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MATHEMATICAL KNOWLEDGE FOR TEACHING IN THE NETHERLANDS

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The implementation of the new mathematical knowledge base in Dutch teacher education institutes for primary education raises a need for curriculum development. Teacher educators have to raise student teachers' subject matter knowledge to a higher level. In working on this aim teacher educators experience that student teachers often feel uncertain about their mathematical skills and are not very interested in formal and abstract mathematics. Student teachers prefer to focus on mathematical pedagogical content knowledge.
This paper presents two design studies that try to tackle this problem. The first one targets the development of student teachers' specialized content knowledge (SCK) and the second one focuses on their horizon content knowledge (HCK). Both studies target developing student teachers' mathematical subject matter knowledge in the perspective of teaching mathematics in primary school. In the studies we established student teachers' learning environments that kept them involved and motivated, even when they found the mathematics hard to do. Primarily, this attitude supported their mathematical growth, while it also developed their pedagogical skills and insight.

INTRODUCTION

Recently, teacher educators in the Netherlands have become involved in implementing the so-called 'Knowledge base for primary teacher education' (Van Zanten et al., 2009). This knowledge base prescribes the mathematical knowledge student teachers should possess before graduation. A considerable part of the knowledge base will be assessed by a nationwide test, to be completed in the last stage of their study by all students who started their teacher education in 2011 or later. This situation leads to major changes in mathematics curricula in Dutch teacher education institutes for primary education, because the knowledge base differs significantly from what was customary in pre-2011 programs. The most important difference is that the new program focuses more on mathematical subject matter knowledge.

The Expertise Centre for the Teacher Education Institutes in the Mathematical Domain (ELWIeR) supports the implementation of the knowledge base in Dutch teacher education. The ELWIeR research program is involved with developing prototypes of activities in teacher education. Design research is the research approach used to construct and evaluate the prototypes (cf. Van den Akker et al., 2006). In this paper we describe two of the pilot studies that are conducted within this research program. The first study focuses on specialized content knowledge (SCK): ‘Developing SCK by reflecting on problem approaches’, the second one aims at developing horizon content knowledge (HCK): ‘Motivating primary student teachers for mathematics’.

THEORETICAL BACKGROUND

Before Shulman’s (1986; 1987) introduction of the notion of pedagogical content knowledge, teacher knowledge was more or less considered to be subject matter knowledge. In introducing this notion, Shulman opened the discussion on the nature of teacher knowledge – especially for mathematics – and in its slipstream a discussion on the relation between teacher knowledge and student achievement in mathematics (Adler et al., 2005; cf. Manizad & Mason, 2010). Analyses of teacher mathematical knowledge and student mathematical achievement led to the conclusion that – at least in the US – knowledge of ‘pure’, higher or advanced mathematics was often inadequate to effectively help students to gain proficiency in mathematics (Ball, et al., 2005). However, mathematical knowledge for teaching does lead to higher student achievements (Hill et al., 2005; Hill et al., 2008).

This paper is about developing mathematical knowledge for teaching. We use the following scheme as a model (figure 1).

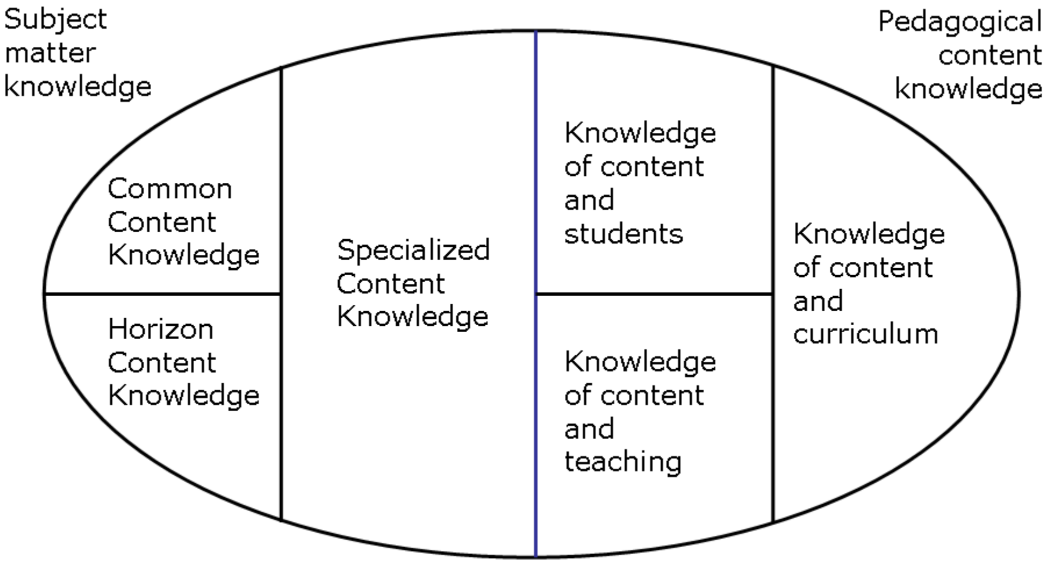


Figure 1. Scheme for ‘Mathematical Knowledge for Teaching’ (Ball et al., 2008, p. 403)

We focus on the left side of the scheme, teachers’ subject matter knowledge and choose developing ‘specialized content knowledge’ (SCK) and ‘horizon content knowledge’ (HCK).

‘(…) specialized content knowledge is the mathematical knowledge that is unique to the teacher.’ (Ball et al., p. 400). ‘Horizon content knowledge is an awareness of how mathematical topics are related over the span of mathematics included in the curriculum’ (Ball et al., 2008, p. 403); it is the mathematics that goes beyond what many teachers consider primary school mathematics. We took these descriptions as starting point for our design activities. The specific Dutch context, where pedagogical content knowledge receives more attention in primary teacher education than subject matter knowledge made that we had to create our own (Dutch) interpretation for these aspects of teacher knowledge (cf. Delaney et al., 2008). We will show this in two studies.

DEVELOPING SCK BY REFLECTING ON PROBLEM APPROACHES

Scope

In the Netherlands realistic mathematics education (RME) plays a part in nearly every primary school (see for example Treffers (1993)). That means for instance that textbooks contain open problems that students can approach in their own way and on their own level. As a consequence, in classroom discourses teachers have to deal with multiple problem approaches by their students, sometimes unconventional ones. They need specialized content knowledge to do this. This SCK is to some degree imbedded in the knowledge base, namely as follows. A teacher must be able:

- to analyze, judge, paraphrase and evaluate mathematical problem approaches
- to explain and clarify problem approaches
- to visualize, and produce written and verbal descriptions of problem approaches
- to produce problem approaches in different ways and on different levels of abstraction.

Student teachers find it hard to deal with children’s problem approaches at their practical schools. Most of the time they themselves can solve the textbook problem in only one way, and find it difficult to interpret and understand problem approaches that differ from their own, especially in the strenuous situation of classroom discourses where immediate teaching is required (Dolk, 1997). Most of the time common mathematical knowledge is not enough to deal with sometimes unconventional problem approaches.

Research question

The need for the development of more SCK in primary student teachers leads to the following research question:

How can student teachers be supported in developing their SCK as described in the knowledge base?

Method

We worked on this question in the context of seminars at the teacher education institute and investigated the effects of materials and the role of other student teachers and the teacher educator in developing SCK. We gave student teachers the opportunity to reflect in an elaborate way on children’s intriguing, various and mathematically challenging problem approaches, both correct and incorrect ones. To support them to do this profoundly and systematically we constructed a reflection format that consists of three steps. First the student teachers must analyze and understand the child’s problem approach, judge whether it was correct, and give a clear representation. Secondly they are challenged to explain or prove why and in which conditions this problem approach will work. And finally they are asked to evaluate if this problem approach is clever, clear and safe, and explain their opinion.

The problem approaches used were from grade 6 students, who worked on problems from their textbook. Each problem that was shown was accompanied by four problem approaches.

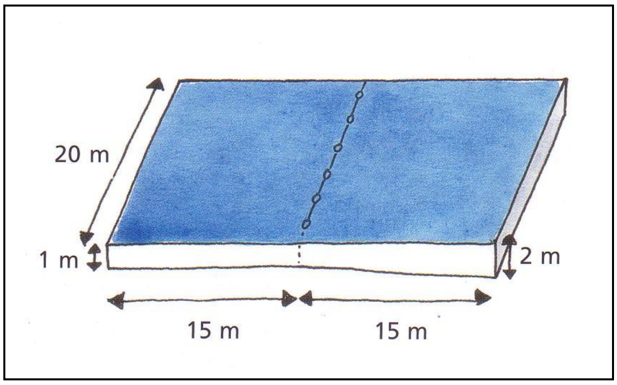
First the student teachers reflected individually on the problem approaches using the reflection format. After some time the teacher educator started a plenary discourse on the children's work. Teacher educators were convinced that this let student teachers construct specialized content knowledge, as they were actively engaged in interpreting mathematics from the child's viewpoint. At the end of each session the student teachers and their educator discussed and evaluated the effects of working on SCK in this way and with these materials. Finally, at the end of the experiment we interviewed each teacher educator and several student teachers. One of the questions in the interview was: 'Do you find this an effective way to develop SCK? Explain your answer.'

We chose this design because reflecting on problem approaches during seminars is less stressful than doing so in practical school. In this way student teachers had time for in-depth reflection. Moreover, student teachers and their teacher educator had the opportunity to challenge each other during classroom discourses and discuss the mathematics in the student approaches more deeply.

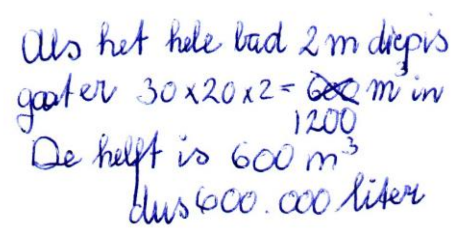
Five groups of student teachers (two first year and two second year fulltime groups and one group with third year part-time students, about one hundred in total), from four different institutes were involved in the experiment. One group had more time available and reflected on all six problems, others only did one or two problems.

Prototypical activity

Swimming pool: How many litres does this swimming pool contain if it is filled to the edge?



A child's problem approach (grade 6)



Translation:

If the whole swimming pool is 2 m deep everywhere, it will contain

30 x 20 x 2 = 1200 m³

Half of it is 600 m³

So 600.000 litres.

Figure 2. Problem and student approach

To give an impression of how student teachers reflected on the given materials we start by showing one of the textbook problems with a grade 6 student problem approach (figure 2).

We noticed that student teachers found it hard to use the reflection format to reflect on students' problem approaches. However, working together in a plenary discourse was helpful. They constructed and discussed several interpretations and gave each other new ideas. For instance in the 'swimming pool example' they realized that the problem approach in figure 2 was wrong. One of the student teachers said: 'It was not correct to take half of 1200 m^3, because then you will have a swimming pool that has a depth of 1 meter everywhere.'

Another student teacher said: 'Or the depth was 0 m on one side and 2 m on the other side. Then you cut the pool diagonally.'

He illustrated both possibilities on the whiteboard (figure 3).

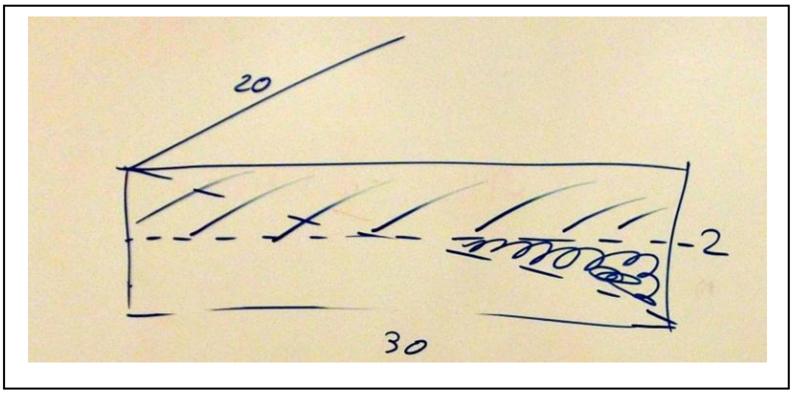


Figure 3. Explanation

The teacher educator asked: 'Which part of the swimming pool is missing if you solve the problem in this way?' The student teacher hatched the missing part with curly lines and recognized that this was 1/8. Another student teacher discovered that you could also solve the problem by calculating 5/8 x 1200. Another one noticed: 'Or subtracting 3/8 x 1200 from 1200.' This shows that children's work inspired the student teachers to construct new ideas and problem approaches. This happened several times and we also saw that student teachers spontaneously asked themselves mathematical questions. It took a lot of time, sometimes twenty minutes to discuss just one problem approach, but the output was rich and valuable.

Results

Reflecting on children's problem approaches in a profound and systematical way with the three steps format is difficult and time-consuming, but both student teachers and educators found it worthwhile. Student teachers were motivated because they were convinced that they have to learn this for their future profession. Teaching mathematics requires more than just checking the students' final answers. Teacher try to understand their students' problem approaches, because that allows them to give better support, and if they try to follow and reflect on students' thoughts and ideas, this can provide information about the students' development. Of course, we realize that in every day classroom management the teacher can invite students to clarify their problem approaches. This wasn't possible during our experiment at the teacher education institute, as we only had the written work of children; we are sure though that reflecting on verbal explanations of children also requires adequate SCK.

Sometimes the student teachers were impressed by a student's ingenious problem approaches. They thus discovered new ideas about solving mathematical problems themselves.

Student teachers and their educators appreciated the reflection format; the three steps in the format supported a profound reflection. The role of the educator during the plenary discourse was also of great importance. Student teachers especially needed support in explaining why a certain approach was correct. Schemes and pictures were helpful. The third year student teachers involved in this research had more skills to make visual representations than those in their first year. The teacher educator's input was also necessary in producing correct and complete notations. He demanded discipline where student teachers were too easy-going. The teacher educator also gave some student teachers more self-confidence; many of them felt extremely uncertain in reflecting on problem approaches used by children. And the role of the teacher educator was crucial during the evaluation of the several approaches. Student teachers are inclined to appreciate their own problem approach most, but during the plenary discourse analyzing, explaining, evaluating and comparing several problem approaches led to more flexibility and a better overview of the approaches. The discourse with other student teachers helped to give the reflection more depth and construct new mathematical ideas. Both the children's problem approaches and the input of other student teachers gave the student teachers' reasoning a boost. They often realized that teachers need to do more than solving a problem in only one way and on one level.

At the end of the experiment all the student teachers and educators were convinced that reflecting on children's problem approaches can support the development of SCK. All aspects mentioned in the knowledge base played a part in the reflections. This was even more so in situations where student teachers started their reflection by themselves solving the problem in different ways and on different levels. On the other hand, we noticed that they are only able to do this if they already possess some SCK.

Teacher educators asked for more material because they realized that one has to practice this approach often, especially when student teachers have to become proficient in interpreting student approaches. Several teacher educators had plans to ask their student teachers to collect problem approaches in their practical schools to use this for reflection in the teacher education institute.

Two experiences in this experiment were less successful:

- One teacher educator did not realize that SCK was a kind of mathematical knowledge. She put much attention on pedagogical knowledge during the plenary discourse on the problem approaches, and as a consequence noticed that her student teachers gained hardly any mathematical knowledge.
- Some student teachers were not able to solve the problems from the textbook. Their common mathematical knowledge was extremely weak. Looking at children's problem approaches was frustrating and confusing for them. Student teachers need to have a basic level of common mathematical knowledge before they can reflect effectively on alternative problem approaches and develop SCK.

In spite of these experiences educators and student teachers were enthusiastic about this method. They are convinced that the method is suitable to develop SCK. They even thought that in-service teachers could learn a lot from reflecting on their students' work in this way.

In the next paragraph we will present a second study – on the development of HCK.

MOTIVATING PRIMARY STUDENT TEACHERS FOR MATHEMATICS

Scope

The knowledge base for primary teachers includes relatively high level mathematics that has no direct link to mathematics in primary school. This is in a sense what Ball et al. describe as ‘horizon content knowledge’ (HCK). As primary student teachers may not see the required mathematical skills as helpful for their teaching, this might lead to student teachers developing two separate ideas on mathematics: one aimed at their teaching and one aimed at scoring for the knowledge base test. Moreover, student teachers might become reluctant to learn the mathematics for the test.

We learned from discussions with teacher educators that these problems are more than hypothetical. Several educators struggled with students that were not motivated to invest in their mathematical knowledge.

Research question

The question that emerged was a developmental one. We tried to find out what approach in primary teacher education would address the problems we observed in connection with the obligation for primary student teachers to develop specific mathematical skills. In other words, our research question is:

How can primary student teachers be supported to develop HCK as required by the knowledge base?

This research question embeds two foci. Student teachers should be motivated to work on their HCK. Moreover, they should be supported to experience this knowledge as useful for their development as a future teacher. The developmental question is approached here from both these perspectives.

Method

We used a design research approach to address this issue, and analyzed the situation as stated above. Next we developed a hypothetic learning trajectory that might help to address the observed problems. To do so, we elaborated ideas and problems from teaching in primary education to student teacher activities. Dutch textbooks for mathematics in primary education are more or less based on realistic mathematics education. As we have seen in the previous paragraph, this means that open problems facilitate students to choose their own approach and work on their own level. We therefore developed open problems. These problems were on the mathematical level of the knowledge base, but had equivalents on a lower level in primary school mathematical education. Three groups of about 23 student teachers in their second year in college worked on these problems that left room for different approaches on several levels. Moreover, this approach meant that we found a means to discuss general ideas about working with challenging open problems in primary education with the student teachers, and thus enriched their ideas about teaching mathematics in primary education. We tried the prototype and reflected on the results of this try-out.

Prototypical activity

Fraction knowledge is one of the subjects in the knowledge base. For example, student teachers are required to show proficiency in relating fractions and decimals, just as children in primary schools have to do at a lower level. However, there is a difference. Student teachers had to learn this in a general manner, especially with repeating decimals (for instance 0.133333…). We wanted student teachers to experience discovery learning and therefore we did not provide them with algorithms to do these transitions. Instead we presented the following open problem:

Do you know a way to find out what fraction corresponds to 0.13333…?

We expected that student teachers would partly recognize this decimal fraction, providing them with a clue to start solving the problem. What we saw is that initially the student teachers had no idea how to solve the problem; therefore we suggested that they name a repeating decimal resembling the one that was shown; one that they could solve. After a moment, one of the student teachers suggested 1.3333… .

Student teacher: 'That is one and a third or four thirds.'

The teacher educator wrote on the whiteboard: 1.3333… = 4/3.

Another student teacher: 'So for 0.13333… you should divide by 10. That is four thirtiest.'

T. wrote on the whiteboard: 0.13333… = 4/30 = 2/15.

One of the other student teachers then said she found yet another way of solving the problem: 'I did a third minus a fifth. A third is 0.333333… and a fifth is 0.2. When you subtract you get 0.13333…'

T. wrote this approach on the whiteboard, while the student teacher told how to make the fraction calculations: 1/3 – 1/5 = 5/15 – 3/15 = 2/15.

The teacher educator further challenged the student teachers to try another one. Then yet another student teacher came forward with her approach: 'I know that 0.11111… is 1/9 as 0.33333… = 1/3. Then I know that 0.011111… is 1/90 and 0,022222… is 2/90 or 1/45. So I add 1/9 and 1/45. That is 5/45 + 1/45 = 6/45 = 2/15.'

As expected, this challenged student teachers to work on various approaches for several other repeating decimals we presented them. Even student teachers who explicitly stated they did not like mathematics were somewhat enthusiastic to solve the problems presented. Two of the better performing student teachers to some extent reconstructed an algorithm, when they were working on finding a fraction corresponding to the decimal 0.285714285714285714… .

Results

Students were surprisingly willing to work on the challenging problems we developed for them. They worked in groups, with students helping each other to grasp the problems, and with possible approaches to solve them, while the teacher educator stimulated heuristic approaches, such as using number references. We observed that the problems' open character made that student teachers could make a start in solving them. They experienced mathematics as problem solving and were able to tell how they could translate this approach to their teaching in primary education. Several students actively redeveloped the problems to enable them to discuss the problems with their students in their own teaching practice. In short: we experienced that key features of RME were also supported in developing teacher education; especially when aimed at supporting students acquiring the knowledge base.

CONCLUSION AND REFLECTION

The implementation of the mathematical knowledge base in teacher education institutes for primary education needs curriculum development because of the challenging level and nature of the required mathematics. Two design studies in Dutch teacher education institutes show that student teachers can be motivated to solve difficult mathematical problems if they experience a connection between their mathematical activities and their future profession; for instance, by letting them puzzle out children's complicated problem approaches or letting them work on open problems, leaving room for different approaches on several levels. Both approaches need time for discourse between student teachers and their educator, but it turned out that both approaches changed the attitude of student teachers towards a more self-confident and motivated one. Although there still remains much work to do, these results are promising, because such an attitude is an excellent foundation for student teachers to develop their mathematical subject matter knowledge.

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