

Why Husserl should have been a strong revisionist in mathematics*

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3rd April 2000

Husserl repeatedly has claimed that (1) mathematics without a philosophical foundation is not a science but a mere technique; (2) philosophical considerations may lead to the rejection of parts of mathematical practice; but (3) they cannot lead to mathematical innovations. My thesis is that Husserl's third claim is wrong, by his own standards.

To explain this thesis, let me first introduce the term 'revisionism'. It is understood here, following Crispin Wright, as the term that applies to 'any philosophical standpoint which reserves the potential right to sanction or modify pure mathematical practice' [Wright 1980, p.117]. I want to make a distinction between weak and strong revisionism. The point of reference is the actual practice of mathematics. Weak revisionism then potentially sanctions a subset of this practice, while strong revisionism potentially not only limits but extends it, in different directions. In strong revisionism, certain combinations of limitation and extension may lead to a mathematics that is no longer compatible with the unrevised one. 'May lead', not 'necessarily leads': it is all a matter of reserving rights; whether there is occasion to exercise them is a further question. To illustrate these categories, let me give examples of non-revisionism, weak revisionism, and strong revisionism.

Non-revisionism can be found in Wittgenstein's *Philosophische Untersuchungen*, where philosophy can neither change nor ground mathematics:

Die Philosophie darf den tatsächlichen Gebrauch der Sprache in keiner Weise antasten, sie kann ihn am Ende also nur beschreiben.
Denn sie kann ihn auch nicht begründen.
Sie läßt alles wie es ist.
Sie läßt auch die Mathematik wie sie ist, und keine mathematische

*I wish to thank Richard Cobb-Stevens, Dirk van Dalen, Menno Lievers, Dieter Lohmar, Charles Parsons, Karl Schuhmann, Richard Tieszen, and Albert Visser for comments on the draft version. Most of this paper was written during a stay at Harvard in 1998, partially made possible by a grant from the Netherlands Organisation for Scientific Research (NWO), which is gratefully acknowledged. I am thankful to the director of the Husserl Archive in Louvain, Rudolf Bernet, who kindly granted permission to quote from an unpublished manuscript in Husserl's *Nachlaß*.

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Entdeckung kann sie weiterbringen. Ein ‘führendes Problem der mathematischen Logik’ ist für uns ein Problem der Mathematik, wie jedes andere. [Wittgenstein 1953, I.124]

Whether Wittgenstein’s claim of non-revisionism is consistent with the rest of his philosophy is a matter of discussion, and a large part of Wright’s study [Wright 1980] from which I took the definition of revisionism given above is devoted to this question. But the quote certainly serves as an illustration of an attitude; the same goes for the examples to follow.

An example of weak revisionism is Hilbert’s Program. In his 1922 lecture ‘Neubegründung der Mathematik’ he said:

Das Ziel, die Mathematik sicher zu begründen, ist auch das meine; ich möchte der Mathematik den alten Ruf der unanfechtbaren Wahrheit, der ihr durch die Paradoxien der Mengenlehre verloren zu gehen scheint, wiederherstellen; aber ich glaube, daß dies bei voller Erhaltung ihres Besitzstandes möglich ist. Die Methode, die ich dazu einschlage, ist keine andere als die axiomatische. [Hilbert 1922, p.160]

This statement clearly shows the two defining characteristics of weak revisionism: the effort to provide mathematics with a foundation and the wish to preserve all of mathematics as actually practiced.

An example of strong revisionism is Brouwer’s intuitionism, which both limits and extends actual practice (in a way that is incompatible with classical mathematics, e.g. in intuitionistic analysis all full functions are continuous). In the overview article of 1952, ‘Historical background, principles and methods of intuitionism’, Brouwer described the effect of introducing intuitionism on mathematics:

In this situation intuitionism intervened with two acts, of which the first seems necessarily to lead to destructive and sterilizing consequences; then however, the second yields ample possibilities for recovery and new developments. [Brouwer 1952, p.140]

More specifically, then, the claim that I want to defend now is that, even though his explicit claims are only for what I would call weak revisionism, from his own general views Husserl should have derived a strong revisionism.¹

My argument is this:

1. Husserl presents a weak revisionism. (Premise)
2. Husserl’s weak revisionism implies a strong revisionism. (Premise)
3. So Husserl’s position in fact warrants a strong revisionism. (1,2)

¹The thesis that genetic phenomenology (usually not recognized as such) performs crucial functions in the conception and revisions of any science, and that analysis of a science should benefit from being aware of these functions, has been formulated sharply and programmatically before by Rota [Rota 1973]. The present argument, although written independently, in effect works out that program for the particular case of mathematics, and refines it by adding the strong-weak distinction.

1 Husserl's weak revisionism

Premise 1 has to be supported by textual evidence, and therefore I will quote amply (the argument for premise 2 is in section 2). We will see that in most of these quotes Husserl discusses not just mathematics but a priori sciences in general; however, for the purposes of the present discussion I will read them only as statements about mathematics.

Such evidence can already be found in Husserl's *Prolegomena* of 1900 [Hua XVIII]. Section 71 of that work is entitled, significantly, 'Teilung der Arbeit. Die Leistung der Mathematiker und die der Philosophen'. In it, Husserl describes how he views the task of philosophy with respect to mathematics:

Sie will dem Spezialforscher nicht ins Handwerk pfuschen, sondern nur über Sinn und Wesen seiner Leistungen in Beziehung auf Methode und Sache zur Einsicht kommen. [...] Erst die philosophische Forschung ergänzt die wissenschaftlichen Leistungen des Naturforschers und Mathematikers so, daß sich reine und echte theoretische Erkenntnis vollendet. Die *ars inventiva* des Spezialforschers und die Erkenntniskritik des Philosophen, das sind ergänzende wissenschaftliche Betätigungen, durch welche erst die volle, alle Wesensbeziehungen umspannende theoretische Einsicht zustande kommt. [Hua XVIII, pp.255–6]²

The revisionism described here is a weak one, as it is not concerned with the content of mathematics but with its epistemological methodology. Philosophy should provide the foundation and insight for mathematics. As we will see in a moment, it is a view that Husserl expressed all through the development and reconceptions of phenomenology in later years. After the *Prolegomena*, he wrote on the philosophy of science most extensively in the posthumously published *Ideen III* [Hua V] (which its editor, Marly Biemel, named 'Die Phänomenologie und die Fundamente der Wissenschaften'); and later detailed discussions can be found in the different versions of the *Encyclopedia Britannica* article [Hua IX] and *Formale und transzendente Logik*.

In *Ideen III*, again the stress is on methodology, as in the *Prolegomena*; but also a new and essential idea is introduced, namely, that of transcendental phenomenology as providing the universal ontology. By this Husserl means that transcendental phenomenology forms a unity with the ontologies of the particular sciences; for example, in section 14 of *Ideen III*, 'Einbezogenheit der Ontologien in die Phänomenologie', he states:

Alles, was uns die Wissenschaften von den Onta, die rationalen und empirischen Wissenschaften (im erweiterten Sinn können sie alle 'Ontologien' heißen, sofern es sich zeigt, daß sie auf Einheiten der 'Konstitution' gehen) darbieten, 'löst sich in Phänomenologisches auf' [...] Was hier zu leisten ist [...], was wir hier unter den Titel

²Lohmar [Lohmar 1989, p.197n21] notes that the 'ars inventiva' is discussed by Descartes and Leibniz.

‘Rückgang auf das konstituierende absolute Bewußtsein’ im Auge haben, setzt voraus eine transzendente Phänomenologie. [Hua V, p.78]

(A brief hint at this idea already appeared in Husserl’s reflections immediately after his lecture series ‘Die Idee der Phänomenologie’ of 1907 [Hua II, pp.13-14].)

To elaborate this point, I want to refer to the Encyclopedia Britannica article of 1927 and *Formale und transzendente Logik* of 1929.³

The interest of the Britannica article for my discussion derives from two facts: first, in versions 3 and 4 of it Husserl gives a direct argument for his conception of transcendental phenomenology as providing the universal ontology; secondly, it is here that he for the first time connects the paradoxes in the foundations of mathematics with this discussion of ontology.⁴ These two issues he takes up again in *Formale und transzendente Logik* [Hua XVII, esp. sections 69–72].

First, the argument. This reconstruction uses Husserl’s own phrases [Hua IX, p.519]:

- 1.1. ‘Die transzendente Phänomenologie ist die Wissenschaft von allen erdenklichen transzendentalen Phänomenen.’ (Premise)
- 1.2. ‘Alles Seiende schöpft sein Seinssinn aus intentionaler Konstitution.’ (Premise)
- 1.3. Therefore, ‘Diese Phänomenologie [ist] *eo ipso* die absolute universale Wissenschaft von allem Seienden’. (1,2)

³These were written at the other end of a period during which Husserl reconceived and developed phenomenology, and accordingly the simultaneous use of these early and late texts may need some justification. Therefore I mention the following details. Husserl’s stenographic manuscript of Ideen III is from 1912; Edith Stein transcribed it in 1916; Ludwig Landgrebe typed her transcription in 1924–1925. Contrary to the manuscript of Ideen II, Husserl hardly made any changes or additions to the manuscript during these years [Hua IV, pp.xv,xvi]. This accords with the judgement one forms from reading these texts, that Husserl had not essentially altered his ideas on this specific subject by the time he wrote the Britannica article and *Formale und transzendente Logik*.

⁴Husserl wrote four versions of the Britannica article, the last of which was published. The first and second did not contain the closing discussion of ontology; therefore I will use the third and the fourth, which is otherwise mainly the result of cutting down the third because of space limitations. All versions, and a discussion of the textual differences, are in Hua IX [Hua IX].

Husserl’s Amsterdamer Vorträge, published in the same volume, were a revision of version 4 of the Britannica article. The part on ontology however, which according to Husserl’s notes was meant to be included [Hua IX, p.349n1], was never written. It is a pity that he did not get round to that, as it is very probable that Brouwer was in the audience (see the draft letter of organiser Pos to Husserl [Husserl 1994, IV:p.441]); the subject may have come up in their long conversations that Husserl enthusiastically mentions in a letter to Heidegger of May 5, 1928:

Mit das Interessanteste waren in Amst[er]dam die langen Gespräche mit Brouwer, der einen ganz bedeutenden Eindruck auf mich machte, den eines völlig originellen, radikal aufrichtigen, echten, ganz modernen Menschen. [Husserl 1994, IV:p.156]

However, neither Husserl nor Brouwer seems to have returned to these conversations in their work since then.

A few lines further on, Husserl restates this conclusion in other words:

1.3*. ‘Unter Heranziehung und Erweiterung des traditionellen Ausdrucks können wir auch sagen: die transzendente Phänomenologie ist die wahre, die wirklich universale Ontologie.’ [Hua IX, pp.519-520]

Premise 1.1 means that transcendental phenomenology is universal in that it deals with all conceivable constitutive possibilities of the transcendental ego.

Premise 1.2 is a version of the fundamental principle of transcendental idealism, as Husserl developed it from 1907 onwards [Hua II]. It singles out a special group among all transcendental phenomena, namely those experiences in which the intentional object is given as itself, as existing. In *Cartesianische Meditationen*, Husserl calls the corresponding group of constitution processes cases of strict (‘prägnant’) constitution [Hua I, section 23]. There are correspondences between the essence of an object and the strict constitution of such an object. That is, the way in which an object is given as an identity through various acts is characterized by a rule that is specific for that kind of object [Hua III/1, p.330], [Hua XVII, section 98]. Strict constitution is constitution according to this rule. Finally, existence is the objective correlate of strict constitution [Hua I, section 26].

By ‘ontology’, in 1.3*, Husserl means the systematic unity of all a priori sciences [Hua IX, p.519]. This unity derives from the fact that all a priori concepts, like everything else, acquire their sense in the one process of constitution. Husserl has in mind here a science of possible being, not just of actual being as a more traditional conception of ontology alluded to in 1.3* would have it, and he traces the idea of such a science back to Leibniz:

Schon Leibniz hatte die fundamentale Einsicht, daß für eine echte theoretische Erkenntnis und Wissenschaft die Erkenntnis der Möglichkeiten derjenigen der Wirklichkeiten vorangehen muß. Demgemäß fordert er für jederlei reale und ideale Seinssphäre zugehörige apriorische Wissenschaften als solche der reinen Möglichkeiten (z.B. auch eine reine Grammatik, reine Rechtslehre usw.). [Hua IX, p.520]

The second point of interest of the Britannica article is its treatment of the paradoxes in the foundations of mathematics. Husserl interprets them as symptoms of the lack of philosophical foundation:

Das Prinzipielle der mathematischen Methode [hat sich] als unzulänglich, die viel bewunderte mathematische Evidenz sich als eine der Kritik und der methodischen Reform bedürftige herausgestellt. [...] Der Kampf um die ‘Paradoxien’, um die rechtmäßige oder Scheinevidenz der Grundbegriffe der Mengenlehre, der Arithmetik, der Geometrie, der reinen Zeitlehre usw. [...] hat es zutage gebracht, daß alle diese Wissenschaften ihrem ganzen Methodentypus nach noch nicht als Wissenschaften im vollen und echten Sinn gelten

können: als Wissenschaften, die bis ins letzte methodisch durchsichtig sind und daher zu vollkommenster Rechtfertigung jedes methodischen Schrittes befähigt und bereit. [Hua IX, p.520–521]

(A similar passage can be found on p.524.) A weak revisionism as described here is called for as the cure to develop mathematics into a true science. Unless that task has been completed, mathematics is, properly speaking, not a science but a useful technique:

Überall beobachten wir, wie bei der erkenntnistheoretischen Problematik sonst, die schon wiederholt erwähnte Verkehrtheit, daß man die Wissenschaften als etwas nimmt, das schon ist; als ob Grundlagenforschung nur eine nachkommende Klärung oder allenfalls eine diesen Wissenschaften selbst nicht wesentlich ändernde Besserung bedeuten sollte. In Wahrheit sind Wissenschaften, die Paradoxien haben, die mit Grundbegriffen operieren, die nicht aus der Arbeit der Ursprungsklärung und Kritik geschaffen sind überhaupt keine Wissenschaften, sondern bei aller ingenüösen Leistung bloß theoretische Techniken. [Hua XVII, p.189]

So the task of philosophy with regard to the sciences is to transform them from techniques to genuine (i.e., insightful) knowledge, by clarification (‘Klärung’) of their concepts:

Es gilt, die Wissenschaften auf ihren Einsicht und strenge Geltung verlangenden Ursprung zurückzuführen und sie in Systeme einsichtiger Erkenntnis zu verwandeln durch klärende, verdeutlichende, letztbegründende Arbeit, die Begriffe und Sätze auf in der Intuition faßbare begriffliche Wesen selbst und die sachlichen Gegebenheiten selbst zurückzuführen, denen sie angemessenen Ausdruck geben, soweit sie wirklich Wahr sind. [Hua V, pp.96–97]

With this clarification comes the possibility of rejecting supposed objects. This is made manifest in the introduction of *Formale und transzendente Logik*:

So bedeutet ursprüngliche Besinnung ineins Näherbestimmung der bloß vage unbestimmten Vorzeichnung, Abhebung der aus assoziativen Überschiebungen herstammenden Vorurteile und Durchstreichung der mit der besinnlichen Erfüllung streitenden; also mit einem Wort Kritik der Echtheit und Unechtheit. [Hua XVII, p.14]

This ‘Kritik der Echtheit und Unechtheit’ is what makes ontological questions decidable. Hence Husserl can claim, in the fourth version of the Britannica article, that in a priori sciences that have a phenomenological foundation, no foundational crises could occur. The clarification that transcendental phenomenology supplies guarantees that there will be no paradoxes [Hua IX, p.297].⁵

⁵The demand that all sciences be grounded in transcendental phenomenology raises the obvious question in what that phenomenology is to be grounded. Grounding is a matter of

Given this function of clarification, evidently Husserl's published philosophy of mathematics can have as domain only current practice, concepts that are pre-given; one cannot clarify something that isn't already there:

Das Ziel der Klärung kann man im Sinn des schon Ausgeführten auch dahin fassen, daß sie den vorgegebenen Begriff gewissermaßen von neuem schaffen, ihn aus der Urquelle der begrifflichen Geltung, der Anschauung, speisen und ihm innerhalb der Anschauung die Teilbegriffe geben will, die zu seinem originären Wesen gehören. [Hua V, p.102]

In *Formale und transzendente Logik*, there is a similar passage that starts with 'Die Grundbegriffe sind immer schon vertraut und zu freier Verfügung' [Hua XVII, p.188]; and in the introduction to the same work, he says of the method of clarification:

Sie geht aus von den theoretischen Gebilden, die uns in der Überschau die historische Erfahrung in die Hand gibt, also von dem, was ihren traditionellen Gehalt ausmacht, und versetzt sie zurück in die lebendige Intention der Logiker, aus der sie als Sinngebilde entsprangen. [Hua XVII, p.14]

Thus, clarification is a retrogressive inquiry back to sense-conferring living intention, a genetic analysis.

In summary of the argument for premise 1 of the main argument, what Husserl presents is a form of revisionism because of the demand for a philosophical foundation of mathematics, without which it could be a technique but not a true science; and it is a weak revisionism because its method, clarification, can only act on the concepts of current mathematical practice. There is the possibility of rejecting parts of existing practice ('Durchstreichung' of intentions); but there is no evidence in his writings for strong revisionism, the introduction of new concepts on philosophical grounds, and indeed Husserl wants to hold to a division of labour between mathematics and philosophy.

2 Husserl's implied strong revisionism

However, that Husserl's weak revisionism does imply a strong revisionism (premise 2 of the main argument) is what I will now argue for. The basis is a certain difference between constitution in the formal and non-formal a priori sciences. I first present the argument as a whole, and then explain terms and comment on the steps.

obtaining evidence, and the science of evidence is precisely transcendental phenomenology itself. So it can only be grounded in itself. Husserl is very much aware of this task, and came to see this need for reflexivity as the one crucial methodological demand. See, for example, *Formale und transzendente Logik* [Hua XVII, pp.294:35–295:6]. A convincing account how transcendental phenomenology indeed is reflexive in the required sense, is in Tito's chapter 5 (g) [Tito 1990].

- 2.1 Formal objects have no hyletic-material binding. (Premise)
- 2.2 Strict ('prägnant') constitution is constitution with evidence and in accordance with laws of essence for that object. (Premise)
- 2.3 Strict constitution of formal objects demands no specific hyletic-material data. (2.1,2.2)
- 2.4 Existence is the objective correlate of strict constitution. (Premise)
- 2.5 For formal objects, transcendental possibility implies existence. (2.3,2.4)
- 2.6 Clarification aims at strict constitution of objects that are already figuring in actual practice. (Premise)
- 2.7 In purely formal sciences, the capacity for clarification is exactly the capacity for strict constitution. (2.5,2.6)
- 2.8 The objects figuring in actual practice need not exhaust the totality of objects that are possible according to essence. (Premise)
- 2.9 So Husserl's weak revisionism implies a strong revisionism. (2.7,2.8)

As I am here trying to make a point about the nature of Husserl's weak revisionism, it will not come as a surprise that three of the five premises already made their appearance in the foregoing exposition of that revisionism. 2.2, (a version of) 2.4 (the principle of transcendental idealism), and 2.6 we already saw in the previous section.

The meaning of the part 2.1–2.5 can be clarified by contrasting the position of Husserl to (one aspect of) that of Nicolai Hartmann in 'Möglichkeit und Wirklichkeit' [Hartmann 1937]. I have chosen Hartmann, because his general position on ideal objects largely coincides with Husserl's, but Husserl wants to add an extra condition.

Hartmann [Hartmann 1937, Kap.41] recognizes two different senses in which an ideal object can be 'possible', in modern language:

logical possibility (Hartmann: 'logische Möglichkeit') An object is logically possible exactly if the corresponding existence statement has a model.

An example considered by Hartmann is a square circle [Hartmann 1937, Kap.41e] If we take 'square' and 'circle' as primitive terms, then a square circle is logically possible because 'there is a square circle' has the form $\exists x(Px \wedge Qx)$, which has a model. This may not be the intended model, but that is no longer a matter of just logic. There is no contradiction in the logical form.⁶

An example of an object that is not logically possible is something that is round and not round: $\exists x(Px \wedge \neg Px)$ has no models, it is a formal contradiction.

⁶ If 'square' and 'circle' are taken as defined rather than primitive terms, a formal contradiction probably would show up. But a definition of these terms (with the intended interpretation in mind) would depend on conceptual analysis of squares and circles, so the appearance of a formal contradiction would also in this case depend on more than only logic.

conceptual possibility (Hartmann: ‘Wesensmöglichkeit’) An object is conceptually possible exactly exactly if it is logically possible and moreover there is no material contradiction involved.

A square circle is not conceptually possible: although it fulfils the condition of logical possibility, it is materially contradictory. A square cannot be a circle at the same time.⁷

Hartmann’s claim is that for ideal entities, conceptual possibility and conceptual existence (Wesenswirklichkeit) are co-extensive.⁸

Husserl, using slightly different terminology, recognizes this distinction between logical and conceptual possibility. In the fourth of the *Logische Untersuchungen*, which is devoted to the theory of pure grammar, he speaks of ‘formal possibility’ and ‘material’ (or ‘synthetic’) possibility [Hua XIX/1, section 14]; but his transcendental turn of a few years later introduces a third sense of possibility:

transcendental possibility (my term) An object is transcendently possible exactly if it is conceptually possible and moreover can be strictly constituted (that is, ideally, with full evidence). For example, the number 3 and the set of all even natural numbers. Conceptually possible, but transcendently impossible entities are, it would seem, non-well founded sets. From the point of view of transcendental idealism, they could therefore not be accepted as existing objects. Hartmann, on the other hand, would ascribe mathematical existence to them, as long as they are not internally inconsistent.

The differences between Hartmann and Husserl arise because Hartmann considers ontology and epistemology as completely independent of each other, also in the case of ideal objects: ‘Denn Erkenntnis kann vom Sein ihres Gegenstandes aus überhaupt nicht impliziert werden. Das ideale Sein ist an sich nicht weniger indifferent gegen die Idealerkenntnis, als das reale Sein gegen die Realerkenntnis.’ [Hartmann 1937, Kap.60a, p.464]. Hence in ontology of ideal objects, he recognizes only what he calls internal constraints, i.e. formal and material consistency.

Husserl, on the other hand, thinks there is a further constraint, the external one implied by transcendental idealism: an object can be said to exist if and only if it can ideally be brought to consciousness [Hua III/1, section 142]—a principle clearly relating ontology and epistemology. This is external because

⁷Hartmann writes: Sehr vieles ist logisch möglich, was nicht wesensmöglich ist. Die Logik muß mit “unmöglichen Gegenständen” rechnen, d.h. mit wesensunmöglichen (viereckiger Kreis); je nachdem, was sie an vorgegebenen Merkmalen (etwa des Kreises) anerkennt, sind solche Gegenstände für sie möglich oder nicht. Die “Unmöglichkeit” solcher Gegenstände ist eben eine Seinsunmöglichkeit (etwa eine geometrische), nicht eine logische. [Hartmann 1937, Kap.41e,p.323] Compare note 6.

⁸[Wirklichkeit und Unwirklichkeit] spielen keine eigene Rolle im idealen Sein neben der Möglichkeit des Seins und Nichtseins. Sie sind mit ihr gesetzt, sind ein selbstverständliches, besagen nicht ‘mehr’ als das Seinkönnen und Nichtseinkönnen. [Hartmann 1937, Kap.41b,p.318]

the constraint arises not from the alleged entities themselves but from properties of consciousness. It is external in exactly the same sense that strong revisionism invokes considerations external to mathematics, namely, philosophical ones. Steps 2.1-2.4 of the argument describe this notion of transcendental possibility. An object cannot be strictly constituted if it is transcendently impossible to do so.

It is against this background that we reach an even stronger version of 2.5: for formal objects, transcendental possibility and existence are co-extensive.⁹ (In the argument 2.1–2.9, I need only one direction of the resulting biconditional.) This claim can be found almost literally in *Erfahrung und Urteil* (provided ‘Möglichkeit’ is interpreted as transcendental possibility):

Alle Existenzialurteile der Mathematik als apriorische Existenzialurteile sind in Wahrheit Existenzialurteile von Möglichkeiten.
[Husserl 1985, section 96c]

How is this claim defended? Husserl views the objects of pure mathematics as formal objects. Correspondingly, the ontological region of pure mathematics is made up of the empty form ‘anything whatever’, and all specializations thereof:

Hieraus erwächst eine universale Wissenschaftsidee, die einer formalen Mathematik im voll umfassenden Sinne, deren Universalgebiet sich fest umgrenzt als Umfang des obersten Formbegriffes Gegenstand-überhaupt oder des in leerster Allgemeinheit gedachten Etwas-überhaupt, mit allen in diesem Feld apriori erzeugbaren und daher erdenkbaren Ableitungsgestalten, die in immer neuer iterativer Konstruktion immer neue Gestalten ergeben. Solche Ableitungen sind neben Menge und Anzahl (endliche und unendliche), Kombination, Relation, Reihe, Verbindung, Ganzes und Teil, usw.
[Hua XVII, p.82]

To say that formal objects have no hyletic-material binding is to say that the hyletic-material is not part of the essence of a formal object, or put differently, that the sensory or the material is not, however indirectly, part of the meaning of the formal. [Hua XVII, p.33]

The non-formal (‘das Sachhaltige’), on the other hand, does have hyletic-material content, and this gives rise to a difference in strict constitution:

Jedes sachhaltige Apriori [...] fordert zur kritischen Herstellung der echten Evidenz den Rückgang auf exemplarische Anschauung von Individuellem [...] Die Evidenz analytisch apriorischer Gesetze bedarf

⁹It should be kept in mind that, phenomenologically, mathematical existence is not shown by proving a formal existential statement, but by obtaining evidence for the objects of the intended interpretation of that statement. So there is no worry that the completeness theorem (which is equivalent to ‘Every consistent theory has a model’) trivializes 2.5, for such a trivialization would be an equivocation of the formal and the phenomenological senses of existence. The models exhibited in proofs of the completeness theorem hardly ever bear any resemblance to our intended interpretation.

solcher bestimmten individuellen Anschauungen nicht, sondern nur irgendwelcher Exempel von Kategorialien, evtl. mit unbestimmt allgemeinen Kernen (wie wenn Sätze über Zahlen als Beispiele dienen), die zwar auf Individuelles intentional zurückweisen mögen, aber in dieser Hinsicht nicht weiter befragt und ausgelegt werden müssen. [Hua XVII, p.221]

These ‘unbestimmt allgemeine Kerne’ justify the inference from 2.3 and 2.4 to 2.5: purely categorial objects may be constituted on the basis of any hyletic-material content whatsoever, all that matters is form.¹⁰ This means, first, that mathematical objects are not arbitrary, but are in their constitution constrained by the laws of categorial formation [Hua XIX/2, section 62]; and, secondly, that one may use spontaneous imagination to start the constitution process. This is all similar to Brouwer, who says that ‘mathematics comes into being when the two-ity created by a move of time is divested of all quality by the subject’, thus leaving ‘the empty form of the common substratum of all two-ities’ [Brouwer 1948, p.1237]. Husserl and Brouwer agree that (in Husserl’s terms) the formal is founded on the hyletic: the constitution of any formal object has to start with some hyletic material, but precisely this material is then abstracted from.

Hence, the particular feature of mathematical essences is that they govern a priori possibilities, possibilities that are independent of particular sense data:

Alle mathematischen Existentialsätze haben diesen modifizierten Sinn: ‘es gibt’ Dreiecke, Vierecke, Polygone aller weiter aufsteigenden Zahlen; ‘es gibt’ regelmäßige Polyeder von 56, aber nicht von allen Zahlen von Seitenflächen. Der wahre Sinn ist nicht schlechthin ein ‘es gibt’, sondern: es ist a priori möglich, das es gibt. [Husserl 1985, p.450]

¹⁰Two remarks:

1. Rosado Haddock arrives, via a different route (a discussion of analyticity) at essentially the same conclusion as 2.5: ‘Actually, the laws of the possibility of constitution of categorial objectualities are precisely the laws of mathematical existence’ [Rosado Haddock 1987, p.97]. He does not connect it to the issue of revisionism. (And whether these ‘laws of possibility’ are of a constructivistic nature (in the sense in which intuitionism is), is another question.) His remark is a special case of one that Husserl makes in the 6. Logische Untersuchung: ‘Die idealen Bedingungen der Möglichkeit kategorialer Anschauung überhaupt sind korrelativ die Bedingungen der Möglichkeit der Gegenstände kategorialer Anschauung und der Möglichkeit von kategorialen Gegenständen schlechthin’ [Hua XIX/2, pp.718–719].
2. Similarly, Gödel writes that one of the distinguishing features of mathematical objects is that

they can be known (in principle) without using the senses (that is, by reason alone) for this very reason, that they don’t concern actualities about which the senses (the inner sense included) inform us, but possibilities and impossibilities. [Gödel 1951, p.312n3]

This remark dates from 1951, which presumably precedes Gödel’s study of Husserl by eight years [Wang 1988, p.121]

Negative Existenzsätze haben die Funktion, die ungültigen Begriffe, die wesenlosen Ausdrücke auszuscheiden. [Hua V, p.83]

As a consequence, to reject a particular mathematical practice it is not necessary (though surely sufficient) that it give rise to contradictions. It may also be that fulfilment of the intentions that make up this practice simply is beyond the power of the laws that govern categorial formation. The point, in both cases, is to show that a meaning can never be fulfilled. (In the second case, we can still have consistency.)

Now consider the following objection to 2.5: ‘Aren’t there many concepts that one would take to be mathematical and that allow for conceptual possibility (i.e., according to essence), but of which we don’t know whether there are objects corresponding to them or not?’¹¹

Answer: the word ‘concept’ is ambiguous between ‘meaning’ and ‘essence’. That we can form a meaning (‘Bedeutung’, or, generalised also to non-linguistic acts, ‘Noema’) does not imply that there is an object falling under it. The origins and formation of a meaning can be studied largely independently of the question whether it refers. If no object corresponding to it exists, there is also no essence of such an object:

Bei logischen Bedeutungen sehen wir nun, daß das Gedachte als solches (logische Bedeutung im noematischen Sinn) ‘widersinnig’ sein kann, es, das doch innerhalb der Seinskategorie ‘logische Bedeutung’ und, allgemeiner, ‘Noema’ ‘existiert’, sein wirkliches Sein hat wie z.B. die Denkbedeutung ‘rundes Viereck’. [...] Das Wesen ist etwas anderes als die Bedeutung. Das Wesen ‘rundes Viereck’ gibt es nicht; aber um das urteilen zu können, ist vorausgesetzt, das ‘rundes Viereck’ eine in dieser Einheitlichkeit seiende Bedeutung ist [...] Bedeutungen setzen und Gegenstände setzen ist zweierlei. [Hua V, pp.85–86,89]

Husserl distinguishes valid meanings from invalid meanings. The valid meanings are those for which the possibility of fulfilment is not a priori excluded, or, in other words, for which fulfilment is ideally possible:

gelten können die Wortbedeutungen als logische Wesen nur dann, wenn nach idealer Möglichkeit das sie in sich aktualisierende ‘logische Denken’ einem ‘entsprechenden Anschauen’ anpaßbar ist; bzw. wenn es ein entsprechendes durch *Intuition* erfaßbares Wesen als entsprechendes Noema gibt, das durch den logischen Begriff seinen getreuen ‘Ausdruck’ findet. [Hua V, p.26; emphases Husserl]

Here is how Husserl applies this criterion to geometry:

Ein geometrisches Urteil *gilt* nur, wenn die Idee, das Wesen Raum und Raumbgestalt *ist*, oder, dem Umfang nach gesprochen, wenn eine Raumbgestalt *möglich* ist. [Hua V, p.82; emphasis Husserl’s]

¹¹This objection was brought to my attention by Richard Tieszen.

Further on in that example he says that an existence proof consists in showing

daß es im Raum seinem Wesen nach ein diesem Gestalt-*begriff* (einer frei gebildeten logischen Bedeutung) entsprechenden geometrisches Wesen wahrhaft gibt. [...] Jedes gültige geometrische Urteil setzt eidetische Einzelheiten (was äquivalent ist mit einer Setzung entsprechender Wesen als Gegenstände), die insgesamt das durch die gültig gesetzte regionale Idee umgrenzte Gebiet der Ontologie ausmachen. [Hua V, pp.82–83, emphasis Husserl's]

Having thus justified 2.5, let us now relate conclusion 2.9 to mathematical practice. Let A be the class of objects that figure in actual practice at a particular moment. From a revisionist point of view (weak or strong), some of them are admissible, others may not be. Let C be the class of objects that will pass the revisionist criteria. Then $A - C$ is the class of objects rejected on revisionist grounds. Strong revisionism allows for the possibility that $C - A$ contains objects, i.e. that there are admissible mathematical objects that are found not by mathematical but by philosophical considerations. For Husserl, making such considerations requires that we leave the natural attitude that the working mathematician is in. The most interesting cases of strong revisionism are those that are not compatible with existing practice.

As we saw in the discussion of the Britannica argument in section 1, Husserl frames his weak revisionism in terms of strict constitution. In *Formale und transzendente Logik*, he describes the activity of his revisionism as a kind of creation (of our awareness of the object, not of that object itself):

Solche Kritik ist schöpferische Konstitution der betreffenden Gegenständlichkeiten in der Einheit einstimmiger Selbstgegebenheit und Schöpfung ihrer Wesen und Wesensbegriffe. [Hua XVII, p.188]

Stated in these terms, 2.7 would read that the capacity to ‘re-create’ formal objects that already figure in practice, is also the capacity for ‘creating’ formal objects that do not. Hence 2.9 says that the same methods that enable Husserl to exercise his weak revisionism in fact make a strong revisionism possible, namely, by shifting attention from A —the objects in current practice—to $C - A$ —the objects that are acceptable from a philosophical point of view, whether they figure in current practice or not. I surmise that much of Brouwer’s development can be seen as such a shift. Brouwer’s introduction of choice sequences, when he realized that his theory of the subject allowed him to do so.

An objection to this argument for strong revisionism in Husserl might be found in the fact that Husserl himself sees the role of phenomenology in mathematics differently. He recognizes that there are different conceptions of mathematics, but he assumes that they all have a common core that is responsible for their being *mathematics*. The task of phenomenology then would be to study this core meaning. This view can be found in *Formale und transzendente Logik* [Hua XVII, pp.12,14]. An even more interesting place is in an as yet unpublished manuscript from 1932 [Husserl 1932], because it is one of the few occasions in his writings where Husserl mentions Brouwer [Schuhmann 1975]:

Soll man sich etwa in der Beurteilung der Mathematik, deren Gesamtsinn ganz und gar von diesen [Grund]begriffen abhängt, an einen Hilbert halten oder an einen Brouwer, oder an wen sonst? Ist es so sicher, trotzdem gerade das heutzutage *communis opinio* ist, daß nicht die klassische Mathematik und ebenso Physik besser beraten war? Aber da werden wir nicht besser fahren. Sie war nie ein Fertiges sondern selbst im Werden, und so wiederholt sich die Schwierigkeit, die Unmöglichkeit einer eindeutigen Auswahl, die uns normbestimmend sein könnte.

Indessen es zeigt sich bald, daß es in der Tat auf eine solche Auswahl durch Entscheidung für irgend eine Partei oder irgend einen maßgebenden Forscher wenig ankommt. Jeder, der Mathematik studiert hat, besitzt das allgemeine Phänomen, das da heißt Mathematik—Mathematik als diese exakte Wissenschaft, die in jeder Zeit im Werden ist und im Lauf der Gesamtzeit, in der sie war und noch ist, die eine im Werden jede Gegenwart gewordene und von Gegenwart zu Gegenwart doch einheitliche im Fortwerden, unerachtet aller nie fehlenden Diskrepanzen der forschenden Personen und ihrer begrifflichen, ihrer theoretischen Leistungsgebilde. [Husserl 1932, 20b-21a]¹²

Husserl argues that phenomenological investigation of the foundations of mathematics should not follow the lead of any particular foundational program. His reasoning here seems to be that, as the science ever grows, there can be no final and univocal selection of concepts. Phenomenology should study the core meaning of mathematics instead; evidently he views *Formale und transzendente Logik* as such a study [Husserl 1932, 24a].

However, although a convincing case could be made that the core meaning of mathematics lies in its formality, a limitation of phenomenological attention to this core does not square with phenomenology as giving the universal ontology as described in section 1 above. In that conception, to which I hold here, any foundational concept proposed in any program is subject to critical investigation. A suspicion that there perhaps never will be a definitive set of all concepts is no reason to stop judging the philosophical merits of any specific concept.

Moreover, from this quote it transpires that Husserl thinks of all the alternative foundational programs as positive sciences, concerned as they are with ‘theoretische Gebilde’. But that makes him overlook the possibility that there might already be a program that is not a positive science, but roots in considerations about the knowing subject. It seems Schmit does the same when he describes Husserl’s investigations as

Untersuchungen, die über die wissenschaftsinterne Begründung, wie die intuitionistische Mathematik sie impliziert, hinausgehen und auf eine Klärung der Subjektivität als solcher abzielen. [Schmit 1981, p.119n262]

¹²I thank Karl Schuhmann for correcting the transcription.

One might argue that Brouwer’s intuitionism, other than what Husserl and Schmit suggest, is a program that begins with the subject. (A source that makes this clear could have been known to Husserl, i.e. Brouwer’s first Vienna lecture of 1928 [Brouwer 1929]; but see note 4.)

Related to this is another objection to my interpretation of phenomenology as a strong revisionism, an objection that comes from Bachelard [Bachelard 1968]. Consider her following paragraph, where she discusses Husserl’s silence on intuitionism (I split it in 3 parts to comment upon):

(1) Interpreting Husserl’s silence on this point, we think that he remains an orthodox mathematician who is obliged to consider intuitionist mathematics as a mathematics on the *fringe* of classical mathematics.

(2) One can rightly object that in wanting to ‘ground’ mathematics through a fundamental reform, intuitionism only succeeded in setting up a new mathematics alongside classical mathematics. The intuitionist conceptions, while giving themselves over to a sort of intentional inquiry, concluded in such an amputation of traditional mathematics that they cannot claim to institute a grounding sense-investigation for mathematics.

(3) Husserl, on the contrary, is unwilling to sacrifice any part of classical mathematics. [Bachelard 1968, p.123; original emphasis]

Concerning (1) and (3), I think that Bachelard is right in ascribing this attitude to Husserl. That would explain how, in *Formale und transzendente Logik*, while claiming, as we saw, to investigate the core of *any* mathematics, he can arrive at conclusions that contradict intuitionism, such as the omnitemporality of all mathematical objects [Hua XVII, p.164]; the unlikely way out would be to deny that intuitionist mathematics exists. But I also argued above that this attitude runs counter to his own general views on ontology.¹³

As for (2), there is a difference between pure meaning-investigation (noematic analysis¹⁴) on the one hand and analysis of essences (eidetic analysis) on the other. Bachelard in effect criticizes intuitionism for not carrying out noematic analyses. And indeed it does not; but to do so has never been its aim in the first place.¹⁵ The project of intuitionism is not to analyse the meaning

¹³At the time of *Formale und transzendente Logik* [Hua XVII, §77], Husserl has come to recognize the idealizations involved in the principle of the excluded middle—the classical mathematician’s fists to box with, as Hilbert once put it [Hilbert 1928, p.80]. That acclaimed law, Husserl says, is not grounded in a subjective law of evidence. But it is not clear that Husserl actually considered the idealizations involved unjustified.

¹⁴Robert Sokolowski [Sokolowski 1984] has argued, against interpretations put forward by Mohanty, Gurwitsch and Føllesdal, that there is a difference between noematic analysis and meaning analysis as normally understood. Namely, the former would require the performance of the phenomenological reduction while the latter does not. For the present argument, the outcome of that discussion does not matter, as both alternatives show the same contrast with the critical eidetic analysis. (Richard Cobb-Stevens told me that Bachelard would have equated noematic analysis and meaning analysis.)

¹⁵But Bachelard sees what Husserl doesn’t, namely, that intuitionism is, in a sense, concerned with intentionality in mathematics.

of concepts in the various philosophies of mathematics. It rather strives to find out, by philosophical means, what is possible in mathematics and what is not. In phenomenological terms: intuitionism attempts eidetic analysis.¹⁶

And that makes all the difference. Eidetic analysis is critical, it separates ‘Wesen’ from ‘Unwesen’ [Hua V, p.89]. Noematic analysis is not critical; it confines itself to a pure investigation of meaning (keep in mind footnote 14), and does not ask the further question whether an intention of that meaning can (ideally) be fulfilled; but, according to the principle of transcendental idealism, it is the answer to that question that is decisive in ontological matters.¹⁷ For example, noematic analysis will not tell us that round squares cannot exist; that judgement requires eidetic investigation (for this and the previous paragraph, [Hua V, section 16]).

Also note how (2) stresses only the negative aspect of intuitionism while neglecting the new possibilities opened up by it. One could say that the first act of intuitionism is an act of weak revisionism, and the second act one of strong revisionism.

To summarize. What is the phenomenological standard for a correct argument in ontology? Such an argument should attempt a strict constitution analysis within a reflexive transcendental phenomenology. I have argued that this standard opens the way to strong revisionism in mathematics.

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¹⁶Dummett’s project [Dummett 1975] is to justify intuitionistic mathematics on purely meaning-theoretical grounds, thus leaving out intuition altogether. From a phenomenological point of view, such an approach must be mistaken, but I will not work out the specific argument here. An argument, independent of phenomenology, to the effect that a meaning-theoretical approach cannot make intuition dispensable, has been put forward by Parsons [Parsons 1986].

¹⁷I cannot agree with Schmit when he gives the following as one of his reasons for not discussing transcendental phenomenology in his work on Husserl’s philosophy of mathematics [Schmit 1981, p.144]:

Im Gegensatz zur wissenschaftsimmanenten Reflexion, die einen positiven Einfluß auf die Methoden der Wissenschaft hat, präjudiziert die transzendente Konstitution in keiner Weise über den Inhalt und die Methoden der Mathematik. Dies erklärt, weshalb die Transzendentalphilosophie keine Berücksichtigung in dieser Arbeit findet, die sich in erster Linie mit Husserls Mathematikbegriff im prägnanten Sinn beschäftigt.

I argue that, because of the entirely formal nature of pure mathematics, transcendental phenomenology does have an a priori contribution to make.

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