ELECTROMAGNETIC PROPERTIES OF THE DEUTERON IN A RELATIVISTIC ONE-BOSON EXCHANGE MODEL

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The deuteron electric electromagnetic form factors are studied in a quasi-potential framework, where relativistic and meson-exchange contributions are treated consistently. At moderate momentum transfer the corrections to the static approximation are found to be significantly less than estimates obtained in the conventional perturbational treatment. It is argued that this is due to the neglect of dynamical corrections to the wave function in the latter approach.

The study of relativistic, meson-exchange and isobar effects in few-nucleon systems such as the deuteron has received considerable attention in the past decade [1]. In many of the calculations, these effects are treated as perturbative corrections to a non-relativistic theory [2–6], whereby it is assumed that the dynamics of the few-nucleon system is not affected. Despite the successes, the lack of a consistent framework is not satisfactory. One way to shed some light into the situation is to consider a model in which the above mentioned effects can be handled appropriately, and still is simple enough to be tractable. For the deuteron such a model is the Bethe–Salpeter equation in the ladder approximation (BSLA) with one-boson exchanges as the driving force [7,8]. Relativistic, meson-exchange and off-shell effects can be treated consistently in this model. Recent results [9] indicate that there is not full agreement with the perturbative calculations done up to now.

Since the BSLA differs so distinctly from the conventional non-relativistic potential theory, it is worthwhile to examine the problem in a more closely related framework like a relativistic quasi-potential approach. The purpose of this note is to present results on the deuteron using the choice due to Gross [10] for the off-mass-shell variable. Our starting point is the BSLA equation for the T-matrix, denoted by $\phi(p,p';P)$, which is a function of the final and initial relative four momenta p and p', and the total momentum P:

$$\phi(p,p';P) = K(p,p') - \frac{i}{4\pi^3}\int d^4p''\, K(p,p'')\, S(p'';P)\, \phi(p'',p';P)\,, \quad (1)$$

where K represents the sum of one-boson exchange terms and

$$S(p;P) = [(\tfrac{1}{2}\not{P}^{(1)} + \not{p}^{(1)}) - M]^{-1} \times [(\tfrac{1}{2}\not{P}^{(2)} - \not{p}^{(2)}) - M]^{-1}$$

is the two-nucleon propagator. The superscripts refer to the particles and M is the nucleon mass. The electromagnetic current of the deuteron, in the impulse approximation, can be written as:

$$J_\mu(q^2) = i\int d^4p\, \psi^\dagger(p';P')\, S^{(1)}(p';P') \times \Gamma_\mu^{(1)}(q)\, S(p;P)\, \psi(p;P)\,, \quad (2)$$

with $p' = p + q/2$, $P' = P + q$, $\Gamma_\mu = F_1(q^2)\gamma_\mu + F_2(q^2) \times \sigma_{\mu\nu}q^\nu/2M$ and where $\psi(p;P)$ satisfies the homogeneous equation corresponding to eq. (1), at $P^2 = M_D^2$, M_D being the deuteron mass. In the CM frame, eq. (1) reduces after partial-wave decomposition to a set of two-dimensional equations for eight coupled spin channels [7]. From eq. (2) it is clear that we need to know ψ in a general frame for $q \neq 0$. For this purpose we make use of the transformation property of ψ under a Lorentz transformation L:

$$\psi(p;P) = \Lambda^{(1)}(L)\,\Lambda^{(2)}(L)\,\psi(L^{-1}p;L^{-1}P)\,. \qquad (3)$$

Λ is the boost operator for spin $\frac{1}{2}$ particles corresponding to L. Defining $L^{-1}P = L'^{-1}P' = (M_D, 0)$, $k' = L'^{-1} \times p'$ and $k = L^{-1}p$, eq. (2) can be written in terms of CM quantities:

$$J_\mu(q^2) = \mathrm{i}\int \mathrm{d}^4k\ \psi^\dagger_{\mathrm{cm}}(k')\,S^{(1)}_{\mathrm{cm}}(k') \times \tilde{\Gamma}_\mu(q)\,S_{\mathrm{cm}}(k)\,\psi(k)\,, \qquad (4)$$

with

$$\tilde{\Gamma}_\mu(q) = \Lambda^{(1)-1}(L')\,\Lambda^{(2)-1}(L') \times \Gamma^{(1)}_\mu(q)\,\Lambda^{(1)}(L)\,\Lambda^{(2)}(L)\,. \qquad (5)$$

The matrix elements of the effective γNN vertex operator $\tilde{\Gamma}_\mu$ between partial wave states have been evaluated using the algebraic program SCHOONSCHIP [10].

In order to have a direct comparison with non-relativistic calculations, we have simplified the equations further. Following Gross [11], we have taken in the integrals of eqs. (1) and (4) only the contributions from the spectator nucleon pole, i.e. $k_0 = \frac{1}{2}M_D - (k^2 + M^2)^{1/2}$. As a consequence the number of channels reduces to four.

The resulting equations have been studied with the exchange of π, η, ϵ, ρ, ω and δ mesons. In the calculations pseudovector (PV) coupling was taken for the πN interaction and a strong form factor $F(t) = \Lambda^2/(\Lambda^2 - t)$, of the dipole form, was used on all meson–nucleon vertices. Taking the coupling parameters, which gave a good fit to the experimental data in the Blankenbecler–Sugar approximation [8], we find that there is more attraction in the ${}^3S_1-{}^3D_1$ channel for the Gross approximation. For example, at 100 MeV labenergy the ${}^3S_1$ phaseshift changes from 40.8° to 49.7°. However, by simply increasing the ω coupling and lowering the ϵ coupling a reasonable fit was found for the ${}^3S_1-{}^3D_1$ channel. The resulting parameters which have been used in our calculations are $g_\pi^2/4\pi = 14.2$, $g_\eta^2/4\pi = 3.09$, $g_\epsilon^2/4\pi = 6.88$, $g_\rho^{v^2}/4\pi = 0.43$, $g_\rho^{T^2}/4\pi = 6.8$, $g_\omega^2/4\pi = 12$ and $g_\delta^2/4\pi = 0.33$; the masses are the same as in ref. [7]; $\Lambda^2 = 1.5$ in units of nucleon mass squared.

The static properties of the deuteron, using the calculated wave functions, are given in table 1. From

Table 1
The bound state energy E_D, quadrupole moment Q, magnetic moment μ and the probabilities of the D-state and the negative energy states ${}^3P_1$ and ${}^1P_1$.

E_D (MeV)	Q (mb)	μ (e/M_D^2)	P_D (%)	$P_{{}^1P_1}$ (%)	$P_{{}^3P_1}$ (%)
−2.225	2.71	0.8694	4.8	0.03	0.07

this table we see that the quadrupole moment is 5% below the experimental value. Leaving out the negative energy states changes the quadrupole moment by less than 0.01 mb, while the value of the magnetic moment is lowered by 2%. The latter is consistent with a lowest order estimate of 1.5% from ref. [12]. A higher D-state probability might improve both the quadrupole and the magnetic moment with experiment.

The results for the electric form factor $A_{\mathrm{el}}(q^2)$, found by evaluating eq. (4) in the Breit frame ($\boldsymbol{P} + \boldsymbol{P}' = 0$) are shown in fig. 1, for higher momentum transfer. They are similar to those given in ref. [13] for the Reid interaction, including relativistic corrections. We used nucleon form factors of the dipole type in our calculations. From fig. 1 we see that the contributions from the negative energy states become important above $q^2 = 130\ \mathrm{fm}^{-2}$. The "static" approximation,

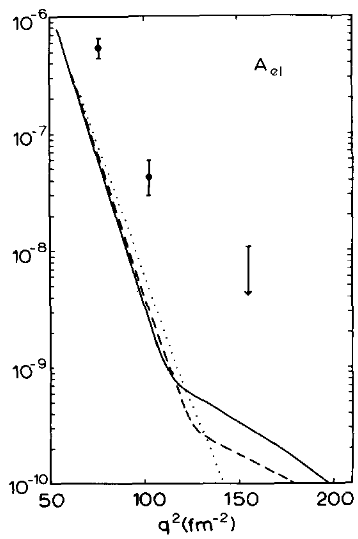

Fig. 1. The electric form factors A_{el} of the deuteron: complete calculation (——), without negative energy states (- - -) and the static approximation (. . .). The experimental data are taken from ref. [14].

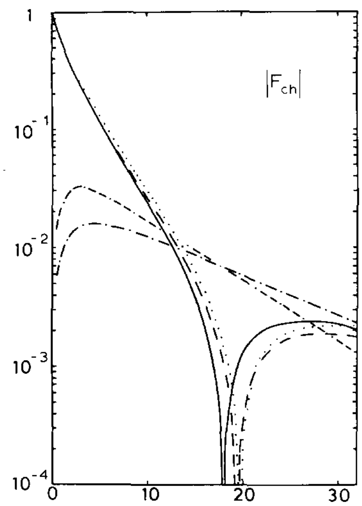

Fig. 2. The charge form factor $|F_{ch}|$ of the deuteron: complete calculation (——), the static approximation (- - -). For comparison the Reid result is given (. . .). Also are shown the absolute values of the difference between diagrams 3c and 3d (–.–.–.) and of diagrams 3b and 3c (——.——.).

which is also shown in the figure, consists of neglecting the boost operators and relativistic kinematics. Comparison with the experimental data [14] shows that we still need other processes like $\rho\pi\gamma$, and isobar admixtures in the deuteron.

We now turn to the charge form factor (CFF), which is shown in fig. 2 for q^2 up to 30 fm^{-2}. Comparing our result with that obtained for the Reid potential we see that the dip is shifted only by 1.8 fm^{-2}. to a lower value. This is essentially in agreement with the CFF calculated from the BSLA equations. On the other hand, estimates of meson-exchange current contributions (MEC) indicate a much larger shift of the dip, and an increase of the secondary maximum by a factor of two [4]. Although the calculations of the one-pion exchange contribution have been carried out for pseudo scalar (PS) coupling, the same behaviour is expected for PV coupling in view of the generalized equivalence theorem [15].

In order to understand our results in some detail, we have first studied the effect of negative-energy states and relativistic kinematics. The negative-energy state contribution to the CFF is less than 1% up to 60 fm^{-2}, while neglect of the boosts in eq. (5) and in the arguments of the wavefunctions give rise to a CFF similar to that of Reid (see fig. 2). The small contribution from the negative-energy states found by us is in agreement with a lowest order perturbational estimate for the pair term with PV coupling. According to the equivalence theorem a large effect should then arise from recoil contributions.

To resolve this we have analysed the conventional perturbation treatment of MEC effects in our model. At the deuteron energy the two-nucleon T-matrix satisfies the homogeneous integral equation

$$\phi(p) = \frac{1}{2\pi^2} \int dp' \, K(p, p_0; p', p_0') S(p') \phi(p') \tag{6}$$

with $p_0 = \frac{1}{2}M_D - (p^2 + M^2)^{1/2}$ and $p_0' = \frac{1}{2}M_D - (p'^2 + M^2)^{1/2}$. The deuteron current in the impulse approximation, shown diagrammatically in fig. 3a, can as well be calculated by replacing one of the ϕ's in the expression of the deuteron current by eq. (6). The corresponding diagram is shown in fig. 3b. In this model the spectator particle is on mass shell, which is indicated by crosses in the diagrams. In the following we shall neglect the small contributions from the negative-energy states. We further consider only the one-pion exchange as in the usual perturbational studies. In a non-relativistic calculation it is assumed that all the particles are on the mass shell (diagram 3d), while the propagator $S(p) = [2(p^2 + M^2)^{1/2} - M_D]^{-1}$ is replaced by the Lippmann–Schwinger propagator $[p^2 - E_D]^{-1}$. Following Drechsel and Weber [6], we can calculate the MEC by taking the intermediate nucleon off mass shell, as shown in diagram 3c, and subtracting the non-relativistic part, diagram 3d. The result is shown in fig. 2. From this we see that the MEC is indeed very large and in accordance with the results of the pair term contribution in the PS case [4]. It should be mentioned that the difference between 3c and 3d vanishes at $q^2 = 0$, so that no wavefunction renormalisation is needed. In the conventional treat-

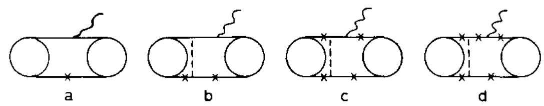

Fig. 3. The diagrams show the steps to be taken from a relativistic quasi-potential theory to the non-relativistic one. 3a is the relativistic impulse approximation, which is equivalent to 3b. 3c is the diagram usually considered in calculating MEC and 3d is the non-relativistic impulse approximation.

ment of the MEC, such as has been carried out above, a crucial step is made in going from a fully relativistic to a non-relativistic theory by tacitly assuming that 3b may be approximated by 3c. In order to get some idea how good this approximation is, we calculated the difference between 3b and 3c by renormalizing the wavefunction for diagram 3b, such that the difference vanishes at $q^2 = 0$. In doing so, we find a contribution opposite in sign to the MEC and of the same order of magnitude. The result is shown in fig. 2. Although the above estimate is rather crude, it indicates that there is a cancellation between the so-called MEC and the contributions which contain intrinsically the detailed dynamics of the two-nucleon system. As a result the CFF is not changed significantly in the momentum transfer range considered.

From this we may conclude that it is important to use a consistent model both for the pure dynamics of the two-nucleon system and for the electromagnetic properties in calculating relativistic and meson-exchange effects.

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