

States of Change: Explaining Dynamics by Anticipatory State Properties*

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Abstract

In Cognitive Science, recently Dynamical Systems Theory (DST) has been advocated as an approach to cognitive modelling that is better suited to the dynamics of cognitive processes than the symbolic/computational approaches are. Often the differences between DST and the symbolic/computational approach are emphasized. However, if two approaches are used also their commonalities can be analysed, and a unifying framework can be sought. In this paper the possibility of such a unifying perspective on dynamics is analysed. The analysis does not only cover dynamics in the cognitive discipline, but also in other disciplines: Physics, Mathematics and Computer Science. The unifying perspective warrants the development of integrated approaches covering both DST aspects and symbolic/computational aspects.

The notion of a state-determined system lies at the heart of DST. This type of system is based on the assumption that properties of a given state fully determine the properties of future states. Taking this assumption as a premise, in this paper the explanatory problem of dynamics is analysed in more detail. The analysis of four cases within different disciplines (Cognitive Science, Physics, Mathematics, Computer Science) shows how in history this perspective has led to a number of often used concepts within these disciplines. In Cognitive Science the concepts desire and intention were introduced, and in classical mechanics the concepts momentum, energy, and force. Similarly, in Mathematics a number of concepts have been developed to formalise the state-determined system assumption. Derivatives (of different orders) of a function, and Taylor approximations are examples of such concepts. Furthermore, also transition systems, a currently (within Computer Science and related areas) popular format for specification of dynamic systems can be interpreted from this perspective. One of the main contributions of the paper is that the case studies provide a unified view on the explanation of dynamics across the chosen disciplines. All approaches to dynamics analysed in this paper share the state-determined system assumption and the (explicit or implicit) use of anticipatory state properties.

Within Cognitive Science realism is one of the problems identified for the symbolic/computational approach, i.e., how do internal states described by symbols relate to the real world in a natural manner. As DST is proposed as an alternative to the symbolic/computational approach, a natural question is whether for DST the realism of the states can be better guaranteed. As a second main contribution the paper provides an evaluation of DST compared to the symbolic/computational approach, which shows that in this respect, i.e., for the realism problem, DST does not provide a better solution than the other approaches. This shows that DST and the symbolic/computational approach not only have the state-determined system assumption and the use of anticipatory state properties in common, but also the realism problem.

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1 Introduction

Due to dynamics the world occurs in different states, i.e., states at different points in time that differ in some of their state properties. In recent years, within Cognitive Science dynamics has been recognized and emphasized as a central issue in describing cognitive processes; for example, (Port and Gelder, 1995; Kelso, 1995). Van Gelder and Port, (1995) emphasize and position dynamics as opposed to the symbolic/computational approach and the connectionist approach. The *Dynamical Systems Theory* (DST) is advocated as a new paradigm that is better suited to the dynamic aspects of cognition.

As will be elaborated further in Section 1.1 below, the notion that lies at the heart of DST is the notion of a state-determined system. This type of system is based on the assumption that properties of a given state fully determine the properties of future states. Taking this assumption as a premise, in this paper the explanatory problem of dynamics is explored in more detail: how to explain properties of a changed state based on properties of the previous state. As discussed further in Section 1.2, the paper analyses how the addition of anticipatory state properties, or potentialities, to the state ontology provides an explanation in terms of properties of the previous state.

The four cases within different disciplines (Cognitive Science, Physics, Mathematics, Computer Science) analysed in this paper show how in history the state-determined system assumption has led to a number of often used concepts within these disciplines. Think of concepts like desire and intention in Cognitive Science, momentum, energy, and force in classical mechanics, and derivatives of a function and Taylor approximations in Mathematics. Derivatives (of different orders) of a function, and Taylor approximations formalise notions of potentiality. Similarly, transition systems (a popular format for specification of dynamic systems) can be interpreted from the perspective of potentialities. The unified view on the explanation of dynamics across different disciplines is one of the main contributions of the paper. The unified view is at a high level of abstraction; of course it does not deny that at more detailed levels there are differences between dynamics within these disciplines as well.

In particular, within Cognitive Science the unifying perspective shows that at a fundamental level DST and the symbolic/computational approach share the state-determined system assumption and the (implicit or explicit) use of anticipatory state properties. A next question is whether the approaches to model cognitive processes also share some of their problems. One of the problems identified for the symbolic/computational approach is the problem of *realism* (i.e., how do internal states described by symbols relate to the real world in a natural manner). As DST is proposed as an alternative to the symbolic/computational approach, a natural

question is whether for DST the realism of the states can be better guaranteed. As anticipatory state properties in the context of the state-determined system assumption are essential to DST, the question of realism has to be answered for these state properties in particular. The analysis made in this paper, however, does not answer this question positively. It shows that even in Physics the foundational question on whether such postulated anticipatory state properties are genuine, real state properties cannot be answered positively. Therefore, as a second main contribution the paper shows that both approaches have the realism problem in common.

Given that the DST and symbolic/computational approaches have much in common at the foundational level, as discussed in this paper, one of the options is to develop integrated approaches, in which both DST aspects and symbolic aspects can be covered. First proposals for such integrated approaches can already be found, for example, in Sun (2002), or Jonker and Treur (2002, 2003) and Jonker, Treur, and de Vries (2002).

1.1 Some of the Assumptions Underlying the Dynamical Systems Theory

Van Gelder and Port (1995) briefly explain a dynamical system is in the following manner. A *system* is a set of changing aspects (or state properties) of the world. A *state* at a given point in time is the way these aspects or state properties are at that time; so a state is characterised by the state properties that hold. The set of all possible states is the *state space*. A *behaviour* of the system is the change of these state properties over time, or, in other words, a succession or sequence of states within the state space. Such a sequence in the state space can be indexed, for example, by natural numbers (*discrete* case) or real numbers (*continuous* case), and is also called a *trace* or *trajectory*. Given these notions, the notion of *state-determined system*, adopted from Ashby (1952) is taken as the basis to describe what a dynamical system is:

A system is state-determined only when its current state always determines a unique future behaviour. Three features of such systems are worth noting.

First, in such systems, the future behaviour cannot depend in any way on whatever states the system might have been in *before* the current state. In other words, past history is irrelevant (or at least, past history only makes a difference insofar as it has left an effect on the current state).

Second, the fact that the current state determines future behaviour implies the existence of some *rule of evolution* describing the behaviour of the system as a function of its current state. For systems we wish to understand we always hope that this rule can be specified in some reasonable succinct and useful fashion. One source of constant inspiration, of course, has been Newton's formulation of the laws of the solar system.

Third, the fact that future behaviours are uniquely determined means that state space sequences can never fork. (Gelder and Port, 1995), p. 6.

According to some a dynamical system is just a state-determined system; e.g., Giunti (1995). For others a dynamical system is a state-determined system for which the state properties are described by numerical values; e.g., Van Gelder and Port (1995). The term ‘dynamical’ is explained as follows:

The word ‘dynamical’ is derived from the Greek word *dynamikos*, meaning ‘forceful’ or ‘powerful’. A system that is dynamical in this sense is one in which changes are a function of the *forces* operating within it. Whenever forces apply we have accelerations or decelerations. (Gelder and Port, 1995), p. 7.

1.2 Dynamical Systems Require Anticipatory State Properties

Assuming that the world (including the mental world) occurs in different states (i.e., states at different points in time that differ in some of their state properties), and given a particular state that just changed with respect to some of its state properties, it is natural to ask for an explanation of why these new state properties occurred. In a state-based approach, as a source for such an explanation, state properties found in the previous state form a first candidate, and for dynamical systems they are assumed to form the only candidate source. Thus, to analyse the claims underlying Dynamical Systems Theory, a main question becomes how to determine for a certain state that it is going to change to a different state, and, more specifically, how to determine (on the basis of some of the state properties in the given state) those state properties for which the new state will differ from the given one. This poses the challenge to identify state properties (or combinations of state properties) occurring in a given state that in some way or the other indicate which of the (other) occurring state properties will be different in a subsequent state; by having these properties the state anticipates on the next state: *anticipatory state properties*. If such state properties (historically sometimes called *potentialities*) are given, anticipation to change is somehow encoded in a state. The existence of such properties is the crucial factor for the validity of the assumptions underlying the Dynamical Systems Theory. In Ashby (1960), a similar claim is expressed as follows:

‘Because of its importance, science searches persistently for the state-determined. As a working guide, the scientist has for some centuries followed the hypothesis that, given a set of variables, he can always find a larger set that (1) includes the given variables, and (2) is state-determined. Much research work consists of trying to identify such a larger set, for when it is too small, important variables will be left out of account, and the behaviour of the set will be capricious. The assumption that such a larger set exists is implicit in almost all science, but, being fundamental, it is seldom mentioned explicitly.’ (Ashby, 1960), p. 28.

Ashby refers to Temple (1942) and Laplace (1825) for support of his claims. He distinguishes phenomena at a macroscopic level for which his claim is assumed to hold from phenomena at the atomic level, for which the claim turns out not to hold.

‘Temple, though, refers to ‘... the fundamental assumption of macrophysics that a complete knowledge of the present state of a system furnishes sufficient data to determine definitely its state at any future time or its response to any future influence.’ Laplace made the same assumption about the whole universe when he stated that, given its state at one instant, its future progress should be calculable. The definition given above makes this assumption precise and gives it in a form ready for use in the later chapters.

The assumption is now known to be false at the atomic level. We, however, will seldom discuss events at this level; and as the assumption has proved substantially true over great ranges of macroscopic science, we shall use it extensively.’

(Ashby, 1960), p. 28.

In this paper the implications of the basic assumptions underlying DST are analysed by adopting (as a premise) some of these basic assumptions and investigating the consequences of this adopted position with respect to the explanatory problem. Most fundamental among these is the state-determined system assumption as described by Ashby (1952). As a consequence of this assumption, the explanatory value of anticipatory state properties has to be considered. Given this premise, possible conceptual analyses are explored and compared to those actually put forward in history to explain change based on some notion of potentiality as an anticipatory state property. As a unifying perspective the following criteria will be considered as characteristic for a perspective based on anticipatory state properties:

- *succession of states*

The world occurs in successive states at different points in time.

- *state-based ontology*

Within an explanation or description only state properties are used, based on a state ontology. In particular, no concepts for actions, events, transitions between states, or processes are used.

- *necessity of anticipatory state properties*

For each state property *a* that occurs in a given state there exists an anticipatory state property *p* related to property *a* that occurs in a preceding state, in conjunction with some additional conditions on specific circumstances in this state (i.e., no obstruction occurs of the actualisation of *p*).

- *sufficiency of anticipatory state properties*

If p is an anticipatory state property related to the occurrence of property a , and property p occurs, then, given suitable further circumstances (i.e., no circumstances obstructing the actualisation of p), in a subsequent state property a will actually occur.

- *grounding of anticipatory state properties*

For each anticipatory state property p there is a specific characteristic in the past and/or current states that guarantees the occurrence of p .

1.3 Overview of the Paper

In this paper often the word potentiality is used as a short name for an anticipatory state property. The paper is structured as follows. Section 2 discusses the perspectives of Zeno and Aristotle, thereby illustrating the problem of explaining changed states in more detail. Sections 3 and 4, demonstrate in some (non-living and living) cases from the areas of early Physics and Cognitive Science the use of assuming potentialities as anticipatory state properties to explain changed states.

A major issue within Cognitive Science and Philosophy of Mind is whether such postulated anticipatory state properties are genuine state properties. For example, as a desideratum it might be posed that they should be identifiable with ‘real’ and perhaps even directly observable state properties. In Section 5 this fundamental issue is addressed in a case study on potentialities for (loco)motion in Physics. More specifically, it is discussed how in classical mechanics as developed by Descartes, Huygens, Newton and Leibniz, among others, a potentiality for ‘*quantity of motion*’ and one for ‘*moving force*’ were developed; in modern physics known as momentum and kinetic energy. These specific types of potentialities are shown to be different. However, both can be related to the concept of velocity, which, in itself, is also not an unproblematic state property. The discussion shows that the mathematical formalisation of this concept (based on the mathematical notion derivative) is inherently based on properties of states at different points in time. Section 6 describes transition systems, a well known representation within Computer Science to specify change in systems, in terms of potentialities.

Assuming that potentialities exist as anticipatory state properties that can explain properties of subsequent, changed states, a next question is how the occurrence or change of a potentiality itself can be explained. This question is discussed in Section 7. The notion of *higher-order potentialities* (i.e., a potentiality to get a potentiality) is discussed to explain changed potentialities. An analysis of a case study within Mathematics relates potentialities to mathematical concepts such as (higher-order) derivatives and Taylor approximations. Section 8 discusses how potentialities can change due to an *interaction* between objects: exchange of

potentialities. It is argued that, if higher-order potentialities are generally assumed to explain changed potentialities, changes due to interaction between objects have to be attributed to higher-order potentialities as well, i.e., such an interaction is characterised by the higher-order potentialities that co-occur with it. Historically, the second-order potentiality for motion was formalised by the notion of ‘force’ in classical mechanics; interactions between objects are characterised by the forces they impress on each other.

2 Explaining Changed States

Following Zeno, Section 2.1 discusses why changed states cannot be explained on the basis of given ‘real’ state properties, and, hence, why dynamics does not exist in reality. Next, assuming successive states, in Section 2.2 options to obtain an explanation of changed states are discussed.

2.1 Why Motion Does Not Exist

An arrow moving from A to B traverses a number of positions between A and B at different time points. Zeno of Elea (about 490-425 BC) asked himself what the difference in state is between a moving arrow in a certain state at some time point t and an arrow at rest at the same position. He came to the answer that there is not any state property that differs for the two states, so there is no difference between a moving arrow and an arrow at rest. From this he concluded that in reality motion does not exist: it is just an illusion, made up by human perception and processing. Zeno summarised his view in the following paradox:

‘If everything is either at rest or moving when it occupies a space equal to itself, while the object moved is in the instant, the moving arrow is unmoved.’

This formulation was taken from Aristotle (translated by Heath, 1931) who incorporated some of Zeno’s work in his writings; a book written by Zeno himself unfortunately disappeared (it is said to be stolen from Zeno). Also Aristotle (384-322 BC) claimed that motion and change do not refer to anything existing in reality:

‘Again, there is no such thing as motion over and above the things. It is always with respect to substance or to quantity or to quality or to place that what changes changes. But it is impossible, as we assert, to find anything common to these which is neither ‘this’ nor quantum nor quale nor any of the other predicates. Hence neither will motion and change have reference to something over and above the things mentioned, for there is nothing over and above them.’

[from (Aristotle, Physics), Book III, Part 1]

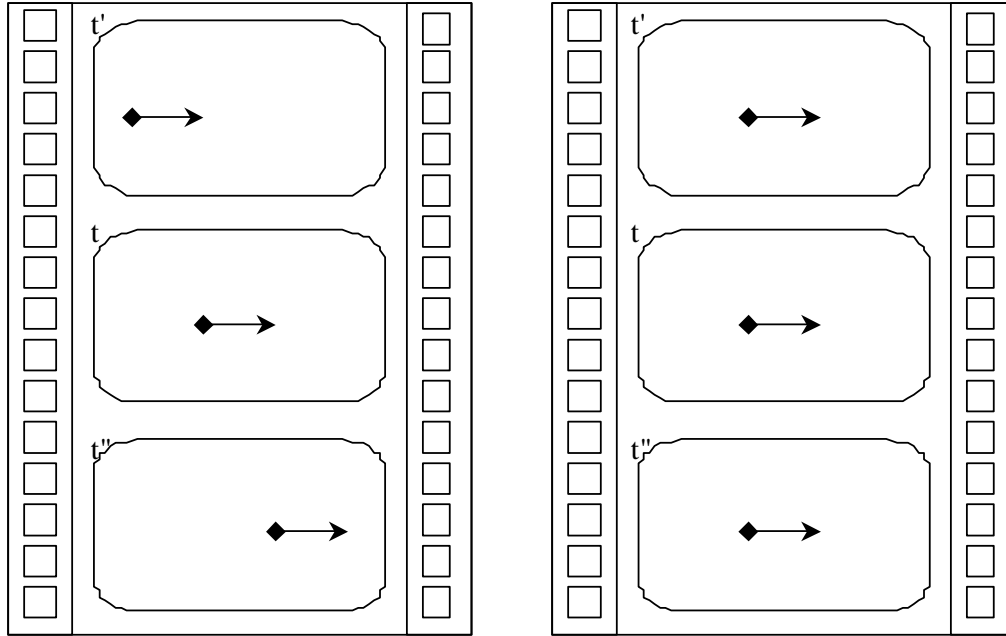


Figure 1 Zeno's two arrows at three different points in time $t' < t < t''$:
the moving arrow (left hand side) and the arrow at rest (right hand side);
at time point t there is no difference in state

In the 19th and 20th century a number of technological developments made it possible to do further experiments. For example, the concept of movie was developed and the technical equipment to implement this concept; a movie is created just by successively displaying a large number of static pictures (e.g., 24 per second); see, for example, (Burns, 2000). Nevertheless humans watching a movie get the impression of motion just like in reality, from which the word 'movie' stems. But nothing really moves in a movie, which is a paradox in words in the same spirit as Zeno's 'the moving arrow is unmoved'. Other examples of 20th century technological developments that support Zeno's view are television, computer animation and virtual reality. All of these experiences support Zeno's view in the sense that having an impression of motion is not an indication that in reality anything like motion exists at all.

These observations suggest the following view on reality. Instead of motion or change, which are human illusions that do not exist in reality, what does exist is the concept of reality as a *succession of states*. Each of these states can be described by the state properties it has; properties make use of language elements for basic state concepts that together form a *state ontology*.

So, for example, to create a virtual reality, i.e., something artificial similar to reality, it is sufficient to display such a succession of states at a frequency of, e.g., 20 pictures per second;

nothing needs to be moved. Indeed recent developments have proven that this works (however, for a range of lower frequencies stroboscopic effects occur, that for viewers lead to disorientation, and for a still lower frequency range the pictures are perceived one-by-one as separate pictures).

2.2 Explaining Properties of Changed States

If motion or change itself was considered part of reality, properties of successive states could be explained by referring to this change. For example:

Why is the arrow at t' at position P' ?

The arrow is at position P' at t' because

at t it was at position P , and

it was moving in the same direction P' has from P , and

nothing was in its way.

However, if motion as such does not exist in the ontology to conceptualise reality, such an explanation is unsatisfactory: this explanation makes use of a concept (for 'motion') that is not 'real', i.e., it does not belong to the ontology that conceptualises reality. Still the different properties of the states in succession exist in reality; they *are* based on the ontology used to conceptualise reality and they ask for an explanation that uses such real concepts.

More specifically, how to explain for a given state, that for one case with a state at rest, for example the arrow at rest, a next state has the same property (same position of the arrow), whereas in another case, for example a snapshot of the moving arrow, a next state has a different property (the arrow at a different position)? How can a new property be explained without involving an unreal concept such as motion? Only taking the usual state properties into account is insufficient to explain differences in properties of subsequent states, because according to Zeno's analysis the state of an object at rest has exactly the same state properties as a snapshot of a moving object. No distinction can be made on the basis of these state properties, and therefore it is inexplicable why in one case a subsequent state has a property different from the properties of such a subsequent state in the other case.

Avoiding the use of a concept for motion itself (1), and the basic state ontology used for state properties apparently being insufficient for such explanations (2), two ways out are possible: *either* extend the assumed state ontology and state properties expressed in terms of them, to be able to discriminate states that are at rest and states that are going to change, *or* keep the basic state ontology and state properties the same but extend the number of states that can be used

in such an explanation to states in the past. So, more specifically, the following two ways can be pursued to solve the explanatory problem:

1. *Extending the state ontology by introducing potentialities*

Assume that to conceptualise a state, the state ontology has more ontological elements and state properties than only the apparent ontology and state properties. In particular, a state can be conceptualised using an additional type of concept in the state ontology: the *potentiality* to subsequently get different state properties, i.e., the potentiality for the state to become changed.

2. *Exploiting temporal relationships: involving states over different time points*

Explaining why a state at time t has a different state property is not possible on the basis of one state at some $t' < t$, but needs to take into account a more extensive *history of different previous states*.

The approach of potentialities to the explanatory problem is discussed in the following sections.

3 Explaining Changed States by Introducing Potentialities

In this section only concepts that relate to states are included in the ontology to conceptualise reality. The assumptions discussed in Section 1.2 focus on the possibility to include concepts in the ontology to conceptualise states that are useful to describe properties of changed states. Aristotle did introduce such a concept; he called it *potentiality* (to move), or movable. The difference between the arrow at rest and the snapshot of the moving arrow at time t at position P is that the former has no potentiality to be at P' , whereas the latter has. This explains why at a next instant t' the former arrow is still where it was, at P , while the latter arrow is at a different position P' :

Why is the arrow at t' at position P' ?

The arrow is at position P' at t' because

at t it was at position P , and

at t it had the potentiality to be at P' , and

at t nothing in the world excluded it to be at P'

Aristotle did not only consider changes of positions (due to locomotion), but also, for example, a young man becoming an old man, and a cold object becoming hot. For each of these types of changes a specific type of potentiality is considered; e.g., the potentiality to be at position P' , the potentiality (of a young man) to be an old man, the potentiality (of a cold object) to be hot. In general, if the potentiality (occurring in a state S) to have state property X has led to a state S' where indeed X holds, then this state property X of state S' is called the *fulfilment* or

actualisation of the potentiality for X occurring in state S. He expresses his view on potentialities and their actualisation as follows:

‘We have now before us the distinctions in the various classes of being between what is full real and what is potential.

Def. The fulfilment of what exists potentially, in so far as it exists potentially, is motion - namely, of what is alterable qua alterable, alteration: of what can be increased and its opposite what can be decreased (there is no common name), increase and decrease: of what can come to be and can pass away, coming to be and passing away: of what can be carried along, locomotion.’

‘The same thing, if it is of a certain kind, can be both potential and fully real, not indeed at the same time or not in the same respect, but e.g. potentially hot and actually cold.’

[from (Aristotle, Physics), Book III, Part I]

In interpretations (and translations) of Aristotle’s work it is sometimes an issue whether he meant that fulfilment of (or actualisation or actuality of) a potentiality is already present during the change itself (especially when the change takes some time) or only in the changed state resulting from the change. For both views phrases can be found that seem to support it. In relation to this issue it is a question whether a potentiality and fulfilment of this potentiality can occur together at the same point in time, or whether a potentiality is replaced (or followed) by its fulfilment; for example, see Kosman (1969) or Sachs (2001).

In the rest of this paper the actuality or fulfilment of a potentiality p (potentiality for state property a) occurring in a state S is assumed to be the property a occurring in the changed state S', so the occurrence of potentiality p as a state property of S does not coincide with its fulfilment which is the occurrence of state property a in state S'. Thus, fulfilment is only defined for the occurrence of a potentiality p in a certain state S that has this potentiality, and the occurrence of a property a that is this fulfilment is in another state S'. Would this state S' also have property p, then the actualisation of the occurrence of p in state S' is the occurrence of property a in yet another state S'', not the occurrence of property a in S'.

A serious question about potentialities is what kind of state properties they are. Zeno claimed that a moving arrow at t does not differ in state from an arrow at rest at the same position. Assuming potentialities as additional state properties, just to make this difference would be a cheap and artificial solution (just defining the problem away) if it is not shown how potentialities can obtain a solid place as genuine state properties.

3.2 Absolute and Relative Potentialities

A difference can be made between a potentiality describing that a subsequent state will have a certain property (*absolute potentiality*), and a potentiality describing that in a subsequent state a certain property has changed in a certain respect (e.g., by a certain amount) with respect to the given state (*relative or parameterised potentiality*). As an example:

- a potentiality to be at position P' is an absolute potentiality
- a potentiality to be 1 meter east from the current position P is a relative potentiality

Relative potentialities can have the advantage that they can be specified in a more generic manner. The criterion '1 meter east from the current position P' can be applied to any point P; within this specification P plays the role of a variable. If all possible instantiations are made for P, a relative potentiality can be viewed as a parameterised family of absolute potentialities. Especially when numbers are used in potentialities, the relative or parameterised form can be very effective. Examples of such relative potentialities in Physics will be shown in Section 5.

4 Mental State Properties as Potentialities

In the previous section the examples refer to motion of non-living objects. Another type of motion to be explained is motion of a living being. Often used explanations of human (or animal) actions refer to internal mental states. For example, for a person B which has the capability to move:

Why is person B at t' at position P'?

Person B is at position P' at t' because

person B was at position P at t, and

at t person B had the desire to be at P', and

at t nothing in the world excluded B from being at P'

The type of explanation discussed in Section 3 has some similarity to an explanation of human behaviour from internal mental state properties such as desires. In the example, the desire (which is often considered as a kind of future-directed mental state property) plays a role similar to that of the potentiality for being at P'. Indeed this similarity can be traced back in history, for example, to Aristotle. For example:

Now we see that the living creature is moved by intellect, imagination, purpose, wish, and appetite. And all these are reducible to mind and desire. (Aristotle, 350 BC, *De Motu Animalium*), Part 6

But how is it that thought (viz. sense, imagination, and thought proper) is sometimes followed by action, sometimes not; sometimes by movement, sometimes not? What happens seems parallel to the case of thinking and inferring about the immovable objects of science.

(...)

here the two premises result in a conclusion which is an action - for example, one conceives that every man ought to walk, one is a man oneself: straightway one walks; or that, in this case, no man should walk, one is a man: straightway one remains at rest. And one so acts in the two cases provided that there is nothing in the one case to compel or in the other to prevent. Again, I ought to create a good, a house is good: straightway I make a house.

(...)

Now that the action is the conclusion is clear. But the premises of action are of two kinds, of the good and of the possible.

And as in some cases of speculative inquiry we suppress one premise so here the mind does not stop to consider at all an obvious minor premise; for example if walking is good for man, one does not dwell upon the minor 'I am a man'. And so what we do without reflection, we do quickly. For when a man actualizes himself in relation to his object either by perceiving, or imagining or conceiving it, what he desires he does at once. For the actualizing of desire is a substitute for inquiry or reflection. I want to drink, says appetite; this is drink, says sense or imagination or mind: straightway I drink. In this way living creatures are impelled to move and to act, and desire is the last or immediate cause of movement, and desire arises after perception or after imagination and conception. And things that desire to act now create and now act under the influence of appetite or impulse or of desire or wish. (Aristotle, 350 BC, *De Motu Animalium*),

Part 7

Here properties of 'mind and desire' are mentioned as the source of motion of a living being. He shows how the occurrence of certain internal (mental) state properties (e.g., desires) within the living being entail or cause the occurrence of an action in the external world; see also (Nussbaum, 1978). Such internal state properties are sometimes called by him 'things in the soul', 'states of character', 'moral states'. In (Aristotle, 350 BC, *Nicomachean Ethics*), this idea is explored in more detail for the case of voluntary human action.

Now the man acts voluntarily; for the principle that moves the instrumental parts of the body in such actions is in him, and the things of which the moving principle is in a man himself are in his power to do or not to do. Such actions, therefore, are voluntary... (Aristotle, 350 BC, *Nicomachean Ethics*), Book III, Part 1

We deliberate not about ends but about means. For a doctor does not deliberate whether he shall heal, nor an orator whether he shall persuade, nor a statesman whether he shall produce law and

order, nor does any one else deliberate about his end. They assume the end and consider how and by what means it is to be attained; and if it seems to be produced by several means they consider by which it is most easily and best produced, while if it is achieved by one only they consider how it will be achieved by this and by what means this will be achieved, till they come to the first cause, which in the order of discovery is last. (Aristotle, 350 BC, *Nicomachean Ethics*), Book III, Part 3

(Aristotle, 350 BC, *Nicomachean Ethics*), Book III, Part 5

This describes what today is often called *means-end reasoning*. He explicitly summarises that ‘things in the soul’ control action:

Now there are three things in the soul which control action and truth - sensation, reason, desire. Of these sensation originates no action; this is plain from the fact that the lower animals have sensation but no share in action. (Aristotle, 350 BC, *Nicomachean Ethics*), Book VI, Part 2

Based on Aristotle’s analysis the pattern to explain motion is as follows:

at any point in time
if A has a desire D
and A has the belief that X is a (or: the best) means to achieve D
then A will do X

This form of analysis has been called ‘practical syllogism’.

Within Philosophy of Mind and Cognitive Science, a well known manner to characterise mental state properties is based on the notion of *functional role* or *causal role*; e.g., (Kim, 1996, 1998).

‘Functionalism takes mental properties and kinds as functional properties, properties specified in terms of their roles as causal intermediaries between sensory inputs and behavioural outputs, and the physicalist form of functionalism takes physical properties as the only potential occupants, or "realizers", of these causal roles. To use a stock example, for an organism to be in pain is for it to be in some internal state that is typically caused by tissue damage, and that typically causes groans, winces, and other characteristic pain behaviour.

(Kim, 1998), pp. 19-20

For example, the mental property ‘pain’ can be characterised in a simplified form by the following functional role description:

tissue damage leads to pain
 pain leads to groans

These ‘leads to’ relationships can be mapped onto causal relationships between the underlying physical state properties (the realizers). But they can also be interpreted in the form of temporal relationships in the following manner:

if at time t' tissue damage occurs
then at a next time point t pain will occur

if at time t pain occurs
then at a next time point t" groans will occur

The analysis above shows that mental state properties usually have a future-directed aspect, which makes that they can be considered special cases of potentialities for (externally observable) behaviour. Leaving the issue of awareness aside, in this example pain can be viewed as the potentiality for groans. This analysis provides a unifying perspective covering both DST approaches and symbolic/computational approaches to cognitive modelling; the state-determined system assumption and anticipatory state properties are common ground for all of these approaches.

The similarity in explanatory pattern for different cases of dynamics in different disciplines may lead to the question what potentialities actually are. It may seem overdone to attribute such invisible state properties to certain (living or nonliving) objects, thereby suggesting that these objects are in a sense similar to living objects. Wouldn't the use of such vague concepts stand in the way of a genuine physical description of the world? Recall, however, that adding the concept potentiality to the state ontology was done to solve an explanatory problem that otherwise was hard to solve: how to explain that two given arrows in exactly the same position, one arrow is in another position in a next state, whereas the other arrow still is in the same position. So, simply putting away such an unclear concept leaves us with this problem. Considered at an abstract level, the discussion in Philosophy of Mind on the existence and place of mental state properties has much in common with the more general discussion on the existence and place of potentialities as genuine state properties.

5 Giving Potentialities a Place in Physics

In later times successors of Aristotle, such as René Descartes (1596-1650), Christiaan Huygens (1629-1695), Isaac Newton (1643-1727) and Gottfried Wilhelm Leibniz (1646-1716), among others, have addressed the question how to further develop the phenomenon of change or dynamics and, in particular, the concept potentiality within physics. Contributions of these will be discussed in this section and in Section 8.3; for more specific references, see there. Indeed they succeeded in giving certain types of potentialities a well-respected place in modern physics (actually in more than one way).

5.1 Potentialities in Physics

To obtain a better understanding of the concept of potentiality (which he called *quantity of motion*, or *tendency to motion*), Descartes did some reflection on objects of different sizes.

‘Now, although this motion in moved matter is nothing other than its mode, nevertheless it has a certain and determinate quantity, which we easily understand to be able to be always the same in the whole universe of things, even though it be changed in its individual parts. So it is evident, as we think, that when one part of matter is moved twice as fast as another, and this second [part of matter] is twice as large as the first, there is as much motion in the smaller as in the larger ...’

(Descartes, Principles of Philosophy, Part II, Paragraph 36)

Descartes took the product of mass and velocity of an object as an appropriate foundation for its potentiality to be in a changed position, or quantity of motion. Thus he related the vague concept potentiality to other, better known concepts. Notice that this anticipatory state property ‘quantity of motion’ is a *relative* potentiality: the actualisation of a given quantity of motion entails being at another position as specified by this quantity relative to the current position (and not as being at some absolutely specified position). Descartes also expresses a law of conservation for this quantity of motion:

‘... it is most wholly in accord with reason that we think on this basis alone that God moved the parts of matter in various ways when He first created them and that He now conserves all of this matter clearly in the same way and for the same reason that He formerly created, and that He also conserves the same amount [tantundem] of motion in it always.’

(Descartes, Principles of Philosophy, Part II, Paragraph 36)

In modern physics this ‘quantity of motion’ concept is called *linear momentum*, or just *momentum*, and the conservation, for example, during elastic collisions, is called the ‘law of momentum conservation’. Newton incorporated this notion in his approach to motion; actually the law of momentum conservation as formulated by Descartes has a strong relationship to

Newton's second and third law (see also in Section 7.3). This is one way in which a concept 'potentiality' was given a well-respected place in physics, in particular in classical mechanics.

Huygens (1629-1695), and later his student Leibniz (1646-1716), used a different way to give a concept 'potentiality' a place in physics. Leibniz called this concept *vis viva* (*living force*) or *motive force*, or *moving force*, or *force of motion*, or simply *force* or *power*. First, Leibniz asserts that like Descartes he also subscribes to a principle of conservation:

'Seeing that velocity and mass compensate for each other in the five common machines, a number of mathematicians have estimated the force of motion by the quantity of motion, or by the product of the body and its velocity.

(...)

Now, since it is reasonable that the same sum of motive force should be conserved in nature, and not be diminished -- since we never see force lost by one body without being transferred to another -- or augmented, a perpetual motion machine can never be successful, because no machine, not even the world as a whole, can increase its force without a new impulse from without. This led Descartes, who held motive force and quantity of motion to be equivalent, to assert that God conserves the same quantity of motion in the world.'

[Adapted from Leibniz (1956), vol. I, pp. 455-458]]

Next, he describes in a thought experiment what Descartes' principles would entail:

'In order to show what a great difference there is between these two concepts, I begin by assuming, on the other hand, that a body falling from a certain altitude, acquires the same force which is necessary to lift it back to its original altitude, if its direction were to carry it back and if nothing external interfered with it.

(...)

I assume also, in the second place, that the same force is necessary to raise a body of 1 kilogram to the height of 4 meters, as is necessary to raise a body of 4 kilograms to the height of 1 meter. Cartesians, as well as other philosophers and mathematicians of our times, admit both of these assumptions. Hence it follows, that the body of 1 kilogram, in falling from a height of 4 meters, should acquire precisely the same amount of force as the body of 4 kilograms, falling from a height of 1 meter. For, in falling 4 meters, the body of 1 kilogram will have there, in its new position, the force required to rise again to its starting point, by the first assumption; that is, it will have the force needed to raise a body of 1 kilogram (namely, itself) to the height of 4 meters. Similarly, the body of 4 kilograms, after falling 1 meter, will have there, in its new position, the force required to rise again to its own starting point, by the first assumption; that is, it will have

the force sufficient to raise a body of 4 kilograms (itself, namely) to a height of 1 meter. Therefore, by the second assumption, the force of the body of 1 kilogram, when it has fallen 4 meters, and that of the body of 4 kilograms, when it has fallen 1 meter, are equal.'

[Adapted from Leibniz (1956), vol. I, pp. 455-458)]

By incorporating results from experimental work of Galileo, Leibniz then shows that motive force and quantity of motion are different concepts:

'Now let us see whether the quantities of motion are the same in both cases. Contrary to expectations, there appears a very great difference here. I shall explain it in this way. Galileo has proven that the velocity acquired in a fall of four meters, is twice the velocity acquired in a fall of one meter. So, if we multiply the mass of the 1-kilogram body, by its velocity at the end of its 4-meter fall (which is 2), the product, or the quantity of motion, is 2; on the other hand, if we multiply the mass of the 4-kilogram body, by its velocity (which is 1), the product, or quantity of motion, is 4. Therefore the quantity of motion of the 1-kilogram body after falling four meters, is half the quantity of motion of the 4-kilogram body after falling 1 meter, yet their forces are equal, as we have just seen. There is thus a big difference between motive force and quantity of motion, and the one cannot be calculated by the other, as we undertook to show.'

[Adapted from Leibniz (1956), vol. I, pp. 455-458)]

Leibniz summarised his results as follows:

'Thus, through the resolution of bodies into parts, the speed, or space and time, being conserved, we had inferred, demonstrated, that given the same speeds the powers were proportional to the bodies. Similarly, we have demonstrated, which is paradoxical, but absolutely true, that, the body being conserved, time and space being resolved jointly (for otherwise the case given could not be divided in several cases congruent with each other while different), given the same bodies, the powers are proportional to the square of speeds.' (Leibniz 1991b, II, §E, p. 816)

So Leibniz claimed that the potentiality 'motive force' was proportional not with velocity as in the case of 'quantity of motion', but with the square of velocity. In this way Leibniz put the foundation for the law of conservation of energy, in this case involving kinetic energy (which actually was later taken $\frac{1}{2} mv^2$) and potential energy, and exchange between the two.

In a broader sense Leibniz aimed at developing what he called a science of power and action, or a *science of dynamics*; in the Specimen præliminare of the Dynamica he states:

'I judged that it was worth the trouble to muster the force of my reasonings through demonstrations of the greatest evidence, so that, little by little, I might lay the foundations for the true elements of the new science of power and action, which one might call dynamics. I have

gathered certain preliminaries of this science for special treatment, and I wanted to select a ready specimen from these in order to excite clever minds to seek truth and to receive the genuine laws of nature, in place of imaginary ones.’ (GM VI, p. 187; Leibniz 1989, p. 107)

5.2 What Kind of State Property is a Potentiality?

Within physics, potentialities have found their place in different manners. Basic concepts in classical mechanics such as momentum, kinetic energy and force can be considered variants of potentialities. In Section 4.1 the first two of these concepts are discussed; the concept force will be discussed in Section 6.3 in the context of higher-order potentialities and exchange of potentialities by interaction. Both for momentum and kinetic energy a conservation law has been found, and both concepts can be expressed in terms of mass and velocity. Does this mean that in these two forms, potentialities have become genuine state properties, which are even definable in terms of other state properties? Even leaving relativity theory aside, this is not a simple question. A straightforward answer would be: indeed, potentialities are genuine state properties because they are defined in terms of mass and velocity which are assumed to be genuine state properties. For the sake of simplicity accepting this claim for mass, a question, however, remains what kind of state property velocity is.

A first approach is to take velocity to be distance traversed divided by time passed over some chosen time interval from t' to t ; i.e.:

$$v = (x(t) - x(t')) / (t - t').$$

This definition involves states at different points in time, so it is not based on one state at one time point. This notion of velocity actually is *velocity over the given time interval from t' to t* , so a property of a sequence of states indexed by the time points of the interval, or, to simplify it a bit, a property of a pair of states for the starting point and the end point of the time interval. This is not what one would call a genuine state property.

A second approach is to identify velocity at some point in time with what a *speedometer* displays. Indeed at a point in time t the position of the pointer of a speedometer is a genuine state property. Would this offer an appropriate solution? A first objection may be that this property is just the position of the pointer, not velocity. For every type of object and speedometer a different state concept would arise: think of speedometers for cars compared to those of airplanes, ships, rockets; and what about the velocity of a bird or an approaching meteor. Even if for a certain class of objects, such as cars, a standardisation would be reached for a speedometer, then still the position of the pointer of the speedometer is itself not velocity; at most it has a *relation* to velocity. In particular, the pointer position itself does not affect the

(changed) position of the car at the next instant; velocity does affect this position. However, for such a relation between pointer position and velocity we are still in need for a genuine state property velocity; apparently this problem was not solved by the pointer position.

As a further objection, the position of the pointer of a speedometer results from (or is affected by) the actual motion, so there is a small time delay between having some velocity and what the pointer displays. Therefore the pointer actually indicates speed over time points $t' < t$, which, although close to t are not equal to t . Thus the pointer position, which is a state property of the state at t , actually relates to states at $t' < t$, so even if it would be related to a certain velocity state property, this would be a state property of the state at some $t' < t$, not a state property of the state at t . This makes clear that the speedometer concept will not help to make velocity a genuine state property of the state at t .

A third approach is what is sometimes called the notion of *instantaneous velocity*. In modern physics and mathematics, for continuous processes that satisfy sufficiently strong conditions of smoothness, this is usually defined as a limit:

$$v(t) = \lim_{t' \rightarrow t} (x(t') - x(t)) / (t' - t)$$

Note that this limit is defined in terms of the whole family or sequence of states around t , i.e., in terms of the state properties $x(t')$ for all t' in a neighbourhood of t . In mathematical terms this limit can be defined as

$$\forall \varepsilon > 0 \exists \delta > 0 \forall t' [0 < |t' - t| < \delta \Rightarrow |(x(t') - x(t)) / (t' - t) - v(t)| < \varepsilon]$$

It is clear that this statement refers to a whole sequence of states for time points t' around t . In this sense the notion of instantaneous velocity does not provide a better solution for a foundation of potentialities as genuine state properties than the other two approaches. Moreover, behind the assumption of smoothness some further assumptions are hidden, namely, to obtain such smoothness, either some inertia of speed has to be assumed (e.g., in free space), or otherwise a type of persistence of other state properties inducing velocity, which may be strongly situation-dependent.

In summary, it turns out that for all three approaches discussed the notion used to define potentiality for a state at one time point t takes into account different states, not only the one state at t . So, do we have to admit that addressing motion and change by extending the state ontology by some form of additional state properties for potentialities has failed? The answer on this question seems to be: yes and no. The answer is 'yes' in the sense that in the three

possibilities considered there has not been found anything physically real in the state as a basis for a potentiality as a genuine state property. The answer is ‘no’ in the sense that the historical developments as discussed in this section have provided quite powerful mathematical means (calculus, differential equations) to model all kinds of problems in diverse application areas. In our daily life we all rely on artefacts constructed using classical mechanics; e.g., bridges, buildings, transportation means. Given that this conceptual machinery works quite well in predictions, makes that the question is still there: what is it that makes this machinery so successful?

6 Transition Systems and Potentialities

An often used method (in Computer Science and related areas) to specify how a state in a system may change is known as ‘transition systems’. These are collections of specifications that each consist of a pair (ϕ, ψ) , also denoted as $\phi \rightarrow \psi$ and sometimes called a *transition rule* with antecedent ϕ and consequent ψ . In this specification:

- the first description ϕ indicates a combination of state properties for the current state
- the second description ψ indicates one or more state properties for the next state

The idea is that if the combination of properties specified in the first description holds in a (current) state, then in a next state the properties specified by the second description will hold. This is illustrated by a simple model of agent behaviour based on beliefs desires and intentions. Consider an agent walking down a street and observing an ice cream sign at the supermarket across the street he believes the supermarket sells ice cream. Based on this belief (b1) the agent generates a desire (d) for ice cream. Given this desire, and the belief (b2) that the supermarket is reachable (by crossing the street) the agent generates the intention (i) of having ice cream. Given this intention and the belief (b3) that no traffic is on the street he actually crosses the street and obtains the ice cream (e). In this case the state ontology is described by six basic state properties: b1, b2, b3, d, i, e. The simple scenario can be described in transition system format as follows:

$$\begin{aligned} b1 &\rightarrow d \\ b2 \wedge d &\rightarrow i \\ b3 \wedge i &\rightarrow e \end{aligned}$$

Based on such a specification a trace of subsequent states is made as follows.

- Given a current state S, take the transition rules for which the antecedent holds in the current state. This is the set of applicable rules.
- Collect the consequents of all applicable rules and obtain the next state S' by modifying S so that all these consequents hold in S' (and the rest of S is persisting).

So, for example, the subsequent states for a given initial state for which the three beliefs hold are as follows:

- 0 [b1, b2, b3]
- 1 [b1, b2, b3, d]
- 2 [b1, b2, b3, d, i]
- 3 [b1, b2, b3, d, i, e]

How can this be interpreted in terms of potentialities. For example, consider state 1. As in the next state, state 2, state property i holds, in state 1 the potentiality for i to hold has to be present. On the other hand, i occurs in state 2 because of the second transition rule. Taken together this means that this transition rule can be interpreted for state 1 as indicating that, due to the occurrence of both $b2$ and d in this state, also the potentiality $p(i)$ for i occurs in state 1. Similarly the other transition rules can be interpreted as indications of which potentialities occur in a given state. In general, according to this interpretation a transition system specifies for each state which potentialities occur: for each transition rule $\phi \rightarrow \psi$, if in a state S its antecedent ϕ holds, then in this state S also the potentialities $p(\psi)$ for ψ occur. Thus a transition rule $\phi \rightarrow \psi$ can be interpreted as an implication $\phi \rightarrow p(\psi)$, describing a logical relationship between state properties in a given state. In the example scenario the subsequent states can be described as follows:

- 0 [b1, b2, b3, $p(d)$]
- 1 [b1, b2, b3, d , $p(i)$]
- 2 [b1, b2, b3, d , i , $p(e)$]
- 3 [b1, b2, b3, d , i , e]

In principle, other variants of transition systems are possible as well, for example, for nondeterministic behaviour where only one (choice of an) applicable transition rule at a time is applied. The analysis of such variants is similar.

7 How to Explain Changed Potentialities

The effect of a potentiality on a future state can be described by relating the present state to the future state. This specification can be viewed as the definition of what it is a potentiality for. A much more complicated question is how to specify when (under which past and present circumstances) a potentiality occurs. For the case of empty space, where an object is assumed to have no interaction with other objects, a potentiality is present because it was present at an earlier point in time and persisted until t (inertia of motion). However, if the potentiality in a new state is different from the earlier one, a question becomes why this is so. This leads to the question addressed in this section of how a *changed potentiality* can be explained.

7.1 Introducing Higher-Order Potentialities: Potentialities for Potentialities

The use of higher-order potentialities is one answer to the question where changed potentialities come from. The idea behind higher-order potentialities is simple. To obtain an explanation of changed state properties over time, potentialities were introduced. Potentialities are also changing over time. If they are genuine state properties themselves, it would be reasonable to treat them just like any other state property that changes over time. This means that for a potentiality $p^{(1)}$ a so-called *second-order potentiality* $p^{(2)}$ is introduced to explain why $p^{(1)}$ may become changed over time. And of course this process can be repeated for $p^{(2)}$, and so on. This leads to an infinite sequence of *higher-order potentialities*,

$$p^{(1)}, p^{(2)}, p^{(3)}, p^{(4)}, \dots$$

where for each natural number n the potentiality $p^{(n)}$ is called an *n-th-order potentiality*. The idea is the following:

- for a certain point in time t_0 the occurrence of a state property can be determined on the basis of the state at a previous time point $t_1 < t_0$ and, in particular, the first-order potentiality at that time point t_1 .
- the first-order potentiality at t_1 can be determined by the state at a time point $t_2 < t_1$ and, in particular the second-order potentiality at t_2 .
- and so on

This process can be visualised as depicted in Figure 2.

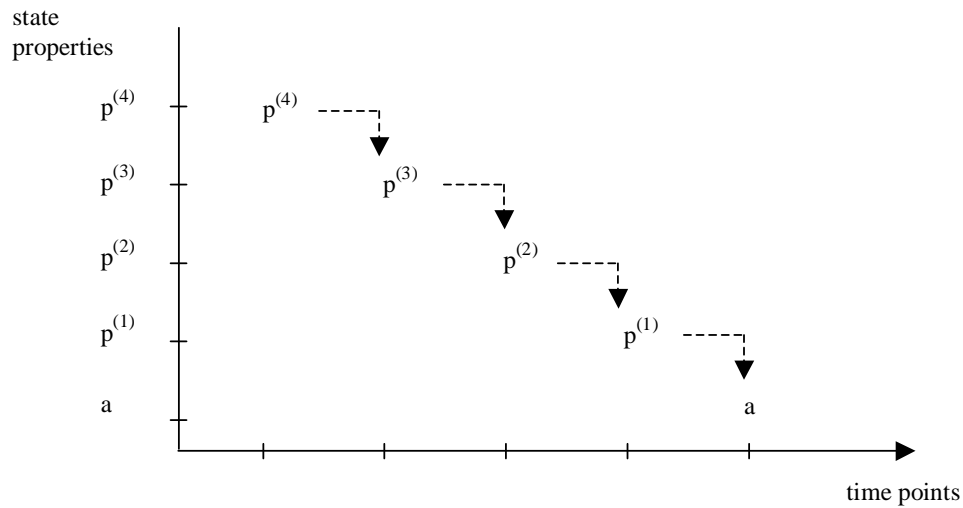


Figure 2 Dynamics based on higher order potentialities

This shows how the concept of potentiality to explain change of a certain basic state property can take the form of a single entity, for example one number, to indicate what a changed property in an immediate subsequent state will be, but this can be extended by a large number of other (higher-order) entities that can explain changed basic state properties in further future states.

7.2 Mathematical Formalisation of Higher-Order Potentialities in Calculus

Strange as the idea of an infinite number of higher-order potentialities may seem at first sight, in mathematical context (in particular in calculus) this has been worked out well. For the discrete case, the idea of difference tables for functions has been developed; see Figure 3. These differences play the role of relative potentialities: they indicate the next value not in an absolute sense, but in comparison to the current value.

time point t	f value	1 st -order difference	2 nd -order difference	3 ^d -order difference	4 th order difference
0	3	1	1	-3	1
1	4	2	-2	-2	
2	6	0	-4		
3	6	-4			
4	2				

Figure 3 Dynamics based on a higher-order difference table

For the continuous approach higher-order potentialities have been formalised in the form of the higher-order derivatives of a function. The well-known Taylor approximation and Taylor series for sufficiently smooth functions (at least infinitely often differentiable) show how changes of the value from t to t' (within some given neighbourhood of t) depend on all (higher-order) derivatives:

$$f(t') = f(t) + \sum_k f^{(k)}(t)(t' - t)^k / k!$$

This expression shows how the (relative) potentiality at t , defined by the combination of all (infinitely many) higher-order potentialities, determines the changed state at the future time points t' .

This analysis places the question of a potentiality as a genuine state property in a different light. Apparently, in the continuous case a potentiality may take the form of kind of infinitary

property (an infinite-dimensional vector of higher-order potentialities); such properties are far remote from what usually are understood as genuine state properties. A possible way out is to consider only changes that involve a finite number of higher-order potentialities. For example, within a constant gravitation field, the second-order potentiality (the acceleration) is constant (9.8 m/sec^2), and hence no third- or higher-order potentiality is needed: they are all zero. However, further away in the universe, if an object is approaching the earth, gravitation will increase over time, so this assumption of constancy will not always be fulfilled.

7.3 Higher-Order Potentialities in Cognitive Models

In earlier sections 4 and 6 the concept of intention was interpreted as a potentiality. Where do intentions come from? A common view is that, given some beliefs, intentions come from desires, by some form of selection process. In this interpretation a desire can be viewed as a potentiality as well, but not a potentiality for some state of the (external) world, but a potentiality for an intention, which itself is also considered a potentiality (for a world state). Therefore this view identifies a desire as a second-order potentiality (for a world state). The next question, of course, is where desires come from. There seems to be no general answer to this question. There may be some form of third-order potentiality involved, e.g., based on observations or beliefs, or it is based on a whole history of experiences.

8 Changed Potentialities Due To Interaction

Potentiality can lead to what Aristotle calls ‘the actuality of the potentiality’, e.g., the actual being at position P', but there may be cases where potentialities are not actualised, but disappear without having their effect. For example, some heavy object can be positioned in such a way that the arrow cannot be at P', due to its interaction with the object. This section addresses how potentialities can be exchanged between objects by interaction. Some examples are used to show that an interaction can lead to changed potentialities (Section 8.1). Since changed potentialities can be explained using higher-order potentialities, an interaction can be characterised by the higher-order potentialities it invokes (Section 8.2). In Section 8.3 Newton's three law of mechanics are considered from this perspective. The notion ‘force’ plays the role of a second-order potentiality for the potentiality ‘quantity of motion’. Within classical mechanics interactions between physical objects are characterised by the forces invoked by the interaction.

8.1 Exchange of Potentialities by Interaction

An intensively studied example is one billiard ball A at t' moving to P, where another, equal billiard ball B is positioned at rest. If ball A reaches P at time t , it has the potentiality to be at a next position P' at a next point in time t'' . However, what actually occurs is that ball A is still at P at time t'' , at rest, and ball B is at P' at time t'' .

How can this be explained? A first part of the explanation is that apparently at time t ball B had a potentiality to be at P' ; assuming the presence of this potentiality, the explanation runs as above:

Why is ball B at t'' at position P' ?

Ball B is at position P' at t'' because

at t it was at position P , and

at t it had the potentiality to be at P' , and

at t nothing in the world excluded it to be at P'

But how can the presence of the potentiality for B at the current time t be explained? Apparently potentialities can be transferred from one object to another one: it seems that the potentiality of ball A was carried over to a same potentiality of ball B; see Figure 4. Section 8 addresses this question of transfer of potentialities between moving objects (without being actualised) in more detail.

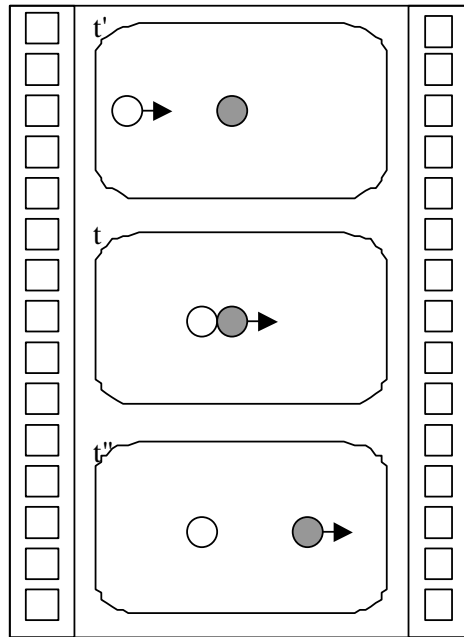


Figure 4 Transfer of potentiality at time t from ball A (white) to ball B (red)

Given the type of explanation discussed in Section 2.1, a next question is: how can the presence of a potentiality (e.g., of ball B at t) be explained? Assuming that the billiard ball experiment takes place in isolation of other possible interactions, the most reasonable candidate for the origin of this potentiality is ball A, because of its intense interaction (collision) with B; a

reasonable explanation is that during this interaction the potentiality that ball A had before t was transferred to a potentiality of ball B at t :

Why has ball B at t the potentiality to be at position P' ?

*At t ball B has the potentiality to be at position P' because
before t ball A had the potentiality to be at position P' , and
at t there was an intense interaction between ball A and ball B.*

This provides an iterated explanation for B's being at P' at time t' , by first asking why B is at P' at time t' , and by next asking, for the explaining potentiality of B, how that in turn can be explained. Such explanations are based on the view that within the universe potentialities can be transferred between objects by interaction.

A similar pattern of iterated explanation occurs in a comparable human case study when asking how the intention of person B can be explained.

Why has person B at t the intention to be at position P' ?

*At t person B has the intention to be at position P' because
before t person A had the intention to be at position P' , and
at t there was contact between person A and person B in which they agreed
that B would replace A at P' .*

For example, being at P' may stand for attending a meeting. In this case a given internal mental property (the intention to be at P') in one person induces an internal mental property in another person by interaction between the two.

8.2 The Role of Higher-Order Potentialities in the Exchange of Potentialities

In Section 7 higher-order potentialities were introduced to explain changed potentialities. The change of first-order potentialities due to interaction between objects was discussed in Section 8.1. In fact two types of circumstances entail the same effect.

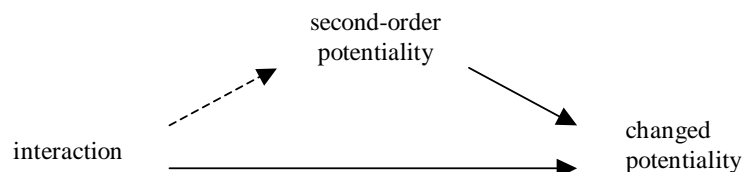


Figure 5 Interaction characterised by second-order potentialities

This suggests the question how interactions can be characterised using second-order potentialities; see Figure 5. Actually, the interaction as described in Section 8.1 abstracts from the interaction process itself. It only considers the two states at a time point t just before and t' just after the interaction and makes up what has changed in the meantime. The interaction process takes place within the time interval from t to t' . To be able to explain the first-order potentiality of ball B at t' it has to be assumed that between t and t' a second-order potentiality has occurred within ball B. Since ball A is the only other entity available, this second-order potentiality of ball B is somehow related to the presence of ball A. Following a similar analysis, the decrease of A's potentiality can be explained if it is assumed that within A a (negative) second-order potentiality occurs between t and t' . This leads to the assumption that a collision between such (elastic) objects generates second-order potentialities during their contact. How is that possible? Close observation reveals that within such a time interval of the interaction between t and t' both balls have some deformation. The idea is that such a deformation entails or co-occurs with a second-order potentiality. In some sense, this deformation realises the second-order potentiality.

8.3 Higher-Order Potentialities to Characterise Interaction: the Case of Mechanics

One of the implications of the conservation law for potentialities in the form of quantity of motion as formulated by Descartes, is that an object in motion and not interacting with other objects remains in (the same quantity of) motion (inertia of motion). This law, already known by Descartes and Galileo, is adopted by Newton (1729) in his *Principia* as the first law:

'LAW I

Every body continues in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed upon it.

Projectiles continue in their motions, so far as they are not retarded by the resistance of the air, or impelled downwards by the force of gravity.'

This law states that the absence of impressed forces entails unchanged (quantity of) motion, which suggests that 'impressed forces' relate in one way or the other to 'change of motion'.

As another implication of the conservation law for quantity of motion formulated by Descartes, the following analysis of an interaction between two objects A and B during a time interval from t to t' can be made. Suppose that p is the potentiality (i.e., quantity of motion) of A at t and q the potentiality of B at t . Moreover, suppose that p' and q' are the potentialities of A and B at t' respectively. Then the conservation law expresses that the sum of the potentialities p and q is the same as the sum of p' and q' . Therefore an increase $\Delta q = q' - q$ from q to q' has to correspond to an equal decrease $\Delta p = p' - p$ of p to p' ; in symbols: $\Delta q = -\Delta p$. So the degree of

change in potentiality is equal (but in opposite direction) for both objects. In the case of the billiard balls above, indeed this is the case: all of A's potentiality is transferred to B.

According to the analysis in Section 8.2, these changes of potentiality can be based on second-order potentialities. Therefore equation $\Delta q = -\Delta p$ expresses that for the interaction interval the second-order potentialities $p^{(2)}$ and $q^{(2)}$ for A and B are equal in value, but opposite: $q^{(2)} = -p^{(2)}$. So, in this case the interaction between A and B can be characterised by the occurrence of equal but opposite second-order potentialities (see Figure 5).

It is assumed that observations like these led Newton (1729) in his *Principia* to reformulate what in principle was already available from, among others, Descartes and Galileo, in the form of what has become known as his third and second law. Newton's third law states the mutual influence of two objects in interaction as follows:

'LAW III

To every action there is always opposed an equal reaction: or, the mutual actions of two bodies upon each other are always equal, and directed to contrary parts.

Whatever draws or presses another is as much drawn or pressed by that other. If you press a stone with your finger, the finger is also pressed by the stone. If a horse draws a stone tied to a rope, the horse (if I may say so) will be equally drawn back towards the stone; for the distended rope, by the same endeavour to relax or unbend itself, will draw the horse as much as it does the stone towards the horse, and will obstruct the progress of the one as much as it advances that of the other. For, because the motions are equally changed, the changes of the velocities made towards contrary parts are inversely proportional to the bodies. This law takes place also in attractions, as will be proved in the next Scholium.'

Here he uses words such as 'action', 'pressing', 'drawing', and 'equal change of the two motions'. In his second law he uses the term 'impressed motive force' to express the change of motion.

'LAW II

The change of motion is proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed.

If any force generates a motion, a double force will generate double the motion, a triple force triple the motion, whether that force be impressed altogether and at once, or gradually and successively. And this motion (being always directed the same way with the generating force), if the body moved before, is added to or subtracted from the former motion, according as they

directly conspire with or are directly contrary to each other; or obliquely joined, when they are oblique, so as to produce a new motion compounded from the determination of both.'

This law expresses that the concept of force used by Newton directly relates to change of motion. Terms like (im)pressing, drawing and action are not further explained. However, for quantity of motion he gives the same definition as Descartes.

'DEFINITION II

The quantity of motion is the measure of the same, arising from the velocity and quantity of matter conjointly.

The motion of the whole is the sum of the motions of all the parts; and therefore in a body double in quantity, with equal velocity, the motion is double; with twice the velocity, it is quadruple.'

For an impressed force the following definition is given:

'DEFINITION IV

An impressed force is an action exerted upon a body, in order to change its state, either of rest, or of uniform motion in a right line.

This force consists in the action only, and remains no longer in the body when the action is over. For a body maintains every new state it acquires by its inertia only. But impressed forces are of different origins, as from percussion, from pressure, from centripetal force.'

This definition refers to 'exerted action', which itself is not further defined, and to the corresponding change of the object's state of motion. In the following definition he shows how this notion applies in the particular case of centripetal (i.e., directed to one point) force:

'DEFINITION VIII

The motive quantity of a centripetal force is the measure of the same, proportional to the motion which it generates in a given time.

Thus the weight is greater in a greater body, less in a less body; and, in the same body, it is greater near to the earth, and less at remoter distances. This sort of quantity is the centripetency, or propension of the whole body towards the centre, or, as I may say, its weight; and it is always known by the quantity of an equal and contrary force just sufficient to hinder the descent of the body.'

These definitions show that the concept ‘force’ used by Newton as an addition to state ontology can be given a definitional relationship to ‘motion generated in a given time’. This ‘motion generated in a given time’ can be considered a second-order potentiality for the first-order potentiality ‘motion’. So, within classical mechanics, after the concepts ‘momentum’ and ‘kinetic energy’ which were added to the state ontology as specific types of (first-order) potentiality, the concept ‘force’ can be considered a third anticipatory state property added to the state ontology, this time as a second-order potentiality. For Newton, initially a force was a discrete event, something that, if repeated, comes in ‘blows’. So, one blow F may lead to one increase Δp in motion, and a number of blows F_1, \dots, F_k relates to a number of increases $\Delta p_1 \dots \Delta p_k$ which can be summed up to obtain the overall increase in p :

$$\Delta p = \Delta p_1 + \dots + \Delta p_k = F_1 + \dots + F_k$$

However, studying the orbits of planets and attempting to explain the circular motion, he had to assume that such blows come all the time with very small time distances between them. To incorporate this and similar phenomena, Newton and also Leibniz developed mathematical techniques of calculus, such as differentiation and integration. Using these techniques, Newton’s second law is formulated as $F = dp/dt$ or $F = d(mv)/dt$. For a mass which is constant over time this is equivalent with $F = ma$ with a the acceleration dv/dt ; in this - most known form - the law was formulated by Euler 65 years after the *Principia* appeared. In 20th century text books such as (Mach, 1942) the concept ‘moving force’ is *defined* in terms of second order potentialities in the following form:

Definition.

Moving force is the product of the mass value of a body with the acceleration induced in that body.

(Mach, 1942), p. 304

This shows the second-order potentiality character of a force, defined in terms of acceleration, which is a second-order potentiality for velocity.

Returning to the billiard balls, the second-order potentiality that occurs during their interaction can be called a motive force. But, this is nothing else than another word for the same, thus providing not any additional explanatory value, it seems. Analysing the motion of planets around the sun, Newton found out that they can only follow their orbit if a second-order potentiality is assumed, in the direction of the sun. Newton calculated (using his calculus under development) in detail that this motive force was proportional to 1 divided by the square of the distance. For example, for an object in space with mass m approaching earth (with mass M),

Newton's law of gravitation for the motive force on the object is as follows (here x is the distance between the object and the earth, and c is a constant):

$$F = c \, m M / x^2$$

But here, the objects being at a large distance, the occurrence of such a second-order potentiality is even more surprising than in the case of the billiard balls, as Nagel states and cites from Newton:

‘Although it was Newton who propounded the theory of gravitation, he did not regard it as ultimately satisfactory because it involved the notion of ‘action at a distance’ - a notion he regarded as ‘so great an absurdity that I believe no man, who has in philosophical matters a competent faculty in thinking, can ever fall into it’. For he maintained that ‘it is inconceivable that inanimate brute matter should, without the mediation of something else which is not material, operate upon and affect other matter without mutual contact’ (Newton, 1958), pp. 302-303; (Nagel, 1961) p. 171.

This shows that from the perspective of explanation, second-order potentialities to characterise the effect of interaction between objects, provide by themselves not a satisfactory description yet. The question ‘Where do they come from?’ remains unanswered.

9 Conclusion

Dynamical Systems Theory (DST) has recently been put forward in Cognitive Science as an approach to the dynamics of cognitive processes, and proposed as an alternative for the symbolic/computational approach; for example, (Port and Gelder, 1995; Kelso, 1995; Gelder and Port, 1995). The notion of a state-determined system (Ashby, 1952) is central for DST; such a system is based on the assumption that properties of a given state fully determine the properties of future states. As DST has been advocated as a better alternative to symbolic/computational approaches, the question may be raised whether this fundamental assumption is what makes it distinctive from these other approaches. The answer to this question has been shown to be ‘no’. It turns out that the state-determined system assumption, and the use of anticipatory state properties it entails, unifies different approaches to cognitive processes that are often seen as mutually exclusive.

As within Cognitive Science one of the problems identified for the symbolic/computational approach is the problem of realism (i.e., how do internal states described by symbols relate to the real world in a natural manner), a further relevant question is in how far DST does better in

this respect than the symbolic/computational approach: in how far is the realism of the states better guaranteed by DST.

In this paper the focus was on a unifying perspective on the explanatory problem of dynamics, in relation to the basic assumptions underlying DST. In the context of the assumptions behind DST (in particular state-determinedness) this can be formulated as: how to explain properties of a changed state based on properties of a previous state. To obtain such an immediate explanation in terms of properties of a previous state, within this previous state the occurrence of anticipatory state properties, or potentialities is assumed. Application of such an explanatory pattern for dynamics was discussed for different contexts varying from moving physical objects (e.g., Newton's laws) to animal or human action (e.g., Aristotle's practical syllogisms) and computer systems. It was shown how in history the perspective based on potentialities has led to a number of often used concepts within classical mechanics, a branch of physics frequently used by engineers today: momentum, energy, and force. Also it was discussed how within mathematics, more specifically in calculus, a number of concepts have been developed to formalise notions of potentiality: in particular derivatives (of different orders) of a function and Taylor approximations. Furthermore, it was shown how transition systems, a currently (within Computer Science and related areas) popular format for (non-quantitative) specification of dynamic systems can be interpreted from the perspective of potentialities.

Thus for the explanatory pattern for dynamics based on anticipatory state properties that is considered here, the commonalities were summarised for applicability in the different scientific disciplines Cognitive Science, Physics, Mathematics and Computer Science. One of the main contributions of the paper is that this provides a unified view on the explanation of dynamics across these different disciplines. This unified view is at a high level of abstraction; it does not deny that at more detailed levels there are differences between these disciplines as well. For example, some of the disciplines (Physics, Mathematics) allow for an appropriate form of quantification, whereas for other disciplines (Cognitive Science, Computer Science) such a quantification is not possible, or at least debatable. Between the disciplines where appropriate quantification is possible differences may also occur. An example of such a difference is that for the considered case from Physics conservation laws are possible, whereas for Mathematics (as such) the perspective is more abstract in the sense that such laws are not essential. Also for the non-quantified cases in Cognitive Science and Computer Science no role is played by conservation laws. For example, a desire may lead to an intention without the desire losing any of its powers. Although at a more detailed level differences between the cases in the different disciplines exist as mentioned, this does not affect that at the higher level of abstraction the dynamics in the different cases can be addressed in a unified manner using the explanatory pattern based on anticipatory state properties.

The different examples in different disciplines show that the postulation of new anticipatory state properties in addition to the basic state ontology is a fruitful approach to describe dynamics based on the state-determined system assumption, as DST does. The question in how far these additional ‘synthetic’ state properties are genuine or ‘real’ state properties was shown to be a hard question that is not simple to answer, even in Physics. Nevertheless, the fruitfulness of having such state properties added is uncontroversial; for example we all trust artefacts in our environment that were constructed based on physics and mathematics using such state properties.

Given the assumption on state-determinedness, anticipatory state properties are essential to DST. In an explanatory context the question of realism of these anticipatory state properties in DST has to be answered. The analysis made in this paper, however, answers this question negatively. It is shown that even in Physics the foundational question on whether such postulated anticipatory state properties are genuine, real state properties cannot be answered positively.

As an evaluation of DST compared to the symbolic/computational approach, this shows that not only the two types of approaches have the assumption of state-determined systems, and the use of anticipatory state properties in common, but also share the realism problem: DST does not provide a better solution than the symbolic/computational approach. However, the investigations in the history of other scientific disciplines also have shown that this shortcoming does not necessarily stand in the way of fruitfulness of any of these approaches within Cognitive Science. It may well be the case that DST will develop further, especially to address cases where the timing aspects are crucial, whereas symbolic/computational approaches will still develop further for other cases, for example, where more complex types of states are relevant, such as in reasoning or language processing.

Given that the approaches are not as exclusive as sometimes suggested, as shown by the commonalities found in this paper, integrated approaches will be developed, in which both DST aspects and symbolic aspects can be covered. Proposals for such integrated approaches can already be found, for example, in Sun (2002), or Jonker and Treur (2002, 2003) and Jonker, Treur, and de Vries (2002). In Jonker and Treur (2002) it is shown how the expressive temporal symbolic language TTL can be used to model cognitive phenomena, including their quantitative and continuous temporal aspects, and it is shown how DST techniques fit in this language. Sun (2002) claims that quantitative, continuous modelling approaches (such as DST) are suitable to address the implicit, non-representational internal agent states, whereas symbolic, discrete approaches are suitable for more explicit, representational internal agent

states. He also shows the crucial role of the interaction between the two types of states, and is an advocate of hybrid modelling approaches where both types of aspects play a role.

Potentialities as postulated state properties often have relationships to other state properties of the state in which they occur (they can be said to be realised by these other state properties). But potentialities usually also relate, in a temporal manner, to state properties in other (past and future) states. These realisation relationships and temporal relationships can be exploited to obtain a further analysis of the foundational question. Such a further analysis is the focus of current research.

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