A logic of natural relations

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Abstract

Predicate logic functions like a distorting mirror in the sense that it often does not faithfully represent reality. As an example, consider Adam's loving Eve. In the state itself, Adam does not occur first or second, but in predicate logic it is represented by an ordered pair. In this paper a new logical system is developed which does not have this artifact and in which we can reason in a natural way about all kinds of relations. The starting point is to view a relation as a network of interrelated states.

KEY WORDS: natural relation, relational complex, substitution

1 Introduction

Much of what is going on in our world is expressed in terms of relations between things. But it is not immediately clear what exactly relations are, nor how we should talk about them. Consider, for example, the love relation. How can we think of the state of Adam's loving Eve?

The *standard view* is to identify this state with the ordered pair $\langle Adam, Eve \rangle$. There is, however, a problem. Of course, Adam is mentioned first in the sentence "Adam loves Eve", but it makes no sense to say that Adam occurs first in the state of Adam's loving Eve. A more natural way of thinking of Adam's loving Eve is offered by the *positionalist view* on relations. On this view, the love relation has unordered positions *lover* and *beloved.* This view, however, is also not without problems; we are inclined not to accept positions on a fundamental metaphysical level.

In 2000, Kit Fine proposed a quite different view on relations [Fin00]. On this *antipositionalist view*, the constituents of the complexes of a relation neither come in a certain order, nor do they occupy positions. Instead, the complexes form a network interrelated by substitutions. For example, the complex of Adam's loving Eve and the complex of Abelard's loving Eloise are interrelated because the second complex can be obtained from the first by substituting Abelard for Adam and Eloise for Eve.

Building and comparing mathematical models for different views on relations provided strong support for the superiority of the antipositionalist view [Leo08;

Leo13]. These metaphysical developments trigger a question of fundamental import: what kind of logic is appropriate for reasoning about relations?

There is general agreement that predicate logic offers a powerful means to reason about mathematical relations, but it seems less suitable for natural relations in which there is no intrinsic order between the constituents of its complexes. In fact, in those cases, standard predicate logic functions like a distorting mirror. It does not faithfully represent reality.

Reality

Traditional Logic

To illustrate, consider the formula $L(a, e)$. The terms a and e come in a certain order, but if this formula is interpreted as Adam's loving Eve, then the order is just a representational artifact. We may, however, be misled into thinking that the order is essential for the underlying relation. A great variety of relations has a similar problem (e.g. the relation of vertical placement and the relation of relative size). Not only in standard predicate logic, but also in other existing logics we impose an order that is not present in the represented facts. How do we get rid of this weakness? Apparently, we need a different kind of logic.

2 Minimalistic logic

We will define a one-sorted logic with no predicate letters—except the equality symbol. The signature of the language will consist only of a set of constants. Furthermore, we will neither assume that the domain is nonempty nor that constants have existential import, i.e. their interpretation may be undefined. As such, it is a *free* logic of a very basic kind.

As we will see, this *minimalistic logic* has despite its simplicity the same expressive power as predicate logic. It will be the backbone of the logic of relations developed later in this paper.

Definition 2.1. A *minimalistic structure* M is a tuple $\langle E, \mathsf{App}, \mathsf{Con}, I\rangle$ with

(i) E a (possibly empty) collection of entities,

(ii) App a partial function from E to the collection of partial functions on E,

(iii) Con a collection of constants,

(iv) I a partial function from Con to E.

The entities may be individuals, like a number or a person, but they may also be complex, like the fact that $2 + 3 = 5$ or something like Adam's loving Eve.

The main idea in defining logic corresponding with this minimalistic structure is to let terms represent all these kinds of entities.

We now define the language of minimalistic logic. The language has the following symbols:

(i) constants: the elements of Con

(ii) variables: $x_0, x_1, x_2, \ldots$

(iii) application symbol: $\cdot(\cdot)$

(iv) equality symbol: $=$

(v) connectives: $\wedge, \vee, \rightarrow, \neg, \forall, \exists$

The application symbol will be used for partial function application. The details will be given shortly.

The definition of the terms is straightforward:

Definition 2.2. The collection of *terms* is defined inductively as follows:

(i) every variable is a term,

(ii) every constant is a term,

(iii) if t, t' are terms, then $t(t')$ is a term.

We have only one kind of atomic formulas, namely those of the form $t = t'$:

Definition 2.3. The collection of *formulas* is defined inductively as follows:

(i) if t, t' are terms, then $t = t'$ is a formula,

(ii) if φ, ψ are formulas, then $(\varphi \wedge \psi), (\varphi \vee \psi), (\varphi \rightarrow \psi)$ are formulas,

(iii) if φ is a formula, then $\neg\varphi$ is a formula,

(iv) if φ is a formula and x is a variable, then $\forall x\ \varphi, \exists x\ \varphi$ are formulas.

As the definitions show, we not only have no predicates (except equality), but also no terms of arity ≥ 2. Therefore, an artifactual ordering of arguments does not occur in minimalistic logic. Of course, this does not yet mean that minimalistic logic is adequate for reasoning about natural relations like the love relation. In Section 3, we will see how it can be used as a basis for a logic of relations.

2.1 Semantics

The semantics of the minimalistic logic can be defined very similar to the semantics of predicate logic. We will give an interpretation by means of assignments.

Let $M = \langle E, \mathsf{App}, \mathsf{Con}, I\rangle$ be a minimalistic structure. Let g : variables $\to E$ be a function if E is nonempty, otherwise let $g(x)$ be undefined.

The terms are interpreted as follows:

$$\begin{array}{ll} [t]_{M,g} = I(t) & \text{if } t \text{ is a constant} \\ [t]_{M,g} = g(t) & \text{if } t \text{ is a variable} \\ [t(t')]_{M,g} = \mathsf{App}([t]_{M,g})([t']_{M,g}) & \end{array}$$

Note that the interpretation of a term may be undefined for the following reasons: (i) I is a partial function, (ii) $g(x)$ is undefined if E is empty, (iii) App is a partial function from E to the partial functions on E.

For interpreting the formulas, let $V_{M,g}$: formulas $\to \{0,1\}$ be a total function such that

$$\begin{array}{ll} V_{M,g}(t = t') = 1 & \text{iff } [t]_{M,g} \text{ and } [t']_{M,g} \text{ are both defined and } [t]_{M,g} = [t']_{M,g} \\ V_{M,g}(\varphi \wedge \psi) = 1 & \text{iff } V_{M,g}(\varphi) = 1 \text{ and } V_{M,g}(\psi) = 1 \\ \ldots & \\ V_{M,g}(\neg\varphi) = 1 & \text{iff } V_{M,g}(\varphi) = 0 \\ V_{M,g}(\forall x\, \varphi) = 1 & \text{iff } V_{M,g[x:e]}(\varphi) = 1 \text{ for every } e \in E \\ V_{M,g}(\exists x\, \varphi) = 1 & \text{iff } V_{M,g[x:e]}(\varphi) = 1 \text{ for some } e \in E \end{array}$$

For terms, the existence predicate $\mathsf{E!}$ may be defined as follows:

$$\mathsf{E!}\, t \;=_{\text{df}}\; t = t$$

A formula φ is valid if every interpretation makes φ true. The next theorem shows that the expressiveness of minimalistic logic and of predicate logic are similar.

Theorem 2.4. *Validity in free first-order predicate logic is effectively reducible to validity in minimalistic logic, and vice versa.*

Proof. We may translate formulas of free first-order logic into minimalistic logic in a straightforward way. Define ind as some constant of the minimalistic logic. For each predicate letter P, let c_P be a constant of the minimalistic logic and define $P(x_1, \ldots, x_n)^*$ as

$$\mathsf{E!}\,\mathsf{ind}(x_1) \wedge \ldots \wedge \mathsf{E!}\,\mathsf{ind}(x_n) \wedge \mathsf{E!}\, c_P(x_1)\ldots(x_n)$$

Furthermore, define $(\varphi \wedge \psi)^*$ as $(\varphi^* \wedge \psi^*)$, $(\neg\varphi)^*$ as $\neg(\varphi^*)$, and $(\forall x\, \varphi)^*$ as

$$\forall x\, (\mathsf{E!}\,\mathsf{ind}(x) \to \varphi^*)$$

It is easy to see, that φ is valid iff φ^* is valid.

Vice versa, define a free first-order logic with a binary partial function symbol F and translate $x(y)$ as $F(x, y)$. This translation preserves validity as well. $\dashv$

The translations given in the proof of Theorem 2.4 show that although minimalistic logic has no predicates, it is very similar to standard predicate logic.

It is interesting to note that an alternative minimalistic logic can be defined with the same expressive power, but without terms of the form $t(t')$. Instead of these terms, it is sufficient to have terms of the form: $\mathsf{part\_fun}(t)$, $\mathsf{arg}(t)$, and $\mathsf{result}(t)$.

2.2 Axiomatization

Because terms may not be defined, it is useful to define *weak equality* between terms:

$$t \simeq t' \;\; =_{\text{df}} \;\; \mathsf{E!}\, t \vee \mathsf{E!}\, t' \rightarrow t = t'$$

So, in words, $t \simeq t'$ means that if either t or t' is defined, then so is the other and their interpretations are the same.

The axioms of minimalistic logic are all generalizations of formulas of the following form:

1. Tautologies
2. $\forall x\ (\varphi \rightarrow \psi) \rightarrow (\forall x\ \varphi \rightarrow \forall x\ \psi)$
3. $\varphi \rightarrow \forall x\ \varphi$, where x does not occur free in φ
4. $\forall x\ \phi(x) \rightarrow (\mathsf{E!}\, t \rightarrow \varphi(t))$, where t is substitutable for x in φ
5. $\forall x\ x = x$
6. $\mathsf{E!}\, t(t') \rightarrow \mathsf{E!}\, t \wedge \mathsf{E!}\, t'$
7. $t \simeq t' \rightarrow (\varphi(t) \leftrightarrow \varphi(t'))$, where $\varphi(t')$ has free t' at zero or more places where $\varphi(t)$ has free t.

We have one rule of inference:

$$\text{from } \varphi \text{ and } \varphi \rightarrow \psi, \text{ infer } \psi.$$

Theorem 2.5. *Minimalistic logic is sound and complete.*

Proof. By translating back and forth from minimalistic logic to free first-order logic, the theorem follows from the soundness and completeness of free first-order logic. $\dashv$

In minimalistic logic we quantify over all entities, including partial functions. As such, it may be conceived as higher-order logic, although it is not more expressive than ordinary first-order logic. To increase the expressiveness, we might do the following. Let $A \subseteq E$ be the collection of entities that are not partial functions. Let us call a structure $M = \langle E, \mathsf{Con}, I \rangle$ *full* if any function f on A belongs to E. Then restricting the semantics to full structures increases the expressiveness.

3 Developing a logic of relations

At the beginning of the twentieth century, Bertrand Russell developed a substitutional theory of classes and relations [Rus73]. The theory has one type of primitive formula denoted as

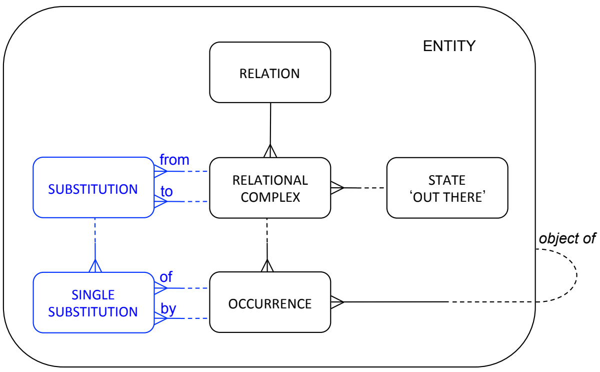
$$p \, \tfrac{x}{a}! \, q$$

which is read as "q results from p by substituting x for a in all those places (if any) where a occurs in p" [Rus73, p. 168]. The theory seemed to offer a solution for Russell's paradox of classes—which was troubling the foundations of mathematics at the time—but the theory soon turned out to be inconsistent as well [Lan03, pp. 271–278].

In this section, a new logic of relations will be developed in which the notion of substitution plays a key role as well. The logic will be defined as a theory in minimalistic logic. My goal is more modest than Russell's with his substitutional theory. Russell's theory was intended as a foundational system for mathematics, but here only a logical framework for relational structures will be created.

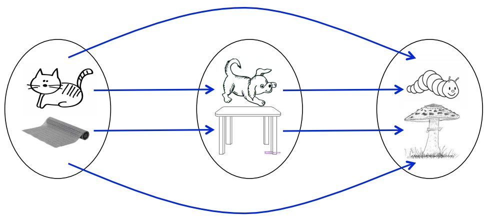
3.1 An antipostionalist view

Before developing a logic of relations, let us first take a closer look at the structure of relations. The diagram below depicts an antipositionalist view on relations.



In this so-called *entity-relationship diagram*, the relationship between RELATION and RELATIONAL COMPLEX should be read as follows: each relation 'has' one or more relational complexes and each relational complex belongs to exactly one relation. The relationship between RELATIONAL COMPLEX and OCCURRENCE should be read as: each relational complex may contain one or more occurrences of entities and each occurrence belongs to exactly one relational complex. Other relationships should be read in a similar way. The diagram also contains *subtypes*, e.g. RELATION and RELATIONAL COMPLEX are subtypes of ENTITY. We do not assume that every instance of ENTITY belongs to one of the given subtypes. (For more details about entity-relationship modelling, see [Bar92] and [Tha00].)

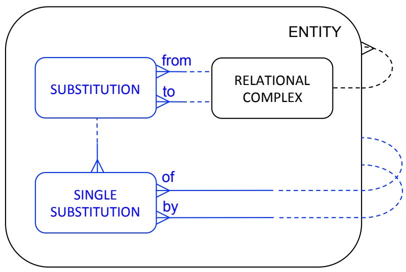
For substitution, we have a composition rule that may be explained with an example.



Consider the complex of a cat on a mat. If we substitute a dog for the cat and a table for the mat, we get the complex of a dog on a table. Then substituting in the last complex a caterpillar for the dog and a mushroom for the table gives the same result as substituting a caterpillar for the cat and a mushroom for the mat in the original complex.

3.2 A simple logic of complexes

To start, let us make some simplifications and assume that we only have complexes interconnected by substitutions:



We define a *simple logic of complexes* as a minimalistic logic with two constants:

$$\mathsf{src},\ \mathsf{tgt}$$

The constants will be interpreted as partial functions that give the *source* and the *target* of a substitution. For example, if a substitution goes from complex A to complex B, then A is called the source and B the target of the substitution.

Before giving the axioms, we give a few definitions:

$$\begin{aligned} t \text{ is a complex} &=_{\text{df}} \exists s\ (t = \mathsf{src}(s)) \\ t \text{ in } t' &=_{\text{df}} \exists s\ (\mathsf{src}(s) = t' \wedge \mathsf{E!}\, s(t)) \\ s : t \twoheadrightarrow t' &=_{\text{df}} \mathsf{src}(s) = t \wedge \mathsf{tgt}(s) = t' \end{aligned}$$

The axioms of the simple logic of complexes are:

Source and target axioms:
$\forall s\ (\mathsf{E!}\,\mathsf{src}(s) \leftrightarrow \mathsf{E!}\,\mathsf{tgt}(s))$
$\forall s\ (\mathsf{E!}\,\mathsf{tgt}(s) \to \mathsf{tgt}(s) \text{ is a complex})$

Constituents axiom:
$\forall x\ \forall s\ (x \text{ in } \mathsf{src}(s) \to \mathsf{E!}\, s(x))$

Extensionality of substitutions axiom:
$\forall s, s'\ (\mathsf{src}(s) = \mathsf{src}(s') \wedge \forall u\ (s(u) \simeq s'(u)) \to s = s')$

Identity of substitutions axiom:
$\forall x\ (x \text{ is a complex } \to \exists s : x \twoheadrightarrow x\ \forall u\ (\mathsf{E!}\, s(u) \to s(u) = u))$

Composition of substitutions axiom:
$\forall x, y, z\ \forall s : x \twoheadrightarrow y,\ s' : y \twoheadrightarrow z\ \exists s'' : x \twoheadrightarrow z\ \forall u\ (s''(u) \simeq s'(s(u)))$

The *source and target axioms* say that any substitution has a source and a target, and that the target is a complex as well. The *constituents axiom* says that for a given complex, all substitutions are defined for the same objects. The *extensionality of substitutions axiom* says that a substitution is uniquely determined by what objects are substituted for the constituents of a complex. The *identity of substitutions axiom* says that the identity substitution results in the same complex. Finally, the *composition of substitutions axiom* says that substitutions can be composed like partial functions.

Note that by the *constituents axiom* and the *extensionality of substitutions axiom* the substitution s in the *identity of substitutions axiom* is unique. We denote it as id_x. Also the s'' in the *composition of substitutions axiom* is unique and is denoted as $s \cdot s'$.

The new logic may appear more cumbersome than predicate logic. However, the next definition of a simultaneous substitution may make it easier:

Definition 3.1. Define $t[u_1 \mapsto v_1, \ldots, u_n \mapsto v_n]$ as $\mathsf{tgt}(s)$ with

$$\begin{aligned} \mathsf{src}(s) = t \wedge \forall x\ (x \text{ in } t \to & (x = u_1 \to s(x) = v_1) \wedge \\ & \ldots \wedge \\ & (x = u_n \to s(x) = v_n) \wedge \\ & (x \neq u_1 \wedge \ldots \wedge x \neq u_n \to s(x) = x)) \end{aligned}$$

So, $t[u_1 \mapsto v_1, \ldots, u_n \mapsto v_n]$ is the result of simultaneously substituting the entities $v_1, \ldots, v_n$ for $u_1, \ldots, u_n$ in the term t.

As an example, we define a theory for natural numbers with ordering:

Example 3.2.
Constants:

$$\mathsf{src},\ \mathsf{tgt},\ \mathsf{suc01},\ \mathsf{ord01},\ \mathsf{IsNum0}$$

Definitions:

$$\begin{array}{lcl} \mathsf{S}(x,y) & : & \mathsf{E}!\,\mathsf{suc01}[0 \mapsto x, 1 \mapsto y] \\ x < y & : & \mathsf{E}!\,\mathsf{ord01}[0 \mapsto x, 1 \mapsto y] \\ \mathsf{IsNum}(x) & : & \mathsf{E}!\,\mathsf{IsNum0}[0 \mapsto x] \end{array}$$

Axioms: The axioms of the simple logic of complexes, plus:

$$\begin{array}{ll} \exists x & (\mathsf{IsNum}(x)) \\ \forall x,y,z & (\mathsf{S}(x,y) \wedge \mathsf{S}(x,z) \to y = z) \\ \forall y & (\mathsf{IsNum}(y) \wedge \neg(y = 0) \to \exists x\ (\mathsf{IsNum}(x) \wedge \mathsf{S}(x,y))) \\ \forall x,y & (\mathsf{IsNum}(x) \wedge \mathsf{IsNum}(y) \to \\ & \qquad (\exists z\ (\mathsf{IsNum}(z) \wedge \mathsf{S}(y,z) \wedge x < z) \leftrightarrow x < y \vee x = y)) \\ \forall x & (\mathsf{IsNum}(x) \to \neg(x < 0)) \\ \forall x,y & (\mathsf{IsNum}(x) \wedge \mathsf{IsNum}(y) \to x < y \vee x = y \vee y < x) \\ \forall x,y & (\mathsf{IsNum}(x) \wedge \mathsf{IsNum}(y) \wedge x < y \to \neg(y < x)) \\ \forall x,y,z & (\mathsf{IsNum}(x) \wedge \mathsf{IsNum}(y) \wedge \mathsf{IsNum}(z) \wedge x < y \wedge y < z \to x < z) \end{array}$$

⊣

The axioms of the example look like formulas of ordinary predicate logic, but there is a fundamental difference. Here the order disappears when the formulas are written in their basic form. It is worthwhile to note that because we are dealing with a finite number of axioms, it is even possible to get an equivalent theory without any constants at all.

As we saw earlier, the expressive power of minimalistic logic is the same as that of predicate logic. So, the expressiveness of the simple logic of complexes is obviously not greater. Nevertheless, certain properties can be expressed very economically in the simple logic of complexes, as shown in the next example.

Example 3.3. In the logic of complexes it can be expressed in a *single* formula that two entities have the same properties and relations:

$$\forall x\ (x \text{ is a complex} \to \mathsf{E}!\,x[a \mapsto b, b \mapsto a])$$

This is generally not possible in predicate logic, in particular not if the totality of predicate letters are of an infinite number of different degrees. ⊣

The next theorem shows that the expressiveness of the simple logic of complexes is not less than that of predicate logic.

Theorem 3.4. *Validity in free first-order predicate logic is effectively reducible to validity in the simple logic of complexes.*

Proof. Translate formulas of free first-order predicate logic into the simple logic of complexes as follows: For each predicate letter P, define $P(x_1, \dots, x_n)^*$ as

$$\mathsf{E}!\,c_P[d_1 \mapsto x_1, \dots, d_n \mapsto x_n] \wedge$$
$$d_1, \dots, d_n \text{ are the objects of } c_P \wedge x_1, \dots, x_n \text{ are individuals}$$

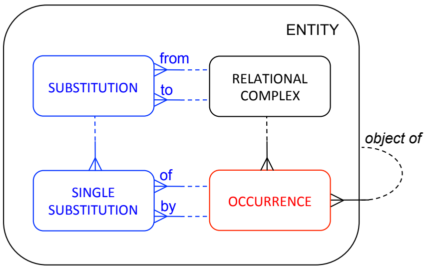
Define $(\varphi \wedge \psi)^*$ as $(\varphi^* \wedge \psi^*)$, $(\neg\varphi)^*$ as $\neg(\varphi^*)$, and $(\forall x\ \varphi)^*$ as

$$\forall x\ (x \text{ is an individual } \rightarrow \varphi^*)$$

Here, *x is an individual* may be defined as $\mathsf{E!}\,\mathsf{ind}[d \mapsto x]$ with constants ind and d. ⊣

3.3 Developing a more advanced logic of complexes

In the simple logic of complexes substitution works directly on the objects of a complex. A more refined logic may be obtained by including *occurrences* of objects:



The logic does not deviate much from the simple logic of complexes. In fact, it can be obtained as an extension. First, add a constant:

$$\mathsf{obj}$$

The constant obj will be interpreted as a partial function, where the intended interpretation of the term '$\mathsf{obj}(x)$' is the object of which the interpretation of x is an occurrence.

Next, add two axioms:

Definedness of occurrences axiom:
$\forall x, y\ (x \text{ in } y \rightarrow \mathsf{E!}\,\mathsf{obj}(x))$

Determinedness of substitutions axiom:
$\forall s, s'\ (\mathsf{src}(s) = \mathsf{src}(s') \wedge \forall u\ (\mathsf{obj}(s(u)) \simeq \mathsf{obj}(s'(u))) \rightarrow \mathsf{tgt}(s) = \mathsf{tgt}(s'))$

The first axiom says that each occurrence is an occurrence of an object, and the second axiom that the result of a substitution is uniquely determined by what objects are substituted for the occurrences in the original complex.

A further extension to a logic involving the notion of *relations* and *states 'out there'* can now be obtained in a fairly straightforward way.

4 Conclusions

The minimalistic logic defined in Section 2 is essentially a stripped version of free predicate logic; in particular, it has no predicate letters. Its main value is as a starting point for developing a logic of relations.

As we have seen, the (simple) logic of complexes has the same expressive power as ordinary predicate logic. But the logic of complexes has a significant advantage: It does not impose an artificial ordering of the arguments of a relation. Now an interesting question is: *Which logic should we use in the future?* Let us make some further comparisons with respect to conceptual simplicity, ease of use, richness, and applicability.

Conceptual simplicity

The logic of complexes seems conceptually not more complicated than predicate logic. Its structures are even simpler; they only consist of a collection of entities, a partial application function, constants and an interpretation for the constants. As a result, the signature also simply consists of constants.

If we compare the framework of the logics, then we see another significant difference. The formulas of the logic of complexes are real 'logical' assertions about the terms, i.e. they are built up only with logical connectives, quantifiers and the equality symbol. On the either hand, in predicate logic we have in addition atomic formulas of the form $P(t_1, \ldots, t_n)$. In my view, this makes the design of the logic of complexes more pure.

Furthermore, the notion of substitution is quite intuitive and the same is true of the axioms of the logic of complexes. Of course we are very used to working with ordered tuples, and this may give us the wrong impression that it is conceptually simple.

Ease of use

At first sight, predicate logic is easier to use. The equivalent of a formula $L(\text{Adam}, \text{Eve})$ in the logic of complexes is a much larger formula involving $\mathsf{src}, \mathsf{tgt}$, $=$, connectives, and quantifiers. However, by making use of abbreviations, the equivalent formula becomes something like

$$\mathsf{E!}\, L_{ab}[a \mapsto \text{Adam}, b \mapsto \text{Eve}]$$

We could even go one step further and simply define $L(x, y)$ as the formula $\mathsf{E!}\, L_{ab}[a \mapsto x, b \mapsto y]$. As suggested by Example 3.2, such definitions may make theories look superficially very similar to ordinary predicate logical theories.

Richness

In higher-order predicate logic quantification is always over predicates, functions and other entities of a *fixed* degree. In the logic of complexes we do not have such a rigid limitation. As a consequence, some properties may be expressed more economically in the logic of complexes. In Example 3.3 we saw that in the logic of complexes it can always be expressed with a *single* formula that two objects have the same relations, but that this is generally not possible in predicate logic.

Another strong point of the logic of complexes is that complexes may have an infinite degree, i.e. that they may have an infinite number of relata. We might, for example, define a complex N for which $\forall x\, (\text{Is\_Number}(x) \leftrightarrow x \text{ in } N)$.

Applicability
The new logic is not only suitable for natural relations 'out there' like the love relation. I expect it is equally suitable for mathematical relations. In my view it is a mistake to say that in mathematics we have both a *less-than* relation and a *greater-than* relation. It seems much better to say that we have a *single* strict ordering relation consisting of complexes in which one relatum fulfills the role of *greater* and the other relatum the role of *lesser*. This can perfectly be expressed in the logic of complexes, as shown in Example 3.2.
The logic of complexes may offer ingredients for the development of programming languages and programming concepts in which the general notion of substitution fulfils a central role. This is of particular interest because it may lead to programs with a simpler internal structure than conventional ones. Maybe *complex-oriented programming* should be the new paradigm.
Another interesting application will be the development of a philosophically driven alternative for set theory. There is a substantial need for this, since we do not live in a world of sets, but in a world of things with relations between them. Although almost everything can be *coded* in set theory, the coding is in some cases quite artificial.[1]

Finally, I would like to mention one more advantage of the new logic. It may facilitate *coordinate-free thinking*. If complex thoughts can be expressed in a (formal) language without reference to any particular coordinate system, then this will be of great heuristic value.

In conclusion, there are strong arguments in favor of the logic of complexes. Further research is needed to investigate the promising possibilities.

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[1] An ordered pair $\langle a, b\rangle$, for example, may be coded in set theory as the set $\{\{a\}, \{a, b\}\}$ (Kuratowski's definition), or as $\{\{\{a\}, \varnothing\}, \{\{b\}\}\}$ (Wiener's definition), or as $\{\{a, 1\}, \{b, 2\}\}$ (Hausdorff's definition).

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