IOP PUBLISHING EUROPEAN JOURNAL OF PHYSICS

Eur. J. Phys. **28** (2007) 415–427 doi:10.1088/0143-0807/28/3/004

Teaching the relation between solar cell efficiency and annual energy yield

Wilfried G J H M van Sark

Department of Science, Technology and Society, Copernicus Institute for Sustainable Development and Innovation, Utrecht University, Heidelberglaan 2, 3584 CS Utrecht, The Netherlands

Received 15 December 2006, in final form 30 January 2007
Published 12 March 2007
Online at stacks.iop.org/EJP/28/415

Abstract

To reach a sustainable world the use of renewable energy sources is imperative. Photovoltaics (PV) is but one of the technologies that use the power of the sun and its deployment is growing very fast. Several master programs have been developed over the world, including Utrecht University, that teach these technologies. Within the framework of a course on energy conversion technologies, we have developed a classroom problem that focuses on the difference between PV efficiency and annual yield for the two locations: the Utrecht University campus and the African Sahara desert. In spreadsheet format, students calculate annual yield, and they find a best method to do so. The exercise can be done in about three hours, and students will learn that the annual yield in the Sahara is only twice that at Utrecht University,

1. Introduction

Reducing the amount of greenhouse gas emissions by 2008–2012 is now official policy in many countries since the Kyoto protocol came into force on 16 February 2005. However, the environmental effects of fossil fuel consumption is but one of the challenges that face us in the coming decades. The most important other issues are oil scarcity, energy security, and the immensely growing energy needs of the developing world [1]. This poses an enormous challenge to mankind in realizing a transition to a fully sustainable energy and materials system. Such a system involves the implementation of many technologies that exploit renewable energy sources [2], e.g., biomass, wind and solar. Some of these conversion technologies, such as photovoltaic (PV) solar energy conversion, are barely out of their research phase and are too expensive to be implemented without the use of financial support schemes. Training young people is a prerequisite to creating a large pool of well-educated people who can generate and apply new knowledge that is needed to realize a sustainable world.

At Utrecht University (UU), The Netherlands, as in many other countries [3, 4], new masters programmes have been developed that educate bachelors to a master (MSc) of sustainable development or a master of energy science. It is our aim to educate natural

0143-0807/07/030415+13$30.00

scientists in such a way that they are well prepared to contribute significantly to the transition towards sustainable energy and material systems, by doing applied research, consultancy work and/or policy advice. The programmes consist of several courses with different focus points covering research methods and policy schemes. Obviously, a course on energy conversion technologies (ECT) is obligatory; energy conversion principles ranging from gas turbine thermodynamics to PV conversion technology are treated. In all courses, 'learning-by-doing-it-yourself' activates the students to acquire the knowledge needed to solve energy technology and related policy problems. In the ECT course this is achieved by doing several problem-solving classes, with topics such as the working principle of a refrigerator, a gas turbine, nuclear fission and PV. In addition, students must write papers on pre-selected topics in energy conversion.

Since the Pearson paper of 1957 [5] on the conversion of solar to electrical energy a few papers have been published on photovoltaics education including various experimental ones, e.g. [6–9]. In addition to using the computer for current–voltage characteristics analysis [10, 11], modelling of p–n junction devices by means of computer simulation has been presented [12]. Some types of thin film solar cells can be made by students themselves using low technology fabrication, e.g., solar cells based on cadmium sulfide and cadmium telluride [13] and the easy-to-fabricate dye sensitized solar cell [14], which has found its way into the undergraduate chemistry curriculum at Utrecht University. Replacing some ingredients by spinach and toothpaste is found to stimulate students' imagination even more [15].

This paper highlights one classroom problem, which addresses PV performance or, more specifically, the relation between solar cell efficiency and annual energy yield. This is to show that although efficiency is an important parameter, the annual energy yield is the parameter that really counts in a sustainable energy world. Efficiency is determined at standard test conditions (STC), which do not reflect the actual irradiance conditions throughout the year. The efficiency of a solar cell or panel is indirectly specified as the rated maximum power that a cell can deliver at STC. The rated power of a solar panel is given in watt peak (Wp), while annual yields are given in kWh. A user of PV energy is more interested in the amount of kWh that he does not have to buy from the utility.

The classroom problem is designed to be solvable in about 3 h. It follows a 2h lecture on principles of photovoltaic solar energy conversions. Teaching assistants help the students, however, to a limited extent; the students should learn to be able to tackle the problem themselves. The remainder of this paper is organized as follows: after a theoretical introduction on solar cell performance and irradiance-dependent efficiency, several methods of calculating annual energy yield are presented. The calculations are to be performed for two locations, the UU campus and the Sahara desert, thereby illustrating the location-dependent differences in annual yield. Throughout the paper, questions are outlined that are to be answered by the students. We close with concluding remarks.

2. Theoretical background

2.1. Solar cell performance

The performance of a solar cell, the building block of solar or PV panels, is characterized by four general parameters [16], which are derived from the current–voltage characteristic (I–V) measured under standard test conditions (STC, Air Mass 1.5 (AM1.5) spectrum, 1000 W m^{-2} irradiation, 25 °C ambient temperature), see also figure 1: open-circuit voltage

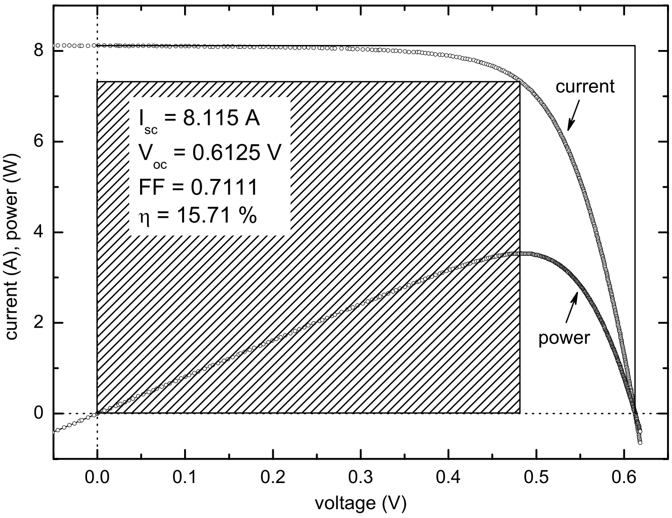

Figure 1. *I–V* characteristics of a 15 × 15 cm^2 crystalline silicon solar cell measured at STC. Performance parameters are $I_{sc} = 8.115$ A; $V_{oc} = 0.6125$ V, and $\eta = 15.71\%$. The fill factor is a measure of the squareness of the *I–V* characteristics and is defined as $FF = V_{mpp} I_{mpp} / V_{oc} I_{sc}$ and equals $FF = 0.7111$. In other words, 71.11% of the area (0, 0) to (V_{oc}, I_{sc}) is filled.

V_{oc}, short-circuit current I_{sc}, fill factor FF, and energy conversion efficiency η. The latter is calculated from

$$\eta = \frac{P_{max}}{A P_{in}} = \frac{V_{mpp} I_{mpp}}{A P_{in}} = \frac{V_{oc} I_{sc} FF}{A P_{in}}, \tag{1}$$

where P_{max} is the maximum generated power, A the cell area, P_{in} the incident power (= 1000 W m^{-2} at STC). P_{max} is given by $P_{max} = V_{mpp} I_{mpp}$, where V_{mpp} and I_{mpp} are the voltage and current, respectively, at the maximum power point. The fill factor thus is defined as $FF = V_{mpp} I_{mpp} / V_{oc} I_{sc}$.

Normal operating conditions of PV systems are rarely STC. Depending on geographical location, season, and time of the day, full sun conditions will prevail or (partly) overcast skies. Incident spectra also differ as a function of longitude and time of day from AM1 to about AM10. Under full sun, the temperature of the module can be much larger than 25 °C, reaching values between 60 °C and 80 °C. This lowers the efficiency, as the open-circuit voltage and, to a lesser extent, the fill factor are functions of temperature. This usually is parametrized by use of temperature coefficients $\mathrm{d}V_{oc}/\mathrm{d}T$ and $\mathrm{d}FF/\mathrm{d}T$. The values differ for different solar cell materials, but in general are negative. A small positive temperature coefficient $\mathrm{d}J_{sc}/\mathrm{d}T$ of the short-circuit current may be present.

Lower irradiances not only lower the power output of the solar cell, but also affect its efficiency, depending on series resistance. Many cells with appreciable series resistance show a maximum efficiency at irradiances lower than 1000 W m^{-2}, peaking between 100 and 500 W m^{-2} [17]. A shunt resistance has an influence especially at irradiances <100 W m^{-2} [18]. If indoor irradiation conditions prevail ($\ll$100 W m^{-2}), the performance drop will be much more dramatic.

2.2. Irradiance-dependent solar cell performance model

In order to make an estimate of the irradiation dependence of the efficiency, we start with the general expression for the current–voltage characteristic which reflects the fact that a solar cell is a single-junction diode [16]

$$I = I_L - I_{01}\left(\exp\frac{qV'}{kT} - 1\right) - I_{0n}\left(\exp\frac{qV'}{nkT} - 1\right) - \frac{V'}{R_{\mathrm{sh}}}, \quad V' = V + IR_{\mathrm{se}} \tag{2}$$

in which I_L is the photocurrent, I_{01} and I_{0n} are diode saturation currents for the diodes with ideality factor 1 and n, V' is the effective voltage, R_{se} and R_{sh} represent series and parallel (shunt) resistances, respectively, k is Boltzmann's constant (1.38×10^{-23} J K^{-1}), T is the temperature (K), and q is the elementary charge (1.602×10^{-19} C). At room temperature kT/q equals to 25.67 mV, and is also denoted as the thermal voltage V_{th}.

For an ideal single junction, equation (2) is simplified to include one diode (with $n = 1$), zero series resistance, and infinite shunt resistance. Then, V_{oc} and I_{sc} are given by

$$V_{\mathrm{oc}} = \frac{kT}{q}\ln\left(\frac{I_L}{I_0} + 1\right), \quad I_{\mathrm{sc}} = I_L. \tag{3}$$

Shockley and Queisser have derived for this idealized case that the fill factor is an implicit function of open circuit voltage only [19]:

$$FF = \frac{v_{\mathrm{m}}^2}{(1 + v_{\mathrm{m}} - \exp(-v_{\mathrm{m}}))(v_{\mathrm{m}} + \ln(1 + v_{\mathrm{m}}))}, \quad v_{\mathrm{oc}} = v_{\mathrm{m}} + \ln(1 + v_{\mathrm{m}}) \tag{4}$$

where the normalized open-circuit voltage v_{oc} is defined as $v_{\mathrm{oc}} = V_{\mathrm{oc}}\frac{q}{kT}$, and the normalized voltage at maximum power v_{m} as $v_{\mathrm{m}} = V_{\mathrm{mpp}}\frac{q}{kT}$. This equation has been approximated empirically by Green [16]:

$$FF_0 = \frac{v_{\mathrm{oc}} - \ln(v_{\mathrm{oc}} + 0.72)}{v_{\mathrm{oc}} + 1}, \tag{5}$$

where FF_0 is the fill factor at zero series resistance. Fill factor loss due to series resistance R_{s} then can be represented by

$$FF = FF_0(1 - r_{\mathrm{s}}), \tag{6}$$

where r_{s} is the normalized series resistance given as $r_{\mathrm{s}} = R_{\mathrm{s}}/R_{\mathrm{CH}}$. The characteristic resistance R_{CH} is defined as $R_{\mathrm{CH}} = V_{\mathrm{oc}}/I_{\mathrm{sc}}$.

Equations (5) and (6) have been thoroughly validated and are found to be accurate for $v_{\mathrm{oc}} > 10$ and $r_{\mathrm{s}} < 0.4$ [16].

Similarly, shunt resistance effects can be estimated using

$$FF = FF_0\left[1 - \frac{(v_{\mathrm{oc}} + 0.7)}{v_{\mathrm{oc}}}\frac{FF_0}{r_{\mathrm{sh}}}\right], \tag{7}$$

in which $r_{\mathrm{sh}} = R_{\mathrm{sh}}/R_{\mathrm{CH}}$ is the normalized shunt resistance. This equation is accurate for $v_{\mathrm{oc}} > 10$ and $r_{\mathrm{sh}} > 2.5$ [16]. The combined effect of series and shunt resistance can be represented by equation (7) if FF_0 is replaced by FF as given in equation (6):

$$FF = FF_0(1 - r_{\mathrm{s}})\left[1 - \frac{(v_{\mathrm{oc}} + 0.7)}{v_{\mathrm{oc}}}\frac{FF_0(1 - r_{\mathrm{s}})}{r_{\mathrm{sh}}}\right]. \tag{8}$$

In the following we will denote all variables as a function of irradiance level G, e.g., $V_{\mathrm{oc}}(G)$, $J_{\mathrm{sc}}(G)$, $FF(G)$ and $\eta(G)$, while only certain variables are assumed constant.

The starting point of the method that calculates irradiance-dependent efficiencies based on the STC parameter set is the availability of the performance parameters V_{oc}, J_{sc}, FF and

η at a certain irradiance level G_0, i.e., $V_{\rm oc}(G_0)$, $J_{\rm sc}(G_0)$, $FF(G_0)$ and $\eta(G_0)$. This level does not necessarily have to be 1000 W m^{-2}; only a known value is needed for the analysis and of course this is usually the STC value. We further assume that the *I*–*V* characteristics can be described by an idealized one-diode model, simplifying equation (2). The normalized open circuit voltage and characteristic resistance are calculated first: $v_{\rm oc}(G_0)$ and $R_{\rm CH}(G_0)$. Further, under the assumption that $I_L(G) = I_{\rm sc}(G)$ the current ratio $I_L(G_0)/I_0(G_0)$ at irradiance level G_0 is calculated using equation (3) followed by $FF_0(G_0)$ with equation (5). Taking into account only series resistance losses, $R_{\rm s}\,(G_0)$ is calculated using equation (6) and $R_{\rm CH}(G_0)$. Shunt resistance losses are expected to be important only at very low irradiance levels, and are not included here; see also section 3.2.2.3.

In order to calculate $V_{\rm oc}(G)$, $J_{\rm sc}(G)$, $FF(G)$ and $\eta(G)$ for values of G in the range of interest, 0.1–1300 W m^{-2}, we first need to assume that both I_0 and $R_{\rm s}$ are not dependent on G. In the series resistance $R_{\rm s}$ many components are lumped together, such as the series resistance from the metal grid, the contact resistance and the emitter and base resistances, which generally are dependent on injection level in the cell. However, for the sake of simplicity, we neglect the irradiance dependence of $R_{\rm s}$. Second, we assume that the short circuit current is linearly dependent on G: $I_{\rm sc}(G) = aG$. It is not necessary to know the constant a; we only use the linear dependence. For example, at $G = 0.1G_0$ the current ratio $I_L(G)/I_0$ equals $0.1I_L(G_0)/I_0$, and $I_{\rm sc}(G) = 0.1I_{\rm sc}(G_0)$. With these values at G, $V_{\rm oc}(G)$, $R_{\rm CH}(G)$, $r_{\rm s}(G_0)$, $v_{\rm oc}(G)$, $FF_0(G)$, $FF(G)$ and $\eta(G)$ are calculated, for instance all in separate columns in a spreadsheet.

3. Problem set-up

The problem is built up from a low level, to let the students get acquainted with solar cell basics and to recapitulate the 2 h lecture; the level then is increased step by step. First, the students will be confronted with simple solar cell efficiency calculations. Second, they are asked to calculate annual yields using three methods, of which two employ irradiance-dependent efficiency. They will learn which method is the most appropriate. The annual yields are calculated for two locations, the Utrecht University campus (The Netherlands) and the Sahara desert, to show the difference between a perceived wet and sunny climate, respectively. All measured *I*–*V* characteristics are from Reich *et al* [20] and are provided to the students in spreadsheet format.

3.1. Solar cell efficiency

The students are asked to calculate the efficiency of a solar cell from a given current–voltage (*I*–*V*) characteristic, see figure 1, which is measured at STC. They must determine the relevant parameters from this graph and use equation (1). Most of the students give the correct answer (15.71%); some, however, forget to include the area of the cell, which is 225 cm^2. The dashed area illustrates the concept of the fill factor: it fills the area of (0, 0) to ($V_{\rm oc}$, $I_{\rm sc}$) only partly, i.e., the fraction is 0.7111.

The students now turn to the analysis of figure 2, where measured *I*–*V* characteristics are shown as a function of irradiance [20]. They are asked to plot the parameters $V_{\rm oc}$, V_{mp}, FF and η, in one graph. This requires some spreadsheet analysis and plotting abilities, which students at this level should have acquired. They arrive at a result that should look like the plot shown in figure 3. They are also asked to explain the observed behaviour. Both FF and to a lesser extent η show a maximum between 200 and 300 W m^{-2}. The students should realize that this is caused by the series resistance. In addition, $V_{\rm oc}$ depends logarithmically on G for

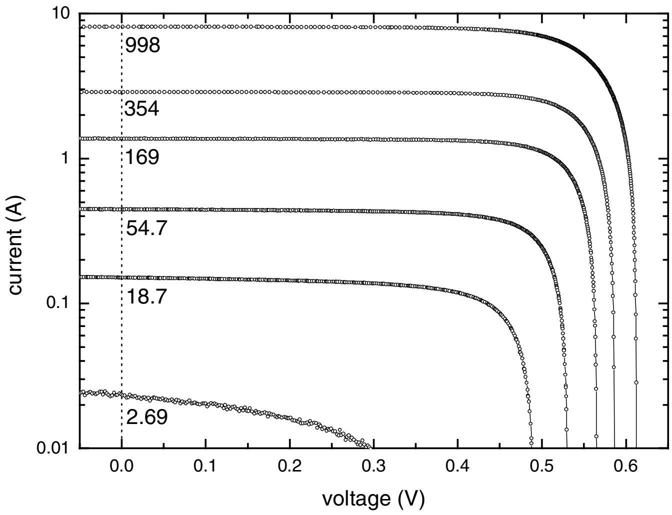

Figure 2. *I–V* characteristics of a 15 × 15 cm^2 solar cell measured at various irradiance values, i.e, 2.69, 18.7, 54.7, 169, 354 and 998 W m^{-2}.

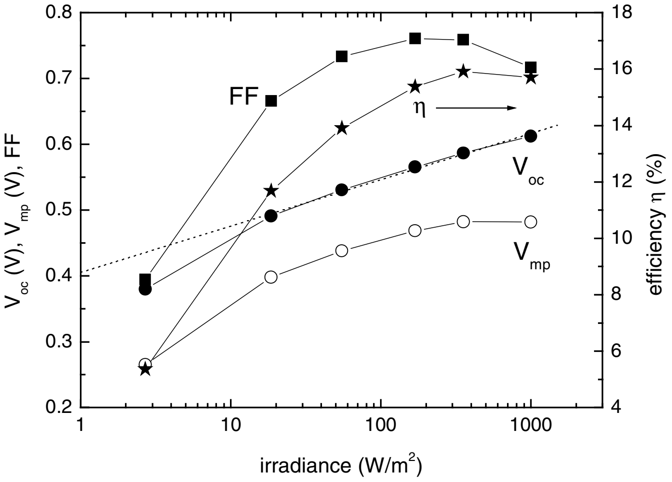

Figure 3. Performance parameters as a function of irradiance, derived from the data in figure 2. The dotted line illustrates a single logarithmic dependence of the open-circuit voltage.

two decades between 10 and 1000 W m^{-2}, as can be seen from the close correspondence of the data points with the dotted line and thus shows the validity of equation (3). Note that we implicitly assume that $I_L \propto G$; students should verify this by plotting I_{sc} versus G on a double logarithmic scale.

3.2. Annual energy yield

PV panels are built up from several solar cells, and range from about 0.01 m^2–2 m^2 in size, depending on the application possibilities and marketability as judged by various

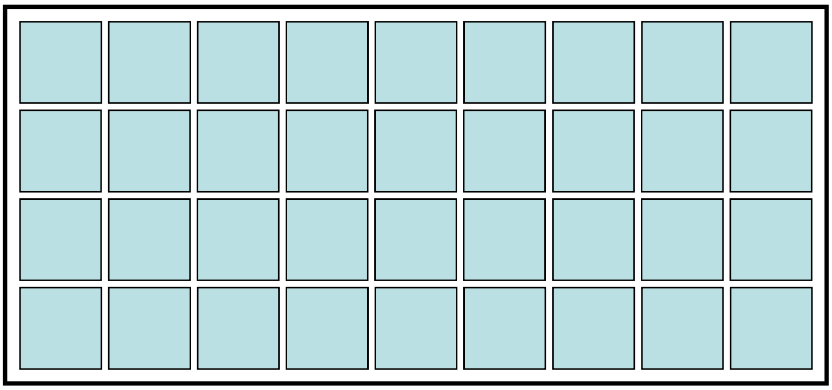

Figure 4. Schematic layout of a PV module with 36 solar cells in a 4 × 9 arrangement. Note the small but significant inter-cell and edge areas.

manufacturers. Building integrated PV and power plant applications usually require the larger size, while for rural electrification projects the smaller size suffices. The rated power depends on the area of the module, while the specific yield, defined as the amount of energy generated per W of rated power, does not.

In the following we assume to have 36 identical solar cells with characteristics as shown above. These cells are mounted together in series in a solar panel, see figure 4, and a small edge area and inter-cell area is present. This panel would deliver a total power of 127.2 W at STC at a current of 7.333 A and a voltage of about 17.35 V. The total cell area is 0.81 m^2. The panel area is larger, and depends on the arrangement of the cells, e.g., in a 4 × 9 way (rectangular module, figure 4), or a 6 × 6 way (square module). Now using an inter-cell distance of 1 cm and an module edge width of 2 cm, as most modules are framed, the area of the rectangular module would be 0.9801 m^2. The area of the square module is somewhat larger at 0.9849 cm^2. Thus the module efficiency is nearly 20% lower than the cell efficiency which is only due to the extra area of the module needed, or, in absolute terms, the *module* efficiency is 13.0%. Using non-identical cells, the module efficiency suffers from mismatch loss, which is minimized by manufacturers by selecting cells from the same efficiency class. After cells are made, they are tested and divided into efficiency classes of 0.1% width. A module that has an efficiency of 13% is built up from cells from the efficiency class 15.65–15.75%.

3.2.1. Sun hour methods

3.2.1.1. Simple approach. A first estimate is to be given by the students on the annual energy yield Y of a 1 m^2 sized PV module rated at $P = 130$ Wp, reflecting the 13% efficiency calculated above. This module comprises 36 identical solar cells of which the I–V characteristic was given in figure 1. As we seek the number of kWh produced, the students may simply multiply the power rating of the module by the number of so-called sun hours h_s. They should realize that $Y = Ph_\mathrm{s}$, where h_s is considerably lower than the amount of hours in one year (8760). The module is rated at 130 Wp, which is determined at STC of 1000 W m^{-2} irradiance. Thus we seek the number of hours for which the following equation holds:

$$h_\mathrm{s} = \frac{\sum_i G\left(h_i\right)}{1000}, \tag{9}$$

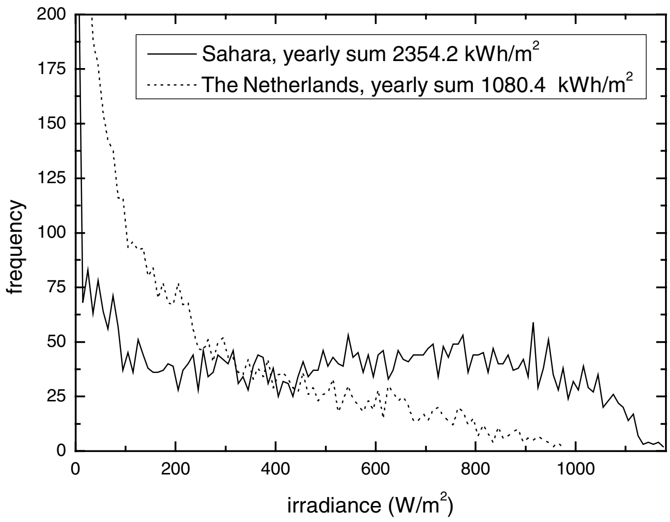

Figure 5. Irradiance distribution calculated from monthly irradiance data from the NASA SSE database for the Sahara (20°N, 10°E) and The Netherlands (52°7′N, 5°10′E). The bin size is 10 W m^{-2}.

where $G\ (h_i)$ is the hourly irradiance at hour h_i and $1 \leqslant i \leqslant 8760$. Students are well aware of the fact that irradiance is a varying parameter, and usually educated guesswork leads to values for h_s that are between 750 and 1500 h, thus yielding $Y = 97.5 - 195$ kWh in the Netherlands. More often used is the specific yield Y_P (kWh Wp^{-1}) defined as

$$Y_P = \frac{Y}{P}, \tag{10}$$

and therefore $Y_P = 0.75 - 1.5$ kWh Wp^{-1} (or by definition: $Y_P = 10^{-3} h_s$). In the following, the students are to discover that this simple guesswork is not that bad.

3.2.1.2. More complex approach. A distribution of hourly irradiance values $G(h_i)$ can be determined from data that are available from meteorological institutes or companies, e.g., in the form of typical reference year (TRY) data or actual measured data for a specific year. If not available, estimates can be generated using the HOMER simulation model [21], which uses monthly irradiance values to generate hourly data based on the Graham algorithm [22]. This has been proven to result in a realistic day-to-day and hour-to-hour variability of the generated data set. The monthly irradiance values can be inputted, or directly taken, even from within HOMER while on-line, from the NASA Surface Solar Energy (SSE) data set [23]. This data set comprises accurate 10 year average irradiance data, which were derived from satellite images that were measured from July 1983 to June 1993.

As an example, figure 5 shows the distribution of irradiance data at the Utrecht University campus (52°7′N, 5°10′E), The Netherlands, which are derived from the NASA SSE dataset. The students are given these data as spreadsheet files with 8760 elements. They have to determine the annual sum and will find that it is 1080.4 kWh m^{-2}, and thus $h_s = 1080.4$ h. The annual specific yield is 1.0804 kWh Wp^{-1}. For comparison, the distribution of irradiance data in the African Sahara is also shown, using the location 20° N, 10° E. We have specifically chosen the Sahara as studies on very large-scale PV power plants have concentrated on such

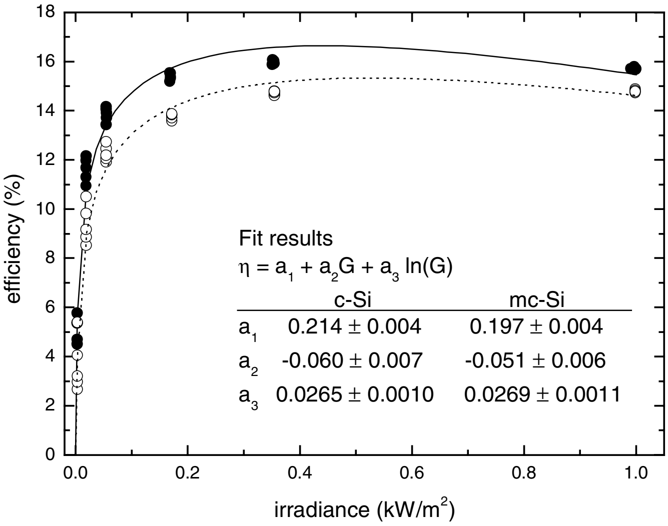

Figure 6. Measured efficiencies of c-Si and mc-Si solar cells as a function of irradiance. For c-Si, the fit shows a maximum efficiency of 16.64% at 0.45 kW m^{-2}, while the measured efficiency at 1 kW m^{-2} (STC) is 15.47%. For mc-Si, the fit shows a maximum efficiency of 15.32% at 0.525 kW m^{-2}, while the measured efficiency at 1 kW m^{-2} (STC) is 14.80%.

desert areas. The annual sum is 2354.2 kWh m^{-2}, and $h_s = 2354.2$ h. The annual specific yield is 2.3542 kWh Wp^{-1}, which is only 2.18 times larger than the value at the Utrecht University campus. This fact usually is found counterintuitive by the students, as the weather in The Netherlands frequently is described as wet.

3.2.2. Solar cell efficiency methods

3.2.2.1. Constant efficiency method. Having available the hourly irradiance values an alternative approach is to find the yield from

$$Y_P = \sum_i \frac{G(h_i)\eta}{P}, \tag{11}$$

where it is assumed that the conversion efficiency η is constant. As expected, the annual yield of the 130 Wp module with $\eta = 13\%$ equals 1.0804 kWh Wp^{-1} and 2.3542 kWh Wp^{-1}, for Utrecht and the Sahara, respectively.

3.2.2.2. Three-parameter fit method. In reality the conversion efficiency is a function of irradiance and we should replace the constant η with the irradiance dependent $\eta(G)$:

$$Y_P = \sum_i \frac{G(h_i)\eta(G(h_i))}{P}. \tag{12}$$

Figure 6 shows two examples of measured efficiencies of a crystalline (c-Si) and a multicrystalline (mc-Si) silicon cell as a function of irradiance [20]. One may parametrize this curve using [24]

$$\eta(G) = a_1 + a_2 G + a_3 \ln G. \tag{13}$$

Table 1. Specific energy yield Y_P (kWh Wp^{-1}) for Utrecht University and the Sahara calculated using various methods for the c-Si and mc-Si solar cells of figure 6.

Y_P (kWh Wp^{-1})	Utrecht University		Sahara	
Method	c-Si	mc-Si	c-Si	mc-Si
Constant η	1.0804	1.0804	2.3543	2.3543
Three-parameter curve $\eta(G)$	1.1055	1.0787	2.3988	2.3784
STC parameter method	1.1139	1.0891	2.4028	2.3745

The measured data are fitted to find for c-Si $a_1 = 0.214 \pm 0.004$, $a_2 = -0.060 \pm 0.007$ and $a_3 = 0.0265 \pm 0.0010$, when G is taken in kW m^{-2}. The parameters for mc-Si are $a_1 = 0.197 \pm 0.004$, $a_2 = -0.051 \pm 0.006$ and $a_3 = 0.0269 \pm 0.0011$. The students now should add a column to the spreadsheet they are already using to calculate $\eta(G)$ for all 8760 h. The specific yield Y_P for all hours in the year is calculated in yet another column, using equation (12). The results are summarized in table 1. The c-Si solar cell delivers 2.17 times more energy when employed in the Sahara than employed in the Netherlands, with a factor of 2.18 more irradiance; the mc-Si cell delivers 2.20 time more energy. The students should realize that these differences are due to the fact that the efficiency curves show larger values at irradiances around 500 W m^{-2}, which is different for c-Si and mc-Si, in combination with the difference in shape of the irradiance distribution.

3.2.2.3. STC parameter method. The irradiance-dependent efficiency curves as shown in figure 6 are usually not available for commercial cells and modules. From specification data sheets of modules only data determined at STC are given. To determine irradiance-dependent performance parameters the calculation method outlined in section 2.2. is used.

The students are now asked to implement the calculation method in their spreadsheet to calculate the irradiation-dependent performance parameters of the c-Si solar cell with STC parameters as shown in figure 1, for values of G in the range 0.1–1300 W m^{-2}. They first determine the constant parameters I_0 and R_s at $G = 1000$ W m^{-2} (STC) and find them to be 2.309×10^{-10} A and 0.010 46 Ω, respectively. Then, for all G, e.g., 25 values logarithmically divided over the range, the students recalculate $V_{oc}(G)$, $R_{CH}(G)$, $r_s(G_0)$, $v_{oc}(G)$, $FF_0(G)$, $FF(G)$ and $\eta(G)$ in separate columns in the spreadsheet. The result should look like the one depicted in figure 7. The fill factor shows a maximum value of 0.807 at 50 W m^{-2}, or about 13% larger than at 1000 W m^{-2}. This small decrease towards high irradiance is only slightly found in the efficiency (i.e., about 4%) as the increases in both short-circuit current and open-circuit voltage counteract the fill factor decrease. The fill factor at zero series resistance equals 0.831. At a light level of 1 W m^{-2} the efficiency is decreased to 78% of its value at 1000 W m^{-2}.

The data in figure 7 clearly show a discrepancy between the constant efficiency and STC method on one side and the three parameter fit method on the other side, which is based on measured data. First, the students need to verify that the assumptions under which the accuracy of equations (4) and (5) is guaranteed are valid, i.e., they should check that for all G the conditions $v_{oc} > 10$ and $r_s < 0.4$ are satisfied. In the case of the data depicted in figure 7, the students should find that the minimum value of v_{oc} is 14 (at $G = 0.1$ W m^{-2}) and the maximum value of r_s is 0.17 (at $G = 1300$ W m^{-2}). Second, shunt resistance losses were not taken into account while they are important at low irradiance levels, i.e., at $G < 100$ W m^{-2} [18]. Figure 7 shows that the methods start to deviate for exactly these lower values for the irradiance. Therefore, including the shunt resistance in the STC method may lead to

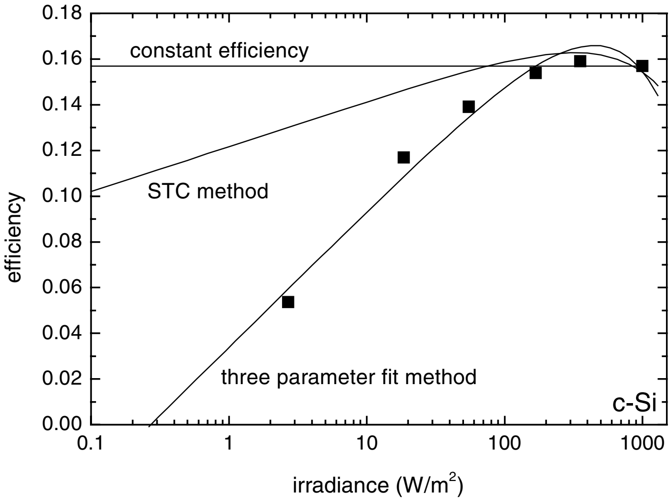

Figure 7. Efficiency of the c-Si solar cell of figure 1 as a function of irradiance for the three methods used: constant efficiency, three-parameter fit and STC method.

smaller discrepancies. However, the value of R_{sh} cannot be determined easily. Of course, it is possible to use equation (7) assuming only shunt effects account for losses. It then is found for many cells that the requirement $r_{\mathrm{sh}} > 2.5$ is not fulfilled for values of G lower than 10–100 W m^{-2}, and calculated values for the fill factor and efficiency even can be negative!

Finally, using equation (12) and the procedure described above for all 8760 values of $G\ (h_i)$, the students are able to calculate the annual yields for the UU campus and the Sahara. The results are shown in table 1.

3.2.2.4. Comparison of methods. From the results in table 1, the students will realize that only small differences in calculated annual yield exist of only a few per cent. Apparently, it does not matter too much which method is used. This clearly has to do with the shape of the irradiance distribution (figure 5): for 94.8% of the amount of hours the irradiance in the Sahara is larger than 10 W m^{-2}, while the differences in calculated efficiencies for the three methods are not that large between 10 and 1000 W m^{-2} (figure 7). The amount of hours for which $G >$ 100 W m^{-2} is 82.5%. Of course, at 1000 W m^{-2} the amount of power delivered is the largest. At 10 W m^{-2} the efficiencies are 15.7, 14.1 and 9.2% for the constant efficiency method, the STC method and the three-parameter fit method, respectively. According to these methods, at 10 W m^{-2} the cell would deliver 1.57, 1.41 and 0.92 W m^{-2}, which is less than a per cent of the power delivered at STC. For the UU campus the differences between the methods are somewhat larger, which is due to the irradiance distribution: only 90.1% (58.4%) of the amount of hours the irradiance at the UU campus is larger than 10 (100) W m^{-2}.

The students may find it frustrating to have found that in terms of annual yield it does not matter which calculation method is used. The much more complex STC method leads to results that are only a few per cent different from those obtained with the simple constant efficiency method. Clearly, the latter method is thus preferred, while the former method gives more insight into solar cell performance parameters.

3.2.2.5. Connection with practice. The present calculations are performed assuming that the PV modules are placed flat on the roof, in other words, that the tilt angle is zero. Usually, modules are tilted toward the south. As a rule of thumb, the tilt angle for optimal yield equals the latitude. For typical houses in the Netherlands, with fixed roofs at typical tilt angles of 30°–45°, the yield is nearly 20% larger than the zero-tilt case. For the Sahara an only 4% larger yield is found for the optimum tilt compared to the zero-tilt case.

The 1 m^2 130 Wp horizontally placed panel used here is calculated to deliver 143 kWh per year at 1.1 kWh Wp^{-1} in the Dutch climate. This is increased by tilting the module by nearly 20%. In practice, however, values around 0.9 kWh Wp^{-1} are found, for installations installed under 30°–45° tilt angle. We have ignored temperature effects, which are especially important at high irradiance, which lowers the cell efficiency. Also cell interconnection losses and other system losses are ignored, which further lower the module efficiency and consequently the system yield by about 70%. Assuming that for a typical roof some 20 m^2 are available for PV panels, they would deliver nearly 2600 kWh per year, including temperature effects. This constitutes nearly 75% of the annual electricity consumption of an average household in the Netherlands (3500 kWh).

4. Conclusion

Photovoltaics is one of many renewable energy technologies to be used in the near future and teaching PV should therefore be part of (under-)graduate courses. PV basics can be taught within the framework of electrical engineering or semiconductor physics classes, but PV performance then often is not treated. We have developed a classroom problem that shows the difference between PV efficiency and annual yield, which is much appreciated by the students.

The methods developed can easily be transferred to spreadsheet format, with which students are familiar. The exercise can be done in about 3 h depending on the students' level. For me, this exercise proves successful when students realize and question me that cell optimization should be done differently: it is the annual yield that counts.

Acknowledgments

This problem was developed for hands-on experience with PV performance in the course 'Energy Conversion Technologies' which is part of the Master of Sustainable Development and Master of Energy Science Programs at Utrecht University. We gratefully acknowledge many students ($>$100) who have taken this class in the past 3 years and have helped to improve earlier versions. Also, discussions with Erik Alsema, Nils Reich, Corry de Keizer (Utrecht University) and Wim Sinke (Energy Research Centre of the Netherlands, ECN) are highly valued. We further acknowledge financial support in part from the Netherlands Organization for Energy and the Environment (NOVEM) and the Netherlands Organization for Scientific Research (NWO) within the framework of their joint programme on Stimulating Energy Research through the project SYN-Energy.

References

[1] Dorian J P, Franssen H T and Simbeck D R 2006 Global challenges in energy *Energy Pol.* **34** 1984–91
[2] UNDP 2004 *World Energy Assessment: Overview 2004 Update*
[3] Infield D G *et al* 2000 Development of a European masters degree course in renewable energy *Proc. Altener 2000 Conf. (WIP, Munich, Germany, 2000)* pp 223–5
[4] Wenham S R *et al* 2002 New undergraduate engineering programs in photovoltaics and renewable energy *Proc. PV in Europe: From PV Technology to Energy Solutions (WIP, Munich, Germany, ETA, Florence, Italy, 2002)* ed J-L Bal, G Silvestrini, A Grassi, W Palz, R Vigotti, M Gamberale and P Helm pp 799–802

[5] Pearson G L 1957 Conversion of solar to electrical energy *Am. J. Phys.* **25** 591–8

[6] Mialhe P and Charette J 1983 Experimental analysis of *I–V* characteristics of solar cells *Am. J. Phys.* **51** 68–70

[7] Hafemeister D 1987 Science and society text X: energy conservation *Am. J. Phys.* **55** 307–15

[8] Morgan M J, Jakovidis G and McLeod I 1994 An experiment to measure the *I–V* characteristics of a silicon solar cell *Phys. Educ.* **29** 252–4

[9] Kraftmakher Y 2000 Photovoltaic cell: efficiency of energy conversion *Eur. J. Phys.* **21** 159–66

[10] Polman A, Van Sark W G J H M, Sinke W C and Saris F W 1986 A new method for the evaluation of solar cell parameters *Sol. Cells* **17** 241–51

[11] Martil I and Gonzalez Diaz G 1992 Determination of the dark and illuminated characteristic parameters of a solar cell from *I–V* characteristics *Eur. J. Phys.* **13** 193–7

[12] Rebello N S, Ravipati C, Zollman D A and Escalada L T 1997 Computer simulation of p–n junction devices *Am. J. Phys.* **65** 765–73

[13] Buckley R W 1992 School photovoltaics *Phys. Educ.* **27** 323–6

[14] O'Regan B and Grätzel M 1991 A low-cost, high-efficiency solar cell based on dye-sensitized colloidal TiO_2 films *Nature* **353** 737–40

[15] Siemsen F, Bunk A, Fischer K, Korneck F, Engel H and Roux D 1998 Solar energy from spinach and toothpaste: fabrication of a solar cell in schools *Eur. J. Phys.* **19** 51–8

[16] Green M A 1982 *Solar Cells: Operating Principles, Technology and Systems Application* (Englewood Cliffs, NJ: Prentice-Hall)

[17] Marion B 2002 A method for modeling the current–voltage curve of a PV module for outdoor conditions *Prog. Photovolt: Res. Appl.* **10** 205–14

[18] Grunov P, Lust S, Sauter D, Hoffmann V, Beneking C, Litzenburger B and Podlowski L 2004 Weak light performance and annual energy yields of PV modules and systems as a result of the basic parameter set of industrial solar cells *Proc. 19th European Photovoltaic Solar Energy Conf. (WIP, Munich, Germany, 2004)* ed W Hoffmann, J-L Bal, H Ossenbrink, W Palz and P Helm pp 2190–3

[19] Shockley W and Queisser H J 1961 Detailed balance limit of efficiency of p–n junction solar cells *J. Appl. Phys.* **32** 510

[20] Reich N H, Van Sark W G J H M, Alsema E A, Kan S Y, Silvester S, Van der Heide A S H, Lof R W and Schropp R E I Weak light performance and spectral response of different solar cell types *Proc. 20th European Photovoltaic Solar Energy Conf. (WIP, Munich, Germany, 2005)* ed W Hoffmann, J-L Bal, H Ossenbrink, W Palz and P Helm pp 2120–3

[21] Lambert T, Gilman P and Lilienthal P 2006 Micropower system modeling with HOMER *Integration of Alternative Sources of Energy* ed F A Farret and M G Simões (New York: Wiley) pp 379–418

[22] Graham V A and Hollands K G T 1990 A method to generate synthetic hourly solar radiation globally *Sol. Energy* **44** 333–41

[23] Chandler W S, Whitlock C H and Stackhouse P W Jr 2004 NASA climatological data for renewable energy assessment *J. Sol. Energy Eng.* **126** 945–9

[24] Beyer H G, Betcke J, Drews A, Heinemann D, Lorenz E, Heilscher G and Bofinger S 2004 Identification of a general model for the MPP performance of PV-modules for the application in a procedure for the performance check of grid connected systems *Proc. 19th European Photovoltaic Solar Energy Conf. (WIP, Munich, Germany, 2004)* ed W Hoffmann, J-L Bal, H Ossenbrink, W Palz and P Helm pp 3073–6