

Experimental and numerical findings on the long-term evolution of migrating alternate bars in alluvial channels

Alessandra Crosato,^{1,2} Frehiwot Beidmariam Desta,³ John Cornelisse,⁴ Filip Schuurman,⁵ and Wim S. J. Uijttewaai⁶

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[1] Migrating alternate bars form in alluvial channels as a result of morphodynamic instability. Extensive literature can be found on their origin and short-term development, but their long-term evolution has been poorly studied so far. In particular, it is not clear whether migrating bars eventually reach a (dynamic) equilibrium, as in previous studies bars were observed to elongate with time. We studied the long-term evolution of alternate bars by performing two independent long-duration laboratory experiments and some numerical tests with a physics-based depth-averaged model. In a straight flume with constant water flow and sediment recirculation, migrating bars followed a cyclic variation. They became gradually longer and higher for a while, then quickly much shorter and lower. In one case, all migrating bars simultaneously vanished almost completely only to reform soon after. At the same time, steady bars, two to three times as long, progressively developed from upstream, gradually suppressing the migrating bars. We also observed simultaneous vanishing of migrating bars in an annular flume experiment, this time at intervals of 6–8 d. Numerical simulations of long alluvial channels with constant flow rate and fixed banks show periodic vanishing of a few migrating bars at a time, occurring at regular spacing. Under constant flow rates, migrating bars appear as a transition phenomenon of alluvial channels having a cyclic character. These observations, however, might hold only for certain morphodynamics conditions, which should be further investigated.

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1. Introduction

[2] The linear stability analyses carried out in the 1970s showed that a flat alluvial bed may develop into a pattern of alternate or multiple bars, depending on the flow width-to-depth ratio and other morphodynamic parameters [e.g., Hansen, 1967; Callander, 1969; Engelund, 1970; Engelund and Skovgaard, 1973; Parker, 1976; Fredsøe, 1978]. Different bar wavelengths may be unstable, but according to theory the wavelength selected corresponds to the bars with

the largest temporal growth rate. This assumption, however, leads to systematic under-prediction of the bar wavelengths, typically by 30%–40% [Nelson, 1990]. This could be ascribed to the observed elongation of migrating bars with time [e.g., Fujita and Muramoto, 1985], since the wavelengths predicted by linear theories represent only the initial phases of the bar development. Nelson [1990] and Defina [2003] confirmed the experimental observations of Fujita and Muramoto using fully nonlinear physics-based models. They also found that bar growth and migration rates decreased with increasing wavelength, which is in agreement with weakly nonlinear theories [e.g., Tubino and Seminara, 1990]. The question remained, therefore, whether migrating bars eventually reach a (dynamic) equilibrium configuration under constant flow rate. Previous work focusing on the evolution of migrating bar amplitude includes the weakly nonlinear analysis by Colombini *et al.* [1987], which is valid for straight channels with width-to-depth ratios that are close to the critical value for bar instability. This theory allows for computing an “equilibrium” amplitude of alternate bars. Tubino [1991] investigated the effects of varying discharge on bar amplitude and concluded that under unsteady conditions such “equilibrium” is hardly attained. These results were confirmed by Welford’s [1994] field investigation. Schielen *et al.* [1993] extended the weakly nonlinear analysis by Colombini *et al.* [1987] and showed that in straight,

¹Section of Hydraulic Engineering, Faculty of Civil Engineering and Geosciences, Delft University of Technology, Delft, Netherlands.

²UNESCO-IHE, Delft, Netherlands.

³Ethiopian Road Authority, Ethiopia and UNESCO-IHE, Delft, Netherlands.

⁴Department of Marine and Coastal Systems, Deltares, Delft, Netherlands.

⁵Department of Physical Geography, Faculty of Geoscience, Utrecht University, Utrecht, Netherlands.

⁶Section of Fluid Mechanics, Faculty of Civil Engineering and Geosciences, Delft University of Technology, Delft, Netherlands.

Corresponding author: A. Crosato, Section of Hydraulic Engineering, Faculty of Civil Engineering and Geosciences, Delft University of Technology, Stevinweg 1, 2628 CN Delft, Netherlands. (a.crosato@tudelft.nl)

infinitely long channels with constant flow rate and fixed banks the bar amplitude may present fluctuations related to the dispersive properties of (bed) wave groups occurring on long spatial scales.

[3] The effects of point bars on the evolution of migrating bars was studied experimentally by *Kinoshita and Miwa* [1974] and theoretically by *Tubino and Seminara* [1990], with the conclusion that point bars in sinuous channels may have the effects of suppressing alternate bars. *Repetto and Tubino* [1999] extended the analysis to sinuous channels with variable width. The experiments by *Struiksmā and Crosato* [1989] and *Lanzoni* [2000a, 2000b] showed that the formation of alternate steady bars in forced systems has the effect of damping migrating bars. *Crosato et al.* [2011] showed that steady alternate bars may develop, on the long term, also in systems without geometric external forcing, also suggesting that migrating bars may be eventually damped in straight systems with uniform inflow.

[4] Here we investigate the long-term evolution of migrating alternate bars both experimentally and numerically. We performed two independent long-duration laboratory experiments, the first one in a straight flume with sand bed and sediment recirculation, the second one in an annular flume with a sand bed. In the straight flume we performed two tests [*Crosato et al.*, 2010, 2011]: the first one was characterized by the presence of a transverse plate obstructing part of the flow at the upstream boundary. This plate created a strong permanent flow perturbation leading to a typical “forced system” [*Struiksmā and Crosato*, 1989; *Lanzoni*, 2000a and 2000b]. The second test was characterized by uniform inflow (a system without a transverse plate). Systems without permanent external forcing are traditionally known as “free systems,” although imposing uniform flow at the upstream boundary should be regarded as external forcing too.

[5] The bed topography of the system with a plate was soon dominated by the presence of steady alternate bars. Migrating bars formed only in the second half of the channel. In contrast, the bed topography of the system without a plate was initially dominated by migrating bars. In this case, however, slowly growing steady bars, characterized by twice as large wavelengths and smaller amplitude, formed too, but became appreciable only after 3 weeks. We analyzed the consequences of steady bar formation in systems with uniform inflow in the work of *Crosato et al.* [2011], whereas here we focus on the evolution of migrating bars and on the interactions between steady and migrating bars. In both tests, migrating bars followed a cyclic variation. They became gradually longer and higher for a while, then quickly much shorter and lower. In one case, all bars simultaneously vanished almost completely only to reform soon after. The steady bars, in contrast, never vanished.

[6] The periodic vanishing of bars was also observed in a 42-d-long experimental test carried out in a circular rotating flume at the Delft University of Technology 30 yr ago, but these observation remained unpublished [*Cornelisse*, 1981]. Finally, we performed numerical investigations using the physics-based morphodynamic model Delft3D (available at <http://www.deltares.nl>), which is known to reproduce well river bar development and behavior [e.g., *Mosselman et al.*, 2003; *Crosato and Desti*, 2009; *Crosato et al.*, 2011]. The results show a cyclic vanishing of a few consecutive bars occurring at regular spacing.

2. Long-Duration Straight Flume Experiment

2.1. Experimental Setup

[7] The experiments consisted of recording the long-term water flow and channel bed evolution in a straight flume with a movable bed, fixed banks, and constant discharge. The total length of the flume was 26 m, the channel width was 60 cm. The bed was covered by an initially smooth layer of sand having a thickness of 25 cm. The median diameter of the sediment, D_{50} , was 0.238 mm. Water and sediment were recirculated, but some water was added regularly, to compensate for small losses due to evaporation. The discharge was kept constant at 6.9 L s^{-1} .

[8] A wire mesh was introduced at the upstream boundary to dissipate the excess energy of the incoming water and to distribute the flow uniformly. A honeycomb flow straightener was placed immediately downstream from it, followed by a floating sponge to further diminish large scale fluctuations and to suppress surface waves.

[9] Longitudinal profiles of the bed and water levels were measured three times per day, at least three times per week. The water and bed levels were measured with a laser sensor mounted on a carriage cart at three different points across the cross section, 5 cm from both the left and right sidewalls and at the channel centerline. Transverse velocity profiles were measured across three sections at intervals of 5 and 10 cm in a transverse direction, using a calibrated electromagnetic liquid velocity meter (EMS) with a 30 mm E-type probe.

[10] Because of the presence of relatively large dunes or ripples (Figure 1, left plate), the raw data had to be filtered to clean out the bar signal. The filter used was based on the MATLAB software ProcessV3 and optimized for bedforms having a wavelength $>1 \text{ m}$ (bars). Figure 2 shows an example of both raw and filtered profiles. Migrating bars were studied by plotting subsequent filtered bed level profiles, which allowed estimating their wavelength, amplitude, and celerity. The values of these variables, which were used to study the development of bars over time, were derived by averaging the characteristics of all of the bars that were simultaneously present in the second half of the flume. Nonmigrating bars were identified by averaging the filtered bed-level profiles over time, which smoothed out most unsteady signals. Turbulence and perturbation smoothing at the upstream boundary and the adopted measuring and postprocessing techniques reduced the effective length of the channel to $\sim 20 \text{ m}$.

[11] Two experimental tests were carried out. In the first one, a transverse plate was placed at the upstream boundary to create a permanent external flow perturbation (Figure 1, central plate). This test was meant to study the formation of steady and migrating bars in “forced systems” as in previous experiments [e.g., *Struiksmā and Crosato*, 1989; *Lanzoni*, 2000a, 2000b]. In the second test, the transverse plate was removed and the initial bed carefully smoothed out to eliminate all perturbations to the flow (Figure 1, right plate). This test was meant to study the evolution of the bed topography in the same system, but without any permanent flow distortion at the upstream boundary to reproduce a “free system.” However, maintaining a constant uniform flow at the upstream boundary can be regarded as another type of permanent forcing, so this experiment represents the morphodynamic response to another type of forcing.

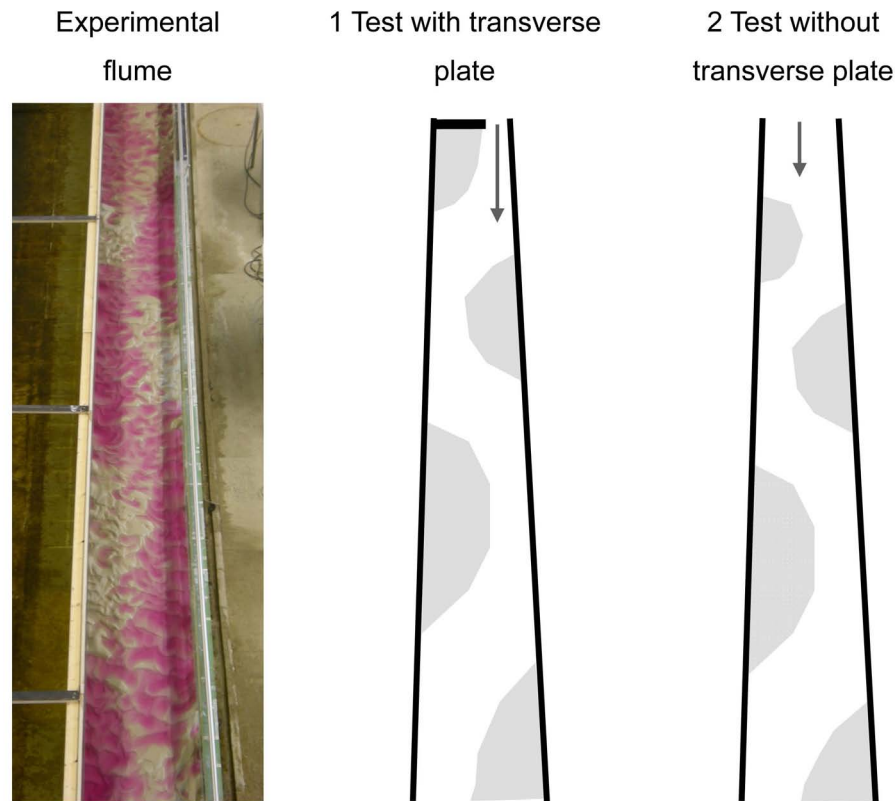


Figure 1. (left) Perspective view of the experimental flume, 60 cm wide, looking from downstream. The presence of alternate bars can be detected from the presence of colored water in the deep parts. (center) Schematized perspective view of the flume during the “forced system” test. (right) Schematized perspective view of the flume during the “free system” test.

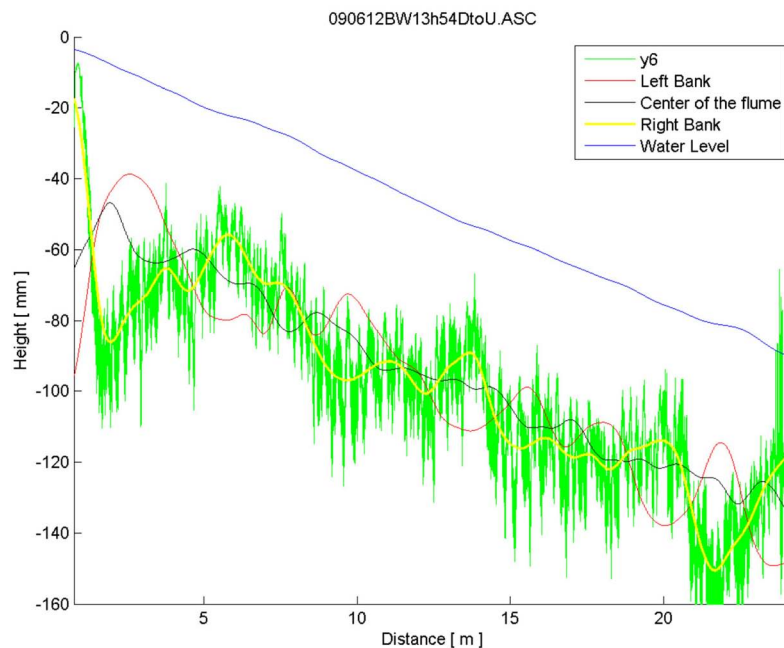


Figure 2. Example of measured profiles. In green, the raw bed level profile (including ripples/dunes) measured at a distance of 5 cm from the left sidewall. In yellow, the corresponding filtered signal. The red and black lines correspond to the filtered bed level profiles along the right sidewall and channel centerline. The blue line shows the measured water level profile.

2.2. System With a Transverse Plate

[12] The experimental test started with the layout shown in Figure 1 (central picture). The transverse plate, placed at the inflow cross-section of the channel near the right sidewall, obstructed two-thirds of the channel width. This created permanent nonuniformity of the inflow. At the start, the bed was flattened and the flume was given a longitudinal slope equal to 3%, but during the first 2 d the longitudinal slope gradually increased until it reached an equilibrium value of 3.54%. The experimental conditions are summarized in Table 1.

[13] During the experiment, the longitudinal profiles of bed and water levels were measured three times per day. The transverse velocity profile was measured across several sections, but with lower frequency.

[14] Nonmigrating alternate bars developed rapidly after the start of the experiment, forced by the presence of the plate, starting from upstream. Their wavelength slowly increased from the initial value of 6.5 m to the final value of ~ 7.5 m (12.5 times the channel width), which was reached after ~ 2 weeks (Figure 3). After that, both the wavelength and amplitude of the steady bars remained constant with time.

[15] Migrating bars were present from the first day on, but only in the second half of the flume (Figure 4). They slowly migrated downstream, disappearing from the flume at the downstream boundary. Because of the increasing dominance of the steady bars, the area in which the migrating bars formed became shorter and shorter with time. Migrating bars did not form where steady bars had a comparable or larger amplitude; this means that steady and migrating bars were present in the flume simultaneously, but at different locations (Figure 4). We can state that alternate steady bars acted as point bars inside sinuous channels, which are known to reduce or even suppress migrating bars [e.g., *Kinoshita and Miwa*, 1974; *Tubino and Seminara* 1990].

[16] The averaged wavelength of migrating bars ranged between 2.6 and 4.1 m, their celerity between 22 and 39 cm h⁻¹, and their (filtered) amplitude between 5 and 18 mm. The ratio between steady and migrating bar wavelengths ranged between 1.8 and 2.9, which is in agreement with previous studies [e.g., *Olesen*, 1984].

[17] Figure 5 shows the temporal evolution of the wavelength and amplitude of migrating alternate bars (daily-averaged values of migrating bars in the second half of the flume).

[18] After 4 d, several bars unexpectedly vanished, only to reform with smaller sizes the day after (Figure 5, drop in averaged bar amplitude). Subsequently, the size of migrating bars increased again. Other drops in averaged bar size

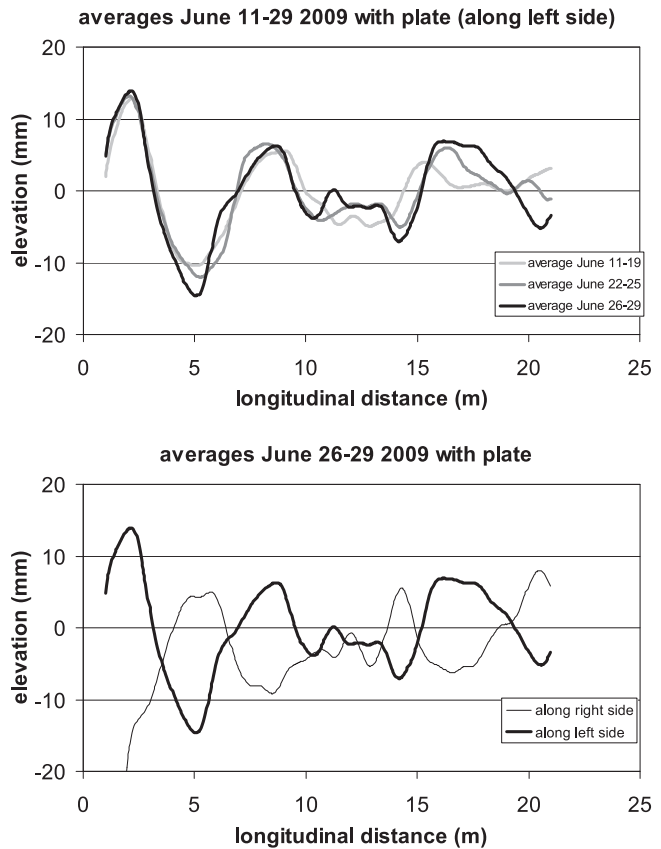


Figure 3. (top) Weekly averaged bed level profiles 5 cm from the left sidewall (values relative to the local cross-sectional averaged bed level). (below) Bed levels along left and right side walls showing the presence of steady alternate bars. System with shown with transverse plate.

were observed on days 8 and 14. The spatially averaged bar amplitude tended to decrease with time (Figure 5) at every new cycle.

2.3. System Without a Transverse Plate

[19] The experimental test started with the layout shown in Figure 1 (right). The duration of this test was ~ 10 weeks, which means that it was much longer than any other published experiment of this type. The starting conditions were a smooth bed with longitudinal slope of 3%, as in the previous experimental test. Subsequently, the bed slope gradually increased, reaching the equilibrium value of 3.74% after ~ 2 d. The experimental conditions are summarized in Table 2.

Table 1. Experimental Conditions Straight Flume^a

Channel Length (m)	Channel Width (m)	Longitudinal Bed Slope (–)	Water Discharge (m ³ s ⁻¹)	Median Sed. Diameter (mm)	Mean Water Depth (m)
26	0.6	3.54%	6.9×10^{-3}	0.238	0.051
Mean flow velocity (m s ⁻¹)	Froude (–)	Shields (–)	Bar mode (–) ^b	Damping coefficient (1 m ⁻¹) ^c	Alternate bar wavelength (m) ^c
0.225	0.33	0.46	1	–0.07	5

^aData based on system with plate.

^bMode according to *Crosato and Mosselman* [2009].

^cData as in the work of *Crosato et al.* [2011], based on simplified linear model by *Struikma et al.* [1985].

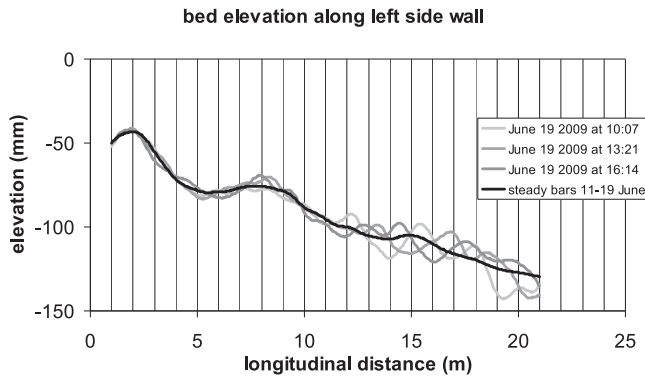


Figure 4. Successive measurements of bed level profile 5 cm from the left sidewall (filtered data) showing the presence of steady and migrating alternate bars 8 d after the start of the experiment. Migrating bars are present only in the second half of the flume. (black line) Longitudinal profile averaged over 1 week (see Figure 3). System shown with transverse plate.

[20] The slightly higher longitudinal slope and flow velocity and the slightly lower water depth with respect to the previous experiment can be attributed to the absence of the transverse plate.

[21] During the first week, the longitudinal profiles of bed and water levels were measured three times per day. The measurements were later carried out twice per day in the following 3 weeks, but only once every 3 d during the second month, but in the last week, the measurements were carried out again with the initial frequency. The last measurement was taken 68 d after the start of the experiment. The final bed configuration is shown in Figure 1 (left).

[22] A steady bar started to appear from the first week, but for the first 3 weeks this bar was weak and not always detectable. Its initial wavelength was 7.0 m, but as in the previous test, the wavelength gradually increased and reached the final value of 7.5 m after ~ 3 weeks. After that, the steady bar continued to grow in amplitude, although slowly, and two more bars started to appear ~ 6 weeks after the start of the experiment. Figure 6 shows the longitudinal profile of the time-averaged bed elevation along the left

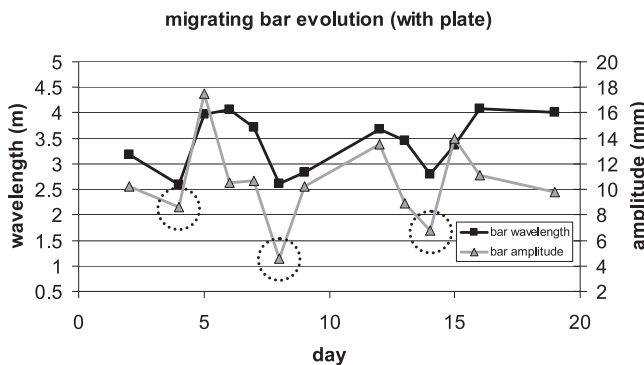


Figure 5. Temporal evolutions of spatially averaged migrating bar wavelength and amplitude. Vanishing/reduction events can be recognized by a drop in the averaged bar amplitude (circles). System shown with transverse plate.

sidewall showing the presence of steady alternate bars. These had a different phase lag than in the preceding test with external forcing, and smaller amplitude, but they eventually had the same wavelength. The steady bars that developed in the channel without the plate are therefore very similar to the steady bars that developed in the channel with the plate.

[23] Migrating bars started to form immediately. As in the previous test, they could freely disappear from the flume at the downstream boundary while they reformed upstream. Their average wavelength ranged between 2.5 and 4.9 m, their celerity between 23 and 40 cm h^{-1} , and their (filtered) amplitude between 5 and 16 mm. The ratio between steady and migrating bar wavelengths ranged between 1.4 and 3.0.

[24] Initially, migrating bars were present along almost the entire flume length; 1 month after, they developed only in the second half of the flume, in a way similar to the previous experiment with a transverse plate and to prior experiments in which steady bars coexisted with migrating bars [Struiksma and Crosato, 1989; Lanzoni, 2000a, 2000b]. This coincided with the emerging of a steady bar in the upstream part of the flume. This means that, even in the absence of a geometrical disturbance, the channel bed topography gradually acquired the characteristics that could be expected with permanent upstream forcing (Figure 7).

[25] As in the system with a plate, the migrating bars had a cyclic behavior: they formed, vanished or strongly reduced in size, and reformed. We observed four events of this type in 2 months. This is visible in Figure 8, which shows the temporal variations of daily-averaged bar wavelengths and amplitude. The period between two successive events was 13–22 d, but it is possible that we missed one or more of these events. Moreover, we observed smaller drops in the averaged bar amplitude with periods of ~ 1 week (Figure 8). Bars vanishing and reforming occurred in < 1 d, as shown in Figure 9; this means that the daily averaged values do not clearly show the vanishing events. As in the system with a plate, the spatially averaged bar amplitude tended to slowly decrease at every new cycle (Figure 8).

[26] Bar celerity is plotted against bar wavelength in Figure 10 (both tests), where it is possible to observe a decrease in the bar celerity if the bar wavelength increases. This is in qualitative agreement with the findings of Fujita and Muramoto [1985], Tubino and Seminara [1990], Nelson [1990], and Defina [2003].

[27] Contrary to migrating bars, the steady bars never vanished. They tended toward an equilibrium configuration, and this also happened in the system without a transverse plate.

3. Long-Duration Rotating Annular Flume Experiment

[28] Annular flumes are often used to study changes of channel bed topography in the laboratory, especially for cohesive sediment [Booij, 1994; Yang *et al.*, 2000]. In annular flumes there is no pump to recirculate the sediment-laden water, because the flow is obtained by the rotation of the top lid. Counter-rotation of the channel is a way to minimize secondary circulation and lateral variability of shear stress, due to the differences in tangential velocity between the inner and the outer sides [Partheniades *et al.*, 1966].

Table 2. Experimental Conditions Straight Flume^a

Channel Length (m)	Channel Width (m)	Longitudinal Bed Slope (–)	Water Discharge (m ³ s ^{–1})	Median Sed. Diameter (mm)	Mean Water Depth (m)
26	0.6	3.74%	6.9×10^{-3}	0.238	0.049
Mean flow velocity (m s ^{–1})	Froude (–)	Shields (–)	Bar mode (–) ^b	Damping coefficient (1 m ^{–1}) ^c	Alternate bar wavelength
0.235	0.33	0.46	1	–0.1	5

^aData based on system without plate.^bMode according to *Crosato and Mosselman* [2009].^cData as in the work of *Crosato et al.* [2011], based on simplified linear model by *Struikma et al.* [1985].

The idea is to approach the idealized concept of an endless straight channel flow. However, complete elimination of the secondary circulation is difficult, if not impossible, to achieve [*Booij and Uijtewaal*, 1999]. For this, the flow in an annular flume finally resembles the flow in an infinitely long mildly curved channel, i.e., with a radius of curvature that is much larger than the radius of curvature of the flume.

[29] A long-duration experiment was carried out at the Laboratory of Fluid Mechanics of Delft University of Technology, in 1981, but its results were never published. The experiment was originally meant to study alternate bar formation and was similar to the one previously carried out by *Engelund* [1975]. One specific test, characterized by long duration, aimed at finding out whether an alluvial channel with migrating alternate bars would eventually develop into a “dynamic equilibrium” configuration, characterized by a constant averaged amplitude. The experimental conditions in the annular flume cannot be quantitatively compared to the conditions in the straight channel. We think, however, that it might be important to show the long-term evolution of migrating alternate bars in a completely different system, with different boundary conditions and, in particular, no sediment recirculation.

3.1. Experimental Setup

[30] The characteristics of the flume are described in Table 3 and the flume layout is shown in Figure 11. In the long-duration test there was no counter-rotation of the channel, the rotational velocity of the top lid, ω_b , being 0.9 rad s^{–1}. The rotational velocity of the channel, ω_b , was therefore equal to zero. This means that the experiment did not reproduce well the flow in a mildly curved infinitely

long channel, but kept some asymmetry due to higher flow velocity near the outer wall.

[31] The experimental test started with a flat bed. Water was regularly added to compensate for any losses due to evaporation. The bed level was measured with an electric field sensor along 10 concentric circles across the channel width, i.e., every 2 cm starting 1 cm from the outer wall. Raw data were filtered to eliminate ripples.

[32] The channel bed soon became dominated by the presence of alternate bars, which were superimposed by ripples or dunes. The amplitude of the bars was systematically larger near the outer side of the flume and smaller near the inner side (Figure 12). This is because of the resulting mildly curved flow, so the basic channel bed was slightly sloping in a transverse direction, with higher level at the inner side of the flume.

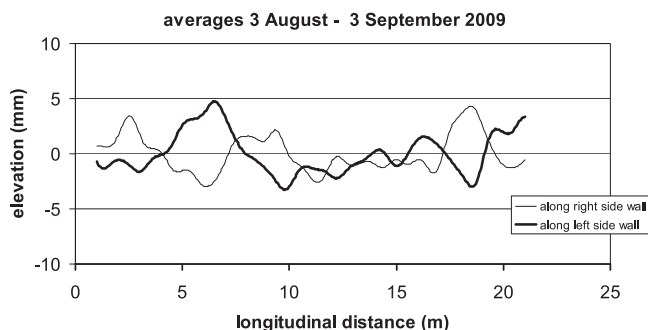


Figure 6. Monthly averaged longitudinal profiles of bed elevation 5 cm from the sidewalls (values relative to the cross-sectional-averaged value of the bed level), showing the presence of steady alternate bars. System shown without transverse plate.

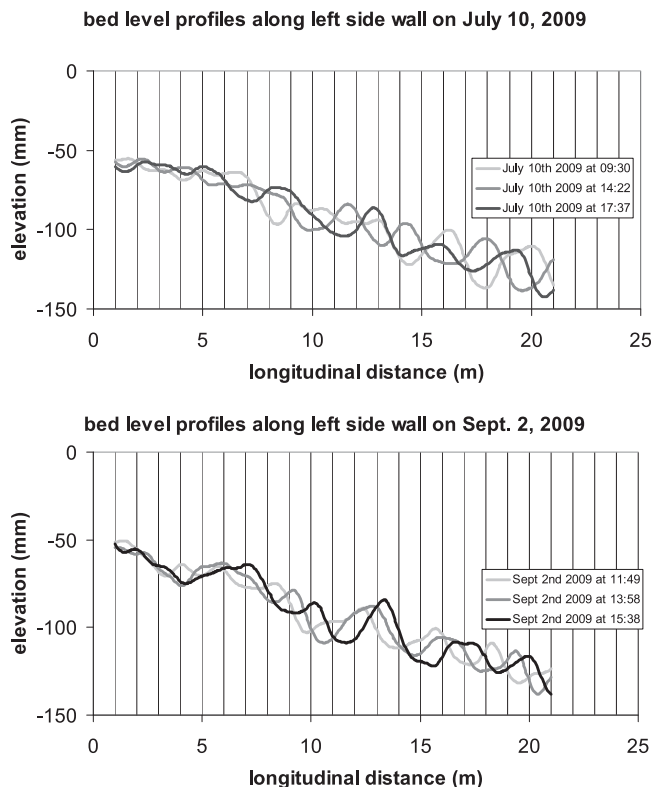


Figure 7. Successive measurements of bed-level profile 5 cm from the left sidewall (filtered data) showing the presence of both steady and migrating bars. It is possible to observe the area in which migrating bars formed moved downstream and grew smaller with time. System shown without transverse plate.

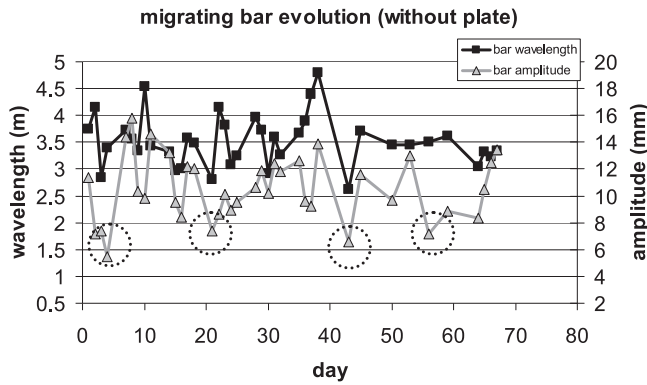


Figure 8. Temporal evolution of spatially averaged migrating bar wavelength and amplitude. Bar vanishing/reduction can be recognized by a drop in bar amplitude (circles). System shown without transverse plate.

[33] As in the straight flume experiments, the bed level signal had to be filtered to smooth out the ripples. The dominant bed wave, defined as the one having the highest energy density, was then determined by means of spectral Fourier analyses of the bed elevation measured along the largest concentric circle, i.e., the one having a diameter equal to 2.32 m, which were carried out at regular intervals of 1 d. It was not possible to correctly determine the bar celerity, because it varied in a radial direction, showing bars that were advancing slightly faster near the outer wall (probably due to no counter-rotation of the channel).

3.2. Experimental Results

[34] A dominant alternate bar with a wavelength of 1.4 m and an amplitude of the order of 5 cm was found to be present 90% of the time, but for 10% of the time the channel bed was almost plane, without a clear dominant wavelength. The photo of the original plot (1981) showing the results of the spectral analysis with time (one measurement per day or less) is given in Figure 13. Almost all of the bars vanished at the same time at intervals of 6–8 d (darkened plots, in which the spectral density is relatively low and without a clear peak). Digitalization of the old drawing (original data are lost) led to the graphs depicted in Figure 14.

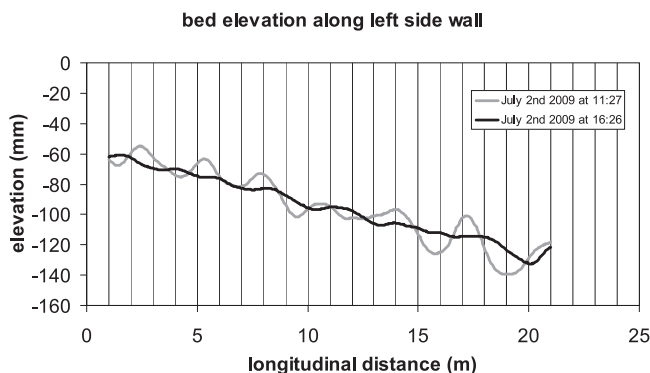


Figure 9. Evolution of bed topography 5 cm from the left sidewall 3 d after the start of the experiment. System shown without transverse plate.

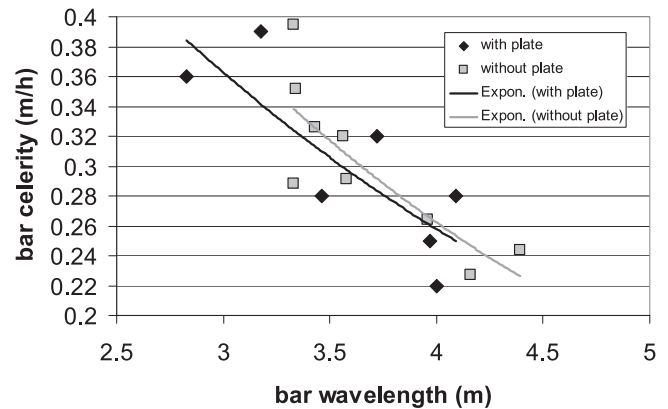


Figure 10. Celerity versus wavelength of migrating bars. System with plate illustrated by black rhombuses and trend line. System without plate illustrated by gray squares and trend line. The trend lines are almost identical.

[35] Table 4 lists the maximum value of spectral density, in arbitrary units (first peak), which is proportional to the bar amplitude to the square. During the bar vanishing events, the averaged spectral density was 247 arbitrary units. The averaged spectral density diminished at every new cycle of bar formation, it started with 928 arbitrary units (day 23), it became equal to 715 arbitrary units during the second cycle (days 25–29), and reduced to 682 arbitrary units in the third cycle (days 32–37). This means that the time-averaged bar amplitude diminished at every cycle. The same phenomenon was also observed in the straight flume experiments.

4. Numerical Simulations

4.1. Numerical Model Setup

[36] The formation of alternate bars has been successfully studied with two-dimensional numerical models since the 1980s [e.g., *Struiksma et al.* 1985; *Shimizu and Itakura*, 1989; *Nelson and Smith*, 1989; *Nelson*, 1990]. Here we used the fully nonlinear physics-based numerical model Delft3D (open source: available at <http://www.deltares.nl>), based on the Reynolds equations for shallow flow [*Lesser et al.*, 2004], to study the long-term behavior of alternate bars in a straight alluvial channel with fixed banks.

[37] The hydrodynamic part of the model is based on the 3-D Reynolds-averaged Navier-Stokes (RANS) equations for incompressible fluid and water (Boussinesq approach). We used a 2-D depth-averaged version of the model with an appropriate parameterization of two relevant 3-D effects of the spiral motion that arises in a curved flow [e.g., *Blanckaert et al.*, 2003]. First, the model corrects the direction of sediment transport through a modification in the direction of the bed shear stress, which would otherwise coincide with the direction of the depth-averaged flow

Table 3. Experimental Conditions in Annular Flume

Inner Wall Diameter ^a (m)	Outer Wall Diameter ^a (m)	Channel Width (m)	Median Sed. Diameter (mm)	Mean Water Depth (m)
1.94	2.34	0.2	0.200	0.06

^aValues relative to the inner side of the wall (inside the flume).

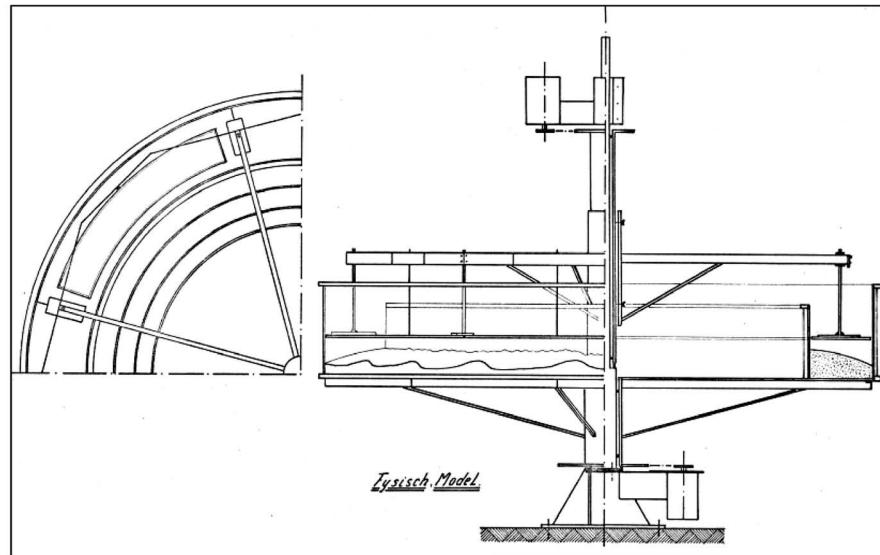


Figure 11. Annular flume lay out.

velocity vector, and taking into account the gravity effects due to a sloping bed. Second, the model includes the transverse redistribution of main flow velocity due to secondary-flow convection, through a correction in the bed friction term. The closure scheme for turbulence is a $k-\varepsilon$ model, in which k is the turbulent kinetic energy and ε is the turbulent dissipation.

[38] The evolution of bed topography is computed from a sediment mass balance and a sediment transport formula. The model accounts for the effects of longitudinal and transverse bed slopes on bed load direction [Bagnold, 1966; Ikeda, 1982].

[39] The equations are formulated in orthogonal curvilinear co-ordinates. The set of partial differential equations in combination with the set of initial and boundary conditions is solved on a finite difference grid.

[40] We simulated the evolution of bed topography in straight river channels with uniform width and non-erodible banks. Sediment characteristics and longitudinal bed slope were loosely based on the Waal River in the Netherlands. The other characteristics were selected such that they meet the requirements to produce a single threaded-bar pattern based on the channel pattern predictor of Kleinhans and Van den Berg [2011].

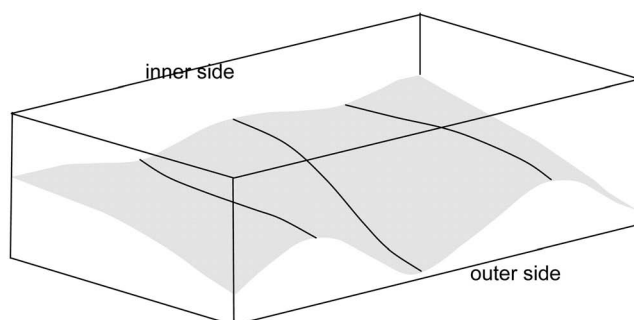


Figure 12. Alternate bars in the annular flume.

[41] The length of the channel was 20 km, the longitudinal bed slope was 2×10^{-4} (0.2 m km^{-1}), the median sediment grain diameter was 2 mm, and the relative submerged mass density of the sediment equaled 1.65 (from a sediment mass density of 2650 kg m^{-3}). The sediment transport rates were computed using Engelund and Hansen's [1967] formula. The other channel characteristics are summarized in Table 5.

[42] The grid cell sizes in the transverse and longitudinal directions were 20 and 50 m, respectively. The time step was 6 s. To speed up the computations we applied a morphological factor equal to 25 [Roelvink, 2006]. This amplifies the bed level changes in each hydrodynamic time step and serves as the ratio between morphological and hydrodynamic time steps. Analyses of the effects of using a morphological factor were carried out by Crosato et al. [2011]. The results showed that using a morphological factor larger than 1 leads to overestimations of bar celerity.

[43] The reproduction of migrating bars in a numerical model without any permanent external forcing requires an unsteady forcing at the upstream boundary [e.g., Struiksma, 1998; Mosselman et al., 2003]. For this reason, a small spatially and temporally varying random perturbation of the inflow was imposed on each grid cell at the upstream boundary. The random perturbation had a standard deviation of 0.5% of the discharge, varying every 2.3 d (morphodynamic simulated time). The downstream boundary condition was a constant water elevation, computed from uniform flow conditions. This allowed bars to freely disappear from the model domain without any distortion in shape during the entire computational period.

[44] The simulations started with a flat channel bed with a continuous longitudinal slope and small random perturbation of maximum 1 cm in each grid cell, with water depths ranging between 6 and 7.4 m. Sensitivity analyses, carried out by repeating run 2 with different random perturbations, showed that the results were unaffected by how the initial bed was perturbed. While bar wavelengths and amplitude varied according to how discharge was perturbed, the phenomena

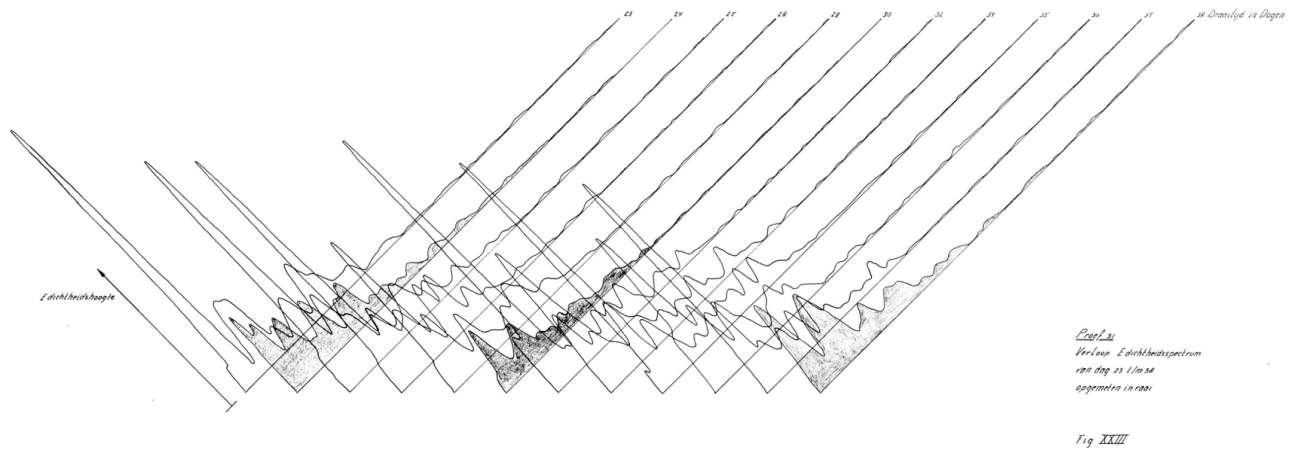


Figure 13. Original drawings showing the results of daily spectral analyses [Cornelisse, 1981]. The days in which the bars vanished (day 24, 30, and 38) were darkened in the figure.

of interest (cyclic variation and bar vanishing) were observed in all cases.

4.2. Model Results

[45] The runs simulated the first 7 yr (morphodynamic simulated time) of bar development, starting from a flat bed. Figures 15A, 15B, and 15C show the time evolution of the longitudinal profiles of near-bank bed elevation, corrected for the longitudinal slope, for runs 1, 2, and 3, respectively,

$$Z_{b,x}(T) = z_{b,x}(T) - z_{b,x}(0), \quad (1)$$

where $Z_{b,x}(T)$ is the relative near-bank bed elevation at location x and time T (m), $z_{b,x}(T)$ is the bed elevation with respect to a fixed level at time T , and $z_{b,x}(0)$ is the same, but at run start (m). Since the initial condition is flat bed, $z_{b,x}(0)$ is equal to the cross-sectionally averaged bed elevation, but dependent on longitudinal slope.

[46] The orange stripes in Figure 15 show the downstream migration of the bar tops, whereas the blue stripes show the downstream migration of the pools. The sloping of the stripes indicates bar celerity: the higher the sloping is, the slower the bars are (less distance covered in greater time). The initial longitudinal profile (flat bed) is visible on top of the figures: it is represented by a white horizontal stripe, because the initial condition is that of a flat bed. Bars start to appear after ~ 7 months in runs 1 and 2, but almost immediately in run 3, in which the longitudinal slope and shear stress were substantially higher than in runs 1 and 2.

[47] In accordance with the flume observations, periodic vanishing of alternate bars also occurred in the numerical tests. It is represented by a drastic decrease of bar amplitude: white sloping stripes in Figures 15A, 15B, and 15C (to help the reader, in Figure 15A, diagonal arrows point along the direction of what becomes a vanishing wave further downstream). Only a few consecutive bars vanished. Vanishing started near the upstream boundary and migrated as a wave in the downstream direction with celerity almost twice as large as the one of alternate bars, the celerity of the alternate bars was $\sim 5 \text{ km yr}^{-1}$ and the celerity of the

vanishing wave was $\sim 9 \text{ km yr}^{-1}$. Bar vanishing occurred approximately every 6 months, which is a different time-scale than the imposed upstream discharge perturbation.

[48] Run 3 represents a system at super-resonant conditions [Blondeaux and Seminara, 1985] (this was determined by computing the longitudinal damping of alternate bars according to the simplified linear theory by Struiksmas *et al.* [1985]). In this case, steady bars are expected to become more pronounced in a downstream direction [Struiksmas and Crosato, 1989; Parker and Johannesson, 1989]. In our computations migrating bars became larger in the longitudinal direction, so in run 3 (Figure 15C) three bars grew toward the water level, resulting in local water depths smaller than the threshold depth for sediment transport. These resulted in a number of inactive grid cells. For this reason, these bars no longer migrate.

[49] Figure 16 shows the temporal evolution of bed elevation near the right and left banks at a fixed location, 15 km downstream from the upstream boundary during run 2. It is possible to observe that the bed topography is indeed characterized by the presence of alternate bars (opposite relative bed elevations near opposing banks) and that bar vanishing has a periodic character.

5. Discussion

[50] We observed periodic migrating-bar vanishing and reforming in a straight flume and in an annular flume, as well as in three 2-D numerical simulations reproducing bar formation in straight channels with fixed banks and under constant flow rate. In all cases, the phenomenon was observable due to the long duration of the tests. In the laboratory experiments, several migrating bars or all bars present in the flume vanished or strongly decreased in size within a short time, at cycles of several days to weeks. The amplitude of the bars that redeveloped decreased after each cycle. In the numerical tests, bar vanishing occurred in the form of a wave, moving downstream with celerity twice as large as the one of bars. In this case, bar vanishing involved only a few consecutive bars. Although cyclic bar vanishing was clearly observable in the three runs described in section 4, we had not observed it in two previous long-term numerical tests (the ones described by Crosato *et al.*

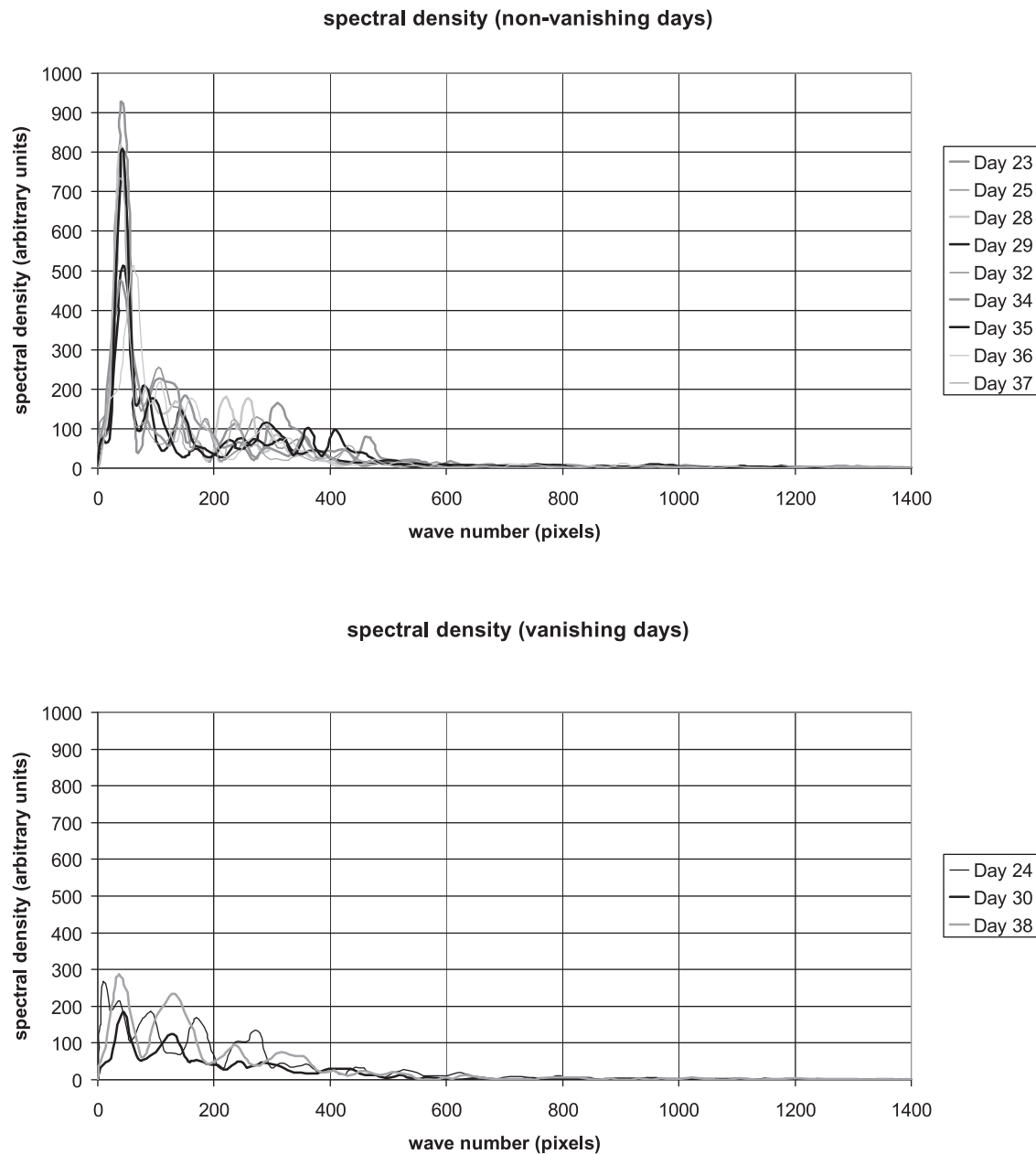


Figure 14. Spectral density as a function of wave number. (top) Days with well-developed bars. (bottom) Days with vanishing bars.

[2011]). This means that this phenomenon does not always occur. This is also true for the formation of steady bars. Contrary to the two runs described by *Crosato et al.* [2011], the three model runs described in section 4 do not show steady bar development. This means that most probably both phenomena, migrating bar vanishing and steady bar formation, occur only within certain ranges of flow and sediment characteristics.

[51] Our study is too limited to define these ranges, for which a fully nonlinear analysis appears necessary, considering the long-term character of these phenomena. However, in the straight flume experiments, as well as in the numerical computations, some parameters appear with similar values: the Shields number ranges between 0.4 and 0.55 and the Froude number between 0.2 and 0.33. Instead, Shields and Froude numbers are much lower, 0.09 and

Table 4. Maximum Values of Spectral Density (Arbitrary Units), First Peak^a

Day	23	24	25	28	29	30	32	34	35	36	37	38
Spectral density	928	268	814	818	512	185	876	476	809	512	735	287

^aData from experiment in annular flume. Boldface entries indicate vanishing bar.

Table 5. Characteristics of the Simulated River Channel

Variables	Run 1	Run2	Run3
Water discharge ($\text{m}^3 \text{s}^{-1}$)	2500	2500	2500
Longitudinal bed slope (–)	0.000186	0.0002	0.0003
Channel width (m)	200	200	200
Mean water depth (m)	7.4	6.85	6.06
Width-to-depth ratio (–)	27	29	33
Mean velocity (m s^{-1})	1.7	1.8	2.1
Froude	0.2	0.22	0.27
Median sed. diameter (mm)	2	2	2
Shields (–)	0.42	0.42	0.55
Steady bar mode ^a (–)	1	1	1
Damping coefficient ^b (m^{-1})	0.0002	0.0002	–0.0001
Alternate bar wavelength ^c (m)	4300	4700	5000
Migrating bar mode ^b (–)	1	1	1
Length simulated reach (m)	20,000	20,000	20,000

^aMode according to *Crosato and Mosselman* [2009].

^bData as in the work of *Crosato et al.* [2011], based on simplified linear model by *Struiksmas et al.* [1985].

^cMode according to *Tubino and Seminara* [1990].

0.14, respectively (Table 6), in the computational tests described by *Crosato et al.* [2011], in which no bar vanishing could be observed. Could this give us an indication of the conditions for which migrating bars have unstable behavior?

[52] In the straight flume experiments, the theoretical longitudinal damping of steady bars, computed according to the linear theory of *Struiksmas et al.* [1985] (in the way described by *Crosato et al.* [2011]), is negative, which means that in both cases the system was at super-resonant conditions [*Blondeaux and Seminara*, 1985; *Parker and Johannesson*, 1989]. Runs 1 and 2 represent subresonant conditions, whereas Run 3 represents super-resonant conditions (Table 6). Finally, the two runs described by *Crosato et al.* [2011] represent both a sub- and a super-resonant system. Therefore, resonance does not appear to be discriminating between unstable and stable migrating bar behavior, although only theoretical research could clarify this point.

[53] We can make some observations about the factors causing the cyclic behavior of bars:

[54] In the annular flume, bar vanishing may be related to the bar wavelength not being an exact submultiple of the flume circumference. In this case, the formation of a transition area may be the cause of periodic bar failure, perhaps taking place as a domino effect. However, the occurrence of this phenomenon should be further investigated.

[55] In the straight flume, periodic bar vanishing/reduction events occurred with different upstream boundary conditions: flow concentration near one side due to the transverse plate, and with uniform inflow. So, upstream flow conditions do not seem to be responsible for the phenomenon. Neither

is sediment recirculation responsible, since the events occurred in two completely different experimental conditions: in the straight flume with sediment recirculation, and in the annular flume with free sediment movement.

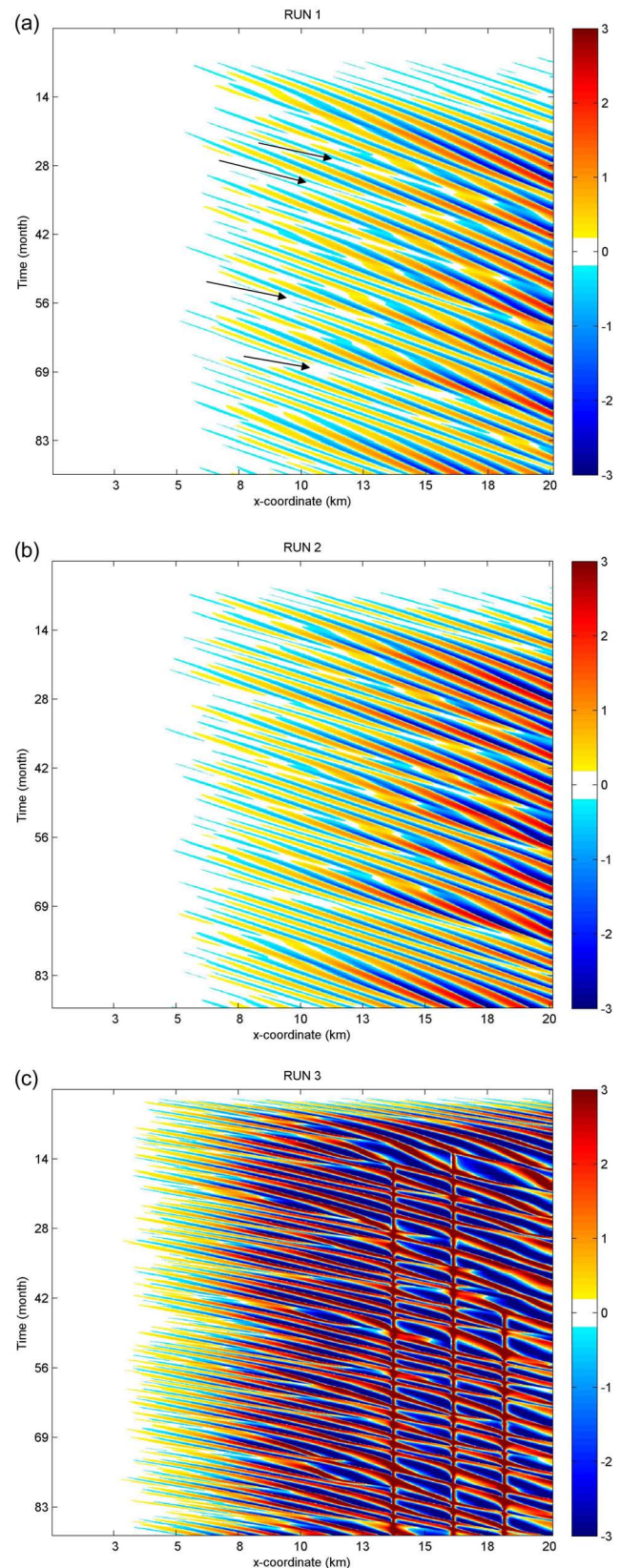


Figure 15. (a) Time series of longitudinal profile of nearside channel bed elevation corrected for the longitudinal slope, in meters (color bar), run 1. Some diagonal arrows point along the direction of what becomes a vanishing wave further downstream. (b) Time series of longitudinal profile of nearside channel bed elevation corrected for the longitudinal slope, in meters (color bar), run 2. (c) Time series of longitudinal profile of nearside channel bed elevation corrected for the longitudinal slope, in meters (color bar), run 3.

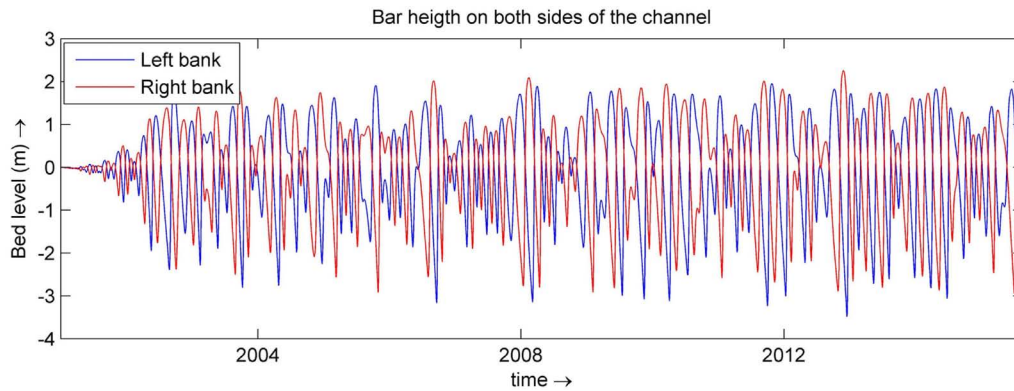


Figure 16. Temporal variation of near-bank bed elevation on both sides at a distance of 15 km from the upstream boundary (run 2).

[56] *Schielen et al.* [1993] studied the stability of a narrow spectrum of migrating bed oscillations near the critical wave number by means of a weakly nonlinear analysis and a simple morphodynamic model. They showed that the resulting Ginzburg-Landau equation describing the evolution of the envelope amplitude of the wave group can lead to the modulation of the bar amplitude, but on large spatial scales. Their mathematical model suggests that this modulation requires a dune-covered bed and can occur only for particular values of the relevant parameters. One condition is surely satisfied by both experiments (straight and annular flume), since the channel bed was covered by ripples or dunes. However, according to *Schielen et al.*, the minimum channel length required to observe modulation of the bar amplitude is 65 times the channel width. This means that, in theory, the straight flume was too short to detect such modulation (26 m against a minimum length of 39 m) or the annular flume. Finally, the modulation detected by *Schielen et al.* [1993], does not explain the vanishing of several bars or even all of the bars at the same time. Instead, in the numerical tests only a few bars vanished at a time and the simulated channel length was long enough. So, the modulation detected by *Schielen et al.* [1993] may provide an explanation to the bar instability observed from the numerical tests. This possibility should be further investigated.

6. Conclusions

[57] The linear analyses carried out in the 1970s showed that migrating bars may form due to morphodynamic instability, but they did not answer the question of whether these

bars would eventually attain an equilibrium configuration. We carried out two long-duration laboratory experiments, one in a straight flume and one in an annular flume, and 2-D numerical tests to investigate the long-term development of alternate bars as well as the interaction between steady and migrating bars in alluvial channels with fixed banks and constant flow rate.

[58] In general, steady and migrating bars appeared as two separate phenomena. In the straight flume experiment without a plate (uniform inflow), the formation of migrating bars preceded the appearance of steady bars. This suggests that the presence of migrating bars may produce the flow perturbation that is needed for the development of steady bars. These observations confirm and refine the results of *Crosato et al.* [2011]. *Hall* [2004] demonstrated theoretically that temporal variations, such as discharge fluctuations, can be the origin of steady bed oscillation in rivers. The straight flume experiment, in which the discharge was kept constant, shows that temporal flow variations due to migrating bars may have the same effect.

[59] In turn, and in agreement with previous observations [*Struiksmas et al.*, 1985; *Struiksmas and Crosato*, 1989; *Lanzoni*, 2000a], the development of steady bars gradually restricted the domain in which increasingly smaller migrating bars formed. In practice, steady alternate bars suppressed migrating bars. The suppression of migrating bars by point bars in sinuous channels was demonstrated by *Kinoshita and Miwa* [1974] and *Tubino and Seminara* [1990]. This means that steady alternate bars have similar effects as point bars in sinuous channels.

Table 6. Summary of Dimensionless Parameters Characterizing the Studied Systems

System	Width/Depth (–)	WL × Damp. ^a (–)	Shields (–)	Froude (–)	Sed. Diam./Depth (–)
Run 1	27	0.86	0.42	0.20	0.0003
Run 2	29	0.85	0.42	0.22	0.0003
Run 3	33	–0.45	0.55	0.27	0.0003
Str. flume with plate	12	–0.35	0.46	0.33	0.047
Str. flume without plate	12	–0.5	0.46	0.33	0.049
Run 1 ^b	30	2.7	0.09	0.14	0.0007
Run 2 ^b	53	–2.1	0.08	0.14	0.0007

^aAlternate bar wavelength times damping coefficient (–) computed according to the simplified linear theory by *Struiksmas et al.* [1985]: negative value, super-resonant conditions; positive value, subresonant conditions.

^bRuns done by *Crosato et al.* [2011] in which migrating bars showed no cyclic behavior.

[60] Both experimental and numerical tests show that migrating bars may be intrinsically unstable, since they cyclically disappeared or drastically reduced in size only to reappear a short time after. The initial growth in bar size and the decrease in bar celerity that we observed in the straight flume experiments confirm the laboratory observations by *Fujita and Muramoto* [1985] and the numerical results by *Nelson* [1990] and *Defina* [2003]. However, only very limited information on unstable behavior of migrating bars can be found in the literature. One single-bar vanishing event was reported by *Takebayashi and Egashira* [2001], who performed some laboratory tests on bar formation with graded sediment. Unstable bar behavior, but without vanishing, was also reported by *Lanzoni* [2000a] in a straight flume experiment with uniform flow, and by *Defina* [2003] in numerical model simulations. The relatively long period of each bar cycle (several days to 2–3 weeks) in both straight and annular flume experiments may explain why the cyclic character of bar behavior had remained unnoticed in previous studies, as they are characterized by durations of several hours to days. However, since our numerical simulations suggest that cyclic bar vanishing may occur only within certain ranges of the morphodynamic parameters, it is also possible that previous investigations were carried out in conditions in which migrating bar vanishing does not occur.

[61] The weakly nonlinear analysis of *Schielen et al.* [1993] shows that alternate bar amplitude may fluctuate, on the long spatial scale, due to the dispersive properties of the wave group. This may explain the amplitude variations that we observed in the numerical tests. However, the length scale of the modulation appears too long to clearly show up in the flume tests. Furthermore, such modulation does not explain the collapse of all of the bars at the same time. We therefore believe that the cyclic bar behavior that we observed in the experiments has another origin.

[62] All of the experiments and numerical tests were carried out under constant flow conditions. This means that the results cannot be simply applied to real rivers. *Tubino* [1991] and *Welford* [1994] demonstrated that discharge variations are important for alternate bar formation and, in particular, for bar amplitude. Moreover, in real rivers, migrating bars may form at certain discharges and not at others. Finally, real rivers normally have a large number of geometrical discontinuities and should be regarded as “forced systems.” In these systems, steady bars tend to quickly dominate the bed topography, as in the straight flume experiment with the plate. This means that in most rivers migrating alternate bars are affected by the presence of steady alternate bars and point bars. Given the results of our investigations, we can assume that in real rivers migrating bars are generally smaller than theories would predict and confined to restricted regions of the channel.

[63] We did not provide mathematical explanations for the observed periodical vanishing of migrating bars, for which we suggest performing fully-nonlinear analyses. Our work may be of interest to readers who take a theoretical approach to river morphodynamics, since it shows the long-term evolution of steady and migrating bars under several controlled conditions. Our observations suggest that under constant flow rates migrating bars may be a mere transition

phenomenon of alluvial channels, present only until steady bars are fully developed.

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