

## ON THE USE OF RAYLEIGH WAVE GROUP VELOCITIES FOR THE ANALYSIS OF CONTINENTAL MARGINS

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### ABSTRACT

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Rayleigh wave group velocities provide a low-cost means for a quick assessment of averaged local properties of the Earth's crust in continental margin regions of the Atlantic type. Sufficiently accurate measurements (with a standard error of 0.3 km/s or less) of group velocities in continental shelf areas at periods between 5 and 30 seconds provide important information about structural parameters. They may resolve the Moho depth to within 4 or 5 km, depending on crustal thickness, and give useful estimates of the average velocities in the upper part of the crust. The group velocities of Rayleigh waves in this period range are influenced most by the shear velocity at all depths and the compressional velocity and the density near the surface. For continental rise regions, the dominating influence of the water layer limits the effectiveness of the method.

### INTRODUCTION

Surface waves have been used for some time with much success for the exploration of the Earth's properties, especially the shear velocity distribution in the upper mantle. Excellent reviews of the methods that have been used and the results that have been obtained may be found in Dziewonski and Hales (1972), Knopoff (1972), Seidl and Mueller (1977) and Kovach (1978). For the theory of surface waves and their excitation, the reader is referred to Takeuchi and Saito (1972) or Vlaar and Nolet (1978).

Lately, interest in the use of surface wave dispersion for crustal studies has been renewed (Tatham, 1975; Braile and Keller, 1975). The main reason for the present analysis on the use of surface waves in the context of continental margin studies, is that surface waves may provide additional information to refraction or reflection studies at very low cost.

For the measurement of surface wave dispersion, use can be made of natural (shallow) earthquakes, recorded with a long-period seismograph. The 15-s instruments of the WWSSN network are well suited for this work, espe-

cially when recording at high speed (3 cm/min) so that short periods can be digitized accurately. Even the cost of installing a temporary station and its maintenance is negligible in comparison to that of a seismic sounding experiment at sea.

Interpretation of surface waves is not hampered by the existence of one or more low-velocity layers, and usually no identification problems occur, as with refraction onsets. By its very nature, the energy of the surface wave is concentrated near the surface of the Earth. The depth of penetration increases with increasing period. When the wave encounters material of higher velocity at depth, the phase velocity increases. As a rule of thumb, both Love and Rayleigh waves at a certain frequency may be considered to penetrate to that depth where the shear velocity equals the phase velocity of the surface wave. The group velocity, which results from the interference of waves with neighbouring frequencies, usually has a somewhat more complicated behaviour than the phase velocity, but is much easier to measure. Neither of the two velocities can be said to represent the velocity of the rock at a specified depth; surface wave dispersion is an integrated effect of the rock properties above the depth of penetration. Therefore, surface waves do not provide as clear a resolution of the interfaces as body waves do. We can only expect to resolve the average velocity in a few layers. Moreover, the dispersion is also the average dispersion over the horizontal pathlength, which is usually much longer than in refraction studies. The width of the path can be estimated as one wavelength ( $= \text{phase velocity} \times \text{period } (T)$ ), or about  $3T$  km). In the presence of lateral inhomogeneity, reflection and mode-conversion is stronger for Love waves than for Rayleigh waves (Drake, 1972). Rayleigh waves provide the additional advantage that they appear isolated from the Love waves on vertical seismograms, and that the fundamental mode is well separated in time from the first and higher modes.

In this paper we analyse the resolving power of Rayleigh group velocities in the period range 5–30 s, for crustal structures representative of continental margins of the Atlantic type. A margin of this type can be divided into three distinct structural elements: continental shelf, continental slope and continental rise. Instead of modelling the margin as a whole, we will investigate separate models for the elements shelf and rise. Continental slope type structures are less suitable for investigation by surface waves owing to their small width and their highly variable and heterogeneous character. Special emphasis will be put on the determination of layer thicknesses in the crust.

## RAYLEIGH WAVE DISPERSION

A limited, though useful, method to analyse the resolving power of Rayleigh group velocities is to use linear perturbation theory to calculate the effect of small perturbations in velocity or density on the dispersion curve. Jeffreys (1961) and others have shown how Rayleigh's Principle may be applied to the problem of surface wave dispersion. Rodi et al. (1975) have

developed a convenient way to extend these results to group velocities.

We assume that parameter changes  $\delta\alpha_i$ ,  $\delta\beta_i$ , and  $\delta\rho_i$  (compressional velocity, shear velocity and density respectively) in layer  $i$  result in group velocity changes  $\delta U_j$  (at frequency  $\omega_j$ ) that can to sufficient accuracy be expressed by a linear relationship:

$$\begin{bmatrix} \delta U_1 \\ \delta U_2 \\ \vdots \\ \delta U_M \end{bmatrix} = \begin{bmatrix} \partial U_1/\partial\beta_N \dots \partial U_1/\partial\beta_1 & \partial U_1/\partial\alpha_N \dots \partial U_1/\partial\alpha_1 & \partial U_1/\partial\rho_N \dots \partial U_1/\partial\rho_1 \\ \partial U_2/\partial\beta_N \dots \partial U_2/\partial\beta_1 & \partial U_2/\partial\alpha_N \dots \partial U_2/\partial\alpha_1 & \partial U_2/\partial\rho_N \dots \partial U_2/\partial\rho_1 \\ \vdots & \vdots & \vdots \\ \partial U_M/\partial\beta_N \dots & & \partial U_M/\partial\rho_1 \end{bmatrix} \begin{bmatrix} \delta\beta_N \\ \vdots \\ \delta\beta_1 \\ \delta\alpha_N \\ \vdots \\ \delta\alpha_1 \\ \delta\rho_N \\ \vdots \\ \delta\rho_1 \end{bmatrix}$$

or:

$$\partial U = A \partial p \quad (1)$$

Here  $N$  is the deepest layer, layer 1 is the (solid) layer at the surface. From (1) it is clear that inspection of the matrix  $A$  will show us the relative importance of each structural parameter. For one of the models that will be studied later in this paper (model 4.4.10, see inset Fig. 2 and Table I), we have calculated matrix  $A$  using spherical Earth calculations and the method of Rodi et al. (1975). The result is listed in Table II, the variational parameters for the crustal layers are shown as a function of depth in Fig. 1.

From Table II we see that the shear velocity in any layer influences at least part of the data, though the influence of the low velocity channel (layer 5) is limited to the longest periods. The influence of layer 1 is highest at the short period of 5 s and negligible above 10 s. For a layer thickness of 8 km this maximum shifts to the datum of 10 s. The influence of the compressional velocity is only large in layer 1, but definitely not negligible in the rest of the crust. The same is true for the density, for which the magnitude of the variational parameters approaches half of those for the shear velocity. The importance of  $\alpha$  and  $\rho$  for the group velocity in crustal structures is surprising in view of their relative unimportance for mantle studies, where they are often ignored.

Eigenvalue analysis of the matrix  $A$  shows that a data set with a precision of 0.1 km/s would resolve about 5 parameters. These are the average compressional velocity in the crust with a precision of about 0.2 km/s; the sub-Moho shear velocity, and the average of the shear velocity in layer 2 and layer 3, also to a precision of about 0.2 km/s; and the shear velocity in layer 1 with a standard error of 0.1 km/s. A fifth parameter is a combination of density parameters. As was noted by Nolet (1977), surface waves are

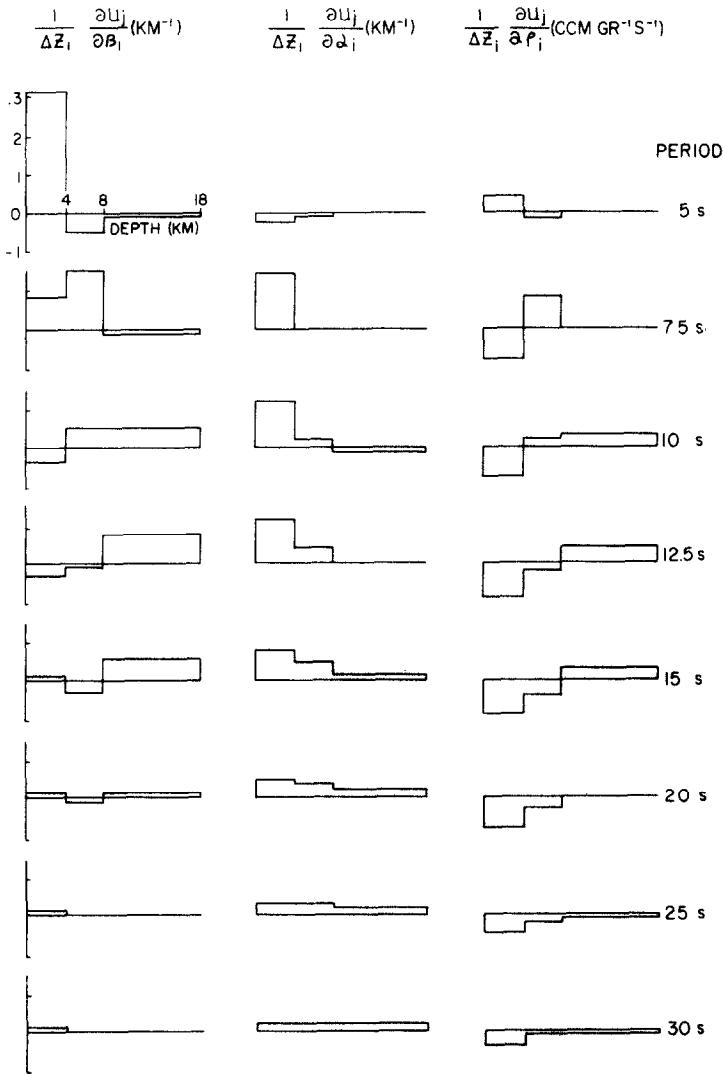


Fig. 1. Variational parameters for Rayleigh wave group velocities as a function of depth in the crust for the 4.4.10 shelf model. All figures are at the same scale as the first. The variational parameters as listed in Table II are normalized by the layer thickness  $\Delta z_i$ .

insensitive to multiplications of  $\rho(z)$  with a constant factor when the velocities are kept fixed. Therefore it is impossible to resolve a local average of  $\rho$  from surface wave data only; one may, however, determine the average slope of  $\rho$ .

The above analysis does not take into account that the thickness of the layers may vary as well.

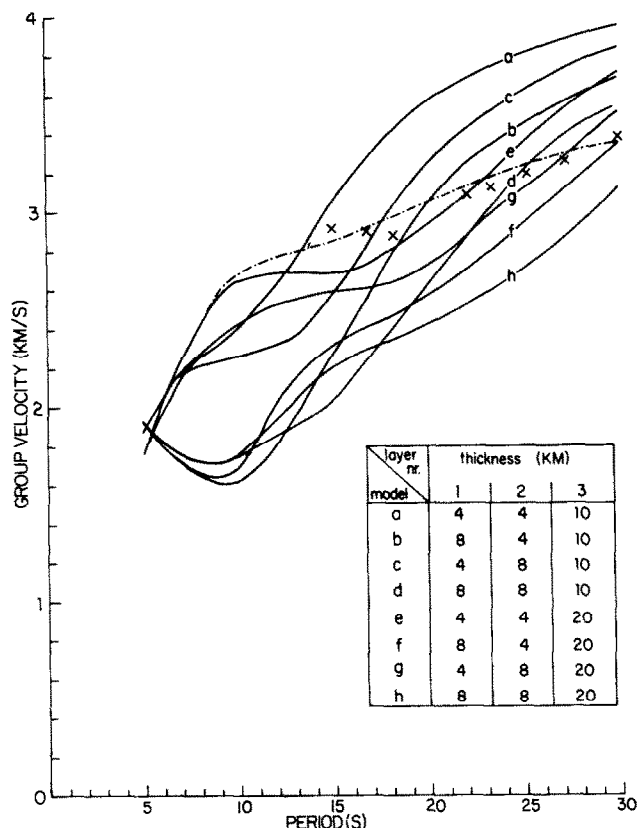


Fig. 2. Group velocity curves for eight different shelf models in the period range 5–30 s. Thicknesses of the crustal layers are given in the inset. Velocities and densities are listed in Table I. Crosses indicate group velocity measurements from a path along the northern Brazilian continental shelf. Epicentral coordinates of the event: 10.58°N 63.40°W. Station coordinates: 5.07°S 35.02°W. Dashed line: the result of the inversion procedure. The model belonging to this curve is shown in Fig. 4.

TABLE I

Velocities and densities \*

	$\alpha$ (km/s)	$\beta$ (km/s)	$\rho$ (g/cm <sup>3</sup> )
<b>Shelf models:</b>			
layer 1	3.90	2.23	2.30
layer 2	5.60	3.20	2.50
layer 3	6.60	3.77	2.70
layer 4	8.10	4.70	3.35
layer 5	8.00	4.20	3.35
<b>Rise models:</b>			
layer 0	1.50	—	1.03
layer 1	3.30	1.89	2.20
layer 2	5.40	3.08	2.50
layer 3	6.80	3.89	2.70
layer 4	8.10	4.70	3.35
layer 5	8.00	4.20	3.35

\* Based on recent compilations (e.g., Burk and Drake, 1974; De Almeida, 1976).

TABLE II  
Matrix **A** calculated for the 4.4.10 shelf model

Period(s)	$\partial U_j / \partial \beta_i$					$\partial U_j / \partial \alpha_i$					$\partial U_j / \partial \rho_i$ (km cm <sup>3</sup> g <sup>-1</sup> s <sup>-1</sup> )				
	layer no.					layer no.					layer no.				
	5	4	3	2	1	5	4	3	2	1	5	4	3	2	1
30	-0.20	0.74	0.00	0.01	0.05	-0.00	0.04	0.14	0.08	0.07	-0.06	0.27	-0.10	-0.06	-0.11
25	-0.12	0.60	0.03	-0.01	0.06	-0.00	0.02	0.16	0.10	0.10	-0.04	0.28	-0.09	-0.08	-0.15
20	-0.03	0.41	0.15	-0.05	0.06	-0.00	0.00	0.17	0.14	0.16	-0.01	0.26	-0.03	-0.12	-0.21
15	-0.00	0.06	0.61	-0.07	0.04	-0.00	-0.02	0.12	0.18	0.28	-0.00	0.16	0.23	-0.17	-0.32
12.5	0.00	-0.20	0.86	0.03	0.01	0.00	-0.02	0.02	0.16	0.33	0.00	-0.01	0.40	-0.12	-0.33
10	0.00	-0.24	0.58	0.29	0.02	0.00	-0.01	-0.06	0.07	0.31	0.00	-0.11	0.31	0.05	-0.27
7.5	0.00	-0.07	-0.02	0.62	0.28	0.00	-0.00	-0.03	0.00	0.31	0.00	-0.04	0.04	0.28	-0.29
5	0.00	-0.00	-0.18	0.06	1.23	0.00	-0.00	-0.00	-0.04	0.10	0.00	-0.00	-0.10	0.08	0.03

## LAYER THICKNESS RESOLUTION

To analyse the resolving power of Rayleigh group velocities we refrain from relying fully on a linearized perturbation theory, such as the generalized matrix theory employed by Braile and Keller (1975) \*. The main reason for this is that the dependence of the group velocities on the location of strong velocity contrasts may occasionally be very nonlinear, making the outcome of the calculations rather doubtful in margin regions, where the range of models is not a priori limited to a small set. Also, calculations of the Backus–Gilbert type, though very useful for one particular data set, are difficult to extend to other data sets or sets with different precision. Master curves for mantle dispersion are given by Knopoff (1972). Mantle dispersion, however, is a linear phenomenon.

For this analysis we have constructed a set of *continental shelf* models, which allows for a more flexible approach. The models all consist of a 3-layer crust on top of a lid and a low velocity channel which invariably starts at a depth of 101 km. Since we restrict ourselves to short period data, the choice of the depth of the channel is of little influence to the results. In the following, we will denote the models as  $(d_1, d_2, d_3)$  (e.g., model 4.8.20), where:

$d_1$  = thickness (in km) of the upper crustal layer with average  $\alpha = 3.9$  km/s;

$d_2$  = thickness of the middle layer with velocity  $\alpha = 5.6$  km/s;

$d_3$  = thickness of the deepest crustal layer,  $\alpha = 6.6$  km/s.

Values for layer thicknesses, velocities and densities as given in the inset of Fig. 2 and in Table I, have been adopted on the basis of recent compilations (e.g., Burk and Drake, 1974; De Almeida, 1976). For continental shelves the velocities and density of layer 1 are representative for a sedimentary sequence. The velocities and density for layer 2 represent either high velocity sedimentary rocks or crystalline basement. For layer 3 we have taken velocities and a density appropriate for the lower crust (see e.g., Smithson et al., 1977).

The group velocity curves for eight different thickness combinations are shown in Fig. 2. A few general characteristics are easily noted: the highest group velocities belong to thin crusts; when layer 1 is thick (8 km), the curves show a minimum near a period of 9 s; all curves with  $d_3 = 20$  km are flatter at  $12 < T < 22$  s than the models with  $d_3 = 10$  km.

When we single out the  $d_3 = 10$  models (steep slopes), we note that these curves have a maximum separation which is at least 0.5 km/s for some period. Consequently, if we know that the velocities adopted for the master-models (Table I) are correct (or if we can correct for the differences using linear perturbation theory), a datum precision of 0.5 km/s would suffice

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\* Contrary to statements in their paper, their approach is not formally equivalent to a Backus–Gilbert inversion since the resolving kernels are not required to be unimodular. An adaptation of Backus–Gilbert theory to discrete systems was developed by Kennett and Nolet (1978).

to distinguish between different models of this type. With this precision we would establish the thicknesses of layer 1 and 2 to within 2 km.

In most cases, however, we will not know the layer velocities. Here the linear perturbation theory may help us in getting an estimate of the resolution for this case. From (1) and the variational parameters in Table II we easily see that a decrease of  $\alpha$  in layer 3 and 4 with 0.17 and of  $\beta$  with 0.10 km/s, will cause the curve of model 4.4.10 to be lowered by roughly 0.1 km/s. From similar calculations made for the other models, this appears to be the right order of magnitude. This extra "uncertainty" of 0.1 km/s is for a large part (0.07) due to the uncertainty in  $\beta$  which will be present even if precise  $\alpha$  values are available, since knowledge on  $\beta$  is commonly not at hand. With the introduction of more ocean bottom seismometers, however, shear velocity data are becoming available (see e.g., Hyndman, this volume).

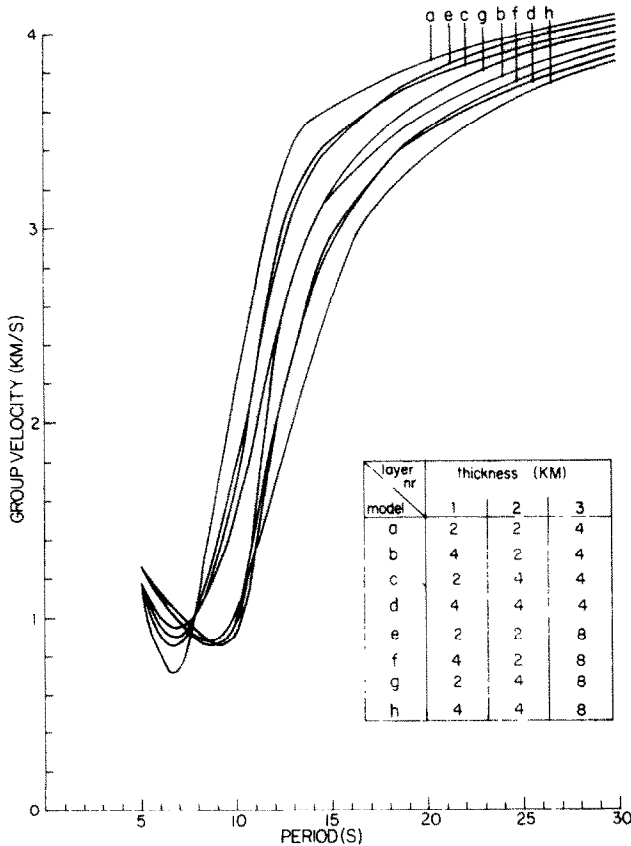


Fig. 3. Group velocity curves for eight different continental rise models in the period range 5–30 s. Thicknesses of the crustal layers are given in the inset. Water depth is 3 km. Velocities and densities are listed in Table I.



A realistic guess of an uncertainty of  $\pm 0.15$  km/s in  $\beta$  (from assumption on Poisson's ratio only) will therefore reduce the maximum separation of the curves by 0.2 km/s. The precision of 2 km in layer thickness will therefore only be reached for data errors less than 0.3 km/s.

The situation is worse when the crust thickens: the same reasoning as before shows that we cannot hope to resolve the thickness of layer 2 when  $d_3 = 20$  km, since curves with  $d_2 = 8$  and  $d_2 = 4$  km are only about 0.2 km/s apart. The thickness of layer 1 is well resolved, however.

The choice between  $d_3 = 10$  or 20 km (or some value in between) must not be made on the ground of curve separation, but of the slope of the curve. Though this is more difficult to express quantitatively, we feel confident that a group velocity precision of 0.3 km/s is sufficient to distinguish between values of 10, 15 (curves not shown) or 20 km for  $d_3$ , or an accuracy of  $\pm 2.5$  km. Taking the r.m.s. sum of the uncertainties we find that we can establish the total thickness of the crust to  $\pm 3.8$  km for thin crusts and to  $\pm 5.1$  km for thicker crusts when the group velocities are known with a precision of 0.3 km/s. In taking a r.m.s. sum we assume for lack of better that the uncertainties are independent. We expect however that errors will usually be of opposite sign, so that our estimate is a pessimistic one.

When studying the structure of *continental rise* areas, a complicating factor turns up: The velocities in the water layer are so low that the wave energy is easily "caught" in this when the thickness of it is a few kilometers. The uniform influence of the water layer dominates the influences of crustal differences. In Fig. 3, group velocity curves for eight different rise models are shown ranging from 4.4.8 to 2.2.4 with a 3 km water layer on top. From this figure it is obvious that one may at most hope to resolve the thickness of the sedimentary layer from the minimum at short periods.

#### PRECISION OF GROUP VELOCITY MEASUREMENTS

In this paragraph we will investigate whether a precision of 0.3 km/s is realizable. To the group velocity data one must assign an error which is the r.m.s. sum of errors with different causes (which we assume independent):

$$\sigma_U = (2E_d^2 + E_{ph}^2 + E_T^2 + E_n^2 + E_{t_0}^2 + E_{inst}^2)^{1/2} \quad (2)$$

where:

$\sigma_U$  is the relative error in group velocity  $U$

$E_d = \delta x / \Delta$  with  $\delta x$  the uncertainty in latitude and longitude of the epicentre (assumed equal), and  $\Delta$  epicentral distance.

$E_{ph} = |\partial \phi / \partial \omega| U / \Delta$ , due to the variation of initial phase with frequency. Although  $|\partial \phi / \partial \omega|$  can reach values of 3 s and more, high values are only found near nodes in the radiation pattern, where amplitudes are small and measurements unlikely to occur. A value  $|\partial \phi / \partial \omega|$  of about 1 s therefore is judged conservative.

$E_T = \delta T (\partial U / \partial T) / U$  due to an error  $\delta T$  in the period  $T$ . The magnitude of  $\delta T$  will depend sharply on the method used for measurement.

$E_n$  = a relative error accounting for the presence of noise. This will be especially important if large microseisms, with periods of about 6 seconds, contaminate the measurements at short periods.

$E_{t_0} = \delta t_0 U / \Delta$  due to uncertainty  $\delta t_0$  in the origin time.

$E_{inst} = \delta t_i U / \Delta$  due to errors in the instrument correction (delay time  $\delta t_i$ ).

Using (2), we may make a calculation of the order of magnitude of  $\sigma_U$ . Assuming an epicentral distance of 400 km, epicentral uncertainty of 20 km, period  $15 \pm 2$  s, group velocity 2.5 km/s,  $\partial U / \partial T = 0.1$  km/s<sup>2</sup>,  $E_n = 5\%$ , we arrive at a relative error of 12%, or 0.3 km/s. With the parameter values chosen, this may be considered as a conservative estimate. This error is dominated by  $E_n$ ,  $E_T$ , and  $E_d$ . The latter will decrease with epicentral distance. Although an epicentral distance of 400 km may be attractive from the point of view of interpretation, we will, when using existing stations, in general have to put up with larger epicentral distances.

As an example we show in Fig. 2 group velocity data obtained along the northern Brazilian continental shelf. This margin may be studied using earthquakes from the Lesser Antilles Arc. The closest WWSSN station that records waves travelling along the margin is Natal (N.E. Brazil), at an epicentral distance of about 3000 km. As a consequence, most of the high frequency part of the signal is lost through scattering and damping. Comparison with the mastercurves shows that the average structure is close to a 4.4.20 model. This choice is primarily based on the slope of the curve at shorter periods.

Using the variational parameters calculated for this model, we have corrected the velocities of the master model (assuming Poisson's ratio 0.25), and arrive at an averaged crustal structure for the northern Brazilian continental shelf shown in Fig. 4. This model fits the data within 0.1 km/s (see Fig. 2). Also shown in the figure is the model found by Houtz et al. (1977) from

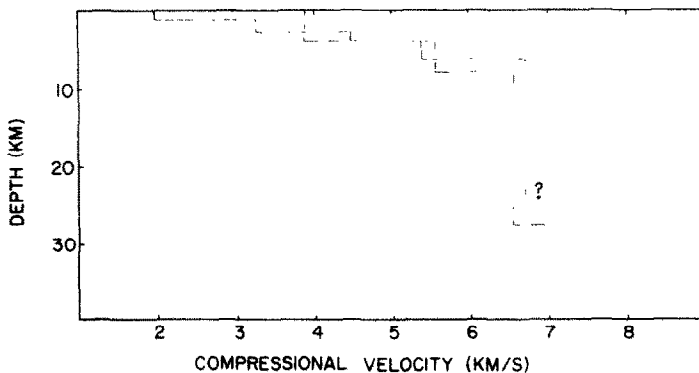


Fig. 4. Compressional velocity—depth profiles for the northern Brazilian continental shelf. Dashed line: the model resulting from the surface wave analyses. Solid line: the model resulting from a refraction study by Houtz et al. (1977).

refraction data. This model represents a local sample of the northern Brazilian shelf. The similarity between both models is reassuring. Whereas the refraction data do not give information about the Moho depth, the surface wave data estimate the Moho depth at 28 km.

## CONCLUSIONS

In this study we have found that Rayleigh waves are a most effective diagnostic tool for the exploration of continental shelf areas. Whenever a proper combination of natural earthquakes and a seismic station with a vertical long-period recording instrument is available, this can be done at negligible cost. Many more regions would become accessible to an application of the method if broadband ocean bottom seismometers (Bolt, 1977) or temporary landbased stations are employed. When natural earthquakes are absent, the group velocity of the wave between two stations along a great-circle path may be inferred from the crosscorrelation of two signals (Dziewonski and Hales, 1972); in this case it would, however, be difficult to measure short periods because of the larger epicentral distances, and the resolving power may be somewhat reduced. For continental rise areas the method is less effective because of the dominating influence of the thick water layer.

We defer a detailed discussion of the inversion method to a second paper (Cloetingh et al., in prep.), which will also contain a complete set of variational tables and master curves for a wide range of continental margin, marginal sea, and continental type structures.

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