-\eta')\,, \tag{A1}$$

where $z = \cos[\angle(\vec{x}, \vec{x}'')]$, $\|\vec{x}-\vec{x}''\| = \sqrt{r^2 + r''^2 - 2rr''z}$, and we have used the second delta-function in the second line of Eq. (12) to integrate over r'', whereby $r'' \to \eta'' - \eta'$. Next, we shall use the delta-function in (A1) to integrate over z. In doing so, first notice that

$$\frac{\delta\left((\eta-\eta'') - \|\vec{x}-\vec{x}''\|\right)}{\|\vec{x}-\vec{x}''\|} = \frac{\delta(z - z_+) + \delta(z - z_-)}{r(\eta'' - \eta')}\,, \tag{A2}$$

where $z = z_\pm$ are the two poles of at which the argument of the delta-function vanishes. Notice that the delta-function gives a contribution to the integral only if the poles lie in the interval of integration,

$$-1 < z_\pm < 1\,, \tag{A3}$$

which can be also written as,

$$r^2 - 2r(\eta''-\eta') + (\eta''-\eta')^2 - (\eta-\eta'')^2 < 0 < r^2 + 2r(\eta''-\eta') + (\eta''-\eta')^2 - (\eta-\eta'')^2\,, \tag{A4}$$

The left inequality is satisfied for $r_- < r < r_+$, where

$$r_- = (\eta''-\eta') - (\eta-\eta'')\,, \qquad r_+ = (\eta''-\eta') + (\eta-\eta'') = \eta-\eta'\,, \tag{A5}$$

while the right inequality in (A4) is satisfied when $r > \tilde{r}_+$ or when $r < \tilde{r}_-$, where

$$\tilde{r}_- = -(\eta''-\eta') - (\eta-\eta'') = -(\eta-\eta')\,, \qquad \tilde{r}_+ = -(\eta''-\eta') + (\eta-\eta'')\,, \tag{A6}$$

and of course $r > 0$ (notice that both z_+ and z_- result in the same constraints on r). This means that

$$-(\eta''-\eta') + (\eta-\eta'') < r < \eta-\eta'\,, \tag{A7}$$

and of course $r > 0$. These conditions can be written as

$$\theta\big(-(\eta''-\eta') + (\eta-\eta'')\big)\big[\theta(r + (\eta''-\eta') - (\eta-\eta'')) - \theta\big(r - (\eta-\eta')\big)\big] + \theta\big((\eta''-\eta') - (\eta-\eta'')\big)\big[\theta(\eta-\eta' - r)\big] \tag{A8}$$

With these remarks in mind, we can perform the z-integral in Eq. (A1),

$$\begin{aligned} A_\mu(x) \;&=\; \frac{e\delta_\mu^0}{4\pi r}\theta(\Delta\eta_0 - r) - (1-\xi)\frac{e}{4\pi}\partial_\mu\partial_0\frac{1}{r}\int_{\eta_0}^{\eta} d\eta''\int_{\eta_0}^{\eta''} d\eta' \\ &\times\Big\{\theta\big(-(\eta''-\eta') + (\eta-\eta'')\big)\big[\theta(r + (\eta''-\eta') - (\eta-\eta'')) - \theta\big(r-(\eta-\eta')\big)\big] + \theta\big((\eta''-\eta') - (\eta-\eta'')\big)\big[\theta(\eta-\eta'-r)\big]\Big\}. \end{aligned} \tag{A9}$$

We can now act with the time derivative to get rid of one of the integrals,

$$\begin{aligned} A_\mu(x) \;&=\; \frac{e\delta_\mu^0}{4\pi r}\theta(\Delta\eta_0 - r) - (1-\xi)\frac{e}{4\pi}\partial_\mu\int_{\eta_0}^{\eta}\frac{d\eta'}{r}\Big\{\theta(\eta-\eta'-r)\Big\}. \\ &- (1-\xi)\frac{e}{4\pi}\partial_\mu\frac{1}{r}\int_{\eta_0}^{\eta} d\eta''\int_{\eta_0}^{\eta''} d\eta'\Big\{\theta\big(-(\eta''-\eta')+(\eta-\eta'')\big)\big[-\delta(r+(\eta''-\eta')-(\eta-\eta'')) + \delta(r-(\eta-\eta'))\big] \\ &\qquad + \theta\big((\eta''-\eta')-(\eta-\eta'')\big)\delta((\eta-\eta')-r)\Big\}. \end{aligned} \tag{A10}$$

The η'-integral in the last two lines can be performed, and the result is

$$\begin{aligned} &-(1-\xi)\frac{e}{4\pi}\partial_\mu\frac{1}{r}\int_{\eta_0}^{\eta} d\eta''\int_{\eta_0}^{\eta''} d\eta'\Big\{-\delta(r+(\eta''-\eta')-(\eta-\eta'')) + \delta(r-(\eta-\eta'))\Big\} \\ &\quad = -(1-\xi)\frac{e}{4\pi}\partial_\mu\frac{1}{r}\int_{\eta_0}^{\eta} d\eta''\Big\{-\theta\big(r\big)\theta\big(r - (\eta-\eta'') + (\eta''-\eta_0)\big) + 2\theta\big(r - (\eta-\eta'')\big) - \theta\big(r-(\eta-\eta_0)\big)\Big\} \end{aligned} \tag{A11}$$

Collecting all the terms together we get,

$$A_\mu(x) = \frac{e\delta_\mu^0}{4\pi r}\theta(\Delta\eta_0 - r) - (1-\xi)\frac{e}{4\pi}\partial_\mu\frac{1}{r}\int_0^{\Delta\eta_0} d\Delta\eta\Big\{\theta(\Delta\eta-r) - \theta(r-2\Delta\eta+\Delta\eta_0) + 2\theta(r-\Delta\eta) - \theta(r-\Delta\eta_0)\Big\}, \tag{A12}$$

where $\Delta\eta = \eta - \eta'$ and $\Delta\eta_0 = \eta - \eta_0$m, $r \geq 0$. This result appears in the main text as (13) and it is then used to calculate the response potential $A_\mu(x)$.

Appendix B: The photon tensor structures

Here we consider how the operator (20) on de Sitter space acts on the photon propagator $\imath[_\mu\Delta_\alpha](x;x')$, where the tensor decomposition (23) suitable for de Sitter space is used.

The first operator (d'Alembertian) in (21) acts as,

$$
\begin{aligned}
&g^{\mu\nu}\Box_x\{(\partial_\nu\partial'_\alpha y)\times f_1(y)+(\partial_\nu y)\times(\partial'_\alpha y)\times f_2(y)\} \\
&\qquad = H^2(\partial^\mu\partial'_\alpha y)\Big\{\big[(4y-y^2)f_1''+D(2-y)f_1'-f_1\big]+\Big[2(2-y)f_2\Big]\Big\} \\
&\qquad = H^2(\partial^\mu y)\times(\partial'_\alpha y)\Big\{\big[-2f_1'\big]+\big[(4y-y^2)f_2''+(D+4)(2-y)f_2'-(D+1)f_2\big]\Big\}\,.
\end{aligned}
\tag{B1}
$$

The *second part* of the operator in (20) acts as

$$
\begin{aligned}
&\nabla^\nu\nabla^\mu\{(\partial_\nu\partial'_\alpha y)\times f_1(y)+(\partial_\nu y)\times(\partial'_\alpha y)\times f_2(y)\} \\
&\qquad = H^2(\partial^\mu\partial'_\alpha y)\Big\{\big[(2-y)f_1'-f_1\big]+\big[(4y-y^2)f_2'+(D+1)(2-y)f_2\big]\Big\} \\
&\qquad + H^2(\partial^\mu y)(\partial'_\alpha y)\Big\{\big[(2-y)f_1''-(D+1)f_1'\big]+\big[(4y-y^2)f_2''+(D+3)(2-y)f_2'-2f_2\big]\Big\}\,.
\end{aligned}
\tag{B2}
$$

Finally, the *third part* of the operator in (20) yields

$$
\begin{aligned}
&\nabla^\mu\nabla^\nu\{(\partial_\nu\partial'_\alpha y)\times f_1(y)+(\partial_\nu y)\times(\partial'_\alpha y)\times f_2(y)\} \\
&\qquad = H^2(\partial^\mu\partial'_\alpha y)\Big\{\big[(2-y)f_1'-Df_1\big]+\big[(4y-y^2)f_2'+(D+1)(2-y)f_2\big]\Big\} \\
&\qquad + H^2(\partial^\mu y)(\partial'_\alpha y)\Big\{\big[(2-y)f_1''-(D+1)f_1'\big]+\big[(4y-y^2)f_2''+(D+3)(2-y)f_2'-(D+1)f_2\big]\Big\}\,.
\end{aligned}
\tag{B3}
$$

Note that subtracting (B3) from (B2) yields

$$
R^{\mu\nu}\{(\partial_\nu\partial'_\alpha y)\times f_1(y)+(\partial_\nu y)\times(\partial'_\alpha y)\times f_2(y)\} \;=\; H^2(\partial^\mu\partial'_\alpha y)\big[(D-1)f_1\big]+H^2(\partial^\mu y)\times(\partial'_\alpha y)\big[(D-1)f_2\big]\,,
\tag{B4}
$$

as it should (see Eq. (20)). Here we made use of $R^{\mu\nu}V_\nu=(\nabla^\nu\nabla^\mu-\nabla^\mu\nabla^\nu)V_\nu$ for any vector field V_ν.

Now upon inserting the results (B1–B3) into the photon propagator equation (21) we get,

$$
\begin{aligned}
&L^{\mu\nu}\{(\partial_\nu\partial'_\alpha y)\times f_1(y)+(\partial_\nu y)\times(\partial'_\alpha y)\times f_2(y)\} \\
&\quad = H^2(\partial^\mu\partial'_\alpha y)\Big\{\Big[(4y-y^2)f_1''+\Big((D-1)+\frac{1}{\xi}\Big)(2-y)f_1'-\frac{D}{\xi}f_1\Big] \\
&\qquad\qquad + \Big[-\Big(1-\frac{1}{\xi}\Big)(4y-y^2)f_2'-\Big(D-1-\frac{D+1}{\xi}\Big)(2-y)f_2\Big]\Big\} \\
&\quad + H^2(\partial^\mu y)(\partial'_\alpha y)\Big\{\Big[-\Big(1-\frac{1}{\xi}\Big)(2-y)f_1''+\Big(D-1-\frac{D+1}{\xi}\Big)f_1'\Big] \\
&\qquad\qquad + \Big[\frac{4y-y^2}{\xi}f_2''+\Big(1+\frac{D+3}{\xi}\Big)(2-y)f_2'-\Big(D-1+\frac{D+1}{\xi}\Big)f_2\Big]\Big\}
\end{aligned}
\tag{B5}
$$

$$
\quad = (\partial^\mu\partial'_\alpha y)\frac{\imath\delta^D(x-x')}{\sqrt{-g}(-2H^2)}\,.
\tag{B6}
$$

This result is used in the main text to obtain Eqs. (25–26) for the scalar structure functions f_1 and f_2.

Appendix C: An alternative method to calculate the scalar structure function A_2

This appendix is an attempt to construct in an alternative manner the second scalar structure function A_2 defined in Eq. (23). Namely, we shall attempt to solve for A_2 by performing the suitable double integral in Eq. (39). For simplicity, we work here in $D=4$. Taking account of the four dimensional form for A_1 and $I[A_1]$ in Eqs. (52–53) and the corresponding source function s_ξ in Eq. (54), it is straightforward to perform the integrals in (39). The (naïve) result is (up to a constant),

$$
\begin{aligned}
\tilde{A}_2(y) \;=\; &\frac{1}{8\pi^2}\Big\{(1-\xi)\Big[\frac{1}{3}\frac{1}{4-y}+\frac{1}{2}\ln\Big(\frac{y}{4}\Big)-\frac{1}{6}\ln\Big(1-\frac{y}{4}\Big)\Big] \\
&-(3-\xi)\Big[-\frac{5}{9}\frac{1}{4-y}+\frac{y}{6(4-y)}\ln\Big(\frac{y}{4}\Big)+\frac{5}{18}\ln\Big(1-\frac{y}{4}\Big)+\frac{1}{3}\mathrm{Li}_2\Big(1-\frac{y}{4}\Big)\Big]\Big\}\,.
\end{aligned}
\tag{C1}
$$

This result does not reproduce the right singular structure of Eq. (39). This can be seen from the near pole (analytic) structure of (C1), which around $y\sim 4$ and $y\sim 0$ is

$$\begin{aligned}\tilde{A}_2(y\sim 4) &= \frac{1}{8\pi^2}\left[\frac{1-\xi}{3}+\frac{5(3-\xi)}{9}\right]\left[\frac{1}{4-y}-\frac{1}{2}\ln\Big(1-\frac{y}{4}\Big)\right]+\mathcal{O}\Big((4-y)^0\Big)\\ \tilde{A}_2(y\sim 0) &= \frac{1}{8\pi^2}\left\{\frac{1-\xi}{2}\ln\Big(\frac{y}{4}\Big)\right\}+\mathcal{O}\Big((4-y)^0\Big)\,,\end{aligned} \tag{C2}$$

such that, when the operator $\Box/H^2=(4y-y^2)(d/dy)^2+4(2-y)(d/dy)$ acts on A_2 and one takes proper account of the singular (pole) structure indicated by the $\imath\epsilon$ prescription in $y=y^{++}(x;x')$, one gets from (C1–C2),

$$\frac{\Box}{H^2}\tilde{A}_2(y(x;x'))=s_\xi(y(x;x'))-\frac{1}{2}\left[\frac{1-\xi}{3}+\frac{5(3-\xi)}{9}\right]\frac{\imath\delta^4(x-\bar{x}')}{H^4\sqrt{-g}}\,, \tag{C3}$$

where we took account of (see Eq. (42)),

$$\Box\imath G_{A_2}(x;x')=\frac{\imath\delta^4(x-x')+\imath\delta^4(x-\bar{x}')}{\sqrt{-g}}\,, \tag{C4}$$

where $\bar{x}^\mu=(-\eta,\vec{x}\,)$ is the antipodal point of x^μ. Adding a homogeneous, de Sitter invariant, solution $\propto G_{A_2}$ to A_2 cannot remove the singular contribution in Eq. (C3). Indeed, from (C4) it immediately follows that adding such a term can only replace the delta function at the antipodal point with that at the light cone $x^\mu=x'^\mu$, such that we have

$$\begin{aligned}A_2 &= \tilde{A}_2+\frac{1}{2H^2}\left[\frac{1-\xi}{3}+\frac{5(3-\xi)}{9}\right]\imath G_{A_2}(x;x')\\ \frac{\Box}{H^2}A_2(y(x;x')) &= s_\xi(y(x;x'))+\frac{1}{2}\left[\frac{1-\xi}{3}+\frac{5(3-\xi)}{9}\right]\frac{\imath\delta^4(x-x')}{H^4\sqrt{-g}}\,.\end{aligned} \tag{C5}$$

Even though this new choice for A_2 has the right (Hadamard) singular structure, the resulting photon propagator (23) does not satisfy the correct equation of motion (21), that is for this propagator the delta function structure on the right hand side of (21) is incorrect. This means that it is impossible to construct a de Sitter invariant solution to A_2 of the form $A_2(y(x;x'))$ with the right singular structure, which is at odds with the statement made in Ref. [1]. Curiously, the singular terms in Eqs. (C3) and (C5) vanish in the gauge $\xi=9/4$, which differs from the Feynman gauge, $\xi=1$, used in Ref. [1]. For simplicity we have here considered the $D=4$ case, but of course the same conclusion can be reached for a general number of space-time dimensions.

---

[1] B. Allen and T. Jacobson, "Vector Two Point Functions in Maximally Symmetric Spaces," Commun. Math. Phys. **103** (1986) 669.

[2] N. C. Tsamis and R. P. Woodard, "A Maximally symmetric vector propagator," J. Math. Phys. **48** (2007) 052306 [gr-qc/0608069].

[3] T. Prokopec, N. C. Tsamis and R. P. Woodard, "Two loop stress-energy tensor for inflationary scalar electrodynamics," Phys. Rev. D **78** (2008) 043523 [arXiv:0802.3673 [gr-qc]].

[4] T. Prokopec, N. C. Tsamis and R. P. Woodard, "Stochastic Inflationary Scalar Electrodynamics," Annals Phys. **323** (2008) 1324 [arXiv:0707.0847 [gr-qc]].

[5] T. Prokopec, N. C. Tsamis and R. P. Woodard, "Two Loop Scalar Bilinears for Inflationary SQED," Class. Quant. Grav. **24** (2007) 201 [gr-qc/0607094].

[6] E. O. Kahya and R. P. Woodard, "Charged scalar self-mass during inflation," Phys. Rev. D **72** (2005) 104001 [gr-qc/0508015].

[7] T. Prokopec, O. Tornkvist and R. P. Woodard, "One loop vacuum polarization in a locally de Sitter background," Annals Phys. **303** (2003) 251 [gr-qc/0205130].

[8] T. Prokopec, O. Tornkvist and R. P. Woodard, "Photon mass from inflation," Phys. Rev. Lett. **89** (2002) 101301 [astro-ph/0205331].

[9] T. Prokopec and R. P. Woodard, "Vacuum polarization and photon mass in inflation," Am. J. Phys. **72** (2004) 60 [astro-ph/0303358].

[10] T. Prokopec and R. P. Woodard, "Dynamics of superhorizon photons during inflation with vacuum polarization," Annals Phys. **312** (2004) 1 [gr-qc/0310056].

[11] T. Prokopec and E. Puchwein, "Photon mass generation during inflation: de Sitter invariant case," JCAP **0404** (2004) 007 [astro-ph/0312274].

[12] T. Prokopec and E. Puchwein, "Nearly minimal magnetogenesis," Phys. Rev. D **70** (2004) 043004 [astro-ph/0403335].
[13] E. T. Akhmedov, A. Roura and A. Sadofyev, "Classical radiation by free-falling charges in de Sitter space-time," Phys. Rev. D **82** (2010) 044035 [arXiv:1006.3274 [gr-qc]].
[14] K. E. Leonard, T. Prokopec and R. P. Woodard, "Covariant Vacuum Polarizations on de Sitter Background," Phys. Rev. D **87** (2013) 4, 044030 [arXiv:1210.6968 [gr-qc]].
[15] K. E. Leonard, T. Prokopec and R. P. Woodard, "Representing the Vacuum Polarization on de Sitter," J. Math. Phys. **54** (2013) 032301 [arXiv:1211.1342 [gr-qc]].
[16] D. Glavan, S. P. Miao, T. Prokopec and R. P. Woodard, "Electrodynamic Effects of Inflationary Gravitons," arXiv:1308.3453 [gr-qc].
[17] K. E. Leonard and R. P. Woodard, "Graviton Corrections to Vacuum Polarization during Inflation," Class. Quant. Grav. **31** (2013) 015010 [arXiv:1304.7265 [gr-qc]].
[18] K. E. Leonard and R. P. Woodard, "Graviton Corrections to Maxwell's Equations," Phys. Rev. D **85** (2012) 104048 [arXiv:1202.5800 [gr-qc]].
[19] S. Deser and D. Brill, "Instability of Closed Spaces in General Relativity," Commun. Math. Phys. **32** (1973) 291.
[20] L. F. Abbott and S. Deser, "Stability of Gravity with a Cosmological Constant," Nucl. Phys. B **195** (1982) 76.
[21] A. Higuchi, "Quantum linearization instabilities of de Sitter space-time. 1," Class. Quant. Grav. **8** (1991) 1961.
[22] N. C. Tsamis and R. P. Woodard, "The Structure of perturbative quantum gravity on a De Sitter background," Commun. Math. Phys. **162** (1994) 217.
[23] T. Prokopec, "Symmetry breaking and the Goldstone theorem in de Sitter space," JCAP **1212** (2012) 023 [arXiv:1110.3187 [gr-qc]].
[24] V. K. Onemli and R. P. Woodard, "Quantum effects can render w < -1 on cosmological scales," Phys. Rev. D **70** (2004) 107301 [gr-qc/0406098].
[25] Gradshteyn and Ryzhik, "Table of Integrals, Series, and Products," Alan Jeffrey and Daniel Zwillinger (eds.), ISBN number: 0-12-373637-4.
[26] P. J. Mora, N. C. Tsamis and R. P. Woodard, "Graviton Propagator in a General Invariant Gauge on de Sitter," J. Math. Phys. **53** (2012) 122502 [arXiv:1205.4468 [gr-qc]].
[27] S. P. Miao, N. C. Tsamis and R. P. Woodard, "The Graviton Propagator in de Donder Gauge on de Sitter Background," J. Math. Phys. **52** (2011) 122301 [arXiv:1106.0925 [gr-qc]].
[28] N. C. Tsamis and R. P. Woodard, "Dimensionally regulated graviton 1-point function in de Sitter," Annals Phys. **321** (2006) 875 [gr-qc/0506056].
[29] P. J. Mora, N. C. Tsamis and R. P. Woodard, "Hartree approximation to the one loop quantum gravitational correction to the graviton mode function on de Sitter," JCAP **1310** (2013) 018 [arXiv:1307.1422, arXiv:1307.1422].
[30] B. Allen and A. Folacci, "The Massless Minimally Coupled Scalar Field in De Sitter Space," Phys. Rev. D **35** (1987) 3771.