

Quantum gravitational effects on massive fermions during inflation

S. P. Miao*

*Institute for Theoretical Physics, Spinoza Institute, University of Utrecht Luevenlaan 4,
Postbus 80.195, 3508TD Utrecht, Netherlands*

(Received 22 July 2012; published 20 November 2012)

We compute the one loop graviton contribution to the self-energy of a very light fermion on a locally de Sitter background. This result can be used to study the effect that a small mass has on the propagation of fermions through the sea of infrared gravitons generated by inflation. We employ dimensional regularization and obtain a fully renormalized result by absorbing all divergences with Bogliubov-Parasuk-Hepp-Zimmermann counterterms. An interesting technical aspect of this computation is the need for two noninvariant counterterms owing to the breaking of de Sitter invariance by our gauge condition.

DOI: [10.1103/PhysRevD.86.104051](https://doi.org/10.1103/PhysRevD.86.104051)

PACS numbers: 04.30.Nk, 04.62.+v, 98.80.Cq, 98.80.Qc

I. INTRODUCTION

In this paper we compute and renormalize the one-loop quantum gravitational corrections to the self-energy of very light fermions on a locally de Sitter background. The physical motivation for this exercise is to facilitate a later study of how inflationary gravitons affect fermions and, in particular, the contrast between the case of exactly massless fermions and those with a small mass. Nonzero mass introduces two competing effects: it changes how fermions propagate, and it also alters how they interact with gravity. The first of these changes tends to suppress the effects of inflationary gravitons because it makes the fermion wave function oscillate so that interactions at different times tend to cancel. However, the new interaction enhances the effect of inflationary gravitons because it does not fall off with time.

The current work can be seen as complementing two previous studies of massless fermions on a de Sitter background. In both cases the technique was to compute the one-loop fermion self-energy $-i[\Sigma_j](x; x')$ and then use it to solve the quantum-corrected Dirac equation for fermion mode functions,

$$\sqrt{-g}i\mathcal{D}_{ij}\psi_j(x) - \int d^4x'[\Sigma_j](x; x')\psi_j(x') = 0. \quad (1)$$

The first model results from Yukawa coupling the fermion to a massless, minimally coupled (MMC) scalar on a non-dynamical de Sitter background [1]. The second model consists of the fermion with dynamical gravity on de Sitter background [2]. Powers of the inflationary scale factor $a = e^{Ht}$ are crucial for understanding the results in both cases. The self-energy from the $\phi\bar{\psi}\psi$ interaction of the first model grows like $a \ln(a)$ relative to the classical term. The resulting fermion mode functions behave as if they had a growing mass. The interactions of the second model all possess derivatives—for example, $\partial h\bar{\psi}\psi$ —which limit the induced self-energy to grow no faster

than $\ln(a)$ relative to the classical term. The resulting fermion mode functions behave as if they had a growing field strength, which could be understood as the random walk that fermions take under buffeting from the sea of inflationary gravitons [2,3]. Although the effect from gravitons is smaller than that from massless scalars, *it is universal*, independent of assumptions about the existence or couplings of unnaturally light scalars. It is even conceivable that the graviton effect might, in a more complicated model, lead to baryogenesis during inflation.

What we expect for massive fermions in dynamical gravity is that the absence of derivatives in the $amh\bar{\Psi}\Psi^1$ interaction will cause the self-energy to grow like $a \ln(a)$ relative to the classical kinetic term, and like $\ln(a)$ relative to the classical mass term,

$$\sqrt{-g}[i\mathcal{D} - am]\Psi_j(x) - \int d^4x'[\Sigma_j](x; x')\Psi_j(x') = 0. \quad (2)$$

When the classical mass is large (relative to the Hubble parameter), we expect at most a small enhancement of the fermion field strength. When the classical mass is small, classical dynamics are mostly controlled by the kinetic term and we expect the quantum correction to have a much larger proportional impact. One might intuitively expect the crossover to come for fermion masses near the Hubble parameter. However, we shall specialize to the case of very light fermions, both because this is where the largest effects should occur, and because expanding in the fermion mass makes an enormous simplification in the computation.

This work also deserves a place in the growing list of studies of quantum infrared effects during inflation. Among these are the following:

- (i) The effects of self-interacting, MMC scalars on non-dynamical de Sitter background [4–7].

*s.miao@uu.nl

¹In Sec. II we rescale fermion fields and the metric $\Psi = a^{\frac{p-1}{2}}\psi$; $g_{\mu\nu} = a^2\tilde{g}_{\mu\nu}$.

- (ii) The effects of a charged, MMC scalar on non-dynamical de Sitter background [8,9].
- (iii) The effects of a nonlinear sigma model on non-dynamical de Sitter background [10,11].
- (iv) The effects of a MMC scalar on gravitons on de Sitter background [12,13].
- (v) The effects of gravitons on a MMC scalar on de Sitter background [14].
- (vi) The effects of gravitons on interacting conformal matter on de Sitter background [15,16].

It should also be noted that the series of leading infrared logarithms can be summed for scalar potential models using the stochastic technique of Starobinsky and Yokoyama [17]. The same resummation can be achieved for Yukawa theory [18], and for scalar QED [19], but it has so far not been accomplished for either the nonlinear sigma model [10,11] or for quantum gravity [20]. Each fully renormalized quantum gravitational result is an important piece of *data* in the search for such a resummation.

Although Dirac + Einstein is not perturbatively renormalizable [21], ultraviolet divergences can always be absorbed in the Bogliubov-Parasuk-Hepp-Zimmermann (BPHZ) sense [22–25]. A widespread misconception exists that no valid quantum predictions can be extracted from such a theory. This is not true: while nonrenormalizability does preclude being able to compute *everything*, that is not the same thing as being able to compute *nothing*. The problem with a nonrenormalizable theory is that no physical principle fixes the finite parts of the escalating series of BPHZ counterterms needed to absorb ultraviolet divergences, order by order in perturbation theory. Hence any prediction of the theory that can be changed by adjusting the finite parts of these counterterms is essentially arbitrary. However, loops of massless particles make nonlocal contributions to the effective action that can never be affected by local counterterms. These nonlocal contributions typically dominate the infrared. Further, they cannot be affected by whatever modification of ultraviolet physics ultimately results in a completely consistent formalism. As long as the eventual fix introduces no new massless particles, and does not disturb the low energy couplings of the existing ones, the far infrared predictions of a BPHZ-renormalized quantum theory will agree with those of its fully consistent descendant.

It is worth mentioning the many studies that have exploited this basic facet of low energy effective field theory. The oldest example is the solution of the infrared problem in quantum electrodynamics by Bloch and Nordsieck [26], long before that theory's renormalizability was suspected. Weinberg [27] was able to achieve a similar resolution for quantum gravity with zero cosmological constant. The same principle was at work in the Fermi theory computation of the long range force due to loops of massless neutrinos by Weinberg and Sucher [28,29]. Matter that is not supersymmetric generates nonrenormalizable corrections to the graviton propagator at one loop, but this did not prevent the

computation of photon, massless neutrino, and massless, conformally coupled scalar loop corrections to the long range gravitational force [30–33]. The same principles of low energy effective field theory have been applied to compute graviton loop corrections to the long range force by Donoghue [34,35] and many others [36–40].

That summarizes why the exercise we have undertaken is both valid and interesting. The necessary Feynman rules are given in Secs. II, III, and IV. Because some of these are the same as for the previous study of massless fermions [2,3], we merely present the relevant old results and reserve discussion for the new features associated with a nonzero fermion mass. Section II covers the fermionic sector that gives the fermion propagator and the interactions. Section III presents the graviton propagator. The BPHZ counterterms necessary for our computations are carefully analyzed in Sec. IV. In Sec. V we evaluate the contributions from diagrams involving a single 4-point interaction. In Sec. VI we evaluate the more difficult contributions that involve two 3-point interactions. Renormalization is accomplished in Sec. VII, and our conclusions are given in Sec. VIII.

II. FEYNMAN RULES FOR A MASSIVE DIRAC THEORY

In this section we derive the Feynman rules to facilitate the computation we are going to perform. To obtain three-point vertices, four-point vertices, and the massive fermion propagator we first start with the Lagrangian of a massive Dirac theory,

$$\mathcal{L}_{\text{Dirac}} \equiv \bar{\psi} e^\mu{}_b \gamma^b i \mathcal{D}_\mu \psi \sqrt{-g} - m \bar{\psi} \psi \sqrt{-g}. \quad (3)$$

Here the vierbein $e^\mu{}_b$ consists of coordinate indices expressed by Greek letters and Lorentz indices denoted by Latin letters; and \mathcal{D}_μ is the covariant derivative that is formed by the spin connection and the Dirac Lorentz representation matrices,

$$\mathcal{D}_\mu \equiv \partial_\mu + \frac{i}{2} A_{\mu cd} J^{cd}, \quad (4)$$

$$A_{\mu cd} = e^\nu{}_c (e_{\nu d, \mu} - \Gamma^\rho{}_{\mu\nu} e_{\rho d}), \quad (5)$$

$$J^{bc} \equiv \frac{i}{4} (\gamma^b \gamma^c - \gamma^c \gamma^b).$$

Because our locally de Sitter background is conformally flat, it is useful to rescale the vierbein by an arbitrary function of spacetime $a(x)$,

$$e_{\beta b} \equiv a \tilde{e}_{\beta b} \Rightarrow g_{\mu\nu} = a^2 \tilde{g}_{\mu\nu}. \quad (6)$$

Hence one can express the old connections in terms of the ones formed from the rescaled fields,

$$\Gamma^\rho{}_{\mu\nu} = a^{-1} (\delta^\rho{}_\mu a_{,\nu} + \delta^\rho{}_\nu a_{,\mu} - \tilde{g}^{\rho\sigma} a_{,\sigma} \tilde{g}_{\mu\nu}) + \tilde{\Gamma}^\rho{}_{\mu\nu}, \quad (7)$$

$$A_{\mu cd} = -a^{-1} (\tilde{e}^\nu{}_c \tilde{e}_{\mu d} - \tilde{e}^\nu{}_d \tilde{e}_{\mu c}) a_{,\nu} + \tilde{A}_{\mu cd}, \quad (8)$$

and redefined fermion fields $\Psi \equiv a^{\frac{D-1}{2}} \psi$ to simplify the Lagrangian,

$$\mathcal{L}_{\text{Dirac}} = \bar{\Psi} \tilde{e}^\mu_b \gamma^b i \tilde{D}_\mu \Psi \sqrt{-\tilde{g}} - am \bar{\Psi} \Psi \sqrt{-\tilde{g}}, \quad (9)$$

where $\tilde{D}_\mu \equiv \partial_\mu + \frac{i}{2} \tilde{A}_{\mu cd} J^{cd}$.

We perturb the metric as

$$\tilde{g}_{\mu\nu} \equiv \eta_{\mu\nu} + \kappa h_{\mu\nu} \quad \text{with} \quad \kappa^2 = 16\pi G. \quad (10)$$

We then fix the local Lorentz gauge freedom by imposing symmetric gauge ($e_{\beta b} = e_{b\beta}$), and solve for the vierbein in terms of the graviton [41],

$$\begin{aligned} \tilde{e}[\tilde{g}]_{\beta b} &\equiv (\sqrt{\tilde{g}} \eta^{-1})_\beta^\gamma \eta_{\gamma b} \\ &= \eta_{\beta b} + \frac{1}{2} \kappa h_{\beta b} - \frac{1}{8} \kappa^2 h_\beta^\gamma h_{\gamma b} + \dots \end{aligned} \quad (11)$$

At this stage there is no more point in distinguishing between Latin letters for local Lorentz indices and Greek letters for vector indices. Other conventions are that graviton indices are raised and lowered with the Lorentz metric ($h^\mu_\nu \equiv \eta^{\mu\rho} h_{\rho\nu}$, $h^{\mu\nu} \equiv \eta^{\mu\rho} \eta^{\nu\sigma} h_{\rho\sigma}$) and that the trace of the graviton field is $h \equiv \eta^{\mu\nu} h_{\mu\nu}$. We also employ the usual Dirac *slash* notation,

$$\not{V}_{ij} \equiv V_\mu \gamma_{ij}^\mu. \quad (12)$$

Therefore one can expand all familiar operators accordingly in powers of graviton field and obtain the perturbed, conformally rescaled Dirac Lagrangian,

$$\begin{aligned} \mathcal{L}_{\text{Dirac}} &= \bar{\Psi} [i\not{\partial} - am] \Psi + \frac{\kappa}{2} \bar{\Psi} [hi\not{\partial} - h^{\mu\nu} \gamma_\mu i\partial_\nu - h_{\mu\rho, \sigma} \gamma^\mu J^{\rho\sigma} - amh] \Psi + \kappa^2 \left\{ \left[\frac{1}{8} h^2 - \frac{1}{4} h^{\rho\sigma} h_{\rho\sigma} \right] \bar{\Psi} i\not{\partial} \Psi \right. \\ &+ \left[-\frac{1}{4} h h^{\mu\nu} + \frac{3}{8} h^{\mu\rho} h_{\rho}^\nu \right] \bar{\Psi} \gamma_\mu i\partial_\nu \Psi + \left[-\frac{1}{4} h h_{\mu\rho, \sigma} + \frac{1}{8} h^\nu_\rho h_{\nu\sigma, \mu} + \frac{1}{4} (h^\nu_\mu h_{\nu\rho})_{, \sigma} + \frac{1}{4} h^\nu_\sigma h_{\mu\rho, \nu} \right] \bar{\Psi} \gamma^\mu J^{\rho\sigma} \Psi \\ &\left. - am \left[\frac{1}{8} h^2 - \frac{1}{4} h^{\alpha\beta} h_{\alpha\beta} \right] \bar{\Psi} \Psi \right\} + \mathcal{O}(\kappa^3). \end{aligned} \quad (13)$$

From the quadratic operator we see that the rescaled massive fermion propagator can be connected to the solution of Candelas and Raine [18,42] up to some powers of scale factors,

$$iS[m](x; x')_{\text{C.R.}} = (aa')^{-\frac{D-1}{2}} iS[m](x; x'). \quad (14)$$

Therefore the conformally rescaled fermion propagator is

$$\begin{aligned} iS[m](x; x') &= \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \Gamma\left(\frac{D}{2} - 1\right) (aa')^{\frac{D-1}{2}} \left(ai\not{\mathcal{D}} \frac{1}{\sqrt{aa'}} + \sqrt{\frac{a}{a'}} mI \right) \\ &\times \left\{ \frac{\Gamma(\frac{D}{2} - 1 + i\frac{m}{H}) \Gamma(\frac{D}{2} - i\frac{m}{H})}{\Gamma(\frac{D}{2} - 1) \Gamma(\frac{D}{2})} {}_2F_1\left(\frac{D}{2} - 1 + i\frac{m}{H}, \frac{D}{2} - i\frac{m}{H}; \frac{D}{2}; 1 - \frac{y}{4}\right) \left(\frac{I - \gamma^0}{2}\right) \right. \\ &\left. + \frac{\Gamma(\frac{D}{2} - 1 - i\frac{m}{H}) \Gamma(\frac{D}{2} + i\frac{m}{H})}{\Gamma(\frac{D}{2} - 1) \Gamma(\frac{D}{2})} {}_2F_1\left(\frac{D}{2} - 1 - i\frac{m}{H}, \frac{D}{2} + i\frac{m}{H}; \frac{D}{2}; 1 - \frac{y}{4}\right) \left(\frac{I + \gamma^0}{2}\right) \right\}. \end{aligned} \quad (15)$$

Here $i\not{\mathcal{D}}$ is just $a^{-(\frac{D+1}{2})} i\not{\partial} a^{(\frac{D-1}{2})}$ and a de Sitter invariant length function y is formed by the following function of the invariant length $\ell(x; x')$ between x^μ and x'^μ ,

$$y(x; x') \equiv 4 \sin^2\left(\frac{1}{2} H \ell(x; x')\right) = aa' H^2 \Delta x^2(x; x'), \quad (16)$$

$$= aa' H^2 (\|\tilde{x} - \tilde{x}'\|^2 - (|\eta - \eta'| - i\delta)^2). \quad (17)$$

It is useful to recast the solution (15) using the transformation formula for hypergeometric functions [43] and then expand it in powers of y ,

$$\begin{aligned} iS[m](x; x') &= \frac{\Gamma(\frac{D}{2} - 1)}{4\pi^{\frac{D}{2}}} [i\not{\partial} + am] \frac{1}{\Delta x^{D-2}} + \frac{(H^2 aa')^{\frac{D-1}{2}}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(\frac{D}{2} - 1) \Gamma(2 - \frac{D}{2}) (i\frac{m}{H})}{\Gamma(1 + i\frac{m}{H}) \Gamma(1 - i\frac{m}{H})} \left[i\not{\partial} + \left(\frac{D}{2} - 1\right) iHa\gamma^0 + am \right] \\ &\times \sum_{n=0}^{\infty} \left\{ \frac{\Gamma(n + \frac{D}{2} - 1 + i\frac{m}{H}) \Gamma(n + \frac{D}{2} - 1 - i\frac{m}{H})}{\Gamma(n + \frac{D}{2} - 1) \Gamma(n + 1)} \left[\frac{i\frac{m}{H}}{(n + \frac{D}{2} - 1)} + \gamma^0 \right] \left(\frac{y}{4}\right)^n \right. \\ &\left. - \frac{\Gamma(n + 1 + i\frac{m}{H}) \Gamma(n + 1 - i\frac{m}{H})}{\Gamma(n + 1) \Gamma(n + 3 - \frac{D}{2})} \left[\frac{i\frac{m}{H}}{(n + 1)} + \gamma^0 \right] \left(\frac{y}{4}\right)^{n+2-\frac{D}{2}} \right\}. \end{aligned} \quad (18)$$

Because we only endow fermions with a very small mass compared with the Hubble parameter, for the computation purpose we simplify the infinite series expansion by only keeping terms at order m ,

$$i[S_j](x; x') = i[S_j]_{\text{cf}}(x; x') + i[S_j]_{\text{fm}}(x; x') - \left(\frac{m}{H}\right) \frac{(H^2 a a')^{\frac{D}{2}-1}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(\frac{D}{2}-1)\Gamma(3-\frac{D}{2})}{(2-\frac{D}{2})} \left[\not{x} \gamma^0 + \left(\frac{D}{2}-1\right) H a \right] \\ \times \sum_{n=0}^{\infty} \left[\frac{\Gamma(n+\frac{D}{2}-1)}{\Gamma(n+1)} \left(\frac{y}{4}\right)^n - \frac{\Gamma(n+1)}{\Gamma(n+3-\frac{D}{2})} \left(\frac{y}{4}\right)^{n+2-\frac{D}{2}} \right] + \mathcal{O}(m^2). \quad (19)$$

The explicit expression for the first two terms in (19) are

$$i[S_j]_{\text{cf}}(x; x') = \frac{\Gamma(\frac{D}{2}-1)}{4\pi^{\frac{D}{2}}} i \not{x}_{ij} \frac{1}{\Delta x^{D-2}}, \quad (20)$$

$$i[S_j]_{\text{fm}}(x; x') = \frac{\Gamma(\frac{D}{2}-1)}{4\pi^{\frac{D}{2}}} \frac{ma}{\Delta x^{D-2}}. \quad (21)$$

Here *cf* stands for *conformal* and *fm* stands for *flat space-time mass*.² Even though the two infinite series expansions in (19) tend to cancel out with each other in $D = 4$, the combinations are still finite owing to the divergent factor $\frac{1}{(2-\frac{D}{2})}$. In addition, they cannot be reduced to an elementary function. These facts complicate the computation.

We now represent the various interaction terms in (13) as vertex operators acting on the fields. At order κ the interactions involve fields, $\bar{\Psi}_i$, Ψ_j , and $h_{\alpha\beta}$, which we number 1, 2, and 3, respectively. Each of the three interactions can be written as some combination $V_{lij}^{\alpha\beta}$ of tensors, spinors, and a derivative operator acting on these fields. For example, the first interaction is

$$\frac{\kappa}{2} h \bar{\Psi}_i \not{x} \Psi = \frac{\kappa}{2} \eta^{\alpha\beta} i \not{x}_{2ij} \times \bar{\Psi}_i \Psi_j h_{\alpha\beta} \\ \equiv V_{lij}^{\alpha\beta} \times \bar{\Psi}_i \Psi_j h_{\alpha\beta}. \quad (22)$$

Hence the 3-point vertex operators are

$$V_{lij}^{\alpha\beta} = \frac{\kappa}{2} \eta^{\alpha\beta} i \not{x}_{2ij}, \quad V_{2ij}^{\alpha\beta} = -\frac{\kappa}{2} \gamma_{ij}^{(\alpha} i \partial_2^{\beta)}, \\ V_{3ij}^{\alpha\beta} = -\frac{\kappa}{2} (\gamma^{(\alpha} J^{\beta)\mu})_{ij} \partial_{3\mu}, \quad V_{4ij}^{\alpha\beta} = -\frac{\kappa}{2} a m \eta^{\alpha\beta} I_{ij}. \quad (23)$$

The order κ^2 interactions define 4-point vertex operators $U_{lij}^{\alpha\beta\rho\sigma}$ similarly, for example,

$$\frac{1}{8} \kappa^2 h^2 \bar{\Psi}_i \not{x} \Psi = \frac{1}{8} \kappa^2 \eta^{\alpha\beta} \eta^{\rho\sigma} i \not{x}_{2ij} \times \bar{\Psi}_i \Psi_j h_{\alpha\beta} h_{\rho\sigma} \\ \equiv U_{lij}^{\alpha\beta\rho\sigma} \times \bar{\Psi}_i \Psi_j h_{\alpha\beta} h_{\rho\sigma}. \quad (24)$$

The ten 4-point vertex operators are given in Table I. Note that we do not bother to symmetrize upon the identical graviton fields.

²This term even at $a = 1$ does not stand for the full mass term in flat space. It is actually the most singular term at order m .

III. GRAVITON PROPAGATOR

In this section we briefly sketch how to obtain the graviton propagator and presently give the explicit expression for it. The low energy effective field theory for gravity is Einstein-Hilbert,

$$\mathcal{L}_{\text{E-H}} \equiv \frac{1}{16\pi G} (R - (D-2)\Lambda) \sqrt{-g}. \quad (25)$$

We follow the same convention as the fermion sector to rescale the metric (6) and connections (7) for gravity even though it is not conformally invariant. In order to obtain the graviton propagator we also need to fix a . We work on the open conformal coordinate patch of de Sitter, which implies,

$$ds^2 = a^2(-d\eta^2 + d\vec{x} \cdot d\vec{x}), \quad \text{where } a(\eta) = -\frac{1}{H\eta}, \quad (26)$$

and the D -dimensional Hubble constant is $H \equiv \sqrt{\Lambda/(D-1)}$. Note that the conformal time η runs from $-\infty$ to zero. For this choice of scale factor we can extract a surface term from the invariant Lagrangian and write it in the form [44],

TABLE I. Vertex operators $U_{lij}^{\alpha\beta\rho\sigma}$ contracted into $\bar{\Psi}_i \Psi_j h_{\alpha\beta} h_{\rho\sigma}$.

Number	Vertex operator
1	$\frac{1}{8} \kappa^2 \eta^{\alpha\beta} \eta^{\rho\sigma} i \not{x}_{2ij}$
2	$-\frac{1}{4} \kappa^2 \eta^{\alpha\rho} \eta^{\sigma\beta} i \not{x}_{2ij}$
3	$-\frac{1}{4} \kappa^2 \eta^{\alpha\beta} \gamma_{ij}^{\rho} i \partial_2^{\sigma}$
4	$\frac{3}{8} \kappa^2 \eta^{\alpha\rho} \gamma_{ij}^{\beta} i \partial_2^{\sigma}$
5	$-\frac{1}{4} \kappa^2 \eta^{\alpha\beta} (\gamma^{\rho} J^{\sigma\mu})_{ij}$
6	$\frac{1}{8} \kappa^2 \eta^{\alpha\rho} (\gamma^{\mu} J^{\beta\sigma})_{ij} \partial_{4\mu}$
7	$\frac{1}{4} \kappa^2 \eta^{\alpha\rho} (\gamma^{\beta} J^{\sigma\mu})_{ij} (\partial_3 + \partial_4)_{\mu}$
8	$\frac{1}{4} \kappa^2 (\gamma^{\rho} J^{\sigma\alpha})_{ij} \partial_4^{\beta}$
9	$-\frac{1}{8} \kappa^2 \eta^{\alpha\beta} \eta^{\rho\sigma} a m$
10	$\frac{1}{4} \kappa^2 \eta^{\alpha\rho} \eta^{\sigma\beta} a m$

$$\begin{aligned}\mathcal{L}_{\text{E-H-Surface}} = & \left(\frac{D}{2} - 1\right) H a^{D-1} \sqrt{-\tilde{g}} \tilde{g}^{\rho\sigma} \tilde{g}^{\mu\nu} h_{\rho\sigma, \mu} h_{\nu 0} \\ & + a^{D-2} \sqrt{-\tilde{g}} \tilde{g}^{\alpha\beta} \tilde{g}^{\rho\sigma} \tilde{g}^{\mu\nu} \\ & \times \left\{ \frac{1}{2} h_{\alpha\beta, \rho} h_{\sigma\mu, \nu} - \frac{1}{2} h_{\alpha\beta, \rho} h_{\sigma\mu, \nu} \right. \\ & \left. + \frac{1}{4} h_{\alpha\beta, \rho} h_{\mu\nu, \sigma} - \frac{1}{4} h_{\alpha\beta, \mu} h_{\rho\sigma, \nu} \right\}. \quad (27)\end{aligned}$$

There has been a long controversy about the graviton propagator in de Sitter space which is discussed in Refs. [45–48]. It turns out that one cannot add a gauge fixing term that preserves all de Sitter symmetries, so we break spatial special conformal transformations³ with the term

$$\begin{aligned}\mathcal{L}_{GF} = & -\frac{1}{2} a^{D-2} \eta^{\mu\nu} F_\mu F_\nu, \\ F_\mu \equiv & \eta^{\rho\sigma} \left(h_{\mu\rho, \sigma} - \frac{1}{2} h_{\rho\sigma, \mu} + (D-2) H a h_{\mu\rho} \delta_\sigma^0 \right).\end{aligned} \quad (28)$$

The quadratic part of $\mathcal{L}_{\text{E-H}} + \mathcal{L}_{GF}$ can be partially integrated to take the form $\frac{1}{2} h^{\mu\nu} D_{\mu\nu}{}^{\rho\sigma} h_{\rho\sigma}$. Hence one can solve the graviton propagator accordingly,

$$D_{\mu\nu}{}^{\rho\sigma} \times i[\rho\sigma\Delta^{\alpha\beta}](x; x') = \delta_\mu^{(\alpha} \delta_\nu^{\beta)} i\delta^D(x - x'). \quad (29)$$

For more details of solving this gauge-fixed graviton propagator equation, one can consult [2,44]. Here we would like to give the result without a derivation. The graviton propagator in this gauge takes the form of a sum of constant index factors times scalar propagators,

$$i[\rho\sigma\Delta^{\alpha\beta}](x; x') = \sum_{I=A,B,C} [\rho\sigma T_I^{\alpha\beta}] i\Delta_I(x; x'), \quad (30)$$

where the index factors are

$$[\rho\sigma T_{\rho\sigma}^A] = 2\bar{\eta}_{\mu(\rho} \bar{\eta}_{\sigma)\nu} - \frac{2}{D-3} \bar{\eta}_{\mu\nu} \bar{\eta}_{\rho\sigma}, \quad (31)$$

$$[\rho\sigma T_{\rho\sigma}^B] = -4\delta_{(\mu}^0 \bar{\eta}_{\nu)(\rho} \delta_{\sigma)}^0, \quad (32)$$

$$\begin{aligned}[\rho\sigma T_{\rho\sigma}^C] = & \frac{2}{(D-2)(D-3)} [(D-3)\delta_\mu^0 \delta_\nu^0 + \bar{\eta}_{\mu\nu}] \\ & \times [(D-3)\delta_\rho^0 \delta_\sigma^0 + \bar{\eta}_{\rho\sigma}],\end{aligned} \quad (33)$$

$$\bar{\eta}_{\mu\nu} \equiv \eta_{\mu\nu} + \delta_\mu^0 \delta_\nu^0, \quad \text{and} \quad \bar{\delta}_\nu^\mu \equiv \delta_\nu^\mu - \delta_\nu^0 \delta_\mu^0. \quad (34)$$

The most singular term for each scalar propagator is the propagator for a massless, conformally coupled scalar [49],

$$i\Delta_{\text{cf}}(x; x') = \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \Gamma\left(\frac{D}{2} - 1\right) \left(\frac{4}{y}\right)^{\frac{D}{2}-1}. \quad (35)$$

³The de Sitter symmetry group in $D = 4$ includes three spatial translations, three spatial rotations, one dilatation, and three spatial special conformal transformations.

The three scalar propagators are

$$\begin{aligned}i\Delta_A(x; x') = & i\Delta_{\text{cf}}(x; x') + \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \\ & \times \left\{ \frac{D}{D-4} \frac{\Gamma^2(\frac{D}{2})}{\Gamma(D-1)} \left(\frac{4}{y}\right)^{\frac{D}{2}-2} - \pi \cot\left(\frac{\pi}{2}D\right) \right. \\ & \left. + \ln(aa') \right\} + \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \sum_{n=1}^{\infty} \left\{ \frac{1}{n} \frac{\Gamma(n+D-1)}{\Gamma(n+\frac{D}{2})} \left(\frac{y}{4}\right)^n \right. \\ & \left. - \frac{1}{n-\frac{D}{2}+2} \frac{\Gamma(n+\frac{D}{2}+1)}{\Gamma(n+2)} \left(\frac{y}{4}\right)^{n-\frac{D}{2}+2} \right\}, \quad (36)\end{aligned}$$

$$\begin{aligned}i\Delta_B(x; x') = & i\Delta_{\text{cf}}(x; x') - \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \sum_{n=0}^{\infty} \left\{ \frac{\Gamma(n+D-2)}{\Gamma(n+\frac{D}{2})} \left(\frac{y}{4}\right)^n \right. \\ & \left. - \frac{\Gamma(n+\frac{D}{2})}{\Gamma(n+2)} \left(\frac{y}{4}\right)^{n-\frac{D}{2}+2} \right\}, \quad (37)\end{aligned}$$

$$\begin{aligned}i\Delta_C(x; x') = & i\Delta_{\text{cf}}(x; x') + \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \\ & \times \sum_{n=0}^{\infty} \left\{ (n+1) \frac{\Gamma(n+D-3)}{\Gamma(n+\frac{D}{2})} \left(\frac{y}{4}\right)^n \right. \\ & \left. - \left(n-\frac{D}{2}+3\right) \frac{\Gamma(n+\frac{D}{2}-1)}{\Gamma(n+2)} \left(\frac{y}{4}\right)^{n-\frac{D}{2}+2} \right\}. \quad (38)\end{aligned}$$

These expressions might seem daunting but they are actually simple to use because the infinite sums vanish in $D = 4$, and each term in these sums goes like a positive power of $y(x; x')$. This means the infinite sums can only contribute when multiplied by a divergent term, and even then only a small number of terms can contribute. Note also that the B -type and C -type propagators agree with the conformal propagator in $D = 4$.

IV. COUNTERTERM ANALYSIS

In this section we deal with the local counterterms we must add, order by order in perturbation theory, to absorb divergences in the sense of BPHZ renormalization. The particular counterterms that renormalize the fermion self-energy must obviously involve a single $\bar{\psi}$ and a single ψ . At one-loop order the superficial degree of divergence of quantum gravitational contributions to the fermion self-energy is 3, so the necessary counterterms can involve zero, one, two or three derivatives. These derivatives can either act upon the fermi fields or upon the metric, in which case they must be organized into curvatures or derivatives of curvatures. We close with a discussion of possible non-invariant counterterms.

All one-loop corrections from quantum gravity must carry a factor of $\kappa^2 \sim \text{mass}^{-2}$. There will be additional dimensions associated with derivatives and with the various fields, and the balance must be struck using the renormalized fermion mass, m . For the purpose of our

computation, we only focus the counterterms at order m . Because superficial degree of divergence is three, the possible expressions at order m must consist of one mass and two derivatives that can either act upon the fermions or else on the metric to produce curvatures,

$$\kappa^2 m \bar{\psi} (i\mathcal{D})^2 \psi \sqrt{-g}, \quad \kappa^2 m R \bar{\psi} \psi \sqrt{-g}. \quad (39)$$

We then specialize the above expressions from the general background to de Sitter. Hence the invariant counter Lagrangian we require at order m is

$$\Delta \mathcal{L}_{\text{inv}} = \lambda_1 \kappa^2 m \bar{\psi} (i\mathcal{D})^2 \psi \sqrt{-g} + \lambda_2 \kappa^2 m R \bar{\psi} \psi \sqrt{-g}, \quad (40)$$

$$\rightarrow \lambda_1 \kappa^2 \bar{\Psi} m (i \not{\partial} a^{-1} i \not{\partial}) \Psi + \lambda_2 \kappa^2 (D-1) D H^2 m a \bar{\Psi} \Psi. \quad (41)$$

Here λ_1 and λ_2 are D -dependent constants that are dimensionless. The associated vertex operators are

$$C_{1ij} \equiv \lambda_1 \kappa^2 \left(\frac{m}{a} \partial^2 + m H \gamma^0 \not{\partial} \right) = \lambda_1 \kappa^2 m \not{\partial} a^{-1} \not{\partial}, \quad (42)$$

$$C_{2ij} \equiv \lambda_2 D(D-1) \kappa^2 H^2 m a. \quad (43)$$

C_1 is the higher derivative counterterm. It will renormalize the most singular terms—coming from the $i\Delta_{\text{cf}}$ part of the graviton propagator—which are unimportant because they are suppressed by powers of the scale factor. The other vertex operator, C_2 , is a sort of dimensionful field strength renormalization in de Sitter background. It will renormalize the less singular contributions that derive physically from inflationary particle production.

Because our gauge fixing functional (necessarily) breaks de Sitter invariance, it is also necessary to consider noninvariant counterterms. These noninvariant counterterms must respect the symmetries of the gauge condition, which are homogeneity, isotropy, and dilatation invariance. As one-loop counterterms, they should also contain a factor of κ^2 , multiplied by a spinor differential operator with the dimension of mass cubed, involving no more than three derivatives and acting between $\bar{\Psi}$ and Ψ . As the only dimensionful constant in our problem, powers of H must be used to make up whatever dimensions are not supplied by derivatives. Homogeneity implies that the spinor differential operator cannot depend upon the spatial coordinate x^i . Similarly, isotropy requires that any spatial derivative operators ∂_i must either be contracted into γ^i or another spatial derivative. Owing to the identity,

$$(\gamma^i \partial_i)^2 = -\nabla^2, \quad (44)$$

we can think of all spatial derivatives as contracted into γ^i . Although the temporal derivative is not required to be multiplied by γ^0 we lose nothing by doing so provided additional dependence upon γ^0 is allowed.

The final residual symmetry is dilatation invariance. It has the crucial consequence that derivative operators can only appear in the form $a^{-1} \partial_\mu$. In addition the entire counterterm must have an overall factor of a , and there can be no other dependence upon η . So the most general order m counterterm consistent with our gauge condition takes the following form:

$$\Delta \mathcal{L}_{\text{non}} = \kappa^2 H^2 m a \bar{\Psi} S((Ha)^{-1} \gamma^0 \partial_0, (Ha)^{-1} \gamma^i \partial_i) \Psi, \quad (45)$$

where the spinor function $S(b, c)$ is at most a second-order polynomial function of its arguments, and it may involve γ^0 in an arbitrary way.

Three more principles constrain the order m noninvariant counterterms. The first of these principles is that the fermion self-energy at order m involves only even powers of gamma matrices. This follows because the three-point vertices, the four-point vertices, and the fermion propagator all consist of an even number of γ 's at order m^1 and an odd number of γ 's at order m^0 . The diagram that consists of one 4-point vertex possesses an even number of gamma matrices at order m . The contribution from any diagram with two 3-point vertices consists of three factors involving gamma matrices: one factor from the fermion propagator and one factor from each of the two vertices. At order m such a product consists of one even and two odd factors, so it contains an even number of gamma matrices. This principle fixes the dependence upon γ^0 and allows us to express the spinor differential operator in terms of just six constants β_i ,

$$\begin{aligned} \kappa^2 H^2 m a S((Ha)^{-1} \gamma^0 \partial_0, (Ha)^{-1} \gamma^i \partial_i) \\ = \kappa^2 m a \{ \beta_1 (a^{-1} \gamma^0 \partial_0)^2 + \beta_2 [(a^{-1} \gamma^0 \partial_0)(a^{-1} \gamma^i \partial_i)] \\ + \beta_3 (a^{-1} \gamma^i \partial_i)^2 + H \gamma^0 (\beta_4 (a^{-1} \gamma^0 \partial_0) \\ + \beta_5 (a^{-1} \gamma^i \partial_i)) + H^2 \beta_6 \}. \end{aligned} \quad (46)$$

In this expansion, but for the rest of this section only, we define noncommuting factors within square brackets to be symmetrically ordered, for example,

$$\begin{aligned} [(a^{-1} \gamma^0 \partial_0)(a^{-1} \gamma^i \partial_i)] \\ \equiv \frac{1}{2} (a^{-1} \gamma^0 \partial_0)(a^{-1} \gamma^i \partial_i) + \frac{1}{2} (a^{-1} \gamma^i \partial_i)(a^{-1} \gamma^0 \partial_0). \end{aligned} \quad (47)$$

The second principle is that our gauge condition (28) becomes Poincaré invariant in the flat space limit of $H \rightarrow 0$, where the conformal time is $\eta = -e^{-Ht}/H$ with t held fixed. In that limit only the three quadratic terms of (46) survive,

$$\begin{aligned} \lim_{H \rightarrow 0} \kappa^2 H^2 m a S((Ha)^{-1} \gamma^0 \partial_0, (Ha)^{-1} \gamma^i \partial_i) \\ = \kappa^2 m a \{ \beta_1 (a^{-1} \gamma^0 \partial_0)^2 + \beta_2 [(a^{-1} \gamma^0 \partial_0)(a^{-1} \gamma^i \partial_i)] \\ + \beta_3 (a^{-1} \gamma^i \partial_i)^2 \}. \end{aligned} \quad (48)$$

Because the entire theory is Poincaré invariant in that limit, these three terms must sum to a term proportional to $(\gamma^\mu \partial_\mu)^2$, which implies,

$$\beta_1 = \frac{1}{2}\beta_2 = \beta_3. \quad (49)$$

But in that case the three quadratic terms sum to give (42),

$$\begin{aligned} \kappa^2 m a \{ (a^{-1} \gamma^0 \partial_0)^2 + 2[(a^{-1} \gamma^0 \partial_0)(a^{-1} \gamma^i \partial_i)] + (a^{-1} \gamma^i \partial_i)^2 \} \\ = \kappa^2 m \not{a} a^{-1} \not{a}. \end{aligned} \quad (50)$$

Because it is the same as one of the invariant counterterms, it need not be included in \mathcal{S} . Besides, the final term in (46) recovers the other invariant counterterm (43). So the two remaining noninvariant counterterms we need to consider in (46) are

$$\Delta \mathcal{L}_{\text{non}} = \bar{\Psi} \{ \kappa^2 m a H \gamma^0 [\beta_4 (a^{-1} \gamma^0 \partial_0) + \beta_5 (a^{-1} \gamma^i \partial_i)] \} \Psi. \quad (51)$$

However, these two terms are not independent of the last term in (42). Therefore we could chose any four independent counterterm operators we need for this computation,

$$\alpha_1 \kappa^2 \frac{m}{a} \partial^2, \quad \alpha_4 \kappa^2 H^2 m a, \quad (52)$$

$$\alpha_2 \kappa^2 m H \partial_0, \quad \alpha_3 \kappa^2 m H \gamma^0 \not{a}. \quad (53)$$

V. CONTRIBUTIONS FROM THE 4-POINT VERTICES

In this section the contributions from the 4-point vertex operators of Table I are evaluated. The generic diagram topology is depicted in Fig. 1. The analytic form is

$$-i[\Sigma_j^{4\text{pt}}](x; x') = \sum_{l=1}^{10} i U_{lij}^{\alpha\beta\rho\sigma} i[\Delta_{\rho\sigma}](x; x') \delta^D(x - x'). \quad (54)$$

And the generic contraction for each of the vertex operators in Table I is given in Table II.

From an inspection of the generic contractions in Table II, it is obvious that we must work out how the three index factors $[\Delta_{\rho\sigma}^I]$ that make up the graviton propagator contract into $\eta^{\alpha\beta}$ and $\eta^{\alpha\rho}$. For the A-type and B-type index factors the various contractions give

$$\eta^{\alpha\beta}[\Delta_{\rho\sigma}^A] = -\left(\frac{4}{D-3}\right) \bar{\eta}_{\rho\sigma}, \quad (55)$$

$$\eta^{\alpha\rho}[\Delta_{\rho\sigma}^A] = \left(D - \frac{2}{D-3}\right) \bar{\eta}_{\beta\sigma},$$

$$\eta^{\alpha\beta}[\Delta_{\rho\sigma}^B] = 0, \quad (56)$$

$$\eta^{\alpha\rho}[\Delta_{\rho\sigma}^B] = -(D-1) \delta_\beta^0 \delta_\sigma^0 + \bar{\eta}_{\beta\sigma}.$$

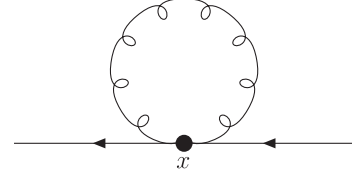


FIG. 1. Contribution from 4-point vertices.

For the C-type index factor they are

$$\begin{aligned} \eta^{\alpha\beta}[\Delta_{\rho\sigma}^C] &= \left(\frac{4}{D-2}\right) \delta_\rho^0 \delta_\sigma^0 + \frac{4}{(D-2)(D-3)} \bar{\eta}_{\rho\sigma}, \\ \eta^{\alpha\rho}[\Delta_{\rho\sigma}^C] &= -2\left(\frac{D-3}{D-2}\right) \delta_\beta^0 \delta_\sigma^0 + \frac{2}{(D-2)(D-3)} \bar{\eta}_{\beta\sigma}. \end{aligned} \quad (57)$$

At order m we actually only require double contractions. For the A-type index factor these are

$$\begin{aligned} \eta^{\alpha\beta} \eta^{\rho\sigma}[\Delta_{\rho\sigma}^A] &= -4\left(\frac{D-1}{D-3}\right), \\ \eta^{\alpha\rho} \eta^{\beta\sigma}[\Delta_{\rho\sigma}^A] &= D(D-1) - 2\left(\frac{D-1}{D-3}\right). \end{aligned} \quad (58)$$

The double contractions of the B-type and C-type index factors are

$$\eta^{\alpha\beta} \eta^{\rho\sigma}[\Delta_{\rho\sigma}^B] = 0, \quad \eta^{\alpha\rho} \eta^{\beta\sigma}[\Delta_{\rho\sigma}^B] = 2(D-1), \quad (59)$$

$$\begin{aligned} \eta^{\alpha\beta} \eta^{\rho\sigma}[\Delta_{\rho\sigma}^C] &= \frac{8}{(D-2)(D-3)}, \\ \eta^{\alpha\rho} \eta^{\beta\sigma}[\Delta_{\rho\sigma}^C] &= 2 \frac{(D^2 - 5D + 8)}{(D-2)(D-3)}. \end{aligned} \quad (60)$$

Table III was generated from Table II by expanding the graviton propagator in terms of index factors,

TABLE II. Generic 4-point contractions.

I	$i[\Delta_{\rho\sigma}](x; x') i U_I^{\alpha\beta\rho\sigma} \delta^D(x - x')$
1	$-\frac{1}{8} \kappa^2 i[\Delta_\alpha^\rho](x; x) \not{a} \delta^D(x - x')$
2	$\frac{1}{4} \kappa^2 i[\Delta_{\alpha\beta}^{\alpha\beta}](x; x) \not{a} \delta^D(x - x')$
3	$\frac{1}{4} \kappa^2 i[\Delta_{\rho\sigma}^{\alpha\alpha}](x; x) \gamma^\rho \partial^\sigma \delta^D(x - x')$
4	$-\frac{3}{8} \kappa^2 i[\Delta_{\alpha\sigma}^{\alpha\sigma}](x; x) \gamma^\beta \partial^\sigma \delta^D(x - x')$
5	$-\frac{i}{4} \kappa^2 \partial'_\mu i[\Delta_{\rho\sigma}^{\alpha\alpha}](x; x') \gamma^\rho J^{\sigma\mu} \delta^D(x - x')$
6	$\frac{i}{8} \kappa^2 \partial'_\mu i[\Delta_{\alpha\sigma}^{\alpha\sigma}](x; x') \gamma^\mu J^{\beta\sigma} \delta^D(x - x')$
7	$\frac{i}{4} \kappa^2 \partial_\mu i[\Delta_{\alpha\sigma}^{\alpha\sigma}](x; x) \gamma^\beta J^{\sigma\mu} \delta^D(x - x')$
8	$\frac{i}{4} \kappa^2 \partial'^\beta i[\Delta_{\rho\sigma}^{\alpha\alpha}](x; x') \gamma^\rho J^{\sigma\alpha} \delta^D(x - x')$
9	$-\frac{i}{8} \kappa^2 a m i[\Delta_\alpha^\rho](x; x) \delta^D(x - x')$
10	$\frac{i}{4} \kappa^2 a m i[\Delta_{\alpha\beta}^{\alpha\beta}](x; x) \delta^D(x - x')$

TABLE III. 4-point contribution from each part of the graviton propagator at order m^1 . The vertices 1–8 could only give the contribution at order m^0 .

I	J	$i[\alpha_\beta T_{\rho\sigma}^J]i\Delta_J(x; x')iU_I^{\alpha\beta\rho\sigma}\delta^D(x - x')$
9	A	$\frac{i}{2}\kappa^2 \frac{(D-1)}{(D-3)}ami\Delta_A(x; x)\delta^D(x - x')$
9	C	$-\frac{i}{(D-2)(D-3)}\kappa^2ami\Delta_C(x; x)\delta^D(x - x')$
10	A	$\frac{i}{4}\kappa^2[D(D-1) - 2\frac{(D-1)}{(D-3)}]ami\Delta_A(x; x)\delta^D(x - x')$
10	B	$\frac{i}{2}\kappa^2(D-1)ami\Delta_B(x; x)\delta^D(x - x')$
10	C	$\frac{i}{2}\kappa^2 \frac{(D^2-5D+8)}{(D-2)(D-3)}ami\Delta_C(x; x)\delta^D(x - x')$

$$i[\alpha_\beta \Delta_{\rho\sigma}](x; x') = [\alpha_\beta T_{\rho\sigma}^A]i\Delta_A(x; x') + [\alpha_\beta T_{\rho\sigma}^B]i\Delta_B(x; x') + [\alpha_\beta T_{\rho\sigma}^C]i\Delta_C(x; x'). \quad (61)$$

We then perform the relevant contractions using the previous identities.

From Table III it is apparent that we require the coincidence limits on each of the scalar propagators. For the A-type propagator these are

$$\lim_{x' \rightarrow x} i\Delta_A(x; x') = \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \left\{ -\pi \cot\left(\frac{\pi}{2}D\right) + 2\ln(a) \right\}. \quad (62)$$

The analogous coincidence limits for the B-type propagator are actually finite in $D = 4$ dimensions,

$$\lim_{x' \rightarrow x} i\Delta_B(x; x') = \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \times -\frac{1}{D-2}. \quad (63)$$

The same is true for the coincidence limits of the C-type propagator,

$$\lim_{x' \rightarrow x} i\Delta_C(x; x') = \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \times \frac{1}{(D-2)(D-3)}. \quad (64)$$

We apply the various coincidence limits to each contraction in Table III and present the order m , 4-point

TABLE IV. The 4-point contributions at order m . All contributions are multiplied by $\frac{\kappa^2 H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})}$. Here $A \equiv \frac{\pi}{2} \times \cot(\frac{D\pi}{2}) - \ln(a)$.

I	J	$iam\delta^D(x - x')$
9	A	$-\frac{(D-1)}{(D-3)}A$
9	B	0
9	C	$-\frac{1}{(D-2)^2(D-3)^2}$
10	A	$-\frac{1}{2}[D(D-1) - \frac{2(D-1)}{(D-3)}]A$
10	B	$-\frac{1}{2}\frac{(D-1)}{(D-2)}$
10	C	$\frac{1}{2}\frac{(D^2-5D+8)}{(D-2)^2(D-3)^2}$

contributions in Table IV. The total summation for these local contributions is quite simple,

$$-i[\Sigma^{4pt}](x; x') = \frac{i\kappa^2 H^2 ma}{2^{D+1}\pi^{\frac{D}{2}}} \frac{H^{D-4}}{(D-4)} \frac{-\Gamma(D+1)}{\Gamma(\frac{D}{2})} \times \delta^D(x - x') + \frac{i\kappa^2 H^2}{16\pi^2} ma \times [12\ln a - 1]\delta^4(x - x'). \quad (65)$$

VI. CONTRIBUTIONS FROM THE 3-POINT VERTICES

In this section we work out the contributions from two 3-point vertex operators. The generic diagram topology is depicted in Fig. 2. The analytic form is

$$-i[\Sigma_j^{3pt}](x; x') = \sum_{I=1}^4 iV_{Iik}^{\alpha\beta}(x)i[S_\ell](x; x') \times \sum_{J=1}^4 iV_{Jlj}^{\rho\sigma}(x')i[\alpha_\beta \Delta_{\rho\sigma}](x; x'). \quad (66)$$

Because there are four 3-point vertex operators in (23), there are 16 vertex products in (66). We label each contribution by the numbers on its vertex pair, for example,

$$[I - J] \equiv iV_I^{\alpha\beta}(x) \times i[S](x; x') \times iV_J^{\rho\sigma}(x') \times i[\alpha_\beta \Delta_{\rho\sigma}](x; x'). \quad (67)$$

Table V gives the generic reductions, before decomposing the graviton propagator.⁴ Most of these reductions are straightforward but one subtlety deserves mention, that is, derivatives on external lines must be partially integrated back on the entire diagram. This happens whenever the second vertex is $J = 1$ or $J = 2$, for example,

$$[2-2] \equiv -\frac{i\kappa}{2}\gamma^\alpha i\partial^\beta \times i[S](x; x') \times -\frac{i\kappa}{2}\gamma^\rho i\partial_{\text{ext}}^{\sigma} \times i[\alpha_\beta \Delta_{\rho\sigma}](x; x'), \quad (68)$$

$$= -\frac{\kappa^2}{4}\partial'^\sigma \{ \gamma^\alpha \partial^\beta i[S](x; x') \gamma^\rho i[\alpha_\beta \Delta_{\rho\sigma}](x; x') \}. \quad (69)$$

Another simplification we might use for later contractions is that the Dirac slash of the conformal part of fermion propagator gives a delta function,

$$i\not{\partial}[S]_{\text{cf}}(x; x') = i\delta^D(x - x'). \quad (70)$$

A. Conformal contributions

The key to accomplishing a tractable reduction of the diagrams of Fig. 2 is that the first term of each of the scalar

⁴We would not consider the 4-4 contraction because it is an order m^2 contribution.

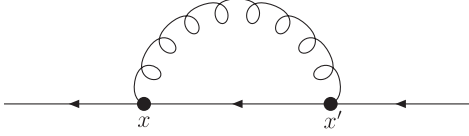


FIG. 2. Contribution from two 3-point vertices.

propagators $i\Delta_I(x; x')$ is the conformal propagator $i\Delta_{\text{cf}}(x; x')$. The sum of the three index factors also gives a simple tensor, so it is very convenient to write the graviton propagator in the form

$$i[\mu\nu\Delta_{\rho\sigma}](x; x') = \left[2\eta_{\mu(\rho}\eta_{\sigma)\nu} - \frac{2}{D-2}\eta_{\mu\nu}\eta_{\rho\sigma} \right] i\Delta_{\text{cf}}(x; x') + \sum_{I=A,B,C} [\mu\nu T^I_{\rho\sigma}] i\delta\Delta_I(x; x'), \quad (71)$$

where $i\delta\Delta_I(x; x') \equiv i\Delta_I(x; x') - i\Delta_{\text{cf}}(x; x')$. In this subsection we evaluate the contribution to (66) using the 3-point vertex operators (23) and the fermion propagator (19) but only the conformal part of the graviton propagator,

$$i[\mu\nu\Delta_{\rho\sigma}](x; x') \rightarrow \left[2\eta_{\mu(\rho}\eta_{\sigma)\nu} - \frac{2}{D-2}\eta_{\mu\nu}\eta_{\rho\sigma} \right] i\Delta_{\text{cf}}(x; x') \equiv [\alpha\beta T^{\text{cf}}_{\rho\sigma}] i\Delta_{\text{cf}}(x; x'). \quad (72)$$

We carry out the reduction in three stages. In the first stage the conformal part (72) of the graviton propagator is substituted into the generic results from Table V and the contractions are performed. We also make use of the following gamma matrix identities:

TABLE V. Generic contributions from the 3-point vertices.

I	J	$iV_I^{\alpha\beta}(x)i[S](x; x')iV_J^{\rho\sigma}(x')i[\alpha\beta\Delta_{\rho\sigma}](x; x')$
1	1	$-\frac{1}{4}\kappa^2\partial'_\mu\{\not{\partial}i[S](x; x')\gamma^\mu i[\alpha^\Delta\rho_\rho](x; x')\}$
1	2	$\frac{1}{4}\kappa^2\partial'^\rho\{\not{\partial}i[S](x; x')\gamma^\rho i[\alpha^\Delta\rho_\sigma](x; x')\}$
1	3	$\frac{1}{4}i\kappa^2\not{\partial}i[S](x; x')\gamma^\rho J^{\sigma\mu}\partial'_\mu i[\alpha^\Delta\rho_\sigma](x; x')$
1	4	$\frac{1}{4}i\kappa^2 am\not{\partial}i[S](x; x')i[\alpha^\Delta\rho_\rho](x; x')$
2	1	$\frac{1}{4}\kappa^2\partial'_\mu\{\gamma^\alpha\partial^\beta i[S](x; x')\gamma^\mu i[\alpha_\beta\Delta^\rho_\rho](x; x')\}$
2	2	$-\frac{1}{4}\kappa^2\partial'^\rho\{\gamma^\alpha\partial^\beta i[S](x; x')\gamma^\sigma i[\alpha_\beta\Delta_{\rho\sigma}](x; x')\}$
2	3	$-\frac{1}{4}i\kappa^2\gamma^\alpha\partial^\beta i[S](x; x')\gamma^\rho J^{\sigma\mu}\partial'_\mu i[\alpha_\beta\Delta_{\rho\sigma}](x; x')$
2	4	$-\frac{1}{4}i\kappa^2 am\gamma^\alpha\partial^\beta i[S](x; x')i[\alpha_\beta\Delta^\rho_\rho](x; x')$
3	1	$-\frac{1}{4}i\kappa^2\partial'_\nu\{\gamma^\alpha J^{\beta\mu}i[S](x; x')\gamma^\nu\partial_\mu i[\alpha_\beta\Delta^\rho_\rho](x; x')\}$
3	2	$\frac{1}{4}i\kappa^2\partial'^\rho\{\gamma^\alpha J^{\beta\mu}i[S](x; x')\gamma^\sigma\partial_\mu i[\alpha_\beta\Delta_{\rho\sigma}](x; x')\}$
3	3	$-\frac{1}{4}\kappa^2\gamma^\alpha J^{\beta\mu}i[S](x; x')\gamma^\rho J^{\sigma\nu}\partial'_\nu i[\alpha_\beta\Delta_{\rho\sigma}](x; x')$
3	4	$-\frac{1}{4}\kappa^2 am\gamma^\alpha J^{\beta\mu}i[S](x; x')\partial_\mu i[\alpha_\beta\Delta^\rho_\rho](x; x')$
4	1	$-\frac{1}{4}i\kappa^2\partial'_\mu\{ami[S](x; x')\gamma^\mu i[\alpha^\Delta\rho_\rho](x; x')\}$
4	2	$\frac{1}{4}i\kappa^2\partial'^\rho\{ami[S](x; x')\gamma^\sigma i[\alpha^\Delta\rho_\sigma](x; x')\}$
4	3	$-\frac{1}{4}\kappa^2 ami[S](x; x')\gamma^\rho J^{\sigma\mu}\partial'_\mu i[\alpha^\Delta\rho_\sigma](x; x')$
4	4	$-\frac{1}{4}\kappa^2 a^2 m^2 i[S](x; x')i[\alpha^\Delta\rho_\rho](x; x')$

 TABLE VI. Contractions from the $i\Delta_{\text{cf}}$ part of the graviton propagator.

I	J	sub	$iV_I^{\alpha\beta}(x)i[S](x; x')iV_J^{\rho\sigma}(x')[\alpha_\beta T^{\text{cf}}_{\rho\sigma}]i\Delta_{\text{cf}}(x; x')$
1	1		$\frac{D}{D-2}\kappa^2\partial'_\mu\{\not{\partial}i[S](x; x')\gamma^\mu i\Delta_{\text{cf}}(x; x')\}$
1	2		$-\frac{1}{D-2}\kappa^2\partial'_\mu\{\not{\partial}i[S](x; x')\gamma^\mu i\Delta_{\text{cf}}(x; x')\}$
1	3		$-\frac{(D-1)}{2(D-2)}\kappa^2\not{\partial}i[S](x; x')\not{\partial}i\Delta_{\text{cf}}(x; x')$
1	4		$-\frac{D}{D-2}\kappa^2 iam\not{\partial}i[S](x; x')i\Delta_{\text{cf}}(x; x')$
2	1		$-\frac{1}{D-2}\kappa^2\partial'_\mu\{\not{\partial}i[S](x; x')\gamma^\mu i\Delta_{\text{cf}}(x; x')\}$
2	2	a	$-\frac{1}{4}\kappa^2\not{\partial}'\{\partial_\mu i[S](x; x')\gamma^\mu i\Delta_{\text{cf}}(x; x')\}$
2	2	b	$-\frac{1}{4}\kappa^2\partial'_\mu\{\partial^\mu\gamma^\beta i[S](x; x')\gamma_\beta i\Delta_{\text{cf}}(x; x')\}$
2	2	c	$\frac{1}{2(D-2)}\kappa^2\partial'_\mu\{\not{\partial}i[S](x; x')\gamma^\mu i\Delta_{\text{cf}}(x; x')\}$
2	3	a	$\frac{1}{8}\kappa^2\gamma^\beta\partial^\mu i[S](x; x')\gamma_\beta\partial'_\mu i\Delta_{\text{cf}}(x; x')$
2	3	b	$\frac{1}{8}\kappa^2\not{\partial}'i\Delta_{\text{cf}}(x; x')\partial_\mu i[S](x; x')\gamma^\mu$
2	3	c	$\frac{1}{4(D-2)}\kappa^2\not{\partial}i[S](x; x')\not{\partial}'i\Delta_{\text{cf}}(x; x')$
2	4		$\frac{1}{(D-2)}\kappa^2 iam\not{\partial}i[S](x; x')i\Delta_{\text{cf}}(x; x')$
3	1		$\frac{(D-1)}{2(D-2)}\kappa^2\partial'_\mu\{\not{\partial}i\Delta_{\text{cf}}(x; x')i[S](x; x')\gamma^\mu\}$
3	2	a	$-\frac{1}{8}\kappa^2\not{\partial}'\{\not{\partial}i[S](x; x')\not{\partial}i\Delta_{\text{cf}}(x; x')\}$
3	2	b	$-\frac{1}{4(D-2)}\kappa^2\partial'_\mu\{\not{\partial}i\Delta_{\text{cf}}(x; x')i[S](x; x')\gamma^\mu\}$
3	2	c	$-\frac{1}{8}\kappa^2\partial'_\mu\{\gamma^\beta i[S](x; x')\gamma_\beta\partial^\mu i\Delta_{\text{cf}}(x; x')\}$
3	3	a	$\frac{1}{16}\kappa^2\gamma^\beta i[S](x; x')\gamma_\beta\partial^\mu\partial'_\mu\Delta_{\text{cf}}(x; x')$
3	3	b	$\frac{1}{16}\kappa^2\gamma^\mu i[S](x; x')\partial'_\mu\not{\partial}i\Delta_{\text{cf}}(x; x')$
3	3	c	$-\frac{(2D-3)}{8(D-2)}\kappa^2\gamma^\mu i[S](x; x')\partial_\mu\not{\partial}'i\Delta_{\text{cf}}(x; x')$
3	4		$-\frac{(D-1)}{2(D-2)}\kappa^2 iam\not{\partial}i\Delta_{\text{cf}}(x; x')i[S](x; x')$
4	1		$\frac{D}{(D-2)}\kappa^2 iam\partial'_\mu\{i[S](x; x')\gamma^\mu i\Delta_{\text{cf}}(x; x')\}$
4	2		$-\frac{1}{(D-2)}\kappa^2 iam\partial'_\mu\{i[S](x; x')\gamma^\mu i\Delta_{\text{cf}}(x; x')\}$
4	3		$-\frac{(D-1)}{2(D-2)}\kappa^2 iam i[S](x; x')\not{\partial}i\Delta_{\text{cf}}(x; x')$
4	4		$\frac{D}{(D-2)}\kappa^2 ia^2 m^2 i[S](x; x')\Delta_{\text{cf}}(x; x')$

$$\gamma^\rho J^{\beta\mu} + \gamma^\beta J^{\rho\mu} = \frac{i}{2}(\gamma^\rho\eta^{\beta\mu} + \gamma^\beta\eta^{\rho\mu}) - i\gamma^\mu\eta^{\rho\beta}, \quad (73)$$

$$\gamma_\alpha J^{\alpha\mu} = -\frac{i}{2}(D-1)\gamma^\mu.$$

At this stage we do not act on any derivatives on the fermion propagator. The results of these reductions are presented in Table VI. The conformal tensor factor $[\alpha_\beta T^{\text{cf}}_{\rho\sigma}]$ consists of three distinct terms, and the factors of $\gamma^\alpha J^{\beta\mu}$ in Table V can contribute different terms with a distinct structure, so we have sometimes broken up the result for a given vertex pair into parts. These parts are distinguished in Table VI and subsequently by subscripts taken from the lower case Latin letters.

In the second stage we substitute the conformal part of the graviton propagator,

$$i\Delta_{\text{cf}}(x; x') = \frac{\Gamma(\frac{D}{2}-1)}{4\pi^{\frac{D}{2}}} \frac{(aa')^{1-\frac{D}{2}}}{\Delta x^{D-2}}, \quad (74)$$

TABLE VII. $i\Delta_{\text{cf}}(x; x') \times i[\mathcal{S}]_{\text{cf}}(x; x')$. All contributions are multiplied by $\frac{\kappa^2}{8\pi^D} \Gamma(\frac{D}{2})\Gamma(\frac{D}{2}-1)ma(aa')^{1-\frac{D}{2}}$.

		Coefficient	Coefficient	Coefficient
I	J	$\partial^2 \frac{1}{\Delta x^{2D-4}}$	$Ha\gamma^0 \not{\partial} \frac{1}{\Delta x^{2D-4}}$	$Ha'\not{\partial}\gamma^0 \frac{1}{\Delta x^{2D-4}}$
1	4	0	0	0
2	4	0	0	0
3	4	$-\frac{1}{4} \frac{(D-1)}{(D-2)^2}$	$-\frac{1}{8} \frac{(D-1)}{(D-2)}$	0
4	1	$-\frac{1}{2} \frac{D}{(D-2)^2}$	0	$\frac{1}{4} \frac{D}{(D-2)}$
4	2	$\frac{1}{2} \frac{1}{(D-2)^2}$	0	$-\frac{1}{4} \frac{1}{(D-2)}$
4	3	$-\frac{1}{4} \frac{(D-1)}{(D-2)^2}$	$\frac{1}{8} \frac{(D-1)}{(D-2)}$	0

and decompose the fermion propagator (19) into the conformal part, the flat spacetime mass term, $n = 0$ part, and $n \geq 1$ part of the infinite series expansion.⁵ In the final stage we act on the derivatives. We start from the most singular contribution in Table VI, which substitutes the conformal parts of the fermion propagator into the contraction 1-4, 2-4, 3-4, 4-1, 4-2, and 4-3.⁶ The contraction 1-4 and 2-4 vanish owing to Eq. (70) and owing to the zero contribution from D powers of the coordinate separation in dimensional regularization. We also must remember that $[\Sigma](x; x')$ will be used inside an integral in the quantum-corrected Dirac equation (2). For that purpose the most singular term at $x'^\mu = x^\mu$ is quadratically divergent in $D = 4$ dimensions. Hence we first conveniently⁷ employ the following identities to express the rest of them as a less singular form,

$$\begin{aligned} \frac{1}{\Delta x^{2D-2}} &= \frac{\partial^2}{2(D-2)^2} \frac{1}{\Delta x^{2D-4}}, \\ \frac{\Delta x_\mu}{\Delta x^{2D-2}} &= \frac{-\partial_\mu}{2(D-2)} \frac{1}{\Delta x^{2D-4}}. \end{aligned} \quad (75)$$

The individual result is quoted in Table VII and collected all terms of this class,

$$\begin{aligned} &\frac{\kappa^2 ma(aa')^{1-\frac{D}{2}}}{8\pi^D} \Gamma\left(\frac{D}{2}\right) \Gamma\left(\frac{D}{2}-1\right) \frac{(D-1)}{(D-2)} \left\{ \frac{-1}{(D-2)} \partial^2 \right. \\ &\quad \left. - \frac{H}{4} (a - a') \partial_0 - \frac{Ha'}{4} \gamma^0 \not{\partial} \right\} \frac{1}{\Delta x^{2D-4}}. \end{aligned} \quad (76)$$

The expression (76) is still logarithmically divergent in $D = 4$ after pulling out various derivatives. To further renormalize this divergence we extract derivatives with respect to the coordinate x^μ again, which can of course

⁵We will explain why we separate the $n = 0$ part from the rest of the infinite series expansion in a later paragraph.

⁶The contraction 4-4 is an order m^2 contribution.

⁷Some individual terms are easier written as a derivative with respect to x^μ acting upon a less singular coordinate separation than taking the derivative directly.

be taken outside the integral in (2) to give a less singular integrand,

$$\frac{1}{\Delta x^{2D-4}} = \frac{1}{2(D-3)(D-4)} \partial^2 \frac{1}{\Delta x^{2D-6}}. \quad (77)$$

Expression (77) is integrable in four dimensions and we could take $D = 4$ except for the explicit factor of $1/(D-4)$. Of course that is how ultraviolet divergences manifest in dimensional regularization. We can segregate the divergence on a local term by employing a simple representation for a delta function,

$$\begin{aligned} &\frac{\partial^2}{D-4} \left(\frac{1}{\Delta x^{2D-6}} \right) \\ &= \frac{\partial^2}{D-4} \left\{ \frac{1}{\Delta x^{2D-6}} - \frac{\mu^{D-4}}{\Delta x^{D-2}} \right\} + \frac{i4\pi^{\frac{D}{2}} \mu^{D-4}}{\Gamma(\frac{D}{2}-1)} \frac{\delta^D(x-x')}{D-4} \\ &= -\frac{\partial^2}{2} \left\{ \mu^{2D-8} \frac{\ln(\mu^2 \Delta x^2)}{\Delta x^2} + O(D-4) \right\} \\ &\quad + \frac{i4\pi^{\frac{D}{2}} \mu^{D-4}}{\Gamma(\frac{D}{2}-1)} \frac{\delta^D(x-x')}{D-4}. \end{aligned} \quad (78)$$

After substituting (77) and (78) into (76) one gets

$$\begin{aligned} &\frac{3\kappa^2}{64\pi^4} \left\{ \frac{1}{2} \frac{m}{a'} \partial^2 + \frac{mH}{4} \left(\frac{a}{a'} - 1 \right) \partial_0 + \frac{mH}{4} \gamma^0 \not{\partial} \right\} \partial^2 \left[\frac{\ln(\mu^2 \Delta x^2)}{\Delta x^2} \right] \\ &\quad + \frac{\kappa^2}{8\pi^{\frac{D}{2}}} \Gamma\left(\frac{D}{2}\right) \frac{(aa')^{1-\frac{D}{2}}}{(D-2)} \frac{(D-1)}{(D-3)} \frac{\mu^{D-4}}{(D-4)} \left\{ \frac{-2}{(D-2)} \frac{m}{a'} \partial^2 \right. \\ &\quad \left. - \frac{mH}{2} \left(\frac{a}{a'} - 1 \right) \partial_0 - \frac{mH}{2} \gamma^0 \not{\partial} \right\} i \delta^D(x-x'). \end{aligned} \quad (79)$$

To reach the same expressions as the counterterms we mentioned in the Sec. III, we make use of the following identities:

$$\begin{aligned} \frac{1}{a'} \partial^2 \delta^D(x-x') &= \left\{ \frac{1}{a} \partial^2 + 2H\partial_0 \right\} \delta^D(x-x'), \\ \frac{1}{a'} \partial_0 \delta^D(x-x') &= \left\{ \frac{1}{a} \partial_0 - H \right\} \delta^D(x-x'), \\ \ln(a') \partial_0 \delta^D(x-x') &= \{ \ln(a) \partial_0 + Ha \} \delta^D(x-x'), \\ \frac{\ln(a')}{a'} \partial_0 \delta^D(x-x') &= \left\{ \frac{\ln(a)}{a} \partial_0 + H(1 - \ln(a)) \right\} \delta^D(x-x'), \\ \frac{\ln(a')}{a'} \partial^2 \delta^D(x-x') &= \left\{ \frac{\ln(a)}{a} \partial^2 + 2H(\ln(a) - 1) \partial_0 + H^2 a \right\} \\ &\quad \times \delta^D(x-x'). \end{aligned} \quad (80)$$

After applying (80) and (79) and expanding out $(aa')^{1-\frac{D}{2}}$, we get the total of this most singular class that is consistent with our counterterm convention,

$$\begin{aligned}
-i[\Sigma^{\text{cfcf}}](x; x') &= \frac{i\kappa^2}{16\pi^{\frac{D}{2}}} \frac{\Gamma(\frac{D}{2}-1)(D-1)\mu^{D-4}}{(D-3)(D-4)} \left\{ \frac{-2}{(D-2)} \frac{m}{a} \partial^2 - \frac{4mH}{(D-2)} \partial_0 - \frac{1}{2} mH \gamma^0 \bar{\not{a}} + \frac{1}{2} mH^2 a \right\} \delta^D(x-x') \\
&+ \frac{i\kappa^2}{8\pi^2} \left\{ \ln(a) \left[\frac{3}{2} \frac{m}{a} \partial^2 + 3mH \partial_0 + \frac{3}{4} mH \gamma^0 \bar{\not{a}} - \frac{3}{4} mH^2 a \right] - \frac{3}{2} mH \partial_0 + \frac{3}{4} mH^2 a \right\} \delta^4(x-x') \\
&+ \frac{\kappa^2}{64\pi^2} \left\{ \frac{3}{2} \frac{m}{a'} \partial^2 + \frac{3}{4} mH \left(\frac{a}{a'} - 1 \right) \partial_0 + \frac{3}{4} mH \gamma^0 \bar{\not{a}} \right\} \partial^2 \left[\frac{\ln(\mu^2 \Delta x^2)}{\Delta x^2} \right]. \tag{81}
\end{aligned}$$

A less singular contribution comes from the flat spacetime mass term of the fermion propagator. Note that the contraction (3-3) involves two derivatives acting upon the conformal graviton propagator, which would produce a delta function,

$$\begin{aligned}
\partial^\mu \partial'_\mu i\Delta_{\text{cf}} &= \frac{-\Gamma(\frac{D}{2})}{4\pi^{\frac{D}{2}}} \frac{(D-2)}{(aa')^{\frac{D}{2}-1}} \left[\frac{H^2 aa' \Delta \eta^2}{\Delta x^D} + \frac{H^2 aa'}{2\Delta x^{D-2}} \right. \\
&\left. + \frac{i2\pi^{\frac{D}{2}}}{\Gamma(\frac{D}{2})} \delta^D(x-x') \right]. \tag{82}
\end{aligned}$$

This delta function would give zero when it is multiplied by D powers of the coordinate separation, which occurs in

this case. Next, in order to facilitate our computation, we make use of (75) and the following identity,

$$\frac{H^2 aa' \Delta \eta^2}{\Delta x^{2D-2}} = \frac{Ha - Ha'}{2(D-2)} \partial_0 \frac{1}{\Delta x^{2D-4}}, \tag{83}$$

and break up $Ha(a')\not{a}\gamma^0$ into $Ha(a')\partial_0$ and $Ha(a')\gamma^0\bar{\not{a}}$. The intermediate results are summarized in Table VIII. When all terms in Table VIII are summed, we employ (77) for making the expression integrable in $D = 4$ and also segregate the ultraviolet divergences into the local terms using (78). The total result we get from this contribution is

$$\begin{aligned}
-i[\Sigma^{\text{cfm}}](x; x') &= \frac{i\kappa^2 mH}{16\pi^{\frac{D}{2}}} \frac{2\Gamma(\frac{D}{2})\mu^{D-4}}{(D-3)(D-4)} \left\{ (b_2 + b_3) \partial_0 + (b_{2a} + b_{3a}) \gamma^0 \bar{\not{a}} + (b_4 - b_2) Ha \right\} \delta^D(x-x') \\
&+ \frac{i\kappa^2 mH}{64\pi^2} \left\{ \ln(a) \left[\frac{1}{4} \gamma^0 \bar{\not{a}} - \frac{1}{2} Ha \right] + \frac{1}{8} Ha \right\} \delta^4(x-x') + \frac{\kappa^2 mH}{64\pi^4} \left\{ \left[\frac{1}{8} \frac{a}{a'} \partial_0 - \frac{3}{32} \partial_0 + \frac{9}{16} \frac{a}{a'} \gamma^0 \bar{\not{a}} \right. \right. \\
&\left. \left. - \frac{17}{32} \gamma^0 \bar{\not{a}} + \frac{1}{4} Ha \right] \partial^2 \left[\frac{\ln(\mu^2 \Delta x^2)}{\Delta x^2} \right] + \left[-\frac{3}{8} \frac{1}{Ha'} \partial^4 + \frac{Ha}{4} \partial_0^2 \right] \frac{1}{\Delta x^2} \right\}. \tag{84}
\end{aligned}$$

b_2, b_{2a}, b_3, b_{3a} and b_4 are D dimension-dependent coefficients,

$$\begin{aligned}
b_2 &= -\frac{(D-1)(D-5)}{8(D-2)^2} (D-4) - \frac{(3D-4)}{32(D-2)}; & b_{2a} &= \frac{(D-4)}{8(D-2)^2} + \frac{(-3D+5)}{2(D-2)^2} + \frac{(6D-17)}{16(D-2)} + \frac{3}{32}; \\
b_3 &= \frac{(4D-7)}{32(D-2)} (D-4) + \frac{(2D-5)}{16(D-2)}; & b_{3a} &= \frac{(D-4)}{32(D-2)} + \frac{(6D-7)}{16(D-2)}; \\
b_4 &= \left[\frac{-D+6}{32} + \frac{1}{8(D-2)} \right] (D-4) - \frac{1}{16}. \tag{85}
\end{aligned}$$

At the next step we are going to consider contributions from the infinite series expansion of the fermion propagator. Because the series carries at least one power of mass, we only need to consider diagrams that do not originate from the mass term in the Lagrangian. Because the infinite series is vastly more complicated than other parts of the fermion propagator it would be desirable to carry out the computation in $D = 4$ dimensions. Whether or not it is legitimate for us to do this entirely depends on whether this kind of contraction is integrable in four dimensions. The dimensionality of the series of the fermion propagator is $\frac{1}{\Delta x^{D-3-2n}}$ and the one from the conformal part of the graviton propagator is $\frac{1}{\Delta x^{D-2}}$. Also remember that all the terms in Table V that derive from two order m^0 vertices carry two derivatives. This means that the total dimensionality in this class is $\frac{1}{\Delta x^{2D-3-2n}}$. Therefore, we shall separate the $n = 0$ part,

which is necessary when working on an arbitrary D dimension, from the rest of the infinite series expansion, which is integrable in four dimensions. Because the $n = 0$ part is not integrable in $D = 4$, it is worth mentioning its simplification from (19) by performing $\frac{m}{H}$ expansion for gamma functions rather than expanding it out around $D = 4$,

$$\begin{aligned}
\Gamma\left(\frac{D}{2} - 1 \pm i\frac{m}{H}\right) &= \Gamma\left(\frac{D}{2} - 1\right) \left[1 \pm i\frac{m}{H} \psi\left(\frac{D}{2} - 1\right) \right] \\
&+ \mathcal{O}(m^2), \tag{86}
\end{aligned}$$

$$\Gamma\left(1 \pm i\frac{m}{H}\right) = \left[1 \pm i\frac{m}{H} \psi(1) \right] + \mathcal{O}(m^2). \tag{87}$$

Here ψ 's stand for digamma functions and they cancel out completely at order m when one substitutes the above equations back to the $n = 0$ part of the series,

TABLE VIII. $i\Delta_{\text{cf}} \times i[S]_{\text{fm}}$. Note that all contributions are multiplied by the factor $\frac{\kappa^2}{16\pi^D} \Gamma(\frac{D}{2})\Gamma(\frac{D}{2}-1)ma(aa')^{1-\frac{D}{2}}$.

(I-J) _{sub}	$\partial^2 \frac{1}{\Delta x^{2D-4}}$	$Ha\partial_0 \frac{1}{\Delta x^{2D-4}}$	$\gamma^0 \not{\partial} \frac{Ha}{\Delta x^{2D-4}}$	$\partial_0 \frac{Ha'}{\Delta x^{2D-4}}$	$\gamma^0 \not{\partial} \frac{Ha'}{\Delta x^{2D-4}}$	$\frac{H^2 aa'}{\Delta x^{2D-4}}$
(1-1)	$\frac{D}{(D-2)^2}$	$\frac{-2D}{(D-2)^2}$	$\frac{-2D}{(D-2)^2}$	$\frac{-D}{2(D-2)}$	$\frac{D}{2(D-2)}$	$\frac{-D}{(D-2)}$
(1-2)	$\frac{-1}{(D-2)^2}$	$\frac{2}{(D-2)^2}$	$\frac{2}{(D-2)^2}$	$\frac{1}{2(D-2)}$	$\frac{-1}{2(D-2)}$	$\frac{1}{(D-2)}$
(1-3)	$\frac{-(D-1)}{2(D-2)^2}$	$\frac{(D-1)}{2(D-2)^2}$	$\frac{(D-1)}{2(D-2)^2}$	$\frac{(D-1)}{4(D-2)}$	$\frac{-(D-1)}{4(D-2)}$	$\frac{(D-1)}{2(D-2)}$
(2-1)	$\frac{-1}{(D-2)^2}$	$\frac{2}{(D-2)^2}$	$\frac{2}{(D-2)^2}$	$\frac{1}{2(D-2)}$	$\frac{-1}{2(D-2)}$	$\frac{1}{(D-2)}$
(2-2) _a	$\frac{-1}{4(D-2)}$	$\frac{1}{2(D-2)}$	$\frac{-1}{2(D-2)}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{4}$
(2-2) _b	$\frac{-D}{4(D-2)}$	$\frac{D}{2(D-2)}$	0	$\frac{D}{8}$	0	$\frac{D}{4}$
(2-2) _c	$\frac{1}{2(D-2)^2}$	$\frac{-1}{(D-2)^2}$	$\frac{-1}{(D-2)^2}$	$\frac{-1}{4(D-2)}$	$\frac{1}{4(D-2)}$	$\frac{-1}{2(D-2)}$
(2-3) _a	$\frac{D}{8(D-2)}$	$\frac{-D}{8(D-2)}$	0	$\frac{-D}{16}$	0	$\frac{-D}{8}$
(2-3) _b	$\frac{1}{4(D-2)^2}$	$\frac{-1}{4(D-2)^2}$	$\frac{-1}{4(D-2)^2}$	$\frac{-1}{8(D-2)}$	$\frac{1}{8(D-2)}$	$\frac{-1}{4(D-2)}$
(2-3) _c	$\frac{1}{8(D-2)}$	$\frac{-1}{8(D-2)}$	$\frac{1}{8(D-2)}$	$\frac{-1}{16}$	$\frac{-1}{16}$	$\frac{-1}{8}$
(3-1) + (3-2) _b	$\frac{(2D-3)}{4(D-2)^2}$	$\frac{(2D-3)}{4(D-2)}$	$\frac{(2D-3)}{4(D-2)}$	$\frac{-(2D-3)}{8(D-2)}$	$\frac{(2D-3)}{8(D-2)}$	$\frac{(2D-3)}{8}$
(3-2) _a	$\frac{-1}{8(D-2)}$	$\frac{-1}{8}$	$\frac{1}{8}$	$\frac{1}{16(D-2)}$	$\frac{1}{16(D-2)}$	$\frac{-1}{16}$
(3-2) _c	$\frac{-D}{8(D-2)}$	$\frac{-D}{8}$	0	$\frac{D}{16}$	0	$\frac{-D(D-2)}{16}$
(3-3) _a	0	$\frac{D}{32}$	0	$\frac{-D}{32}$	0	$\frac{D(D-2)}{32}$
(3-2) _b	0	$\frac{1}{32}$	$\frac{-1}{32}$	$\frac{-1}{32}$	$\frac{-1}{32}$	$\frac{(D-2)}{32}$
(3-3) _c	0	$\frac{-(2D-3)}{16(D-2)}$	$\frac{-(2D-3)}{16(D-2)}$	$\frac{(2D-3)}{16(D-2)}$	$\frac{-(2D-3)}{16(D-2)}$	$\frac{-(2D-3)}{16}$

$$\begin{aligned}
i[S](x; x')_{n=0} &= \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(\frac{D}{2}-1)}{(2-\frac{D}{2})} \left(i\frac{m}{H}\right) (aa')^{\frac{D}{2}-1} \left[i\not{\partial}\gamma^0 + i\left(\frac{D}{2}-1\right)Ha \right] \left[\Gamma\left(3-\frac{D}{2}\right)\Gamma\left(\frac{D}{2}-1\right) - \left(\frac{y}{4}\right)^{2-\frac{D}{2}} \right] \\
&= \Gamma\left(\frac{D}{2}-1\right) \left\{ \frac{mHa a'}{16\pi^{\frac{D}{2}}} \left[\frac{2\gamma^\nu \gamma^0 \Delta x_\nu}{\Delta x^{D-2}} + \frac{1}{(2-\frac{D}{2})} \frac{Ha}{\Delta x^{D-4}} \right] - \frac{mH^{D-3}(aa')^{\frac{D}{2}-1}}{(4\pi)^{\frac{D}{2}}} \Gamma\left(\frac{D}{2}\right)\Gamma\left(2-\frac{D}{2}\right)Ha \right\}. \quad (88)
\end{aligned}$$

Note that the final two terms that have the same $D = 4$ limit in (88) tend to cancel each other out. The first derivative of (88) has the same pattern,

$$\begin{aligned}
\partial_\mu [S]_{n=0} &= \Gamma\left(\frac{D}{2}-1\right) \left\{ \frac{mHa a'}{8\pi^{\frac{D}{2}}} \left[\frac{\gamma_\mu \gamma^0}{\Delta x^{D-2}} - (D-2) \frac{\gamma^\nu \gamma^0 \Delta x_\nu \Delta x_\mu}{\Delta x^D} + \frac{Ha \delta_\mu^0 \gamma^\nu \gamma^0 \Delta x_\nu}{\Delta x^{D-2}} + \frac{Ha \Delta x_\mu}{\Delta x^{D-2}} - \frac{2H^2 a^2 \delta_\mu^0}{(D-4)\Delta x^{D-4}} \right] \right. \\
&\quad \left. - \frac{m(H^2 aa')^{\frac{D}{2}-1}}{H(4\pi)^{\frac{D}{2}}} \Gamma\left(\frac{D}{2}+1\right)\Gamma\left(2-\frac{D}{2}\right)H^2 a^2 \delta_\mu^0 \right\}. \quad (89)
\end{aligned}$$

The final two terms of Eqs. (88) and (89) would give a nonzero contribution when they are multiplied by the divergent term.⁸

The results derived from this class are lengthy, and we tabulate them separately based on their distinctive characteristics. Some contractions would produce at least one $(D-4)$ factor. One source of $(D-4)$ is from total derivatives acting upon $(aa')^{2-\frac{D}{2}}$. This factor can arise when one power of (aa') comes from the fermion propagator and the rest of it, $(aa')^{1-\frac{D}{2}}$, originates from the conformal part of

the graviton propagator, i.e., (1-1), (1-2), (2-1), (2-2), (3-1), (3-2). Another source of $(D-4)$ comes from the following peculiar gamma function contraction:

$$\gamma^\beta \gamma^\nu \gamma^0 \gamma_\beta = (D-4)\gamma^0 \gamma^\nu + 2(D-2)\eta^{0\nu}, \quad (90)$$

which occurs in the contractions (2-2)_b, (2-3)_a, (3-2)_c, and (3-3)_a. We summarized the terms without $(D-4)$ ⁹ in Tables IX and X whereas the terms with the $(D-4)$ factor are presented in Table XI.

⁸One can consult the various gamma function contractions with (89) in the Appendix.

⁹The contractions (1-3), (2-3)_b, (2-3)_c, (3-3)_b, and (3-3)_c produce no $(D-4)$ factor.

TABLE IX. $i\Delta_{\text{cf}} \times i[S]_{\eta=0}$. Note that all contributions are multiplied by the factor $\frac{\kappa^2 m H}{32\pi^2} \Gamma(\frac{D}{2})\Gamma(\frac{D}{2}-1)(aa')^{2-\frac{D}{2}}$.

(I-J) _{sub}	$\partial_0 \frac{1}{\Delta x^{2D-4}}$	$\gamma^0 \bar{\not{\partial}} \frac{1}{\Delta x^{2D-4}}$	$\frac{Ha}{\Delta x^{2D-4}}$	$\frac{Ha'}{\Delta x^{2D-4}}$	$\frac{H^2 aa' \gamma^0 \gamma^k \Delta x_k}{\Delta x^{2D-4}}$
(1-1)	$\frac{4D}{(D-2)^2}$	$\frac{4D}{(D-2)^2}$	0	0	0
(1-2)	$\frac{-4}{(D-2)^2}$	$\frac{-4}{(D-2)^2}$	0	0	0
(1-3)	$\frac{-(D-1)}{(D-2)^2}$	$\frac{-(D-1)}{(D-2)^2}$	$\frac{-(D-1)(D-3)}{(D-2)^2}$	0	0
(2-1)	$\frac{-4}{(D-2)^2}$	$\frac{-4}{(D-2)^2}$	0	0	0
(2-2) _a	$\frac{(D-3)}{2(D-2)}$	0	$\frac{(2D-9)}{2(D-2)}$	0	0
(2-2) _b	$\frac{(D-5)}{2}$	0	$\frac{D(D-4)}{2(D-2)}$	0	0
(2-2) _c	$\frac{2}{(D-2)^2}$	$\frac{2}{(D-2)^2}$	0	0	0
(2-3) _a	$\frac{-(D-3)}{4}$	0	$\frac{-1}{4}$	$\frac{-(D-2)}{4}$	0
(2-3) _b	$\frac{1}{2(D-2)^2}$	$\frac{1}{2(D-2)^2}$	$\frac{(D-3)}{2(D-2)^2}$	0	0
(2-3) _c	$\frac{-1}{4}$	0	$\frac{-1}{4}$	$\frac{-3}{8}$	$\frac{-1}{4}$
(3-1) + (3-2) _b	$\frac{-(2D-3)}{2(D-2)}$	$\frac{-(2D-3)}{2(D-2)}$	$\frac{-(D-5)(2D-3)}{4(D-2)}$	$\frac{-(2D-3)}{4(D-2)}$	$\frac{(2D-3)}{4(D-2)}$
(3-2) _a	$\frac{-1}{4(D-2)}$	$\frac{(D-3)}{4(D-2)}$	$\frac{-(D-3)}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
(3-2) _c	$\frac{D-3}{4}$	0	$\frac{-D}{8}$	$\frac{D}{8}$	0
(3-3) _a	0	0	$\frac{-(D-2)(D-3)}{16}$	$\frac{(D-2)(D-3)}{16}$	0
(3-3) _b	0	$\frac{-1}{8}$	$\frac{(D-2)}{32}$	$\frac{(3D-8)}{32}$	$\frac{D-2}{32}$
(3-3) _c	0	$\frac{(2D-3)}{4(D-2)}$	$\frac{(2D-3)(3D-7)}{16(D-2)}$	$\frac{(2D-3)}{16}$	$\frac{-(2D-3)}{16}$

After segregating the divergences into the local terms, the total result from Tables IX and X is

$$\begin{aligned}
 -i[\Sigma^{\text{cfn0-1}}](x; x') = & \frac{i\kappa^2 H^2}{32\pi^2} \frac{2\Gamma(\frac{D}{2})}{(D-3)} \frac{\mu^{D-4}}{(D-4)} \left\{ d_1 \frac{m}{H} \gamma^0 \bar{\not{\partial}} + (d_2 + d_3 + d_4) ma \right\} \delta^D(x - x') \\
 & + \frac{i\kappa^2 H^2}{32\pi^2} \ln(a) \left[-2 \frac{m}{H} \gamma^0 \bar{\not{\partial}} + \frac{9}{4} ma \right] \delta^4(x - x') + \frac{\kappa^2 H^2}{64\pi^4} \left[\left[-\frac{1}{2} \frac{m}{H} \gamma^0 \bar{\not{\partial}} + \frac{9m(a+a')}{32} \right] \partial^2 \left[\frac{\ln(\mu^2 \Delta x^2)}{\Delta x^2} \right] \right. \\
 & + \left[\frac{-5}{16} \frac{m}{H} \partial_0 + \frac{3}{4} ma + \frac{13}{32} ma' \right] \partial^2 \frac{1}{\Delta x^2} + \frac{9m(a+a')}{16} \partial_0^2 \frac{1}{\Delta x^2} \\
 & \left. + \frac{3m(5a+a')}{16} \gamma^0 \partial_0 \bar{\not{\partial}} \frac{1}{\Delta x^2} - \frac{1}{4} m H a a' \gamma^0 \bar{\not{\partial}} \frac{1}{\Delta x^2} \right]. \quad (91)
 \end{aligned}$$

Here d_1 , d_2 , d_3 , and d_4 are dimension-dependent coefficients,

$$\begin{aligned}
 d_1 &= \frac{-(D-8)(3D-4)}{8(D-2)^2}; \\
 d_2 &= -\frac{1}{4} - \frac{1}{(D-2)} + (D-4) \left[\frac{-D}{16} + \frac{9}{32} - \frac{3}{8(D-2)} \right]; \\
 d_3 &= \frac{(D-6)}{8} \left[\frac{1}{2} - \frac{(2D-3)}{(D-2)^2} \right]; \\
 d_4 &= -\frac{3}{16}(D-1). \quad (92)
 \end{aligned}$$

Because the contributions from Table XI carries a factor of $(D-4)$, they would survive only if they combine with a

$\frac{1}{(D-4)}$ after making them integrable in $D=4$ dimensions. The result from this part is finite so we give an expression for the $D=4$ dimensions,

$$\begin{aligned}
 -i[\Sigma^{\text{cfn0-2}}](x; x') = & \frac{\kappa^2 H^2}{64\pi^4} \left\{ \frac{1}{8} \frac{m}{H} \gamma^0 \bar{\not{\partial}} - \frac{17}{32} ma \right. \\
 & \left. + \frac{35}{32} ma' \right\} \partial^2 \frac{1}{\Delta x^2}. \quad (93)
 \end{aligned}$$

The contributions that originate from the final two terms of (88) and (89) tend to cancel. They are taken special care of first before they are tabulated because most of the contributions from this class give finite results in $D=4$ dimensions except for a few divergent terms from the double derivatives on $\Delta_{\text{cf}}(x; x')$. The strategy to deal with

TABLE X. $i\Delta_{\text{cf}} \times i[S]_{n=0}$. Note that all contributions are multiplied by the factor $\frac{\kappa^2 m H}{32\pi^2} \Gamma(\frac{D}{2})\Gamma(\frac{D}{2}-1)(aa')^{2-\frac{D}{2}}$.

(I-J) _{sub}	$Ha\partial_0^2 \frac{1}{\Delta x^{2D-6}}$	$\partial_0^2 \frac{Ha'}{\Delta x^{2D-6}}$	$Ha\gamma^0 \partial_0 \not{\partial} \frac{1}{\Delta x^{2D-6}}$	$\gamma^0 \partial_0 \not{\partial} \frac{Ha'}{\Delta x^{2D-6}}$	$\frac{H^2 a^2 Ha'}{\Delta x^{2D-6}}$
(1-1)	$\frac{2D}{(D-2)^2(D-3)}$	0	$\frac{2D}{(D-2)^2(D-3)}$	0	$\frac{2D}{(D-2)^2}$
(1-2)	$\frac{-2}{(D-2)^2(D-3)}$	0	$\frac{-2}{(D-2)^2(D-3)}$	0	$\frac{-2}{(D-2)^2}$
(1-3)	$\frac{-(D-1)}{2(D-2)^2(D-3)}$	0	$\frac{-(D-1)}{2(D-2)^2(D-3)}$	0	0
(2-1)	$\frac{-2}{(D-2)^2(D-3)}$	0	$\frac{-2}{(D-2)^2(D-3)}$	0	$\frac{-2}{(D-2)^2}$
(2-2) _a	0	0	0	0	$\frac{-1}{2(D-2)}$
(2-2) _b	$\frac{-1}{2(D-3)}$	0	0	0	$\frac{-D}{2(D-2)}$
(2-2) _c	$\frac{1}{(D-2)^2(D-3)}$	0	$\frac{1}{(D-2)^2(D-3)}$	0	$\frac{1}{D-2}$
(2-3) _a	$\frac{1}{8(D-3)}$	0	0	0	0
(2-3) _b	$\frac{1}{4(D-2)^2(D-3)}$	0	0	$\frac{1}{4(D-2)^2(D-3)}$	0
(2-3) _c	0	$\frac{1}{16(D-3)}$	0	0	0
(3-1) + (3-2) _b	$\frac{-(2D-3)}{4(D-2)(D-3)}$	0	$\frac{-(2D-3)}{4(D-2)(D-3)}$	0	$\frac{-(2D-3)}{8(D-2)}$
(3-2) _a	0	0	0	0	$\frac{1}{16}$
(3-2) _c	$\frac{D-2}{8(D-3)}$	0	0	0	$\frac{D}{16}$
(3-3) _a	$\frac{-(D-2)}{32(D-3)}$	$\frac{(D-2)}{32(D-3)}$	0	0	0
(3-3) _b	0	$\frac{1}{32(D-3)}$	0	$\frac{1}{32(D-3)}$	0
(3-3) _c	$\frac{(2D-3)}{16(D-2)(D-3)}$	0	$\frac{(2D-3)}{16(D-2)(D-3)}$	0	0

TABLE XI. $i\Delta_{\text{cf}} \times i[S]_{n=0}$ with a $(D-4)$ prefactor. All contributions are multiplied by $\frac{\kappa^2 m H}{32\pi^2} \Gamma(\frac{D}{2})\Gamma(\frac{D}{2}-1)(aa')^{2-\frac{D}{2}} \frac{(D-4)}{2}$.

(I-J) _{sub}	$\partial_0 \frac{1}{\Delta x^{2D-4}}$	$\gamma^0 \not{\partial} \frac{1}{\Delta x^{2D-4}}$	$\frac{Ha}{\Delta x^{2D-4}}$	$\frac{Ha'}{\Delta x^{2D-4}}$	$\frac{H^2 aa' \gamma^0 \not{\partial} \Delta x_k}{\Delta x^{2D-4}}$
(1-1)	0	0	$\frac{4D}{(D-2)^2}$	0	0
(1-2)	0	0	$\frac{-4}{(D-2)^2}$	0	0
(1-3)	0	0	0	0	0
(2-1)	0	0	$\frac{-4}{(D-2)^2}$	0	0
(2-2) _a	0	0	$\frac{-1}{(D-2)}$	$\frac{3}{2(D-2)}$	$\frac{1}{(D-2)}$
(2-2) _b	$\frac{-(D-5)}{2(D-2)}$	$\frac{-(D-5)}{2(D-2)}$	$\frac{-(3D-4)}{2(D-2)}$	$\frac{3}{2}$	0
(2-2) _c	0	0	$\frac{2}{(D-2)^2}$	0	0
(2-3) _a	$\frac{(D-3)}{4(D-2)}$	$\frac{(D-3)}{4(D-2)}$	$\frac{1}{4(D-2)}$	$\frac{3}{8}$	$\frac{1}{4}$
(2-3) _b	0	0	0	0	0
(2-3) _c	0	0	0	0	0
(3-1) + (3-2) _b	0	0	$\frac{-(2D-3)}{4(D-2)}$	$\frac{-(2D-3)}{4(D-2)}$	$\frac{(2D-3)}{4(D-2)}$
(3-2) _a	0	0	$\frac{-1}{8}$	$\frac{-(D-4)}{8(D-2)}$	$\frac{1}{8}$
(3-2) _c	$\frac{-(D-3)}{4(D-2)}$	$\frac{-(D-3)}{4(D-2)}$	$\frac{-(D-2)}{4}$	$\frac{(D-3)}{4}$	0
(3-3) _a	0	0	$\frac{(D-3)}{16}$	$\frac{-(D-3)}{16}$	$\frac{-(D-2)}{16}$
(3-3) _b	0	0	0	0	0
(3-3) _c	0	0	0	0	0

TABLE XII. $i\Delta_{\text{cf}} \times i[S]_{n=0}$ form the two terms that tend to cancel. All contributions are multiplied by $\frac{\kappa^2 m H}{64\pi^4}$.

(I-J) _{sub}	$H^2 a^2 \partial_0 \frac{1}{\Delta x^2}$	$\gamma^0 \vec{\gamma} \frac{H^2 a^2}{\Delta x^2}$	$\frac{H^2 a^2 H a'}{\Delta x^2}$	$H a \partial^2 \left[\frac{\ln(\mu^2 \Delta x^2)}{\Delta x^2} \right]$
(1-1)	$-2 - 4 \ln(\frac{y}{4})$	$-2 - 4 \ln(\frac{y}{4})$	0	0
(1-2)	$\frac{1}{2} + \ln(\frac{y}{4})$	$\frac{1}{2} + \ln(\frac{y}{4})$	0	0
(1-3)	$\frac{9}{4} + \frac{3}{2} \ln(\frac{y}{4})$	$\frac{9}{4} + \frac{3}{2} \ln(\frac{y}{4})$	$\frac{9}{4} + \frac{3}{2} \ln(\frac{y}{4})$	0
(2-1)	$\frac{1}{2} + \ln(\frac{y}{4})$	$\frac{1}{2} + \ln(\frac{y}{4})$	0	0
(2-2) _a	$\frac{1}{4} + \frac{1}{2} \ln(\frac{y}{4})$	$-\frac{1}{4} - \frac{1}{2} \ln(\frac{y}{4})$	0	0
(2-2) _b	$1 + 2 \ln(\frac{y}{4})$	0	0	0
(2-2) _c	$-\frac{1}{4} - \frac{1}{2} \ln(\frac{y}{4})$	$-\frac{1}{4} - \frac{1}{2} \ln(\frac{y}{4})$	0	0
(2-3) _a	$-\frac{3}{2} - \ln(\frac{y}{4})$	0	$-\frac{3}{2} - \ln(\frac{y}{4})$	0
(2-3) _b	$-\frac{3}{8} - \frac{1}{4} \ln(\frac{y}{4})$	$-\frac{3}{8} - \frac{1}{4} \ln(\frac{y}{4})$	$-\frac{3}{8} - \frac{1}{4} \ln(\frac{y}{4})$	0
(2-3) _c	$-\frac{3}{8} - \frac{1}{4} \ln(\frac{y}{4})$	$\frac{3}{8} + \frac{1}{4} \ln(\frac{y}{4})$	$-\frac{3}{8} - \frac{1}{4} \ln(\frac{y}{4})$	0
(3-1) + (3-2) _b	$\frac{5}{8} \ln(\frac{y}{4})$	$\frac{5}{8} \ln(\frac{y}{4})$	0	$\frac{5}{8}$
(3-2) _a	$-\frac{1}{8} \ln(\frac{y}{4})$	$-\frac{1}{8} \ln(\frac{y}{4})$	0	$-\frac{1}{8}$
(3-2) _c	$-\frac{1}{2} \ln(\frac{y}{4})$	0	0	$-\frac{1}{2}$
Subtotal	0	$\frac{3}{4} - \ln(\frac{y}{4})$	0	0

the finite part of the contributions is to perform the $(D-4)$ expansion and make use of (78) and the following key identities:

$$\partial_\mu \frac{1}{\Delta x^{D-2}} = \mu^{D-4} \left[1 + \frac{1}{2}(D-4)(1 - \ln \Delta x^2) \right] \partial_\mu \frac{1}{\Delta x^2}, \quad (94)$$

$$\partial_\mu \frac{1}{\Delta x^{2D-6}} = \mu^{2D-8} \left[1 + \frac{1}{2}(D-4)(2 - 2 \ln \Delta x^2) \right] \partial_\mu \frac{1}{\Delta x^2}, \quad (95)$$

$$\partial^2 \frac{1}{\Delta x^{D-2}} = \frac{i4\pi^{\frac{D}{2}}}{\Gamma(\frac{D}{2}-1)} \delta^D(x-x'). \quad (96)$$

Here we present one example from (3-2)_c,

$$\begin{aligned} & \Gamma\left(\frac{D}{2}\right) \Gamma\left(\frac{D}{2}-1\right) \left(\frac{m}{H}\right) \frac{\kappa^2}{8} \left\{ \frac{H^2 (aa')^{2-\frac{D}{2}}}{32\pi^D \frac{1}{2}(D-4)} \left[\frac{DHa}{2(D-3)} \partial^2 \right. \right. \\ & \quad \left. \left. + \frac{D}{2} H^2 a^2 \partial_0 \right] \frac{1}{\Delta x^{2D-6}} - \frac{H^{D-2}}{2^{D+1} \pi^D} \Gamma\left(\frac{D}{2}\right) \frac{\Gamma(3-\frac{D}{2})}{\frac{1}{2}(D-4)} \right. \\ & \quad \left. \times \left[\frac{DHa}{(D-2)} \partial^2 + \frac{D}{2} H^2 a^2 \partial_0 \right] \frac{1}{\Delta x^{D-2}} \right\}. \end{aligned} \quad (97)$$

There are three distinctive contributions in (97). We extract out the prefactor $\Gamma(\frac{D}{2})\Gamma(\frac{D}{2}-1)\frac{\kappa^2 m H}{64\pi^D}$ to avoid the repeated

and lengthy expressions. The first kind comes from the two single derivative terms,

$$\begin{aligned} & \frac{\frac{1}{4}\mu^{2D-8}}{\frac{1}{2}(D-4)} \frac{DH^2 a^2}{2} \left\{ \left[1 + \frac{1}{2}(D-4)(-\ln aa' + 2 - 2 \ln \mu^2 \Delta x^2) \right] \right. \\ & \quad \left. + \left[-1 + \frac{1}{2}(D-4) \left(-\ln \frac{H^2}{4\mu^2} - 1 - 1 + \ln \mu^2 \Delta x^2 \right) \right] \right\} \\ & \times \partial_0 \frac{1}{\Delta x^2} \rightarrow \frac{DH^2 a^2}{8} \left[-\ln \frac{y}{4} \right] \partial_0 \frac{1}{\Delta x^2}, \end{aligned} \quad (98)$$

and we present the temporal and spatial contributions separately in Table XII. The second kind is the contributions that produce a delta function originated from the two double derivative terms,

$$\begin{aligned} & \frac{\frac{1}{4}\mu^{D-4}}{\frac{1}{2}(D-4)} \frac{DHa}{2} \left\{ \left[1 + \frac{1}{2}(D-4)(-\ln aa' - 2) \right] \right. \\ & \quad \left. + \left[-1 + \frac{1}{2}(D-4) \left(-\ln \frac{H^2}{4\mu^2} - 1 + 1 \right) \right] \right\} \\ & \times \frac{i4\pi^{\frac{D}{2}} \delta^D(x-x')}{\Gamma(\frac{D}{2}-1)} \\ & \rightarrow -DHa \left[\ln \frac{Ha}{2\mu} + 1 \right] \frac{i\pi^{\frac{D}{2}} \mu^{D-4}}{\Gamma(\frac{D}{2}-1)} \delta^D(x-x'), \end{aligned} \quad (99)$$

and the final results are displayed in the first column of Table XIV. The third kind is the residual term from $\partial^2 \frac{1}{\Delta x^{2D-6}}$,

$$\frac{\mu^{2D-8}}{\frac{1}{2}(D-4)} \times -\frac{1}{2}(D-4) \frac{DH a}{8} \partial^2 \left[\frac{\ln \mu^2 \Delta x^2}{\Delta x^2} \right], \quad (100)$$

and we give the results in the final column of Table XII.

The final contribution in this category is from (3-3) that consists of some finite terms and local divergent terms. Recall in (82) that two derivatives acting on the conformal part of the graviton propagator would produce a delta function. In general, it would be zero in dimensional regularization when it acts on dimension-dependent power of the coordinate separation. However, the last term with the divergent coefficient in (88) does not possess any dimension-dependent power of Δx^2 . As a result, when $i[S]_{n=0}$ are multiplied by $\partial \cdot \partial \Delta_{\text{cf}}$, no other terms in the calculation can be used to cancel this particular divergent local term. Here we present (3-3)_a as an example,

$$\begin{aligned} & \Gamma\left(\frac{D}{2}\right) \Gamma\left(\frac{D}{2}-1\right) \frac{-(D-2)}{16} \frac{m}{H} \frac{\kappa^2}{2} \left\{ \frac{H^2 (aa')^{2-\frac{D}{2}}}{32\pi^D \frac{1}{2}(D-4)} \right. \\ & \times \left[\frac{DH^3 a^2 a' \Delta \eta^2}{\Delta x^{2D-4}} + \frac{D}{2} \frac{H^3 a^2 a'}{\Delta x^{2D-6}} \right] \\ & - \frac{H^{D-2} \Gamma(\frac{D}{2})}{2^{D+1} \pi^D} \frac{\Gamma(3-\frac{D}{2})}{\frac{1}{2}(D-4)} \left[\frac{DH^3 a^2 a' \Delta \eta^2}{\Delta x^D} \right. \\ & \left. \left. + \frac{D}{2} \frac{H^3 a^2 a'}{\Delta x^{D-2}} + \frac{DH a i 2 \pi^{\frac{D}{2}}}{\Gamma(\frac{D}{2})} \delta^D(x-x') \right] \right\}. \quad (101) \end{aligned}$$

We employ the same trick to deal with the finite part, extract out the prefactor $\Gamma(\frac{D}{2}) \Gamma(\frac{D}{2}-1) \frac{\kappa^2 m H}{64 \pi^D} \frac{-(D-2)}{16}$,

$$\begin{aligned} & \left\{ \left[1 + \frac{1}{2}(D-4)(-\ln aa' - 2 \ln \mu^2 \Delta x^2) \right] \right. \\ & + \left[-1 + \frac{1}{2}(D-4) \left(-\ln \frac{H^2}{4 \mu^2} - 1 + \ln \mu^2 \Delta x^2 \right) \right] \left. \right\} \\ & \times \frac{\mu^{2D-8}}{\frac{1}{2}(D-4)} \left[\frac{DH^3 a^2 a' \Delta \eta^2}{\Delta x^2} + \frac{D}{2} \frac{H^3 a^2 a'}{\Delta x^2} \right], \quad (102) \end{aligned}$$

TABLE XIII. $i \Delta_{\text{cf}} \times i[S]_{n=0}$ from the two terms that tend to cancel. All contributions are multiplied by $\frac{\kappa^2 m H}{64 \pi^D} \times (1 + \ln(\frac{y}{4}))$.

(I-J) _{sub}	$\frac{H^2 a^2 \Delta \eta}{\Delta x^4}$	$\frac{H^2 a a' \Delta \eta}{\Delta x^4}$	$\frac{H^2 a^2 \gamma^0 \gamma^k \Delta x_k}{\Delta x^4}$	$\frac{H^2 a a' \gamma^0 \gamma^k \Delta x_k}{\Delta x^4}$	$\frac{H^2 a^2 H a'}{\Delta x^2}$
(3-3) _a	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	$\frac{1}{4}$
(3-3) _b	$\frac{1}{8}$	$-\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{16}$
(3-3) _c	$-\frac{5}{8}$	$\frac{5}{8}$	$\frac{5}{8}$	$\frac{5}{8}$	$-\frac{5}{16}$
Subtotal	0	0	$\frac{3}{4}$	$\frac{3}{4}$	0

and tabulate the result for $D = 4$ in Table XIII. The divergent term can be read off directly and we present it in the second column of Table XIII. Finally we enclose this subclass by summing up all the terms from Tables XII, XIII, and XIV,

$$\begin{aligned} & -i[\Sigma^{\text{cf}0-3}](x; x') \\ & = \frac{i \kappa^2 H^{D-2}}{2^{D+1} \pi^{\frac{D}{2}}} \frac{\Gamma(\frac{D}{2}) \Gamma(\frac{D}{2}-1) \Gamma(3-\frac{D}{2}) (2D-3)}{2(D-4)} m a \delta^D(x-x') \\ & - \frac{i \kappa^2 H^2}{32 \pi^2} \frac{m a}{4} \delta^4(x-x') + \frac{\kappa^2 H^2}{64 \pi^4} \left\{ \left[\frac{3}{8} - \frac{11}{8} \ln\left(\frac{y}{4}\right) \right] m H a^2 \right. \\ & \left. - \frac{3}{8} \left[1 + \ln\left(\frac{y}{4}\right) \right] m H a a' \right\} \gamma^0 \not{x} \frac{1}{\Delta x^2}. \quad (103) \end{aligned}$$

The last computation in this subsection involves the rest of the infinite series expansion that are all integrable and hence we can compute it in $D = 4$ dimensions directly. The fermion propagator for the $n \geq 1$ of the series in four dimensions we employed is

$$\begin{aligned} i[S]_{n \geq 1} & = \frac{m H a a'}{16 \pi^2} \sum_{n=1}^{\infty} \left\{ \frac{2 \gamma^\nu \gamma^0 \Delta x_\nu}{\Delta x^2} \left[1 + n \ln\left(\frac{y}{4}\right) \right] \right. \\ & \left. + H a \left[1 + (n+1) \ln\left(\frac{y}{4}\right) \right] \right\} \left(\frac{y}{4} \right)^n, \quad (104) \end{aligned}$$

and its derivative¹⁰ is

$$\begin{aligned} \partial_\mu i[S]_{n \geq 1} & = \frac{m H a a'}{16 \pi^2} \Sigma_{n=1}^{\infty} \left(\frac{y}{4} \right)^n \left\{ \frac{4 \gamma^\nu \gamma^0 \Delta x_\nu \Delta x_\mu}{\Delta x^4} \right. \\ & \times \left[(2n-1) + (n^2-n) \ln \frac{y}{4} \right] + \frac{2 \gamma_\mu \gamma^0}{\Delta x^2} \\ & \times \left[1 + n \ln \frac{y}{4} \right] + \frac{2 H a (\delta_\mu^0 \gamma^\nu \gamma^0 \Delta x_\nu + \Delta x_\mu)}{\Delta x^2} \\ & \times \left[(2n+1) + (n^2+n) \ln \frac{y}{4} \right] + H^2 a^2 \delta_\mu^0 \\ & \left. \times \left[(2n+3) + (n^2+3n+2) \ln \frac{y}{4} \right] \right\}. \quad (105) \end{aligned}$$

One interesting pattern is that taking the derivative of the coefficient of the logarithm term with respect to n gives the coefficient for the term without logarithms. Before we table the result from each lengthy expression, we present the result from the contraction (1-1),

¹⁰One can find various gamma functions contracted with (105) in the Appendix.

TABLE XIV. $i\Delta_{\text{cf}} \times i[S]_{n=0}$. The delta function contribution from the two terms that tend to cancel.

(I-J) _{sub}	$\frac{i\kappa^2 H^2}{64\pi^2} ma\delta^4(x-x')$	$\frac{i\kappa^2 H^{D-2}}{2^{D+2}\pi^2} \Gamma(\frac{D}{2}) \frac{\Gamma(3-\frac{D}{2})}{\Gamma(D-4)} ma\delta^D(x-x')$
(3-1) + (3-2) _b	$5[1 + \ln(\frac{Ha}{2\mu})]$	0
(3-2) _a	$-[1 + \ln(\frac{Ha}{2\mu})]$	0
(3-2) _c	$-4[1 + \ln(\frac{Ha}{2\mu})]$	0
(3-3) _a	0	$\Gamma(\frac{D}{2}) \times \frac{D}{4}$
(3-3) _b	0	$\Gamma(\frac{D}{2}) \times (1 - \frac{D}{2})$
(3-3) _c	0	$\Gamma(\frac{D}{2} - 1) \times \frac{(2D-3)}{2}$

$$\begin{aligned}
2\kappa^2 \partial'_\mu \{ \not{\partial} i[S](x; x') \gamma^\mu i\Delta_{\text{cf}}(x; x') \} = & \frac{\kappa^2 mH}{32\pi^4} \left\{ 8 \left[\frac{\gamma^0 \gamma^\mu \Delta x_\mu}{\Delta x^6} + \frac{Ha \gamma^0 \gamma^\mu \Delta \eta \Delta x_\mu}{\Delta x^6} \right] \left[(3n^2 - 2n - 2) + (n^3 - n^2 - 2n) \ln\left(\frac{y}{4}\right) \right] \right. \\
& - \frac{4Ha}{\Delta x^4} \left[(3n^2 - 1) + (n^3 - n) \ln\left(\frac{y}{4}\right) \right] - \frac{2H^2 a^2 \gamma^0 \gamma^\mu \Delta x_\mu}{\Delta x^4} [(3n^2 + 4n - 1) \\
& + (n^3 + 2n^2 - n - 2) \ln\left(\frac{y}{4}\right)] + \frac{H^2 a^2 Ha'}{\Delta x^4} [(3n^2 + 6n + 2) \\
& \left. + (n^3 + n^2 + 2n) \ln\left(\frac{y}{4}\right)] \right\} \left(\frac{y}{4}\right)^n. \quad (106)
\end{aligned}$$

The same pattern here happened again. We should keep it in mind that this pattern might remain after summing up each individual contribution. We also separated temporal terms with spatial ones for our conventional choice of counterterms and summarized the results in Tables XV, XVI, XVII, and XVIII, before many summations were performed. Table XVI (Table XVIII) is the partner of Table XV (Table XVII) with the extra logarithm, $\ln \frac{y}{4}$. From the coefficient of each individual term at the bottom of each table one might already notice that the pattern we mentioned above still exists. The benefit for postponing the infinite summation for each individual contraction is not only because it is a less complicated procedure but also because the pattern serves as one consistent check whether

or not we have done the computation correctly for such a long and complicated calculation.

We can easily read off the contribution from each distinctive term. For example, the total coefficient of $\frac{\gamma^0 \gamma^k \Delta x_k}{\Delta x^6}$ from Tables XV and XVI is

$$\frac{\kappa^2 mH}{64\pi^4} \sum_{n=1}^{\infty} \left[8(3n^2 - 2n - 1) + 8(n^3 - n^2 - n) \ln\left(\frac{y}{4}\right) \right] \left(\frac{y}{4}\right)^n. \quad (107)$$

After collecting all of the terms from the four tables, we obtain

$$\begin{aligned}
& \frac{\kappa^2 mH}{64\pi^4} \sum_{n=1}^{\infty} \left(\frac{y}{4}\right)^n \left\{ \frac{\gamma^0 \gamma^k \Delta x_k}{\Delta x^6} \left[8(3n^2 - 2n - 1) + 8(n^3 - n^2 - n) \ln\left(\frac{y}{4}\right) \right] + \left[5 \frac{Ha \gamma^0 \Delta \eta \gamma^k \Delta x_k}{\Delta x^6} + \frac{Ha' \gamma^0 \Delta \eta \gamma^k \Delta x_k}{\Delta x^6} \right. \right. \\
& - 3 \frac{Ha \Delta \eta^2}{\Delta x^6} - 3 \frac{Ha' \Delta \eta^2}{\Delta x^6} \left. \right] \left[\left(6n^2 - 4n - \frac{3}{2} \right) + \left(2n^3 - 2n^2 - \frac{3}{2}n \right) \ln\left(\frac{y}{4}\right) \right] - 3 \left[\frac{Ha}{\Delta x^4} + \frac{Ha'}{\Delta x^4} \right] \\
& \times \left[(6n^2 + 2n) + (2n^3 + n^2) \ln\left(\frac{y}{4}\right) \right] + \frac{H^2 a^2 \gamma^0 \gamma^k \Delta x_k}{\Delta x^4} \left[- \left(9n^2 + 12n + \frac{3}{4} \right) + \left(-3n^3 - 6n^2 - \frac{3}{4}n + \frac{9}{4} \right) \ln\left(\frac{y}{4}\right) \right] \\
& \left. + \frac{H^2 aa' \gamma^0 \gamma^k \Delta x_k}{\Delta x^4} \left[\left(-3n^2 - 2n + \frac{5}{4} \right) + \left(-n^3 - n^2 + \frac{5}{4}n + \frac{3}{4} \right) \ln\left(\frac{y}{4}\right) \right] \right\}. \quad (108)
\end{aligned}$$

One might notice that the infinite series could be summed easily using the following identities:

$$\sum_{n=1}^{\infty} Y^n = \frac{Y}{1-Y}; \quad \sum_{n=1}^{\infty} nY^n = \frac{Y}{(1-Y)^2}; \quad (109)$$

TABLE XV. $i\Delta_{\text{cf}} \times i[S]_{n \geq 1} - \text{I}$. All contributions are multiplied by $\frac{\kappa^2 m H}{64 \pi^4} \sum_{n \geq 1}^{\infty} \left(\frac{\gamma}{4}\right)^n$. Here $f'(n) = (6n^2 - 4n - \frac{3}{2})$.

(I-J) _{sub}	$\frac{\gamma^0 \gamma^k \Delta x_k}{\Delta x^6}$	$\frac{\Delta \eta}{\Delta x^6}$	$\frac{H a \gamma^0 \Delta \eta \gamma^k \Delta x_k}{\Delta x^6}$	$\frac{H a' \gamma^0 \Delta \eta \gamma^k \Delta x_k}{\Delta x^6}$	$\frac{H a \Delta \eta^2}{\Delta x^6}$	$\frac{H a' \Delta \eta^2}{\Delta x^6}$
(1-1)	$16(3n^2 - 2n - 2)$	$-16(3n^2 - 2n - 2)$	$16(3n^2 - 2n - 2)$	0	$-16(3n^2 - 2n - 2)$	0
(1-2)	$-4(3n^2 - 2n - 2)$	$4(3n^2 - 2n - 2)$	$-4(3n^2 - 2n - 2)$	0	$4(3n^2 - 2n - 2)$	0
(1-3)	$6(2n + 1)$	$-6(2n + 1)$	$6(2n + 1)$	0	$-6(2n + 1)$	0
(2-1)	$-4(3n^2 - 2n - 2)$	$4(3n^2 - 2n - 2)$	$-4(3n^2 - 2n - 2)$	0	$4(3n^2 - 2n - 2)$	0
(2-2) _a	$-2(3n^2 - 2n - 1)$	$2(3n^2 - 2n + 1)$	0	$2(3n^2 - 2n)$	0	$-2(3n^2 - 2n)$
(2-2) _b	0	$4(6n^2 - 4n - 1)$	0	0	$4(3n^2 - 2n - 2)$	$-4(3n^2 - 2n)$
(2-2) _c	$2(3n^2 - 2n - 2)$	$-2(3n^2 - 2n - 2)$	$2(3n^2 - 2n - 2)$	0	$-2(3n^2 - 2n - 2)$	0
(2-3) _a	0	$2(4n - 1)$	0	0	$2(2n + 1)$	$-2(2n - 1)$
(2-3) _b	$-(2n + 1)$	$(2n + 1)$	$-(2n + 1)$	0	$(2n + 1)$	0
(2-3) _c	$-2n$	$(2n - 2)$	0	$(2n - 1)$	0	$-(2n - 1)$
(3-1)	$-6(2n - 2)$	$6(2n - 2)$	$-6(2n - 2)$	0	$6(2n - 2)$	0
(3-2) _a	$(2n - 1)$	$-(2n + 1)$	0	$-2n$	0	$2n$
(3-2) _b	$(2n - 2)$	$-(2n - 2)$	$(2n - 2)$	0	$-(2n - 2)$	0
(3-2) _c	0	$-2(4n - 1)$	0	0	$-2(2n - 2)$	$4n$
(3-3) _a	0	0	0	0	-1	+1
(3-3) _b	1	0	0	$-\frac{1}{2}$	0	$\frac{1}{2}$
(3-3) _c	-5	0	$-\frac{5}{2}$	0	$\frac{5}{2}$	0
Total _{sub}	$8(3n^2 - 2n - 1)$	0	$5f'(n)$	$f'(n)$	$-3f'(n)$	$-3f'(n)$

TABLE XVI. $i\Delta_{\text{cf}} \times i[S]_{n \geq 1} - \text{I}'$. All contributions are multiplied by $\frac{\kappa^2 m H}{64 \pi^4} \sum_{n \geq 1}^{\infty} \left(\frac{\gamma}{4}\right)^n \ln\left(\frac{\gamma}{4}\right)$. Here $f(n) = 2n^3 - 2n^2 - \frac{3}{2}n$.

(I-J) _{sub}	$\frac{\gamma^0 \gamma^k \Delta x_k}{\Delta x^6}$	$\frac{\Delta \eta}{\Delta x^6}$	$\frac{H a \gamma^0 \Delta \eta \gamma^k \Delta x_k}{\Delta x^6}$	$\frac{H a' \gamma^0 \Delta \eta \gamma^k \Delta x_k}{\Delta x^6}$	$\frac{H a \Delta \eta^2}{\Delta x^6}$	$\frac{H a' \Delta \eta^2}{\Delta x^6}$
(1-1)	$16(n^3 - n^2 - 2n)$	$-16(n^3 - n^2 - 2n)$	$16(n^3 - n^2 - 2n)$	0	$-16(n^3 - n^2 - 2n)$	0
(1-2)	$-4(n^3 - n^2 - 2n)$	$4(n^3 - n^2 - 2n)$	$-4(n^3 - n^2 - 2n)$	0	$4(n^3 - n^2 - 2n)$	0
(1-3)	$6(n^2 + n)$	$-6(n^2 + n)$	$6(n^2 + n)$	0	$-6(n^2 + n)$	0
(2-1)	$-4(n^3 - n^2 - 2n)$	$4(n^3 - n^2 - 2n)$	$-4(n^3 - n^2 - 2n)$	0	$4(n^3 - n^2 - 2n)$	0
(2-2) _a	$-2(n^3 - n^2 - n)$	$2(n^3 - n^2 + n)$	0	$2(n^3 - n^2)$	0	$-2(n^3 - n^2)$
(2-2) _b	0	$4(2n^3 - 2n^2 - n)$	0	0	$4(n^3 - n^2 - 2n)$	$-4(n^3 - n^2)$
(2-2) _c	$2(n^3 - n^2 - 2n)$	$-2(n^3 - n^2 - 2n)$	$2(n^3 - n^2 - 2n)$	0	$-2(n^3 - n^2 - 2n)$	0
(2-3) _a	0	$2(2n^2 - n)$	0	0	$2(n^2 + n)$	$-2(n^2 - n)$
(2-3) _b	$-(n^2 + n)$	$(n^2 + n)$	$-(n^2 + n)$	0	$(n^2 + n)$	0
(2-3) _c	$-n^2$	$(n^2 - 2n)$	0	$(n^2 - n)$	0	$-(n^2 - n)$
(3-1)	$-6(n^2 - 2n)$	$6(n^2 - 2n)$	$-6(n^2 - 2n)$	0	$6(n^2 - 2n)$	0
(3-2) _a	$(n^2 - n)$	$-(n^2 + n)$	0	$-n^2$	0	n^2
(3-2) _b	$(n^2 - 2n)$	$-(n^2 - 2n)$	$(n^2 - 2n)$	0	$-(n^2 - 2n)$	0
(3-2) _c	0	$-2(2n^2 - n)$	0	0	$-2(n^2 - 2n)$	$2n^2$
(3-3) _a	0	0	0	0	$-n$	$+n$
(3-3) _b	n	0	0	$-\frac{1}{2}n$	0	$\frac{1}{2}n$
(3-3) _c	$-5n$	0	$-\frac{5}{2}n$	0	$\frac{5}{2}n$	0
Total _{sub}	$8(n^3 - n^2 - n)$	0	$5f(n)$	$f(n)$	$-3f(n)$	$-3f(n)$

TABLE XVII. $i\Delta_{\text{cf}} \times i[S]_{n \geq 1} - \text{II}$. All contributions are multiplied by $\frac{\kappa^2 m H}{64\pi^4} \sum_{n \geq 1} \left(\frac{\gamma}{4}\right)^n$.

(I-J) _{sub}	$\frac{Ha}{\Delta x^4}$	$\frac{Ha'}{\Delta x^4}$	$\frac{H^2 a^2 \gamma^0 \gamma^k \Delta x_k}{\Delta x^4}$	$\frac{H^2 a a' \gamma^0 \gamma^k \Delta x_k}{\Delta x^4}$	$\frac{H^2 a^2 \Delta \eta}{\Delta x^4}$	$\frac{H^2 a^2 H a'}{\Delta x^2}$
(1-1)	$-8(3n^2 - 1)$	0	$-4(3n^2 + 4n - 1)$	0	$4(3n^2 + 4n - 1)$	$2(3n^2 + 6n + 2)$
(1-2)	$2(3n^2 - 1)$	0	$(3n^2 + 4n - 1)$	0	$-(3n^2 + 4n - 1)$	$-\frac{1}{2}(3n^2 + 6n + 2)$
(1-3)	$-3(2n + 1)$	0	$-\frac{3}{2}(2n + 3)$	0	$\frac{3}{2}(2n + 3)$	$\frac{3}{4}(2n + 3)$
(2-1)	$2(3n^2 - 1)$	0	$(3n^2 + 4n - 1)$	0	$-(3n^2 + 4n - 1)$	$-\frac{1}{2}(3n^2 + 6n + 2)$
(2-2) _a	$-(3n^2 + 2n)$	$-2(3n^2 + n)$	$-\frac{1}{2}(3n^2 + 4n - 1)$	$-(3n^2 + 2n)$	$-\frac{1}{2}(3n^2 + 4n - 1)$	$-\frac{1}{4}(3n^2 + 6n + 2)$
(2-2) _b	$-2(2n + 1)$	$-2(6n^2 + 2n)$	0	0	$-2(3n^2 + 4n - 1)$	$-(3n^2 + 6n + 2)$
(2-2) _c	$-(3n^2 - 1)$	0	$-\frac{1}{2}(3n^2 + 4n - 1)$	0	$\frac{1}{2}(3n^2 + 4n - 1)$	$\frac{1}{4}(3n^2 + 6n + 2)$
(2-3) _a	0	$-(4n + 1)$	0	0	$-(2n + 3)$	$-\frac{1}{2}(2n + 3)$
(2-3) _b	$\frac{1}{2}(2n + 1)$	0	$\frac{1}{4}(2n + 3)$	0	$-\frac{1}{4}(2n + 3)$	$-\frac{1}{8}(2n + 3)$
(2-3) _c	$-\frac{1}{2}(2n + 1)$	$-(2n + \frac{1}{2})$	$-\frac{1}{4}(2n + 3)$	$-\frac{1}{2}(2n + 1)$	$-\frac{1}{4}(2n + 3)$	$-\frac{1}{8}(2n + 3)$
(3-1)	$6(n - \frac{3}{4})$	$-\frac{3}{2}$	$3n$	$\frac{3}{2}$	$-3n$	$-\frac{3}{4}(2n + 1)$
(3-2) _a	$\frac{1}{4}(4n + 1)$	$(2n + \frac{1}{4})$	$\frac{1}{2}n$	$\frac{1}{4}(4n + 1)$	$\frac{1}{2}n$	$\frac{1}{8}(2n + 1)$
(3-2) _b	$-(n - \frac{3}{4})$	$\frac{1}{4}$	$-\frac{1}{2}n$	$-\frac{1}{4}$	$\frac{1}{2}n$	$\frac{1}{8}(2n + 1)$
(3-2) _c	2	$(4n + 1)$	0	0	$2n$	$\frac{1}{2}(2n + 1)$
(3-3) _a	-1	1	0	0	$\frac{1}{2}$	$\frac{1}{4}$
(3-3) _b	0	$\frac{1}{2}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$
(3-3) _c	$\frac{5}{2}$	0	$\frac{5}{8}$	0	$-\frac{5}{8}$	$-\frac{5}{16}$
Total _{sub}	$-6(3n^2 + n)$	$-6(3n^2 + n)$	$-(9n^2 + 12n + \frac{3}{4})$	$(-3n^2 - 2n + \frac{5}{4})$	0	0

TABLE XVIII. $i\Delta_{\text{cf}} \times i[S]_{n \geq 1} - \text{II}'$. All contributions are multiplied by $\frac{\kappa^2 m H}{64\pi^4} \sum_{n \geq 1} \left(\frac{\gamma}{4}\right)^n \ln\left(\frac{\gamma}{4}\right)$.

(I-J) _{sub}	$\frac{Ha}{\Delta x^4}$	$\frac{Ha'}{\Delta x^4}$	$\frac{H^2 a^2 \gamma^0 \gamma^k \Delta x_k}{\Delta x^4}$	$\frac{H^2 a a' \gamma^0 \gamma^k \Delta x_k}{\Delta x^4}$	$\frac{H^2 a^2 \Delta \eta}{\Delta x^4}$	$\frac{H^2 a^2 H a'}{\Delta x^2}$
(1-1)	$-8(n^3 - n)$	0	$-4(n^3 + 2n^2 - n - 2)$	0	$4(n^3 + 2n^2 - n - 2)$	$2(n^3 + 3n^2 + 2n)$
(1-2)	$2(n^3 - n)$	0	$(n^3 + 2n^2 - n - 2)$	0	$-(n^3 + 2n^2 - n - 2)$	$-\frac{1}{2}(n^3 + 3n^2 + 2n)$
(1-3)	$-3(n^2 + n)$	0	$-\frac{3}{2}(n^2 + 3n + 2)$	0	$\frac{3}{2}(n^2 + 3n + 2)$	$\frac{3}{4}(n^2 + 3n + 2)$
(2-1)	$2(n^3 - n)$	0	$(n^3 + 2n^2 - n - 2)$	0	$-(n^3 + 2n^2 - n - 2)$	$-\frac{1}{2}(n^3 + 3n^2 + 2n)$
(2-2) _a	$-(n^3 + n^2)$	$-(2n^3 + n^2)$	$-\frac{1}{2}(n^3 + 2n^2 - n - 2)$	$-(n^3 + n^2)$	$-\frac{1}{2}(n^3 + 2n^2 - n - 2)$	$-\frac{1}{4}(n^3 + 3n^2 + 2n)$
(2-2) _b	$-2(n^2 + n)$	$-2(2n^3 + n^2)$	0	0	$-2(n^3 + 2n^2 - n - 2)$	$-(n^3 + 3n^2 + 2n)$
(2-2) _c	$-(n^3 - n)$	0	$-\frac{1}{2}(n^3 + 2n^2 - n - 2)$	0	$\frac{1}{2}(n^3 + 2n^2 - n - 2)$	$\frac{1}{4}(n^3 + 3n^2 + 2n)$
(2-3) _a	0	$-(2n^2 + n)$	0	0	$-(n^2 + 3n + 2)$	$-\frac{1}{2}(n^2 + 3n + 2)$
(2-3) _b	$\frac{1}{2}(n^2 + n)$	0	$\frac{1}{4}(n^2 + 3n + 2)$	0	$-\frac{1}{4}(n^2 + 3n + 2)$	$-\frac{1}{8}(n^2 + 3n + 2)$
(2-3) _c	$-\frac{1}{2}(n^2 + n)$	$-(n^2 + \frac{1}{2}n)$	$-\frac{1}{4}(n^2 + 3n + 2)$	$-\frac{1}{2}(n^2 + n)$	$-\frac{1}{4}(n^2 + 3n + 2)$	$-\frac{1}{8}(n^2 + 3n + 2)$
(3-1)	$(3n^2 - \frac{9}{2}n)$	$-\frac{3}{2}n$	$\frac{3}{2}(n^2 - 1)$	$\frac{3}{2}n$	$-\frac{3}{2}(n^2 - 1)$	$-\frac{3}{4}(n^2 + n)$
(3-2) _a	$\frac{1}{4}(2n^2 + n)$	$(n^2 + \frac{1}{4}n)$	$\frac{1}{4}(n^2 - 1)$	$\frac{1}{4}(2n^2 + n)$	$\frac{1}{4}(n^2 - 1)$	$\frac{1}{8}(n^2 + n)$
(3-2) _b	$-(\frac{1}{2}n^2 - \frac{3}{4}n)$	$\frac{1}{4}n$	$-\frac{1}{4}(n^2 - 1)$	$-\frac{1}{4}n$	$\frac{1}{4}(n^2 - 1)$	$\frac{1}{8}(n^2 + n)$
(3-2) _c	$2n$	$(2n^2 + n)$	0	0	$(n^2 - 1)$	$\frac{1}{2}(n^2 + n)$
(3-3) _a	$-(n + \frac{1}{2})$	$(n + \frac{1}{2})$	0	0	$\frac{1}{2}(n + 1)$	$\frac{1}{4}(n + 1)$
(3-3) _b	$-\frac{1}{8}$	$\frac{1}{2}n + \frac{1}{8}$	$\frac{1}{8}(n + 1)$	$\frac{1}{8}(2n + 1)$	$\frac{1}{8}(n + 1)$	$\frac{1}{16}(n + 1)$
(3-3) _c	$\frac{5}{2}n + \frac{5}{8}$	$-\frac{5}{8}$	$\frac{5}{8}(n + 1)$	$\frac{5}{8}$	$-\frac{5}{8}(n + 1)$	$-\frac{5}{16}(n + 1)$
Total _{sub}	$-3(2n^3 + n^2)$	$-3(3n^3 + n^2)$	$(-3n^3 - 6n^2 - \frac{3}{4}n + \frac{9}{4})$	$(-n^3 - n^2 + \frac{5}{4}n + \frac{3}{4})$	0	0

$$\sum_{n=1}^{\infty} n^2 Y^n = \frac{Y(Y+1)}{(1-Y)^3}; \quad \sum_{n=1}^{\infty} n^3 Y^n = \frac{Y(Y^2+4Y+1)}{(1-Y)^3}. \quad (110)$$

Here Y stands for $\frac{Y}{4}$. After the summation, the total contribution from $i\Delta_{\text{cf}}i[S]_{n \geq 1}$ is

$$\begin{aligned} -i[\Sigma^{\text{cf}1}](x; x') = & \frac{\kappa^2 m H}{64\pi^4} \left\{ 8 \frac{\gamma^0 \gamma^k \Delta x_k}{\Delta x^6} \left[\frac{(-Y^2+7Y)}{(1-Y)^3} + \frac{(Y^3+6Y^2-Y)}{(1-Y)^4} \ln(Y) \right] + \left[5 \frac{Ha \gamma^0 \Delta \eta \gamma^k \Delta x_k}{\Delta x^6} \right. \right. \\ & + \frac{Ha' \gamma^0 \Delta \eta \gamma^k \Delta x_k}{\Delta x^6} - 3 \frac{Ha \Delta \eta^2}{\Delta x^6} - 3 \frac{Ha' \Delta \eta^2}{\Delta x^6} \left. \right] \left[\frac{-3Y^3+26Y^2+Y}{2(1-Y)^3} + \frac{3Y^3+22Y^2-3Y}{2(1-Y)^4} \ln(Y) \right] \\ & + \left[\frac{Ha}{\Delta x^4} + \frac{Ha'}{\Delta x^4} \right] \left[\frac{-12(Y^2+2Y)}{(1-Y)^3} + \frac{-3(Y^3+8Y^2+3Y)}{(1-Y)^4} \ln(Y) \right] \\ & + \frac{H^2 a^2 \gamma^0 \gamma^k \Delta x_k}{\Delta x^4} \left[\frac{-3(Y^2-6Y+29)}{4(1-Y)^3} + \frac{-3(3Y^4-12Y^3+23Y^2+10Y)}{4(1-Y)^4} \ln(Y) \right] \\ & + \frac{H^2 a a' \gamma^0 \gamma^k \Delta x_k}{\Delta x^4} \left[\frac{(5Y^3-14Y^2-15Y)}{4(1-Y)^3} + \frac{(-3Y^4+14Y^3-35Y^2)}{4(1-Y)^4} \ln(Y) \right] \left. \right\}. \quad (111) \end{aligned}$$

B. Subleading contributions from $i\delta\Delta_A$

In this subsection we compute the contribution from substituting the residual A -type part of the graviton propagator in Table V,

$$i[\alpha_\beta \Delta_{\rho\sigma}](x; x') \rightarrow \left[\bar{\eta}_{\alpha\rho} \bar{\eta}_{\sigma\beta} + \bar{\eta}_{\alpha\sigma} \bar{\eta}_{\rho\beta} - \frac{2}{D-3} \bar{\eta}_{\alpha\beta} \bar{\eta}_{\rho\sigma} \right] i\delta\Delta_A(x; x'). \quad (112)$$

As with the conformal contributions of the previous section, we first make the requisite contractions and then act on the derivatives. The result of this first step is displayed in Tables XIX and XX. We have sometimes decomposed the result for a single vertex pair into as many as five terms because the three different tensors in (112) can make distinct contributions, and because distinct contributions also come from breaking up factors of $\gamma^\alpha J^{\beta\mu}$. These distinct contributions are tagged with subscripts a, b, c , etc.

TABLE XIX. Contractions from the $i\delta\Delta_A$ part of the graviton propagator-I.

I	J	sub	$iV_I^{\alpha\beta}(x)i[S](x; x')iV_J^{\rho\sigma}(x')[\alpha_\beta T_{\rho\sigma}^A]i\delta\Delta_A(x; x')$
1	4		$-\frac{(D-1)}{(D-3)} i\kappa^2 am \not{\partial} i[S](x; x') i\delta\Delta_A(x; x')$
2	4		$\frac{1}{(D-3)} i\kappa^2 am \not{\partial} i[S](x; x') i\delta\Delta_A(x; x')$
3	4	a	$-\frac{(D-1)}{2(D-3)} i\kappa^2 am \gamma^0 \partial_0 i\delta\Delta_A(x; x') i[S](x; x')$
3	4	b	$-\frac{(D-2)}{2(D-3)} i\kappa^2 am \not{\partial} i\delta\Delta_A(x; x') i[S](x; x')$
4	1		$\frac{(D-1)}{(D-3)} i\kappa^2 am \partial'_\mu \{i[S](x; x') \gamma^\mu i\delta\Delta_A(x; x')\}$
4	2		$\frac{1}{(D-3)} i\kappa^2 am \partial_k \{i[S](x; x') \gamma_k i\delta\Delta_A(x; x')\}$
4	3	a	$-\frac{(D-1)}{2(D-3)} i\kappa^2 am i[S](x; x') \gamma^0 \partial_0 i\delta\Delta_A(x; x')$
4	3	b	$-\frac{(D-2)}{2(D-3)} i\kappa^2 am i[S](x; x') \not{\partial} i\delta\Delta_A(x; x')$

TABLE XX. Contractions from the $i\delta\Delta_A$ part of the graviton propagator-II.

I	J	sub	$iV_I^{\alpha\beta}(x)i[S](x; x')iV_J^{\rho\sigma}(x')[\alpha_\beta T_{\rho\sigma}^A]i\delta\Delta_A(x; x')$
1	1		$\frac{(D-1)}{(D-3)} \kappa^2 \partial'_\mu \{i[S](x; x') \gamma^\mu i\delta\Delta_A(x; x')\}$
1	2		$\frac{1}{(D-3)} \kappa^2 \partial_k \{i[S](x; x') \gamma_k i\delta\Delta_A(x; x')\}$
1	3	a	$-\frac{(D-1)}{2(D-3)} \kappa^2 \not{\partial} i[S](x; x') \gamma^0 \partial'_0 i\delta\Delta_A(x; x')$
1	3	b	$\frac{(D-2)}{2(D-3)} \kappa^2 \not{\partial} i[S](x; x') \not{\partial} i\delta\Delta_A(x; x')$
2	1		$-\frac{1}{(D-3)} \kappa^2 \partial'_\mu \{i[S](x; x') \gamma^\mu i\delta\Delta_A(x; x')\}$
2	2	a	$\frac{1}{4} \kappa^2 \not{\partial} \{i[S](x; x') \gamma_k i\delta\Delta_A(x; x')\}$
2	2	b	$+\frac{1}{4} \kappa^2 \partial_\ell \{i[S](x; x') \gamma_k i\delta\Delta_A(x; x')\}$
2	2	c	$-\frac{1}{2(D-3)} \kappa^2 \partial_k \{i[S](x; x') \gamma_k i\delta\Delta_A(x; x')\}$
2	3	a	$\frac{1}{2(D-3)} \kappa^2 \not{\partial} i[S](x; x') \gamma^0 \partial'_0 i\delta\Delta_A(x; x')$
2	3	b	$-\frac{1}{4} \kappa^2 \gamma_k \partial_\ell i[S](x; x') \gamma_{(k} \partial_{\ell)} i\delta\Delta_A(x; x')$
2	3	c	$-\frac{1}{4(D-3)} \kappa^2 \not{\partial} i[S](x; x') \not{\partial} i\delta\Delta_A(x; x')$
3	1	a	$\frac{1}{2} \frac{(D-1)}{(D-3)} \kappa^2 \partial'_\mu \{i[S](x; x') \gamma^0 \partial_0 i\delta\Delta_A(x; x') i[S](x; x') \gamma^\mu\}$
3	1	b	$\frac{(D-2)}{2(D-3)} \kappa^2 \partial'_\mu \{i[S](x; x') \not{\partial} i\delta\Delta_A(x; x') i[S](x; x') \gamma^\mu\}$
3	2	a	$\frac{1}{2(D-3)} \kappa^2 \partial_k \{i[S](x; x') \gamma^0 \partial_0 i\delta\Delta_A(x; x') i[S](x; x') \gamma_k\}$
3	2	b	$\frac{1}{4(D-3)} \kappa^2 \partial_k \{i[S](x; x') \not{\partial} i\delta\Delta_A(x; x') i[S](x; x') \gamma_k\}$
3	2	c	$\frac{1}{8} \kappa^2 \not{\partial} \{i[S](x; x') \not{\partial} i\delta\Delta_A(x; x')\}$
3	2	d	$\frac{1}{8} \kappa^2 \partial_k \{i[S](x; x') \gamma_\ell \partial_k i\delta\Delta_A(x; x')\}$
3	3	a	$-\frac{1}{4} \frac{(D-1)}{(D-3)} \kappa^2 \gamma^0 i[S](x; x') \gamma^0 \partial_0 \partial'_0 i\delta\Delta_A(x; x')$
3	3	b	$\frac{(D-2)}{4(D-3)} \kappa^2 \gamma^0 i[S](x; x') \partial_0 \not{\partial} i\delta\Delta_A(x; x')$
3	3	c	$-\frac{(D-2)}{4(D-3)} \kappa^2 \gamma_k i[S](x; x') \partial_k \gamma^0 \partial'_0 i\delta\Delta_A(x; x')$
3	3	d	$\frac{3D-7}{16(D-3)} \kappa^2 \gamma_k i[S](x; x') \partial_k \not{\partial} i\delta\Delta_A(x; x')$
3	3	e	$-\frac{1}{16} \kappa^2 \gamma_k i[S](x; x') \gamma_k \nabla^2 i\delta\Delta_A(x; x')$

TABLE XXI. The local terms from $i\delta\Delta_A \times i[S]_{\text{cf}}$.

(I-J) _{sub}	$\frac{i\kappa^2 H^2}{16\pi^2} \frac{\Gamma(\frac{D}{2}+1)}{2} \frac{ma}{(D-4)} \delta^D(x-x')$	$\frac{i\kappa^2 H^2}{16\pi^2} ma \delta^4(x-x')$
(1-4)	$\frac{32}{\sqrt{\pi}} \frac{H^{D-4}}{(D-3)}$	$-12 \ln(a)$
(2-4)	$-\mu^{D-4} \frac{(D-1)}{(D-3)^2}$	$3 \ln(a)$
(3-4) _a	$-\mu^{D-4} \frac{(D-1)}{2(D-3)^2}$	$\frac{3}{2} \ln(a)$
(3-4) _b	$-\mu^{D-4} \frac{(D-1)(D-2)}{2(D-3)^2}$	$3 \ln(a)$
(4-1)	0	$9 + 6 \ln(\frac{H^2}{4\mu^2})$
(4-2)	0	0
(4-3) _a	$-\mu^{D-4} \frac{(D-1)}{2(D-3)^2}$	$\frac{3}{2} \ln(a)$
(4-3) _b	$-\mu^{D-4} \frac{(D-1)(D-2)}{2(D-3)^2}$	$3 \ln(a)$
Total	$\frac{32}{\sqrt{\pi}} \frac{H^{D-4}}{(D-3)} - \mu^{D-4} \frac{D(D-1)}{(D-3)^2}$	$9 + 6 \ln(\frac{H^2}{4\mu^2})$

The next step is to act on the derivatives, and it is of course necessary to have an expression for $i\delta\Delta_A(x; x')$. From (36) one can deduce

$$\begin{aligned}
 i\delta\Delta_A(x; x') &= \frac{H^2}{16\pi^{\frac{D}{2}}} \frac{\Gamma(\frac{D}{2}+1)}{\frac{D}{2}-2} \frac{(aa')^{2-\frac{D}{2}}}{\Delta x^{D-4}} + \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \\
 &\times \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \left\{ -\pi \cot\left(\frac{\pi}{2}D\right) + \ln(aa') \right\} \\
 &+ \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \sum_{n=1}^{\infty} \left\{ \frac{1}{n} \frac{\Gamma(n+D-1)}{\Gamma(n+\frac{D}{2})} \left(\frac{y}{4}\right)^n \right. \\
 &\left. - \frac{1}{n-\frac{D}{2}+2} \frac{\Gamma(n+\frac{D}{2}+1)}{\Gamma(n+2)} \left(\frac{y}{4}\right)^{n-\frac{D}{2}+2} \right\}.
 \end{aligned} \tag{113}$$

In $D=4$ the most singular contributions to (66) have the form, $i\delta\Delta_A/\Delta x^4$. Because the infinite series terms in (113) behave like positive powers of Δx^2 these terms make integrable contributions to the quantum-corrected Dirac

equation (2). We can therefore take $D=4$ for those terms, at which point all the infinite series terms drop. Hence it is only necessary to keep the first line of (113) and that is all we need to use.

The generic contraction from Table XIX only consists of one derivative acting on a propagator; therefore, the order m contributions must be and could only be from the most singular part of the fermion propagator. In reducing these terms the following derivatives occur many times:

$$\begin{aligned}
 \partial_i i\delta\Delta_A(x; x') &= -\frac{H^2}{8\pi^{\frac{D}{2}}} \Gamma\left(\frac{D}{2}+1\right) (aa')^{2-\frac{D}{2}} \frac{\Delta x^i}{\Delta x^{D-2}} \\
 &= -\partial'_i i\delta\Delta_A(x; x'),
 \end{aligned} \tag{114}$$

$$\begin{aligned}
 \partial_0 i\delta\Delta_A(x; x') &= \frac{H^2}{8\pi^{\frac{D}{2}}} \Gamma\left(\frac{D}{2}+1\right) (aa')^{2-\frac{D}{2}} \left\{ \frac{\Delta \eta}{\Delta x^{D-2}} \right. \\
 &\left. - \frac{aH}{2\Delta x^{D-4}} \right\} + \frac{H^{D-2}}{2^D \pi^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} aH,
 \end{aligned} \tag{115}$$

$$\begin{aligned}
 \partial'_0 i\delta\Delta_A(x; x') &= \frac{H^2}{8\pi^{\frac{D}{2}}} \Gamma\left(\frac{D}{2}+1\right) (aa')^{2-\frac{D}{2}} \left\{ -\frac{\Delta \eta}{\Delta x^{D-2}} \right. \\
 &\left. - \frac{a'H}{2\Delta x^{D-4}} \right\} + \frac{H^{D-2}}{2^D \pi^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} a'H.
 \end{aligned} \tag{116}$$

We also make use of the following identities to simplify the contributions:

$$\begin{aligned}
 \frac{\Delta \eta^2}{\Delta x^{2D-2}} &= \frac{1}{4(D-2)(D-3)} \partial_0^2 \frac{1}{\Delta x^{2D-6}} \\
 &- \frac{1}{2(D-2)} \frac{1}{\Delta x^{2D-4}},
 \end{aligned} \tag{117}$$

TABLE XXII. The nonlocal terms from $i\delta\Delta_A \times i[S]_{\text{cf}}$. All contributions are multiplied by $\frac{i\kappa^2 H^2}{32\pi^4} ma$.

(I-J) _{sub}	$\partial^2 \frac{\ln(\mu\Delta x)^2}{\Delta x^2}$	$\gamma^0 \partial_0 \tilde{\partial} \frac{\ln(\mu\Delta x)^2}{\Delta x^2}$	$\nabla^2 \frac{\ln(\mu\Delta x)^2}{\Delta x^2}$	$\partial_0^2 \frac{1}{\Delta x^2}$	$\gamma^0 \partial_0 \tilde{\partial} \frac{1}{\Delta x^2}$	$\nabla^2 \frac{1}{\Delta x^2}$
(1-4)	0	0	0	0	0	0
(2-4)	$\frac{3}{4}$	-1	-1	0	$-2 - \ln(\frac{H^2}{4\mu^2})$	$-2 - \ln(\frac{H^2}{4\mu^2})$
(3-4) _a	$\frac{3}{8}$	0	0	$\frac{3}{4}$	$\frac{3}{4}$	0
(3-4) _b	$\frac{3}{4}$	0	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$
(4-1)	3	0	0	0	0	0
(4-2)	0	1	-1	0	$\frac{3}{2} + \ln(\frac{H^2}{4\mu^2})$	$-\frac{3}{2} - \ln(\frac{H^2}{4\mu^2})$
(4-3) _a	$\frac{3}{8}$	0	0	$\frac{3}{4}$	$-\frac{3}{4}$	0
(4-3) _b	$\frac{3}{4}$	0	0	0	$\frac{1}{2}$	$-\frac{1}{2}$
Total	6	0	-2	$\frac{3}{2}$	$-\frac{1}{2}$	$-\frac{9}{2} - 2 \ln(\frac{H^2}{4\mu^2})$

$$\frac{\overline{\Delta x^2}}{\Delta x^{2D-2}} = \frac{1}{4(D-2)(D-3)} \nabla^2 \frac{1}{\Delta x^{2D-6}} + \frac{(D-1)}{2(D-2)} \frac{1}{\Delta x^{2D-4}}, \quad (118)$$

$$\frac{\gamma^0 \Delta \eta \gamma^k \Delta x_k}{\Delta x^{2D-2}} = \frac{-1}{4(D-2)(D-3)} \gamma^0 \partial_0 \bar{\not{\epsilon}} \frac{1}{\Delta x^{2D-6}}, \quad (119)$$

$$\begin{aligned} \frac{\Delta \eta}{\Delta x^{2D-4}} &= \frac{1}{2(D-3)} \partial_0 \frac{1}{\Delta x^{2D-6}}, \\ \frac{\gamma^k \Delta x_k}{\Delta x^{2D-4}} &= \frac{-1}{2(D-3)} \bar{\not{\epsilon}} \frac{1}{\Delta x^{2D-6}}. \end{aligned} \quad (120)$$

Note that the contraction (1-1) produces a delta function through (70) and picks up one divergent and one finite local

term that do not possess any dimension-dependent powers of Δx . We tabulate this kind of result in Table [XXI](#). One might already notice that the first two terms of (113)¹¹ and the final two terms of (115) and (116) tend to cancel each other in $D = 4$. We indeed encounter this kind of cancellation entirely for the leading divergent terms in contractions (4-1) and (4-2), and hence we present the remaining finite results in Table [XXII](#). For the rest of the contractions, the sum of the leading divergent contributions do not vanish after we apply (117)–(120), (94)–(96), (77), and (78). Therefore we give the local terms in Table [XXI](#) and the finite nonlocal terms in Table [XXII](#). After collecting all contributions from Tables [XXI](#) and [XXII](#), we obtain

$$\begin{aligned} -i[\Sigma^{\text{idAcf}}](x; x') &= \frac{i\kappa^2 H^2}{16\pi^{\frac{D}{2}}} \frac{\Gamma(\frac{D}{2} + 1)}{2} \frac{ma}{(D-4)} \left[\frac{32}{\sqrt{\pi}} \frac{H^{D-4}}{(D-3)} - \frac{\mu^{D-4} D(D-1)}{(D-3)^2} \right] \delta^D(x - x') \\ &+ \frac{i\kappa^2 H^2}{16\pi^2} \left[9 + 6 \ln\left(\frac{H^2}{4\mu^2}\right) \right] ma \delta^4(x - x') + \frac{\kappa^2 H^2}{32\pi^4} ma \left\{ [6\partial^2 - 2\nabla^2] \left[\frac{\ln(\mu^2 \Delta x^2)}{\Delta x^2} \right] \right. \\ &\left. + \left[\frac{3}{2} \partial_0^2 - \frac{1}{2} \gamma^0 \partial_0 \bar{\not{\epsilon}} - \left(\frac{9}{2} + 2 \ln\left(\frac{H^2}{4\mu^2}\right) \right) \nabla^2 \right] \frac{1}{\Delta x^2} \right\}. \end{aligned} \quad (121)$$

With the generic contractions from Table [XX](#), which are comprised of two derivatives acting on the propagators, the order m contributions could either come from the flat spacetime mass term or from the infinite series expansion of the fermion propagator.

We first deal with the contributions from the flat spacetime mass term of the fermion propagator. At $D = 4$ these

terms have a dimension $\frac{i\delta\Delta_A}{\Delta x^4}$ that are not integrable when we substitute back to the quantum-corrected Dirac equation and working on an arbitrary D is required. The contractions (1-1), (1-2), (2-1), and (2-2) tend to cancel through (113). It turns out that the sum of divergent terms vanish and we are left only with the finite terms. Hence we present the results for $D = 4$ at the end. Here we show (2-1) as an example,

$$\begin{aligned} \frac{-\kappa^2}{(D-3)} \partial'_\mu \{ \bar{\not{\epsilon}} i[S] \gamma^\mu i\delta\Delta_A \} &= \frac{\kappa^2 H^2}{32\pi^D} \frac{\Gamma(\frac{D}{2})\Gamma(\frac{D}{2} + 1)}{(D-3)} ma \partial'_\mu \left\{ \frac{(aa')^{2-\frac{D}{2}}}{\frac{1}{2}(D-4)} \frac{\gamma^k \Delta x_k \gamma^\mu}{\Delta x^{2D-4}} \right\} \\ &+ \frac{\kappa^2 H^{D-2}}{2^{D+1}\pi^D} \frac{\Gamma(D-1)}{(D-3)} ma \partial'_\mu \left\{ \left[\frac{-2}{(D-4)} + \ln(aa') \right] \frac{\gamma^k \Delta x_k \gamma^\mu}{\Delta x^D} \right\} \\ &= \frac{\kappa^2 H^2}{32\pi^D} \frac{\mu^{D-4} ma}{(D-3)} \left\{ \left[\frac{2}{(D-4)} - \frac{2}{(D-4)} - \frac{1}{2} - \ln\frac{H^2}{4\mu^2} - 1 \right] \bar{\not{\epsilon}} \not{\epsilon} \frac{1}{\Delta x^2} - \bar{\not{\epsilon}} \not{\epsilon} \frac{\ln(\mu^2 \Delta x^2)}{\Delta x^2} \right\} \\ &= \frac{\kappa^2 H^2}{32\pi^4} ma \left\{ \left[-\frac{3}{2} - \ln\frac{H^2}{4\mu^2} \right] \bar{\not{\epsilon}} \not{\epsilon} \frac{1}{\Delta x^2} - \bar{\not{\epsilon}} \not{\epsilon} \frac{\ln(\mu^2 \Delta x^2)}{\Delta x^2} \right\}. \end{aligned} \quad (122)$$

The most singular terms in the contractions (2-3)_a, (3-1)_a, and (3-2)_a are integrable after exacting out derivatives.¹² Because the contraction (3-3) involves various double derivatives directly acting on $i\delta\Delta_A$, we give their expressions here:

$$\begin{aligned} \partial_0 \partial'_0 i\delta\Delta_A &= \frac{H^2 \Gamma(\frac{D}{2} + 1)}{8\pi^{\frac{D}{2}} (aa')^{\frac{D}{2}-2}} \left\{ \frac{-1}{\Delta x^{D-2}} - \frac{(D-2)\Delta \eta^2}{\Delta x^D} \right. \\ &\left. + \frac{1}{2}(D-4) \left[\frac{H^2 aa' \Delta \eta^2}{\Delta x^{D-2}} + \frac{1}{2} \frac{H^2 aa'}{\Delta x^{D-4}} \right] \right\}, \end{aligned} \quad (123)$$

$$\begin{aligned} \partial_k \partial_0 i\delta\Delta_A &= \frac{H^2 \Gamma(\frac{D}{2} + 1)}{8\pi^{\frac{D}{2}} (aa')^{\frac{D}{2}-2}} \left\{ \frac{-(D-2)\Delta \eta \Delta x_k}{\Delta x^D} \right. \\ &\left. + \frac{1}{2}(D-4) \frac{Ha \Delta x_k}{\Delta x^{D-2}} \right\}, \end{aligned} \quad (124)$$

¹¹ $\pi \cot(\frac{D\pi}{2}) = \frac{1}{\frac{1}{2}(D-4)}$.

¹² Some less singular terms in the contractions (2-3)_a, (3-1)_a, and (3-2)_a get cancelled through the property of (115) and (116).

TABLE XXIII. $i\delta\Delta_A \times i[S]_{\text{fin}}$. All contributions are multiplied by $\frac{\kappa^2 H^2}{32\pi^4} ma$.

(I-J)	$\partial^2 \frac{\ln(\mu\Delta x)^2}{\Delta x^2}$	$\gamma^0 \partial_0 \tilde{\not{\partial}} \frac{\ln(\mu\Delta x)^2}{\Delta x^2}$	$\nabla^2 \frac{\ln(\mu\Delta x)^2}{\Delta x^2}$	$Ha\partial_0 \frac{\ln(\mu\Delta x)^2}{\Delta x^2}$	$Ha\gamma^0 \tilde{\not{\partial}} \frac{\ln(\mu\Delta x)^2}{\Delta x^2}$
(1-1)	-3	0	0	3	3
(1-2)	0	-1	1	0	-1
(2-1)	0	1	1	0	0
(2-1)	0	0	$\frac{1}{2}$	0	0
Total	-3	0	$\frac{5}{2}$	3	2

TABLE XXIV. $i\delta\Delta_A \times i[S]_{\text{fin}}$. All contributions are multiplied by $\frac{\kappa^2 H^2}{32\pi^4} ma$.

(I-J) _{sub}	$\partial^2 \frac{1}{\Delta x^2}$	$\gamma^0 \partial_0 \tilde{\not{\partial}} \frac{1}{\Delta x^2}$	$\nabla^2 \frac{1}{\Delta x^2}$	$Ha\partial_0 \frac{1}{\Delta x^2}$	$Ha\gamma^0 \tilde{\not{\partial}} \frac{1}{\Delta x^2}$
(1-1)	$-\frac{3}{2} - 3\ln\frac{H^2}{4\mu^2}$	0	0	$\frac{3}{2} + 3\ln\frac{H^2}{4\mu^2}$	$\frac{3}{2} + 3\ln\frac{H^2}{4\mu^2}$
(1-2)	0	$-\frac{3}{2} - \ln\frac{H^2}{4\mu^2}$	$\frac{3}{2} + \ln\frac{H^2}{4\mu^2}$	0	$-\frac{1}{2} - \ln\frac{H^2}{4\mu^2}$
(2-1)	0	$\frac{3}{2} + \ln\frac{H^2}{4\mu^2}$	$\frac{3}{2} + \ln\frac{H^2}{4\mu^2}$	0	0
(2-2)	0	0	$\frac{3}{4} + \frac{1}{2}\ln\frac{H^2}{4\mu^2}$	0	0
(2-3) _a	0	$+\frac{1}{4}$	0	0	0
(3-1) _a	$+\frac{3}{2}$	$-\frac{3}{2}$	$-\frac{3}{2}$	0	0
(3-1) _b	0	-1	-1	0	0
(3-2)	0	$\frac{1}{2}$	$-\frac{3}{4}$	0	0
(3-3) _{b+c}	0	0	0	0	0
Total	$-3\ln\frac{H^2}{4\mu^2}$	$-\frac{7}{4}$	$\frac{1}{2} + \frac{5}{2}\ln\frac{H^2}{4\mu^2}$	$\frac{3}{2} + 3\ln\frac{H^2}{4\mu^2}$	$1 + 2\ln\frac{H^2}{4\mu^2}$

$$\partial_k \partial'_0 i\delta\Delta_A = \frac{H^2 \Gamma(\frac{D}{2} + 1)}{8\pi^{\frac{D}{2}} (aa')^{\frac{D}{2}-2}} \left\{ \frac{(D-2)\Delta\eta\Delta x_k}{\Delta x^D} + \frac{1}{2}(D-4) \frac{Ha'\Delta x_k}{\Delta x^{D-2}} \right\}, \quad (125)$$

$$\partial_k \partial_l i\delta\Delta_A = \frac{H^2 \Gamma(\frac{D}{2} + 1)}{8\pi^{\frac{D}{2}} (aa')^{\frac{D}{2}-2}} \left\{ \frac{(D-2)\Delta x_k \Delta x_l}{\Delta x^D} - \frac{\delta_{kl}}{\Delta x^{D-2}} \right\}. \quad (126)$$

Note that the terms with a factor of $(D-4)$ in Eqs. (123)–(125) have dimensionality, which is either $\frac{1}{\Delta^{D-4}}$ or $\frac{1}{\Delta^{D-3}}$. When they combine with $i[S]_{\text{fin}}$, whose dimensionality is $\frac{1}{\Delta^{D-2}}$, one can see that those contributions are integrable in four dimensions and hence could only give the contributions of the order $(D-4)$. Therefore, we can drop the terms we mentioned above from (123) to (125) when we compute the contraction (3-3). In addition, the contractions (3-3)_b and (3-3)_c are both finite in $D=4$ dimensions after performing the partial integration.¹³ The finite contributions mentioned above are displayed in Tables XXXIII and XXXIV and they can be read off immediately:

¹³Partial integration is not standard usage in physics. Here we mean extracting the derivatives outside the quantum-corrected Dirac equation.

$$\begin{aligned} \frac{\kappa^2 H^2 ma}{32\pi^4} & \left\{ \left[-3\partial^2 + \frac{5}{2}\nabla^2 + 3Ha\partial_0 + 2Ha\gamma^0 \tilde{\not{\partial}} \right] \frac{\ln\mu^2 \Delta x^2}{\Delta x^2} \right. \\ & + \left[-3\ln\frac{H^2}{4\mu^2} \partial^2 - \frac{7}{4}\gamma^0 \partial_0 \tilde{\not{\partial}} + \left(\frac{1}{2} + \frac{5}{2}\ln\frac{H^2}{4\mu^2} \right) \nabla^2 \right. \\ & \left. \left. + \left(\frac{3}{2} + 3\ln\frac{H^2}{4\mu^2} \right) Ha\partial_0 + \left(1 + 2\ln\frac{H^2}{4\mu^2} \right) Ha\gamma^0 \tilde{\not{\partial}} \right] \frac{1}{\Delta x^2} \right\}. \end{aligned} \quad (127)$$

The rest of the contractions that require further renormalization are summarized in Table XXV. We then apply the same formalism to partially integrate, extract the local divergences, and take $D=4$ for the remaining, integrable, and ultraviolet finite nonlocal terms. The subtotal from Table XXV could be obtained as follows:

$$\begin{aligned} & \frac{i\kappa^2 H^2}{64\pi^{\frac{D}{2}}} \Gamma\left(\frac{D}{2} + 1\right) \frac{\mu^{D-4}(D-1)(D^3 - 9D^2 + 20D - 4)}{8(D-2)(D-3)^2(D-4)} \\ & \times ma\delta^D(x-x') + \frac{i\kappa^2 H^2}{64\pi^2} \frac{3ma}{2} \ln a \delta^4(x-x') \\ & + \frac{\kappa^2 H^2 ma}{32\pi^4} \left\{ \frac{3}{32} \partial^2 \frac{\ln\mu^2 \Delta x^2}{\Delta x^2} + \left[\frac{3}{4} \partial_0^2 + \frac{1}{8} \gamma^0 \partial_0 \tilde{\not{\partial}} \right. \right. \\ & \left. \left. - \frac{3}{16} \nabla^2 + \frac{3}{4} Ha\partial_0 + Ha\gamma^0 \tilde{\not{\partial}} \right] \frac{1}{\Delta x^2} \right\}. \end{aligned} \quad (128)$$

TABLE XXV. The contributions from $i\delta\Delta_A \times i[S]_{\text{fm}}$ that are not integrable in four dimensions. Note that all contributions are multiplied by the factor $\frac{\kappa^2 H^2}{64\pi^D} \Gamma(\frac{D}{2} - 1) \Gamma(\frac{D}{2} + 1) ma(aa')^{2-\frac{D}{2}}$.

(I-J) _{sub}	$\frac{\Delta\eta^2}{\Delta x^{2D-2}}$	$\frac{\Delta x^2}{\Delta x^{2D-2}}$	$\frac{1}{\Delta x^{2D-4}}$	$\frac{\gamma^0 \Delta\eta \gamma^k \Delta x_k}{\Delta x^{2D-2}}$	$\frac{Ha\Delta\eta}{\Delta x^{2D-4}}$	$\frac{Ha\gamma^0 \gamma^k \Delta x_k}{\Delta x^{2D-4}}$
(1-3) _a	$-(D-1)$	0	0	$(D-1)$	$\frac{(D-1)}{(D-2)}$	0
(1-3) _b	0	$\frac{-(D-2)^2}{(D-3)}$	0	$\frac{-(D-2)}{(D-3)}$	0	$\frac{-(D-2)}{(D-3)}$
(2-3) _{b+c}	0	$\frac{(D-1)(D-2)^2}{4(D-3)}$	0	0	0	0
(3-3) _a	$\frac{(D-1)(D-2)}{2(D-3)}$	0	$\frac{(D-1)}{2(D-3)}$	0	0	0
(3-3) _{d+c}	0	$\frac{(D-2)^2(D-5)}{8(D-3)}$	$\frac{-(D-1)(D-2)(D-5)}{8(D-3)}$	0	0	0
Total	$\frac{(D-1)(3D-8)}{2(D-3)}$	$\frac{3(D-2)^2(D-5)}{8(D-3)}$	$\frac{-(D-1)^2(D-6)}{8(D-3)}$	$\frac{-1}{(D-3)}$	$\frac{(D-1)}{(D-2)}$	$\frac{-(D-2)}{(D-3)}$

Combining (127) and (128) gives

$$\begin{aligned}
-i[\Sigma^{\text{idAfm}}](x; x') &= \frac{i\kappa^2 H^2}{64\pi^{\frac{D}{2}}} \Gamma\left(\frac{D}{2} + 1\right) \frac{\mu^{D-4}(D-1)(D^3 - 9D^2 + 20D - 4)}{8(D-2)(D-3)^2(D-4)} \times ma\delta^D(x - x') + \frac{i\kappa^2 H^2}{64\pi^2} \frac{3ma}{2} \ln a \delta^4(x - x') \\
&+ \frac{\kappa^2 H^2 ma}{32\pi^4} \left\{ \left[-\frac{93}{32} \partial^2 + \frac{5}{2} \nabla^2 + 3Ha\partial_0 + 2Ha\gamma^0 \not{\partial} \right] \left[\frac{\ln(\mu^2 \Delta x^2)}{\Delta x^2} \right] + \left[\left(\frac{3}{4} + 3 \ln \frac{H^2}{4\mu^2} \right) \partial_0 - \frac{13}{8} \gamma^0 \partial_0 \not{\partial} \right. \right. \\
&\left. \left. + \left(\frac{5}{16} - \frac{1}{2} \ln \frac{H^2}{4\mu^2} \right) \nabla^2 + \left(\frac{9}{4} + 3 \ln \frac{H^2}{4\mu^2} \right) Ha\partial_0 + \left(2 + 2 \ln \frac{H^2}{4\mu^2} \right) Ha\gamma^0 \not{\partial} \right] \frac{1}{\Delta x^2} \right\}. \quad (129)
\end{aligned}$$

The final class is comprised of terms that come from the infinite series expansion. These contributions are integrable in $D = 4$ and do not require any further renormalizations, so we set $D = 4$ right at the beginning without alternating the final result. Therefore we shall apply the infinite series expansion of the fermion propagator for $D = 4$ to this calculation. It is actually the same expression as (104). In addition, one can also use (105) and all related identities in the Appendix but remember that the series is summed up from $n = 0$ instead of $n = 1$. The residual A -type graviton propagator and its various derivatives in four dimensions occurred very frequently,

$$i\delta\Delta_A(x; x') = \frac{-H^2}{8\pi^2} \left[\ln\left(\frac{H^2 \Delta x^2}{4}\right) + \frac{1}{2} \right], \quad (130)$$

$$\partial_i i\delta\Delta_A(x; x') = -\partial_i i\delta\Delta_A(x; x') = \frac{-H^2}{4\pi^2} \frac{\Delta x_i}{\Delta x^2}, \quad (131)$$

$$\partial_0 i\delta\Delta_A(x; x') = -\partial'_0 i\delta\Delta_A(x; x') = \frac{H^2}{4\pi^2} \frac{\Delta\eta}{\Delta x^2} \quad (132)$$

$$\partial_0 \partial'_0 i\delta\Delta_A(x; x') = \frac{-H^2}{4\pi^2} \left[\frac{1}{\Delta x^2} + \frac{2\Delta\eta^2}{\Delta x^4} \right], \quad (133)$$

$$\partial_0 \partial_i i\delta\Delta_A(x; x') = -\partial'_0 \partial_i i\delta\Delta_A(x; x') = \frac{H^2}{4\pi^2} \frac{-2\Delta\eta \Delta x_i}{\Delta x^4}, \quad (134)$$

$$\partial_i \partial_l i\delta\Delta_A(x; x') = \frac{H^2}{4\pi^2} \left[\frac{-\delta_{il}}{\Delta x^2} + \frac{2\Delta x_i \Delta x_l}{\Delta x^4} \right]. \quad (135)$$

We also make use of the following identities to facilitate our computation more effectively:

$$\gamma^l \gamma^k \gamma_l = \gamma^k, \quad \gamma^\mu \gamma^0 \gamma_\mu = 2\gamma^0, \quad (136)$$

$$\begin{aligned}
\gamma^l \gamma^k \gamma^0 \gamma^l \Delta x_k &= \gamma^0 \gamma^k \Delta x_k, \\
\gamma^k \gamma^l \gamma^0 \gamma^l \Delta x_k &= -3\gamma^0 \gamma^k \Delta x_k,
\end{aligned} \quad (137)$$

$$\gamma^j \gamma^\nu \gamma^0 \gamma^j \Delta x_\nu = 3\Delta\eta + \gamma^0 \gamma^k \Delta x_k, \quad (138)$$

$$\gamma^0 \gamma^\mu \gamma^0 \gamma^\nu \Delta x_\mu \Delta x_\nu = \Delta x^2 - 2\gamma^0 \Delta\eta \gamma^\mu \Delta x_\mu, \quad (139)$$

$$\begin{aligned}
\gamma^k \gamma^\mu \gamma^0 \gamma^\nu \Delta x_k \Delta x_\mu \Delta x_\nu &= -\gamma^0 \gamma^k \Delta x_k \Delta x^2 \\
&- 2\Delta\eta \gamma^k \Delta x_k \gamma^\mu \Delta x_\mu,
\end{aligned} \quad (140)$$

$$\gamma^0 \gamma^\nu \gamma^0 \gamma^k \Delta\eta \Delta x_\nu \Delta x_k = \Delta\eta \overline{\Delta x^2} - \gamma^0 \Delta\eta^2 \gamma^k \Delta x_k, \quad (141)$$

$$\gamma^k \gamma^\nu \gamma^0 \gamma^l \Delta x_k \Delta x_l \Delta x_\nu = \Delta\eta \overline{\Delta x^2} - \gamma^0 \gamma^k \Delta x_k \overline{\Delta x^2}. \quad (142)$$

Note that any derivatives acting on $i\delta\Delta_A$ would eliminate $\ln(\frac{H^2 \Delta x^2}{4})$ and that the exceptions are the generic contractions (1-1), (1-2), (2-1), and (2-2). We list these results in Tables XXVI, XXVII, and XXVIII. The rest of the contributions without $\ln \frac{H^2 \Delta x^2}{4}$ are summarized in Tables XXIX, XXX, and XXXI. From these tables one can see that the derivative of the coefficient with $\ln \frac{\nu}{4}$ is the coefficient without $\ln \frac{\nu}{4}$. Based on the characteristic of (104) we should not be too surprised at this peculiar pattern occurring here again as in the previous subsection.

TABLE XXVI. $i\delta\Delta_A \times i[S]_{n \geq 0} - \text{IA}$. The factor $\frac{i\kappa^2 H^2}{2^6 \pi^4} \frac{mHaa'}{2} (\ln \frac{H^2 \Delta x^2}{4}) \sum_{n=0}^{\infty} Y^n$ multiplies all contributions. Here $Y = \frac{y}{4}$; $\ln Y$ and 1 are the multiplicative factors for each individual row.

I-J _{sub}		$\frac{\Delta \eta^3}{\Delta x^6}$	$\frac{\gamma^0 \Delta \eta^2 \gamma^k \Delta x_k}{\Delta x^6}$	$\frac{\Delta \eta}{\Delta x^4}$	$\frac{\gamma^0 \gamma^k \Delta x_k}{\Delta x^4}$
(1-1)	1	0	0	$24(3n^2 + 4n + 1)$	$-24(3n^2 + 4n + 1)$
(1-1)	$\ln Y$	0	0	$24(n^3 + 2n^2 + n)$	$-24(n^3 + 2n + n)$
(1-2)	1	0	0	0	$8(3n^2 - 1)$
(1-2)	$\ln Y$	0	0	0	$8(n^3 - n)$
(2-1)	1	$-16(3n^2 - 6n + 2)$	$16(3n^2 - 6n + 2)$	$-8(6n^2 - 6n + 1)$	$8(3n^2 - 2n)$
(2-1)	$\ln Y$	$-16(n^3 - 3n^2 + 2n)$	$16(n^3 - 3n^2 + 2n)$	$-8(2n^3 - 3n^2 + n)$	$8(n^3 - n^2)$
(2-2) _a	1	$-2(3n^2 - 6n + 2)$	$2(3n^2 - 6n + 2)$	$-(6n^2 - 6n + 1)$	$(6n^2 - 2n - 1)$
(2-2) _a	$\ln Y$	$-2(n^3 - 3n^2 + 2n)$	$2(n^3 - 3n^2 + 2n)$	$-(2n^3 - 3n^2 + n)$	$(2n^3 - n^2 - n)$
(2-2) _b	1	$-6(3n^2 - 6n + 2)$	$-2(3n^2 - 6n + 2)$	$-3(6n^2 - 6n + 1)$	$-(6n^2 - 2n - 1)$
(2-2) _b	$\ln Y$	$-6(n^3 - 3n^2 + 2n)$	$-2(n^3 - 3n^2 + 2n)$	$-3(2n^3 - 3n^2 + n)$	$-(2n^3 - n^2 - n)$
(2-2) _c	1	$4(3n^2 - 6n + 2)$	$-4(3n^2 - 6n + 2)$	$2(6n^2 - 6n + 1)$	$-2(6n^2 - 2n - 1)$
(2-2) _c	$\ln Y$	$-4(n^3 - 3n^2 + 2n)$	$-4(n^3 - 3n^2 + 2n)$	$2(2n^3 - 3n^2 + n)$	$-2(2n^3 - n^2 - n)$
Total	1	$-20(3n^2 - 6n + 2)$	$12(3n^2 - 6n + 2)$	$2(6n^2 + 78n + 7)$	$-6(6n^2 + 18n + 5)$
Total	$\ln Y$	$-20(n^3 - 3n^2 + 2n)$	$12(n^3 - 3n^2 + 2n)$	$2(2n^3 + 39n^2 + 7n)$	$-6(2n^3 + 9n^2 + 5n)$

The final result for $-i[\Sigma^{\text{dAn}0}](x; x')$ could be computed using (109) and (110) and then adding the lowest order constant to each distinct contribution because the summation here starts from $n = 0$ rather than $n = 1$. Finally we tabulate the lengthy results in Tables XXXII and XXXIII.

C. Subleading contributions from $i\delta\Delta_B$

In this subsection we evaluate the contribution from substituting the residual B -type part of the graviton propagator in Table V,

$$i[\alpha_\beta \Delta_{\rho\sigma}] \rightarrow -[\delta_\alpha^0 \delta_\sigma^0 \bar{\eta}_{\beta\rho} + \delta_\alpha^0 \delta_\rho^0 \bar{\eta}_{\beta\sigma} + \delta_\beta^0 \delta_\sigma^0 \bar{\eta}_{\alpha\rho} + \delta_\beta^0 \delta_\rho^0 \bar{\eta}_{\alpha\sigma}] i\delta\Delta_B. \quad (143)$$

As in the two previous subsections we first make the requisite contractions and then act on the derivatives. The result of this first step is presented in Table XXXIV.

Because the four different tensors in (143) can make distinct contributions, and because distinct contributions also come from breaking up factors of $\gamma^\alpha J^{\beta\mu}$, we have sometimes decomposed the result for a single vertex pair into parts. These distinct parts in Table XXXIV are subsequently labeled by subscripts a, b, c , etc.

After the conformal contribution has been subtracted, $i\delta\Delta_B(x; x')$ is the residual of the B -type propagator (37)

$$i\delta\Delta_B(x; x') = \frac{H^2 \Gamma(\frac{D}{2})}{16\pi^{\frac{D}{2}}} \frac{(aa')^{2-\frac{D}{2}}}{\Delta x^{D-4}} - \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-2)}{\Gamma(\frac{D}{2})} + \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \sum_{n=1}^{\infty} \left\{ \frac{\Gamma(n + \frac{D}{2})}{\Gamma(n+2)} \left(\frac{y}{4}\right)^{n-\frac{D}{2}+2} - \frac{\Gamma(n+D-2)}{\Gamma(n+\frac{D}{2})} \left(\frac{y}{4}\right)^n \right\}. \quad (144)$$

TABLE XXVII. $i\delta\Delta_A \times i[S]_{n \geq 0} - \text{IB}$. The factor $\frac{i\kappa^2 H^2}{2^6 \pi^4} \frac{mHaa'}{2} (\ln \frac{H^2 \Delta x^2}{4}) \sum_{n=0}^{\infty} Y^n$ multiplies all contributions. Here $Y = \frac{y}{4}$; $\ln Y$ and 1 are the multiplicative factors for each individual row.

I-J _{sub}		$\frac{Ha\Delta\eta^2}{\Delta x^4}$	$\frac{Ha'\Delta\eta^2}{\Delta x^4}$	$\frac{Ha\gamma^0\Delta\eta\gamma^k\Delta x_k}{\Delta x^4}$	$\frac{Ha'\gamma^0\Delta\eta\gamma^k\Delta x_k}{\Delta x^4}$	$\frac{Ha}{\Delta x^2}$
(1-1)	1	$24(3n^2 + 4n + 1)$	0	$-24(3n^2 + 4n + 1)$	0	$12(3n^2 + 6n + 2)$
(1-1)	$\ln Y$	$24(n^3 + 2n^2 + n)$	0	$-24(n^3 + 2n^2 + n)$	0	$12(n^3 + 3n^2 + 2n)$
(1-2)	1	0	0	$8(3n^2 - 1)$	0	0
(1-2)	$\ln Y$	0	0	$8(n^3 - n)$	0	0
(2-1)	1	$4(3n^2 - 1)$	$-4(3n^2 - 1)$	$-4(3n^2 - 1)$	$4(3n^2 - 1)$	$2(6n^2 + 6n + 1)$
(2-1)	$\ln Y$	$4(n^3 - n)$	$-4(n^3 - n)$	$-4(n^3 - n)$	$4(n^3 - n)$	$2(2n^3 + 3n^2 + n)$
(2-2) _a	1	$(3n^2 - 1)$	0	0	0	$\frac{1}{2}(6n^2 + 6n + 1)$
(2-2) _a	$\ln Y$	$(n^3 - n)$	0	0	0	$\frac{1}{2}(2n^3 + 3n^2 + n)$
(2-2) _b	1	$3(3n^2 - 1)$	0	0	0	$\frac{3}{2}(6n^2 + 6n + 1)$
(2-2) _b	$\ln Y$	$3(n^3 - n)$	0	0	0	$\frac{3}{2}(2n^3 + 3n^2 + n)$
(2-2) _c	1	$-2(3n^2 - 1)$	0	0	0	$-(6n^2 + 6n + 1)$
(2-2) _c	$\ln Y$	$-2(n^3 - n)$	0	0	0	$-(2n^3 + 3n^2 + n)$
Total	1	$6(15n^2 + 16n + 3)$	$-4(3n^2 - 1)$	$-4(15n^2 + 24n + 7)$	$4(3n^2 - 1)$	$9(6n^2 + 10n + 3)$
Total	$\ln Y$	$6(5n^3 + 8n^2 + 3n)$	$-4(n^3 - n)$	$-4(5n^3 + 12n^2 + 7n)$	$4(n^3 - n)$	$9(2n^3 + 5n^2 + 3n)$

TABLE XXVIII. $i\delta\Delta_A \times i[S]_{n \geq 0}$ - IC. The factor $\frac{i\kappa^2 H^2}{2^6 \pi^4} \frac{mHaa'}{2} (\ln \frac{H^2 \Delta x^2}{4}) \sum_{n=0}^{\infty} Y^n$ multiplies all contributions. Here $Y = \frac{v}{4}$; $\ln Y$ and 1 are the multiplicative factors for each individual row.

I-J _{sub}		$\frac{Ha'}{\Delta x^2}$	$\frac{H^2 a^2 \Delta \eta}{\Delta x^2}$	$\frac{H^2 a^2 \gamma^0 \gamma^k \Delta x_k}{\Delta x^2}$	$\frac{H^2 aa' \gamma^0 \gamma^k \Delta x_k}{\Delta x^2}$	$H^3 a^2 a'$
(1-1)	1	0	$-6(3n^2 + 6n + 2)$	$6(3n^2 + 6n + 2)$	0	$-3(3n^2 + 8n + 5)$
(1-1)	$\ln Y$	0	$-6(n^3 + 3n^2 + 2n)$	$6(n^3 + 3n^2 + 2n)$	0	$-3(n^3 + 4n^2 + 5n + 2)$
(1-2)	1	0	0	$-2(3n^2 + 6n + 2)$	0	0
(1-2)	$\ln Y$	0	0	$-2(n^3 + 3n^2 + 2n)$	0	0
(2-1)	1	$-2(6n^2 + 6n + 1)$	0	0	$-2(3n^2 + 4n + 1)$	0
(2-1)	$\ln Y$	$-2(2n^3 + 3n^2 + n)$	0	0	$-2(n^3 + 2n^2 + n)$	0
(2-2) _a	1	0	0	0	0	0
(2-2) _a	$\ln Y$	0	0	0	0	0
(2-2) _b	1	0	0	0	0	0
(2-2) _b	$\ln Y$	0	0	0	0	0
(2-2) _c	1	0	0	0	0	0
(2-2) _c	$\ln Y$	0	0	0	0	0
Total	1	$-2(6n^2 + 6n + 1)$	$-6(3n^2 + 6n + 2)$	$4(3n^2 + 6n + 2)$	$-2(3n^2 + 4n + 1)$	$-3(3n^2 + 8n + 5)$
Total	$\ln Y$	$-2(2n^3 + 3n^2 + n)$	$-6(n^3 + 3n^2 + 2n)$	$4(n^3 + 3n^2 + 2n)$	$-2(n^3 + 2n^2 + n)$	$-3(n^3 + 4n^2 + 5n + 2)$

As was the case for the $i\delta\Delta_A(x; x')$ contributions considered in the previous subsection, for the infinite series terms from $i\delta\Delta_B(x; x')$ this diagram is not sufficiently singular enough to make a nonzero contribution in the $D = 4$ limit. Unlike $i\delta\Delta_A(x; x')$, even the $n = 0$ terms of $i\delta\Delta_B(x; x')$ vanish for $D = 4$. This means that they only survive when multiplied by a singular term.

Because most of the contractions involve at least one derivative of $i\delta\Delta_B$, it is worth working out its various derivatives and observing their behaviors in the $D = 4$ limit,

$$\begin{aligned} \partial_i i\delta\Delta_B(x; x') &= -\frac{H^2 \Gamma(\frac{D}{2})}{16\pi^{\frac{D}{2}}} (D-4)(aa')^{2-\frac{D}{2}} \frac{\Delta x^i}{\Delta x^{D-2}} \\ &= -\partial'_i \delta\Delta_B(x; x'), \end{aligned} \quad (145)$$

$$\begin{aligned} \partial_0 i\delta\Delta_B(x; x') &= \frac{H^2 \Gamma(\frac{D}{2})}{16\pi^{\frac{D}{2}}} (D-4)(aa')^{2-\frac{D}{2}} \\ &\quad \times \left[\frac{\Delta \eta}{\Delta x^{D-2}} - \frac{aH}{2\Delta x^{D-4}} \right], \end{aligned} \quad (146)$$

TABLE XXIX. $i\delta\Delta_A \times i[S]_{n \geq 0}$ - IIA. The factor $\frac{i\kappa^2 H^2}{2^6 \pi^4} \frac{mHaa'}{2} \sum_{n=0}^{\infty} Y^n$ multiplies all contributions. Here $Y = \frac{v}{4}$; $\ln Y$ and 1 are the multiplicative factors for each individual row.

I-J		$\frac{\Delta \eta^3}{\Delta x^6}$	$\frac{\gamma^0 \Delta \eta^2 \gamma^k \Delta x_k}{\Delta x^6}$	$\frac{\Delta \eta}{\Delta x^4}$	$\frac{\gamma^0 \gamma^k \Delta x_k}{\Delta x^4}$
(1-1)	1	0	0	$12(3n^2 + 8n + 3)$	$-12(3n^2 + 8n + 3)$
(1-1)	$\ln Y$	0	0	$12(n^3 + 4n^2 + 3n)$	$-12(n^3 + 4n^2 + 3n)$
(1-2)	1	0	0	0	$4(3n^2 + 4n + 1)$
(1-2)	$\ln Y$	0	0	0	$4(n^3 + 2n^2 + n)$
(1-3)	1	0	0	$-6(2n + 1)$	$8(2n + 1)$
(1-3)	$\ln Y$	0	0	$-6(n^2 + n)$	$8(n^2 + n)$
(2-1)	1	$-8(3n^2 - 2n)$	$8(3n^2 - 2n)$	$-4(6n^2 + 2n)$	$4(3n^2 + 2n + 1)$
(2-1)	$\ln Y$	$-8(n^3 - n^2)$	$8(n^3 - n^2)$	$-4(2n^3 + n^2)$	$4(n^3 + n^2 + n)$
(2-2)	1	$-2(3n^2 - 2n)$	$-2(3n^2 - 2n)$	$-(6n^2 + 2n - 3)$	$-(6n^2 + 6n - 1)$
(2-2)	$\ln Y$	$-2(n^3 - n^2)$	$-2(n^3 - n^2)$	$-(2n^3 + n^2 - 3n)$	$-(2n^3 + 3n^2 - n)$
(2-3)	1	$10(2n - 1)$	$-6(2n - 1)$	$2(10n - 2)$	$-2(2n + 1)$
(2-3)	$\ln Y$	$10(n^2 - n)$	$-6(n^2 - n)$	$2(5n^2 - 2n)$	$-2(n^2 + n)$
(3-1)	1	$-8(2n - 2)$	$8(2n - 2)$	$2(4n - 1)$	$-2(8n - 3)$
(3-1)	$\ln Y$	$-8(n^2 - 2n)$	$8(n^2 - 2n)$	$2(2n^2 - n)$	$-2(4n^2 - 3n)$
(3-2)	1	$-2(2n - 2)$	$-2(2n - 2)$	$-(4n - 1)$	$(4n - 3)$
(3-2)	$\ln Y$	$-2(n^2 - 2n)$	$-2(n^2 - 2n)$	$-(2n^2 - n)$	$(2n^2 - 3n)$
(3-3)	1	-5	3	$-\frac{7}{2}$	$-\frac{1}{2}$
(3-3)	$\ln Y$	-5n	3n	$-\frac{7}{2}n$	$-\frac{1}{2}n$
Total	1	$-5(6n^2 - 4n - 1)$	$3(6n^2 - 4n - 1)$	$(6n^2 + 98n + \frac{49}{2})$	$-(18n^2 + 78n + \frac{37}{2})$
Total	$\ln Y$	$-5(2n^3 - 2n^2 - n)$	$3(2n^3 - 2n^2 - n)$	$(2n^3 + 49n^2 + \frac{49}{2}n)$	$-(6n^3 + 39n^2 + \frac{37}{2}n)$

$$\partial_0' i\delta\Delta_B(x; x') = \frac{H^2\Gamma(\frac{D}{2})}{16\pi^{\frac{D}{2}}}(D-4)(aa')^{2-\frac{D}{2}} \times \left\{ -\frac{\Delta\eta}{\Delta x^{D-2}} - \frac{a'H}{2\Delta x^{D-4}} \right\}, \quad (147)$$

$$\partial_0\partial_0' i\delta\Delta_B(x; x') = \frac{H^2\Gamma(\frac{D}{2})}{16\pi^{\frac{D}{2}}}(D-4)(aa')^{\frac{D}{2}-2} \left\{ \frac{-(D-2)\Delta\eta^2}{\Delta x^D} - \frac{1}{\Delta x^{D-2}} \right\} + \mathcal{O}[(D-4)^2], \quad (148)$$

TABLE XXX. $i\delta\Delta_A \times i[S]_{n \geq 0} - \text{IIB}$. The factor $\frac{i\kappa^2 H^2}{2^6 \pi^4} \frac{mHaa'}{2} \sum_{n=0}^{\infty} Y^n$ multiplies all contributions. Here $Y = \frac{y}{4}$; $\ln Y$ and 1 are the multiplicative factors for each individual row.

I-J		$\frac{Ha\Delta\eta^2}{\Delta x^4}$	$\frac{Ha'\Delta\eta^2}{\Delta x^4}$	$\frac{Ha\gamma^0\Delta\eta\gamma^k\Delta x_k}{\Delta x^4}$	$\frac{Ha'\gamma^0\Delta\eta\gamma^k\Delta x_k}{\Delta x^4}$	$\frac{Ha}{\Delta x^2}$
(1-1)	1	$12(3n^2 + 8n + 3)$	0	$-12(3n^2 + 8n + 3)$	0	$6(3n^2 + 6n + 2)$
(1-1)	$\ln Y$	$12(n^3 + 4n^2 + 3n)$	0	$-12(n^3 + 4n^2 + 3n)$	0	$6(n^3 + 3n^2 + 2n)$
(1-2)	1	0	0	$4(3n^2 + 4n + 1)$	0	0
(1-2)	$\ln Y$	0	0	$4(n^3 + 2n^2 + n)$	0	0
(1-3)	1	$-6(2n + 1)$	0	$8(2n + 1)$	0	0
(1-3)	$\ln Y$	$-6(n^2 + n)$	0	$8(n^2 + n)$	0	0
(2-1)	1	$2(3n^2 + 4n + 1)$	$-2(3n^2 - 1)$	$-2(3n^2 + 4n + 1)$	$2(3n^2 - 1)$	$(6n^2 + 14n + 5)$
(2-1)	$\ln Y$	$2(n^3 + 2n^2 + n)$	$-2(n^3 - n)$	$-2(n^3 + 2n^2 + n)$	$2(n^3 - n)$	$(2n^3 + 7n^2 + 5n)$
(2-2)	1	$(3n^2 + 4n + 1)$	0	0	0	$\frac{1}{2}(6n^2 + 14n + 5)$
(2-2)	$\ln Y$	$(n^3 + 2n^2 + n)$	0	0	0	$\frac{1}{2}(2n^3 + 7n^2 + 5n)$
(2-3)	1	$-3(2n + 1)$	0	$2(2n + 1)$	0	$-3(2n + 1)$
(2-3)	$\ln Y$	$-3(n^2 + n)$	0	$2(n^2 + n)$	0	$-3(n^2 + n)$
(3-1)	1	$4n$	$-2(2n + 1)$	$-2(2n)$	$2(2n + 1)$	$-(2n + 3)$
(3-1)	$\ln Y$	$(2n^2 - 2)$	$-2(n^2 + n)$	$-2(n^2 - 1)$	$2(n^2 + n)$	$-(n^2 + 3n + 2)$
(3-2)	1	$6n$	0	$2(2n)$	0	$6n + \frac{9}{2}$
(3-2)	$\ln Y$	$3(n^2 - 1)$	0	$2(n^2 - 1)$	0	$3n^2 + \frac{9}{2}n + \frac{3}{2}$
(3-3)	1	$\frac{5}{2}$	0	0	0	$\frac{7}{4}$
(3-3)	$\ln Y$	$\frac{5}{2}(n + 1)$	0	0	0	$\frac{7}{4}(n + 1)$
Total	1	$5(9n^2 + 20n + \frac{13}{2})$	$-2(3n^2 + 2n)$	$-2(15n^2 + 34n + 12)$	$2(3n^2 + 2n)$	$27n^2 + 55n + \frac{79}{4}$
Total	$\ln Y$	$5(3n^3 + 10n^2 + \frac{13}{2}n - \frac{1}{2})$	$-2(n^3 + n^2)$	$-2(5n^3 + 17n^2 + 12n)$	$2(n^3 + n^2)$	$9n^3 + \frac{55}{2}n^2 + \frac{77}{4}n + \frac{5}{4}$

TABLE XXXI. $i\delta\Delta_A \times i[S]_{n \geq 0} - \text{IIC}$. The factor $\frac{i\kappa^2 H^2}{2^6 \pi^4} \frac{mHaa'}{2} \sum_{n=0}^{\infty} Y^n$ multiplies all contributions. Here $Y = \frac{y}{4}$; $\ln Y$ and 1 are the multiplicative factors for each individual row.

I-J		$\frac{Ha'}{\Delta x^2}$	$\frac{H^2 a^2 \Delta\eta}{\Delta x^2}$	$\frac{H^2 a^2 \gamma^0 \gamma^k \Delta x_k}{\Delta x^2}$	$\frac{H^2 aa' \gamma^0 \gamma^k \Delta x_k}{\Delta x^2}$	$H^3 a^2 a'$
(1-1)	1	0	$-3(3n^2 + 10n + 8)$	$3(3n^2 + 10n + 8)$	0	$-\frac{3}{2}(3n^2 + 8n + 5)$
(1-1)	$\ln Y$	0	$-3(n^3 + 5n^2 + 8n + 4)$	$3(n^3 + 5n^2 + 8n + 4)$	0	$-\frac{3}{2}(n^3 + 4n^2 + 5n + 2)$
(1-2)	1	0	0	$-(3n^2 + 10n + 8)$	0	0
(1-2)	$\ln Y$	0	0	$-(n^3 + 5n^2 + 8n + 4)$	0	0
(1-3)	1	0	$\frac{3}{2}(2n + 3)$	$-2(2n + 3)$	0	0
(1-3)	$\ln Y$	0	$\frac{3}{2}(n^2 + 3n + 2)$	$-2(n^2 + 3n + 2)$	0	0
(2-1)	1	$-(6n^2 + 6n + 1)$	0	0	$-(3n^2 + 4n + 1)$	0
(2-1)	$\ln Y$	$-(2n^3 + 3n^2 + n)$	0	0	$-(n^3 + 2n^2 + n)$	0
(2-2)	1	0	0	0	0	0
(2-2)	$\ln Y$	0	0	0	0	0
(2-3)	1	0	0	0	0	0
(2-3)	$\ln Y$	0	0	0	0	0
(3-1)	1	$(2n - 2)$	0	0	$2(2n + 2)$	0
(3-1)	$\ln Y$	$(n^2 - 2n - 3)$	0	0	$2(n^2 + 2n + 1)$	0
(3-2)	1	0	0	0	0	0
(3-2)	$\ln Y$	0	0	0	0	0
(3-3)	1	0	0	0	0	0
(3-3)	$\ln Y$	0	0	0	0	0
Total	1	$-(6n^2 + 4n + 3)$	$-3(3n^2 + 9n + \frac{13}{2})$	$2(3n^2 + 8n + 5)$	$-(3n^2 - 3)$	$-\frac{3}{2}(3n^2 + 8n + 5)$
Total	$\ln Y$	$-(2n^3 + 2n^2 + 3n + 3)$	$-3(n^3 + \frac{9}{2}n^2 + \frac{13}{2}n + 3)$	$2(n^3 + 4n^2 + 5n + 2)$	$-(n^3 - 3n - 2)$	$-\frac{3}{2}(n^3 + 4n^2 + 5n + 2)$

TABLE XXXII. The total result for $i\delta\Delta_A \times i[S]_{n \geq 0}$. The factor $\frac{i\kappa^2 H^2}{2^6 \pi^4} \frac{m H a a'}{2}$ multiplies all contributions. Here $Y = \frac{\gamma}{4}; \ln \frac{H^2 \Delta x^2}{4}$ and 1 are the multiplicative factors for the each individual column. The various functions $f_i(Y)$ and $g_i(Y)$ are presented in Table XXXIII.

	$\ln \frac{H^2 \Delta x^2}{4}$	1
$\frac{\Delta \eta^3}{\Delta x^6}$	$-20f_1(Y) - 20f_2(Y) \ln Y$	$-5g_1(Y) - 5g_2(Y) \ln Y$
$\frac{\gamma^0 \Delta \eta^2 \gamma^k \Delta x_k}{\Delta x^6}$	$12f_1(Y) + 12f_2(Y) \ln Y$	$3g_1(Y) + 3g_2(Y) \ln Y$
$\frac{\Delta \eta}{\Delta x^4}$	$2f_3(Y) + 2f_4(Y) \ln Y$	$g_3(Y) + g_4(Y) \ln Y$
$\frac{\gamma^0 \gamma^k \Delta x_k}{\Delta x^4}$	$-6f_5(Y) - 6f_6(Y) \ln Y$	$-g_5(Y) - g_6(Y) \ln Y$
$\frac{H a \Delta \eta^2}{\Delta x^4}$	$6f_7(Y) + 6f_8(Y) \ln Y$	$5g_7(Y) + 5g_8(Y) \ln Y$
$\frac{H a' \Delta \eta^2}{\Delta x^4}$	$-4f_9(Y) - 4f_{10}(Y) \ln Y$	$-2g_9(Y) - 2g_{10}(Y) \ln Y$
$\frac{H a \gamma^0 \Delta \eta \gamma^k \Delta x_k}{\Delta x^4}$	$-4f_{11}(Y) - 4f_{12}(Y) \ln Y$	$-2g_{11}(Y) - 2g_{12}(Y) \ln Y$
$\frac{H a' \gamma^0 \Delta \eta \gamma^k \Delta x_k}{\Delta x^4}$	$4f_9(Y) + 4f_{10}(Y) \ln Y$	$2g_9(Y) + 2g_{10}(Y) \ln Y$
$\frac{H a}{\Delta x^2}$	$9f_{13}(Y) + 9f_{14}(Y) \ln Y$	$g_{13}(Y) + g_{14}(Y) \ln Y$
$\frac{H a'}{\Delta x^2}$	$-2f_{15}(Y) - 2f_{16}(Y) \ln Y$	$-g_{15}(Y) - g_{16}(Y) \ln Y$
$\frac{H^2 a^2 \Delta \eta}{\Delta x^2}$	$-6f_{17}(Y) - 6f_{18}(Y) \ln Y$	$-3g_{17}(Y) - 3g_{18}(Y) \ln Y$
$\frac{H^2 a^2 \gamma^0 \gamma^k \Delta x_k}{\Delta x^2}$	$4f_{17}(Y) + 4f_{18}(Y) \ln Y$	$2g_{19}(Y) + 2g_{20}(Y) \ln Y$
$\frac{H^2 a a' \gamma^0 \gamma^k \Delta x_k}{\Delta x^2}$	$-2f_{19}(Y) - 2f_{20}(Y) \ln Y$	$-g_{21}(Y) - g_{22}(Y) \ln Y$
$H^3 a^2 a'$	$-3f_{21}(Y) - 3f_{22}(Y) \ln Y$	$-\frac{3}{2}g_{19}(Y) - \frac{3}{2}g_{20}(Y) \ln Y$

$$\begin{aligned}
\partial_0 \partial_k i\delta\Delta_B(x; x') &= -\partial_0' \partial_k \delta\Delta_B(x; x') \\
&= \frac{H^2 \Gamma(\frac{D}{2})}{16\pi^{\frac{D}{2}}} \frac{(D-4)(D-2)}{(aa')^{\frac{D}{2}-2}} \left\{ \frac{-\Delta \eta \Delta x_k}{\Delta x^D} \right\} \\
&\quad + \mathcal{O}[(D-4)^2], \tag{149}
\end{aligned}$$

$$\begin{aligned}
\nabla^2 i\delta\Delta_B(x; x') &= \frac{H^2 \Gamma(D)2}{16\pi^{\frac{D}{2}}} \frac{(D-4)}{(aa')^{\frac{D}{2}-2}} \\
&\quad \times \left\{ \frac{(D-2)\overline{\Delta x^2}}{\Delta x^D} - \frac{(D-1)}{\Delta x^{D-2}} \right\}. \tag{150}
\end{aligned}$$

The fact that the first line of $i\delta\Delta_B(x; x')$ and its various derivatives are all of the order $(D-4)$ means that they would only contribute when they are multiplied by a divergence. Note that the contractions consisting of mass interaction vertices all vanish through (143), so no order m contributions come from $i\delta\Delta_B \times i[S]_{\text{cf}}$. The potential non-zero, order m contributions could either come from the flat spacetime mass term or from the infinite series expansion of the fermion propagator. Remember that the only term in $i[S]_{\text{fm}}$ behaves like $\frac{1}{\Delta x^{D-2}}$ and that the most singular one in $i[S]_{n \geq 0}$ goes like $\frac{1}{\Delta x^{D-3}}$. In addition, the generic contractions in Table XXXIV are comprised of two derivatives. Therefore one can count that the dimensionality of most singular contribution from $i[S]_{\text{fm}}$ is $\frac{1}{\Delta x^{2D-4}}$ whereas the one from $i[S]_{n \geq 0}$ is $\frac{1}{\Delta x^{2D-3}}$. The former is logarithmically divergent in $D=4$ before performing the partial integration, so one still needs to keep arbitrary D for the computation; the

latter is entirely integrable in $D=4$ so that one could compute this part in four dimensions and the result turns out to be zero owing to the cancellation of the first two series of $i\delta\Delta_B(x; x')$ and owing to the $(D-4)$ factor from its various derivatives. Therefore the only class we need to work out in this subsection is $i\delta\Delta_B(x; x') \times i[S]_{\text{fm}}(x; x')$.

We take special care of the contraction (2-2) because it is the only contraction in Table XXXIV that derivatives might have a chance not to act upon $i\delta\Delta_B$. We also break up $\gamma^{(0)}\partial^k$ into $\frac{1}{2}\gamma^0\partial^k$ and $\frac{1}{2}\gamma^k\partial^0$ for each subcontraction and tabulate the results in Table XXXV. These expressions are integrable in four dimensions and the contributions from the left column cancel out exactly with that from the right column in $D=4$. Table XXXVI gives the rest of our results for the most singular contributions, those in which all derivatives act upon the conformal coordinate separation. There is no net contribution when one or more of the derivatives acts upon a scale factor. Those expressions are also integrable in $D=4$ dimensions, at which point we can take $D=4$ and the result vanishes on account of the overall factor of $(D-4)$ or $(D-4)^2$. Finally we read off the net contribution from Table XXXVI and take $D=4$,

$$\begin{aligned}
-i[\Sigma^{\text{idBfm}}](x; x') &= \frac{\kappa^2 H^2}{2^{10} \pi^4} m a \left\{ \frac{3}{2} \partial^2 \frac{1}{\Delta x^2} \right\} \\
&= \frac{\kappa^2 H^2}{2^8 \pi^4} m a \left\{ \frac{3}{8} \partial^2 \frac{1}{\Delta x^2} \right\}. \tag{151}
\end{aligned}$$

TABLE XXXIII. The coefficient functions for Table XXXII.

$f_i(Y)$	$g_i(Y)$
$f_1(Y) = \frac{Y(2Y^2+5Y-1)}{(1-Y)^3} + 2$	$g_1(Y) = \frac{-Y(Y^2-12Y-1)}{(1-Y)^3} - 1$
$f_2(Y) = \frac{6Y^3}{(1-Y)^4}$	$g_2(Y) = \frac{Y(3Y^2+10Y-1)}{(1-Y)^4}$
$f_3(Y) = \frac{Y(7Y^2-86Y+91)}{(1-Y)^3} + 7$	$g_3(Y) = \frac{Y(49Y^2-282Y+257)}{2(1-Y)^3} + \frac{49}{2}$
$f_4(Y) = \frac{-6Y(5Y^2+Y-8)}{(1-Y)^4}$	$g_4(Y) = \frac{Y(-45Y^2-82Y+151)}{2(1-Y)^4}$
$f_5(Y) = \frac{Y(5Y^2-22Y+29)}{(1-Y)^3} + 5$	$g_5(Y) = \frac{Y(37Y^2-194Y+229)}{2(1-Y)^3} + \frac{37}{2}$
$f_6(Y) = \frac{2Y(-Y^2-Y+8)}{(1-Y)^4}$	$g_6(Y) = \frac{Y(-29Y^2-26Y+127)}{2(1-Y)^4}$
$f_7(Y) = \frac{Y(3Y^2-7Y+34)}{(1-Y)^3} + 3$	$g_7(Y) = \frac{Y(13Y^2-48Y+71)}{2(1-Y)^3} + \frac{13}{2}$
$f_8(Y) = \frac{2Y(7Y+8)}{(1-Y)^4}$	$g_8(Y) = \frac{Y(Y^3-4Y^2+Y+38)}{2(1-Y)^4} - \frac{1}{2}$
$f_9(Y) = \frac{Y(-Y^2+5Y+2)}{(1-Y)^3} - 1$	$g_9(Y) = \frac{Y(Y+5)}{(1-Y)^3}$
$f_{10}(Y) = \frac{6Y^2}{(1-Y)^4}$	$g_{10}(Y) = \frac{2Y(2Y+1)}{(1-Y)^4}$
$f_{11}(Y) = \frac{Y(7Y^2-23Y+46)}{(1-Y)^3} + 7$	$g_{11}(Y) = \frac{Y(12Y^2-43Y+61)}{(1-Y)^3} + 12$
$f_{12}(Y) = \frac{6Y(Y+4)}{(1-Y)^4}$	$g_{12}(Y) = \frac{-2Y(2Y-17)}{(1-Y)^4}$
$f_{13}(Y) = \frac{Y(3Y^2-10Y+19)}{(1-Y)^3} + 3$	$g_{13}(Y) = \frac{Y(79Y^2-270Y+407)}{4(1-Y)^3} + \frac{79}{4}$
$f_{14}(Y) = \frac{2Y(Y+5)}{(1-Y)^4}$	$g_{14}(Y) = \frac{Y(-5Y^3+20Y^2-29Y+230)}{4(1-Y)^4} + \frac{5}{4}$
$f_{15}(Y) = \frac{Y(Y^2-2Y+13)}{(1-Y)^3} + 1$	$g_{15}(Y) = \frac{Y(3Y^2-4Y+13)}{(1-Y)^3} + 3$
$f_{16}(Y) = \frac{6Y(Y+1)}{(1-Y)^4}$	$g_{16}(Y) = \frac{Y(-3Y^3+12Y^2-7Y+10)}{(1-Y)^4} + 3$
$f_{17}(Y) = \frac{Y(2Y^2-7Y+11)}{(1-Y)^3} + 2$	$g_{17}(Y) = \frac{Y(13Y^2-38Y+37)}{2(1-Y)^3} + \frac{13}{2}$
$f_{18}(Y) = \frac{6Y}{(1-Y)^4}$	$g_{18}(Y) = \frac{3Y(-Y^3+4Y^2-6Y+5)}{(1-Y)^4} + 3$
$f_{19}(Y) = \frac{Y(Y^2-3Y+8)}{(1-Y)^3} + 1$	$g_{19}(Y) = \frac{Y(5Y^2-15Y+16)}{(1-Y)^3} + 5$
$f_{20}(Y) = \frac{2Y(Y+2)}{(1-Y)^4}$	$g_{20}(Y) = \frac{-2Y(Y^3-4Y^2+6Y-6)}{(1-Y)^4} + 2$
$f_{21}(Y) = \frac{Y(5Y^2-15Y+16)}{(1-Y)^3} + 5$	$g_{21}(Y) = \frac{-3Y^2(Y-3)}{(1-Y)^3} - 3$
$f_{22}(Y) = \frac{-2Y(Y^3-4Y^2+6Y-6)}{(1-Y)^4} + 2$	$g_{22}(Y) = \frac{2Y(Y^3-4Y^2+8Y-2)}{(1-Y)^4} - 2$

TABLE XXXIV. Contractions from the $i\delta\Delta_B$ part of the graviton propagator. The contributions from (1-1)-(1-4), (4-1)-(4-4), (2-4), and (3-4) are zero.

I	J	sub	$iV_I^{\alpha\beta}(x)i[S](x;x')iV_J^{\rho\sigma}(x')[L_{\alpha\beta}T_{\rho\sigma}^B]i\delta\Delta_B(x;x')$
2	1		0
2	2	a	$-\frac{1}{2}\kappa^2\partial'_0\{\gamma^{(0)\partial k}i[S](x;x')\gamma_k i\delta\Delta_B(x;x')\}$
2	2	b	$-\frac{1}{2}\kappa^2\partial_k\{\gamma^{(0)\partial k}i[S](x;x')\gamma^0 i\delta\Delta_B(x;x')\}$
2	3	a	$-\frac{1}{8}\kappa^2\gamma_k\partial_0 i[S](x;x')\gamma^k\partial'_0 i\delta\Delta_B(x;x')$
2	3	b	$\frac{1}{8}\kappa^2\gamma^0\partial'_0 i\delta\Delta_B(x;x')\partial_k i[S](x;x')\gamma^k$
2	3	c	$-\frac{1}{8}\kappa^2\gamma^k\partial_k i\delta\Delta_B(x;x')\partial_0 i[S](x;x')\gamma^0$
2	3	d	$\frac{1}{8}\kappa^2\gamma^0\partial^k i[S](x;x')\gamma^0\partial_k i\delta\Delta_B(x;x')$
3	1		0
3	2	a	$\frac{1}{8}\kappa^2\partial'_0\{\gamma^k i[S](x;x')\gamma_k\partial_0 i\delta\Delta_B(x;x')\}$
3	2	b	$\frac{1}{8}\kappa^2\gamma^k\partial_k i[S](x;x')\gamma^0\partial_0 i\delta\Delta_B(x;x')$
3	2	c	$-\frac{1}{8}\kappa^2\gamma^0\partial'_0 i[S](x;x')\gamma^k\partial_k i\delta\Delta_B(x;x')$
3	2	d	$-\frac{1}{8}\kappa^2\partial_k\{\gamma^0 i[S](x;x')\gamma^0\partial^k i\delta\Delta_B(x;x')\}$
3	3	a	$-\frac{1}{16}\kappa^2\gamma_k i[S](x;x')\gamma^k\partial_0\partial'_0 i\delta\Delta_B(x;x')$
3	3	b	$\frac{1}{16}\kappa^2\gamma^0 i[S](x;x')\gamma^k\partial_k\partial'_0 i\delta\Delta_B(x;x')$
3	3	c	$-\frac{1}{16}\kappa^2\gamma^k i[S](x;x')\gamma^0\partial_0\partial_k i\delta\Delta_B(x;x')$
3	3	d	$\frac{1}{16}\kappa^2\gamma^0 i[S](x;x')\gamma^0\nabla^2 i\delta\Delta_B(x;x')$

D. Subleading contributions from $i\delta\Delta_C$

The point of this subsection is to work out the contribution from replacing the graviton propagator in Table V by its residual C -type part,

$$i[\alpha_\beta\Delta_{\rho\sigma}] \rightarrow 2\left[\frac{\bar{\eta}_{\alpha\beta}\bar{\eta}_{\rho\sigma}}{(D-2)(D-3)} + \frac{\delta_\alpha^0\delta_\beta^0\bar{\eta}_{\rho\sigma} + \bar{\eta}_{\alpha\beta}\delta_\rho^0\delta_\sigma^0}{D-2} + \left(\frac{D-3}{D-2}\right)\delta_\alpha^0\delta_\beta^0\delta_\rho^0\delta_\sigma^0\right]i\delta\Delta_C. \quad (152)$$

As in the previous subsections we first make the requisite contractions and then act on the derivatives. The result of this first step is displayed in Tables XXXVII and XXXVIII. The four different tensors in (152) can make distinct contributions, and distinct contributions also come from breaking up factors of $\gamma^\alpha J^{\beta\mu}$, so we have sometimes decomposed the result for a single vertex pair into parts. These distinct contributions are tagged with subscripts a, b, c , etc.

Here $i\delta\Delta_C(x; x')$ is the residual of the C -type propagator (38) after the conformal contribution has been subtracted,

$$i\delta\Delta_C(x; x') = \frac{H^2}{16\pi^{\frac{D}{2}}}\left(\frac{D}{2}-3\right)\Gamma\left(\frac{D}{2}-1\right)\frac{(aa')^{2-\frac{D}{2}}}{\Delta x^{D-4}} + \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \times \frac{\Gamma(D-3)}{\Gamma(\frac{D}{2})} - \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \sum_{n=1}^{\infty} \left\{ \left(n - \frac{D}{2} + 3\right) \times \frac{\Gamma(n + \frac{D}{2} - 1)}{\Gamma(n+2)} \left(\frac{y}{4}\right)^{n-\frac{D}{2}+2} - (n+1) \times \frac{\Gamma(n+D-3)}{\Gamma(n+\frac{D}{2})} \left(\frac{y}{4}\right)^n \right\}. \quad (153)$$

The only way $i\delta\Delta_C(x; x')$ can give a nonzero contribution in $D = 4$ dimensions is for it to multiply a divergence as with the contributions from $i\delta\Delta_B(x; x')$ considered in the previous subsection. That means only the $n = 0$ term can possibly contribute. Even for the $n = 0$ term, both derivatives must act upon the coordinate separation to make a nonzero contribution in $D = 4$ dimensions.

Because all the vertex pairs involve one or more derivatives of $i\delta\Delta_C$, here we list them as follows:

$$\partial_i i\delta\Delta_C = \frac{H^2\Gamma(\frac{D}{2}-1)}{16\pi^{\frac{D}{2}}}\left(\frac{D}{2}-3\right)(D-4)(aa')^{2-\frac{D}{2}}\frac{-\Delta x^i}{\Delta x^{D-2}} = -\partial'_i i\delta\Delta_C, \quad (154)$$

$$\partial_0 i\delta\Delta_C = \frac{H^2\Gamma(\frac{D}{2}-1)}{16\pi^{\frac{D}{2}}}\left(\frac{D}{2}-3\right)(D-4)(aa')^{2-\frac{D}{2}} \times \left\{ \frac{\Delta\eta}{\Delta x^{D-2}} - \frac{aH}{2\Delta x^{D-4}} \right\}, \quad (155)$$

$$\partial'_0 i\delta\Delta_C = \frac{H^2\Gamma(\frac{D}{2}-1)}{16\pi^{\frac{D}{2}}}\left(\frac{D}{2}-3\right)(D-4)(aa')^{2-\frac{D}{2}} \times \left\{ -\frac{\Delta\eta}{\Delta x^{D-2}} - \frac{a'H}{2\Delta x^{D-4}} \right\}, \quad (156)$$

TABLE XXXV. The contributions for the contraction (2-2) from $i\delta\Delta_B \times i[S]_{\text{fm}}$.

Prefactor	$\frac{\kappa^2 H^2}{2^8 \pi^D} \Gamma(\frac{D}{2}) \Gamma(\frac{D}{2} - 1) \frac{ma}{(aa')^{\frac{D}{2}-2}}$	$\frac{\kappa^2 H^{D-2}}{2^{D+4} \pi^D} \frac{\Gamma(D-2) \Gamma(\frac{D}{2}-1)}{\Gamma(\frac{D}{2})} ma$
$(2-2)_{a_1}$	$(D-1) \left[\frac{(D-2)}{2(D-3)} \partial_0^2 + Ha \partial_0 \right] \frac{1}{\Delta x^{2D-6}}$	$-(D-1) [\partial_0^2 + Ha \partial_0] \frac{1}{\Delta x^{D-2}}$
$(2-2)_{a_2}$	$\left[\frac{(D-2)}{2(D-3)} \gamma^0 \partial_0 \tilde{\theta} - Ha \gamma^0 \tilde{\theta} \right] \frac{1}{\Delta x^{2D-6}}$	$[\gamma^0 \partial_0 \tilde{\theta} + Ha \gamma^0 \tilde{\theta}] \frac{1}{\Delta x^{D-2}}$
$(2-2)_{b_1}$	$\frac{(D-2)}{2(D-3)} \gamma^0 \partial_0 \tilde{\theta} \frac{1}{\Delta x^{2D-6}}$	$-\gamma^0 \partial_0 \tilde{\theta} \frac{1}{\Delta x^{D-2}}$
$(2-2)_{b_2}$	$\frac{-(D-2)}{2(D-3)} \nabla^2 \frac{1}{\Delta x^{2D-6}}$	$\nabla^2 \frac{1}{\Delta x^{D-2}}$

TABLE XXXVI. $i\delta\Delta_B \times i[S]_{\text{fm}}$ terms. All contributions are multiplied by $\frac{\kappa^2 H^2}{2^{10} \pi^D} ma \Gamma(\frac{D}{2}) \Gamma(\frac{D}{2} - 1) (aa')^{2-\frac{D}{2}}$.

I-J _{sub}	Contributions
$(2-3)_a$	$\frac{(D-1)}{2(D-3)} [\partial^2 - (D-4) \partial_0^2] \frac{1}{\Delta x^{2D-6}}$
$(2-3)_b$	$-\frac{(D-4)}{2(D-3)} \gamma^0 \partial_0 \tilde{\theta} \frac{1}{\Delta x^{2D-6}}$
$(2-3)_c$	$\frac{(D-4)}{2(D-3)} \gamma^0 \partial_0 \tilde{\theta} \frac{1}{\Delta x^{2D-6}}$
$(2-3)_d$	$\frac{(D-1)}{2(D-3)} [\partial^2 + (D-4)(D-1) \nabla^2] \frac{1}{\Delta x^{2D-6}}$
$(3-2)_a$	$\frac{(D-4)(D-1)}{(D-3)} \partial_0^2 \frac{1}{\Delta x^{2D-6}}$
$(3-2)_b$	$-\frac{(D-4)}{(D-3)} \gamma^0 \partial_0 \tilde{\theta} \frac{1}{\Delta x^{2D-6}}$
$(3-2)_c$	$\frac{(D-4)}{(D-3)} \gamma^0 \partial_0 \tilde{\theta} \frac{1}{\Delta x^{2D-6}}$
$(3-2)_d$	$-\frac{(D-4)}{(D-3)} \nabla^2 \frac{1}{\Delta x^{2D-6}}$
$(3-3)_a$	$\frac{-(D-1)}{4(D-3)} \partial^2 \frac{1}{\Delta x^{2D-6}}$
$(3-3)_b$	$\frac{-(D-4)}{4(D-3)} \gamma^0 \partial_0 \tilde{\theta} \frac{1}{\Delta x^{2D-6}}$
$(3-3)_c$	$\frac{(D-4)}{4(D-3)} \gamma^0 \partial_0 \tilde{\theta} \frac{1}{\Delta x^{2D-6}}$
$(3-3)_d$	$\left[\frac{-(D-1)}{4(D-3)} \partial^2 + \frac{(D-4)}{4(D-3)} \nabla^2 \right] \frac{1}{\Delta x^{2D-6}}$
total	$\left\{ \frac{(D-1)}{2(D-3)} \partial^2 + (D-4) \left[\frac{(D-1)}{(D-3)} \partial_0^2 \right] - \frac{3}{4(D-3)} \nabla^2 \right\} \frac{1}{\Delta x^{2D-6}}$

TABLE XXXVII. Contractions from the $i\delta\Delta_C$ part of the graviton propagator-I.

I	J	sub	$iV_I^{\alpha\beta}(x) i[S](x; x') iV_J^{\rho\sigma}(x') [\alpha_\beta T_{\rho\sigma}^C] i\delta\Delta_C(x; x')$
1	4		$\frac{2}{(D-3)(D-2)} \kappa^2 iam \tilde{\theta} i[S](x; x') i\delta\Delta_C(x; x')$
2	4	a	$\frac{1}{(D-2)} \kappa^2 iam \gamma^0 \partial_0 i[S](x; x') i\delta\Delta_C(x; x')$
2	4	b	$-\frac{1}{(D-3)(D-2)} \kappa^2 iam \tilde{\theta} i[S](x; x') i\delta\Delta_C(x; x')$
3	4	a	$\frac{(D-1)}{2(D-3)(D-2)} \kappa^2 iam \gamma^0 \partial_0 i\delta\Delta_C(x; x') i[S](x; x')$
3	4	b	$\frac{1}{2(D-3)(D-2)} \kappa^2 iam \tilde{\theta} i\delta\Delta_C(x; x') i[S](x; x')$
4	1		$-\frac{2}{(D-3)(D-2)} \kappa^2 iam \partial'_\mu \{ i[S](x; x') \gamma^\mu i\delta\Delta_C(x; x') \}$
4	2	a	$-\frac{1}{(D-2)} \kappa^2 iam \partial'_0 \{ i[S](x; x') \gamma^0 i\delta\Delta_C(x; x') \}$
4	2	b	$-\frac{1}{(D-3)(D-2)} \kappa^2 iam \partial_k \{ i[S](x; x') \gamma^k i\delta\Delta_C(x; x') \}$
4	3	a	$\frac{(D-1)}{2(D-3)(D-2)} \kappa^2 iam i[S](x; x') \gamma^0 \partial_0 i\delta\Delta_C(x; x')$
4	3	b	$\frac{1}{2(D-3)(D-2)} \kappa^2 iam i[S](x; x') \tilde{\theta} \delta\Delta_C(x; x')$

TABLE XXXVIII. Contractions from the $i\delta\Delta_C$ part of the graviton propagator-II.

I	J	sub	$iV_I^{\alpha\beta}(x) i[S](x; x') iV_J^{\rho\sigma}(x') [\alpha_\beta T_{\rho\sigma}^C] i\delta\Delta_C(x; x')$
1	1		$-\frac{2}{(D-3)(D-2)} \kappa^2 \partial'_\mu \{ \tilde{\theta} i[S](x; x') \gamma^\mu i\delta\Delta_C(x; x') \}$
1	2	a	$-\frac{1}{(D-2)} \kappa^2 \partial'_0 \{ \tilde{\theta} i[S](x; x') \gamma^0 i\delta\Delta_C(x; x') \}$
1	2	b	$-\frac{1}{(D-3)(D-2)} \kappa^2 \partial_k \{ \tilde{\theta} i[S](x; x') \gamma^k i\delta\Delta_C(x; x') \}$
1	3	a	$\frac{(D-1)}{2(D-3)(D-2)} \kappa^2 \tilde{\theta} i[S](x; x') \gamma^0 \partial'_0 i\delta\Delta_C(x; x')$
1	3	b	$-\frac{1}{2(D-3)(D-2)} \kappa^2 \tilde{\theta} i[S](x; x') \tilde{\theta} i\delta\Delta_C(x; x')$
2	1	a	$-\frac{1}{(D-2)} \kappa^2 \partial'_\mu \{ \gamma^0 \partial_0 i[S](x; x') \gamma^\mu i\delta\Delta_C(x; x') \}$
2	1	b	$\frac{1}{(D-3)(D-2)} \kappa^2 \partial'_\mu \{ \tilde{\theta} i[S](x; x') \gamma^\mu i\delta\Delta_C(x; x') \}$
2	2	a	$-\frac{(D-3)}{2(D-2)} \kappa^2 \partial'_0 \{ \gamma^0 \partial_0 i[S](x; x') \gamma^0 i\delta\Delta_C(x; x') \}$
2	2	b	$-\frac{1}{2(D-2)} \kappa^2 \partial_k \{ \gamma^0 \partial_0 i[S](x; x') \gamma^k i\delta\Delta_C(x; x') \}$
2	2	c	$\frac{1}{2(D-2)} \kappa^2 \partial'_0 \{ \tilde{\theta} i[S](x; x') \gamma^0 i\delta\Delta_C(x; x') \}$
2	2	d	$\frac{1}{2(D-3)(D-2)} \kappa^2 \partial_k \{ \tilde{\theta} i[S](x; x') \gamma^k i\delta\Delta_C(x; x') \}$
2	3	a	$\frac{1}{4} \frac{(D-1)}{(D-2)} \kappa^2 \gamma^0 \partial_0 i[S](x; x') \gamma^0 \partial'_0 i\delta\Delta_C(x; x')$
2	3	b	$-\frac{1}{4(D-2)} \kappa^2 \gamma^0 \partial_0 i[S](x; x') \tilde{\theta} i\delta\Delta_C(x; x')$
2	3	c	$-\frac{(D-1)}{4(D-3)(D-2)} \kappa^2 \tilde{\theta} i[S](x; x') \gamma^0 \partial'_0 i\delta\Delta_C(x; x')$
2	3	d	$\frac{1}{4(D-3)(D-2)} \kappa^2 \tilde{\theta} i[S](x; x') \tilde{\theta} i\delta\Delta_C(x; x')$
3	1	a	$-\frac{(D-1)}{2(D-3)(D-2)} \kappa^2 \partial'_\mu \{ \gamma^0 \partial_0 i\delta\Delta_C(x; x') i[S](x; x') \gamma^\mu \}$
3	1	b	$-\frac{1}{2(D-3)(D-2)} \kappa^2 \partial'_\mu \{ \tilde{\theta} i\delta\Delta_C(x; x') i[S](x; x') \gamma^\mu \}$
3	2	a	$-\frac{1}{4} \frac{(D-1)}{(D-2)} \kappa^2 \partial'_0 \{ \gamma^0 \partial_0 i\delta\Delta_C(x; x') i[S](x; x') \gamma^0 \}$
3	2	b	$-\frac{1}{4(D-2)} \kappa^2 \partial'_0 \{ \tilde{\theta} i\delta\Delta_C(x; x') i[S](x; x') \gamma^0 \}$
3	2	c	$-\frac{(D-1)}{4(D-3)(D-2)} \kappa^2 \partial_k \{ \gamma^0 \partial_0 i\delta\Delta_C(x; x') i[S](x; x') \gamma^k \}$
3	2	d	$-\frac{1}{4(D-3)(D-2)} \kappa^2 \partial_k \{ \tilde{\theta} \delta\Delta_C(x; x') i[S](x; x') \gamma^k \}$
3	3	a	$\frac{(D-1)^2}{8(D-3)(D-2)} \kappa^2 \gamma^0 i[S](x; x') \gamma^0 \partial_0 \partial'_0 i\delta\Delta_C(x; x')$
3	3	b	$-\frac{(D-1)}{8(D-3)(D-2)} \kappa^2 \gamma^0 i[S](x; x') \gamma^k \partial_0 \partial_k i\delta\Delta_C(x; x')$
3	3	c	$\frac{(D-1)}{8(D-3)(D-2)} \kappa^2 \gamma^k i[S](x; x') \gamma^0 \partial_k \partial'_0 i\delta\Delta_C(x; x')$
3	3	d	$-\frac{1}{8(D-3)(D-2)} \kappa^2 \gamma^k i[S](x; x') \gamma^l \partial_k \partial_l i\delta\Delta_C(x; x')$

TABLE XXXIX. Contributions from Table XXXVII for $i\delta\Delta_C \times i[S]_{\text{cf}}(x; x')$ -I.

I-J _{sub}	$\frac{\kappa^2 H^2}{2^5 \pi^D} \Gamma(\frac{D}{2}) \Gamma(\frac{D}{2} - 1) \frac{(3-\frac{D}{2})}{(aa')^{\frac{D}{2}-2}} \frac{ma}{(aa')^{\frac{D}{2}-2}}$	$\frac{\kappa^2 H^{D-2}}{2^{D+1} \pi^D} \Gamma(D-3) ma$
$(4-1)$	$\frac{-1}{(D-2)(D-3)} \partial^2 \frac{1}{\Delta x^{2D-6}}$	$\frac{2}{(D-2)^2(D-3)} \partial^2 \frac{1}{\Delta x^{D-2}}$
$(4-2)_a$	$\frac{1}{2(D-2)} \tilde{\theta} \gamma^0 \partial_0 \frac{1}{\Delta x^{2D-6}}$	$\frac{-1}{(D-2)^2} \tilde{\theta} \gamma^0 \partial_0 \frac{1}{\Delta x^{D-2}}$
$(4-2)_b$	$\frac{-1}{2(D-2)(D-3)} \tilde{\theta} \tilde{\theta} \frac{1}{\Delta x^{2D-6}}$	$\frac{1}{(D-2)^2(D-3)} \tilde{\theta} \tilde{\theta} \frac{1}{\Delta x^{D-2}}$

TABLE XL. Contributions from Table XXXVII for $i\delta\Delta_C \times i[S]_{\text{cf}}(x; x')\text{-II}$. All the terms are multiplied by $\frac{\kappa^2 H^2}{2^5 \pi^D} \Gamma(\frac{D}{2}) \Gamma(\frac{D}{2} - 1) \frac{(3-\frac{D}{2})(D-1)}{(D-2)^2(D-3)^2} \frac{ma}{(aa')^{\frac{D}{2}-2}}$.

I-J _{sub}	$\partial^2 \frac{1}{\Delta x^{2D-6}}$
(2-4) _b	$-\frac{1}{4}$
(3-4) _a	$-\frac{1}{8}$
(3-4) _b	$-\frac{1}{8}$
(4-3) _a	$-\frac{1}{8}$
(4-3) _b	$-\frac{1}{8}$

$$\begin{aligned} \partial_0 \partial'_0 i\delta\Delta_C &= \frac{H^2 \Gamma(\frac{D}{2} - 1)}{16\pi^{\frac{D}{2}}} \left(\frac{D}{2} - 3\right) \frac{(D-4)}{(aa')^{\frac{D}{2}-2}} \\ &\times \left\{ \frac{-(D-2)\Delta\eta^2}{\Delta x^D} - \frac{1}{\Delta x^{D-2}} \right\} + \mathcal{O}[(D-4)^2], \end{aligned} \quad (157)$$

$$\begin{aligned} \partial_k \partial_0 i\delta\Delta_C &= -\partial_k \partial'_0 i\delta\Delta_C \\ &= \frac{H^2 \Gamma(\frac{D}{2} - 1)}{16\pi^{\frac{D}{2}}} \left(\frac{D}{2} - 3\right) \frac{(D-2)(D-4)}{(aa')^{\frac{D}{2}-2}} \\ &\times \frac{-\Delta\eta\Delta x_k}{\Delta x^D} + \mathcal{O}[(D-4)^2], \end{aligned} \quad (158)$$

$$\begin{aligned} \partial_k \partial_l i\delta\Delta_C &= \frac{H^2 \Gamma(\frac{D}{2} - 1)}{16\pi^{\frac{D}{2}}} \left(\frac{D}{2} - 3\right) \frac{(D-4)}{(aa')^{\frac{D}{2}-2}} \\ &\times \left\{ \frac{(D-2)\Delta x_k \Delta x_l}{\Delta x^D} - \frac{\delta_{kl}}{\Delta x^{D-2}} \right\}. \end{aligned} \quad (159)$$

Note that $i\delta\Delta_C$ and its various derivatives have the same behaviors as $i\delta\Delta_B$. The propagator itself tends to cancel in $D = 4$ dimensions and its various derivatives all carry the $(D-4)$ factor. This means that they could give the nonzero contributions only when they are multiplied by the singular terms. For the generic contraction I in Table XXXVII, the only order m contribution must come

from the conformal part of the fermion propagator whereas in the generic contraction II in Table XXXVIII the order m contribution could be either from the flat spacetime mass term or from the infinite series expansion of the fermion propagator. The fortunate thing is that the contribution from the infinite series expansion of the fermion propagator vanishes. The most singular terms from this particular contribution have dimensionality $\frac{1}{\Delta x^{2D-5}}$, which are integrable in $D = 4$. Therefore they completely vanish in $D = 4$ dimensions owing to the behaviors of the residual part of the C -type graviton propagator we mentioned above.

We first work out the contributions from Table XXXVII, which contain a logarithmic divergence. The contraction (1-4) and (2-4)_a are simple owing to the special property of the conformal part of the fermion propagator (70) and hence they only pick up the constant part of $i\delta\Delta_C$,

$$[1-4] = \frac{\kappa^2 H^{D-2}}{2^D \pi^{\frac{D}{2}}} \frac{\Gamma(D-3)}{\Gamma(\frac{D}{2})} \frac{2}{(D-2)(D-3)} ima\delta^D(x-x'), \quad (160)$$

$$[2-4]_a = \frac{\kappa^2 H^{D-2}}{2^D \pi^{\frac{D}{2}}} \frac{\Gamma(D-3)}{\Gamma(\frac{D}{2})} \frac{1}{(D-2)} ima\delta^D(x-x'). \quad (161)$$

The same procedure is employed to make the expressions integrable in $D = 4$. We summarized the contractions (4-1), (4-2)_a, and (4-2)_b in Table XXXIX and listed the rest of this category in Table XL. The two series in Table XXXIX from each contraction cancel out with each other precisely in four dimensions. Finally the total summation from (160) and (161), Tables XXXIX and XL is quite simple in $D = 4$,

$$\begin{aligned} -i[\Sigma^{\text{idCcf}}](x; x') &= \frac{i\kappa^2 H^2}{16\pi^2} \frac{3}{2} ma\delta^4(x-x') \\ &+ \frac{\kappa^2 H^2}{32\pi^4} ma \left\{ \frac{-9}{16} \partial^2 \frac{1}{\Delta x^2} \right\}. \end{aligned} \quad (162)$$

TABLE XLI. Contributions from Table XXXVIII for $i\delta\Delta_C \times i[S]_{\text{fm}}(x; x')\text{-I}$.

I-J _{sub}	$\frac{\kappa^2 H^2}{2^6 \pi^D} \Gamma^2(\frac{D}{2} - 1)(3 - \frac{D}{2}) \frac{ma}{(aa')^{\frac{D}{2}-2}}$	$\frac{\kappa^2 H^{D-2}}{2^{D+2} \pi^D} \Gamma(D-3) ma \frac{1}{(D-2)}$
(1-1)	$[\frac{1}{(D-3)^2} \partial^2 - \frac{2Ha\gamma^0}{(D-2)(D-3)} \not\partial] \frac{1}{\Delta x^{2D-6}}$	$\frac{2[-\partial^2 + Ha\gamma^0 \not\partial]}{(D-2)(D-3)} \frac{1}{\Delta x^{D-2}}$
(1-2) _a	$[\frac{-1}{2(D-3)} \not\partial \gamma^0 \partial_0 - \frac{Ha}{(D-2)} \partial_0] \frac{1}{\Delta x^{2D-6}}$	$\frac{[\not\partial \gamma^0 \partial_0 + Ha \partial_0]}{(D-2)} \frac{1}{\Delta x^{D-2}}$
(1-2) _b	$[\frac{1}{2(D-3)^2} \not\partial + \frac{Ha\gamma^0}{(D-2)(D-3)}] \tilde{\not\partial} \frac{1}{\Delta x^{2D-6}}$	$\frac{-[\not\partial + Ha\gamma^0]}{(D-2)(D-3)} \tilde{\not\partial} \frac{1}{\Delta x^{D-2}}$
(2-1) _a	$[\frac{-1}{2(D-3)} \gamma^0 \partial_0 - \frac{Ha\gamma^0}{(D-2)} \not\partial] \frac{1}{\Delta x^{2D-6}}$	$\frac{[\gamma^0 \partial_0 + Ha\gamma^0]}{(D-2)} \not\partial \frac{1}{\Delta x^{D-2}}$
(2-1) _b	$\frac{1}{2(D-3)^2} \tilde{\not\partial} \not\partial \frac{1}{\Delta x^{2D-6}}$	$\frac{-1}{(D-2)(D-3)} \tilde{\not\partial} \not\partial \frac{1}{\Delta x^{D-2}}$
(2-2) _a	$[-\frac{1}{4} \partial_0^2 - \frac{(D-3)}{2(D-2)} Ha \partial_0] \frac{1}{\Delta x^{2D-6}}$	$\frac{(D-3)}{2(D-2)} [\partial_0^2 + Ha \partial_0] \frac{1}{\Delta x^{D-2}}$
(2-2) _b	$[\frac{1}{4(D-3)} \gamma^0 \partial_0 + \frac{Ha\gamma^0}{2(D-2)}] \tilde{\not\partial} \frac{1}{\Delta x^{2D-6}}$	$\frac{-[\gamma^0 \partial_0 + Ha\gamma^0]}{2(D-2)} \tilde{\not\partial} \frac{1}{\Delta x^{D-2}}$
(2-2) _c	$\frac{-1}{4} \gamma^0 \partial_0 \tilde{\not\partial} \frac{1}{\Delta x^{2D-6}}$	$\frac{1}{2(D-2)} \gamma^0 \partial_0 \tilde{\not\partial} \frac{1}{\Delta x^{D-2}}$
(2-2) _d	$\frac{1}{4(D-3)^2} \nabla^2 \frac{1}{\Delta x^{2D-6}}$	$\frac{-1}{2(D-2)(D-3)} \nabla^2 \frac{1}{\Delta x^{D-2}}$

TABLE XLII. Contribution from Table XXXVIII for $i\delta\Delta_C \times i[S]_{\text{fin}}(x; x')$ -II. All the terms are multiplied by $\frac{\kappa^2 H^2}{2^8 \pi^4} \Gamma^2(\frac{D}{2} - 1) \times \frac{(3-\frac{D}{2})(D-1)}{(D-2)(D-3)} \frac{ma}{(aa')^{\frac{D}{2}-2}}$.

I-J _{sub}	$\partial^2 \frac{1}{\Delta x^{2D-6}}$
$(1-3)_a$	$\frac{-1}{2(D-3)}$
$(1-3)_b$	$\frac{-1}{2(D-3)}$
$(2-3)_a$	$\frac{-1}{4}$
$(2-3)_d$	$\frac{1}{4(D-3)}$
$(3-3)_a$	$\frac{1}{8} \frac{(D-1)}{(D-3)}$
$(3-3)_d$	$\frac{1}{8(D-3)}$
total	$\frac{(D-8)}{8(D-3)} - \frac{(D-4)}{4(D-3)}$

The final class we need to complete is the contributions from Table XXXVIII. Very similarly to what happened with $i\delta\Delta_B$, the contractions from (1-1), (1-2), (2-1), (2-2) tend to cancel and we summarized them in Table XLI. We also tabulate the rest of the contributions

that do not vanish in $D = 4$ in Table XLII. As already explained, terms for which one or more derivatives act upon a scale factor make no contribution in $D = 4$ dimensions, so the final nonzero contribution comes from the derivatives only acting upon the coordinate separation, Δx^2 . The net contributions from Table XLI do vanish completely in $D = 4$ dimensions and the only nonzero contributions of this class come from Table XLII,

$$-i[\Sigma^{\text{idCfm}}](x; x') = \frac{\kappa^2 H^2}{2^8 \pi^4} ma \left\{ \frac{-3}{4} \partial^2 \frac{1}{\Delta x^2} \right\}. \quad (163)$$

VII. RENORMALIZATION

Except for the finite results from (111), and the results from Tables XXXII and XXXIII, each of which possesses distinctive expressions, the rest of the regulated result we have worked so hard to compute derives from summing expressions (65), (81), (84), (91), (93), (103), (121), (151), (162), and (163)

$$\begin{aligned}
i\kappa^2 \left\{ \beta_1 \frac{m}{a} \partial^2 + \beta_2 m H \partial_0 + \beta_3 m H \gamma^0 \bar{\not{\partial}} + \beta_4 m H^2 a \right\} \delta^D(x - x') &+ \frac{i\kappa^2}{16\pi^2} \left\{ \left(3 \ln a - \frac{3}{8} \right) \frac{m}{a} \partial^2 + \left(\frac{97}{16} \ln a - \frac{63}{16} \right) m H \partial_0 \right. \\
&+ \left(\frac{9}{16} \ln a + \frac{1}{8} \right) m H \gamma^0 \bar{\not{\partial}} + \left(\frac{95}{8} \ln a + \frac{195}{32} \right) H^2 m a \left. \right\} \delta^4(x - x') + \frac{\kappa^2}{64\pi^4} \left\{ \left[\frac{3}{2} \frac{m}{a'} \partial^2 + \left(\frac{7}{8} \frac{a}{a'} - \frac{27}{32} \right) m H \partial_0 \right. \right. \\
&+ \left(\frac{9}{16} \frac{a}{a'} - \frac{9}{32} \right) m H \gamma^0 \bar{\not{\partial}} + H^2 m \left(\frac{215}{32} a + \frac{9}{32} a' \right) \left. \right] \partial^2 + H^2 m a [\nabla^2 + 6 H a \partial_0 + 4 H a \gamma^0 \bar{\not{\partial}}] \left[\frac{\ln(\mu^2 \Delta x^2)}{\Delta x^2} \right] \\
&+ \frac{\kappa^2 H^2}{64\pi^4} \left\{ \frac{9}{16} m a' \partial_0^2 + \left(\frac{-53}{16} m a + \frac{3}{16} m a' \right) \gamma^0 \partial_0 \bar{\not{\partial}} + \left(\frac{-49}{16} + \ln \frac{H^2}{4\mu^2} \right) m a \nabla^2 + \left(\frac{9}{2} + 6 \ln \frac{H^2}{4\mu^2} \right) H m a^2 \partial_0 \right. \\
&+ \left. \left(\frac{35}{8} - \frac{11}{8} \ln \frac{y}{4} + 4 \ln \frac{H^2}{4\mu^2} \right) H m a^2 \gamma^0 \bar{\not{\partial}} + \left(\frac{-5}{8} - \frac{3}{8} \ln \frac{y}{4} \right) H m a a' \gamma^0 \bar{\not{\partial}} \right\} \frac{1}{\Delta x^2}. \quad (164)
\end{aligned}$$

The various D -dependent constants in (164) are

$$\beta_1 = \frac{\mu^{D-4}}{16\pi^{\frac{D}{2}}} \frac{\Gamma(\frac{D}{2} - 1)}{(D-3)(D-4)} \left\{ \frac{-2(D-1)}{(D-2)} \right\}, \quad (165)$$

$$\beta_2 = \frac{\mu^{D-4}}{16\pi^{\frac{D}{2}}} \frac{\Gamma(\frac{D}{2} - 1)}{(D-3)(D-4)} \left\{ \frac{-4(D-1)}{(D-2)} + (D-2)(b_2 + b_3) \right\}, \quad (166)$$

$$\beta_3 = \frac{\mu^{D-4}}{16\pi^{\frac{D}{2}}} \frac{\Gamma(\frac{D}{2} - 1)}{(D-3)(D-4)} \left\{ \frac{-(D-1)}{2} + (D-2)(b_{2a} + b_{3a}) + \frac{(D-2)}{2} d_1 \right\}, \quad (167)$$

$$\begin{aligned}
\beta_4 = \frac{\mu^{D-4}}{16\pi^{\frac{D}{2}}} \frac{\Gamma(\frac{D}{2} - 1)}{(D-3)(D-4)} \left\{ \frac{(D-1)}{2} + (D-2)(b_4 - b_2) + \frac{(D-2)(d_2 + d_3 + d_4)}{2} + \frac{D(D-1)}{8(D-3)} \left[-D(D-2) - \frac{1}{4} \right. \right. \\
\left. \left. - \frac{1}{4}(D-4) \right] \right\} + \frac{H^{D-4}}{2^D \pi^{\frac{D}{2}} (D-4)} \left\{ \frac{-\Gamma(D+1)}{2\Gamma(\frac{D}{2})} + \frac{(2D-3)}{4} \Gamma(\frac{D}{2}) \Gamma(\frac{D}{2} - 1) \Gamma(3 - \frac{D}{2}) + \frac{\Gamma(D)}{\Gamma(\frac{D}{2})} \frac{2}{(D-3)} \right\}. \quad (168)
\end{aligned}$$

Here $b_2, b_{2a}, b_3, b_{3a}, b_4, d_1, d_2, d_3$, and d_4 are defined in (85) and (92). In obtaining these expressions we have always chosen to convert finite, $D = 4$ terms with ∂^2 acting on $1/\Delta x^2$, into delta functions,

$$\partial^2 \left[\frac{1}{\Delta x^2} \right] = i4\pi^2 \delta^4(x - x'). \quad (169)$$

All such terms have then been included in those that are proportional to $\delta^4(x - x')$.

The local divergences in this expression can be absorbed by the BPHZ counterterms enumerated at the end of Sec. III. The generic diagram topology is depicted in Fig. 3, and the analytic form is

$$-i[\Sigma^{\text{ctm}}](x; x') = \sum_{I=1}^4 iC_{Iij} \delta^D(x - x'), \quad (170)$$

$$= i\kappa^2 \left\{ \alpha_1 \frac{m}{a} \partial^2 + \alpha_2 m H \partial_0 + \alpha_3 m H \gamma^0 \bar{\not{p}} + \alpha_4 H^2 m a \right\} \delta^D(x - x'). \quad (171)$$

In comparing (164) and (171) it would seem that the simplest choice for the coefficients α_i is



FIG. 3. Contribution from counterterms.

$$\begin{aligned} \alpha_1 &= 3F_1, & \alpha_2 &= \frac{97}{16}F_1, \\ \alpha_3 &= \frac{9}{16}F_1, & \text{and } \alpha_4 &= \frac{85}{8}F_2. \end{aligned} \quad (172)$$

Here

$$F_2 = \frac{\mu^{D-4}}{16\pi^2(D-4)}, \quad F_1 = \frac{\Gamma\left(\frac{D}{2} - 1\right)}{(D-3)} \times F_2. \quad (173)$$

Except for the distinctive contributions from (111), Tables XXXII and XXXIII, the rest of our final result for the renormalized self-energy is

$$\begin{aligned} -i[\Sigma^{\text{ren}}](x; x') &= \frac{i\kappa^2}{16\pi^2} \left\{ \left[3 \ln a + \frac{1}{8} \right] \frac{m}{a} \partial^2 + \left[\frac{97}{16} \ln a - \frac{39}{16} \right] m H \partial_0 + \left[\frac{9}{16} \ln a + \frac{5}{16} \right] m H \gamma^0 \bar{\not{p}} \right. \\ &+ \left[\frac{95}{8} \ln a - \frac{29}{32} - \frac{85}{16} \psi(1) + \frac{5}{8} \ln \frac{H^2}{4\mu^2} \right] H^2 m a \left. \right\} \delta^4(x - x') + \frac{\kappa^2}{64\pi^4} \left\{ \left[\frac{3}{2} \frac{m}{a'} \partial^2 + \left(\frac{7}{8} \frac{a}{a'} - \frac{27}{32} \right) m H \partial_0 \right. \right. \\ &+ \left(\frac{9}{16} \frac{a}{a'} - \frac{9}{32} \right) m H \gamma^0 \bar{\not{p}} + H^2 m \left(\frac{215}{32} a + \frac{9}{32} a' \right) \left. \right\} \partial^2 + H^2 m a [\nabla^2 + 6H a \partial_0 + 4H a \gamma^0 \bar{\not{p}}] \\ &\times \left[\frac{\ln(\mu^2 \Delta x^2)}{\Delta x^2} \right] + \frac{\kappa^2 H^2}{64\pi^4} \left\{ \frac{9}{16} m a' \partial_0^2 + \left[\frac{-53}{16} m a + \frac{3}{16} m a' \right] \gamma^0 \partial_0 \bar{\not{p}} + \left[\frac{-49}{16} + \ln \frac{H^2}{4\mu^2} \right] m a \nabla^2 \right. \\ &+ \left[\frac{9}{2} + 6 \ln \frac{H^2}{4\mu^2} \right] H m a^2 \partial_0 + \left[\frac{35}{8} - \frac{11}{8} \ln \frac{y}{4} + 4 \ln \frac{H^2}{4\mu^2} \right] H m a^2 \gamma^0 \bar{\not{p}} + \left[\frac{-5}{8} - \frac{3}{8} \ln \frac{y}{4} \right] H m a a' \gamma^0 \bar{\not{p}} \left. \right\} \frac{1}{\Delta x^2}. \end{aligned} \quad (174)$$

VIII. DISCUSSION

Dimensional regularization has been used to compute quantum gravitational corrections to the fermion self-energy at one-loop order in a locally de Sitter background. Our regulated result is (164). Although Dirac + Einstein is not perturbatively renormalizable [21] a finite result (174) is obtained by absorbing the divergences with BPHZ counterterms.

At first order in m only four counterterms are necessary for this one-loop one-particle-irreducible function. None of them represents redefinitions of terms in the Lagrangian of Dirac + Einstein. Two de Sitter invariant counterterm operators (52) come from generally coordinate invariant fermion bilinears (40). The other two counterterm operators (53) are from other fermion bilinears (51) that respect the symmetries of our de Sitter noninvariant gauge (28).

BPHZ renormalization does not yield a complete theory because no physical principle fixes the finite part of these

counterterms. Hence our renormalized result could be changed by altering the finite parts of the four BPHZ counterterms. It is simple to be quantitative about this. Were we to make finite shifts $\Delta\alpha_i$ in our counterterms (172) the induced change in the renormalized self-energy would be

$$\begin{aligned} -i[\Delta\Sigma^{\text{ren}}](x; x') &= i\kappa^2 \left\{ \Delta\alpha_1 \frac{m}{a} \partial^2 + \Delta\alpha_2 m H \partial_0 \right. \\ &+ \Delta\alpha_3 m H \gamma^0 \bar{\not{p}} + \Delta\alpha_4 H^2 m a \left. \right\} \delta^4(x - x'). \end{aligned} \quad (175)$$

However, at late times [which access the far infrared because all momenta are redshifted by $a(t) = e^{Ht}$] the local part of the renormalized self-energy (174) is dominated by the large logarithms,

$$\frac{i\kappa^2}{16\pi^2} \left\{ 3 \ln a \frac{m}{a} \partial^2 + \frac{97 \ln a}{16} m H \partial_0 + \frac{9 \ln a}{16} m H \gamma^0 \not{\partial} + \frac{95 \ln a}{8} H^2 m a \right\} \delta^4(x - x'). \quad (176)$$

The coefficients of these logarithms are finite and completely fixed by our calculation. As long as the shifts $\Delta\alpha_i$ are finite, their impact (175) must eventually be dwarfed by the large logarithms (176).

It does not seem too surprising that the leading behavior in the far infrared cannot be disturbed by the nonrenormalizability of quantum gravity. Loops of massless particles make finite, nonanalytic contributions that cannot be changed by local counterterms and that dominate the far infrared. Further, no matter how general relativity is corrected to fix the ultraviolet problem, it cannot involve any new massless particle or else we would have seen a new long range force. In addition, the correction also cannot change how the existing massless particle interacts at low energy, otherwise we would have detected classical violation of general relativity. Therefore these effects must occur as well, “with precisely the same numerical values,” in whatever fundamental theory ultimately resolves the ultraviolet problem of quantum gravity. The concept we have just emphasized is known as the low energy effective field theory and has a very old and distinguished pedigree [26–40].

So we can use (174) reliably in the far infrared. The point of this exercise has been to study the effect of breaking conformal invariance with a small fermion mass. Obtaining (174) completes the first part of our study. What remains is to use our result to solve the quantum-corrected Dirac equation (2). We shall undertake that in a subsequent paper. However, it seems clear that the dominant effect must come from the terms that possess large logarithms in local terms and in (111), Tables XXXII and XXXIII.¹⁴

As predicted in the Introduction, these terms are enhanced by a factor of $\ln(a)$ relative to the classical mass term and $a \ln(a)$ relative to the classical kinetic term. When the classical mass term is much smaller than the Hubble parameter, classical dynamics are dominated by the kinetic term. Therefore a larger enhancement of the fermion field strength $a \ln(a)$ is expected in contrast with the only $\ln(a)$ enhancement that soft virtual gravitons induce on massless fermions [2,3].

Loop corrections from massless, minimally coupled scalars and gravitons during inflation have attracted more and more attention recently. The interesting time-growing effects of infrared logarithms might have a chance to eventually overcome the smallness of the loop counting

parameter of $GH^2 \lesssim 10^{-10}$ and yield significant results. It is not even possible to exclude the possibility that infrared logarithms can contaminate the power spectrum of cosmological density perturbations [51–54]! However, the logarithms would only start to grow at horizon crossing, and must cease growing when the mode reenters the horizon after inflation. Hence the largest enhancement for a currently observable mode would be $\ln(a) \lesssim 100$ that must be set against $GH^2 \lesssim 10^{-10}$. Therefore the proportional correction in these studies are still too small to be detected by current measuring techniques.

The enormously superhorizon modes that have not experienced the second horizon crossing would give more significant corrections. Although they are also down by the constant GH^2 , the time-dependent enhancement factor $\ln(a)$ could be arbitrarily big so that perturbation theory eventually breaks down. One must develop the nonperturbative technique to follow what happens. Starobinsky has advocated gaining quantitative control over this regime by summing the leading infrared logarithms to each order [55]. With Yokoyama he has given a complete solution for the case of a minimally coupled scalar with arbitrary potential that is a spectator to de Sitter inflation [17]. This powerful nonperturbative technique has been successfully generalized to Yukawa theory [18], which showed that the system decays in a big rip singularity, and to scalar quantum electrodynamics (SQED) [19], which confirmed the conjecture by Davis, Dimopoulos, Prokopec, and Tornkvist that superhorizon photons acquire mass during inflation [56].

The asymptotic late time effect is small in the simple scalar models, and in SQED for which the series of leading infrared logarithms has been summed. However, the same kind of effect from Yukawa is huge. Therefore, it is by no means clear what might be the outcome for more complicated theories¹⁵ that also show infrared logarithms such as quantum gravity [57–59]. Another application of our result (174) is to serve as *data* in checking the validity of the new, more general rule [60,61] for reproducing the leading logarithms of massive Dirac + Einstein theory. This might serve as an important intermediate point in the difficult task of generalizing Starobinsky’s techniques to full blown quantum gravity.

It is fitting to close with a comment on whether or not the infrared logarithm that appears in (174) is a gauge artifact. One obvious way of checking this is to redo the computation in a different gauge. Recently two graviton propagators have been constructed, respectively, by imposing the exact de Donder gauge [47] and a general one parameter gauge [62]. Both gauges respect de Sitter invariance but the same de Sitter breaking factor $\ln(a)$ shows up even in the transverse-traceless sector of the propagator [62,63].

¹⁴Without an explicit calculation we cannot determine whether or not the $\ln(\frac{y}{4})$ in the nonlocal terms will produce infrared enhancements because the same term occurring in Ref. [50] fails to give them.

¹⁵By which we mean theories that possess derivative interactions that cannot be avoided by imposing a special gauge as was done in SQED.

Hence the infrared logarithm must be universal because the spin two part of the graviton propagator cannot be altered by changing gauges. We therefore conjecture that the leading infrared logarithm at the one-loop order in quantum gravity might be gauge independent.

ACKNOWLEDGMENTS

The author would like to thank R. P. Woodard for useful discussions. This work was supported by NWO

Veni Project No. 680-47-406, by the Institute for Theoretical Physics at the University of Utrecht, and by the Department of Physics at the University of Crete.

APPENDIX: THE REDUCED FERMION PROPAGATOR AND ITS RELATED IDENTITIES

Here we list some identities we have used for various gamma functions contracted with the first derivative of the $n = 0$ part of the fermion propagator (89),

$$\not{\partial} i[S]_{n=0} = \Gamma\left(\frac{D}{2} - 1\right) \left\{ \frac{mHaa'}{4\pi^{\frac{D}{2}}} \left[\frac{-\gamma^0}{\Delta x^{D-2}} - \frac{Ha\gamma^0 \Delta \eta}{\Delta x^{D-2}} - \frac{H^2 a^2 \gamma^0}{(D-4)\Delta x^{D-4}} \right] - \frac{mH^{D-3}}{(4\pi)^{\frac{D}{2}}} (aa')^{\frac{D}{2}-1} \Gamma\left(\frac{D}{2} + 1\right) \Gamma\left(2 - \frac{D}{2}\right) H^2 a^2 \gamma^0 \right\}, \quad (A1)$$

$$\partial_\mu i[S] \gamma^\mu = \Gamma\left(\frac{D}{2} - 1\right) \left\{ \frac{mHaa'}{4\pi^{\frac{D}{2}}} \left[\frac{(D-2)\Delta \eta \gamma^\mu \Delta x_\mu}{\Delta x^D} + \frac{Ha\gamma^\mu \Delta x_\mu}{\Delta x^{D-2}} - \frac{H^2 a^2 \gamma^0}{(D-4)\Delta x^{D-4}} \right] - \frac{mH^{D-3}}{(4\pi)^{\frac{D}{2}}} (aa')^{\frac{D}{2}-1} \Gamma\left(\frac{D}{2} + 1\right) \Gamma\left(2 - \frac{D}{2}\right) H^2 a^2 \gamma^0 \right\}. \quad (A2)$$

To facilitate the calculation from the infinite series expansion of the fermion propagator for $D = 4$, we might employ the following identities:

$$\not{\partial} i[S] = \frac{mHaa'}{16\pi^2} \sum_{n=0}^{\infty} \left(\frac{y}{4}\right)^n \left\{ \left[\frac{-4\gamma^0}{\Delta x^2} - \frac{4Ha\gamma^0 \Delta \eta}{\Delta x^2} \right] \left[(2n+1) + (n^2+n) \ln \frac{y}{4} \right] + H^2 a^2 \delta_\mu^0 \left[(2n+3) + (n^2+3n+2) \ln \frac{y}{4} \right] \right\}, \quad (A3)$$

$$\partial_\mu i[S] \gamma^\mu = \frac{mHaa'}{16\pi^2} \sum_{n=0}^{\infty} \left(\frac{y}{4}\right)^n \left\{ \frac{4\gamma^0}{\Delta x^2} \left[2n + n^2 \ln \frac{y}{4} \right] - \frac{8\Delta \eta \gamma^\mu \Delta x_\mu}{\Delta x^4} \left[(2n-1) + (n^2-n) \ln \frac{y}{4} \right] + \frac{4Ha\gamma^\mu \Delta x_\mu}{\Delta x^2} \left[(2n+1) + (n^2+n) \ln \frac{y}{4} \right] + H^2 a^2 \delta_\mu^0 \left[(2n+3) + (n^2+3n+2) \ln \frac{y}{4} \right] \right\}, \quad (A4)$$

$$\bar{\not{\partial}} i[S] = \frac{mHaa'}{16\pi^2} \sum_{n=0}^{\infty} \left(\frac{y}{4}\right)^n \left\{ -4 \left[\frac{\Delta \eta \gamma^k \Delta x_k}{\Delta x^4} + \frac{\gamma^0 \bar{\Delta} x^2}{\Delta x^4} \right] \left[(2n-1) + (n^2-n) \ln \frac{y}{4} \right] - \frac{6\gamma^0}{\Delta x^2} \left[1 + n \ln \frac{y}{4} \right] + \frac{2Ha\gamma^k \Delta x_k}{\Delta x^2} \left[(2n+1) + (n^2+n) \ln \frac{y}{4} \right] \right\}, \quad (A5)$$

$$\partial_k i[S] \gamma^k = \frac{mHaa'}{16\pi^2} \sum_{n=0}^{\infty} \left(\frac{y}{4}\right)^n \left\{ 4 \left[\frac{-\Delta \eta \gamma^k \Delta x_k}{\Delta x^4} + \frac{\gamma^0 \bar{\Delta} x^2}{\Delta x^4} \right] \left[(2n-1) + (n^2-n) \ln \frac{y}{4} \right] + \frac{6\gamma^0}{\Delta x^2} \left[1 + n \ln \frac{y}{4} \right] + \frac{2Ha\gamma^k \Delta x_k}{\Delta x^2} \left[(2n+1) + (n^2+n) \ln \frac{y}{4} \right] \right\}. \quad (A6)$$

The formulas keep the same form for the summation starting from $n = 1$ as long as we are working in $D = 4$.

-
- [1] T. Prokopec and R. P. Woodard, *J. High Energy Phys.* **10** (2003) 059.
[2] S. P. Miao and R. P. Woodard, *Classical Quantum Gravity* **23**, 1721 (2006).

- [3] S. P. Miao and R. P. Woodard, *Phys. Rev. D* **74**, 024021 (2006).
[4] V. K. Onemli and R. P. Woodard, *Classical Quantum Gravity* **19**, 4607 (2002).

- [5] V. K. Onemli and R. P. Woodard, *Phys. Rev. D* **70**, 107301 (2004).
- [6] T. Brunier, V. K. Onemli, and R. P. Woodard, *Classical Quantum Gravity* **22**, 59 (2005).
- [7] E. O. Kahya and V. K. Onemli, *Phys. Rev. D* **76**, 043512 (2007).
- [8] T. Prokopec, N. C. Tsamis, and R. P. Woodard, *Classical Quantum Gravity* **24**, 201 (2007).
- [9] T. Prokopec, N. C. Tsamis, and R. P. Woodard, *Phys. Rev. D* **78**, 043523 (2008).
- [10] H. Kitamoto and Y. Kitazawa, *Phys. Rev. D* **83**, 104043 (2011).
- [11] H. Kitamoto and Y. Kitazawa, *Phys. Rev. D* **85**, 044062 (2012).
- [12] S. Park and R. P. Woodard, *Phys. Rev. D* **83**, 084049 (2011).
- [13] S. Park and R. P. Woodard, *Phys. Rev. D* **84**, 124058 (2011).
- [14] E. O. Kahya and R. P. Woodard, *Phys. Rev. D* **76**, 124005 (2007).
- [15] H. Kitamoto and Y. Kitazawa, [arXiv:1204.2876](https://arxiv.org/abs/1204.2876).
- [16] H. Kitamoto and Y. Kitazawa, [arXiv:1203.0391](https://arxiv.org/abs/1203.0391).
- [17] A. A. Starobinsky and J. Yokoyama, *Phys. Rev. D* **50**, 6357 (1994).
- [18] S. P. Miao and R. P. Woodard, *Phys. Rev. D* **74**, 044019 (2006).
- [19] T. Prokopec, N. C. Tsamis, and R. P. Woodard, *Ann. Phys. (N.Y.)* **323**, 1324 (2008).
- [20] R. P. Woodard, *Nucl. Phys. B, Proc. Suppl.* **148**, 108 (2005).
- [21] S. Deser and P. van Nieuwenhuizen, *Phys. Rev. D* **10**, 411 (1974).
- [22] N. N. Bogoliubov and O. Parasiuk, *Acta Math.* **97**, 227 (1957).
- [23] K. Hepp, *Commun. Math. Phys.* **2**, 301 (1966).
- [24] W. Zimmermann, *Commun. Math. Phys.* **11**, 1 (1968); **15**, 208 (1969).
- [25] W. Zimmermann, in *Lectures on Elementary Particles and Quantum Field Theory*, edited by S. Deser, M. Grisaru, and H. Pendleton (MIT Press, Cambridge, MA, 1971), Vol. I.
- [26] F. Bloch and H. Nordsieck, *Phys. Rev.* **52**, 54 (1937).
- [27] S. Weinberg, *Phys. Rev. B* **140**, B516 (1965).
- [28] G. Feinberg and J. Sucher, *Phys. Rev.* **166**, 1638 (1968).
- [29] S. D. H. Hsu and P. Sikivie, *Phys. Rev. D* **49**, 4951 (1994).
- [30] D. M. Capper, M. J. Duff, and L. Halperin, *Phys. Rev. D* **10**, 461 (1974).
- [31] D. M. Capper and M. J. Duff, *Nucl. Phys.* **B82**, 147 (1974).
- [32] D. M. Capper, *Nuovo Cimento A* **25**, 29 (1975).
- [33] M. J. Duff and J. T. Liu, *Phys. Rev. Lett.* **85**, 2052 (2000).
- [34] J. F. Donoghue, *Phys. Rev. Lett.* **72**, 2996 (1994).
- [35] J. F. Donoghue, *Phys. Rev. D* **50**, 3874 (1994).
- [36] I. J. Muzinich and S. Kokos, *Phys. Rev. D* **52**, 3472 (1995).
- [37] H. Hamber and S. Liu, *Phys. Lett. B* **357**, 51 (1995).
- [38] A. Akhundov, S. Belucci, and A. Shiekh, *Phys. Lett. B* **395**, 16 (1997).
- [39] I. B. Kriplovich and G. G. Kirilin, *J. Exp. Theor. Phys.* **98**, 1063 (2004).
- [40] I. B. Kriplovich and G. G. Kirilin, *J. Exp. Theor. Phys.* **95**, 981 (2002).
- [41] R. P. Woodard, *Phys. Lett. B* **148**, 440 (1984).
- [42] P. Candelas and D. J. Raine, *Phys. Rev. D* **12**, 965 (1975).
- [43] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series and Products* (Academic Press, New York, 1965), 4th ed.
- [44] N. C. Tsamis and R. P. Woodard, *Commun. Math. Phys.* **162**, 217 (1994).
- [45] S. P. Miao, N. C. Tsamis, and R. P. Woodard, *J. Math. Phys. (N.Y.)* **50**, 122502 (2009).
- [46] S. P. Miao, N. C. Tsamis, and R. P. Woodard, *J. Math. Phys. (N.Y.)* **51**, 072503 (2010).
- [47] S. P. Miao, N. C. Tsamis, and R. P. Woodard, *J. Math. Phys. (N.Y.)* **52**, 122301 (2011).
- [48] S. P. Miao, N. C. Tsamis, and R. P. Woodard, *Classical Quantum Gravity* **28**, 245013 (2011).
- [49] N. D. Birrell and P. C. W. Davies, *Quantum Fields in Curved Space* (Cambridge University Press, Cambridge, England, 1982).
- [50] L. D. Duffy and R. P. Woodard, *Phys. Rev. D* **72**, 024023 (2005).
- [51] S. Weinberg, *Phys. Rev. D* **74**, 023508 (2006).
- [52] D. Boyanovsky, H. J. de Vega, and N. G. Sanchez, *Nucl. Phys. B* **747**, 25 (2006); *Phys. Rev. D* **72**, 103006 (2005); M. Sloth, *Nucl. Phys. B* **748**, 149 (2006); **775**, 78 (2007); A. Bilandžić and T. Prokopec, *Phys. Rev. D* **76**, 103507 (2007); M. van der Meulen and J. Smit, *J. Cosmol. Astropart. Phys.* **11** (2007) 023; D. H. Lyth, *J. Cosmol. Astropart. Phys.* **12** (2007) 016; D. Seery, *J. Cosmol. Astropart. Phys.* **11** (2007) 025; **02** (2008) 006; **05** (2009) 021; N. Bartolo, S. Matarrese, M. Pietroni, A. Riotto, and D. Seery, *J. Cosmol. Astropart. Phys.* **01** (2008) 015; Y. Urakawa and K. I. Maeda, *Phys. Rev. D* **78**, 064004 (2008); A. Riotto and M. Sloth, *J. Cosmol. Astropart. Phys.* **04** (2008) 030; K. Enqvist, S. Nurmi, D. Podolsky, and G. I. Rigopoulos, *J. Cosmol. Astropart. Phys.* **04** (2008) 025; P. Adshead, R. Easther, and E. A. Lim, *Phys. Rev. D* **79**, 063504 (2009); J. Kumar, L. Leblond, and A. Rajaraman, *J. Cosmol. Astropart. Phys.* **04** (2010) 024; C. P. Burgess, R. Holman, L. Leblond, and S. Shandera, *J. Cosmol. Astropart. Phys.* **03** (2010) 033; L. Senatore and M. Zaldarriaga, *J. High Energy Phys.* **12** (2010) 008; [arXiv:1203.6354](https://arxiv.org/abs/1203.6354); *J. Cosmol. Astropart. Phys.* **08** (2012) 001; D. Seery, *Classical Quantum Gravity* **27**, 124005 (2010); S. B. Giddings and M. S. Sloth, *J. Cosmol. Astropart. Phys.* **01** (2011) 023; **07** (2010) 015; *Phys. Rev. D* **84**, 063528 (2011); *Phys. Rev. D* **86**, 083538 (2012); G. L. Pimental, L. Senatore, and M. Zaldarriaga, *J. High Energy Phys.* **07** (2012) 166; K. Feng, Y.-F. Cai and Y.-S. Piao, [arXiv:1207.4405](https://arxiv.org/abs/1207.4405).
- [53] S. P. Miao and R. P. Woodard, *J. Cosmol. Astropart. Phys.* **07** (2012) 008.
- [54] E. O. Kahya, V. K. Onemli, and R. P. Woodard, *Phys. Lett. B* **694**, 101 (2010).
- [55] A. A. Starobinskiĭ, in *Field Theory, Quantum Gravity and Strings*, edited by H. J. de Vega and N. Sanchez (Springer-Verlag, Berlin, 1986), pp. 107–126.
- [56] A. C. Davis, K. Dimopoulos, T. Prokopec, and O. Törnkvist, *Phys. Lett. B* **501**, 165 (2001); K. Dimopoulos, T. Prokopec, O. Törnkvist, and A. C. Davis, *Phys. Rev. D* **65**, 063505 (2002).
- [57] N. C. Tsamis and R. P. Woodard, *Nucl. Phys. B* **474**, 235 (1996).

- [58] N. C. Tsamis and R. P. Woodard, [Ann. Phys. \(N.Y.\)](#) **253**, 1 (1997).
- [59] N. C. Tsamis and R. P. Woodard, [Phys. Rev. D](#) **54**, 2621 (1996).
- [60] N. C. Tsamis and R. P. Woodard, [Nucl. Phys.](#) **B724**, 295 (2005).
- [61] S. P. Miao and R. P. Woodard, [Classical Quantum Gravity](#) **25**, 145009 (2008).
- [62] P. J. Mora, N. C. Tsamis, and R. P. Woodard, [arXiv:1205.4468](#).
- [63] E. O. Kahya, S. P. Miao, and R. P. Woodard, [J. Math. Phys. \(N.Y.\)](#) **53**, 022304 (2012).