

Department of Philosophy - Utrecht University

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Frans Voorbraak

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Department of Philosophy
University of Utrecht
Heidelberglaan 8
3584 CS Utrecht
The Netherlands

A PREFERENTIAL MODEL SEMANTICS FOR DEFAULT LOGIC

Frans Voorbraak

Department of Philosophy, University of Utrecht

P.O. Box 80.126, 3508 TC Utrecht

Abstract

Shoham proposed a uniform approach to systems for nonmonotonic reasoning, which consists in considering standard logics augmented with a preference relation on the interpretations. Circumscription can easily be seen to be a special case of this preference logic framework, but capturing default logic turned out to be more difficult, and is even thought to be impossible by some researchers of nonmonotonic reasoning. In this paper a preferential model semantics for default logic is given, by defining a preference relation on partial models called hypervaluations. Alternatively, Kripke models for **K45** could have been used instead of hypervaluations. The given semantics slightly strengthens Shoham's notion of preferential entailment, and reflection on the question why such a strengthening is necessary provides some insight into the relation between default logic and circumscription.

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2 Hypervaluations

L_{PL} is a propositional language built on a set of propositional letters PL and the logical connectives \neg for negation and \vee for disjunction. \wedge , \supset , \equiv are assumed to be defined in the usual way.

DEFINITION 2.1

- (i) An *hyperinterpretation* I is a set of 2-valued interpretations $J : PL \rightarrow \{T, F\}$.
- (ii) For any 2-valued interpretation J , $V_J : L_{PL} \rightarrow \{T, F\}$ denotes the usual 2-valued valuation induced by J . $V_J(\varphi) = T$ and $V_J(\varphi) = F$ will sometimes be written as $\models_{2,J} \varphi$ and $\models_{2,J} \neg \varphi$, respectively.
- (iii) The *hypervaluation* $V_I : L_{PL} \rightarrow \{T, F, U\}$ induced by the hyperinterpretation I is given by

$$\begin{aligned} V_I(\varphi) &= T \quad \text{iff } \forall J \in I \ V_J(\varphi) = T \\ &= F \quad \text{iff } \forall J \in I \ V_J(\varphi) = F \\ &= U \quad \text{otherwise.} \end{aligned}$$

$V_I(\varphi) = T$ and $V_I(\varphi) = F$ will sometimes be written as $\models_{h,I} \varphi$ and $\models_{h,I} \neg \varphi$, respectively.

The definition of hypervaluations is strongly reminiscent of the definition of van Fraassen's supervaluations. In fact, a supervaluation corresponds with a hypervaluation whose elements are the complete extensions of some 3-valued interpretation. The *consequence relation* \models_x induced by $\models_{x,I}$ is defined by $\Gamma \models_x \Delta$ iff $\forall I (\forall \psi \in \Gamma \ \models_{x,I} \psi \Rightarrow \exists \varphi \in \Delta \ \models_{x,I} \varphi)$. We do *not* require that $\forall I (\forall \varphi \in \Delta \ \models_{x,I} \varphi \Rightarrow \exists \psi \in \Gamma \ \models_{x,I} \psi)$, as some authors do (cf. Blamey (1986)). This stronger definition would for example render the inference from $\{p \vee q, \neg q\}$ to p invalid. (A counterexample would be $I = \{J, J'\}$, where $J(p) = J'(p) = F$, $J(q) = T$, and $J'(q) = F$.)

It is well-known that the supervaluation consequence relation \models_s is strictly weaker than the ordinary consequence relation. (See e.g. Langholm (1988).) The hypervaluation consequence relation \models_h is weaker still:

PROPOSITION 2.2 $\models_h \subset \models_s \subset \models_2$.

Proof. $\models_h \subseteq \models_s \subseteq \models_2$ is trivial. $\models_s \not\subseteq \models_h$ follows from the fact that $\{p \vee q\} \models_s \{p, q\}$, whereas $\{p \vee q\} \not\models_h \{p, q\}$. $\models_2 \not\subseteq \models_s$ follows from the fact that $\models_2 \{p, \neg p\}$, whereas $\not\models_s \{p, \neg p\}$.

It is easy to see that \models_h , \models_s , and \models_2 coincide in case the consequence set Δ is \emptyset or a singleton. It is also clear that $\Gamma \models_h \Delta$ iff for every *non-empty* hyperinterpretation I ($\forall \psi \in \Gamma \ \models_{h,I} \psi \Rightarrow \exists \varphi \in \Delta \ \models_{h,I} \varphi$). Further, \models_h is closely related to the consequence relations of the modal logics **K45**, **KD45**, and **S5**:

PROPOSITION 2.3 Let $\Gamma, \Delta \subseteq L_{PL}$, let for any $\Omega \subseteq L_{PL}$ $\Box\Omega$ denote the set $\{\Box\varphi \mid \varphi \in \Omega\}$, and let $S \in \{\mathbf{K45}, \mathbf{KD45}, \mathbf{S5}\}$. Then $\Gamma \models_h \Delta \Leftrightarrow \Box\Gamma \models_S \Box\Delta$.

Proof. It is easy to see that there exists a 1-1 correspondence μ between non-empty hyperinterpretations and distinguishable connected **S5** models such that $V_I(\varphi) = T$ iff $\mu(I) \models \Box\varphi$. The equivalence of $\models_{\mathbf{S5}}$, $\models_{\mathbf{KD45}}$, and $\models_{\mathbf{K45}}$ restricted to boxed formulas is well-known.

The above proposition justifies to some extent the epistemic reading of the hypervaluation consequence relation which was mentioned in the introduction, since **K(D)45** is widely regarded to be a good candidate for the logic of the beliefs of a rational agent, whereas **S5** is often mentioned as a suitable logic of knowledge. Below we mention some properties of \models_h .

PROPOSITION 2.4 Suppose $\Gamma \models_h \Delta$ and $\Delta \neq \emptyset$. Then $\exists \varphi \in \Delta$ s.t. $\Gamma \models_h \varphi$.

COROLLARY 2.5 Suppose $\Delta \neq \emptyset$. Then $\Gamma \models_h \Delta$ iff $\exists \varphi \in \Delta$ s.t. $\Gamma \models_2 \varphi$.

COROLLARY 2.6 (Compactness) If $\Gamma \models_h \Delta$, then there exists finite sets Γ' , Δ' such that $\Gamma' \subseteq \Gamma$, $\Delta' \subseteq \Delta$, and $\Gamma' \models_h \Delta'$.

COROLLARY 2.7 $\Gamma \models_h \Delta$ is decidable for finite Γ and Δ .

Hypervaluations are generalized to first-order languages in the obvious way: First-order hyperinterpretations are just sets of ordinary first-order interpretations, and the first-order hypervaluation induced by a first-order hyperinterpretation I is defined as in the propositional case. Propositions 2.2 and 2.4 remain valid in the first-order case, and so do corollaries 2.5 and 2.6. For corollary 2.7 the decidability of propositional \models_2 is of course essential. The first-order analogue of proposition 2.3 will only be valid for a suitable first-order generalization of the propositional Kripke models. Such a generalization is given in Voorbraak (forthcoming).

3. Default logic and preferential model semantics

For convenience, we repeat some basic definitions of default logic. For a proper introduction, see e.g. Reiter (1980), Etherington (1988), or Lukaszewicz (1990). Our presentation of the preferential model semantics relies heavily on Makinson (1989).

Let L be some (standard) first-order language and let $Th : \wp L \rightarrow \wp L$ be given by $Th(\Sigma) = \{\varphi \mid \Sigma \models_2 \varphi\}$. α, β, \dots denote formulas of L .

DEFINITION 3.1 A *default rule* (or simply a *default*) is an expression of the form $\alpha : \beta_1, \dots, \beta_n / \omega$ ($n \geq 1$). α is called the *prerequisite*, β_1, \dots, β_n the *justifications*, and ω the *consequent* of the rule. A default rule $\alpha : \beta_1, \dots, \beta_n / \omega$ is called *closed* iff the set of its free variables $FV\{\alpha, \beta_1, \dots, \beta_n, \omega\} = \emptyset$.

DEFINITION 3.2 A *default theory* is a pair $\langle D, W \rangle$, where D is a set of defaults and W is a set of closed formulas of L . A default theory is called *closed* iff every default in D is closed.

DEFINITION 3.3 E is an extension of a closed default theory $\langle D, W \rangle$ iff $E = \cup_{i \geq 0} E_i$, where

- $E_0 = W$
- $E_{i+1} = \text{Th}(E_i) \cup \{ \omega \mid \alpha : \beta_1, \dots, \beta_n / \omega \in D, \alpha \in E_i \text{ and } \forall j \in \{1, \dots, n\} \neg \beta_j \notin E_i \}$.

An extension of a default theory $\langle D, W \rangle$ is intended to represent a reasonable state of belief based on the defaults in D and on the propositions in W . As extensions of a default theory $\langle D, W \rangle$ which is not closed one simply takes the extensions of the closed default theory $\langle D', W \rangle$, where, roughly speaking, D' is obtained from D by taking all closed instances of D . (Since the defaults of D' may contain Skolem constants, the extensions of $\langle D', W \rangle$ have to be restricted to L . Details can be found in e.g. Lukaszewicz (1990).)

DEFINITION 3.4 Let $\delta = \alpha : \beta_1, \dots, \beta_n / \omega$ be a closed default and let Γ be a set of first-order models. The *preference relation corresponding to δ* , \geq_δ , over $\wp \Gamma$ is defined as follows:

$$\begin{aligned} \Gamma_1 \geq_\delta \Gamma_2 \text{ iff } & \forall M \in \Gamma_2 \ M \models_2 \alpha, \\ & \forall i \in \{1, \dots, n\} \ \exists M_i \in \Gamma_2 \ M_i \models_2 \beta_i, \\ & \text{and } \Gamma_1 = \Gamma_2 - \{M \mid M \not\models_2 \omega\}. \end{aligned}$$

Intuitively, $\Gamma_1 \geq_\delta \Gamma_2$ means that on account of δ the (partial) world-description Γ_1 is preferred to the (partial) world-description Γ_2 . The preference relation \geq_D corresponding to a set of defaults D is simply the transitive closure of the union of the preference relations corresponding to the elements of D . Notice that, in spite of the suggestive notation, \geq_δ and \geq_D are not necessarily partial orderings, since in general neither of them is reflexive. Using the reflexive closures of \geq_δ and \geq_D instead of the relations themselves would have worked as well, but we stick to the definition given in Etherington (1988).

Let $\text{MOD}(W)$ be the set of first-order models of W . The extensions of a closed default theory $\langle D, W \rangle$ correspond with the \geq_D -maximal elements of $\wp(\text{MOD}(W))$ which have some additional property called stability:

DEFINITION 3.5 Let $\Delta = \langle D, W \rangle$ be a closed default theory and let $\Gamma \subseteq \text{MOD}(W)$.

Γ is called *stable for Δ* iff $\exists D' \subseteq D$ such that $\Gamma \geq_{D'} \text{MOD}(W)$ and for every justification β of a default of D' $\exists M \in \Gamma \ M \models_2 \beta$.

PROPOSITION 3.6 (Etherington (1988)) Let $\Delta = \langle D, W \rangle$ be a closed default theory.

- (i) If E is an extension of Δ , then $\{M \mid M \models_2 E\}$ is stable for Δ and a \geq_D -maximal element of $\wp(\text{MOD}(W))$.
- (ii) If Γ is stable for Δ and a \geq_D -maximal element of $\wp(\text{MOD}(W))$, then $\{\varphi \mid \forall M \in \Gamma \ M \models_2 \varphi\}$ is an extension of Δ .

In the above proposition, \geq_D can be replaced by \geq_Δ , defined by $\Gamma_1 \geq_\Delta \Gamma_2$ iff $\Gamma_1 \geq_D \Gamma_2 \geq_D \text{MOD}(W)$ or $\Gamma_1 \geq_D \Gamma_2 = \text{MOD}(W)$.

DEFINITION 3.7 A *preferential model structure* (p.m.s.) for L is a tuple $\text{pr} = \langle \mathfrak{M}, \models, \sqsubset \rangle$, where \mathfrak{M} is a set of models, $\models \subseteq \mathfrak{M} \times L$ is a satisfaction relation, and $\sqsubset \subseteq \mathfrak{M} \times \mathfrak{M}$. We often write $M \models \varphi$ for $\langle M, \varphi \rangle \in \models$.

For technical reasons, Makinson (1989) imposes no constraints on \models and \sqsubset . However, Shoham (1987,1988) seems to allow only partial orderings \sqsubset and models of standard logics. He does not make precise what he understands by 'standard' and just assumes standard logics to be propositional or predicate (modal) logics, but a reasonable interpretation of his intentions is to require that \models induces a (standard) consequence relation satisfying inclusion, idempotency, and monotony. The preferential model structures proposed below are also preferential model structures in this stricter sense.

DEFINITION 3.8 Let $\text{pr} = \langle \mathfrak{M}, \models, \sqsubset \rangle$ be a p.m.s. for L .

- (i) $M \in \mathfrak{M}$ *preferentially satisfies* $\Gamma \subseteq L$, notation $M \models_{\text{pr}} \Gamma$, iff $M \models \Gamma$ (i.e., $\forall \varphi \in \Gamma M \models \varphi$) and $\neg \exists M' \sqsupset M M' \models \Gamma$. The set of *preferred models* $\mathfrak{M}_{\text{pr}} = \{M \in \mathfrak{M} \mid \exists \Gamma \subseteq L M \models_{\text{pr}} \Gamma\}$. We will usually write $M \models_{\text{pr}} \varphi$ instead of $M \models_{\text{pr}} \{\varphi\}$.
- (ii) The operation $\text{Th}_{\text{pr}} : \wp L \rightarrow \wp L$ of *preferential entailment* is given by $\varphi \in \text{Th}_{\text{pr}}(\Gamma)$ iff $\forall M (M \models_{\text{pr}} \Gamma \Rightarrow M \models \varphi)$.
- (iii) Γ is called *preferentially satisfiable* iff $\exists M M \models_{\text{pr}} \Gamma$.
- (iv) Γ is called *preferentially valid* iff $\Gamma \subseteq \text{Th}_{\text{pr}}(\emptyset)$.

Notice that φ is preferentially valid iff φ is true in every preferred model of \emptyset , or, in other words, iff for every \sqsubset -maximal model $M M \models \varphi$. Shoham gives a slightly different, and in our opinion less intuitive, definition of preferential validity, namely φ is preferentially valid iff $\neg\varphi$ is not preferentially satisfiable. In general, both definitions are *not* equivalent.

DEFINITION 3.9 Let $\Delta = \langle D, W \rangle$ be a closed default theory.

- (i) m is called Δ -*bounded* iff $\exists m' \geq_\Delta m$ such that m' is \geq_Δ -maximal and stable for Δ .
- (ii) The *p.m.s. associated with* Δ is the p.m.s. $\text{pr}(\Delta) = \langle \mathfrak{M}, \models, \sqsubset \rangle$, where $\mathfrak{M} = \wp(\text{MOD}(W))$, $\models = \{\langle m, \varphi \rangle \mid m \in \mathfrak{M}, \varphi \in L, \text{ and } \models_{h,m} \varphi\}$, and \sqsubset is defined as follows:
 - $m \sqsupset m'$ iff m is Δ -bounded and m' is not Δ -bounded,
 - or m and m' are not Δ -bounded and $m' \neq m = \emptyset$,
 - or m and m' are Δ -bounded, $m \neq m'$, and $m \geq_\Delta m'$.

$\text{Th}_\Delta, \models_\Delta$ and \mathfrak{M}_Δ abbreviate $\text{Th}_{\text{pr}(\Delta)}, \models_{\text{pr}(\Delta)}$, and $\mathfrak{M}_{\text{pr}(\Delta)}$, respectively, and \sqsubset_Δ denotes the preference relation of $\text{pr}(\Delta)$. \sqsubset_Δ is a strict p.o. and $\mathfrak{M}_\Delta = \{\emptyset\} \cup \{m \in \mathfrak{M} \mid \neg \exists m' \sqsupset_\Delta m\}$. Hence the only preferred model which is not necessarily \sqsubset_Δ -maximal is the trivial model \emptyset .

PROPOSITION 3.10 Let $\Delta = \langle D, W \rangle$ be a closed default theory and $\text{pr}(\Delta) = \langle \mathcal{M}, \models, \sqsubset \rangle$ its associated p.m.s.

- (i) If E is an extension of Δ , then $\{M \mid M \models_2 E\} \models_{\Delta} E$.
- (ii) If $m \in \mathcal{M}_{\Delta}$ and $m \neq \emptyset$, then $\{\varphi \mid m \models \varphi\}$ is a consistent extension of Δ .
- (iii) If $\mathcal{M}_{\Delta} = \{\emptyset\}$, then Δ has no consistent extension.
- (iv) $\text{Th}_{\Delta}(\Gamma) = \bigcap \{E \mid E \text{ is an extension of } \Delta \text{ such that } \Gamma \subseteq E\}$.

Proof. (i) If E is an extension of Δ , then, by proposition 3.6, $m = \{M \mid M \models_2 E\}$ is \geq_D -maximal and stable for Δ . Hence $\neg \exists m' \sqsupset m$. Since $m \models E$, we have $m \models_{\Delta} E$.

(ii) If $m \in \mathcal{M}_{\Delta}$ and $m \neq \emptyset$, then m is \geq_D -maximal and stable for Δ . Hence, by proposition 3.6, $\{\varphi \mid m \models \varphi\}$ is an extension of Δ . Since $m \neq \emptyset$, this extension is consistent.

(iii) If $\mathcal{M}_{\Delta} = \{\emptyset\}$, then either there are no Δ -bounded models, and therefore no extensions for Δ , or \emptyset is Δ -bounded, and thus $\{\varphi \mid \emptyset \models \varphi\}$ is the unique, inconsistent extension of Δ .

(iv) Assume $\varphi \in \text{Th}_{\Delta}(\Gamma)$ and let E be an extension of Δ such that $\Gamma \subseteq E$. Then, as in (i), $\neg \exists m' \sqsupset m = \{M \mid M \models_2 E\}$. Since $m \models \Gamma$, we have $m \models_{\Delta} \Gamma$. Hence $m \models \varphi$, and thus $\varphi \in E$.

On the other hand, assume $\varphi \notin \text{Th}_{\Delta}(\Gamma)$. Then $\exists m$ such that $m \models_{\Delta} \Gamma$ and $m \not\models \varphi$. By (ii), $E = \{\psi \mid m \models \psi\}$ is an extension of Δ and clearly $\Gamma \subseteq E$. Since $\varphi \notin E$, $\varphi \notin \bigcap \{E \mid E \text{ is an extension of } \Delta \text{ such that } \Gamma \subseteq E\}$.

Notice that \emptyset being the only preferred model corresponds to Δ having an inconsistent extension or having no extension at all. It is possible to give an alternative definition of \sqsubset_{Δ} such that $\emptyset \in \mathcal{M}_{\Delta}$ iff Δ has an inconsistent extension and $\mathcal{M}_{\Delta} = \emptyset$ iff Δ has no extension. However, this alternative \sqsubset_{Δ} will in general not be a strict partial ordering, since for any $\text{pr} = \langle \mathcal{M}, \models, \sqsubset \rangle$ with finite \mathcal{M} and s.p.o. \sqsubset we have $\mathcal{M}_{\text{pr}} \neq \emptyset$. The collapsing of the cases that Δ has an inconsistent extension and that Δ has no extension can be defended by pointing out that both are boundary cases added for technical convenience, rather than representations of belief states of truly rational agents. (In both cases a rational agent would have to revise his belief state.)

An immediate corollary of proposition 3.10(iv) is the monotony of Th_{Δ} . Hence as long as you keep the default theory constant, the reasoning is monotonic. Default consequence is nonmonotonic because default theories are updated in the light of new information. To capture this in terms of preferential model semantics, we propose a strengthening of the notion of preferential consequence and we define a preferential model semantics for a class of default theories with the same defaults. The strong notion of preferential consequence is equivalent to the usual notion in case the models are two-valued. Applied to hypervaluations, the strong notion takes account of the intuition that default extensions—the preferred models of default theory—have to be grounded on the facts. The definition of a p.m.s. associated with a class of defaults is a global version of the corresponding definition for a single default theory.

DEFINITION 3.11 Let $\text{pr} = \langle \mathcal{M}, \models, \sqsubset \rangle$ be a p.m.s. $M \in \mathcal{M}$ *strongly preferentially satisfies* Γ , notation $M \models_{\text{pr}^*} \Gamma$, iff $M \models_{\text{pr}} \Gamma$ and $\forall M' (\Gamma \subseteq \{\varphi \mid M' \models \varphi\} \subset \{\varphi \mid M \models \varphi\} \Rightarrow M' \sqsubset M)$. $\varphi \in \text{Th}_{\text{pr}^*}(\Gamma)$ iff $\forall M (M \models_{\text{pr}^*} \Gamma \Rightarrow M \models \varphi)$

DEFINITION 3.12 Let D be a set of closed defaults. The *p.m.s. associated with D* is the p.m.s. $\text{pr}(D) = \langle \mathfrak{M}, \models, \sqsubset \rangle$, where $\mathfrak{M} = \wp(\text{MOD}(\emptyset))$, $\models = \{ \langle m, \varphi \rangle \mid m \in \mathfrak{M}, \varphi \in L, \text{ and } \models_{h,m} \varphi \}$, and \sqsubset is defined as follows: $m \sqsubset m'$ iff $m \sqsubset_{\Delta} m'$, where $\Delta = \langle D, \{ \varphi \mid m' \models \varphi \} \rangle$.

We write \sqsubset_D for the preference relation of $\text{pr}(D)$ and we use obvious abbreviations, such as Th_D^* and \models_D^* for $\text{Th}_{\text{pr}(D)^*}$ and $\models_{\text{pr}(D)^*}$, respectively. Th_D^* corresponds with C_D of Makinson (1989), since $\text{Th}_D^*(\Gamma)$ is the intersection of all extensions of $\langle D, \Gamma \rangle$.

PROPOSITION 3.13 $\text{Th}_D^*(\Gamma) = \bigcap \{ E \mid E \text{ is an extension of } \langle D, \Gamma \rangle \}$.

Proof. Assume $m \models_D^* \Gamma$. Then $m \models_D \Gamma$ and $m \sqsubset_D \text{MOD}(\Gamma)$, and thus $\neg \exists m' \sqsubset_{\langle D, \{ \varphi \mid m \models \varphi \} \rangle} m' \models \Gamma$, and $m \sqsubset_{\langle D, \Gamma \rangle} \text{MOD}(\Gamma)$. But then also $\neg \exists m' \sqsubset_{\langle D, \Gamma \rangle} m' \models \Gamma$. Hence $\{ \varphi \mid m \models \varphi \}$ is an extension of $\langle D, \Gamma \rangle$. On the other hand, assume that $\{ \varphi \mid m \models \varphi \}$ is an extension of $\langle D, \Gamma \rangle$. Then $m \models \Gamma$ and $\neg \exists m' \sqsubset_D m' \models \Gamma$. Thus $m \models_D \Gamma$. Let m' be such that $\Gamma \subseteq \{ \varphi \mid m' \models \varphi \} \subset \{ \varphi \mid m \models \varphi \} \Rightarrow$ Then $m \sqsubset_D m'$, since each default that is applied to get from $\text{MOD}(\Gamma)$ to m can be applied to get from m' to m . Hence $m \models_D^* \Gamma$. We can conclude that $m \models_D^* \Gamma$ iff $\{ \varphi \mid m \models \varphi \}$ is an extension of $\langle D, \Gamma \rangle$, and the proposition follows immediately.

In view of proposition 2.4 it is no surprise that one can also obtain a modal preferential model semantics for default logic. There is a quite obvious reformulation of the above results in terms of **K45** models instead of hypervaluations. The use of **KD45** or **S5** is less straightforward since these logics lack models matching inconsistent extensions. If one assumes extensions to be consistent, then, from a technical point of view, default rules can be interpreted as well in terms of (**KD45**-)belief as in terms of (**S5**-)knowledge. See Voorbraak (forthcoming).

4. Discussion

We have given a preferential model semantics for default consequence understood in the 'meet' or 'sceptical' sense. (φ is a sceptical default consequence of Δ iff φ is true in every extension of Δ .) Makinson (1989) has shown that credulous default consequence (φ is a credulous default consequence of Δ iff φ is true in some (arbitrarily chosen) extension of Δ) does not satisfy cumulative transitivity ($\Gamma \subseteq \Delta \subseteq \text{Cn}(\Gamma) \Rightarrow \text{Cn}(\Delta) \subseteq \text{Cn}(\Gamma)$) and can therefore not be captured in a p.m.s. (Makinson's result that Th_{pr} satisfies cumulative transitivity also holds for Th_{pr^*} .) Sceptical default consequence does satisfy cumulative transitivity, although it is still not a cumulative consequence operation, since it does not satisfy cumulative monotony ($\Gamma \subseteq \Delta \subseteq \text{Cn}(\Gamma) \Rightarrow \text{Cn}(\Gamma) \subseteq \text{Cn}(\Delta)$).

A failure of cumulative monotony implies that (implicitly) facts and derived formulas have a different status. But nonmonotonic formalisms which distinguish facts from derived formulas may very well be cumulative. The failure of cumulative monotony in default logic

seems to be a corollary of the requirement that extensions have to be grounded on the facts. Learning new facts, even previously derivable ones, can result in more grounded (partial) world descriptions. This might be defended by taking the difference between facts and derived formulas serious. Alternatively, one could argue that one should not require that the *logic* is cumulative, but rather that the state of belief of an ideally rational agent should not contain a set of defaults D such that Th_{D^*} fails cumulative monotony.

In a forthcoming paper (Voorbraak (forthcoming)) we argue that both default and superstrongly autoepistemic extensions can be obtained by applying essentially only two different filters, which can roughly be described as taking justification-minimal models and taking grounded models, respectively. A model is called justification-minimal iff the set of false justifications is minimal. Justification minimization is implemented in default logic by requiring the \geq_D -maximal elements to be stable, it is implemented in autoepistemic logic by strengthening minimal AE extensions to superstrongly grounded AE extensions, and it is closely related to the minimization in circumscription.

The groundedness filter requires the extensions of a default theory $\langle D, W \rangle$ to be \geq_D MOD(W) and AE extensions to be minimal. This groundedness filter is not applied in circumscription. This more or less explains why we need a stronger version of preferential consequence for default logic than for circumscription.

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