Annex 2. Forty years of working on mathematics education, seeing mathematics as a human activity for all – The Freudenthal Institute

(With the collaboration of Marja van den Heuvel-Panhuizen[2])

Since the late 1960's the Freudenthal Institute of Utrecht University has worked on the improvement of mathematics education. The inspiration for this work lays in the profound belief that the global community of mathematics education developers and researchers –including the Freudenthal Institute staff – has the responsibility for providing students of all ages – to begin with the very young children in pre-school settings – with the best possible learning environments for developing mathematical skills and concepts. The relevance of learning mathematics is not only that it is necessary for supporting life on earth with sustainable technology and economy, but also that acquiring mathematical competence is of high personal value. Mathematics is, together with reading and writing, one of the key human competences through which people can express themselves as human beings and can understand the world around them. Freudenthal's idea of "mathematics as a human activity" and his quintessential and ambitious goal of "mathematics for all" have always been the Institute's guiding principles for researching and developing mathematics education. This resulted in a domain-specific didactical theory that nowadays is called "Realistic Mathematics Education" for which Treffers and his colleagues of the former "Wiskobas" group laid the foundation.

"Mathematics as a human activity" means the opposite of the traditional transition approach to teaching mathematics. Instead of teaching students ready-made mathematics, they should be given the opportunity to develop mathematics by a process of guided-reinvention. This means that the teacher plays a strong pro-active role by creating a stimulating and supportive learning environment. "Mathematics for all" means making mathematics accessible for students at all intellectual levels. This requires a deep awareness of mathematics as more than just the most abstract way of thinking. Even students with lower potential in learning mathematics can use mathematics to solve problems by using informal context-connected strategies. The

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heterogeneity of students poses a real challenge to education. "Mathematics for all" also includes the high-ability students.

The work at the Freudenthal Institute characterizes a synthesis of theoretical expertise, empirical knowledge and practice-based experience in the area of learning and teaching mathematics. A multi-disciplinary approach in connection with a strong intertwinement of theoretical and practical perspectives can be found in the design and research methods, as well as in the qualities of the staff involved. Many Freudenthal Institute staff members are researchers as well as curriculum and software designers or are still active as teacher educators or teacher advisors. Moreover, in many projects, staff members collaborate with mathematics teachers and others from school practice. These projects cover a wide scope of educational settings ranging from pre-school education, primary school education, general and vocational secondary school education and higher education. They even include outsideschool settings where learning takes place. Essential is that the Freudenthal Institute has chosen for an integrated approach in which research and development, as well as teacher education and implementation through professional development of in-service teachers are closely connected. Moreover, the Freudenthal Institute has always had a good relationship with textbook authors who were free to use the Institute's ideas for teaching mathematics and who, in turn, contributed to the reform movement in mathematics education which has come into being in the last decennia.

1 In 2006 the previous Freudenthal Institute which had its focus only on mathematics education merged with the groups for physics education, chemistry education and biology education which resulted in the "Freudenthal Institute for Science and Mathematics Education" as part of the Faculty of Science of Utrecht University. The present text discusses the mathematics education department of this new Freudenthal Institute.

2 The ultimate goal of the work at the Freudenthal Institute is the enhancement of the quality of learning and teaching mathematics. The quality of learning signifies that the subject matter knowledge and skills that students have to learn should be meaningful. This means that students have to acquire relevant knowledge and skills, both for the present and for future studies and employment. Another aspect that indicates the quality of learning is whether that learning is effective in terms of its results. Important is also that the learning enables transfer and that students can apply the acquired knowledge and skills to new problems and new situations in and outside school. The quality of education implies having adequate instructional settings, didactical models and tools available in order to achieve a high output in terms of students' competences and attitudes in mathematics.

Last but not least, the quality of teachers is an important factor in mathematics education, both in realizing education and in innovating education. The former is more related to educating prospective teachers and the latter to counselling experienced teachers, however both should be considered in close connection to each other.

Although the reform of mathematics education was quite successful in the Netherlands in terms of results in international comparisons of students achievements (TIMSS and PISA), it was quite remarkable that the process of renewal took place with almost no direct intervention of the Dutch government. Instead, it was the community of mathematics education developers and researchers, mathematics educators and school advisors, textbook authors, and school inspectors who supported the reform and who all made this process happen together with the teachers. If the government played a part in this process, then it was that they funded the setting and maintenance of the infrastructure in mathematics education. This gave teachers and all the others involved in the reform process conferences to meet and websites and journals to share ideas, experiences and materials.

Creating an opportunity for the mathematics education community to work on their own development did not only bring in ownership, but also resulted in a positive cost-benefit ratio. Despite high yields in terms of the students' achievements, for many years the Dutch government made very low expenditures available for education compared to the governments in other countries. This self-contained process of educational reform is not without dangers. For example, the reform process did not generate a national system for professional development of in-service teachers. With respect to this, the TIMSS 2007 report came with hard facts. The professional development in the Netherlands, in contrast to many other countries, ended up at a very bottom position. Not having such a system for professional development is a serious threat of the quality of education. Especially in primary school, where teachers are all subject teachers, it is an absolute necessity that teachers take refresher courses. This is particularly true with respect to the use of new technology in mathematics education.

More than any other development, Information and Communication Technology (ICT) has proven to make a true contribution to the further development of mathematics education. In fact, ICT lies at the heart of mathematics teaching because it gives didactical models such as the number line and the fraction bar a new dynamic life. Moreover, ICT offers students possibilities for modelling problem situations by themselves and creates an environment in which students are encouraged to ask themselves and others questions, to test ideas and find proofs for their conjectures. These activities are what 'doing mathematics' is truly about. They give students a more active role in learning mathematics. This is especially the case where (mini-) games are brought into action in

mathematics education. Software with game characteristics has powerful opportunities to change students' attitudes to mathematics and can bring in rehearsal in a natural way which is crucial for raising the students' mastery level of basic mathematical skills. Finally, internet and mobile technology open a new avenue for distance learning and assessment, and can be a powerful tool to make the latest didactical developments accessible for teachers. Therefore, at the Freudenthal Institute, ICT is a key theme in researching and developing mathematics education. Using ICT to overcome geographical barriers opens a whole novel range of possibilities to realize the idea of mathematics for all.

3 The work of the Freudenthal Institute did not escape international notice. Many colleagues from abroad showed interest in the work of the Institute. Often this led to joint projects and collaboration. Since Freudenthal's time, the Institute has been an open research community that collaborates with researchers from various countries and that has a steady stream of international visitors. Except sharing ideas and experiences and jointly carrying out research or development projects, this collaboration also implies that the Freudenthal Institute makes available instructional materials, such as longitudinal learning-teaching trajectories, models of lesson series and ICT-based mini-games, to be used in other countries. Of course, this is not a matter of translation and distribution. Teaching materials developed for Dutch schools cannot be just simply dropped in another country. Adaptation is necessary and this requires a careful process in which the country's culture-specific approach to education is taken into account. This is also the case in the following examples of "Realistic Mathematics Education in transit". Mathematics in Context was one of the major international projects of the Freudenthal Institute that was carried out with the Wisconsin Center for Education Research (WCER) of the University of Wisconsin-Madison in the USA. In this project the Freudenthal Institute developed the draft version of a mathematics textbook series for the American middle grades. This textbook series got its final form through a process of piloting and revision in which the researchers from Madison took the lead.

For more information see:
http://info.eb.com/html/print_math_in_context.html;
http://www.project2061.org/publications/textbook/mgmth/report/2context/info.htm;
http://www.marketwire.com/press-release/Encyclopaedia-Britannica-Inc-985121.html.

The TAL project which the Freudenthal Institute carried out commissioned by the Dutch Ministry of Education was aimed at developing longitudinal learning-teaching trajectories for mathematics education in primary school in order to give teachers an overview of how the learning of mathematics proceeds over the primary grades and to bring coherence in the curriculum. Because several countries attached significance to these trajectories the materials were translated into English. This resulted in new projects.

Together with the University of Cape Town (UCT) and the Cape Peninsula University of Technology (CAPUT), the Count One Count All (COCA) project was started (http://www.fi.uu.nl/coca/). One of the goals of this project—which was funded by the South Africa-Netherlands Research Programme on Alternatives in Development (SANPAD)—was to provide South African teachers in the foundation phase of primary school with a document that describes the learning pathway for number. The development of this learning pathway was based on TAL and is connected to the South African National Curriculum Statement.

Furthermore, an educational publisher in Mexico is working on a Spanish version of TAL which, among other things, is already used for professional development of teachers in Argentina (http://www.fi.uu.nl/nl/Poster_TAL-Alta-final.pdf).

Another example of "Realistic Mathematics Education in transit" is worked out at the Development Institute of Pendidikan Matematika Realistik Indonesia (http://www.pmri.or.id/). The main mission of IP-PMRI is to improve the quality of mathematics education in Indonesia by implementing an Indonesian version of Realistic Mathematics Education. In the DO-PMRI project, IP-PMRI works together with the Freudenthal Institute, APS, and the Indonesian Ministry of Education to enhance the quality of mathematics teachers in a participative way, providing them with teaching tools and methodologies that give Indonesian students better chances to appreciate and understand mathematics.

A last example of a project that goes beyond the boundaries of the Netherlands is the Th!nklets project in which mini-games are developed for primary and secondary education. These mini-games can be downloaded for free (http://www.fi.uu.nl/thinklets/) and are very popular, especially among children aged 8-12. Presently the Freudenthal Institute is working on making some of these mini-games suitable for playing on the XO laptop.

For more information on the work of the Freudenthal Institute, see http://www.fi.uu.nl/nl/brochureFIsme.pdf.