Make it count!

Numerical development and the role of working memory

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Make it count!

Numerical development and the role of working memory

Reken erop! Ontwikkeling van numerieke vaardigheden en de rol van werkgeheugen

(met een samenvatting in het Nederlands)

Proefschrift

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General introduction

“Numbers rule the universe” – Pythagoras

Numbers play an important role in our lives. They are used to specify times, in planning activities, in money transfers and are intertwined with our daily activities. Therefore, it is important for children to understand the number words and number symbols that are so overwhelmingly present in their lives. Children have to learn that number symbols (‘3’) equal number words (‘three’) and that both match a set of objects (‘***’). These number-to-quantity connections are an important prerequisite of math performance. It has been frequently shown that the ability to solve addition or subtraction problems arises from the ability to understand that numbers are connected to a specific quantity. Not only in basic math skills but also in more advance math problems (for example in fractions) it has been shown that number-quantity connections play an important role. Little is known, however, about the *development* of number-to-quantity connections and important precursors. By examining the domain specific and domain general precursors of number-to-quantity connections, this dissertation will provide new insights in the development of these skills.

Number-to-quantity mapping

In this dissertation, the ability to make connections between number symbols and their corresponding quantities is referred to as ‘mapping’ (Geary, 2013; Holloway & Ansari, 2009). The development of mapping skills starts early in childhood when children have not yet developed connections between number words and the quantities they denote. For example, at some point in the preschool years, children are able to recite the counting sequence. When 2- and 3-year-olds are asked, however, to get a specific number of toys out of a larger set, they fail (Wynn, 1992). When connections between numbers and quantities start to develop, these connection are not yet accurate: 5 year olds can understand the difference between for example the quantities of the number symbols ‘2’ and ‘4’ but still have troubles with understanding that for example the number symbols ‘22’ and ‘24’ have the same distance. Older children (8 years old), however, are more capable of understanding differences between small but also large number symbols (e.g. Laski & Siegler, 2007).

A common idea about mapping skills is that they originate from a specialized cognitive system in which numbers and quantities are spatially ordered from left to right often referred to as a 'mental number line'. In this system, smaller numbers and their quantities are associated with the left side of space whereas larger numbers are associated with the right side of space. This has been frequently referred to as the 'Spatial Numerical Association of Response Codes' (SNARC) effect (Dehaene, Bossini & Giraux, 1993; Wood, Willmes, Nuerk, & Fischer, 2008). Moreover, numbers close to each other (e.g. 2 and 3) are also close to each other on the mental number line. For example when the distance between numbers is small (e.g. 2 and 3) it is more difficult to distinguish between these numbers than when the distance between numbers is larger (e.g. 2 vs. 13; Dehaene et al., 1993; Fischer, Castel, Dodd, & Pratt, 2003; Gevers, Lammertyn, Notebaert, Verguts, & Fias, 2006). There are, however, other accounts of the spatial ordering of numbers and quantities. Besides a horizontal ordering from left to right, it has also been found that the SNARC effect occurs in a vertical and close/far dimension. Adults spontaneously associate small numbers with bottom responses (Gevers et al., 2006) and close responses (Santens & Gevers, 2008) and large numbers are associated with top responses (Gevers et al., 2006) and far responses (Santens & Gevers, 2008). According to some researchers vertical and close/far orderings can be traced back to a spatial left-to-right ordering (Gevers et al., 2006).

Although there may be different forms of the spatial ordering of numbers and quantities, it is assumed that this ordering influences our cognitions about distances, heights, numbers and quantities. For example when adults were asked to estimate the height of the Eiffel Tower, their estimates were smaller when they were leaning (without knowing) towards the left (Eerland, Guadalupe, & Zwaan, 2011).

It is assumed that during development, this spatial ordering becomes more precise. Evidence for the increase in accuracy of the spatial ordering of numbers and quantities can be found in studies examining the development of mapping skills. Tasks that are commonly used to measure the development of mapping skills are number comparison tasks (e.g. Durand, Hulme, Larkin, & Snowling, 2005) and number-to-position tasks (e.g. Siegler & Booth, 2004). In both types of tasks, information about the connections between numbers and quantity is needed for accurate performance. When, for example, the connection between numbers and quantities is not yet fully developed, children have more difficulty with deciding which of two numbers is larger in a comparison task and will be less accurate in estimating the correct positions of numbers on the number-to-position task. It is assumed that during development, the spatial ordering of numbers and quantities becomes more precise resulting in an increase in accuracy on the comparison task (Durand et al., 2005; Holloway & Ansari, 2009) and on the number-to-position task (Booth & Siegler, 2006; Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; Siegler & Booth, 2004).

Several studies have shown that mapping skills are related to performance on math performance tasks. Using a number-to-position task, it was demonstrated that individual differences in mapping skills were strongly related to math achievement test scores (Siegler & Booth, 2004). Moreover, math performance could be *predicted* from mapping skills (Booth & Siegler, 2008). Performance on the number-to-position task also differed between typically achieving students and children with math learning difficulties (Geary, Hoard, Nugent, & Byrd-Craven, 2008). Also studies in which a number comparison task was used to assess mapping skills found that performance on the mapping task was related to math ability (Holloway & Ansari, 2009) and could predict math achievement scores (De Smedt, Verschaffel, & Ghesquière, 2009).

Although previous research has focused on mapping skills as precursors of math performance, little is known about the abilities that are needed for development of mapping skills. To come to a full understanding of early numerical learning it is important, however, to gain more insight in processes that underlie the development of mapping skills in young children. On the one hand, previous research suggests that skills such as the ability to compare quantities and the ability to recite the counting sequence form domain-specific precursors of mapping skills (e.g. Dehaene, 2001). On the other hand, it has been proposed that domain-general working memory skills also contribute to the development of mapping (e.g. Geary, Hoard, Byrd-Craven, & Nugent, 2007; Van Dijck & Fias, 2011). The main goal of this dissertation, therefore, is to unravel the processes that contribute to the development of number-to-quantity mapping (see Figure 1). The developmental approach that is utilized in this thesis provides insights in the abilities that contribute to the development of mapping skills by investigating domain specific and domain general precursors of mapping.

Domain-specific precursors

One of the most influential models that serves as a framework for examining the domain-specific precursors of mapping development is the 'triple-code model' proposed by Dehaene (1992; Dehaene & Cohen, 1995). In this model, three codes for numerical processing in adults are presented: 1) an analogue quantity code, 2) a visual code and 3) a verbal code.The analogue code involves the ability to understand and manipulate quantities. Dehaene (2001) suggested that this non-symbolic quantity knowledge is based on a cognitive system dedicated to processing quantity information. Although these skills are in some studies referred to as quantity-to-space mapping (De Hevia & Spelke, 2010; Opfer & Thompson, 2006), in this dissertation the term non-symbolic skills is used to refer to the analogue quantity code. Evidence for the existence of such a system was found in animals and infants (Barth et al., 2006; Dehaene, 2001; Wood & Spelke, 2005; Xu, Spelke, & Goddard, 2005). There is also brain evidence showing that the intraparietal sulcus (IPS) might be a candidate neurological substrate of the non-symbolic quantity processor. However, the evidence is controversial (Ansari, 2008).

This first non-symbolic code for understanding quantities is believed to be innate whereas the second and third code for processing numerical information are

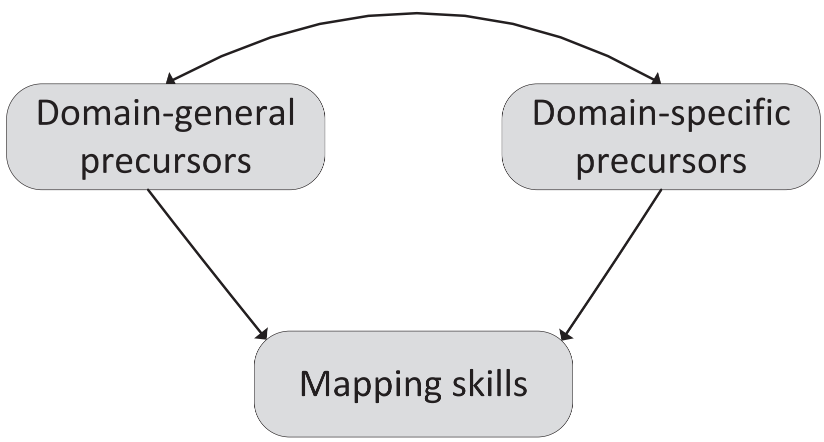

Figure 1. Outline of this dissertation.

believed to be culturally acquired skills. These acquired numerical codes concern the processing of verbal numbers as strings of words and the processing of visual numbers as strings of Arabic number symbols (or other writing systems), but do as such not contain any information about the meaning of the number words and symbols (Dehaene & Cohen, 1995). Between the ages of 3.5 and 8 years children not only gain knowledge in reciting the counting sequence but they also show progress in focusing on numerosity in a set of objects which results in new understandings that number symbols are connected to quantities (Aunio, Hautamäki, Heiskari, & Van Luit, 2006; Hannula & Lehtinen, 2005; Wynn, 1992). Dehaene (2001) suggested that this development is facilitated by transcoding processes enabling information to be translated from non-symbolic to symbolic format and vice versa, providing the number symbols with a non-symbolic quantity meaning. These processes eventually lead to a flexible integration of the different codes, resulting in numerical skills in which number words and symbols are mapped on their corresponding quantity (Dehaene, 2001; Mundy & Gilmore, 2009).

Thus, based on the triple-code-model two domain-specific skills can be identified as precursors of mapping skills: non-symbolic quantity skills and symbolic verbal and visual number skills. Dehaene (2001) proposed that the innate non-symbolic quantity system is presupposed to have a spatial format with spatially ordered positions to which symbolic numbers become connected in the course of development (Dehaene et al., 1993; Fischer et al., 2003; Gevers et al., 2006). This suggests that the spatial ordering of numbers and quantities supposed to underlie mapping skills arises from an innate spatial ordering of quantities. This claim is also supported by Von Aster and Shalev (2007). They proposed that non-symbolic understandings of quantity form a necessary precondition for learning to associate a perceived number of objects with symbolic number words or number symbols. Acquired symbolic skills, in turn, constitute a precondition for the development of mapping skills. Although longitudinal evidence for these developmental models is scarce, evidence for the importance of non-symbolic skills in learning math is provided by several studies from different research lines. A first line of evidence comes from research focusing on identifying the determinants of dyscalculia showing that a deficit in understanding and processing non-symbolic numerical information might underlie the math problems of dyscalculic children (Landerl, Fussenegger, Moll, & Willburger, 2009; Mussolin, Mejias, & Noël, 2010; Piazza et al., 2010). Other evidence for the importance of non-symbolic skills in early numerical development comes from studies showing that non-symbolic skills underlie performance in several numerical tasks (Desoete, Ceulemans, De Weerdt, & Pieters, 2010; Gilmore, McCarthy, & Spelke 2007; Gilmore, McCarthy, & Spelke, 2010; Inglis, Attridge, Batchelor, & Gilmore, 2011).

There are, however, claims against the idea that innate non-symbolic skills form the foundations upon which mapping skills are built. Nùñez (2011) provides evidence that historically number lines are a rather recent way of modeling numbers and suggests that thus particular forms of ordering numbers and quantities depend on particular (Western) cultural practices in mathematics. In addition, several studies have shown that that the spatial ordering of numbers and quantities depend on cultural factors such as reading habits (Shaki & Fischer, 2008; Opfer, Thompson, & Furlong, 2010). Moreover, it has also been shown that the SNARC effect can be easily modified through intervention (Fischer, Mills, & Shaki, 2010). This evidence suggests that the specific spatial ordering of numbers and quantities may depend on particular practices with numbers and on educational approaches. In this light, it might be argued that the spatial ordering of numbers and quantities might not originate from an innate ordering of non-symbolic quantities. Rather, cultural dependent skills related to number symbols and number words might provide a base for the spatial system underlying number-to-quantity mapping.

A possibility is that symbolic skills 'calibrate' non-symbolic skills. It has been proposed that the spatial ordering of numbers and quantities is facilitated by the culture-based ordering of the number words of the counting sequence (Helmreich et al., 2011; Opfer & Furlong, 2011). These symbolic skills might help children to understand the cardinal value of the number words and facilitate the understanding that number words refer to exact quatities. This in turn, leads to a connection between number symbols and non-symbolic quantity values (Noël & Rouselle, 2011). Indeed several studies have demonstrated that symbolic number skills, related to counting and knowledge about number symbols, play a dominant role in (the development of) mapping skills (LeFevre et al., 2010; Sasanguie, Göbel, Moll, Smets, & Reynvoet, 2012). Moreover, it has been demonstrated that mastering of symbolic verbal counting skills precedes mapping skills (Krajewski & Schneider, 2009a; Le Corre & Carey, 2007; Lipton & Spelke; 2005). In addition, it has been demonstrated that children with math difficulties do not show deficits in processing non-symbolic quantity information (De Smedt & Gilmore, 2011; Iuculano, Tang, Hall, & Butterworth, 2008; Rousselle & Noel, 2007). These findings indicate that the role of non-symbolic skills might be not as straightforward as put forward by previous studies assuming that these innate non-symbolic skills provide the foundation upon which mapping skills are built.

The findings of a spatial ordered system for processing numbers and quantities may also suggest that a specific spatial ordering arises from a general purpose spatial system supporting the one-dimensional mapping of numbers to quantities and also the ordering of quantities to space. Mapping skills (and also non-symbolic quantity skills) could arise from such a general purpose spatial system (Ansari, 2008). According to this idea, domain-general skills, such as working memory, might play an important role.

Domain-general precursors

A cognitive framework that is commonly used to examine the role of domain-general skills in relation to school learning is the multi-component model of working memory (WM) proposed by Baddeley and Hitch (1974). This model comprises three distinct components: the visuo-spatial sketchpad (VSSP), the phonological loop (PL) and the central executive (CE). The VSSP and the PL are conceived as modality specific 'slave-systems' and are involved in short-term storage of respectively visual-spatial (and kinesthetic) and verbal-phonological information. The CE is involved in storing and processing modality independent information, using the slave systems. The CE can be further differentiated into sub-processes: the executive functions (EF; Baddeley, 1996). The dominant view today is that there are three interrelated but separate executive functions (Baddeley, 1996; Miyake et al., 2000): updating (monitoring, coding, and revising) of memory representations upon incoming information, shifting of attention between different rules and strategies, and inhibition of prepotent responses and task-irrelevant information. Evidence for both the WM-

structure and the three-factor structure of EF has been found in children (Alloway, Gathercole & Pickering, 2006; Espy et al., 2004; Gathercole, Pickering, Ambridge, & Wearing, 2004; Hughes, 1998; Letho, Juujarvi, Kooistra, & Pulkkinen, 2003).
In the existing literature, however, the use and interpretation of the terms 'working memory' and 'updating' is not straightforward. Although some authors have made a distinction between WM and updating as being different concepts (e.g. Letho et al., 2003), others use the term WM to refer only to the CE, or even more specifically to updating. In this dissertation, the term WM refers to the internal workspace that is used for (modality specific) storage and processing as described by Baddeley and Hitch (1974).

Considering the role of WM in numerical learning, a large body of research has focused on the role of executive functions, the PL and the VSSP in math performance. Several studies have found that, although relations are demonstrated between math and shifting and inhibition, updating seems to play a dominant role in math performance (Andersson, 2008; Boonen, Kolkman, & Kroesbergen, 2011; Bull & Scerif, 2001; St.Clair-Thompson & Gathercole, 2006; Van der Ven, Kroesbergen, Boom, & Leseman, 2012). It is argued that performance on numerical tasks involves simultaneous and sequential processes of perceiving, coding, interpreting, and comparing information in different modalities which is facilitated by the updating function (e.g., De Smedt et al., 2009; Geary et al., 2007). Considering the role of the PL and the VSSP, it has been found that the age of the children under investigation influences the role of the PL and the VSSP. Although the results are not straightforward, the PL is frequently found to play a role in numerical performance of older children (aged 8 to 10; De Smedt et al., 2009; McKenzie et al., 2003), whereas in younger children (aged 4 to 7) the VSSP has been found to be related to numerical performance (Bisanz, Sherman, Rasmussen, & Ho, 2005; De Smedt et al., 2009; Holmes & Adams, 2006; McKenzie, Bull, & Gray, 2003). A possible explanation is that the phonological loop comes into play after a skills has been learned whereas in younger children visual-spatial skills may be recruited for the learning of new early numerical abilities such as the construction of a spatial representation of the number quantity connection (Ragubar, Barnes, & Hecht, 2010). In line with this, it can be argued that visual-spatial working memory might facilitate the construction of a spatial representation of the number quantity connection. For example, during the execution of mapping tasks, such as a number-to-position task, consecutive trials are used to build a 'mental number line' in working memory (Van Dijck & Fias, 2011). Presented stimuli are processed and stored in working memory and used to construct cognitions about the numerical size of consecutive stimuli. Another approach is presented by Henik, Leibovich, Naparstek, Diesendruck and Rubinsten (2011) who suggest that general spatial skills involved in processing non-countable dimensions like distance and height facilitate the spatial ordering on number-quantity connections. A combination of the ideas of Van Dijck and Fias (2011), Raghubar et al., (2010) and Henik et al. (2011) might implicate that visual-spatial working memory skills, rather than innate quantity skills, underlie performance on tasks measuring mapping skills.

Evidence for the involvement of working memory in developing mapping skills in children is found in different types of studies. Studies focusing on math learning difficulties showed that in comparing children with math learning disabilities with their typically achieving peers, working memory contributed to group differences in performance on the number-to-position task (Geary, et al., 2007). Moreover, Geary at al. (2008) demonstrated that children with math learning difficulties had general deficits in working memory and measures of mapping skills. Studies of the relations between working memory and mapping skills provided longitudinal evidence for the role of visual spatial working memory measured at age 5 in number comparison skills measured at age 8 (Krajewski & Schneider, 2009b). Cross-sectional studies have shown that across grade performance of mapping tasks could be explained by working memory. On a number-to-position task, working memory accounted for differences between first- and second-graders (Geary et al., 2007) and was an important predictor over and above age-related changes (Friso-van den Bos, Kolkman, Kroesbergen, & Leseman, 2013). Also on a number comparison task, changes in performance of 6- to 8-year-olds reflected changes in domain-general mechanisms rather than changes in domain-specific number cognitions (Holloway & Ansari, 2008).

The current study

To summarize, previous research has demonstrated that there are three important precursors of mapping skills: 1) non-symbolic skills, 2) symbolic skills and 3) working memory skills. In this dissertation the focus is on the co-development of these skills in children who are at a stage for developing accurate mapping skills. This enables us to examine the contribution of non-symbolic, symbolic and working memory skills to the development of mapping skills.

Starting point of this thesis was the triple-code-model (Dehaene, 1992, Dehaene & Cohen, 1995). Although this model provides insight in the skills that are needed for understanding the connections between numbers and quantities, it does not provide insight in the *development* of these skills. The first aim of this dissertation therefore was to examine the development of non-symbolic, symbolic and mapping skills. By using the triple-code-model as a framework for unraveling the numerical precursors of mapping skills, it is investigated how the different codes co-develop. In doing so, two contrasting hypotheses are tested. In the first hypothesis it is argued that non-symbolic skills contribute to the development of mapping skills whereas in the second hypothesis it is assumed that symbolic skills contribute to the development of mapping skills.

The second aim of this dissertation was to shed light on the contribution of working memory to the development of mapping skills. Although WM has been frequently studied in relation to math performance, little is known about the involvement of WM skills in the development of mapping skills. Moreover, based on previous findings, a differentiation between the different WM components considering their role in numerical learning seems plausible. By examining the co-development of WM skills and numerical skills, it is investigated how the different WM components are involved in the development of mapping skills. It was expected that different components contributed differently to mapping skills.

Outline of this dissertation

In different studies the domain specific and domain general precursors of numerical understanding are investigated.

Chapter 2 focuses on domain specific precursors of mapping skills and uses structural equation modeling techniques to: (a) investigate the structure of non-symbolic, symbolic and mapping skills; and (b) examine the role of non-symbolic versus symbolic numerical skills. Moreover it is examined how domain specific precursors are related to math performance. In Chapter 3, domain general executive functions are examined in relation to mapping skills (in this chapter referred to as *numerical quantity skills*). Bayesian analyses showed that updating, and not shifting or inhibition, was the most important predictor of performance and growth on numerical quantity tasks. These results set the stage for the studies reported in the next chapters were we further concentrated on the updating component of the working memory model and the supporting slave systems. Chapter 4 reports on a growth modeling approach to investigate the contribution of the phonological loop, the visual-spatial sketchpad and the central updating to development of mapping skills (in this chapter referred to as *number-quantity skills*). Chapter 5 examines co-development of precursors of mapping by relating growth models of both domain-specific and domain-general skills. In Chapter 6, the general discussion, the results of the four empirical chapters are discussed.

2

Early numerical development and the role of non-symbolic and symbolic skills

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Abstract

For learning math, non-symbolic quantity skills, symbolic skills and the mapping between number symbols and non-symbolic quantities are all important precursors. Little is known, however, about the interrelated development of these skills. The current study focuses on numerical development by: (a) investigating the structure of non-symbolic, symbolic and mapping skills; and (b) examining the role of non-symbolic versus symbolic numerical skills. Non-symbolic, symbolic and mapping skills of 69 children were assessed at age 4, 5 and 6. Results provided evidence for: (a) the developmental course of all numerical skills showing distinguishable skills at a younger age versus an integration of skills in older children; and (b) the predominant role of symbolic skills versus the subordinate role of non-symbolic skills in the development of mapping skills. Moreover, symbolic and mapping skills were found to be important predictors for math performance. These results provide new insights in early numerical development.

Introduction

In early numerical development children have to learn the connection between number symbols and their corresponding quantities; they have to understand that '3' matches a set of three objects (whether they are fingers, apples or toys) and that '89' is a bigger number than '23.' These skills are referred to as mapping skills. Mapping skills become more accurate during development and underlie a wide range of math tasks (Booth & Siegler, 2008; Siegler & Booth, 2004) but little is known about the early development and precursors of these mapping skills. A common idea in the existing literature is that both *non-symbolic* quantity skills and *symbolic* skills are prerequisites for the mapping of number symbols to non-symbolic quantities (e.g. Gilmore, McCarthy, & Spelke, 2010; Jordan, Glutting, & Ramineni, 2010). Longitudinal evidence for this idea, however, is scarce and, moreover, it is unclear how non-symbolic and symbolic skills contribute to the (early) development of mapping skills. The aim of the current study, therefore, is twofold. First, the structure of a range of numerical skills is examined at different ages in order to examine the developmental course of non-symbolic, symbolic and mapping skills. Second, the contribution of both non-symbolic and symbolic numerical skills to the development of mapping between number symbolic and quantities is examined. By focusing on the developmental relations between non-symbolic, symbolic and mapping skills this longitudinal study will provide new insights in early numerical development.

Numerical Development: Non-symbolic, Symbolic, and Mapping Skills

Current understandings of numerical development assume that both non-symbolic and symbolic skills and the mapping of numbers to quantities are important skills for math learning. In this first part of our paper, we will provide a description of these concepts and we will discuss their interrelations.

Non-symbolic understandings of numerical quantities are demonstrated to be already present in infants (Barth et al., 2006; Dehaene, 2001; Wood & Spelke, 2005; Xu, Spelke, & Goddard, 2005). Dehaene (2001) suggested that this non-symbolic magnitude knowledge is based on a cognitive system dedicated to processing quantity information. Within this system, quantities seem to be ordered spatially on a metaphorical mental number-line with increasing acuity throughout development (Halberda & Feigenson, 2008). On this mental number-line, each quantity has a specific range or position with smaller quantities placed at the left end of the line and bigger quantities placed on the right (Dehaene, Bossini, & Giraux, 1993; Fischer, Castel, Dodd, & Pratt, 2003; Gevers, Lammertyn, Notebaert, Verguts, & Fias, 2006). The ability to understand and manipulate numerical magnitudes is referred to as *non-symbolic skills* in the current study.

Although these non-symbolic skills are believed to be innate, *symbolic skills* are believed to be culturally based acquired skills. These acquired numerical skills concern the ability to represent numbers verbally as strings of words and visually as strings of Arabic number symbols but do not contain any semantic information about the meaning of the number words and symbols (Dehaene & Cohen, 1995). These verbal and visual skills together are referred to as *symbolic skills* in the current study and address the ability to recite the counting sequence or identify number symbols without connection to the corresponding quantities.

Between the ages of 3.5 and 8 years children not only gain knowledge in reciting the counting sequence but they also show progress in focusing on numerosity in a set of objects which results in new understandings that number symbols are connected to quantities (Aunio, Hautamäki, Heiskari, & Van Luit, 2006; Hannula & Lehtinen, 2005; Wynn, 1992). Dehaene (2001) suggested that this development is facilitated by transcoding processes enabling information to be translated from non-symbolic to symbolic format and vice versa, providing the number symbols with a non-symbolic magnitude meaning. These processes eventually lead to a flexible integration of the different codes, resulting in numerical skills in which number words and symbols are mapped on their corresponding magnitude (Dehaene, 2001; Mundy & Gilmore, 2009). A frequently used task to measure mapping skills is a number-line task in which children need to estimate the position of a given number on a horizontal number-line. The idea behind this number-to-position task is that non-symbolic quantity information is needed for accurate placement of symbolic number symbols. When the mapping between number symbols and non-symbolic quantities is not yet fully developed, children have trouble with estimating the correct positions of numbers on a number-line. Evidence for this idea is provided by a large amount of studies focusing on performance on the number-line task. Various studies found evidence for a developmental shift from inaccurate logarithmic placements to accurate linear placements on the number-line task (Booth & Siegler, 2006; Siegler & Booth, 2004; Siegler & Opfer, 2003). Older children, in whom mapping skills were more developed, were able to estimate number position more accurately than younger children in whom the integration of the different codes into mapping skills might not have taken place yet. The development of accurate mapping between number symbols and non-symbolic quantities is important for learning more advanced math operations such as addition or subtraction (Booth & Siegler, 2008; Geary, Hoard, Nugent, & Byrd-Craven, 2008; Siegler & Booth, 2004).

Although previous studies provided evidence for the importance of non-symbolic, symbolic and mapping skills in math learning and revealed their developmental trajectories, studies examining the developmental relations between these skills are scarce. An assumption that can be made based on the existing literature is that numerical skills develop from separate systems for the processing of non-symbolic and symbolic information to an integrated system which enables adults to process numerical information automatically (Dehaene, 2001). The first question that is addressed in the current study is how numerical skills develop in young children. Are non-symbolic, symbolic and mapping skills distinguishable in young children, indicating that there are different systems for symbolic and non-symbolic numerical processing? And do these skills become gradually integrated into one system for numerical processing in the course of development? Therefore the first aim of this paper is to examine the developmental course of non-symbolic, symbolic and mapping skills.

Precursors in the Development of Mapping Skills

The second aim of this paper is to gain insight into the relative role of both non-symbolic and symbolic numerical skills in the development of mapping skills. Although several studies showed that both *non-symbolic* and *symbolic* skills are important in developing accurate mapping between number symbols and quantities (e.g. Gilmore et al., 2010; Jordan et al., 2010), others advocate that either *non-symbolic* skills (e.g. Dehaene, 2001) or *symbolic* skills (e.g. De Smedt & Gilmore, 2011) play a dominant role in this development.

Our first Hypothesis is that non-symbolic magnitude knowledge provides a meaning to number symbols and it is assumed, therefore, that *non-symbolic* skills have a major effect on mapping skills and underlie all further math development. An influential advocate of the importance of non-symbolic skills in the development of mapping skills is Dehaene (2001). Based on the triple-code model (Dehaene, 1992; Dehaene & Cohen, 1995) in which it is assumed that numerical information can be processed mentally in three formats or ‘codes’, Dehaene (2001) formulated a developmental hypothesis in which it was proposed that all children are born with a system for processing non-symbolic quantity information (‘analgue-code’). Exposure to language and math education leads to the acquisition of number words (‘verbal code’) and number symbols (‘viusal code’) which are eventually mapped onto their corresponding quantity, assuming that the non-symbolic skills provide meaning to number symbols which are needed in all math tasks. This claim, emphasizing the importance of non-symbolic skills, is also supported by Von Aster and Shalev (2007). They proposed that non-symbolic understandings of magnitude form a necessary precondition for learning to associate a perceived number of objects with symbolic number words or number symbols. Acquired symbolic skills, in turn, constitute a precondition for the development of mapping skills. Although longitudinal evidence for these developmental models is scarce, evidence for the importance of non-symbolic skills in learning math is provided by several studies from different research lines.

A first line of evidence comes from research focusing on identifying the determinants of dyscalculia. Several authors confirmed the ‘number module deficit hypothesis’ assuming that math problems arise from deficits in non-symbolic skills. In these studies, symbolic and non-symbolic comparison tasks are used to examine differences in performance between dyscalculic children and their typically achieving peers. Group differences on both tasks indicated that a deficit in understanding and processing non-symbolic numerical information might underlie the math problems of dyscalculic children (Landerl, Fussenegger, Moll, & Willburger, 2009; Mussolin, Mejias, & Noël, 2010; Piazza et al., 2010).

Other evidence for the importance of non-symbolic skills in early numerical development comes from studies showing that non-symbolic skills underlie performance in several numerical tasks. On a symbolic task, 5-year-olds spontaneously used non-symbolic skills to manipulate symbolic stimuli, indicating that the non-symbolic knowledge was used for a symbolic task (Gilmore, McCarthy, & Spelke, 2007). Moreover, relations were found between non-symbolic skills and math performance within children 5 to 8 years old (Desoete, Ceulemans, De Weerdt, & Pieters, 2010; Gilmore et al., 2010; Inglis, Attridge, Batchelor, & Gilmore, 2011).

Nonetheless, some studies also question the role of non-symbolic skills (e.g. De Smedt & Gilmore, 2011; Krajewski & Schneider, 2009a; Landerl & Kölle, 2009; Rousselle & Noel, 2007). Gilmore et al. (2010) formulated two hypotheses based on their finding that non-symbolic and symbolic skills were associated with the development of mapping skills: one favoring the role of non-symbolic skills but the second indicating that children’s learning of number symbols and words could sharpen their non-symbolic abilities. Moreover, Inglis et al. (2011) demonstrated that, in adults, no relations were found between math performance and non-symbolic skills. They questioned the key role of non-symbolic skills in explaining variability in math performance throughout development.

Indeed, there is evidence advocating the importance of symbolic skills in learning mathematics. In the following section longitudinal and behavioral arguments will be discussed for the alternative Hypothesis that symbolic skills ‘calibrate’ non-symbolic skills, thereby facilitating mapping and therefore assumed to play a more important role in math learning than non-symbolic skills.

The first argument is provided by longitudinal studies examining math precursors in 6- to 9-year-olds. It was demonstrated that, in numerical development, mastering of verbal counting skills precede mapping skills (Krajewski & Schneider, 2009a). Moreover, non-symbolic skills seemed to play a subordinate role in learning math in contrast to the important role of symbolic skills (LeFevre et al., 2010). These results suggest that the symbolic numerical skills of children might be more important than their non-symbolic quantitative abilities. Moreover, in studies focusing on younger children aged 3 to 5 years old, evidence indicates that skills such as object counting and understanding the cardinality principle are necessary to link non-symbolic skills to number symbols (Le Corre & Carey, 2007; Lipton & Spelke; 2005).

The second argument for the importance of symbolic skills in early numerical development is provided by studies focusing on determinants of dyscalculia. Several studies demonstrated the validity of the ‘access deficit hypothesis.’ In this hypothesis it is assumed that math difficulties can be attributed to problems in mapping skills. In these studies it was demonstrated that children with math difficulties were only impaired on the symbolic comparison tasks but not on the non-symbolic comparison task (De Smedt & Gilmore, 2011; Iuculano, Tang, Hall, & Butterworth, 2008; Rousselle & Noel, 2007). This suggests impairment in symbolic skills rather than impairments in the non-symbolic skills, indicating that symbolic skills play an important role in math learning.

Another argument advocating the importance of symbolic skills comes from behavioral studies examining the relations between symbolic skills and math performance. Several studies showed that symbolic skills predict math performance in children aged 6 to 8 (Holloway & Ansari, 2009; Jordan et al., 2010). Moreover, even if an effect of non-symbolic skills on math performance was found, this effect was mediated by symbolic skills (Cirino, 2010) indicating that symbolic skills play an important role in performing math tasks.

The Current Study

To summarize, empirical evidence for the developmental course of non-symbolic skills, symbolic skills and mapping skills is lacking and the evidence concerning the importance of non-symbolic and symbolic skills within this development is inconclusive. Moreover, very few studies examined non-symbolic skills and symbolic skills and mapping skills in a longitudinal design and no conclusions can be drawn about the interrelated development of these skills. The present study is the first to address these developmental issues. A longitudinal design is used in which non-symbolic, symbolic and mapping skills are measured in the kindergarten years and in first grade. This study addresses the following research questions: (a) how are non-symbolic, symbolic and mapping skills related in the early school years? And (b) what is the role of non-symbolic versus symbolic skills in the development of mapping skills? By answering these questions this study will provide insight into the early development of non-symbolic, symbolic and mapping skills thereby contributing to the debate on which skill is most important for (early) numerical development.

For the first research question three competing hypotheses were formulated. First, it was hypothesized that there are three distinguishable numerical skills in children: non-symbolic skills, symbolic skills and integration of non-symbolic and symbolic skills (Hypothesis 1a). The second hypotheses makes a distinction between non-symbolic skills on the one hand and symbolic and mapping skills on the other, indicating that the processing of numbers in symbolic and mapping tasks takes place in a different system than the processing of quantities (Hypothesis 1b). In the third hypothesis, it is formulated that non-symbolic, symbolic and mapping skills are integrated abilities and thus form one unitary system for numerical processing (Hypothesis 1c).

The second research question addresses the role of non-symbolic and symbolic skills in the development of mapping skills and early math performance. The following competing hypotheses were formulated: (1) non-symbolic skills provide a magnitude meaning to symbolic skills and thus the effect of non-symbolic skills on mapping skills will be stronger than the effect of symbolic skills (Hypothesis 2a); and (2) experience with symbolic skills enables fine-tuning of the non-symbolic system and thus the effect of symbolic on mapping skills will be stronger than the effect of non-symbolic skills (Hypothesis 2b).

Method

Participants and Procedure

The children in this study (*N* = 69), were Dutch-speaking children who had all just entered kindergarten at measurement time 1 (M_{age} = 4.09 years; *SD* = 0.28). Measurement time 2 took place in the second kindergarten year (M_{age} = 5.07 years; *SD* = 0.26) and at measurement time 3, they were all in first grade (M_{age} = 5.96 years; *SD* = 0.21). Performance on the Raven's Colored Progressive Matrices (Raven, 1962) was used as measure for cognitive abilities (*M* = 25.46; *SD* = 5.53). Since only outdated norms are available, no intelligence quotient (IQ) scores were computed. Parents from 63 children (95.3%) filled out a questionnaire on their educational background. Of these parents, 9.4% had finished only lower secondary education or less, 25% had completed upper secondary education or vocational training and 60.9% completed higher vocational training or university. Educational background of the parents was not related to any of the measures.

Children completed two non-symbolic tasks. These included a non-symbolic number-line task and a non-symbolic dot comparison task. Two tasks were used to assess the symbolic skills of the children. These included a number-naming task and a counting task in which children's ability to recite the counting sequence was measured. Mapping skills were assessed using a symbolic number-line task and a symbolic comparison task. In the mapping tasks, the experimenter read aloud the number symbols that were used. The children were tested individually in a quiet room inside the school by trained undergraduate students. All non-symbolic, symbolic and mapping tasks were administered on a laptop computer using E-Prime 1.2 software (Psychological Software Tools, http://www.pstnet.com). Math performance was assessed in first grade using a standardized Dutch math test. This test was presented in a booklet and administered group-wise by the teacher.

Measures

Multiple tasks were used to assess each construct. Internal consistencies were satisfactory (see Table 2.1). All tasks were administrated in the range from 1–10 and in the range from 1–100. In the 1–10 range, however, a strong ceiling effect was visible at measurement time 1 and time 2. Therefore it was decided to only use the tasks in the 1–100 range in the current study. To provide a valid measure of test–retest reliabilities we used a larger sample (*N* = 167) from a related project with the same age group (see Table 1). Given the relatively large period (6 months) between the tests, the reliabilities are acceptable to good. Although the internal consistency of the non-symbolic comparison task was good (see Table 2.2), the test–retest reliability is relatively low which could indicate a measurement problem.

Non-symbolic skills

Non-symbolic number-lines. An adjusted version of the number-to-position task (Laski & Siegler, 2007) was used to measure non-symbolic quantity skills. A computerized version was used in which the child was presented with a horizontal line ranging from 1–100 (shown by boxes with 1 and 100 dots at the beginning and end of the line). To introduce the task, the experimenter demonstrated the position of the quantities '1' and '100' (in dots). Then children were presented with ten trials and asked to point with the mouse to the position they thought each quantity belonged on the number-line. Linear fit scores were computed by fitting the answers of each individual child to a linear curve (Geary et al., 2008). To the authors' knowledge, this task was not reported in previous studies.

Non-symbolic comparison. This instrument was used to measure quantity discrimination. Children were asked to compare two arrays with dots and had to indicate the array with the highest amount of dots. The dots not only varied in amount, but also in size (Barth et al., 2006; Gebuis, Kadosh, & de Haan, 2009). To control for non-numerical parameters (such as dot size, covered surface, density) three conditions can be distinguished: (1) congruent condition in which the area with the largest

Table 2.1

Internal consistency of each task and test-retest correlations in a larger sample (N = 167) within a time range of six months.

	r	*p*	*α*
Number-lines NS	.59	< .01	.73
Comparison NS	.24	.01	.84
Number-naming	.70	< .01	.84
Counting	.39	< .01	.69
Number-lines S	.48	< .01	.79
Comparison S	.61	< .01	.61

Note. NS = Non-symbolic, S = Symbolic

amount of dots was also physically larger; (2) incongruent condition in which the area with the largest amount of dots was physically smaller; and (3) a neutral condition in which the physical size of the dots in both areas was the same but the amount varied. In each condition five trials were presented in the range from 1–100. For each of the 15 items that the child made successful, one point was given. Although few studies reported the reliability of the dot comparison task, the task is frequently used as a measure of non-symbolic numerical skills (De Smedt & Gilmore, 2011; Iuculano et al., 2008; Landerl, Bevan, & Butterworth, 2004; Rousselle & Noel, 2007).

Symbolic skills

Number-naming. In this task, the child was presented with numerals on a computer screen and was asked to identify each number. The numerals were presented in a randomized order in the range from 1–100 (12 trials). The number of correctly identified numerals was used as a score for number-naming.

Counting sequence. The Early Numeracy Test-Revised (ENT-R; Van Luit & Van de Rijt, 2009) was used to measure verbal counting skills. The original ENT-R consists of nine subscales and has two analogous versions, version A (used at measurement time 1 and time 2) and version B (used at measurement time 3). In this study, the items from one subscale were used in which children were asked to recite the counting sequence backwards and forwards and by skipping numbers. The items are scored with 0 for a wrong answer and 1 for a correct answer. The subscale contains five items; the sum of correct answers was used as a measure for counting sequence. Reliability score of the original ENT-R was $\alpha = .93$ (Van Luit & Van de Rijt, 2009).

Mapping

Symbolic number-lines. In this task children were asked to estimate the position of a given number (in the range from 1–100) on a horizontal line. The features of this task were the same as in the non-symbolic number-line task except for the stimuli: instead of using dots, in the symbolic tasks number symbols were used. Linear fit scores of each child were computed and used as measure for performance. As shown by Friso-van den Bos, Kolkman, Kroesbergen and Leseman (2012), individual linearity scores provide a valid measure for examining development in mapping skills in young children. Booth and Siegler (2006) reported consistency scores of individual differences across a number-line estimation task and a computational estimation task (estimating the answer to addition problems) between $r = .38$ and $r = .66$ (p-values between <.01 and .10) for children aged between 5.8 and 9.1 years old.

Symbolic comparison. In this task, children were asked to indicate the number with the highest numerical value out of two numbers. Again, the features were the same as in the non-symbolic task except for the stimuli: instead of using dots, number symbols were used. Note that this task was not administered at measurement time 1. Due to the high working memory load when two numbers need to be perceived, remembered and compared this would not provide a reliable measure of symbolic comparison skills.

Mathematics

Math skills were assessed with a standardized math test (Janssen, Scheltens, & Kraemer, 2005). The math problems that were presented in this test involved numbers, number relations and simple addition and subtraction. A picture accompanied each problem. This picture was sometimes, but not always, needed to solve the problems. The test contained 50 problems which were read aloud by the teacher. Each correct answer was awarded one point. The total of correct answers was converted into competence scores that were used in the analyses. Reliability of this test is considered good ($\alpha = .92$; Jansen, Verhelst, Engelen, & Scheltens, 2010).

Analysis Procedure

Before testing the hypotheses, univariate outliers (scores two standard deviation below or above the mean) were converted into scores within the normal range (with a standard deviation of 2 or −2). This procedure was used on 5.14% of the data points. Post-hoc power analysis revealed a satisfactory power level of .85 (Field, 2005) to find medium effects ($R^2 = .25$) with a sample size of $N = 69$ and an alpha-level of $\alpha = .05$.

Confirmatory factor analyses (CFA) were conducted to examine the structure of numerical skills. For each measurement time we evaluated the fit of three models: (1) a model including a non-symbolic, a symbolic and a mapping factor; (2) a model in which a non-symbolic and a symbolic factor were specified, with the latter containing symbolic and mapping tasks; and (3) a model which included one general numerical factor. Because of the relatively small sample size, additional constraints were added on the errors of the measured variables (Marsh, 1999). Moreover, a bootstrap-analysis was performed to validate the findings of the CFA. This bootstrap-procedure enables us to estimate model fit-indices (and parameters) by producing multiple (1,000) datasets based on our own sample. The results of bootstrap-analyses provide means of the model-fit indices and parameters based on all these datasets and also provide the mean-discrepancy between the fit-indices and parameters of all these datasets. For a bootstrap-CFA, the model with the smallest mean-discrepancy is the most reliable model. This procedure renders valid results, even in small samples of $N = 50$ (Yung & Chan, 1999). In the next section we will present the fit-indices of each model: chi-square (X^2) with its p-value, CFI, RMSEA and AIC. Chi-square is a discrepancy measure between the current model and the saturated model, and should be as low as possible, with a p-value that is as high as possible. CFI compares the fit of the model to the independence model and is considered good if > .95 and acceptable if > .90. RMSEA is the parsimony measure favoring simpler models and is considered good if < .05 and acceptable if < .08. AIC is not an absolute fit index but is used to compare models: the lowest AIC value is preferred (Blunch, 2008).

Structural equation modeling (SEM) was used to examine the relations between the different numerical concepts and math performance over time. Two different models were examined using SEM: (A) a model including all relations between all numerical tasks (but not math performance); and (B) a model only including the significant relations between all numerical tasks and math performance. The first model was used to examine the role of non-symbolic versus symbolic skills in the development of mapping between number symbols and quantities. The second model was used to examine the effect of non-symbolic, symbolic and mapping skills on math performance. Also in this SEM-analysis a bootstrap procedure was used to validate the findings. Since the aim of this last analysis was to estimate parameters (and it was not the aim to compare models) mean-discrepancies (and AIC values) were not reported.

Results

Descriptive Statistics

Table 2.2 presents the descriptive statistics of each task after converting outlying scores. On the non-symbolic (NS) and symbolic (S) comparison task, one-sample *t*-tests showed significant differences between the chance level accuracy and the actual accuracy rate, indicating that children performed well above chance-level at time 1 (NS: $t(65) = 30.71$; $p < .001$; $d = 1.19$) time 2 (NS: $t(67) = 117.05$; $p < .001$; $d = 6.01$; S: $t(44) = 44.04$; $p < .001$; $d = 2.44$) and time 3 (NS: t (67)= 137.81; $p < .001$; $d = 7.10$; S: t (67)= 108.91; $p < .001$; $d = 5.68$). Correlations between the different numerical tasks are presented in Table 2.3. Note that these correlations were calculated within each measurement to describe the data and were not used to test the hypotheses that were formulated in the introduction.

At each measurement time, associations were found between tasks measuring the same concepts and between tasks measuring different concepts. Concerning relations between numerical skills and math performance, a stronger association was found at all measurement times between symbolic and mapping skills and math than between non-symbolic skills and math performance. This was even the case when the relations between math and comparable tasks (non-symbolic and symbolic number-line task) were examined.

Table 2.2
Mean scores (M) and standard deviations (SD) of the numerical tasks at each measurement time.

	Time 1		Time 2		Time 3			
	M	*SD*	*M*	*SD*	*M*	*SD*	Min.	Max.
Number-lines NS	0.33	0.27	0.61	0.22	0.69	0.18	0.00	0.98
Comparison NS	11.70	3.10	13.83	0.97	13.90	0.83	5.02	15.00
Number-naming	2.78	1.86	6.32	3.24	9.90	2.78	0.00	12.00
Counting	1.04	0.97	2.97	1.35	3.82	0.85	0.00	5.00
Number-lines S	0.16	0.17	0.38	0.29	0.57	0.27	0.00	0.96
Comparison S			12.77	1.95	14.03	1.06	8.66	15.00
Math					36.07	10.72	12.36	60.32

Note. The symbolic comparison task was not administered at time 1. NS = Non-symbolic; S = Symbolic

Structure of Numerical Skills

CFA were performed for each measurement time. It was examined whether the numerical tasks were best represented by one, two or three latent factors. In the first model that was tested, three interrelated non-symbolic, symbolic and mapping factors were included. The second model included an interrelated non-symbolic and symbolic factor and in the third model only one factor was included. For each measurement time, the fit indices of the three different models are presented in Table 2.4. At time 1 and time 2, the three-factor model (model 1) shows the smallest mean discrepancy in the bootstrap analyses. This model renders also the best fit-indices when a CFA was performed in the original sample (time 1: $X^2(4) = 2.71$; $p = .61$; CFI = 1.00;

Table 2.3
Correlations between numerical tasks and math performance.

	Number-lines NS		Comparison NS		Number-naming		Counting		Number-lines S		Comparison S	
	r	*p*	*r*	*p*	*r*	*p*	*r*	*p*	*r*	*p*	*r*	*p*
							Time 1					
Comparison NS	.09	.24										
Number-naming	.27	.02	.20	.06								
Counting	.11	.19	.08	.26	.27	.01						
Number-lines S	.35	< .01	-.01	.48	.29	.01	.20	.06				
Math	-.06	.33	.14	.13	.19	.06	.46	< .01	.23	.03		
							Time 2					
Comparison NS	.23	.03										
Number-naming	.23	.03	.18	.07								
Counting	.14	.13	-.01	.48	.41	< .01						
Number-lines S	.26	.02	.28	.01	.40	< .01	.26	.02				
Comparison S	.16	.15	.08	.29	.39	< .01	.18	.12	.27	.04		
Math	.18	.08	.05	.34	.21	.05	-.04	.38	.35	< .01	-.01	.47
							Time 3					
Comparison NS	.10	.22										
Number-naming	.26	.02	-.17	.09								
Counting	.31	.01	-.10	.21	.76	< .01						
Number-lines S	.34	< .01	.04	.38	.30	.01	.43	< .01				
Comparison S	.31	.01	-.03	.41	.28	.01	.19	.06	.46	< .01		
Math	.25	.02	.16	.10	.19	06	.17	.09	.38	< .01	.23	.03

Note. Non-symbolic tasks = Number-lines NS & Comparison NS; Symbolic tasks = Number-naming & Counting; Mapping tasks = Number-lines S & Comparison S; NS = Non-symbolic; S = Symbolic

RMSEA = .00; time 2: $X^2(8) = 6.64$; $p = .58$; CFI = 1.00; RMSEA = .00). The standardized factor loadings are reported in Figure 2.1. At time 3, the one-factor model (model 3) shows the smallest mean discrepancy in the bootstrap analyses. This model renders also the best fit-indices when CFA in the original sample was performed, $X^2(10) = 7.21$; $p = .71$; CFI = 1.00; RMSEA = .00. Note that at time 1 and time 2, the fit-indices of the two-factor model (model 2) also show a good fit. Based on the results of the bootstrap-analyses it was decided to choose the three-factor model as the best-fitting model.

Table 2.4
Fit indices for each model at the different measurement time in the original sample (N=69) and based on Bootstrap-analyses.

	N=69			Bootstrap-procedure		
	model 1	model 2	model 3	model 1	model 2	model 3
			Time 1			
X^2	2.71	4.01	7.60	3.15	4.50	8.50
df	4	5	6	4	5	6
p	.61	.55	.27	.53	.47	.20
CFI	1.00	1.00	.85	1.00	1.00	.85
RMSEA	.00	.00	.06	.00	.00	.08
AIC	34.71	34.01	35.60	25.15	25.55	26.50
Mean-discrepancy				7.59	9.62	15.09
SD				.14	.18	.23
			Time 2			
X^2	6.64	9.26	41.79	6.88	9.86	38.79
df	8	10	11	8	10	11
p	.58	.51	< .01	.55	.45	< .01
CFI	1.00	1.00	.00	1.00	1.00	.11
RMSEA	.00	.00	.20	.00	.00	.19
AIC	44.64	43.26	73.79	32.88	31.86	58.79
Mean-discrepancy				14.98	19.78	50.32
SD				.22	.25	.53
			Time 3			
X^2	25.14	25.15	7.21	25.50	25.51	7.30
df	8	10	10	8	10	10
p	< .01	< .01	.71	< .01	< .01	.70
CFI	.50	.56	1.00	.57	.62	1.00
RMSEA	.18	.15	.00	.18	.15	.00
AIC	63.14	59.15	41.21	51.49	47.51	29.30
Mean-discrepancy				34.15	37.05	17.47
SD				.35	.38	.23

The Effect of Non-symbolic and Symbolic Skills on the Development of Mapping and their Effect on Math Performance

Based on the CFA, factor scores were computed (standardized factor loading × standardized variable score) for three factors at time 1 and time 2 and one factor at time 3. These factor scores were used in an SEM analysis to examine the effect of non-symbolic and symbolic skills on the development of mapping skills (model A) and their relations with math performance (model B). Fit indices for these models were satisfactory (see Figure 2.2). Non-symbolic skills at time 1 did not predict either symbolic or mapping skills at time 2. Symbolic skills at time 1, however, predicted both non-symbolic and mapping skills at time 2. Performance on the numerical factor at time 3 was predicted only by mapping skills at time 2. Figure 2 demonstrates that, at time 1, non-symbolic, symbolic and mapping skills were related to each other. At time 2, however, only symbolic skills and mapping skills showed a significant correlation. These results were confirmed by the bootstrap-analyses.

The results of the second model (model B; see Figure 2.2) demonstrated that math performance is predicted by time 2 mapping skills but not by time 2 symbolic or non-symbolic skills. Moreover, numerical performance at time 3 was related to math performance. Together these variables explain 12% of the variance in math performance. These results were confirmed by the bootstrap-analyses.

The low correlations between the non-symbolic tasks, however, might cause a low reliability of the non-symbolic factor score. To take this into account, another SEM analysis was performed in which the separate non-symbolic tasks were included as separate variables in both model A and model B. These analyses led to the same conclusions for both tasks.

Discussion

The current study is the first to provide both empirical and longitudinal evidence for: (a) the developmental course of non-symbolic, symbolic and mapping skills showing separate skills at a younger age versus an integration of skills in older children; and (b) the predominant role of symbolic skills versus the subordinate role of non-symbolic skills in the development of mapping skills. Moreover, the mapping skills

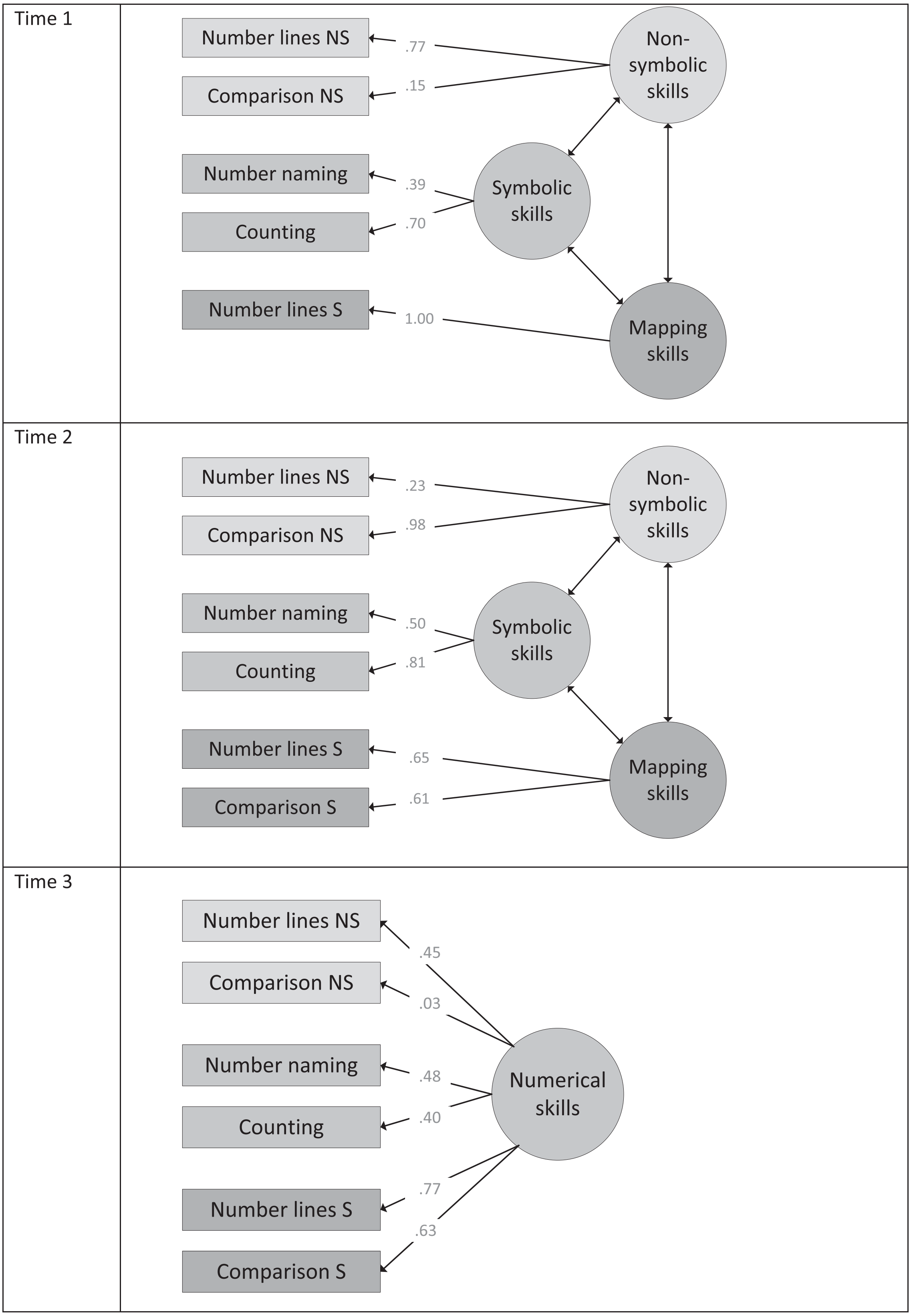

Figure 2.1
Best fitting model with the standardized factor loadings at the different measurement times, NS = non-symbolic, S = symbolic.

were found to be important for math performance. By providing not only a description of the developmental course of non-symbolic skills, symbolic skills and the mapping between number symbols and non-symbolic quantities but also examining the contribution of non-symbolic and symbolic skills to the development of mapping skills, the results of this study provide a valuable insight into the early development of learning math.

It should be noted that the sample size of this study is relatively small. Although the fit-indices of the models are good and the data show significant results, the generalizability of the results is relatively low. Although we tried to compose a representative sample, the chance that the results will differ in another sample cannot be neglected. We therefore stress the need for further research in this area with larger samples.

The first aim of this study was to examine the structure of non-symbolic, symbolic and mapping skills in a developmental perspective. Confirmatory factor analysis confirmed that three numerical factors can be distinguished at age 4 and 5: a non-symbolic factor, a symbolic factor and a mapping factor (Hypothesis 1a). At age 6, however, the data was best represented by one numerical factor (Hypothesis 1c). These results can be explained by using the triple-code model (Dehaene, 2001) which assumes initially separate systems for processing non-symbolic and symbolic information. Although this model was initially proposed as a model for numerical processing in adults, the empirical findings of the current study show that this model is also applicable to numerical processing in young children as was suggested by other research proposing models for numerical development in children (Cirino, 2010; Krajewski & Schneider, 2009a; LeFevre et al., 2010). Moreover, the current results show that at the age of 4 and 5, not only non-symbolic and symbolic numerical information is processed differently, but also there appears to be a third important skill, namely the mapping between number symbols and non-symbolic quantities. This indicates that tasks measuring mapping skills appeal to different abilities than task measuring non-symbolic or symbolic skills. This implies that, when children are presented with a number symbol, it can be processed in two ways: either only symbolic, or mapped onto the non-symbolic numerosity. It can be hypothesized that this depends on task characteristics: is the child asked to just recite the counting

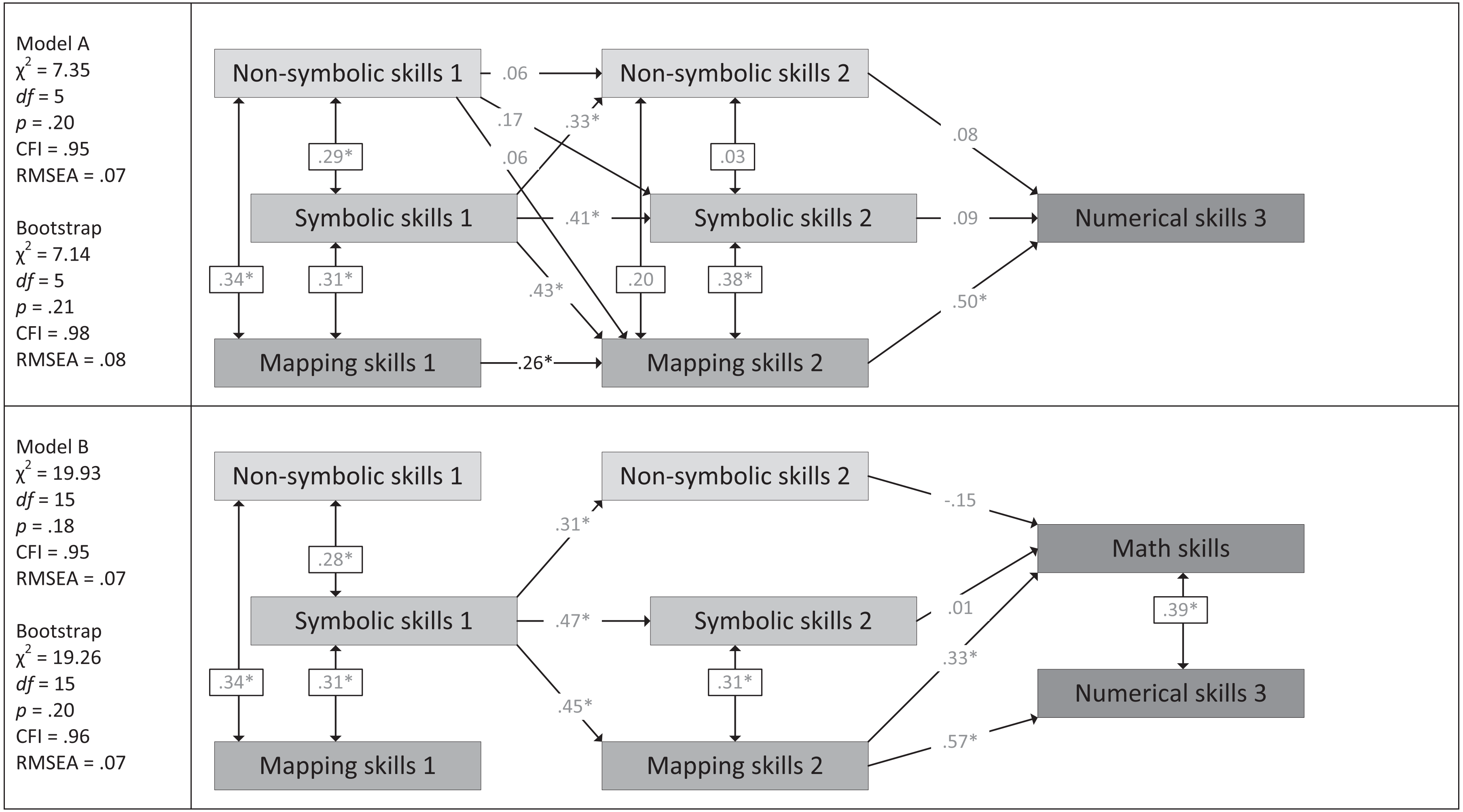

Figure 2.2
Fit índices, standardized regression coefficients and covariances (boxes with borders) for models A and B. Note. * $p < .05$

sequence or is it asked to count a set of objects? The finding that non-symbolic, symbolic and mapping skills load on one factor for numerical processing at age six indicates that, when presented with a number symbol, older children can easily connect this to the corresponding non-symbolic quantity. In the triple-code model this is referred to as transcoding processes enabling information to be translated from one system to another system facilitating automatic processing of number symbols, an important prerequisite for learning later math skills. Thus in concordance with the triple-code model, the evidence in the current study shows that children start the school system with separable systems for numerical processing, which become more integrated during the first years of formal schooling.

The second aim of the current study was to examine the role of non-symbolic and symbolic skills in early numerical development. The results of the SEM-analyses showed that symbolic skills had an effect on non-symbolic skills (but not vice versa) and on mapping skills. Non-symbolic skills did not predict mapping skills. Instead of confirming the well-established hypothesis that non-symbolic skills set up the basis for all further math development (Dehaene, 2001; Hypothesis 2a), the results of the current study provide evidence for the alternative hypothesis that symbolic skills 'calibrate' non-symbolic skills (Hypothesis 2b) as proposed by former research emphasizing the role of symbolic skills in math development (e.g. De Smedt & Gilmore, 2011; LeFevre et al., 2010). The present results fit in with the idea that connections between non-symbolic and symbolic skills (or representations) restructure non-symbolic mathematical intuitions to more general and conventionalized mathematical knowledge. In older children, this would imply that symbolic skills in addition to experience with concrete non-symbolic numerical problems might facilitate abstraction of the embedded rule or principle which can be translated to other math problems (Kaminsky, Sloutsky, & Heckler, 2008). In younger children, symbolic skills such as reciting the counting sequence might help children to understand the cardinal value of a number word and to discover that number words refer to exact quantities which in turn lead to a connection between number symbols and non-symbolic quantity values (Noël & Rouselle, 2011). Thus symbolic skills could shape or 'calibrate' quantity knowledge at the non-symbolic level facilitating development of accurate mapping skills. Moreover, this suggests that symbolic skills play a key-role in early numerical development. Based on the current results it can be argued that symbolic skills facilitate mapping skills and thereby have an indirect effect on the integration of skills when children enter grade 1. In concordance with previous studies (e.g. Booth & Siegler, 2008; Siegler & Booth, 2004), the performance on both the mapping factor in kindergarten year 2 and the numerical factor in grade 1 are in their turn related to math performance in grade 1.

Important points of concern of the current study are the relatively low test–retest correlation on the non-symbolic comparison task and the reliability of the non-symbolic factor. Although the internal reliability figures of both the non-symbolic number-line task and the non-symbolic comparison task are satisfactory and results based on analyses performed with separate tasks versus factor scores show comparable results, a closer look on possible explanations for these correlations is desirable. The low test-retest correlations on the non-symbolic comparison task might be due to a ceiling effect causing less variation in scores at different measurement times. It is thus important in future research to include a more sensitive non-symbolic comparison measure for example by measuring reaction times or including more items in this task.

The low correlations between the non-symbolic comparison and number line task could imply that the tasks intended to measure the same construct address different skills. Non-symbolic quantity information is believed to be processed in an approximate system: there are no exact codes to rely on when performing a task with (large) quantities. In the non-symbolic comparison task, an approximate understanding of the presented quantities is sufficient for accurate performance. In the non-symbolic number-line task, however, children need to indicate an exact position of a given quantity which cannot be derived easily from their approximate non-symbolic system causing children to perform differently on both tasks. This needs to be further examined in future research.

Another important aspect that is not addressed in the current study is the relation between working memory skills and math development. As various studies demonstrated that working memory skills also are related to early math development (e.g. Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; Kolkman, Hoijtink, Kroesbergen, & Leseman, 2012; Kroesbergen, Van ‘t Noordende, & Kolkman, 2012;

Krajewski & Schneider, 2009b; Passolunghi & Cornoldi, 2008), these skills could also play an important role in performance on all numerical tasks examined in the current study. In all numerical tasks simultaneous and sequential processing of perceiving, coding and comparing information is needed to perform accurately (Kolkman, et al., 2012) which places a (high) load on working memory. In numerical development, working memory might explain why some children have trouble with processing numerical information which might not be a problem that can be attributed to understanding numerical information but might rather be caused by problems in more general working memory skills. This needs to be addressed in future studies.

Nevertheless, the current study adds to the existing literature by providing empirical and longitudinal evidence for the development of numerical skills and for the dominant role of symbolic skills in early numerical development, although the conclusions are tentative because of the small sample size. The results suggest that, when children enter the school system, they use different skills for processing non-symbolic and symbolic numerical information. During the early school years they learn to integrate both types of numerical information. The results also show that symbolic skills might be more important than non-symbolic skills in learning math. By providing a (new) developmental view on numerical development in the early school years, this study adds to the current understandings of early numerical development.

3

The role of executive functions in numerical magnitude skills

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Abstract

Executive functions (EF) are closely related to math performance. Little is known, however, about the role of EF in numerical magnitude skills (NS), although these skills are widely acknowledged to be important precursors of math learning. The current study focuses on the different roles of updating, shifting, and inhibition in NS. EF and NS were assessed in 47 five-year old children. Furthermore, 21 children were presented with six training sessions aimed at improving NS. Both pre-test and improvement scores were used to investigate the role of EF in NS. Bayesian analyses show that updating is a more important predictor of individual differences in NS than shifting and inhibition. Moreover, children with better updating skills showed more improvement in number line estimation after the training. It is argued that NS rely on the processing of multiple sources of information and, therefore, may be dependent on updating skills.

Introduction

Past research has shown that executive functions are closely related to math performance and relations have been demonstrated between updating, shifting, inhibition, and both math and precursors of math such as counting (Bull & Scerif 2001; Bull, Espy, & Wiebe, 2008). Little is known, however, about the role of executive functions in numerical skills such as the understanding of numerical magnitudes. Recent studies have shown that these numerical skills provide a strong basis for math learning (e.g. Booth & Siegler, 2008) and it is therefore important to understand individual differences in the development of these skills. The current study will provide more insight into this development by examining the relation between executive functions and numerical magnitude skills.

Executive functions and math learning

One of the most influential cognitive frameworks that have been used for studying executive functions is Baddeley's model of working memory (Baddeley, 1996; 2000). In this multi-component model, the central executive is responsible for control and regulation of cognitive processes in which executive functions are involved (Miyake et al., 2000). Although executive functions have been viewed as a unitary system in the past (e.g. Duncan, Johnson, Swales, & Freer, 1997), the dominant view today is that there are three interrelated but separate executive functions (Baddeley, 1996;

Miyake et al., 2000): updating (monitoring, coding, and revising) of memory representations upon incoming information, shifting of attention between different rules and strategies, and inhibition of prepotent responses and task-irrelevant information. Evidence for this three factor structure of the central executive was found in factor-analytic studies with adults (Miyake et al., 2000). Although the evidence for a similar structure in young children is less straightforward, several studies did confirm the existence of a three factor structure in children (Espy et al., 2004; Hughes, 1998; Letho, Juujarvi, Kooistra, & Pulkkinen, 2003).[1]

Executive functions are frequently studied in relation to math skills (e.g. addition and subtraction) and precursors of math (e.g. counting). Studies which include measures of all three executive functions provide strong indications for the predominant role of updating in math learning. In these studies, updating was found to be related to both math (Anderson, 2008; Bull & Scerif, 2001; St. Clair-Thompson & Gathercole, 2006; Van der Sluis, De Jong, & Van der Leij, 2007; Van der Ven, Kroesbergen, Boom, & Leseman, 2011) and math precursors such as counting, number recognition, and subitizing (Bull et al., 2008; Epsy et al., 2004; Kroesbergen, Kolkman, & Bolier, 2009). Updating therefore seems to play an important role in different aspects of math learning.

Evidence concerning the role of shifting and inhibition in studies including all three executive functions, however, is less straightforward. In most of the studies, no relations were found between shifting and math (Anderson, 2008; Bull & Scerif, 2001; St Clair-Thompson & Gathercole, 2006; Van der Sluis et al., 2007; Van der Ven et al., 2011) or shifting and counting (Bull et al., 2008; Espy et al., 2004). One study, however, did demonstrate the role of shifting in counting as a precursor of math (Kroesbergen, Van Luit, Van Lieshout, Van Loosbroek, & Van der Rijt, 2009). Concerning inhibition, relations with math were found in one study (St Clair-

[1] Note that some studies distinguish between the concept of working memory and updating (e.g. Letho et al., 2003). 'Working memory' is the ability to store and process information simultaneously, whereas 'updating' is the ability to change stored information in the light of incoming information (Van der Ven, Kroesbergen, Boom & Leseman, 2012). Although the concepts of working memory and updating are not identical, the empirical distinction of 'working memory' and 'updating' seems small (Van der Sluis, De jong & Van der Leij, 2007) and measurements used for 'working memory' and 'updating' are closely related and load on the same factor (St Clair-Thompson & Gathercole, 2006). The current study therefore includes studies referring to both concepts and the term 'updating' may also be interpreted as 'working memory'.

Thompson & Gathercole, 2006), whereas others did not demonstrate these relations (Anderson, 2008; Bull & Scerif, 2001; Van der Sluis et al., 2007; Van der Ven et al., 2011). Some studies focusing on counting, number recognition, and subitizing as math precursors, however, did find relations with inhibition (Bull et al., 2008; Espy et al., 2004). These contradictory results concerning shifting and inhibition might be attributed to task requirements (Friso-van den Bos, Van der Ven, Kroesbergen, & Van Luit, 2012) or indicate that executive functions are less important in math learning than updating.

Numerical magnitude skills

Although the above-mentioned studies shed some light on the roles of updating, shifting, and inhibition in math and precursors of math, little is known about the role of executive functions in numerical skills such as magnitude understanding. Magnitude understanding comprises the knowledge that each number symbol is connected to a numerical magnitude. Numerous studies have demonstrated increasing accuracy throughout development in understanding magnitudes using a number line estimation task (Booth & Siegler, 2006; Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; Siegler & Booth, 2004; Siegler & Opfer, 2003). This task requires children to estimate the position of a given number on a horizontal number line. The idea behind this type of task is that magnitude understanding is needed for accurate placement: to place a given number symbol on its corresponding position on the number line, information about the connection between number symbols and their corresponding quantity is needed. Thus, if children have not developed accurate magnitude understandings, their estimations on the number line are imprecise. Various studies showed that younger children have less accurate placements that often form a logarithmic pattern. The estimates of older children, however, often form a linear pattern and are thus more accurate (Booth & Siegler, 2006; Friso – van den Bos, Kolkman, Kroesbergen, & Leseman, 2012; Siegler & Booth, 2004; Siegler & Opfer, 2003). Depending on the age of the child and the scale of the number line, estimations become more accurate (Booth & Siegler, 2006; Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; Siegler & Booth, 2004; Siegler & Opfer, 2003). These magnitude understandings are related to several aspects of early numerical learning, such as number comparison and number categorization (Laski & Siegler, 2007) and can be improved by training (Laski & Siegler, 2007; Opfer & Thompson, 2008; Ramani & Siegler, 2008; Siegler & Ramani, 2008). Furthermore, the development of accurate (linear) magnitude knowledge is positively related to more advanced math skills (Booth & Siegler, 2008; Geary, Hoard, Nugent, & Byrd-Craven, 2008; Siegler & Booth, 2004).

Numerical magnitude skills are thus important for math learning and, therefore, it is important to understand which underlying factors influence these numerical magnitude skills in order to explain individual differences. Examining these skills in relation to executive functions may provide new insights into individual differences in numerical magnitude skills. Although studies focusing on these relations are scarce, two recent studies examined the relation between updating and numerical magnitude skills. Geary et al. (2008) demonstrated that updating skills were related to performance on a number line estimation task in 7-year olds. Updating skills were also found to be related to numerical magnitude skills in younger children aged 5-8 years old. In a longitudinal study by Krajewski and Schneider (2009) updating skills predicted number comparison and number-quantity linkages.

The current study

Although the studies of Geary et al. (2008) and Krajewski and Schneider (2009) indicate the importance of updating in explaining individual differences in numerical magnitude learning, measures of shifting and inhibition are not included and conclusions about the different roles of each executive function in numerical magnitude understanding cannot be drawn. Moreover, previous studies which included measures of all executive functions did not include measures of numerical skills such as magnitude understandings. Therefore, the focus of the current study is on the relations between all three executive functions and numerical magnitude understanding in young children. To examine these relations, four competing hypotheses are formulated based on the literature discussed above. Formulating and testing different competing hypotheses will provide more insight into the relative importance of each executive function rather than just testing whether each executive function plays a role in understanding numerical magnitudes at all.

Previous research showed that updating seems more important than shifting and inhibition since all numerical tasks involve simultaneous and sequential processes of perceiving, coding, interpreting, and comparing information in different modalities. To confirm this assumption, two competitive hypotheses were formulated: 1) updating, shifting, and inhibition are equally important predictors in all numerical tasks (H1: updating = shifting = inhibition) and 2) updating is a more important predictor than shifting and inhibition (H2: updating > shifting = inhibition). Support for the second hypothesis will confirm that updating is indeed a stronger predictor of understanding numerical magnitudes than shifting an inhibition.

Considering the contradictory findings in previous research concerning the role of shifting in relation to inhibition, two different arguments can be posed. First, it can be argued that shifting plays a more important role than inhibition in understanding numerical magnitudes: flexibly shifting attention between different number modalities (Dehaene, 2001; from a verbal digit to a nonverbal magnitude code) is needed in all numerical tasks, whereas the selection of a correct response out of competing alternative responses or the suppression of irrelevant information is not needed in all numerical tasks. Second, it can also be argued that inhibition is more important than shifting: inhibition of irrelevant information in for example extensive task instructions is needed in all numerical tasks, whereas flexibly shifting between number modalities might be an automatic process and therefore plays a minor role. To explore the validity of both arguments the following competitive hypotheses were formulated: 3) updating is a more important predictor than shifting, which in turn is a more important predictor than inhibition (H3: updating > shifting > inhibition), and 4) updating is a more important predictor than inhibition, which in turn is a more important predictor than shifting (H4: updating > inhibition > shifting). Note that it is assumed, based on previous findings, that updating is still more important than shifting and inhibition in these last two hypotheses.

These hypotheses are tested by examining the relations between executive functions and numerical magnitude skills and by using the results of a numerical training to examine the relation between executive functions and the gain in numerical magnitude skills.

Method

Participants & procedure

The children participating in this study (N = 47; 22 girls) were all randomly selected Dutch-speaking children in their second year of kindergarten (mean age = 5.94 years; SD = 0.49 at pre-test). Each child completed two individual testing sessions lasting 20 to 30 minutes each. Executive functions were assessed in the first session and in the second session children completed the numerical tests. At post-test, after six weeks, all children completed the same numerical tasks again. Twenty-one children (10 girls) were randomly selected to participate in a numerical training. The training consisted of six training sessions outside the classroom provided once a week during six successive weeks. Trained undergraduate students in psychology and education conducted the standardized training sessions with the same group of three to four children for six weeks. The children in the control group did not participate in extra activities. Two children in the control group missed the post-test session due to illness.

Measures

Executive functions.

Updating. The Dutch version of the Listening Recall task from the AWMA test battery (Alloway, 2007) was administered to assess *updating* (test-retest: $r = .81$; Gathercole at al., 2008). This verbal measure of updating was used since several studies demonstrated a relation with early math such as comparison skills (Krajewski & Schneider, 2009). In this test, an increasing number of sentences were judged on their proposition after which the first word of each sentence needed to be recalled. The sum of correct answers was used as a measure for updating ($\alpha = .93$).

Shifting. The Dimensional Change Card Sorting task (Zelazo, 2006) was used to assess shifting. This task is widely used as a measure of shifting abilities in young children and is considered age appropriate (Carlson, 2005). Cards were subsequently sorted based on their color (pre-switch phase) or shape (post-switch phase). In the border phase, the cards were sorted on shape or color based on a black border that did or did not appear around the figure to be sorted. The sum of correctly answered trials in the post-switch phase (α = .80) and the border-phase (α = .75) were used as measure of shifting. The reliability of this sum score was sufficient (α = .83).

Inhibition. The Day/Night-Stroop was used to measure inhibition (Gerstadt, Hong, & Diamond, 1994; test-retest: r = .84; Thorell, 2006). Children had to respond to a sun-card by saying 'moon' and to a moon-card by saying 'sun'. The number of correct responses was used as measure of inhibition (α = .87).

Numerical magnitude skills.

Three tasks based on the tasks developed by Laski and Siegler (2007) were used. In all numerical tasks, including the training tasks, the same 22 numbers ranging from 1 to 100 were used (2, 3, 5, 8, 12, 17, 21, 26, 34, 39, 42, 46, 54, 58, 61, 67, 73, 78, 82, 89, 92, and 97).

Numerical magnitude comparison. Half the numbers from each decade that were used on the other two tasks were used on this task. Out of all possible combinations, 10 pairs of numbers were randomly selected: 2 vs. 8, 54 vs. 42, 8 vs. 34, 67 vs. 12, 34 vs. 42, 26 vs. 89, 2 vs. 54, 67 vs. 89, 73 vs. 97 and 8 vs. 73. These pairs were read aloud in a fixed order, although the bigger number was randomly allocated to the left or right. Children had to indicate the number with the largest numerical value. The sum of correct trials was used as a measure of numerical comparison skills. Although the reliability of this task in the current study was rather low (α = .46), reliability figures of the same task using the same numerals but including more items (15) in a larger sample (N = 239) within the same age-range (mean age = 5.56 years) were satisfactory (α = .68).

Categorization task. Twenty-two same-sized cards with printed numbers were shown and read aloud. These needed to be sorted in the correct category (verbally labeled from left to right: 'really small', 'small', 'medium', 'big'and 'really big'). The sum of correctly sorted cards was used as measure of categorization skill (α = .79).

Number line task. The position of a given number (shown and read aloud by the experimenter) needed to be estimated on a horizontal number line. Linear fit scores were computed by fitting the answers of each individual child to a linear curve (see also Geary et al., 2008; α = .83).

Training.

The activities in the training sessions were adapted from the study of Laski and Siegler and will be briefly described below (for a full description of the tasks, see Laski & Siegler, 2007). Feedback was given during all sessions and activities. Incorrect answers were corrected together with an explanation of the correct answer. In the first three sessions the activities *midpoint categorization* and *triad task* were used. In the last three sessions the activities *variable number categorization* and the *triad task* were used.

Midpoint categorization task. Cards displaying category midpoints (10, 30, 50, 70, and 90) needed to be sorted in the correct category. The categories were labeled with pictures presenting a quantity corresponding to the category midpoint; these pictures were removed after the instruction phase. Children were presented with 30 randomized trials.

Variable numbers categorization. Cards displaying varying numbers ranging from 1 to 100 needed to be sorted in the correct category. The instruction was the same as in the midpoint categorization task and children were presented with 22 randomized trials.

Triad task. Out of three cards with numbers, children needed to indicate the two numbers from the same category. Children were presented with 20 randomized triad problems, four trials per category.

Analyses

Multivariate outliers based on Cook's distance were removed from the dataset. Before testing the hypotheses, the correlations between the executive function tasks and the numerical magnitude tasks and the effects of the numerical training were examined in order to justify further analyses.

Bayesian evaluation of informative hypotheses was used to examine the relations between executive functions and numerical magnitude skills (for a detailed introduction see Klugkist, Laudy, & Hoijtink, 2005; Van de Schoot et al., 2011a, 2011). This type of analysis enables the testing of multiple hypotheses (as is the case in this study) without the loss of power (due to, for example, Bonferroni-type of corrections, as elaborated on in Van de Schoot et al., 2011a and; Hoijtink, Klugkist, &

Boelen, 2008). Moreover, this type of analysis is confirmative and renders a quantification of support in the data for each of the hypotheses under consideration and is therefore especially useful to evaluate hypotheses in small samples. The *posterior model probability* (PMP) is computed (see http://tinyurl.com/informative hypotheses) for each hypothesis to quantify the support in the data. If the PMP of one hypothesis is larger than the PMP of the unconstrained hypothesis (H_u), the constraints used to specify the hypothesis are supported by the data; and if the PMP of a first hypothesis is larger than the PMP of a second hypothesis, the support in the data is larger for the first than for the second hypothesis. Note that the sum of the PMPs for a set of competitive hypotheses is always one. Next, the Bayes-factor was computed, which is a measure for the degree of support for each hypothesis compared with a hypothesis without constraints. This Bayes-factor was computed by dividing the PMP-value of a hypothesis by the PMP-value of the unconstrained hypothesis, which results in a value showing the evidence in favor of one hypothesis compared with the unconstrained hypothesis (see Appendix for a detailed description of this approach).

Table 3.1
Descriptive Statistics of the Executive Functions and Numerical Estimation Skills

	Control group			Experimental group				Differences between groups		Performance above chance level	
	N	M	SD	N	M	SD	Range	*t*	*df*	*t*	*df*
Pre-test											
Updating	24	15.92	5.45	16	17.88	4.29	0 - 28	-1.21	38	-	-
Shifting	24	11.79	4.76	16	11.56	5.06	0 - 18	.15	38	15.46*	43
Inhibition	24	14.29	1.90	16	14.31	3.63	0 - 16	.02	38	30.73*	43
Comparison	24	8.17	1.61	16	8.81	1.11	0 - 10	-1.40	38	15.08*	43
Categorization	24	7.08	4.19	16	6.38	3.32	0 - 22	.57	38	3.84*	43
Number lines	24	.49	.28	16	.53	.21	0 - 1	- .47	38	-	-
Post-test											
Comparison	22	8.45	1.26	16	8.88	1.03	0 - 10	-1.10	36		
Categorization	22	9.36	5.11	16	12.75	4.57	0 - 22	-2.11*	36		
Number lines	22	.54	.30	16	.57	.27	0 - 1	- .32	36		
Gain											
Comparison	20	.28	1.39	19	.07	1.18	0 - 22	- .24	37		
Categorization	20	2.20	4.19	19	5.84	5.53	0 - 10	-2.35*	37		
Number lines	20	.03	.16	19	.09	.15	0 - 1	-1.30	37		

Results

Descriptive statistics

Descriptive statistics (after removal of outliers) are presented in Table 1. No differences were found at pre-test between the control group and experimental group. One-sample *t*-tests showed that children performed above chance level on tasks for which answer categories were provided facilitating a guess for the correct answer (see Table 3.1).

Correlations were found between updating and performance on all three numerical tasks. Shifting was related to the performance on the comparison and number line task and marginally significantly related to the categorization task. No relations were found between inhibition and comparison and categorization, although the relation between inhibition and the number line task was marginally significant (see Table 3.2).

Gain scores were computed to examine the effect of the training. Comparing the mean gain scores of the control and the experimental group revealed a medium to strong effect for the categorization task (Cohen's $d = .68$), a small to medium effect for the number line task ($d = .39$), and no effect for the comparison task ($d = .08$; see Table 3.1). Within the experimental group, gain scores on the categorization task were not related to updating, shifting, and inhibition. Gain scores on the number line task were related to updating but not to shifting and inhibition. Within the control group no significant correlations were found between the gain scores and the executive functions (see Table 3.2).

Table 3.2
Correlations (1-tailed) between the executive functions and the numerical tasks at pre-test (N = 40) and the gain scores within the experimental group (N = 19) and control group (N = 18)

	Both groups Pre-test			Experimental group Gain		Control group Gain	
	Comp.	Cat.	Nl.	Cat.	Nl.	Cat.	Nl.
Updating	.38**	.28*	.64**	-.07	.59**	.26	-.10
Shifting	.27*	.22†	.40**	.32	.01	.26	.03
Inhibition	.01	.15	.26†	-.07	.31	-.03	-.11

Note. ** $p < .01$; * $p < .05$; † $p < .10$; Comp. = Comparison, Cat. = Categorization, Nl. = Number lines

Relation between executive functions and numerical magnitude skills.

A confirmatory Bayesian approach was used to test the hypotheses concerning the relation between executive functions and numerical magnitude skills. In these analyses, the pre-test scores of both the control and experimental group were used. Table 3.3 displays the support in the data for each hypothesis (PMP-values and Bayes-factor). Concerning the comparison and number line task, most support was found for H2, indicating that updating was a more important predictor than shifting and inhibition. On the comparison task, the difference in PMP-value between H2 and H3, however, was very small indicating two plausible hypotheses regarding the role of shifting and inhibition.

Concerning the categorization task, the results are less straightforward. The PMP-values showed a small difference between H1 and H2 indicating that the role of each executive function might be equally important or indicating that updating is a more important predictor than shifting and inhibition. As can be seen in Table 3.4, the inequality constraints specified in the hypotheses were supported by the data: the parameter estimate (unconstrained regression coefficient) for updating was larger than

Table 3.3

PMP-values and Bayes-factor for each hypothesis under consideration for the pre-test scores in the control and experimental group (N = 40) and for the gain scores in the experimental group (N = 19)

		Control & experimental group			Experimental group	
		Pre-test Comp.	Pre-test Cat.	Pre-test Nl.	Gain Cat.	Gain Nl.
				PMP-values		
H_u	unconstrained hypothesis	.12	.12	.12	**.35**	.16
H1	Upd. = Shi. = Inh.	.16	**.30**	.08	.34	.08
H2	Upd.> Shi. = Inh.	**.32**	**.27**	**.36**	.14	.26
H3	Upd.> Shi. > Inh.	**.31**	.14	.20	.12	.11
H4	Upd. > Inh. > Shi.	.09	.16	.24	.06	**.39**
				Bayes-factor		
H1	Upd. = Shi. = Inh.	1.33	2.50	.67	.97	.50
H2	Upd. > Shi. = Inh.	2.67	2.25	3.00	.40	1.63
H3	Upd. > Shi. > Inh.	2.58	1.17	1.67	.34	.69
H4	Upd. > Inh. > Shi.	.75	1.33	2.00	.17	2.44

Note. The PMP-values printed in bold indicate the hypothesis with the highest support in the data; Upd. = Updating, Shi. = Shifting, Inh. = Inhibition, Comp. = Comparison, Cat. = Categorization, Nl.= Number lines the estimates for shifting and inhibition on the comparison and number line task. On the categorization tasks, these differences were smaller although the estimate for updating was larger than the estimate for shifting and inhibition.

Role of executive functions within a training focusing on improving numerical magnitude skills.

To test the hypotheses concerning the effect of executive function within the training, we used the gain scores on the numerical tasks in the experimental and training group. The PMP-values and Bayes-factor for each hypothesis under consideration are displayed in Table 3.3. On the categorization task, the PMP-values of the hypotheses were smaller than the PMP-value of the unconstrained hypothesis, indicating that the constraints which were used in the hypotheses were not supported by the data: for the gain scores on the categorization task, none of the executive functions was a predictor. For the gain scores on the number line task, however, the PMP-values for H2 and H4 were larger than the other PMP-values, indicating that for the gain scores on the number line task, updating was a stronger predictor than shifting and inhibition, although the importance of the role of shifting versus inhibition is unclear.

The parameter estimates in Table 3.4 show that on the categorization tasks none of the hypothesized inequality constraints were supported by the data. On the number line task, however, the parameter estimate (unconstrained regression coefficient) for updating was larger than the estimates for shifting and inhibition, indicating that the inequality constraints specified in the hypotheses were supported by the data.

Table 3.4
Unconstrained parameter estimates for the predictor variables at pre-test (N = 40) and in the gain scores (N = 19)

Parameter	Control & experimental group			Experimental group	
	Pre-test Comp.	Pre-test Cat.	Pre-test Nl.	Gain Cat.	Gain Nl.
0	.00	.00	.00	.00	.00
1 Updating	.33	.23	.62	-.34	.64
2 shifting	.08	.07	-.01	.45	-.22
3 Inhibition	-.04	.11	.19	.01	.20

Note. Comp. = Comparison, Cat. = Categorization, Nl.= Number lines

ANCOVA-analyses were performed to examine whether the predicted utility of the EF-tasks was the same in the control and the experimental group for each of the numerical tasks (see Table 3.5). Updating, shifting, and inhibition were used as covariates and interaction effects between these covariates and condition were examined to test if executive functioning had a different effect on numerical gain in the experimental versus the control condition. For the categorization task, no interaction effects were found. For the number line task, however, an interaction effect was found for updating. This indicates that updating affected the gain on the number line task in the experimental condition but not in the control condition.

Discussion

The current study is one of the first to shed light on the different roles of updating, shifting, and inhibition in numerical magnitude skills. The main analyses of this study led to two important conclusions: 1) the three executive functions seem to contribute differently to performance on numerical magnitude tasks and 2) the importance of each executive function seems to depend on the characteristics of the numerical tasks. The results showed that updating might be a more important predictor of numerical magnitude skills than shifting and inhibition. This effect was found for the comparison and number line task, and although the evidence on the categorization task for the predominant role of updating was less convincing, the parameter estimates showed that also on the categorization task, the estimate for updating was larger than for

Table 3. 5
ANCOVA analyses for the gain in categorization and number lines.

	Gain Categorization				Gain Number lines			
Source	F	*df*	*p*	η^2	F	*df*	*p*	η^2
Intercept	0.20	1.00	.66	.01	0.97	1.00	.33	.03
Group	0.49	1.00	.49	.02	2.53	1.00	.12	.08
Updating	0.54	1.00	.47	.02	0.83	1.00	.37	.03
Shifting	0.58	1.00	.45	.02	0.53	1.00	.47	.02
Inhibition	0.18	1.00	.67	.01	2.29	1.00	.14	.07
Group * Upd.	3.03	1.00	.09	.09	4.38	1.00	.04	.12
Group * Shi.	0.89	1.00	.35	.03	2.99	1.00	.09	.09
Group * Inh.	0.07	1.00	.79	.00	1.54	1.00	.22	.05

shifting and inhibition. Moreover, updating played a more important role than shifting and inhibition in improvement on the number line task. This could indicate that among children in the experimental group, those with better updating skills showed greater gains in number line estimation than children with lower updating skills. Moreover, the interaction effect between updating and training group for the number line task implies that updating is important in *learning* more accurate understandings of numerical magnitude. Concerning the improvement on the categorization task, none of the executive functions were involved.

The importance of updating was also suggested by former research focusing on math (e.g. Bull & Scerif, 2001) and math precursors (e.g. Bull et al., 2008). Moreover, the current results are in line with studies demonstrating a relation between updating and numerical magnitude skills (Geary et al., 2008; Krajewski & Schneider, 2009). The prominent role of updating demonstrated in former research, but also in the current study, might be explained by the characteristics of the numerical tasks. In the current study, all tasks demand both simultaneous and sequential processes of perceiving, coding, interpreting, and comparing information in different modalities, explaining the strong involvement of updating. Moreover, a possible explanation for the importance of updating within the numerical training is that children with better updating skills were better learners, who acquired more number knowledge with the same training as peers with lower updating skills.

As in former studies reporting different results on the role of shifting and inhibition in math and precursors of math (Bull & Scerif , 2001, Espy et al., 2004; St Clair-Thompson & Gathercole, 2006), the current results do not render a straightforward conclusion concerning the role of shifting and inhibition. By using different numerical magnitude tasks, however, the current results can be related to the characteristic of the numerical tasks which provide possible explanations for the contradictory results concerning the role of shifting and inhibition. The high task demands in the number line and categorization task might cause inhibition and shifting to play an equally important role. These tasks require processing of complex numerical information about the relative size of a given number using categories or a line in which focusing on relevant information is important besides shifting between different number modalities. In the comparison task, less attention is needed to suppress irrelevant information which might cause that inhibition is less important than shifting in this task. Given that the associations between the numerical tasks and inhibition and shifting are rather low, these possible explanations need to be addressed in further research.

The lack of support for all three executive functions in the improvement on the categorization task may be explained by the nature of the training program, which intensively and exclusively focused on number categorization. It is suspected that the direct effect of the training on the closely related number categorization task suppressed individual differences in performance related to executive functions.

Besides these main findings, this study also showed that basic numerical skills can be trained (Laski & Siegler, 2007). Training effects were found on the categorization and number line task but not on the comparison task. These effects differ from the results of Laski and Siegler (2007; who found training effects on all the three tasks) which might be due to a) a ceiling-effect on the comparison task, which may be prevented by using a more sensitive measure of comparison skills including reaction times as a measure, b) the similarity of the physical and behavioral features of the categorization and number line task and differences in these features between the categorization and comparison task, and c) the differences in sample characteristics such as age and preschool curriculum.

The present study has several limitations. First, the sample size was rather small. We dealt with this problem by using Bayesian evaluation of informative hypotheses which increased the power of our analysis. Although behavioral scientists may be unfamiliar with this approach, evaluation of informative hypotheses has an important advantage over traditional null-hypothesis testing: it may be an appropriate method to determine which hypothesis is most likely given the data while protecting against statistical power loss and by avoiding dichotomous decisions about hypotheses. Second, the internal consistency of the comparison task was rather low, which may have caused the non-significant relations between this task and the executive function measures. A comparison task including more items would have provided a more valid measure of magnitude comparison skills. Another limitation concerns the limited number of tasks that were used to assess each executive function. To deal with the 'impurity problem' in measuring executive functions, it would be preferable to assess each executive functions using different tasks (Van der Ven et al., 2011). This could also have prevented the ceiling effects on the inhibition task. Moreover, the updating task that was used in the current study focused on storage and processing of verbal information though former research also emphasized the importance of visual-spatial skills in early (Bachot, Gevers, Fias, & Rouyers, 2005; De Smedt et al., 2009) and later (Krajewski & Schneider, 2009; Passolunghi & Cornoldi, 2008) math skills. Examination of both verbal and visual-spatial updating skills could provide more insight into the relative contribution of both systems to early math learning.

The current study suggests that future research on the development of numerical magnitude skills in relation to math should include measures of executive functions to deepen the understanding of individual differences in early math learning. Though the role of updating within a numerical training was not consistently found in all tasks, the results suggest that in addition to training numerical skills, training of particular updating skills may also have added value (see Holmes, Gathercole, & Dunning, 2009; Kroesbergen, Van ‘t Noordende, & Kolkman, 2012; Kroesbergen, Van ‘t Noordende, & Kolkman, 2012). Especially in the case of children at risk of math learning disability, an additional intervention effort focusing on executive functions may be necessary to obtain relevant and sustainable results in view of the fact that in math learning disabilities executive functioning is often impaired.

Appendix

The Bayesian approach differs from traditional null-hypothesis testing in two important ways. First, in contrast to the Bayesian approach, traditional null-hypothesis testing often results in loss of power when multiple hypotheses are examined (due to, for example, Bonferroni-type of corrections, as elaborated on in Hoijtink, Klugkist, & Boelen, 2008 and Van de Schoot et al., 2011a). Second, traditional null-hypothesis testing renders a dichotomous decision to either accept or reject an informative hypothesis, based on the criterion of statistical significance. The Bayesian approach is confirmative and straightforwardly renders a quantification of support in the data for each of the hypotheses under consideration and is therefore especially useful for evaluating hypotheses in small samples.

The mixed evidence discussed in the introduction of this paper was translated into a set of competing hypotheses. This set of hypotheses consisted of four competitive hypotheses and an unconstrained hypothesis (H_u), without specification of a relationship between the variables, used as a reference. The aim of Bayesian analysis is to determine the relative support in the data for each hypothesis in this set of competitive hypotheses. Within this procedure, each hypothesis is informative by specifying, based on theory, the relative size and direction of the relationship between the involved variables (Hoijtink, Klugkist, & Boelen, 2008). For example, we expected that updating was the most important predictor for numerical skills. Therefore, we formulated that the size of the regression coefficient between updating and numerical skills is larger than the size of the regression coefficient for shifting and inhibition. The direction of the relationship thus involves the expectation that the regression coefficient of updating is larger than the regression coefficient of shifting and inhibition.

Next, the *posterior model probability* (PMP) that quantifies the support in the data for each hypothesis was computed. Two rules can be used for the interpretation of PMP's: if the PMP of an informative hypothesis is larger than the PMP of the unconstrained hypothesis (H_u), the constraints used to specify the hypothesis are supported by the data; and if the PMP of a first informative hypothesis is larger than the PMP of a second informative hypothesis, the support in the data is larger for the first than for the second hypothesis. Note that the sum of the PMPs for a set of competitive hypotheses is always 1. Unlike traditional null-hypothesis significance testing, no statistical method for accept-reject decisions are used in the Bayesian approach. The interested reader is referred to Cohen (1994) for a critique of such dichotomous decisions. In fact, the best hypothesis in a set of competing hypotheses does not have to be the only hypothesis that is supported by the data. If the PMPs of two hypotheses are similar (e.g., .49 and .51) this implies that the support in the data for both hypotheses is rather similar, and that it cannot be concluded that one is to be preferred over the other. If the PMPs of two hypotheses are dissimilar (e.g., .90 and .10) this implies that the support in the data is nine times larger for the first than for the second hypothesis. If the PMPs are, for example, .40 and .38 and .22, it is clear that the first two hypotheses receive more support in the data than hypothesis 3. Moreover, the first receives more support than the second, but the second hypothesis cannot yet be ruled out completely. As can be seen, using PMPs the summary of the results of hypothesis evaluation is more nuanced than the accept/reject decisions rendered by traditional null-hypothesis significance testing.

The interested reader is referred to Hoijtink (2012), who provides a detailed discussion of the Bayesian approach.

4

Involvement of working memory in longitudinal development of number-quantity skills

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Abstract

The ability to connect numbers and magnitudes is an important prerequisite for math learning, here referred to as number-magnitude skills. It has been proposed that working memory plays an important role in constructing these connections. The aim of the current study was to examine if working-memory accounts for constructing these connections by testing whether development of number-magnitude skills can be explained by different components of working memory. Number-magnitude skills and working memory skills of 69 children were assessed at age 5, 5.5 and 6. Different results were found for different components of working memory. Whereas no effects were found for the phonological loop in development of number-magnitude skills, the visual spatial sketchpad and the central executive predicted variance in intercept and slope of number-magnitude skills. The results of this study are in line with the idea that a general cognitive system is needed in constructing knowledge about the connections between numbers and magnitudes.

Introduction

Skills that enable children to make the connections between numbers and their magnitude form an important prerequisite for math learning. For example, it is important for children to learn that the number '3' is connected to the magnitude '***' in order to understand math problems such as addition and subtraction (De Smedt, Verschaffel, & Ghesquière, 2009; Holloway & Ansari, 2009; Booth & Siegler, 2008). Tasks that are commonly used to measure these number-magnitude skills are number comparison tasks (e.g. Durand, Hulme, Larkin, & Snowling, 2005) and number-to-position tasks (e.g. Siegler & Booth, 2004). In both types of tasks, information about the connections between numbers and magnitude is needed for accurate performance. When, for example, the connection between numbers and magnitudes is not yet fully developed, children have more difficulty with deciding which of two numbers is larger in a comparison task and will be less accurate in estimating the correct positions of numbers on the number-to-position task. It has been found that both tasks are interrelated measures of number-magnitude skills in children (Laski & Siegler, 2007; Ramani & Siegler, 2008).

An increasing amount of studies has examined the cognitive characteristics of these number-magnitude skills. A common idea in the recent literature is that these skills are based on a cognitive system in which numbers and magnitudes are spatially ordered from left to right often referred to as a 'mental number line'. In this ordering, smaller numbers and their magnitudes are associated with the left side of space whereas larger numbers are associated with the right side of space ('SNARC'- effect; Dehaene, Bossini, & Giraux, 1993; Wood, Willmes, Nuerk, & Fischer, 2008). Moreover, there seems to be a unique position for each number on this mental number line: when the distance between numbers is small (e.g. 2 and 3) numerical positions are harder to distinguish and processing these numbers is more difficult than when the distance between numbers is larger (e.g. 2 vs. 13; Dehaene et al., 1993; Fischer, Castel, Dodd, & Pratt, 2003; Gevers, Lammertyn, Notebaert, Verguts, & Fias 2006). It is assumed that during development, this spatial ordering becomes more precise resulting in an increase in accuracy on the comparison task (Durand et al., 2005; Holloway & Ansari, 2009) and on the number-to-position task (Booth & Siegler, 2006; Friso-van den Bos, Kolkman, Kroesbergen, & Leseman, 2013; Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; Siegler & Booth, 2004). It has been theorized that a conceptual change in connections between numbers and magnitudes might underlie this development (Opfer & Siegler, 2007; Siegler & Opfer, 2003). On the number-to-position task, for example, several studies have shown a developmental shift from inaccurate logarithmic estimations to accurate linear estimations depending on the age of the child and the scale of the number-to-position task (Booth & Siegler, 2006; Siegler & Booth, 2004; Siegler & Opfer, 2003).

There is, however, some debate about the origins of the spatial ordering of numbers and magnitudes. Some studies suggest that this spatial ordering is based on an innate system specifically dedicated to process numerical information (e.g. Dehaene, 2001). Others, however, suggest that this spatial ordering of numbers and magnitudes is constructed during experiences with numerical tasks, and at least to some extent dependent on working memory. It is proposed by Van Dijck and Fias (2011) that during performance on a task aimed at measuring number-magnitude skills, such as a number comparison task or a number-to-position task, consecutive trials are used to build a 'mental number line' in working memory. On a number comparison task, for example, the first pair of stimuli is processed and stored in working memory and each following pair of stimuli is added to the existing information, resulting in a mental number line that is adjusted to the task demands. This idea could also explain findings showing that the SNARC-effect depends on the numerical range of the presented task (Fias, Brysbaert, Geypens, & d'Ydewalle, 1996) and the impact of reading habits on the SNARC effect (Shaki & Fischer, 2008). The involvement of general mechanisms in number-magnitude skills is also demonstrated by Van Dijck and Fias (2011). They instructed adults to memorize a sequence of five digits after which a parity judgment task was presented. Participants were instructed to respond only to the digits they had to memorize. Results of this study showed that not the numerical size of the digits, but rather the position in the sequence-to-be-remembered was related to reaction time: For items at the beginning of the sequence faster and more accurate responses were given with the left hand and for items at the end of the sequence the opposite was found. Van Dijck and Fias (2011) concluded that it is not numerical information stored in long term memory that underlies performance on parity judgment. They suggest that temporary position-space associations in working memory underlies performance on such tasks.

Evidence for this working memory account can also be found in dual-task studies demonstrating that both visual and verbal components of working memory are involved in number-magnitude skills. Herrera, Macizo and Semenza (2008) conducted a dual-task study in which adults were asked to compare digits while maintaining unrelated visuospatial or phonological information. Their results demonstrated that the SNARC effect disappeared when visuopatial information had to be memorized although accuracy rates did not change. This effect was not found for the maintenance of phonological information. These results suggest that the spatial ordering of number-magnitude information depends on general mechanisms for processing visuospatial information rather than on mechanisms for processing phonological information. Based on these results, the authors proposed that besides a system for processing magnitude information, there is a more general spatial system involved when numbers are processed. Moreover, Henik, Leibovich, Naparstek, Diesendruck and Rubinsten (2011) proposed that a general spatial system for processing non-countable dimensions like distance and height rather than an innate numerical core system underlies performance on tasks measuring number-magnitude skills.

Although there are no reports of dual-task studies in children, there is evidence for the involvement of working memory in number-magnitude skills in younger age groups. The first line of evidence is provided by studies focusing on math learning difficulties in children in the early school years. Geary, Hoard, Nugent and Byrd-Craven (2008) compared first- and second-graders with math learning disabilities with their typically achieving peers on a number-to-position task and on working memory measures. It was found that the central executive contributed to across grade improvements and to group differences in performance on the number-to-position task. Moreover, Geary et al. (2009) demonstrated that children with math learning difficulties had general deficits in working memory and number-magnitude skills. The second line of evidence is provided by studies focusing on the relations between working memory and number-magnitude skills. Krajewski and Schneider (2009) provided longitudinal evidence for the role of visual spatial working memory measured at age 5 in number comparison skills measured at age 8. In a cross-sectional study, it has been demonstrated that changes in performance on a number comparison task in 6- to 8-year-olds reflect changes in domain-general mechanisms rather than changes in domain-specific number cognitions (Holloway & Ansari, 2008). Moreover, it was found in another cross-sectional study including 4- to 8-year-old children, that the central executive was an important predictor of the accuracy of placements on the number-to-position task over and above age-related development (Friso-van den Bos et al., 2013). Thirdly, in a training study aiming at improving number-magnitude skills, Kolkman, Kroesbergen, Hoijtink, & Leseman (2013) demonstrated that the central executive facilitated the impact of the training. Children showed more improvement on a number-to-position task when they were better able to store and process information, indicating involvement of working memory in learning number-magnitude skills.

Thus, previous studies have found indications for the involvement of working memory in number-magnitude skills. These studies, however, commonly used one type of task to measure number-magnitude skills and one type of task for measuring working memory skills. Moreover, longitudinal measures of these skills are scarce. In the current longitudinal study, the working memory account for number-magnitude skills is examined by testing the hypothesis whether developmental changes in number-magnitude skills can be predicted by working memory. This study contributes to the existing knowledge by examining a sample of young children at a crucial age for developing accurate number-magnitude skills, which allows conclusions about the involvement of working memory in the *development* of number-magnitude skills. Moreover, two different types of tasks are used to measure number-magnitude skills. In addition, different components of working memory are taken into account enabling us to differentiate between the phonological loop (PL), the visual-spatial sketch path (VSSP) and the central executive (CE) and their involvement in number-magnitude skills.

Method

Participants and Procedure

The children in this study were Dutch speaking children recruited from 15 primary schools in the Netherlands. Children who celebrated their fifth birthday between October and May were invited to participate, informed parental consent was received for 83 children. During the course of the study 21 children dropped out due to moving, grade retention or accelerating a grade. The remaining 62 children were included in the analyses in this study. At measurement time 1, children were all halfway their second year of kindergarten (M_{age} = 62.91 months; SD = 2.44). At measurement time 2, six months later, children were all at the end of kindergarten and at measurement time 3, one year after the first measurement, children were all in first grade.

Performance on the Raven's Colored Progressive Matrices (Raven, 1962) was used as measure of cognitive abilities (M = 25.46; SD = 5.53). Following Dutch norms from 1982 (Raven, Court, & Raven, 1995), this mean score corresponds to a percentile score between 75 and 90 indicating that the children in this study performed above average on this test. It should be mentioned, however, that this norms are not very reliable, because they are relatively old, and may lead to overestimation of children's IQs (due to the Flynn-effect). The raw scores of this measure were included in the analyses as a control variable.

Parents of 63 children (95.3%) filled out a questionnaire on their educational background. Of these parents, 9.4% had finished only lower secondary education or less, 25% had completed upper secondary education or vocational training and 60.9% completed higher vocational training or university. Compared to national measures of educational background (lower secondary education: 28%; upper secondary

education: 42%; higher educational education: 27%; CBS Statistics Netherlands, 2007) the educational level of the parents in the current study was above average. Educational background of the parents was not related to any of the measures.
At each measurement time, two tasks were used to measure number-magnitude skills. At the first and last measurement time, children completed four tasks aimed at measuring working memory skills. These tasks assessed skills related to the PL, the VSSP and the CE.
The children were tested individually in a quiet room inside the school by trained undergraduate students. All working memory and number-magnitude tasks were administered on a laptop computer using E-Prime 1.2 software (Psychological Software Tools, http://www.pstnet.com).

Measures

Number-magnitude skills.

Two tasks were used to assess the number-magnitude skills of the children. In the *Number-to-position task*, children were asked to estimate the position of a given number (in the range from 1 to 100) on a horizontal line. A computerized version was used in which the child was presented with a horizontal line ranging from 1-100. To introduce the task, the experimenter demonstrated the position of the numbers '1' and '100'. Then children were asked to point to the position they thought each number belonged on the number line. Each child completed 10 randomized trials. Linear fit scores were computed by fitting the answers of each individual child to a linear curve (Geary et al., 2008). As shown by Friso-van den Bos et al. (2012), individual linearity scores provide a valid measure for examining development in numerical magnitude skills in young children. Previously reported reliability measures are satisfactory (α = .79; Kolkman, et al., 2013).

In the *Comparison task* children were asked to indicate the number with the highest numerical value out of two numbers. We used an adapted version of the tasks used by Barth et al. (2006) and Gebuis, Kadosh, De Haan and Henik (2008), who controlled for non-numerical parameters by including three conditions: (1) a congruent condition in which the number with the largest numerical value was also physically larger, (2) an incongruent condition in which the number with the largest numerical value was physically smaller and (3) a neutral condition in which the physical size of the numbers in both areas was the same but the numerical value varied. In each condition 10 trials were presented in the range from 1-100. For each of the 30 items that the child passed, one point was given. Reliability figures of this type of task are satisfactory (α = .69; Kolkman et al., 2013). The scores on the Comparison task were transformed in proportion scores, resulting in a score between 0 and 1. The scores on the number-to-position task also ranged from 0 to 1 indicating the percentage of variance that could be explained by a linear fit. Because previous studies found that both tasks loaded on the same factor and are thus assumed to measure the same construct (Kolkman et al., 2013) we combined the two scores into one variable for number-magnitude skills based on the mean proportion score. Note that the correlations between both tasks in the current study ranged between r = .24 and r = .54 (see Table 3).

Working memory skills.

Tasks from the Automated Working Memory Assessment [AWMA] battery (Alloway, 2007) were used to assess working memory skills. Test-retest reliability of these tasks for children aged 4.5 years was good (Word recall forward: r = .76; Dot matrix: r = .83; Word recall backward: r = .74; Odd one out: r = .81; Alloway, Gathercole, Kirkwood, & Elliott, 2008).

Phonological loop. *Word Recall* was used to assess verbal storage skills. In this task, a recorded voice names a sequence of semantically unrelated words, after which the child is asked to repeat the words in the same order. The sequence of words becomes longer after a child recalled four sequences of the same length correctly. The task was automatically terminated when a child gave three incorrect answers within a set of items of the same length. The number of correct recalled sequences was used as measure of the PL.

Visual-spatial sketch path. The capacity of the VSSP was assessed with the *Dot Matrix* task. The child was shown a 4 x 4 grid of empty white squares. In one square a red dot appeared. After disappearing, the dot reappeared in another white square. The child had to reproduce the sequence of the dot by pointing out the same sequence in an empty grid. The number of dots in a sequence gradually increased. The child progressed to a longer sequence when four trials of the same length had been remembered correctly. The task was discontinued when three trials of the same length had been reproduced incorrectly. For each completed length six points were given, plus one additional point for each completed sequence of the length that was not finished. The test started with a practice session.

Central executive. For the assessment of the CE, two tasks were used. *Word Recall Backward* was used to assess the storage and processing of verbal information. In this task children were asked to recall a sequence of words in the reversed order. Other features of this tasks were the same as the in the PL task, except for the stimuli; different semantically unrelated words were used.
Visual storage and processing was measured using the *Odd One Out* task. A series of stimuli was shown consecutively. Each stimulus consisted of three boxes with shapes presented next to each other. One of the shapes differed from the other two. The child was asked to point at the deviant shape. Then the next stimulus was presented. At the end of each trial three empty boxes appeared; the child had to point at the locations of the previously shown deviant shapes in the same order in which they had appeared. An answer was correct if each location was recalled correctly in the right order. The task started with only one stimulus; after three correct answers of the same length, the sequence increased by one. When two mistakes were made in trials of the same length, the task was discontinued. The number of correct sequences was used as a final score.

4

Analyses

Before testing the hypotheses, univariate outliers (scores two standard deviations below or above the mean) were converted into scores at a standard deviation of -2 or 2 (as suggested by Field, 2005). We used this procedure on 5.61% of the numerical data points and 6.55% of the working memory data points.

Following the working memory structure proposed by Baddeley (1996; 2000), we performed a confirmatory factor analysis on the working memory scores. The tasks that were used in this study are aimed at measuring the PL (word recall forward), the VSSP (dot matrix) and the CE (word recall backwards and odd one out). The CE tasks, however, do not provide an isolated measure of processing skills but also include a storage component (Cowan, 2010). Therefore we included a factor model of working memory in which word recall forward and backward at both time points loaded on the PL-factor, and dot matrix and odd one out at both time points loaded on the VSSP-factor. Word recall backwards and odd one out at both time points loaded on the CE-factor (see Table 4.1). Moreover, working memory measures of both time points were included in the model. This was done to (1) minimize the amount of variables in our model considering the relatively small sample size, (2) provide a more valid measure of working memory by including measures from different time points and (3) decrease measurement errors by using more tests to predict the latent factor. Moreover, the basic modular structure of working memory has proven to be stable and assessable by the age of four (e.g. Alloway et al., 2008; Alloway, Gathercole, Willis, & Adams, 2004; Gathercole, Pickering, Ambridge, & Wearing, 2004). The standardized factor loadings as a result of the CFA were presented in Table 4.1. Note that the aim of the present study was not to validate latent variables as these were explored in previous studies (Alloway, Gathercole, & Pickering, 2006). The CFA was rather used to construct latent variables for testing our hypotheses in further analyses. The standardized factor loadings of this model were used to compute scores for the variable representing the latent factors that were used in further analyses (factor loadings * standardized score).

A growth curve model was used to examine the development of number-magnitude skills. Three different models were tested: 1) a model including the combined score on the number line and comparison task, 2) a model including the number-to-position

Table 4.1
Standardized factor loadings as a result of the CFA on working memory skills at measurement time 1 and time 3.

	VSSP	PL	CE
Dot Matrix T1	.46*		
Odd One Out T1	.48*		.08
Word recall forward T1		.71*	
Word recall backward T1		.03	.52*
Dot Matrix T3	.46*		
Odd One Out T3	.18		.62*
Word recall forward T3		.63*	
Word recall backward T3		.09	.44*

Note. * $p < .05$

task only, and 3) a model including the comparison task only. Growth curve analysis models the development of a specific skill by estimating the best growth curve for each child. The slope and the intercept are estimated as latent variables, with the scores at each measurement time as observed indicators. The loadings of the intercept are fixed at 1, since the intercept does not change over time. The loadings of the slope reflect the growth. Since we wanted to model the best fitting growth curve, the slope loadings for the successive measurement times were fixed at 0 for time 1 and fixed at 1 for time 3. For time 2, the slope loading was not specified, allowing the estimation of the best fitting slope loading (a procedure described by Welch, 2007). For both the intercept and the slope, means and variances are estimated reflecting the average intercept and slope and individual differences. A significant covariance between intercept and slope reflects that children with higher level develop faster if the covariance is positive or the reverse if it is negative.

Structural Equation Modeling was used to examine the effects of PL, VSSP and CE on the slope and intercept of number-magnitude skills. In the next section, we will present the fit-indices of this model: chi- square (X^2) with its *p*-value, CFI and RMSEA. Chi-square is a discrepancy measure; between the current model and the saturated model, and should be as low as possible, with a *p*-value that is as high as possible. CFI compares the fit of the model to the independence model and is considered good if > .95 and acceptable if > .90. RMSEA is a measure of parsimony, favoring simpler models and is considered good if < .05 and acceptable if < .08 (Blunch, 2008). Then, the parameter estimates of the model were examined in order to test our hypothesis.

All analyses were performed using IBM SPSS Statistics for Windows, Version 20.0 and AMOS version 17.

Results

Descriptive statistics

Table 4.2 presents the mean scores and standard deviations for the measures used in this study. For the number-to-position task linear fit scores are presented, whereas for the comparison task and working memory measures respectively proportion scores

Table 4.3
Correlations between all tasks (1-tailed)

	Number lines T1	Comparison T1	Number lines T2	Comparison T2	Number lines T3	Comparison T3	Dot Matrix T1	Odd One Out T1	Word recall forward T1	Word recall backward T1	Dot Matrix T3	Odd One Out T3	Word recall forward T3
Number-magnitude tasks													
Comparison T1	.24†												
Number lines T2	.40**	.31*											
Comparison T2	.30*	.42**	.33*										
Number lines T3	.48**	.42**	.61**	.38**									
Comparison T3	.36**	.27*	.18†	.31*	.54**								
Working memory tasks													
Dot Matrix T1	.06	.52**	-.08	.15	.14	.21*							
Odd One Out T1	.10	.36**	.23*	.18	.25*	.19†	.30**						
Word recall forward T1	.15	.23†	.08	-.01	.24*	.17†	.02	.12					
Word recall backward T1	.03	.47**	-.05	.10	.13	.27*	.23*	.24*	.16†				
Dot Matrix T3	.05	.23†	.23*	.18	.12	.20*	.20*	.24*	.00	.20†			
Odd One Out T3	.05	.32*	.27*	.26*	.37**	.32**	.23*	.34**	.13	.36**	.32**		
Word recall forward T3	.07	.07	.03	-.06	.00	.08	-.05	-.12	.46**	.08	-.05	.10	
Word recall backward T3	-.07	.09	.03	.08	.16†	.26*	.11	.14	.14	.25*	.02	.38**	.15

Note. * $p < .05$; ** $p < .01$

and mean accuracy scores are presented. Note that the scores for the comparison task are near the maximum score at Time 2, indicating a possible ceiling effect. Correlations between working memory and number-magnitude skills at the different time points are presented in Table 4.3Note that we use these correlations to describe our data and not to test our hypothesis. At the first measurement time, correlations between all working memory measures were found except for word recall forward, which was only related to word recall backward. At the last measurement time, correlations were only found between dot matrix and odd one out and between odd one out and word recall backward. Moreover, the same tasks were related over time. Concerning the number-magnitude skills, significant correlations were found between all tasks within and between measurement times.

Development of number-magnitude skills

First, the growth of the combined number-magnitude score was examined (see Table 4.4). The fit of this model was good (χ^2 (1) = .002, p = .96, CFI = 1.00, RMSEA = .00).

The unstandardized slope loading for Time 2 was .44 (p < .001), indicating that the growth on performance on the number-to-position task was almost linear although the children in this study showed a slightly steeper growth between time 2 and time 3 compared to the growth between time 1 and time 2.The mean (M = .42) and the variance (SD = .04) of the intercept were both significant (p < .001) indicating that

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Table 4.2

Descriptive statistics of working memory and numerical measures.

	Time 1		Time 2		Time 3		Empirical	
	M	*SD*	*M*	*SD*	*M*	*SD*	Min.	Max.
Number lines (linear fit scores)	.28	.25	.38	.29	.56	.27	.00	.96
Comparison (proportion scores)	.73	.13	.89	.08	.92	.07	.48	1.00
Dot matrix (sum of accuracy)	9.21	3.03			11.79	2.75	2.84	17.86
Odd One Out (sum of accuracy)	9.39	2.23			11.68	2.09	4.80	16.22
Word Recall Forward (sum of accuracy)	13.21	2.10			14.69	1.90	9.00	18.74
Word Recall Backward (sum of accuracy)	4.23	1.81			5.49	1.59	0.36	9.06

children had an average level that was different from zero and that children differed in this average level. The mean (M = .33) and the variance (SD = .05) of the slope were also significant ($p < .001$) indicating that children developed significantly and that they also differed in this development. Note that the covariance between intercept and slope is negative ($r = -.73$; $p < .01$), demonstrating that children with higher number-magnitude skills showed less growth than children with lower numerical magnitude skills.

Secondly, the number-to-position task and comparison task were used to build two separate growth models (see Table 4.4). The fit of these models was good (number-to-position: χ^2 (1) = .003, $p = .96$, CFI = 1.00, RMSEA = .00; comparison: χ^2 (1) = .006, $p = .96$, CFI = 1.00, RMSEA = .00). The unstandardized slope loadings for Time 2 were respectively .37 ($p < .001$) and .82 ($p < .001$). This indicates that on the number-to-position task children show most growth between time 2 and time 3, whereas on the comparison task children show most growth between time 1 and time 2.On both tasks, significant means and variances for the slope and intercept were found indicating that children developed significantly and that they also differed in this development. Considering the covariance between slope and intercept, no relation was found between the slope and intercept on the number-to-position task. On the comparison task a negative covariance between intercept and slope was found ($r = -.94$; $p < .01$).

Table 4.4
Estimates for the growth models of the combined number-magnitude model, the number line model and the comparison-only model.

	Combined model	Number-to-position -only model	Comparison-only model
Unstandardized factor loading Time 2	.44**	.37**	.82**
Intercept Mean	.42**	.28**	.22**
Intercept Variance	.04**	.03*	.001**
Slope Mean	.33**	.28**	.06**
Slope Variance	.05**	.04*	.001*
Covariance intercept & slope	-.73*	.19	-.94*

Note. * $p < .05$; ** $p < .01$

Working memory skills as predictor of development of number-magnitude skills

SEM was used to test the predictive value of working memory on development of number-magnitude skills. The unstandardized estimated slope values for measurement time 2 based on the analyses described above were included in these analyses. In these models also performance on Raven's Colored Progressive Matrices was included as a control variable. The results of these analyses are presented in Table 4.5.

The combined model showed a good fit (χ^2 (6) = 1.36, p = .97, CFI = 1.00, RMSEA = .00). The models including only the number-to-position task or the comparison task also showed a good fit (number-to-position: χ^2 (7) = 0.75, p = .99, CFI = 1.00, RMSEA = .00; comparison: χ^2 (7) = 3.25, p = .86, CFI = 1.00, RMSEA = .00). In the combined model, the estimates of the regression coefficients demonstrated that the VSSP had an effect on the intercept of number-magnitude skills (β = .33, p =.05). This was not found for the number-to-position-only model. For the comparison-only model, however, it was found that the VSSP had an effect on both the intercept

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Table 4.5
Estimates for the regression coefficient and covariance in the combined number-magnitude model, the number line-only model and the comparison only model.

			Combined model		Number-to-position -only model		Comparison-only model	
			Regression weights					
			β	*p*	*β*	*p*	*β*	*p*
CE	→	Intercept	-.19	.26	-.24	.27	.11	.53
PL	→	Intercept	.13	.39	.13	.54	.19	.25
VSSP	→	Intercept	.33	.05	.28	.21	.58	< .01
Raven	→	Intercept	-.01	.94	.14	.56	-.12	.50
CE	→	Slope	.36	.03	.51	< .01	.04	.85
PL	→	Slope	-.01	.95	.12	.49	-.18	.36
VSSP	→	Slope	-.17	.31	.05	.81	-.56	< .01
Raven	→	Slope	-.06	.75	-.29	.16	.14	.51
			Covariances					
			r	*P*	*r*	*p*	*r*	*p*
Intercept	↔	Slope	-.74	< .01	.28	.48	-.93	< .01
CE	↔	PL	.24	.02	.24	.02	.24	.02
CE	↔	VSSP	.56	< .001	.56	< .001	.57	< .001
Raven	↔	CE	.42	< .01	.42	< .01	.42	< .01
Raven	↔	PL	.39	< .01	.39	< .01	.39	< .01
Raven	↔	VSSP	.44	< .01	.44	< .01	.44	< .01

and slope of comparison skills (intercept: $\beta = .58$, $p < .001$; slope: $\beta = -.56$, $p < .01$). Note that this negative effect is probably caused by the negative covariance between the slope and the intercept of comparison skills.

Regarding the CE, the results of the combined model showed that the CE had an effect on the slope of number-magnitude skills ($\beta = .36$, $p < .05$). This was also found for the number-to-position-only model ($\beta = .51$, $p < .01$). For the comparison-only model, however, no effects were found for the CE.

In none of the models effects were found for the PL. Moreover, no effects were found on number-magnitude skills for the measure of general cognitive ability (Raven). In none of the models, this measure had an effect on the intercept or slope of number-magnitude skills. Relations were found, however, between measures of general cognitive ability and the working memory measures (see Table 5).

Discussion

In the current study, it was examined to what extent the distinct components of working memory explain variance in the development of number-magnitude skills. In a sample of children at a crucial age for developing accurate number-magnitude skills, evidence was found for the hypothesis that working memory, at least to some extent, can predict the development of number-magnitude skills. Moreover, different results were found for different components of working memory. Whereas no effects were found for the PL in the development of number-magnitude skills, the VSSP and CE predicted variance in intercept and slope of number-magnitude skills. These findings provide evidence for the idea that, as in adults, in performance of young children on number-magnitude tasks working memory might be used to build a task dependent 'mental number line' as proposed by Van Dijck and Fias (2011).

The results of this longitudinal study are in line with previous correlational studies that demonstrated relations between the VSSP and the CE and number-magnitude skills (Friso-van den Bos et al., 2013; Geary et al., 2009; Krajewski & Schneider, 2009; Kolkman et al., 2012). By examining the effects of working memory in two different tasks, an important asset of the current study is that we can also draw conclusions about the involvement of working memory in number-to-position versus number comparison tasks. Below we will discuss our findings in the light of task characteristics that might explain differential involvement of different working memory components. In discussing these findings the small sample size needs to be taken into account. Although the fit-indices of the models are good and the data shows significant results, generalizability of the results may be relatively low.

The results of this study showed that when both tasks were combined, effects on the growth were found for the CE. For the number-to-position task, the CE also predicted part of the variance in growth whereas for the comparison task growth was partly predicted by the VSSP. The latter effect was negative due to the negative covariance between the intercept and slope in the comparison-only model. Children with higher levels of comparison skills at the beginning of the study had less room for growth. The children that were better at comparison had higher VSSP levels as indicated by the negative effect of the VSSP on the slope of comparison.

Concerning the initial level (intercept) of number-magnitude skills, it was found that the VSSP had an effect of the intercept of comparison skills and on the intercept of the combined measure. No effects on the intercept of the number-to-position task were found. We hypothesize that these differences between tasks might arise from differences in the information that is needed for task performance. In the number-to-position task we suspect that previously stored, existing information about numbers and their quantity is needed for task performance. For example, when children are presented with the number 6, they seem to need previously stored knowledge about the relative position of this number. During task performance, this information might be activated and incoming information is assimilated. A process that seems to involve both storage and processing of information in which the CE is strongly involved. Since CE-effects were only found on the growth of the number-to-position task, we argue that this process might be prominent in development of performance on the number-to-position task (a conclusion that is also supported by the findings for the combined model). In the comparison task, however, the presented stimuli provide all the information that is needed for solving the task. For example, when children have to compare the number 4 and 9 they need knowledge about the magnitudes that are connected to these numbers but they do not need previously stored information about other numbers (which is the case in the number-to-position task). We argue that in comparing numbers, children might build a temporary cognition about the presented stimuli that is spatial in nature as proposed by Van Dijck and Fias (2011) and Henik et al. (2011). Since effects were found of the VSSP on both the intercept and growth of the comparison task, we argue that the better visuospatial skills lead to better initial levels of comparing numbers and, moreover, these skills also facilitate development of number comparison skills.

No effects were found for the PL. We hypothesize that the verbal component of working memory might not play a role in development of number-magnitude skills because these skills are based on a *spatial* association between numbers and magnitudes (e.g. Dehaene et al., 1993). This however does not rule out the role of the PL in overall math learning as previous studies have found relations between the PL and math performance (Meyer, Salimpoor, Wu, Geary, & Menon, 2010; Noël, 2009; Passolunghi, Mammarella, & Altoè, 2008). Moreover, the PL could be involved in early development of math learning through skills that were not taken into account in the current study. For example, the PL could be involved in verbal counting skills that are found to be an important domain-specific precursor of number-magnitude skills (e.g. Kolkman et al., 2013). This needs to be further addressed in future research. Another aspect that needs to be taken into account in future studies, is the role of these domain specific skills related to counting. As briefly mentioned in the introduction, some authors argue that domain-specific numerical systems underlie the development of number-magnitude skills. For example, Dehaene (2001) has proposed that a specific innate system dedicated to processing numerical magnitudes underlies all further math development. In order to test this hypothesis against the working memory account, it is needed to assess both domain-specific as domain-general precursors of number-magnitude associations.

The results of this study also show that although working memory and measures of general cognitive ability were closely related, only working memory was a predictor of the development of number-magnitude skills. These findings are in line with previous studies reporting that measures of working memory provide a better prediction of early numerical abilities compared to measures of general intelligence, (Bull & Scerif, 2011; Kroesbergen, Van de Rijt, & Van Luit, 2007).

There are several limitations of the current study that need to be taken into account. First, although correlations are found, firm conclusions about the direction of the relation between working memory and number-magnitude skills cannot be drawn. It is needed to design an experimental study in which the causal relations between visual sketchpad and number-magnitude skills are directly addressed. For example a dual-task study in which a number-magnitude task is performed while visual-spatial information is maintained in working memory. Concerning the comparison task, the presented results show a ceiling effect on the scores of this task (see Table 1 and Table 4). Although relations were found between this task and the number-to-position task and moreover working memory could explain the variance in this task, a more sensitive measure of comparison skills is needed in future research to address this problem. Thirdly, the CFA showed low factor loadings for the Odd one out task for the CE at time 1 and for the Word recall backward tasks for the PL at both measurement times. For the Odd one out task, this might indicate that most children at time 1 were not able to reach higher levels of this task in which the CE was involved. For Word recall backwards this could indicate that this task was difficult for the children implying a larger involvement of the CE then phonological processing. Although these tasks are found to be reliable measures of working memory in young children (Alloway, Gathercole, & Pickering, 2006), the current results showed that there might be large individual differences. A careful consideration of these tasks is needed in future research.

To conclude, the current study demonstrates that the early development of number-magnitude skills seems to be facilitated by general cognitive resources related to the storage of visual information (VSSP) and general abilities to process information (CE). The results of this study are in line with the idea that a general cognitive system is needed in constructing knowledge about the skills between numbers and magnitudes.

5

The role of spatial working memory in the development of number-to-quantity mapping

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Abstract

The ability to connect numbers to their corresponding quantity is often referred to as 'mapping'. The ability to map numbers to quantities is assumed to arise from spatial associations between numbers and quantities. Some authors have proposed that this spatial ordering is built on domain-specific non-symbolic skills for processing quantities. In this study, however, it is proposed that a domain-general spatial system provides the foundation upon which mapping skills are built. In a longitudinal design, the mapping skills, domain-specific skills and domain-general skills of 162 children were assessed at age 5, 6 and 7. Multivariate growth models were used to examine the co-development of the different variables. These results showed that there is a spatial domain-general 'workspace' which facilitates the development of mapping skills and non-symbolic skills providing evidence for the idea that the foundation of numerical development needs to be sought in domain-general skills for spatial processing.

Introduction

The understanding that each number symbol is connected to a numerical magnitude is an important prerequisite for a wide range of math abilities (Booth & Siegler, 2008; De Smedt, Verschaffel, & Ghesquière, 2009; Holloway & Ansari, 2009). In numerical development children thus have to learn that the number '3' is connected to the quantity '***'. The ability to connect numbers to their corresponding magnitude is often referred to as 'mapping' (e.g. Geary, 2013; Holloway & Ansari, 2009). Previous studies have shown that the accuracy of mapping skills increases during development (e.g. Booth & Siegler, 2006; Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007). Although it is commonly assumed that innate domain-specific spatial skills for processing quantities underlie the development of accurate mapping skills (e.g. Dehaene, 2001), there is evidence that contradicts this idea. For example it has been demonstrated that these non-symbolic skills are not related to mapping (e.g. Kolkman, Kroesbergen, & Leseman, 2013). Moreover, several studies have shown that cultural factors such as reading habits are related to mapping (Opfer, Thompson, & Furlong, 2010; Shaki & Fischer, 2008). This evidence indicates that the role of innate non-symbolic quantity skills in mapping skills is not that straightforward as has been proposed in previous studies. In the present study we therefore propose that mapping does not arise from innate domain-specific non-symbolic skills. Instead we examine the idea of a domain-general spatial system as the fundament upon which mapping skills are built (Ansari, 2008; Van Dijck & Fias, 2011). In this study, longitudinal evidence for this idea is presented in a sample of young children in an age-range in which mapping skills are assumed to develop rapidly.

Development of mapping

A widely used task to investigate the development of mapping skills is the number-to-position task (Booth & Siegler, 2006; Geary et al., 2007; Siegler & Booth, 2004; Siegler & Opfer, 2003). This task requires children to estimate the position of a given number on a horizontal line. The idea behind this type of task is that mapping is needed for accurate placement: to place a given number symbol on its corresponding position on the line, information about the connection between number symbols and their corresponding quantity is needed.

It has been proposed that performance on this type of task relies on a specialized cognitive system in which numbers and quantities are spatially ordered from left to right often referred to as a 'mental number line'. In this system, smaller numbers and their quantities are associated with the left side of space whereas larger numbers are associated with the right side of space. This phenomenon is referred to as the 'Spatial Numerical Association of Response Codes' (SNARC) effect (Dehaene, Bossini, & Giraux, 1993; Wood, Willmes, Nuerk, & Fischer, 2008). Moreover, numbers close to each other (e.g. 2 and 3) are also close to each other on the mental number line. For example when the distance between numbers is small (e.g. 2 and 3) it is more difficult to distinguish between these numbers than when the distance between numbers is larger (e.g. 2 vs. 13; Dehaene et al., 1993; Fischer, Castel, Dodd, & Pratt, 2003; Gevers, Lammertyn, Notebaert, Verguts, & Fias, 2006). There are, however, other accounts of the spatial ordering of numbers and quantities. Besides a horizontal ordering from left to right, it has also been found that the SNARC effect occurs in a vertical and close/far dimension. Adults spontaneously associate small numbers with bottom responses (Gevers et al., 2006) and close responses (Santens & Gevers, 2008) and large numbers are associated with top responses (Gevers et al., 2006) and far responses (Santens & Gevers, 2008). According to some researchers vertical and close/far orderings can ultimately be traced back to a spatial left-to-right ordering (Gevers et al., 2006).

It is assumed that during development, this spatial ordering becomes more precise resulting in an increase of accuracy-placement in the number-to-position task. Various studies indeed have shown that younger children have less accurate placements characterized by a logarithmic pattern in which the distances between smaller numbers are overestimated and the distances between larger numbers are underestimated. The estimates of older children, however, often form a linear pattern and are thus more accurate (Booth & Siegler, 2006; Friso - van den Bos, Kolkman, Kroesbergen, & Leseman, 2013; Siegler & Booth, 2004; Siegler & Opfer, 2003). Depending on the age of the child and the scale of the number line, estimations become more accurate over time (Booth & Siegler, 2006; Geary et al., 2007; Siegler & Booth, 2004; Siegler & Opfer, 2003).

The ability to map numbers to their corresponding quantities is an important precursor of math performance. Several studies have shown that mapping skills are related to performance on math tasks. Using a number-to-position task, it was demonstrated that individual differences in mapping skills were strongly related to math achievement test scores (Siegler & Booth, 2004). Moreover, math performance could be *predicted* from mapping skills (Booth & Siegler, 2008). Performance on the number-to-position task also differed between typically achieving students and children with math learning difficulties (Geary, Hoard, Nugent, & Byrd-Craven, 2008). Some studies used a number comparison task to assess mapping skills (in which children are asked to indicate the number symbol with the largest numerical value) and have found that performance on this task was related to math ability (Holloway & Ansari, 2009) and could predict math achievement scores (De Smedt et al., 2009).

Precursors of mapping skills

The view that has dominated the literature on development of mapping emphasized the importance of non-symbolic quantity skills (e.g. Dehaene, 2001). Evidence for these quantity skills found in young children has led to the assumption that these skills are based on a biologically wired-in, innate system for processing quantities (Barth et al., 2006; Dehaene, 2001; Wood & Spelke, 2005; Xu, Spelke, & Goddard, 2005). Although these skills are in some studies referred to as quantity-to-space mapping (De

Hevia & Spelke, 2010; Opfer & Thompson, 2006), in this paper the term non-symbolic skills is used to refer to the system for processing quantity information. The non-symbolic quantity system is presupposed to have a spatial format with spatially ordered positions to which symbolic numbers become connected in the course of development (Dehaene et al., 1993; Fischer et al., 2003; Gevers et al., 2006; Halberda & Feigenson, 2008). Indeed several studies presented empirical evidence for this idea showing that non-symbolic quantity skills are related to (problems in learning) math (Desoete, Ceulemans, De Weerdt, & Pieters, 2010; Gilmore, McCarthy, & Spelke, 2010; Inglis, Attridge, Batchelor, & Gilmore, 2011; Landerl, Fussenegger, Moll, & Willburger, 2009; Mussolin, Meijas, & Noël, 2010; Piazza et al., 2010;).

Recently, however, an increasing body of evidence has demonstrated that non-symbolic quantity skills, perhaps, are *not* the presumed key-precursors of number-to-magnitude mapping skills. A number of recent studies failed to find (developmental) relations between non-symbolic skills and mappings skills (Kolkman et al., 2013) or math (De Smedt & Gilmore, 2011; Rouselle & Noël, 2007). Moreover, it was demonstrated that the spatial ordering of numbers and quantities depends on cultural factors such as reading habits as well (Opfer, Thompson, & Furlong, 2010; Shaki & Fischer, 2008). These studies indicate that the relation between non-symbolic skills and mapping skills is not that straightforward as has been put forward by previous studies. It has also been suggested that a specific spatial ordering of numbers and quantities arises from a general purpose spatial system supporting the one-dimensional mapping of numbers to quantities and also the ordering of quantities to a one-dimensional space. Mapping skills (and also non-symbolic quantity skills) could arise from such a general purpose spatial system (Ansari, 2008). According to this idea, domain-general skills, such as working memory, might play an important role. We therefore suggest that domain-specific spatial ordered non-symbolic quantity skills might thus not be sufficient to explain development of mapping skills.

Another candidate for providing the fundament foundation upon which number-to-quantity mapping is built, has been proposed in recent studies focusing on the involvement of working memory in constructing the spatial ordering of numbers and quantities. Van Dijck and Fias (2011) demonstrated that the position of an item in a larger sequence is associated with space in working memory. For example items at the beginning of a verbal sequence elicited faster and more accurate responses with the left hand then with the right. For items at the end of a verbal sequence the exact opposite was found. This phenomenon was demonstrated in number stimuli but also in non-numerical stimuli. Apparently, adults use temporary position-space associations to order information. These position-space connections might be also important when connections between numbers and quantities or quantities and space are constructed. This idea is closely related to the proposal of Henik, Leibovich, Naparstek, Diesendruck, and Rubinsten (2011) who suggest that general spatial skills involved in processing non-countable dimensions like distance and height facilitate the spatial ordering on number-quantity connections. Considering that spatial skills are important facilitators of the spatial ordering of numbers and quantities, an implication can be that general spatial working memory skills, rather than specific quantity processing skills underlie performance on tasks measuring mapping skills.

Evidence for the involvement of working memory in developing mapping skills in children is found in different types of studies. Studies focusing on math learning difficulties and comparing children with math learning disabilities with their typically achieving peers, have revealed that working memory contributes importantly to group differences in performance on the number-to-position task (Geary, et al., 2007). Moreover, Geary et al. (2008) have demonstrated that children with math learning difficulties had general deficits in working memory and measures of number-magnitude skills. Studies focusing on the relations between working memory and number-magnitude skills have provided longitudinal evidence for the role of visual spatial working memory measured at age 5 in number comparison skills measured at age 8 (Krajewski & Schneider, 2009). Cross-sectional studies have shown that across grade performance of mapping tasks could be explained by working memory. On a number-to-position task working memory accounted for differences between first- and second-graders (Geary et al., 2007). Also on a number comparison task, changes in performance of 6- to 8-year-olds reflect changes in domain-general mechanisms rather than changes in domain-specific number cognitions (Holloway & Ansari, 2008). Moreover, it was found in another cross-sectional study including 4- to 8-year-old children, that working memory was an important predictor of the accuracy of placements on the number-to-position task over and above age-related development

(Friso-van den Bos et al., 2013). Also training studies showed that improvement on a number-to-position task was facilitated by working memory (Kolkman, Hoijtink, Kroesbergen, & Leseman, 2013).

To summarize, there are several indications for the involvement of working memory in (the development of) mapping skills. If working memory is indeed a facilitator of establishing number-to-quantities connections in a spatial format it might also be a good candidate for providing a base for the spatial ordering of *non-symbolic* quantities . In this study, it is hypothesized that working memory offers a 'workspace' which facilitates the processing of spatial ordered information in general. Clearly, the idea of such a workspace is strongly related to current concepts of (spatial) working memory that define (spatial) working memory as the activated part of (spatial) memory (Cowan, 2010).We suspect that this workspace is modality specific, that is, it is dedicated to processing spatial ordered information. This workspace is also involved when information about quantities and numbers needs to be processed. We suggest that it is this general workspace, rather than a specific wired-in spatial system for processing quantity (as proposed by Dehaene, 2001), that facilitates development of mapping skills. To test this idea, we formulated the following hypotheses: 1) spatial working memory facilitates the development of both non-symbolic quantity skills and mapping skills; and 2) the development of mapping skills is not related to the development of non-symbolic quantity skills.

Another important aspect of number-to-quantities mapping has been demonstrated in studies into the role of symbolic number skills. To establish the connection between number and quantities is has been proposed that symbolic counting skills function as a facilitator of this process. The cultural ordering of the number words of the counting sequence is thought to facilitate the association of numbers and magnitudes in a spatial structure (Helmreich et al., 2011; Opfer & Furlong, 2011). Applying knowledge of the count row can help children to understand the cardinal value of the number words and facilitate the understanding that number words refer to exact quantities. This, in turn, strengthens the connection between number symbols and non-symbolic quantity values (Noël & Rouselle, 2011). Indeed several studies have demonstrated that symbolic number skills, related to counting and knowledge about number symbols, play a dominant role in (development of) mapping skills (Kolkman et al., 2013; LeFevre et al., 2010; Sasanguie, Göbel, Moll, Smets, & Reynvoet, 2012). Based on these previous results we argue that symbolic skills are also important in the development of mapping. Since mapping relies on the processing of verbal information, it is hypothesized that also a verbal workspace is involved in learning the verbal counting sequence and in using counting skills to develop number-to-quantity mappings. Therefore a third hypothesis was formulated: 3) verbal working memory facilitates the development of verbal counting skills which in turn contribute to the development of mapping.

The aim of the current study is thus to examine the precursors of number-to-magnitude mapping. By examining the co-development of a constellation of domain-general and domain-specific precursors in a longitudinal design in children at a crucial age for developing accurate mapping skills the current study will contribute to the debate on which skills are needed for developing number-to-magnitude mappings.

Method

Participants

All children that participated in this study, were Dutch speaking children recruited from 15 primary schools in the Netherlands. At the start of this study, informed parental consent was received for 222 children. During the course of the study 27.02% of the children dropped out due to moving, grade retention or accelerating a grade. Note that in The Netherlands, grade retention or accelerating a grade are quite common for high or low performing kindergartners. All analyses were performed on the remaining 162 children.

The first measurement took place when children were in kindergarten year 1 (M_{age} = 59.31 months; *SD* = 3.08) and the last measurement took place halfway first grade (M_{age} = 75.31 months; *SD* = 3.08). At all measurement times the same battery of tests was administered: children completed one task measuring mapping skills, two non-symbolic tasks, two symbolic number tasks and two working memory tasks. The children were tested individually in a quiet room inside the school by trained undergraduate students. All tasks were administered on a laptop computer using E-Prime 1.2 software (Psychological Software Tools, http://www.pstnet.com).

Performance on the Raven's Colored Progressive Matrices (Raven, 1962) was used as measure for cognitive abilities (*M* = 25.84; *SD* = 4.81). Following Dutch norms from 1982 (Raven, Court, & Raven, 1995), this mean score corresponds to a percentile score between 75 and 90 indicating that the children in this study performed above average on this test. It should be mentioned, however, that these norms are not very reliable, because they are relatively old, and may overestimate children's IQs (due to the Flynn-effect). Parents of 129 children (79.62%) filled out a questionnaire on their educational background. Of these parents, 9.30 % had finished only lower secondary education or less, 65.90 % had completed upper secondary education or vocational training and 24.81 % completed higher vocational training or university. Compared to national measures of educational background (lower secondary education: 28%; upper secondary education: 42%; higher educational education: 27%; CBS Statistics Netherlands, 2007) the educational level of the parents in the current study was representative for the average educational level in The Netherlands. General measure of cognitive ability and educational background were used to describe the sample and not included in the analyses.

Measures

Numerical skills.

Mapping skills. In the number-to-position task children were asked to estimate the position of a given number (in the range from 1–100) on a horizontal line. To introduce the task, the experimenter demonstrated the position of the numbers '1' and '100'. Then children were presented with ten trials and asked to point with the mouse to the position they thought each number belonged on the number-line. On each trial the numbers-to-be-estimated were read aloud by the experimenter. Linear fit scores were computed by fitting the answers of each individual child to a linear curve (Geary et al., 2008). As shown by Friso-van den Bos et al. (2013), individual linearity scores provide a valid measure for examining development in mapping skills in young children. Previously reported reliability measures (α = .79) are satisfactory (Kolkman et al., 2013).

Non-symbolic skills. Two tasks were used to assess the non-symbolic skills of the children. The first task was a quantity-to-position task based on the number-to-position task (Laski & Siegler, 2007). The features of this task were the same as the number-to-position task except for the stimuli: in the quantity-to-position task dots instead of numbers were used. Linear fit scores were computed by fitting the answers of each individual child to a linear curve (Geary et al., 2008). Reported reliability measures of this task are good (α = .73; Kolkman et al., 2013) and scores on this task provided a valid measure for examine development in young children (Sasanguie, Göbel, Moll, Smets, & Reynvoet, 2013). A *comparison* task was used to measure quantity discrimination. Children were asked to compare two arrays with dots and had to indicate the array with the highest amount of dots. We used an adapted version of the tasks used by Barth et al. (2006) and Gebuis, Kadosh, De Haan, and Henik (2008), who controlled for non-numerical parameters by including three conditions: (1) congruent condition in which the area with the largest amount of dots was also physically larger; (2) incongruent condition in which the area with the largest amount of dots was physically smaller; and (3) a neutral condition in which the physical size of the dots in both areas was the same but the amount varied. In each condition ten trials were presented in the range from 1-100. For each of the 30 items that the child made successful, one point was given. Although few studies reported the reliability of the dot comparison task, the task is frequently used as a measure of non-symbolic numerical skills (De Smedt & Gilmore, 2011; Iuculano, Tang, Hall, & Butterworth, 2008; Landerl, Bevan, & Butterworth, 2004; Rousselle & Noël, 2007). Reported reliability measures of this task are good (α = .84; Kolkman et al., 2013). The scores on the Comparison task were transformed in proportion scores, resulting in a score between 0 and 1. The scores on the quantity-lines task also ranged from 0 to 1 indicating the percentage of variance that could be explained by a linear fit. Internal consistency of a scale consisting of both the quantity-to-position task and the comparison task was satisfactory (α = .67). This was in line with previous studies demonstrating that both tasks loaded on the same factor and are thus assumed to measure the same construct (Kolkman et al., 2013). Therefore we combined the two scores into one variable for number-magnitude skills based on the mean proportion score.

Symbolic skills. Two tasks were used to assess the symbolic skills of the children. In the *number-naming* task, the child was presented with numerals on a computer screen and was asked to identify each number. The numerals were presented in a randomized order in the range from 1–100 (12 trials). The number of correctly identified numerals was used as a score for number-naming. The Early Numeracy Test-Revised (ENT-R; Van Luit & Van de Rijt, 2009) was used to measure *counting skills*. The original ENT-R consists of nine subscales and has two parallel versions, version A (used at measurement time 2 and time 4) and version B (used at measurement time 1 and time 3). In this study, the items from one subscale were used in which children were asked to recite the counting sequence backwards and forwards and by skipping numbers. The items are scored with 0 for a wrong answer and 1 for a correct answer. The subscale contains five items; the sum of correct answers was used as a measure for counting sequence. Reliability score of the original ENT-R was α = .93 (Van Luit & Van de Rijt, 2009).

Internal consistency of a scale consisting of both the number naming task and the counting task was good (α = .87).The scores on the number-naming task and counting task were therefore transformed in proportion scores and were then combined in one measure for symbolic skills (following Kolkman et al., 2013).

Working memory skills.

Tasks from the Automated Working Memory Assessment battery (AWMA; Alloway, 2007) were used to assess working memory skills. Test-retest reliability of these tasks for children aged 4.5 was good (Word recall backward: r = .74; Odd one out: r = .81; Alloway, Gathercole, Kirkwood, & Elliott, 2008).

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Spatial WM. Spatial WM was measured using the *Odd One Out* task. A series of stimuli was shown consecutively. Each stimulus consisted of three boxes with shapes presented next to each other. One of the shapes differed from the other two. The child was asked to point out the deviant shape. Then the next stimulus was presented. At the end of each trial three empty boxes appeared; the child had to point at the locations of the previously shown deviant shapes in the same order in which they had appeared. An answer was correct if each location was recalled correctly in the right order. The task started with only one stimulus; after three correct answers of the same length, the sequence increased by one. When two mistakes were made in trials of the same length, the task was discontinued. The number of correct sequences was used as a final score.

Verbal WM. For the assessment of verbal WM, *Word Recall Backward* was used. In this task, a recorded voice names a sequence of semantically unrelated words, after which the child is asked to repeat the words in the reversed order. The sequence of words becomes longer after a child recalled four sequences of the same length correctly. The task was
automatically terminated when a child gave three incorrect answers within a set of items of the same length. The number of correct recalled sequences was used as measure.

Analyses

Before testing the hypotheses, univariate outliers (scores two standard deviations below or above the mean) were converted into scores at a standard deviation of -2 or 2 (Field, 2005). We used this procedure on 5.08% of the numerical data points and 6.14% of the working memory data points.

In the first step of the analyses, we examined the univariate growth of the non-symbolic skills, symbolic skills, verbal working memory skills, visual working memory skills and mapping skills. This type of analysis models the development of a specific skill by estimating the best growth curve for each child. The slope and the intercept are estimated as latent variables, with the scores at each measurement time as observed indicators. The loadings of the intercept are fixed at 1, since the intercept does not change over time. The loadings of the slope reflect the growth. Since we wanted to model the best fitting growth curve, the slope loadings for the successive measurement times were fixed at 0 for time 1 and fixed at 1 for time 4. For time 2 and time 3, the slope loading were not specified, allowing the estimation of the best fitting slope loading (a procedure described by Welch, 2007). For both the intercept and the slope, means and variances are estimated reflecting the average intercept and slope and individual differences. A significant covariance between intercept and slope reflects that children with higher level develop faster if the covariance is positive or the reverse if it is negative. In the next section, we will present the fit-indices of these models: chi- square (χ^2) with its *p*-value and CFI and RMSEA. Chi-square is a discrepancy measure; between the current model and the saturated model, and should be as low as possible, with a *p*-value that is as high as possible. Note that the interpretation of the *p*-value is confounded by sample size (Gerbring & Anderson, 1993) and therefore other fit-indices (CFI, RMSEA) should be considered when evaluating a model. Another guideline is the ratio of χ^2 / *df* which should not exceed 2 (Kline, 2005). CFI compares the fit of the model to the independence model and is considered good if > .95 and acceptable if > .90. RMSEA is a pasimony-adjusted index in that its fomula includes a built-in correction for model complexity, favoring simpler models (Kline, 2005) and is considered good if < .05 and acceptable if < .08 (Blunch, 2008).

In the second step of the analyses, multivariate growth models were examined to test our hypotheses. Three multivariate growth models were examined (see Figure 5.1, 5.2 and 5.3): (1) the co-development of visual working memory, mappings skills and non-symbolic skills (without paths between mapping skills and non-symbolic skills), (2) the co-development of visual working memory, mappings skills and non-symbolic skills (including paths between mapping skills and non-symbolic skills)and (3) the co-development of verbal working memory, mapping skills and symbolic number skills.

Due to the directional nature of the hypotheses, one-tailed tests are used in all analyses.

Tabel 5.1
Means and standard deviations of the working memory tasks and all the numerical measures.

	Time 1		Time 2		Time 3		Time 4	
	M	SD	M	SD	M	SD	M	SD
Odd one out	7.58	2.64	8.73	2.83	9.65	2.90	10.37	2.70
Word recall backward	2.97	2.28	3.94	1.88	4.64	1.59	5.25	1.49
Comparison	.80	.14	.86	.09	.89	.07	.91	.07
Quantity-to-position	.44	.27	.43	.27	.54	.27	.57	.25
Number naming	.25	.16	.36	.23	.52	.29	.61	.30
Counting	.35	.18	.36	.27	.55	.24	.61	.30
Number-to-position	.16	.16	.26	.23	.38	.30	.44	.30

Results

Descriptive statistics

Table 5.1 presents the mean scores and standard deviations for the measures used in this study. For the number-to-position task and quantity-to-position task linear fit scores are presented. Proportion scores are presented for the comparison task, the

Table 5.2
Correlations among all variables (part 1)

			1	2	3	4	5	6	7	8	9	10	11	12
2	Number-to-position	Time 2	.21**											
3		Time 3	.13+	.20**										
4		Time 4	.03	.26**	.47**									
5	Odd one out	Time 1	.06	.10	.15*	.34**								
6		Time 2	.01	.11+	.23**	.25**	.52**							
7		Time 3	.05	.18**	.28**	.19**	.37**	.53**						
8		Time 4	.03	.05	.32**	.24**	.43**	.45**	.59**					
9	Word recall backward	Time 1	.20**	.20**	.30**	.14*	.28**	.36**	.53**	.42**				
10		Time 2	.06	.13+	.31**	.20**	.30**	.39**	.34**	.36**	.48**			
11		Time 3	.12+	.24**	.21**	.23**	.22**	.16*	.32**	.27**	.36**	.31**		
12		Time 4	-.02	.12+	.25**	.07	.12+	.15*	.28**	.24**	.35**	.27**	.27**	
13	Quantity-to-position	Time 1	.08	.05	.20**	.14*	.28**	.17*	.29**	.21**	.21**	.25**	.06	.05
14		Time 2	.15*	.26**	.33**	.23**	.25**	.39**	.39**	.32**	.33**	.39**	.11+	.15*
15		Time 3	.04	.08	.26**	.17*	.30**	.30**	.27**	.40**	.24**	.28**	.03	.15*
16		Time 4	.07	.24**	.24**	.26**	.27**	.26**	.34**	.40**	.21**	.28**	.08	.15*
17	Comparison	Time 1	.04	.05	.14*	.17*	.24**	.21**	.36**	.22**	.26**	.04	.09	.16*
18		Time 2	.10	.34**	.21**	.22**	.20**	.18*	.25**	.23**	.29**	.26**	.17*	.19**
19		Time 3	.09	.18*	.31**	.13+	.15*	.06	.27**	.19**	.25**	.12+	.33**	.31**
20		Time 4	.08	.15*	.26**	.14*	.06	.09	.18*	.17*	.17*	.04	.10	.13+
21	Number naming	Time 1	.12+	.30**	.31**	.28**	.30**	.39**	.34**	.24**	.30**	.21**	.15*	.25**
22		Time 2	.25**	.37**	.30**	.38**	.27**	.34**	.37**	.22**	.43**	.22**	.24**	.18*
23		Time 3	.25**	.36**	.44**	.40**	.18*	.22**	.29**	.22**	.31**	.09	.30**	.31**
24		Time 4	.14*	.31**	.47**	.45**	.23**	.31**	.33**	.29**	.27**	.14*	.34**	.31**
25	Counting	Time 1	.21**	.20**	.19*	.24**	.28**	.29**	.24**	.21**	.35**	.18*	.25**	.01
26		Time 2	.19**	.27**	.34**	.37**	.31**	.34**	.40**	.21**	.37**	.34**	.30**	.20**
27		Time 3	.18*	.12+	.33**	.27**	.34**	.32**	.35**	.36**	.33**	.20**	.25**	.26**
28		Time 4	.03	.17*	.38**	.28**	.13+	.21**	.25**	.31**	.21**	.24**	.23**	.24**

Note. $^{*}p < .05$; $^{+}p < .10$ (1-tailed) number naming task and the counting task. Scores on the working memory measures are presented as mean accuracy scores. Bivariate correlations between all variables were examined before the growth models were built (see Table 5.2). These correlations demonstrated (a) relations between subsequent measures of the same task indicating stability of the measurement, (b) relations between mapping skills and counting and number naming skills at all measurement times, (c) relations between mapping skills and visual and verbal working memory skills from time 2 onwards (d) relations between mapping skills and quantity-to-position task and comparison task from time 2 onwards and (e) relations between all measurement times between the tasks measuring non-symbolic skills and between the tasks measuring symbolic skills.

Univariate growth

As a first step in the analyses, separate growth models were built for mapping skills (measured with the number-to-position task), non-symbolic skills (combined measure of quantity-to-position task & comparison task), symbolic skills (combined measure

Table 5.2
Correlations among all variables (part 2)

			13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
14	Quantity-to-position	Time 2	.26**														
15		Time 3	.21**	.36**													
16		Time 4	.21**	.45**	.59**												
17	Comparison	Time 1	.18*	.07	.16*	.30**											
18		Time 2	.14*	.30**	.23**	.25**	.16*										
19		Time 3	.11^{+}	.12^{+}	.19**	.18*	.24**	.28**									
20		Time 4	-.07	.24**	.24**	.21**	.11	.26**	.40**								
21	Number naming	Time 1	14*	.25**	.17*	.19*	.28**	.13^{+}	.13^{+}	.19**							
22		Time 2	.19**	.29**	.16*	.12	.30**	.29**	.18*	.15*	.68**						
23		Time 3	.09	.21**	.15*	.09	.30**	.20**	.29**	.25**	.62**	.64**					
24		Time 4	.15*	.21**	.15*	.15*	.30**	.15*	.25**	.17*	.50**	.58**	.70**				
25	Counting	Time 1	.18*	.25**	.15*	.15*	.20**	.16*	.03	.08	.39**	.46**	.36**	.41**			
26		Time 2	18*	.32**	.20**	.23**	.22**	.33**	.20**	.12^{+}	.40**	.61**	.45**	.44**	.33**		
27		Time 3	23**	.21**	.19**	.14*	.22**	.08	.15*	.08	.46**	.46**	.45**	.43**	.32**	.46**	
28		Time 4	08	.21**	.20**	.27**	.17*	.04	.25**	.09	.23**	.21**	.35**	.33**	.10	.36**	.26**

Note. * $p < .05$; $^{+}$ $p < .10$ (1-tailed)

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of number naming task & counting task), verbal working memory (measured with word recall backward) and visual working memory (measured with odd one out). The fit of the growth models is presented in Table 5.3. The fit of the growth model for spatial working memory was satisfactory. The fit for the other univariate growth models was good. The parameter estimates are presented in Table 5.4. As described above, the factor loadings for time 2 and time 3 were estimated.

The variances in the slope and intercept of each model were examined to investigate whether there was a significant difference between individuals in their initial level and in their growth on each variable. At all variables the intercept variance

Table 5.3
Fit indices of the univariate growth models and multivariate growth models.

	χ^2/df	CFI	RMSEA
OOO	2.28	.98	.09
WRB	0.52	1.00	.00
Non-symbolic skills	2.01	.97	.08
Symbolic skills	1.18	.99	.03
Mapping	0.04	1.00	.00
Model 1	1.37	.95	.05
Model 2	1.20	.97	.04
Model 3	2.17	.88	.09

Table 5.4
Parameter estimates for the univariate growth models.

model	Un-Standard-ized	Standard-ized	*p*-value	SE	Un-Standard-ized	Standard-ized	*p*-value	SE
	factor loadings time 2				factor loadings time 3			
OOO	0.41	.26	< .001	0.07	0.78	.49	< .001	0.07
WRB	0.44	.25	< .001	0.06	0.76	.50	< .001	0.06
Non-symbolic skills	0.30	.24	< .001	0.08	0.70	.53	< .001	0.06
Symbolic skills	0.21	.14	< .001	0.04	0.76	.55	< .001	0.05
Mapping	0.35	.37	< .001	0.06	0.78	.65	< .001	0.08
	Intercept variance				Slope variance			
OOO	4.19		< .001	0.90	3.19		< .001	1.30
WRB	2.70		< .001	0.61	1.15		.07	0.78
Non-symbolic skills	0.007		< .01	0.002	0.014		< .001	0.004
Symbolic skills	0.02		< .001	0.002	0.03		< .001	0.005
Mapping	0.01		< .05	0.005	0.06		< .001	0.002

was significant indicating individual differences in initial level for all variables under consideration. Significant individual variances in slope were found for each task except for the word recall backward task.

Multivariate growth

In this step of the analyses, two multivariate growth models were examined to test our hypotheses. In these analyses, the slope values found in the univariate analyses were included.

The results of these analyses are presented in Table 5.3 and Figures 5.1, 5.2 and 5.3.

Hypothesis 1: general visuospatial processing skills underlie development of non-symbolic skills and mapping skills.

In this first model, the co-development of visual working memory, non-symbolic skills and mapping skills was examined. The fit of this model was good (χ^2 / df = 1.38; CFI = .95; RMSEA = .05). Regarding the intercept of spatial working memory, a negative effect was found on the slope of non-symbolic skills (β = -.22; $p < .05$) and a positive relation was found with the intercept of non-symbolic skills (r = .94; $p < .001$). Moreover, the intercept of spatial working memory had a positive effect on the slope of mapping skills (β = .59; $p < .001$) and was negatively related to the intercept of mapping skills (r = -.18; $p < .05$). To further examine the negative effect of spatial working memory on non-symbolic skills and the negative relation between spatial working memory and mapping skills, two additional models were tested: the first including multivariate growth of spatial working memory and non-symbolic skills and in the second multivariate model spatial working memory and mapping skills were included. In the first model, again a negative effect of the intercept of spatial working memory on the slope of non-symbolic skills and a positive relation between the intercept of spatial working memory and the intercept non-symbolic skills was found. When the relation between both intercepts, however, was fixed at ‘0’, the effect of the intercept of spatial working memory skills on the slope of non-symbolic skills became positive. In the second model, the effect of spatial working memory on the slope of mapping skills was positive as was the relation between the both intercepts. These explorative analyses indicated that the negative effect and relation between the

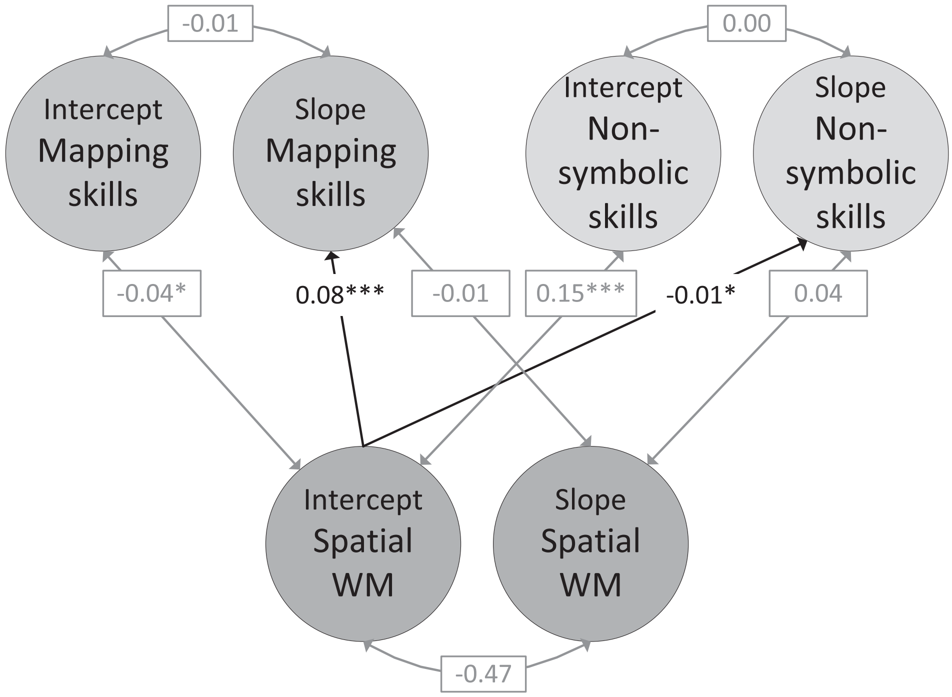

Figure 5.1
Standardized regression coefficients and covariances (boxes with borders) for model 1.
Note. * $p < .05$, ** $p < .01$, *** $p < .001$

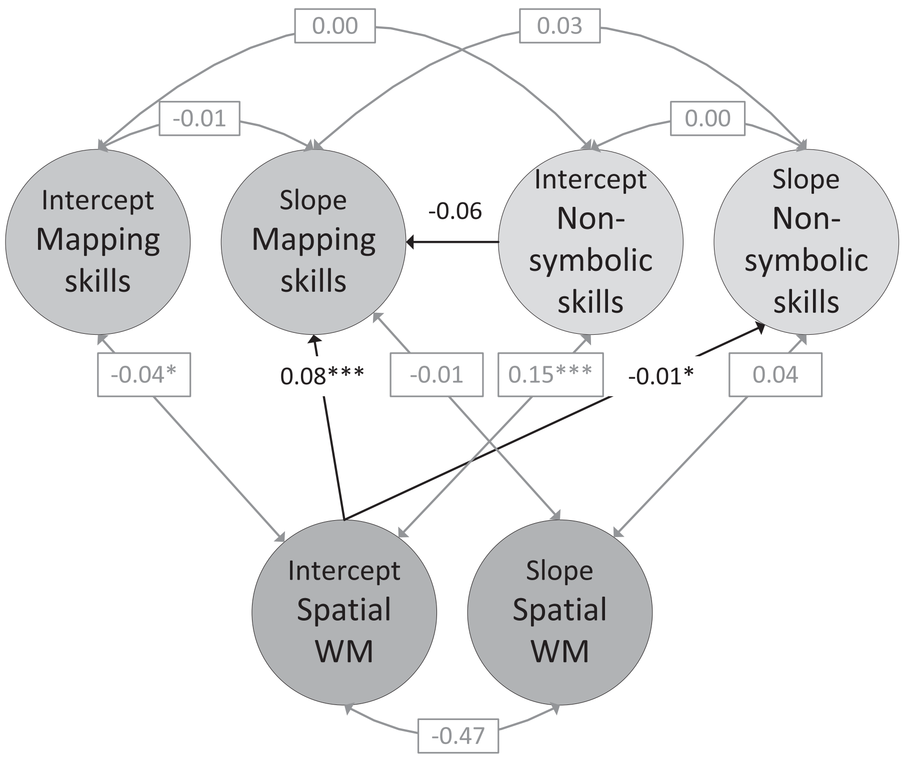

Figure 5.2
Standardized regression coefficients and covariances (boxes with borders) for model 2.
Note. * $p < .05$, ** $p < .01$, *** $p < .001$

intercept of spatial working memory and respectively the slope of non-symbolic skills and the intercept of mapping skills were possibly the result of a model artifact and were not indicative of the direction of the effect or relation in the sample.
To examine the modality specificity of the workspace facilitating non-symbolic skills and mapping skills, these analyses were also performed including word recall backward as a measure of verbal working memory (instead of spatial working memory). The fit of this model was unacceptable (χ^2 / *df* = 2.67; CFI = .78; RMSEA = .09) and thus parameters were not further examined.

Hypothesis 2: Non-symbolic skills and mapping skills are distinguishable abilities.

In this model, we preserved the unstandardized parameter estimates of model 1 and included an effect from the intercept of non-symbolic skills on the slope of mapping skills and relations between intercepts of non-symbolic and mapping skills and between slopes of non-symbolic and mapping skills. The fit of this model was good (χ^2 / *df* = 1.19; CFI = .97; RMSEA = .04). No effect was found between the intercept of non-symbolic skills and the slope of mapping skills. Neither was there a relation found between intercept of non-symbolic skills and intercept of mapping skills, nor between the slopes of non-symbolic skills and mapping skills.

Hypothesis 3: verbal working memory facilitates the development of verbal counting skills which in turn contribute to development of mapping.

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In this model the co-development of verbal working memory, symbolic number skills and mapping was examined. The fit of this model was acceptable (χ^2 / *df* = 2.17; p <.001; CFI = .88; RMSEA = .09). The intercept of verbal working memory predicted the slope of symbolic skills but not of mapping skills (resp. $\beta = .21$; $p < .05$ and $\beta = .10$; $p = .15$). The intercept of symbolic skills predicted the slope of mapping skills ($\beta = .30$; $p < .05$). Examination of the co-variances demonstrated that the intercept of verbal working memory was related to the intercept of symbolic skills but not to the intercept of mapping skills (resp. $r = .20$; $p <.001$ and $r = .08$; $p = .22$). The slope of verbal working memory, however, was related to the slopes of both symbolic skills and mapping skills (resp. $r = .28$; $p <.001$ and $r = .11$; $p <.05$). The intercept and slope

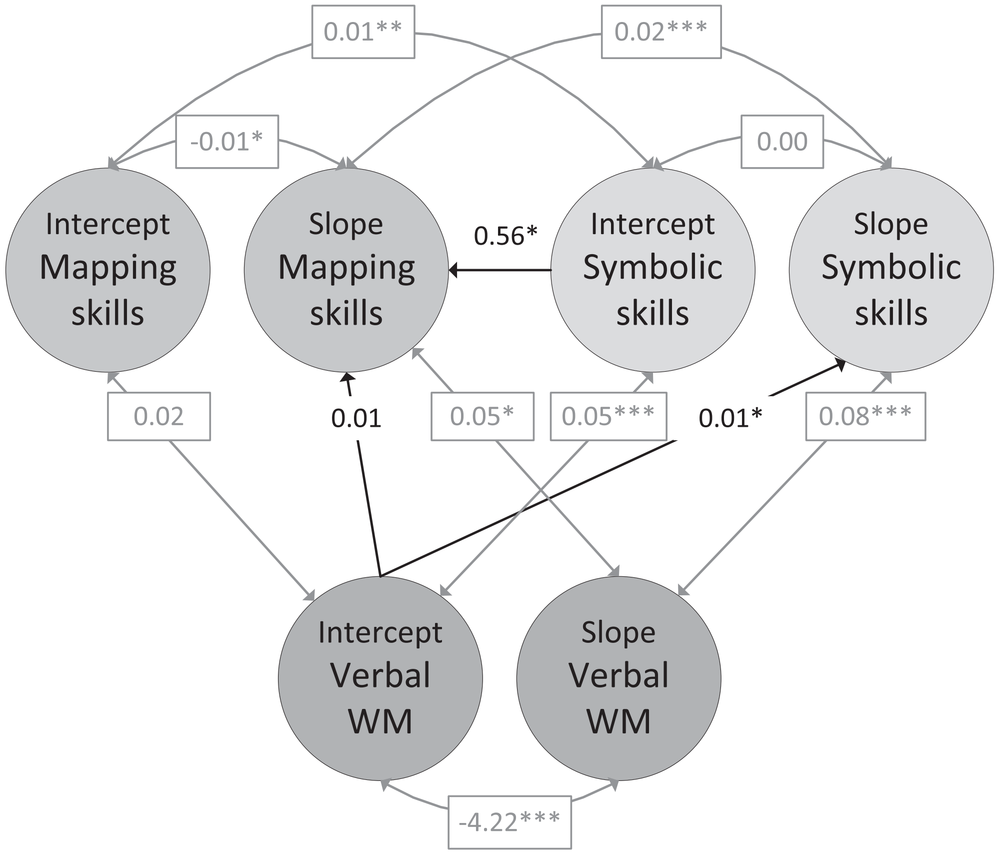

Figure 5.3
Standardized regression coefficients and covariances (boxes with borders) for model 3.
Note. * $p < .05$, ** $p < .01$, *** $p < .001$

of symbolic skills were related to respectively the intercept and slope of mapping skills ($r = .45$; $p <.01$ and $r = .67$; $p <.001$).

Discussion

This paper examined the spatial skills that are presumed to underlie the mapping of numbers to quantities. Previous studies have suggested that a highly domain-specific spatial system for processing quantities underlies the development of accurate mapping (e.g. Dehaene, 2001). Other studies, however, proposed that these spatial ordering of numbers and quantities arises from a general purpose spatial processing system that supports the construction of a one-dimensional mapping of numbers to quantities and also the ordering of quantities in a one-dimensional space (e.g. Henik et al., 2011; Van Dijck & Fias, 2011). Following this idea, it was proposed in this paper that working memory provides a domain-general spatial workspace which facilitates the development of mapping skills. The results of this paper provided longitudinal evidence for this idea. Moreover it was found that this workspace is indeed modality specific: spatial associations between numbers and quantities were facilitated by spatial working memory but not by verbal working memory. Furthermore, the spatial workspace was found to be related to both mapping skills and non-symbolic skills providing evidence for the idea that the foundation of numerical development needs to be sought in domain-general skills for visuospatial processing.

The first hypothesis that was tested in this study assumed that spatial working memory facilitated the development of both non-symbolic quantity skills and mapping skills. The current results provided evidence for this hypothesis, showing that the intercept of visual working memory predicted the growth on non-symbolic quantity skills and mapping skills and was also related to the intercept of these skills. The same analyses including verbal (instead of spatial) working memory yielded a bad fitting model, which could not be accepted. Therefore, we can conclude that it is specifically the processing of spatial information, and not just processing skills in general, that is related to the development of non-symbolic skills and mapping skills. The absence of relations between the slope of visual-spatial working memory and the slope of non-symbolic quantity skills and mapping skills indicates that it is rather the overall level of visual-spatial working memory (intercept) than the growth of visual working memory skills that is related to the development of mapping skills and non-symbolic skills. Following the second hypothesis, it was expected that the development of non-symbolic quantity skills was not related to the development of mapping skills. Results were found in favor of this hypothesis showing no relations between the intercepts and slopes of these two skills indicating that the processing of non-symbolic quantity information might not be a key-precursor of mapping. The third hypothesis assumed that verbal working memory facilitated the development of symbolic counting skills which in turn was a precursor of mapping skills. The results provided evidence for this hypothesis by showing that the intercept of verbal working memory predicted the slope of symbolic counting skill. Moreover, relations were found between verbal working memory and symbolic counting skills. Although no effect was found for the intercept of verbal working memory on the growth of mapping, there was a relation between the intercept of verbal working memory and the intercept of mapping skills. Taken together, these findings suggest that mapping skills and non-symbolic quantity skills arise from a shared ability to process spatial information. Based on findings with adults that position-space associations are built in working memory (Van Dijck & Fias; 2011), we argue that these position-space associations in working memory enable children to order numerical (and non-numerical) information. Working memory might indeed serve as a workspace for spatially ordering information about numbers and quantities. The differential results concerning the different components of working memory showed that, in order to establish a spatial ordering, modality specific processes are involved. In combination with the finding that non-symbolic quantity skills were found not to be related to mapping, it can be concluded that these spatial processing skills provide the basic starting points for numerical learning. In contrast to the common idea that an innate non-symbolic quantity system provides the base for number-quantity mapping (Dehaene, 2001), the current results show that it is rather a system for processing spatial information. This idea is supported by previous findings demonstrating a relation between working memory and mapping (e.g. Geary et al., 2007; Holloway & Ansari, 2008; Krajewski & Schneider, 2009), and also in line with previous studies showing relations between spatial working memory and mapping skills (Kolkman, Kroesbergen, & Leseman, submitted; Krajewski & Schneider, 2009). Moreover, this idea can also explain findings of previous studies in which no relation between non-symbolic skills and mapping skills have been found (Kolkman et al., 2013).

The current findings also suggest that symbolic skills, in contrast to non-symbolic skills, *are* related to mapping development and that symbolic skills require involvement of verbal working memory. It is not surprising that the learning of the counting sequence, which is a verbally oriented task, is related to working memory. However, it is interesting to speculate about how these symbolic skills contribute to the development of mapping in cooperation with a system for ordering and representing information in a spatial format. As has been demonstrated in previous studies, culture-based skills influence the spatial ordering of numbers and quantities. Skills such as being able to recite the counting sequence and our left-to-right reading habits influence the construction of a spatially ordered representation of the number-to- quantity relationships. Indeed, studies in countries with a right-left reading system do show the SNARC effect and other phenomena relating to the spatial representation of number-to-quantity, but in the opposite direction, consistent with the reading system (Shaki, Fischer, & Petrusic, 2009).It might be that symbolic cultural skills, in interaction with a system for spatial processing, facilitate children to construct connections between numbers and quantities. To construct these connections, symbolic knowledge about the counting sequence (a child learns that the number 4 precedes the number 5) provides knowledge about the ordering of the number symbols but does not in itself contain spatial information about the distances between numbers. In a spatial workspace, position-space connection provide the ordering of numbers with spatial information about the distances between these numbers which will eventually result in connections between numbers and quantities. The spatial workspace might thus play a key-role in the spatial ordering of numbers and quantities. Indeed Bachot, Gevers, Fias, and Roeyers (2005) demonstrated that visual-spatial WM is important in constructing number-to-quantity relations as they showed that there was no SNARC effect in children with visual-spatial disabilities.

To summarize, it can be argued that children thus make use of the inherent ordinal structure of the number system to systematically map numbers as a function of their quantity (Van Dijck & Fias, 2011). A workspace that facilitates the construction of a spatial ordering of numbers and quantities, thus might underlie two types of skills: symbolic skills connected to an exact quantity and approximate non-symbolic quantity skills (e.g., Lyons, Ansari, & Beilock, 2012; Noël & Rouselle, 2011). Although these skills arise from a general ability to spatially process and order numerical information (whether this is symbolic or non-symbolic), only the exact skills might be important for math performance in the first grades of primary school. Thus, for the development of mapping skills we propose that there is an interplay between general abilities for visual-spatial processing on the one hand and symbolic counting skills on the other hand. Spatial abilities aid children to understand the cardinality principle of the numbers in the counting sequence by linking the count words to a spatial representation that matches with the ordered structure of the counting sequence, but adds a quantity representation that is based in sensorimotor cognition.

Further research is needed to address these ideas in for example experimental studies and brain imaging studies. Future experimental studies can shed light on the causal relations between working memory and mapping skills and non-symbolic skills. A recommendation for future research is also to include multiple measure for the working memory construct. Inclusion of several tasks increases the validity of the construct measurement (as was done in this study for the domain-specific precursors). Brain imaging studies can provide information about the brain structures that are involved in processing numbers and quantities but also the structures that are involved in processing non-numerical spatial information.

6

Summary and general discussion

Introduction

The aim of this dissertation was to unravel the processes that contribute to the development of number-to-quantity mapping skills. According to a widely shared view, mapping skills originate from a specialized cognitive system in which numbers and the quantities they denote are spatially ordered from left to right with smaller quantities associated with the left side of the space and larger quantities with the right side of space, often referred to as the 'mental number line'. It is assumed that this spatial ordering becomes more precise during development (Booth & Siegler, 2006; Durand, Hurme, Larkin, & Snowling, 2005; Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; Holloway & Ansari, 2009; Siegler & Booth, 2004). An important debate in the recent literature concentrates on the origins of the spatial ordering of numbers and quantities. Some authors assume that the spatial ordering is a highly domain-specific phenomenon that is based on an innate system specifically dedicated to processing approximate non-symbolic quantity information (Dehaene, Bossini, & Giraux, 1993; Wood, Willmes, Nuerk, & Fischer, 2008). Others, however, suggest that a domain-general system for processing spatial information provides the fundament for the spatial representation of numbers and quantities (Henik, Leibovich, Naparstek, Diesendruck, & Rubinsten; 2011). In addition, several studies demonstrate that culturally acquired skills such as counting help children to order their knowledge about quantities, which eventually gives rise to the development of mapping skills (Helmreich et al., 2011; Noël & Rouselle, 2011; Opfer & Furlong, 2011).

The present dissertation contributes to this debate by examining the longitudinal relations between mapping skills and domain-specific and domain-general precursors. The results of the different studies provide evidence consistent with the presupposition that mapping skills are not founded on an innate domain-specific system for ordering of quantities. Rather, the results suggest that mapping skills arise from the interplay of a domain-general ability for processing visual-spatial information and acquired symbolic skills that result from cultural learning, more specifically from-learning the verbal counting sequence. This conclusion is addressed in more detail in this discussion. At the end of this chapter, suggestions for future research and practical implications will be discussed.

Domain-specific precursors

In this dissertation first the role of domain-specific numerical skills in the development of mapping skills was examined. Following the triple-code-model proposed by Dehaene (1992; Dehaene & Cohen, 1995), we investigated the relationships between non-symbolic quantity skills, symbolic number skills and number-to-quantity mapping. For each of these concepts, multiple tasks were used to create valid and reliable measures. Moreover, the children involved in the studies were at an age crucial for developing increasingly accurate mapping skills. The first measurements were taken shortly after the children were introduced to kindergarten, at age four in the Dutch education system. They had little or no experience with explicit number-to-quantity mapping tasks as used in (pre)schools. However, based on previous research, they were expected to develop number-to-quantity mappings with increasing accuracy in the years from kindergarten to first grade. Applying a longitudinal design allowed us to examine the development of number-to-quantity mapping.

Based on previous research two hypotheses were formulated. The first hypothesis stated that non-symbolic quantity skills provide the base for development of mapping skills. This hypothesis was derived from the theory that the innate non-symbolic quantity system has a spatial format with spatially ordered positions to which symbolic numbers become connected in the course of development resulting in a spatial ordering of numbers on a so-called ‘mental number line’, representing the distances between numbers and the quantities numbers denote (Dehaene et al., 1993; Fischer, Castel, Dodd, & Pratt, 2003; Gevers, Lammertyn, Notebaert, Verguts, & Fias, 2006). The second hypothesis stated that symbolic counting skills facilitate mapping development. This hypothesis was based on the idea that the spatial ordering of numbers and quantities is facilitated by the culture-based ordering of the number words of the counting sequence (Helmreich et al., 2011; Opfer & Furlong, 2011). Information about quantities is connected to this ordering resulting in a spatial ordering of numbers and quantities in which count words ‘calibrate’ quantities. In the study reported in Chapter 2 these hypotheses were explicitly tested using a design in which non-symbolic skills, symbolic skills and mapping skills were assessed at three consecutive measurement occasions in kindergarten year 1, kindergarten year 2 and first grade. Analyses of a cross-lagged panel model showed that symbolic skills had an effect on non-symbolic skills (but not vice versa) and on mapping skills. Non-symbolic skills did not predict mapping skills. Instead of confirming the well-established hypothesis that non-symbolic skills set the stage for mapping development (Dehaene, 2001), the results of this study provided evidence for the alternative hypothesis that symbolic skills calibrate non-symbolic skills as proposed by former research emphasizing the role of symbolic skills in math development (e.g. De Smedt & Gilmore, 2011; LeFevre et al., 2010). Symbolic skills such as being able to recite the counting sequence thus might help children to acquire the cardinal value of a number word, to accurately assess the differences between numbers and to discover that number words refer to exact quantities which in turn leads to a connection between number symbols and non-symbolic quantity values (Noël & Rouselle, 2011).

In a second study, reported in Chapter 5, these results were replicated in a different sample. Multivariate growth models demonstrated that non-symbolic quantity skills were not related to number-to-quantity mapping, whereas relations were found between symbolic skills and mapping. Findings from these two studies thus suggest that non-symbolic quantity skills are not as important as has been put forward by previous studies. The results presented in this dissertation rather emphasize the importance of culturally acquired counting skills.

An explanation of these results can be found in the function of numbers in our culture and school system. Exact calculation is in our culture an important skill that children need to have mastered by the time they start in first grade. It is thus important for children to learn to associate exact quantities to numbers. The approximate characteristics of the proposed innate system for processing non-symbolic quantities are, as such, simply insufficient for developing skills for processing exact quantities needed in math performance (Carey, 2001). Because symbolic numbers provide tools to assess exact quantities, symbolic number skills are considered to play a dominant role in the development of accurate number-to-quantity mappings needed for math performance. Number words enable children to construct a generalized concept of number that involves relations between relations: relations between numbers are linked with relations between objects (Wiese, 2003). This idea is supported by previous studies demonstrating that the spatial representation of numbers and quantities is indeed facilitated by acquiring number words and counting skills (Helmreich et al., 2011; Opfer & Furlong, 2011). In line with this, previous research emphasizing the role of symbolic skills in math development has also shown that symbolic skills are important precursors of math learning (e.g. De Smedt & Gilmore, 2011; LeFevre et al., 2010). In younger children, symbolic skills such as reciting the counting sequence can help children to understand the cardinal value of a number word and to discover that number words refer to exact quantities which in turn lead to a connection between number symbols and non-symbolic quantity values (Noël & Rouselle, 2011).

Thus, considering the domain-specific precursors of mapping development, the results of this dissertation provide evidence for the idea that symbolic skills are more important than non-symbolic quantity skills for constructing connections between numbers and quantities. But how should we interpret these findings in the light of the non-symbolic quantity skills that are proven to be present in infants? Previous studies have demonstrated that preverbal infants use spatial representations when presented with, for example, quantity comparison tasks (De Hevia & Spelke, 2010; Opfer & Thompson, 2006). Although these findings are commonly explained within the framework of an innate mental number line (e.g. Dehaene, 2001), the domain-general working memory skills that were investigated in this dissertation might provide an alternative explanation.

Domain-general precursors

To investigate the role of domain-general precursors in development of mapping skills, the working memory model proposed by Baddeley (1996; 2000) was used as a starting point. It was expected that the different components of working memory proposed in this model play different roles in the development of mapping skills.

Considering the involvement of executive functions in the development of mapping skills previous research has indicated that updating is more important than shifting and inhibition since all numerical tasks (including mapping tasks) involve both simultaneous and sequential processes of perceiving, coding, interpreting, and comparing information in different modalities (e.g. symbolic number information and non-symbolic quantity information; Andersson, 2008; Bull & Scerif, 2001; St.Clair-Thompson & Gathercole, 2006; Van der Ven, Kroesbergen, Boom, & Leseman, 2012). Considering the role of shifting in relation to inhibition, previous results are less straightforward. An experimental study reported in Chapter 3 examined to what extent executive functions explained variance in performance on mapping tasks. Bayesian statistical analyses showed that updating was a more important predictor of individual differences in mapping skills than shifting and inhibition. Moreover, children with better updating skills appeared to profit more from the training of mapping skills. These results can be explained in the perspective of on-task involvement of executive functions. Performing a mapping task, requires strong involvement of updating by both simultaneous and sequential processes of perceiving, coding, interpreting, and comparing information in different modalities. Therefore, a possible explanation for the importance of updating during training of mapping skills is that children with better updating skills were better learners, who acquired more accurate mapping skills with the same amount of training than peers with worse updating skills. Another possible explanation, however, is that updating skills are involved because they facilitate the spatial ordering of numbers and quantities that is needed in mapping tasks. Updating skills are needed to construct this spatial ordering for accurate task performance since the ordering of information needs to be adapted to the information presented in the task at hand. This idea was further explored in the studies reported in Chapter 4 and Chapter 5 in which we focused on the updating component of the working memory model. In addition, we also examined the role of the storage components of working memory in mapping development.

In the study of Chapter 4 we examined the involvement of the different components of the working memory model of Baddeley in the storing and processing of information. In this longitudinal study, mapping skills were assessed at three measurement times, allowing modeling of the latent growth of mapping skills. The working memory components phonological loop (PL), visual-spatial sketchpad (VSSP) and central executive (CE) were measured at two time points. Valid and reliable measures of the PL, VSSP and CE were obtained by pooling scores across both measurement times. In this study it was investigated to what extent the different components of working memory could predict the *development* of mapping skills. Evidence was found for the hypothesis that working memory, predicts the development of number-quantity mapping skills. Moreover, the different components of working memory appeared to be differentially related to mapping development. Whereas the PL was not related to the development of mapping skills, the VSSP and CE predicted variance in both the intercept and slope of number-to-quantity mapping.

These findings indicate that working memory (more specifically the visual-spatial components of working memory) facilitates the construction of connections between numbers and quantities. As proposed by Van Dijck and Fias (2011) positions of items in a sequence are in working memory associated with positions in a spatial representation (e.g., number line): items at the beginning of a sequence are associated to the left side of space whereas numbers and the end of a sequence are associated with the right side of space. These temporary position-space connections provide the foundation upon which number-to-quantity connections are built. The idea is closely related to the proposal of Henik et al. (2011), who suggest that general spatial skills involved in processing non-countable dimensions like distance and height facilitate the spatial ordering of number-quantity connections. Consistent with the presupposition that spatial skills are important facilitators of the spatial ordering of numbers and quantities, an implication is that general spatial working memory skills, rather than a specialized cognitive system for quantity processing underlie performance on tasks measuring mapping skills. This hypothesis was further investigated in Chapter 5.

In Chapter 5, we introduced the concept of a spatial ‘workspace’ which facilitates the processing of spatially ordered information in general. We presupposed that this workspace is modality specific, that is, it is dedicated to processing of visual-spatial information. Clearly, the idea of such a workspace is strongly related to current concepts of (spatial) working memory that define (spatial) working memory as the activated part of (spatial) memory (Cowan, 2010). Therefore, frequently used measures of visual-spatial working memory are likely valid indicators of the spatial workspace. We further assumed that this workspace is involved in the processing of information about quantities. We suggested that it is this general workspace (indicated by visual-spatial working memory measures), rather than a specific system for processing quantity information only (as proposed by Dehaene, 2001), that facilitates development of mapping skills.

In addition, it was argued that symbolic skills are also important in the development of mapping. Since mapping involves numbers, it requires the processing of verbal information. It was presupposed that verbal working memory is involved in learning the verbal counting sequence and in processing verbal (counting) information during task performance which promotes the development of mapping skills. To test these ideas we examined the co-development of visual-spatial working memory, verbal working memory, non-symbolic quantity skills, symbolic number skills and mapping. We formulated the following hypotheses: 1) spatial working memory facilitates the development of both non-symbolic quantity skills and mapping skills, 2) the development of mapping skills is not related to the development of non-symbolic quantity skills and 3) verbal working memory facilitates the development of verbal counting skills which in turn contribute to development of mapping. Multivariate growth modeling was applied to test these hypotheses. The results provided evidence for the first hypothesis by showing that spatial working memory predicted the growth of non-symbolic quantity skills and mapping skills, and was also related to the intercept of these skills. The same analyses, but including verbal instead of spatial working memory yielded a bad fitting model that had to be rejected. We, therefore, concluded that it is specifically the processing of spatial information, and not processing skills in general, that is related to the development of non-symbolic skills and mapping skills. Results were found in favor of the second hypothesis revealing the absence of developmental relations between mapping skills and non-symbolic skills. The results also provided evidence for the third hypothesis showing that verbal working memory predicted the growth of symbolic counting skills. The results of Chapter 5, thus, support the notion of a domain-general workspace which facilitates the development of mapping skills. Moreover it was found that this workspace is modality specific: spatial associations between numbers and quantities were facilitated by spatial working memory but not by verbal working memory. Furthermore, the visual-spatial workspace was found to be related to both mapping skills and non-symbolic skills providing evidence for the idea that the basis of numerical development needs to be sought in domain-general skills for visual-spatial processing. In the next section of this chapter we integrate these findings with the results of Chapters 2, 3 and 4 to construct a theory on mapping development.

Development of mapping skills

The findings of this dissertation suggest that both establishing connections between symbolic numbers and the quantities they denote and establishing connections between non-symbolic stimulus configurations and the quantities they contain in a specific spatial format, i.e., the 'mental number line', arise from a general ability to process spatial information. Based on findings with adults that position-space associations are built in working memory (Van Dijck & Fias, 2011), we argue that these position-space associations in working memory enable children to order numerical (and non-numerical) information in a spatial way, for example on a mental number line. Working memory, according to this view, indeed serves as a workspace for spatially ordering information about numbers and quantities. The differential results found for the different components of working memory revealed that, in order to establish a spatial ordering, modality specific spatial processing is involved. In combination with the finding that non-symbolic quantity skills were not related to number-to-quantity mapping, we concluded that spatial processing provides the basis for numerical learning. In contrast to the common idea that a highly specialized non-symbolic quantity system provides the basis for number-quantity mapping (Dehaene, 2001), this thesis supports the alternative account that number-to-quantity mapping results from the cooperation of a general system for processing spatial information and acquired symbolic skills, in particular the ability to use the verbal counting sequence. In line with this, Núñez (2011) reviews evidence showing that the mental number line is not based in a pre-existing, biologically wired-in cognitive system that specifically underlies further numerical development, but rather emerges as a product of mechanisms that are based in cultural and educational practices. By approaching math problems such as, for instance, 7-3, in education as if it were movements along a path, the conceptual metaphor of a number line emerges. In this view, the mental number line is based in sensorimotor cognition (part of what was earlier called the spatial workspace) and gets the function of representing numbers in a spatial ordering as a consequence of particular cultural learning practice with math. The alternative account, outlined here, is consistent with the findings of several previous studies demonstrating a relation between working memory and mapping (e.g. Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; Holloway & Ansari, 2008; Krajewski & Schneider, 2009) and with previous studies showing relations between spatial working memory and mapping skills (Chapter 4; Krajewski & Schneider, 2009). Moreover, the alternative account can also explain findings of previous studies in which no relation between non-symbolic skills and mapping skills have been found (Chapter 2).

The findings reported in this dissertation also suggest that symbolic skills, in contrast to non-symbolic skills, *are* involved in the development of number-to-quantity mapping. Moreover, these symbolic skills, in particular counting, require involvement of verbal working memory, both in learning to count and in applying counting to particular tasks. It is not surprising that the learning of the counting sequence, which is a verbal task, is related to verbal working memory. However, it is interesting to speculate about how these symbolic skills contribute to the development of mapping in cooperation with a system for ordering and representing information in a spatial format. As has been demonstrated in previous studies, culture-based skills influence the spatial ordering of numbers and quantities. Skills such as being able to recite the counting sequence and our left-to-right reading habits influence the construction of a spatially ordered representation of the number-to- quantity relationships. Indeed, studies in countries with a right-left reading system do show the SNARC effect and other phenomena relating to the spatial representation of number-to-quantity, but in the opposite direction, consistent with the reading system (Shaki, Fischer, & Petrusic, 2009).It might be that symbolic cultural skills, in interaction with a system for spatial processing, facilitate children to construct connections between numbers and quantities. To construct these connections, symbolic knowledge about the counting sequence (a child learns that the number 4 precedes the number 5) provides knowledge about the ordering of the number symbols but does not in itself contain spatial information about the distances between numbers. In a spatial workspace, position-space connection provide the ordering of numbers with spatial information about the distances between these numbers which will eventually result in connections between numbers and quantities. The spatial workspace might thus play a key-role in the spatial ordering of numbers and quantities.. Indeed Bachot, Gevers, Fias, and Roeyers (2005) demonstrated that visual-spatial WM is important in constructing number-to-quantity relations as they showed that there was no SNARC effect in children with visual-spatial disabilities.

To summarize, it can be argued that children thus make use of the inherent ordinal structure of the number system to systematically map numbers as a function of their quantity (Van Dijck & Fias, 2011). A workspace that facilitates the construction of a spatial ordering of numbers and quantities, thus might underlie two types of skills: symbolic skills connected to an exact quantity and approximate non-symbolic quantity skills (e.g., Lyons, Ansari, & Beilock, 2012; Noël & Rouselle, 2011). Although these skills arise from a general ability to spatially process and order numerical information (whether this is symbolic or non-symbolic), only the exact skills might be important for math performance in the first grades of primary school.

Thus, for the development of mapping skills we propose that there is an interplay between general abilities for visual-spatial processing on the one hand and symbolic counting skills on the other hand. Spatial abilities aid children to understand the cardinality principle of the numbers in the counting sequence by linking the count words to a spatial representation that matches with the ordered structure of the counting sequence, but adds a quantity representation that is based in sensorimotor cognition. The findings reported in this dissertation lead to a reconsideration of the theoretical model proposed in the introduction of this dissertation (see Figure 6.1).

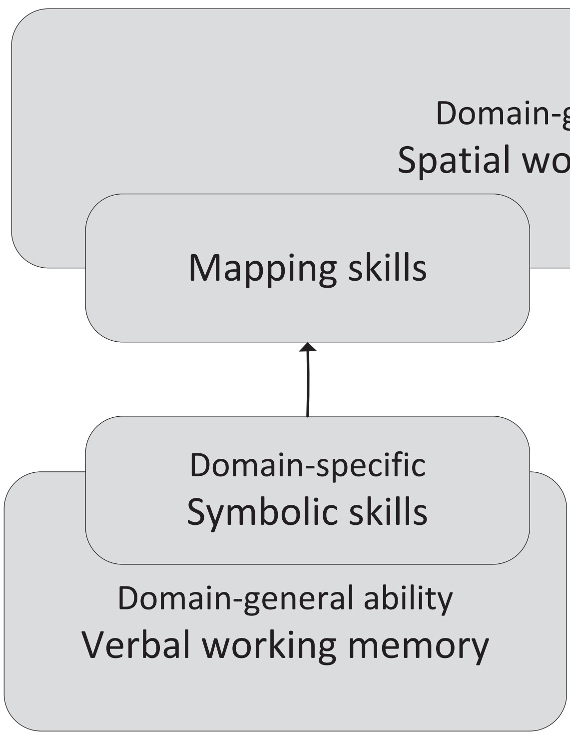

Figure 6.1
Reconsideration of the theoretical model proposed in the introduction of this dissertation

6

Practical implications

Although the main aim of this dissertation was to contribute to theory development, there are implications for practice as well.

The main finding of this dissertation concerns the importance of the cooperation of symbolic number skills (e.g. verbal counting) and visual-spatial working memory in the development of increasingly accurate number-to-quantity mapping. Although the studies reported in this dissertation examined the development of mapping in typically developing kindergartners, the results suggest that children with poor symbolic number skills or poor visual working memory might be at risk for developing math learning difficulties. The findings of this dissertation can inform diagnostic procedures for early identification of these at-risk children and can provide insight in the processes that cause these math learning difficulties. Training symbolic number skills is a promising way of assisting these at-risk children to prevent serious math learning problems. Previous studies have shown that training these skills in young children is effective (e.g. Jordan, Glutting, Dyson, Hassinger-Das, & Irwin, 2012) and can help them to develop accurate mapping skills (e.g. Ramani & Siegler, 2008; Wilson, Dehaene, Dubois, & Fayol, 2009).

Another focus of intervention for children at-risk for math learning difficulties is working memory itself. Although the effects of working memory training are questioned (see for a review Hulme & Melby-Lervåg, 2012), some studies indeed demonstrated the effectiveness of working memory training in children with poor working memory and also demonstrated generalization of working memory training to math learning (Holmes, Gathercole, & Dunning, 2009; Kroesbergen, Van 't Noordende, & Kolkman, 2012).

The findings reported in this dissertation contribute to further understanding of the nature and the causes of the problems children with dyscalculia encounter. It has been repeatedly demonstrated that the number-to-quantity mapping skills of dyscalculic children are much less accurate than those of their typically achieving peers (Landerl & Kölle, 2009;Geary, Hoard, Nugent, Byrd-Craven, 2008). The results of this dissertation suggest that problems with processing spatial information or from a lack of ability to learn symbolic number skills might underlie the inaccurate performance of children with dyscalculia. When the verbal ordering of the count sequence is hard to understand, it becomes difficult to grasp the this ordinal structure of numbers. Moreover, problems with processing spatial information might lead to problems in learning the spatial ordering of numbers and quantities which will in turn result in a lack of knowledge about the meaning of number symbols. This needs to be further addressed in future research.

Directions for future research

The results of this dissertation give rise to a number of potentially interesting questions for future research. First, most of the current research into number-to-quantity mapping and its precursors is correlational. Although we moved a step forwards by applying a longitudinal design, future research is needed to provide stronger evidence for the presumed causal relations. Experimental studies in which, for example, symbolic number skills and non-symbolic quantity skills are trained, can provide insight in the causal relations between these domain-specific skills and mapping performance. Based on the present results we would expect that training symbolic skills is effective in improving mapping skills and training non-symbolic skills is not. Another design that can be used to examine causal relations is a dual-task design. In this type of design, part of the participants are required to perform two tasks simultaneously and their performance is compared with the performance of participants in single-task conditions. For example, performance on a mapping task combined with a task placing a load on (spatial) working memory can be compared with performance on a regular mapping task to investigate whether (spatial) working memory is needed for accurate mapping. This design is used in adults to examine relations between working memory and mapping performance (e.g. Van Dijck & Fias, 2011). In children, however, this type of study is rare. Although two studies did apply a dual-task paradigm to study the role of working memory in simple arithmetic (e.g. addition problems; McKenzie, Bull, & Geary, 2003) and in strategy use (Imbo & Vandierendonck, 2007), a similar study relating working memory to mapping is lacking. Based on the present results we would expect that dual-task studies in children would reveal that a load on working memory interferes with performance on mapping tasks.

A second recommendation for future research is to develop and implement more sensitive measure of non-symbolic quantity skills. As has been demonstrated by previous studies, it is important to take into account the numerical distances between the quantities that are presented as stimuli (Holloway & Ansari, 2009). When the distance between numbers is small (e.g. 2 and 3) numerical positions are harder to distinguish and processing these numbers is more difficult than when the distance between numbers is larger (e.g. 2 vs. 13; Dehaene et al., 1993; Fischer, Castel, Dodd, & Pratt, 2003; Gevers et al., 2006). Examination of this so-called distance effect would provide insight in the spatial ordering of quantities in a more direct way: the presence of a distance effect in young children on a non-symbolic quantity task might indicate that these quantities are indeed spatially ordered. Moreover, it is recommended to include measures of reaction times as these will provide a more sensitive measure of individual differences on a comparison task (whether this task is non-symbolic or symbolic). New technologies such a touch-screen devices support reliable measurement of reaction times in young children.

Third, it would be interesting to examine the contribution of non-symbolic skills, symbolic skills and working memory skills to the performance on mapping tasks in other age groups and to the performance on more complex math tasks. Although this dissertation sheds light on the precursors of mapping skills in an early stage of the development of mapping skills, little is known about the role these precursors play in, for example, adult mapping performance. An assumption that can be addressed in future research is that in adults the spatial ordering of numbers and quantities has become a stable system that is no longer ‘under construction’, which, therefore, will no longer reveal individual differences that relate to working memory and counting skills in normal task-performance. . It would also be interesting to examine the performance of children in a broader age range to further explore the predictive value of mapping to math performance but more importantly, the predictive value of the precursors identified in this study to performance on mapping tasks in older children. This could also help to identify the characteristics of children at risk for developing math learning problems or dyscalculia.

Based on the model that is introduced in this Chapter, we end with two final recommendations. According to the model, the spatial ordering of quantities and numbers arises from a shared ability to order spatially presented information. For the ordering of symbolic information it is suggested that skills such as counting and number naming, involving verbal working memory, are an important contribution. For the ordering of non-symbolic quantity information there might also be contributors which, however, were not taken into account in this study. General properties of the human perception systems are probably involved in the direct perception of quantity information when presented with a particular configuration of stimuli (e.g. dots), or to be more precise, the perception of purely physical characteristics such as amount of surface covered, contour length and configuration density. Although especially the findings with infants have been taken as evidence for the existence of a special biologically wired-in system for quantity processing, discrimination between sets of stimuli as such does not prove that infants and other subjects use approximate stimulus-to-quantity mapping to perform the task. Recent evidence indicates that infants may not be attending to the number of objects in such tasks, but to continuous characteristics like contour length (Ansari & Karmiloff-Smith, 2002). This is supported by findings that infants were able to judge the amount of liquid that was added to a glass cylinder, indicating a basic sensitivity to amount, whether it is amount of liquid, contour, or dots (Clearfield & Mix, 2001; Newcombe, 2002). An alternative explanation for non-symbolic quantity skills is thus that performance reflects the operation of early domain-general perceptual abilities. This explanation needs to be further addressed in future research.

Conclusion

To end, two main conclusions can be drawn. The first conclusion is that non-symbolic quantity skills assumed to be based on a spatial ordering of quantity information are not the main starting point for the development of mapping skills. The development of mapping skills is rather based on general skills for processing visual-spatial information in cooperation with symbolic number skills. The second conclusion is that this general system for processing spatial ordered information (that by definition is not exclusively about number or quantity information) is the fundament upon which spatial orderings of quantities and numbers are built. In interaction with culture based symbolic counting skills, this system facilitates the development of mapping skills.

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Samenvatting
(Summary in Dutch)

Introductie

Een belangrijke voorwaarde om te kunnen leren rekenen is het begrijpen van de koppeling tussen cijfersymbolen en hoeveelheden. Kinderen moeten leren dat het cijfer '3' gelijk is aan het telwoord 'drie' en dat beide gelijk zijn aan de hoeveelheid '***'. De koppeling tussen cijfersymbolen en hoeveelheden wordt in dit proefschrift aangeduid met de term 'mapping'. Uit verschillende onderzoeken is gebleken dat de kennis over cijfersymbolen en hoeveelheden ruimtelijk geordend is. Zo is bijvoorbeeld vaak aangetoond dat cijfers en hoeveelheden met een kleine numerieke waarde gerelateerd zijn aan links terwijl cijfers en hoeveelheden met een grotere numerieke waarde gerelateerd zijn aan rechts. Hoewel wordt aangenomen dat deze ruimtelijk ordening van cijfers en hoeveelheden ten grondslag ligt aan de koppeling tussen cijfers en hoeveelheden, is nog weinig bekend over het ontstaan van die ruimtelijke ordening.

In de afgelopen jaren zijn hierover verschillende theorieën geformuleerd. De meest invloedrijke theorie gaat uit van het idee dat de ruimtelijke ordening van cijfers en hoeveelheden voortkomt uit een aangeboren systeem speciaal uitgerust voor het verwerken van hoeveelheden. Dit domein-specifieke systeem voorziet in een ruimtelijke ordening van hoeveelheden waaraan uiteindelijk de cijfersymbolen worden gekoppeld. De basis voor mapping ligt volgens deze theorie dus bij een *aangeboren* domein-specifiek systeem voor het verwerken van hoeveelheden. Twee andere theorieën gaan hier echter tegenin. De eerste theorie benadrukt het belang van cultureel bepaalde vaardigheden zoals tellen. Volgens deze theorie ontstaat er een ruimtelijke ordening van cijfers en hoeveelheden doordat kinderen de telwoorden van de telrij leren kennen. De cultureel bepaalde ordinale kenmerken van deze telrij zorgen ervoor dat kinderen uiteindelijk leren dat een bepaald telwoord (of cijfer-symbool) gekoppeld is aan een bepaalde hoeveelheid. De basis voor mapping ligt volgens deze theorie dus in een *aangeleerde* vaardigheid, namelijk het leren van de telrij. Dan is er nog een andere theorie die ingaat tegen het idee dat mapping gebaseerd is op aangeboren vaardigheden. Volgens deze derde theorie speelt het werkgeheugen een belangrijke rol in het ontstaan van de ruimtelijke ordening van cijfers en hoeveelheden. Het concept werkgeheugen verwijst naar het systeem voor het opslaan en verwerken van visueel ruimtelijke en verbale informatie. Verondersteld wordt dat er binnen het werkgeheugen verschillende componenten zijn voor het opslaan van visueel ruimtelijke versus verbale informatie: het visueel ruimtelijke schetsblok versus de fonologische lus. Werkgeheugen zou volgens de derde theorie een 'workspace' kunnen bieden voor het ruimtelijk ordenen van numerieke informatie (maar ook voor niet numerieke informatie) waarbij een koppeling gemaakt wordt tussen positie en ruimte. Dat wil zeggen: items aan het begin van een reeks informatie worden geassocieerd met links terwijl items aan het einde van een reeks geassocieerd worden met rechts. Deze positie-ruimte koppelingen zouden volgens de derde theorie wel eens de basis kunnen vormen voor de cijfer-hoeveelheid koppelingen.

In deze dissertatie werden deze drie verschillende verklaringen voor het ontstaan van ruimtelijke ordening van cijfers en hoeveelheden onderzocht. Om een goed beeld te kunnen krijgen van de ontwikkeling van de ruimtelijk georiënteerde koppeling tussen cijfersymbolen en hoeveelheden, is gekozen voor een steekproef bestaande uit jonge kinderen met een leeftijd waarop wordt aangenomen dat de accuratesse van mapping (en dus van de ruimtelijke ordening tussen cijfers en hoeveelheden) een sterke groei laat zien.

Aangeboren systeem of aangeleerde vaardigheid?

Ten eerste is gekeken naar de domein-specifieke factoren die mogelijk ten grondslag liggen aan mapping. Er werden twee typen domein-specifieke vaardigheden onderzocht: non-symbolische hoeveelheid vaardigheden en vaardigheden gerelateerd aan het begrijpen van cijfersymbolen en telwoorden (symbolische vaardigheden). Er werden twee hypothesen onderzocht. De eerste hypothese was gebaseerd op het idee dat non-symbolische vaardigheden, voortkomend uit een aangeboren ruimtelijk ordening van hoeveelheden, ten grondslag liggen aan mapping. Er werd dan ook verwacht dat juist non-symbolische vaardigheden een belangrijke rol spelen in mapping. De tweede hypothese was gebaseerd op het idee dat symbolische vaardigheden, zoals het kennen van de telrij, kinderen ondersteunen bij het ontwikkelen van het begrip dat cijfersymbolen geordend zijn. De verwachting was dat symbolische vaardigheden een belangrijke rol spelen in mapping. In Hoofdstuk 2 zijn deze hypothesen onderzocht. De resultaten lieten zien dat niet non-symbolische vaardigheden maar juist symbolische vaardigheden belangrijk zijn in mapping en dus belangrijk zijn voor het ontwikkelen van het begrip dat hoeveelheden en cijfersymbolen ruimtelijk geordend zijn. Dit werd bevestigd in Hoofdstuk 5 waarin de longitudinale ontwikkeling van non-symbolische en symbolische vaardigheden werd gerelateerd aan mapping. Uit die analyses bleek dat de ontwikkeling van non-symbolische vaardigheden niet was gerelateerd aan mapping. De ontwikkeling van symbolische vaardigheden was juist wel gerelateerd aan mapping. De uitkomsten in beide hoofdstukken lijken er dus op te wijzen dat het leren van cijfersymbolen belangrijk is voor het ontwikkelen van de koppeling tussen cijfers en hoeveelheden. Een aangeboren systeem voor het verwerken van hoeveelheden lijkt hier geen grote rol in te spelen. Deze resultaten kunnen verklaard worden vanuit de functie die cijfersymbolen hebben in onze cultuur. Exacte rekenvaardigheid is een belangrijke vaardigheid die kinderen in onze cultuur moeten leren beheersen. Het is daarom belangrijk dat zij exacte kennis ontwikkelen over de hoeveelheden die gekoppeld zijn aan de cijfersymbolen. De ontwikkeling van deze kennis wordt gefaciliteerd door exacte woorden en symbolen voor cijfers, oftewel symbolische vaardigheden faciliteren het ontstaan van exact begrip van cijfersymbolen. De non-symbolische vaardigheden, waarvan wordt aangenomen dat ze ruimtelijk geordend zijn, zijn simpelweg niet toereikend om exacte berekeningen op te kunnen baseren.

De rol van werkgeheugen

Ten tweede is gekeken naar de domein-algemene werkgeheugen vaardigheden die mogelijk een rol spelen in de ontwikkeling van mapping. Daarbij hebben we allereerst gekeken naar de invloed van de verschillende executieve functies updating (gelijktijdig opslaan en verwerken van informatie), shifting (wisselen van strategie) en inhibitie (negeren van irrelevante stimuli) op mapping. In de experimentele studie gepresenteerd in Hoofdstuk 3 hebben we onderzocht wat de rol was van executieve functies tijdens het verbeteren van mapping door middel van een training. De resultaten lieten zien dat updating belangrijker was in mapping dan shifting en inhibitie. Niet alleen scoorden kinderen met betere updating vaardigheden hoger op de voor- en nameting maar zij lieten ook tijdens de training meer groei zijn. Dit zou verklaard kunnen worden vanuit het idee dat het verwerken van informatie over hoeveelheden en cijfersymbolen (wat centraal staat in mapping taken) een beroep doet op het gelijktijdig opslaan en verwerken van informatie in verschillende modaliteiten (telwoorden, cijfersymbolen en hoeveelheden). Dit proces lijkt gefaciliteerd te worden door updating. Een andere verklaring voor deze resultaten is dat de ruimtelijke ordening die ten grondslag ligt aan mapping gefaciliteerd wordt door updating. Updating zou nodig kunnen zijn om die ruimtelijke ordening te kunnen construeren op basis van inkomende en bestaande informatie met betrekking tot de ordening van cijfers en hoeveelheden. Dit idee was het uitgangspunt voor de studies die gepresenteerd zijn in Hoofdstuk 4 en Hoofdstuk 5.

Er werden in Hoofdstuk 4 drie verschillende werkgeheugen componenten onderzocht: de fonologische lus (opslag van verbale informatie), het visueel-ruimtelijk schetsblok (opslag van visuele informatie) en de central executive (verwerken van verbale en visuele informatie). De resultaten lieten zien dat het visueel-ruimtelijk schetsblok (en niet de fonologische lus) gerelateerd was aan de ontwikkeling van mapping. Daarnaast werd ook gevonden dat de central executive gerelateerd was aan de ontwikkeling van mapping. Deze resultaten impliceren dat het werkgeheugen (en meer specifiek de visueel-ruimtelijke componenten van het werkgeheugen) het ontstaan van connecties tussen cijfers en hoeveelheden faciliteren. Het zou zo kunnen zijn dat de posities van items in een reeks in het werkgeheugen worden gerelateerd aan ruimte: items aan het begin van de reeks worden geassocieerd met links en items aan het einde van de reeks worden geassocieerd met rechts. Deze tijdelijke connecties tussen posities en ruimte zouden de basis kunnen vormen voor het ontwikkelen van connecties tussen cijfers en hoeveelheden. Dit impliceert dat het met name visueel-ruimtelijke (en niet verbale) werkgeheugen vaardigheden zijn die een rol spelen in het ontstaan van mapping. Dit idee is verder onderzocht in Hoofdstuk 5.

In Hoofdstuk 5 werd het concept van een ruimtelijke ‘workspace’ geïntroduceerd. Deze workspace is verondersteld gespecialiseerd te zijn in het verwerken van ruimtelijk geordende informatie. Daarmee ligt dit workspace concept dicht aan tegen het idee van een visueel ruimtelijk werkgeheugen. De hypothese die in Hoofdstuk 5 getest werd, was dat visueel ruimtelijk werkgeheugen ten grondslag ligt aan de ruimtelijke ordening van zowel hoeveelheden als symbolische vaardigheden. Het werd verwacht dat juist deze domein algemene werkgeheugen vaardigheden (en dus niet een domein specifiek systeem voor het verwerken van hoeveelheden) de ontwikkeling van mapping faciliteren. Daarnaast werd verwacht dat het verbale werkgeheugen juist een rol speelt in de ontwikkeling van symbolische vaardigheden die op hun beurt weer van belang zijn voor de ontwikkeling van mapping. De resultaten lieten zien dat de ontwikkeling van mapping gerelateerd was aan de ontwikkeling van visueel ruimtelijk werkgeheugen. Herhaling van de analyses met een verbale component in plaats van een visueel ruimtelijke component resulteerden in een slecht model. Dit betekent dat het dus niet algemene informatie verwerkingsprocessen zijn die ten grondslag liggen aan mapping, maar juist de specifieke visueel ruimtelijke verwerkingsvaardigheden. Daarnaast lieten de analyses zien dat symbolische vaardigheden gerelateerd zijn aan de ontwikkeling van mapping en dat de ontwikkeling van symbolische vaardigheden gefaciliteerd worden door verbaal werkgeheugen. Deze resultaten leverden bewijs voor het idee dat de basis voor numerieke ontwikkeling gezocht moet worden in domein algemene vaardigheden voor het verwerken van visueel ruimtelijke informatie.

Conclusie

Hoewel er een breed geaccepteerd idee is dat numerieke ontwikkeling ontstaat vanuit een aangeboren domein specifiek systeem voor het verwerken van hoeveelheden, laten de resultaten van dit proefschrift een ander beeld zien. In de verschillende hoofdstukken is bewijs geleverd voor het idee dat de koppeling tussen cijfersymbolen en hoeveelheden het resultaat is van de interactie tussen een domein algemeen systeem voor de verwerking van visueel ruimtelijke informatie en aangeleerde symbolische (tel)vaardigheden. Maar hoe ontstaat vanuit deze interactie een numeriek systeem waarin cijfersymbolen zijn geordend van links naar rechts?

Verschillende studies hebben eerder laten zien dat het ordenen van numerieke informatie van klein naar groot een product is van onze cultuur en nauw verweven is met leesgewoontes. In Arabische landen, bijvoorbeeld, waar de leesrichting tegengesteld is aan de westerse leesrichting, is ook de ordening van cijfersymbolen rechts-links georiënteerd. Deze symbolische cultureel afhankelijke vaardigheden voorzien kinderen van kennis over de ordening van cijfersymbolen (4 gaat voor 5) maar geven niet direct informatie over de ruimtelijke afstanden tussen deze cijfersymbolen. Daarvoor is een visueel ruimtelijke workspace nodig. Doordat reeksen van cijfersymbolen altijd in dezelfde volgorde worden aangeboden en aangeleerd, krijgen deze cijfersymbolen een vaste positie in de ruimte. Deze positie-ruimte koppelingen voorzien uiteindelijk in een koppeling tussen cijfersymbolen en hoeveelheden. Het kan dus worden beargumenteerd dat kinderen gebruik maken van de ordinale structuur van cijfers om cijfers uiteindelijk te ordenen op basis van hun numerieke waarde. Daarbij is een visueel ruimtelijke workspace van belang om de constructie van een ruimtelijke ordening van cijfers en hoeveelheden te faciliteren.

Hoewel dit promotieonderzoek theoretisch van aard was, kunnen er op basis van de resultaten ook praktische aanbevelingen gedaan worden. Daarbij moet wel worden opgemerkt dat het huidige onderzoek zich richtte op de ontwikkeling van numerieke vaardigheden binnen een typisch ontwikkelende steekproef. De resultaten suggereren echter dat kinderen met een zwakke symbolische (tel)vaardigheden of een zwak visueel ruimtelijk werkgeheugen mogelijk een verhoogd risico hebben op het ontwikkelen van rekenproblemen. De resultaten zouden dus gebruikt kunnen worden voor het ontwikkelen van diagnostische procedures voor vroege herkenning van deze risicogroep en zouden bovendien inzicht kunnen bieden in de oorzaken van de rekenproblemen die deze groep ervaart. Het verbeteren van symbolische (tel)vaardigheden en visueel ruimtelijk werkgeheugen lijkt vervolgens een veelbelovende manier om de vaardigheden van kinderen in deze risicogroep te versterken. Toekomstig onderzoek is echter nodig om deze praktische implicaties verder te onderzoeken.

Er kunnen op basis van de resultaten van dit proefschrift twee belangrijke conclusies worden getrokken. Ten eerste lijken de non-symbolische vaardigheden, die worden verondersteld te zijn ontstaan vanuit een ruimtelijke ordening van hoeveelheden, niet het startpunt voor de ontwikkeling van mapping. Ten tweede lijkt het algemene systeem voor het verwerken van visueel ruimtelijke informatie het fundament voor de ontwikkeling van ruimtelijke ordeningen van hoeveelheden en cijfersymbolen. In interactie met cultureel bepaalde telvaardigheden, faciliteert dit systeem de ontwikkeling van mapping vaardigheden. De ontwikkeling van mapping lijkt dus gebaseerd te zijn op een combinatie van algemene vaardigheden die belangrijk zijn bij het verwerken van visueel ruimtelijke informatie en vaardigheden met betrekking tot cijfersymbolen.

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Curriculum vitae

Meijke Kolkman was born on November 25, 1982 in Enschede, the Netherlands. She obtained her high school degree (VWO) in 2001 from 'De Heemgaard' in Apeldoorn. After pursuing a career in performing arts at the 'Theaterschool' in Amsterdam, she began her scientific career at Utrecht University. She obtained a bachelor degree in pedagogics in 2005 and obtained a master degree in education & learning with honours in 2006. After working as a school psychologist (orthopedagoog) and a teacher at the university, she worked as a PhD student from 2008 until 2013 at the faculty of social and behavioral sciences at Utrecht University. During these years she was involved in several bachelor- and master-degree courses as a teacher, she represented her fellow PhD-students in the PhD-counsil of ISED and she spent a month in New York at Colombia University.

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