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HOW REPRESENTATION AND COMMUNICATION INFRASTRUCTURES CAN ENHANCE MATHEMATICS TEACHER TRAINING

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Information and Communication Technology (ICT) is used more and more in pre-service or in-service mathematics teacher training according to different approaches: face to face, blended, on-line only. Based on some experiences from different countries (Italy, New Zealand, The Netherlands), the paper points out some problems, which emerge in such cases, and poses a few related questions that will be discussed during the Workshop.

Key words: e-learning, teacher training, pedagogical technology knowledge.

INTRODUCTION

The main goal of the WSG is to discuss how Information and Communication Technology (ICT), conceived as Representation and Communication Infrastructures (Hegedus & Moreano-Armella, 2009), can be used in pre-service or in-service mathematics teacher training.

More precisely, the paper introduces some issues arising from projects and experiences developed in three different cultural and social realities (Italy, New Zealand, The Netherlands), where ICT are used to train mathematics teachers. During the workshop, the presenters will ask for feedback on such issues based on each one's personal or professional experience.

The paper is divided into three parts: first some historical reflections on the developing of ICT and e-learning; then a short summary of the experiences from the three countries; finally a list of issues, which emerge from those experiences.

ICT AND E-LEARNING: AN HISTORICAL EVOLUTION

The technologies began to have a widespread use in teaching in the late nineties under different names, and particularly with a relatively fast diffusion of e-learning methods.
The initial approach was essentially characterized by access to multimedia and hypertext content through Internet, therefore rather crude in terms of learning problems. Successively, some researches followed, which led to new practices in using technologies for teaching, all characterized by a greater focus on the learning processes (for example the so called instrumental approach: see below).
As discussed in Arzarello et al. (2012) and following G. Marconato (2009), this progressive shift can be divided into four phases:

- The past: e-learning or delivery mode (construction, organization and distribution of information/content).
- The present: collaborative and networked learning (communication, collaboration and construction of artefacts).
- The near future: connected learning (social networks, sharing resources, knowledge construction).
- The distant future: immersive learning (visualization, manipulation, interaction and construction).

Practices have moved from a static posting of documents on the web to a progressive dynamic management of documents, arriving at a strong interaction between people and various technologies. Corresponding to each of these four phases, different models were used, which differ greatly from each other, both in terms of how the technology is exploited in them, as well as the educational assumptions on which they are implemented:

- From formal to informal: from learning models based on the school setting with well-defined roles, with well formalized curricula and assessment systems, the new learning environments evolve towards informal learning models based on experience, reflection, conversation, on the solution to problems, on doing activities.
- From structured to unstructured: the structured, formal, sequential, rigidly planned learning environments become unstructured, open, with objectives to achieve but with no predetermined path, continuously adapting to the context.
- From static to dynamic: while the core values of traditional environments are the rigid structure and the planned programming, the new environments are dynamic, with an initial programming representing the intentions, but always ready to adapt to the real conditions in which the process is taking place.
- From certain to uncertain: uncertainty prevails regarding both the outcome, aware that it is not possible to precisely predict the outcome of a complex process such as learning, and the actual process itself.
- From generic to situated: the knowledge objectives and the knowledge itself abandon vagueness and abstractness, focussing increasingly on the context, the

specific situation of use, in line with the principle that the more generic the education is, the less effective.

- From conformity to divergence: before students were asked to comply with rules dictated by the subject, but now preference is given to divergent thinking, creativity, the pursuit of personal goals by students.
- From dependency to responsibility: the attitude of substantial dependence on external choices that the learner in traditional environments is expected to make, becomes an attitude of personal responsibility for the outcome even at the risk of not achieving the hoped-for product.

Instead, the shifts at the practical level are:

- From the content to didactic interaction, construction: the primitive models for the educational use of technology simply replicated the traditional school model, based on content and its transmission. It is now trying to find uses of technology by supporting a richer interaction between the parties involved, a communication in order to share knowledge and experiences, to argue, to address and solve problems, to build artefacts.
- From the centrality of the teacher to the centrality of the learner: in traditional learning environments the teacher, being the repository of content, represented centrality, in the sense that the knowledge was communicated by the teacher and his interpretation of the facts was what the students had to conform to. Now the control of the learning process is in the hands of the learner.
- The teacher's role as expert in the content, to facilitator of the interaction between the learners: with the learner and his learning process having become central, the role of teacher becomes that of a learning support, to the point of disappearing altogether (learning without teaching).
- From the classroom to the community of learning and practice: the classroom is no longer the only place where one learns, in fact one increasingly learns by drawing on the resources in the community.
- From visible technologies to invisible technologies: the initial approach to technology gave the technology itself a strong visibility. With the shift to the use of technology to improve learning processes, the characteristics of these processes have gained centrality and visibility, putting technology into the background, that is, making it transparent and giving it a value not for its own sake (the risk it ran in the past) but for the precise educational and learning purposes for which it is used.

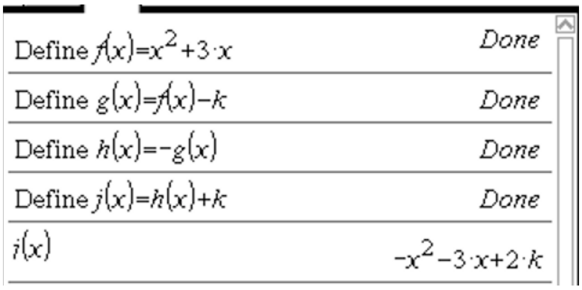
In order to summarize all these shifts, we could say that it is as if there had been a shift from learning with the e-prefix to learning without the prefix, that is a loss of the centrality of technology and a re-appropriation of the centrality of learning processes. Technologies as they are used nowadays can add value to the learning process, based on the methodology, or in their use as a tool to support well-defined teaching strategies, focused on the learning process of students or teachers engaged in professional training. Moreover, since teacher training through technologies has started later, at a riper stage of ICT evolution in teaching, more sophisticated approaches are available (e.g. the instrumented orchestration model: see below) and a specific deeper analysis is needed. We will develop it considering different experiences in three different parts of the World.

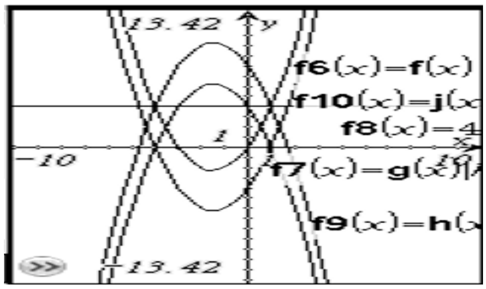
ICT AND TEACHER TRAINING: PERSPECTIVES AND EXAMPLES

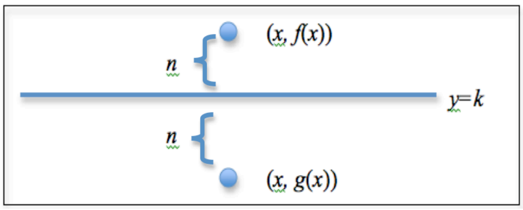
This section is divided into three subsections, where three different experiences will be sketched, each pointing out some different issues about the problems that grow up using ICT in mathematics teacher training. At the end of each subsection some questions arising from it will be pointed out in order to define the issues that will be discussed during the Workshop.

Pedagogical technology knowledge: an example from New Zealand

Consideration of the independent teacher implementation of ICT in school classrooms (rather than by those involved in research) (e.g. Thomas & Hong, 2005) suggests that student activity with ICT often involves its pragmatic value, centred on obtaining answers and checking by-hand work, rather than its epistemic value. Thus, a crucial question teachers could ask when considering implementation of ICT is “How will this ICT use influence student conceptualisation?” (Thomas, Monaghan, & Pierce, 2004). One way to address this is an active, deeper investigation of concepts and an emphasis on those constructs that might not otherwise be prominent. A recent chapter by Heid, Thomas and Zbiek (in press) considers possible activities, with epistemic value, in which CAS could be used to extend student engagement with mathematical constructs in a manner that promotes new knowledge of the mathematics.







$$g(x) = k - n = k - (f(x) - k) = 2k - f(x)$$

Figure 1. Using different representations to improve generalisations and understanding.

Two principles they propose that could be employed by teachers using ICT to assist student learning are: to consider the construction of mathematical generalisations; and to encourage mathematical ways of thinking and ways of understanding (Harel, 2008).

One possible activity of this nature could be: Can we find the function we obtain when the graph of $y = f(x)$ is reflected in a general line $y = k$? You might want to find an algorithm first. Can you justify it algebraically? What would be the result for a reflection in the straight line $y = mx$? A lesson on this might involve students using simple polynomial functions to work out an algorithm to perform the reflection (translate by $-k$ to the x-axis, reflect in the x-axis using $-f(x)$, and then translating back by k). Then a conjecture might be formulated about a general result, and finally this could be proved using algebraic techniques (see Figure 1). The final generalisation to reflection in $y = mx$ is, of course, much more difficult, although producing the algorithm: rotate the line $y = mx$ until it lies on the x-axis, reflect in the x-axis using $-f(x)$, and rotate back through the same angle, would be a good final step, unless students are familiar with matrices.

It is not easy for many teachers to orchestrate the various components of the classroom milieu involving ICT in such a way that the result is what Brousseau calls a cognitive epistemological learning situation. They face many constraints or obstacles, such as their previous experience in using technology, lack of time, few opportunities for professional development (PD), poor access to technology, limited availability of classroom teaching materials, little support from colleagues, pressures of curriculum and assessment requirements, and inadequate technical support (Thomas & Chinnappan, 2008). However, most importantly it may necessitate a change in thinking and attitude for teachers.

Addressing such teacher-related issues is crucial in the successful implementation of ICT in mathematics learning. Recognition that didactical use of ICT requires a complex set of skills and attitudes on the part of teachers is a first step. A consideration of this led Thomas (Hong & Thomas, 2006; Thomas & Hong, 2005) to propose the notion of pedagogical technology knowledge (PTK) as a construct that could be a useful indicator of how ICT might be used. A developed PTK implies understanding of the principles and techniques required to build didactical situations incorporating ICT, enabling a focus on mathematical learning through the technology. Thus a number of teacher factors combine to produce PTK, including: instrumental genesis; mathematical knowledge for teaching (Hill & Ball, 2004); teacher orientations and goals (Schoenfeld, 2011), especially beliefs about the value of technology and the nature of learning mathematical knowledge, and other affective aspects, such as confidence. Thus with active PTK specific mathematical conceptions are placed firmly at the centre of classroom activity directing teachers to structure content and classroom discourse and activities into didactical situations.

Following research (Thomas, Hong, Bosley & delos Santos, 2008) that suggested improved confidence in classroom use of technology as a key driver of the growth of PTK, a recent study sought to examine whether confidence and PTK might be linked (Palmer, 2011). Measures of 42 female teachers' confidence in using technology and PTK level were produced. The latter was obtained using a composite score for belief in the value of technology in teaching mathematics, the level of instrumental genesis and the kinds of pedagogical approach employed. Three distinct groups with low, medium and high levels of confidence emerged, with statistically significant differences in means. There was also evidence of a significant correlation between the confidence and PTK measures (Palmer, 2011), with the mean PTK scores in the three confidence groups differing significantly. Although this evidence does not admit talk about causation, either way, it does suggest the possibility that confidence and PTK are related. This raises the question of how we might simultaneously develop further mathematics teachers' confidence in using technology and their PTK. There seems to be a continuing need for high-quality classroom-based resources that will assist in building didactical situations, along with the corresponding professional development (PD) to implement them. We suggest here that focused and supportive in-service professional development is a critical element. How this PD might be organised using a supportive community is discussed below.

This example can be used to introduce the following questions:

- What additional values does ICT promote compared with more traditional approaches?
- Does the use of ICT modify the way teachers look at mathematics?

Professional development in a community: an example from The Netherlands

Nowadays, mathematics teachers are confronted with a myriad of technological tools to integrate in their teaching. While this availability offers opportunities to improve teaching and learning, it may also confuse teachers and question their practices. A professional development is needed to fully exploit the affordances technology offers. The question, therefore, is what such a professional trajectory might look like, and how technology may act as a vehicle in a collaborative professional development process.

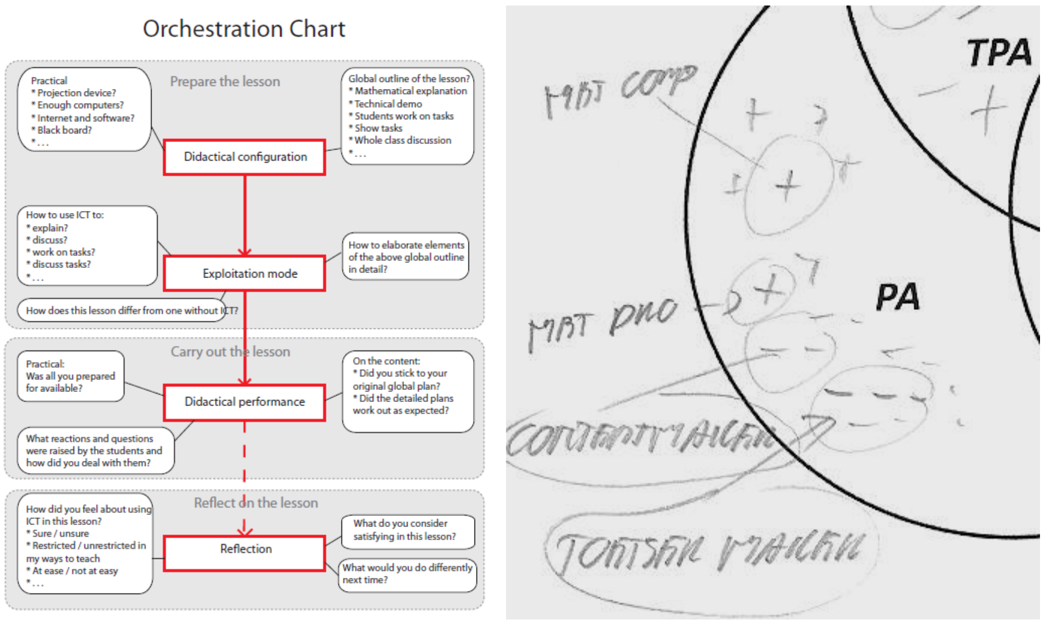


Figure 2. Orchestration Chart (left) and part of a filled-in TPACK Chart.

To answer these questions, a theoretical framework including notions of instrumental orchestration (Drijvers et al. 2010, Drijvers 2012, Trouche 2004, Trouche & Drijvers 2010) and TPACK (Koehler, Mishra & Yahya 2007) underpins the setting-up of a small-scale community of 12 mathematics teachers.

To invite professional development, technology-rich teaching materials are provided that beg for restructuring teaching practice, as well as support for face-to-face meetings and virtual communication through a Moodle environment.

The notion of instrumental orchestration guided the design of an 'orchestration chart' (see left part of Figure 2) and a blog template; TPACK inspired us to develop a TPACK chart that was filled in by the participants three times throughout the project period (see right part of Figure 2 for an example). Data includes blogs written by the teachers, self-reports on their professional development needs, student work and lesson observations.

The results suggest that teachers indeed benefit from being engaged in a mutual process of professional development and that technology indeed offers a vehicle to support this; meanwhile, exchange within the community through digital means remained limited, which suggests that face-to-face meetings remain crucial in such a community.

This example can be used to introduce the following question:

- How to use ICT to support teacher professional development in a community?

Professional development in a community: an example from Italy

In Italy since 2006 there is a project of blended e-learning for teachers of mathematics (lower and higher secondary school), called M@t.abel, an acronymous for "Matematica: apprendimenti di base con e-learning" (= Mathematics: basic learning through e-learning), see Arzarello et al. (2012) and: http://risorsedocentipon.indire.it/home_piattaforma/.
M@t.abel implements and supports the curriculum that has been proposed in past years (2001-2005) by the Italian Commission for the Learning of Mathematics (CIIM), a permanent commission of the Italian Mathematical Union (UMI). The curriculum of the UMI-CIIM, called "Mathematics for the Citizen" concerns the whole Italian pre-university school, from 6 to 19 years and contains also 200 examples of learning situations. It was presented at ICME 10 (see: Materiali per la Scuola in: http://umi.dm.unibo.it/area_download--37.html). This curriculum has partially inspired the Italian Ministry of Education for designing the new National Curriculum of Mathematics. It proposes an integrated use of the mathematics laboratory in the teaching of mathematics. After 2005, the "Mathematics for the Citizen" has been developed into a project of blended e-learning for the Italian teachers of Mathematics and this activity has generated the M@t.abel project, which has involved up to now more 5000 mathematics teachers (grades 6-10) and is now extending to all grades (1-13) teachers. The main goal of the project is to encourage and support the teachers to use the UMI materials and particularly the mathematics laboratory in their classrooms. Hence M@t.abel presents as a concrete help for the Italian teachers to bridge the gap between the old fashioned curriculum and the new one.

The mathematics laboratory is seen as a structured set of activities for the construction of meanings of mathematical objects. It involves people (students and teachers), facilities (classrooms, suitable tools, organization of areas and time), ideas (projects, plans for educational activities, experiments). The tools to be used can be of various kinds, from the poorest and most traditional materials such as paper, squared and transparent sheets, pins, ruler and compass, to the most recent tools like software for geometry and symbolic manipulation, spreadsheets, Internet links. The mathematics laboratory environment is in some way comparable to that of a Renaissance workshop where apprentices learned by doing and seeing, communicating with each other and with teachers. With an evocative metaphor we can say that the classroom becomes like a concert hall where the students are the musicians, the teacher is the conductor and the materials that are used are the musical instruments.

The m@t.abel project is carried out according to a blended e-learning, which consists of face to face meetings, online meetings, experimentation, document production; teachers are divided in virtual classrooms of about 15 people, who work under the supervision of a more expert tutor specifically trained for this task. The e-learning activities are planned in different phases:

a) Initial face to face training. Presentation of the project; illustration of the m@t.abel methodology of teaching-learning: the mathematics laboratory; presentation of didactic material: objectives, conceptual nodes, methodology; in-depth analysis of one of the activities from the conceptual and methodological point of view with the support of the e-learning environment; worksheet reading for the course member: discussion on the concrete use in the classroom of the learning activities available in the platform; knowledge of the e-learning environment and its use; definition of an experimentation protocol.

b) On-line training. This aims at making teachers acquainted with the didactic materials on the platform and at sharing of the experimentation programme. Each course member performs a careful analysis of the proposed activities, discusses and share them with the course colleagues in a virtual classroom. From these activities, the teachers choose two of them and experiment them in the classroom following a shared protocol.

c) Experimentation in the classroom. During the experimentation, the group interacts with the support of the platform and discusses on-line the teaching and technical problems that gradually arise interacting with the tutor and the colleagues.

d) Document production. While experimenting with each activity, the course member drafts a log-book according to a shared protocol. In the log book the course member: explains the main conceptual nodes that the activity refers to; describes the experience in class and the methodology used (worksheets, group work, discussion of mathematics in the classroom, used software, etc...); assesses how the activity was understood by the students and how they dealt with the assigned task; notes the difficulties encountered by students in understanding the various mathematical concepts and methods for overcoming them; comments on the proposed assesment tests and their results.

This example can be used to introduce the following questions:

- How ICT can be concretely used in teachers' training?
- Does the use of ICT modify teachers' practices in their classroom?

UP-AND-COMING ISSUES

Based on the three examples above, the presenters will ask people participating to the Workshop to share ideas about the following issues concerning teacher training:

- What additional values does ICT promote compared with more traditional approaches? (Example from New Zealand)
- Does the use of ICT modify the way teachers look at mathematics? (Example from New Zealand)
- How ICT can support the growing of Community of practice in teachers? (Example from The Netherlands)
- How ICT can be concretely used in teachers' training? (Example from Italy)
- Does the use of ICT modify teachers' practices in their classroom? (Example from Italy)

The Workshop will be organized as follows:

- Sketchy presentations of the three experiences, each illustrating some of the issues above and asking specific questions to the ground (30 min).
- Subdivision in 3 small groups: each discusses some of the questions (40 min).
- Summary of the discussion in the groups (20 min).

References

Arzarello F., & Robutti O. (2010). Multimodality in multi-representational environments. *ZDM, The International Journal on Mathematics Education*, *42*, 715-731, ISSN: 1863-9690.

Arzarello, F., Ciarrapico, L., Robutti, O., & Anichini, G. (2012). *M@t.abel: Mathematics for students on the threshold of the third millennium*, Italian Ministry of Education, University and Research (MIUR).

Arzarello, F., Ferrara, F., & Robutti, O. (2012). Mathematical modelling with technology: the dynamic role of representations, *Teaching Mathematics and its Applications*, *31*(1), 20-30. ISSN 1471-6976.

Drijvers, P. (2012). Teachers transforming resources into orchestrations. In G. Gueudet, B. Pepin, & L. Trouche (Eds.), *From text to 'lived' resources: mathematics curriculum materials and teacher development* (pp. 265-281). New York/Berlin: Springer.

Drijvers, P., Doorman, M., Boon, P., Reed, H., & Gravemeijer, K. (2010). The teacher and the tool: instrumental orchestrations in the technology-rich mathematics classroom. *Educational Studies in Mathematics, 75*(2), 213-234.

Harel, G. (2008). DNR perspectives on mathematics curriculum and instruction, Part I: Focus on proving. *ZDM International Journal of Mathematics Education, 40*(3), 487–500. doi:10.1007/s11858-008-0104-1.

Hegedus, S., & Moreno-Armella, L. (2009). Intersecting representation and communication infrastructures. *ZDM: The International Journal on Mathematics Education: Transforming Mathematics Education through the Use of Dynamic Mathematics Technologies, 41*(4), 399-412.

Heid, M. K., Thomas, M. O. J., & Zbiek, R. M. (In press). How might computer algebra systems change the role of algebra in the school curriculum? In A. J. Bishop, M. A. Clements, C. Keitel, J. Kilpatrick, & F. K. S. Leung (Eds.) *The third international handbook of mathematics education*. Dordrecht: Kluwer Academic Publishers.

Hill, H., & Ball, D. L. (2004). Learning mathematics for teaching: Results from California's mathematics professional development institutes. *Journal for Research in Mathematics Education, 35*(5), 330–351. doi: 10.2307/30034819.

Hong, Y. Y. & Thomas, M. O. J. (2006). Factors influencing teacher integration of graphic calculators in teaching. *Proceedings of the 11th Asian Technology Conference in Mathematics*, Hong Kong, 234-243.

https://php.radford.edu/~ejmt/ContentIndex.php and http://www.radford.edu/ejmt.

Koehler, M. J., Mishra, P., & Yahya, K. (2007). Tracing the development of teacher knowledge in a design seminar: Integrating content, pedagogy and technology. *Computers and Education, 49*, 740-762.

Marconato G. (Ed.) (2009). *Le tecnologie nella didattica. Lo stato dell'arte nel nuovo millennio*, Erickson, Trento, 2009.

Palmer, J. M. (2011). *Examining relationship of teacher confidence to other attributes in mathematics teaching with graphics calculators.* Unpublished MSc Thesis, The University of Auckland.

Schoenfeld, A. H. (2011). *How we think. A theory of goal-oriented decision making and its educational applications*. Routledge: New York.

Thomas, M. O. J., & Chinnappan, M. (2008). Teaching and learning with technology: Realising the potential. In H. Forgasz, A. Barkatsas, A. Bishop, B. Clarke, S. Keast, W-T. Seah, P. Sullivan, & S. Willis (Eds.), *Research in Mathematics Education in Australasia 2004-2007* (pp. 167-194). Sydney: Sense Publishers.

Thomas, M. O. J., & Hong, Y. Y. (2005). Learning mathematics with CAS calculators: Integration and partnership issues. *The Journal of Educational Research in Mathematics, 15*(2), 215–232.

Thomas, M. O. J., Hong, Y. Y., Bosley, J., & delos Santos, A. (2008). Use of calculators in the mathematics classroom. *The Electronic Journal of Mathematics and Technology* (eJMT), [On-line Serial] 2(2), Available at:

https://php.radford.edu/~ejmt/ContentIndex.php and http://www.radford.edu/ejmt.

Trouche, L. & Drijvers, P. (2010). Handheld technology: Flashback into the future. *ZDM, The International Journal on Mathematics Education, 42*(7), 667-681.