Diagram Techniques for Confluence

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Abstract

We develop diagram techniques for proving confluence in abstract reductions systems. The underlying theory gives a systematic and uniform framework in which a number of known results, widely scattered throughout the literature, can be understood. These results include Newman's Lemma (1942), Lemma 3.1 of Winkler and Buchberger (1985), the Hindley-Rosen Lemma (1964), the Request Lemmas of Staples (1975), the Strong Confluence Lemma of Huet (1980), and the Lemma of De Bruijn (1978).

Introduction

The concept of Term Rewriting System (TRS) is paradigmatic for the study of computational procedures. TRSs play an important role in various areas, such as abstract data type specifications, implementations of functional programming languages and automated deduction.

Usually, rewriting is concerned with syntactical objects like terms, strings, term graphs, equivalence classes of terms, or other structured objects. Terms may be first-order or higher-order, such as λ-terms or proofs in some deduction system. However, many of the basic definitions and facts can already be stated on a more abstract level, where the structure of the objects to be rewritten is not yet of relevance. To express this level of abstraction we use the neutral term 'reduction' instead of 'rewriting'. In the next section we give the necessary elementary definitions and basic facts about Abstract Reduction Systems (ARSs).

We develop diagram techniques for proving confluence in abstract reductions systems. The underlying theory gives a systematic and uniform framework in which a number of known results, widely scattered throughout the literature, can be understood. These results include Newman's Lemma (1942), Lemma 3.1

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of Winkler and Buchberger (1985), the Hindley-Rosen Lemma (1964), the Request Lemmas of Staples (1975), the Strong Confluence Lemma of Huet (1980), and the Lemma of De Bruijn (1978).

The present paper extends van Oostrom (1994b, 1994a) in the following ways: all results about reduction diagrams are new, and the concept of trace-decreasing diagram refines the concept of decreasing diagram, has a clearer visualization and yields a new proof of the main theorem. With over 30 figures the approach here is more geometric, as opposed to the more algebraic approach of van Oostrom (1994a).

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