NOTE ON SPIN RELAXATION OF THE ISING CHAIN

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This note corrects an inaccuracy in our previous paper: B. U. Felderhof, Reports Math. Phys. 1 (1970), 215.

We would like to correct an inaccuracy in our article. In equation (6.4) the definition of the operators should be modified as follows

$$\begin{aligned}\xi_q^\dagger(\beta)&=\rho_\pm^{-\frac{1}{2}}\xi_q^\dagger\rho_\pm^{\frac{1}{2}},\\ \xi_q(\beta)&=\rho_\pm^{-\frac{1}{2}}\xi_q\rho_\pm^{\frac{1}{2}},\end{aligned}\tag{N.1}$$

where the upper sign + should be employed for the q-values (4.13a) and the lower sign − for the q-values (4.13b). The operators $\rho_\pm$ are defined by

$$\rho_\pm=\exp[-\beta\mathscr{H}_N^\pm]/Z_N^\pm(\beta),$$

where the commuting operators $\mathscr{H}_N^+$ and $\mathscr{H}_N^-$ are given in (6.5) and $Z_N^\pm(\beta)$ are the corresponding partition functions. With the definition (N.1) the expressions (6.16) remain valid but lead to the conclusion that the state $|0\rangle$ is not the vacuum state for ξ_q-particles with q-values (4.13b). Rather one has

$$\xi_q|0\rangle=0 \quad \text{for} \quad q=\pm(2m-1)\pi/N, \tag{N.2a}$$

$$\xi_q|0^-\rangle=0 \quad \text{for} \quad q=\pm 2m\pi/N, \tag{N.2b}$$

where

$$|0^-\rangle=C|0\rangle$$

with

$$\begin{aligned}C&=\rho_-^{\frac{1}{2}}\rho_+^{-\frac{1}{2}}=A_N[\cosh\beta J+(c_N^\dagger-c_N)(c_1^\dagger+c_1)\sinh\beta J],\\ A_N&=[Z_N^+(\beta)/Z_N^-(\beta)]^{\frac{1}{2}}=(1+a^N)^{\frac{1}{2}}(1-a^N)^{-\frac{1}{2}},\\ a&=\tanh\beta J.\end{aligned}$$

Using (4.5) one has

$$|0^-\rangle = A_N(\cosh \beta J - \sigma_N^z \sigma_1^z \sinh \beta J)\,|0\rangle$$

and one easily shows

$$\langle 0^-|0^-\rangle = 1\,, \qquad \langle 0|0^-\rangle = A_N(1+a^N)^{-1}(\cosh \beta J)^{-1}\,.$$

As a consequence of (N.2b) the excited states $|\boldsymbol{q}_k\rangle$, in case $\boldsymbol{q}_k \equiv (q_1, q_2, \ldots, q_k)$ is a set of q-values (4.13b), must be defined with respect to the vacuum state $|0^-\rangle$ rather than $|0\rangle$. Correspondingly, in this case in (8.5) and (8.9) the state $|0\rangle$ should be replaced by $|0^-\rangle$. For k odd, $|\boldsymbol{q}_k\rangle$ is an eigenstate of $W(\beta)$. In contradiction to the statement following (4.25) we now have

$$W^+(\beta)\,|0\rangle = 0\,,$$

$$W^-(\beta)\,|0^-\rangle = 0\,.$$

With the above interpretation of the states $|\boldsymbol{q}_k\rangle$ the remaining formalism needs no alteration.

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