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SPATIAL SUMMATION FOR COMPLEX BAR PATTERNS

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Abstract—We have obtained contrast detection thresholds for spatial grating patterns of limited coherence length for square target fields subtending 2 × 2′ up to 4 × 4°. The results are consistent with a form of physiological summation in which the spatial numerical energy determines the detectability of the pattern. The limited coherence length permits us to uncouple the relationship between the target extent and the number of "coherent" periods in the pattern. Threshold is determined by the target extent, not by the number of coherent periods. A small effect of coherence length has been observed, coherent patterns being more detectable than less coherent ones, even for very long coherence lengths.

INTRODUCTION

It has been known for some time that the detection threshold for extended complex targets depends on the extent of the target field (Robson, in Mostafavi and Sakrison, 1976; Hoekstra *et al.*, 1974). However, the mechanism of this spatial summation effect is still unclear. It is known that the magnitude of the effect for gratings is more or less equal whether the extent of the pattern is varied parallel or perpendicular to the stripes (Howell and Hess, 1978; Koenderink *et al.*, 1978c). In the peripheral visual field a complex relationship between the magnitude of the effect and the shape of the target field has been reported (Koenderink *et al.*, 1978c). It is known that the exact way in which the target field is terminated is important (Estévez and Cavonius, 1976). On the other hand next to nothing is known about the influence of the spatial structure of the stimulus. This is important in view of the summation mechanism: we do not know whether to regard the summation as coherent or incoherent (summation of amplitudes or for example power?). If one regards the effects as the result of probability summation then a form of incoherent summation appears mandatory. Incoherent summation has been suggested by Quick *et al.* (1976) on theoretical grounds. They also published experimental evidence that seems to favour the numerical energy (that is the incoherent sum) of the signal as the psychophysical variable corresponding to perceived contrast. Mayhew and Frisby (1978) present corroborative evidence. This points to a physiological, incoherent mechanism. Mostafavi and Sakrison (1976) cite unpublished experimental evidence by Halter which points also to incoherent summation. For the case of purely *temporal* modulations Koenderink and van Doorn (1978d) have shown that the summation is both incoherent and physiological.

Graham (1977) remarks that, on grounds of the available evidence, we cannot yet decide whether the effect is the result of probability or physiological summation. Even formally these mechanisms lead to similar results. Physiological summation has been discussed by Kelly (1975). Kelly argues for a form of coherent summation. Coherent summation could lead to psychologically sharp spatial frequency discrimination whereas the underlying physiological units are spatially broadly tuned. One can only decide between coherent and incoherent mechanism if it is possible to uncouple the extent and the bandwidth of the stimulus pattern. This is possible if one uses signals of which the extent and the coherence length are separately adjustable. Then one can decide whether the "critical number of cycles" mentioned by Hoekstra *et al.* (1974) relates to the target extent or to the extent of a coherent wave-train. Such measurements are similar to the determination of the critical bandwidth by means of noise bands in audiology.

Another problem with the summation mechanism is that it is not clear what to take as the summands. For instance, Wilson and Giese (1977) sum contributions of small equal incremental areas in the visual field. On the other hand Koenderink and van Doorn (1978e) show that the summation should extend not over equal incremental areas, but over the stimulated ganglion cells. The notion that the effect depends on the number of active perceptive fields has been related to the fact that the effect seems to depend on the number of cycles and not on the spatial frequency of the stimulus (Koenderink, 1977; Howell and Hess, 1978).

METHODS

Apparatus

The patterns were generated on a modified commercial TV monitor. The interlacing was removed, the frame repetition frequency raised to 61 Hz, the number of lines reduced to 256. The modulation signal was applied directly to the cathode of the CRT. We modulated always perpendicular to the direction of the lines, that is, the intensity of any line was variable. The phosphor was the normal white P4. Linearity was checked photometrically and was within 10% over the range used. The mean retinal illuminance was 200 td. The viewing distance was 3.5 m, the target subtended 2 × 2' up to 4 × 4°.

The extent of the target field was regulated by means of masks made of black opaque material placed directly in front of the screen. If the diameter of the target field exceeded 18 minutes of arc a small black fixation mark (dia 2' min arc) was pasted on the center of the screen.

We employed a chin and forehead rest, an artificial pupil, diameter 2 mm, and optimal correction lenses. The subjects were experienced observers (the authors): JK an emmetrope, 35 years; AD 2 D myope, 30 years. The threshold was found by the method of limits, we used steps of 0.05 log units. The measurements were performed over a time span of several weeks. We present mean values of thresholds obtained on five different days. The standard error (in this case indicative of the long-term repeatability of the measurements) amounts to 0.05–0.1 log units.

Structure of the stimulus

In our experiments we always used a dark surround. Whereas it seems certain that the effects obtained with different surrounds are at least similar (Savoy and McCann, 1975; McCann *et al.*, 1978), it also seems certain that the effects can quantitatively differ in varying circumstances (Estévez and Cavonius, 1976; Howell and Hess, 1978; McCann *et al.*, 1978). We did not extend our investigation to different circumstances although this should be of interest. The dark surround situation seems especially important in view of practical applications. (View into the eye piece of various optical instruments etc.)

We use stimulus patterns with the following dependence of the retinal illuminance I on the relevant spatial coordinate x (m denotes the modulation depth):

$$I(x) = I[1 + m \cdot \sigma_1(x) \cdot \sigma_2(x)] \quad \sigma_{1,2} = \pm 1.$$

The signs $\sigma_{1,2}$ are obtained as follows. The screen is divided into stripes of width D, and alternate signs ($\sigma_1 = +1, -1, +1, -1, \ldots$) are assigned to consecutive stripes. As a result we obtain a regular rectangular grid with bar width D if σ_2 is taken constant (+1 or −1). For every frame σ_1 is modulated in the same way. We also divide the screen into strips of width C (C an integral multiple of D). We assign for any strip a new random value ±1 to σ_2 with probability one half. (If $\sigma_2 = -1$ the contrast of the rectangular grid reverses, if $\sigma_2 = +1$ this does not happen.) For any new frame the assignment of random signs is done anew, thus every 1/61 sec an independent sample is displayed.

The resulting spatial pattern consists of pieces of length C of periodic square wave patterns of wave-length $2D$. The phase of these pieces changes at random. We obtain coherent wave-trains with a length of $C/2D$ periods contained in a target that subtends $W \times W$ degrees (Fig. 1). In general C is unequal to W so that the length of the coherent wave-train is not necessarily equal to the target extent, this is the main difference between our stimuli and those used by previous investigators.

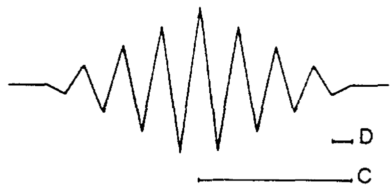

Fig. 1. The spatial one-dimensional autocorrelation function of a pattern with the coherence length (C) equal to eight times the bar width (D). This function is independent of the total width of the pattern. The spatial power spectrum of these patterns peaks at a frequency of $1/2D$ and the harmonics $(2n + 1)/2D$ ($n = 1, 2, 3, \ldots$). More than 98% of the power is in the fundamental. The relative width of the spectral line at $1/2D$ is proportional to the reciprocal of the coherence length and is independent of the total field width.

RESULTS

As mentioned above with the present choice of stimuli we are left with three independent parameters that describe the stimulus structure. We vary these parameters each between 1 and 294' of arc in equal logarithmic steps (powers of two) with the obvious restriction $W \geqslant C \geqslant D$. (For technical reasons the upper value of 294 had to be replaced by 240' of arc; the next lower value is 294/2' of arc etc.) This program resulted in more data than it is possible to show here. However, the results are sufficiently clear so that representative selection will suffice.

For the two extreme cases of minimum ($C = D$) and maximum ($C = W$) coherence length the sensitivity (the reciprocal of the threshold modulation depth) is shown as a function of the target width for both subjects in Figs 2–5. It is clear that the functional dependence does not differ very much for the extreme cases. For complete coherence ($C = W$) the summation does not extend over more than 30' for the finest gratings in accord with expectation (Koenderink, 1977; Howell and Hess, 1978). For both cases the sensitivity is almost proportional to the field diameter.

In Figs 6 and 7 we present the dependence of the sensitivity on the width of the stripes with field width as parameter for the extreme cases. (The results for subject JK are similar.) In the completely coherent case ($C = W$) we obtain results similar to the usual MTF measurements. For the completely incoherent case ($C = D$) we obtain the not unexpected result that the sensitivity does not diminish very much for very fine stripes (see discussion). It proves that the lowest

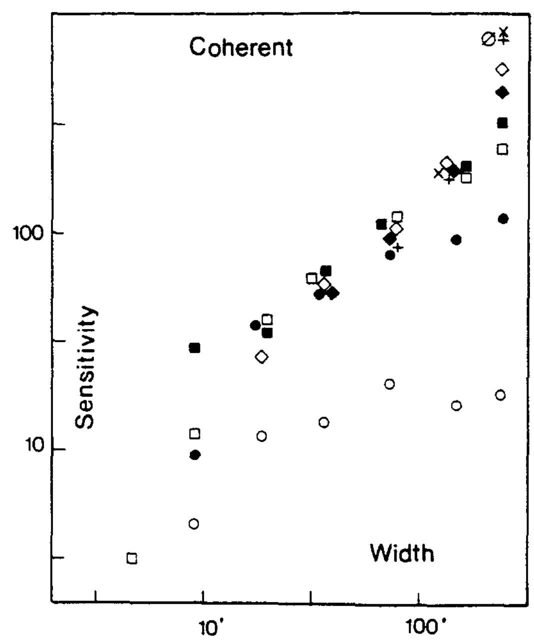

Fig. 2. The sensitivity (the reciprocal of the threshold modulation depth in percent) as a function of the total field width. The coherence length equals the total field width (so we have regular rectangular grating patterns). Subject JK. The bar width (*D*, half a wavelength) is: ∅ *D* = 240′. × *D* = 147′. + *D* = 73′. ◆ *D* = 37′. ◇ *D* = 18′. ■ *D* = 9′. □ *D* = 5′. ● *D* = 2′. ○ *D* = 1′.

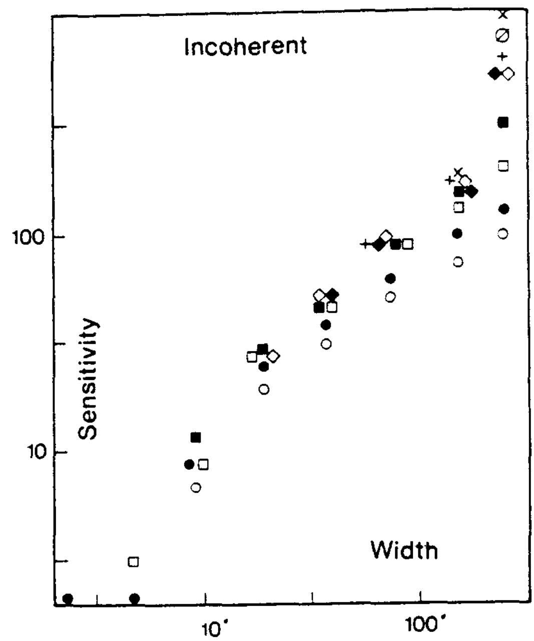

Fig. 4. The sensitivity (the reciprocal of the threshold modulation depth in percent) as a function of the total field width. The coherence length equals the bar width (random light or dark rectangular bars). Subject JK. The bar width (*D*, half a wavelength) is: ∅ *D* = 240′. × *D* = 147′. + *D* = 73′. ◆ *D* = 37′. ◇ *D* = 18′. ■ *D* = 9′. □ *D* = 5′. ● *D* = 2′. ○ *D* = 1′.

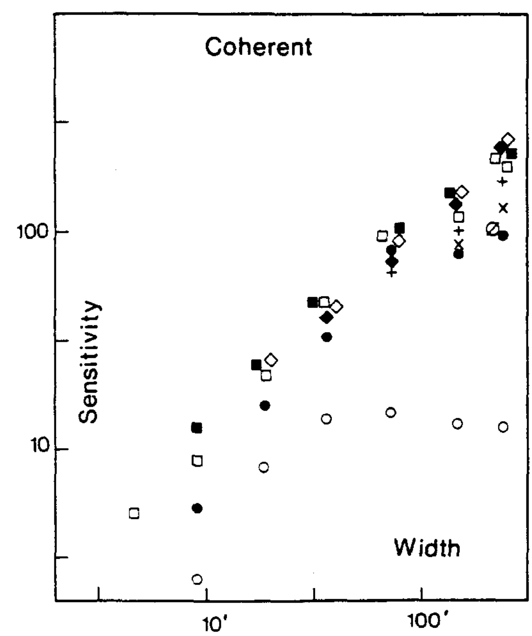

Fig. 3. The sensitivity (the reciprocal of the threshold modulation depth in percent) as a function of the total field width. The coherence length equals the total field width (so we have regular rectangular grating patterns). Subject AD. The bar width (*D*, half a wave-length) is: ∅ *D* = 240′. × *D* = 147′, + *D* = 73′, ◆ *D* = 37′, ◇ *D* = 18′, ■ *D* = 9′, □ *D* = 5′, ● *D* = 2′, ○ *D* = 1′.

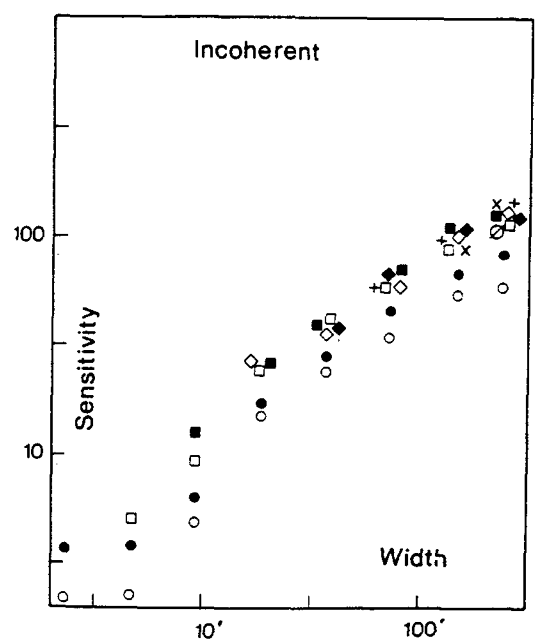

Fig. 5. The sensitivity (the reciprocal of the threshold modulation depth in percent) as a function of the total field width. The coherence length equals the bar width (random light or dark rectangular bars). Subject AD. The bar width (*D*, half wavelength) is: ∅ *D* = 240′. × *D* = 147′. + *D* = 73′. ◆ *D* = 37′. ◇ *D* = 18′. ■ *D* = 9′. □ *D* = 5′. ● *D* = 2′. ○ *D* = 1′.

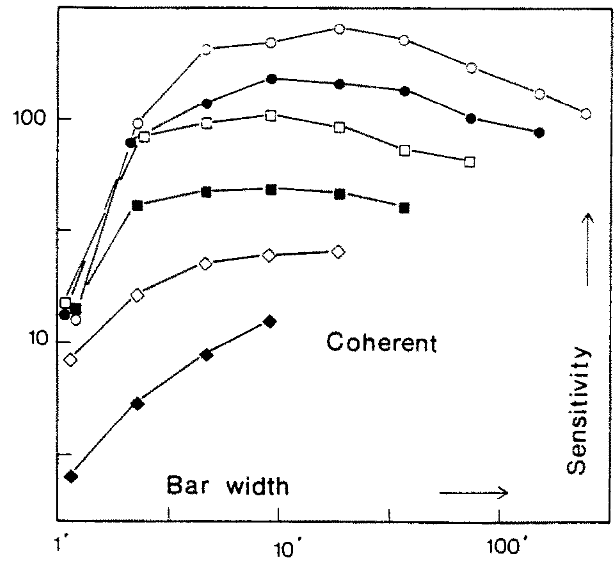

Fig. 6. The sensitivity (the reciprocal of the threshold modulation depth in percent) as a function of the bar width for alternately light and dark stripes (the coherence length equals the total field width). Subject AD. The total field width (W) is: ○ $W = 240'$. ● $W = 147'$. □ $W = 73'$. ■ $W = 37'$. ◇ $W = 18'$. ◆ $W = 9'$.

thresholds are generally obtained for the coherent case.

The dependence on the coherence length is represented in Figs 8 and 9 for the largest target and in Fig. 10 for some intermediate cases. As can be seen from Fig. 10 results obtained with smaller targets are very similar to the results obtained with the largest target. We have repressed most of this material because the effects can be demonstrated for a larger parameter variation in the latter case. We find that for the smallest stripe width ($D = 1'$) the sensitivity is inversely related to the length of the wave-train. That means that the signal is easier to detect if phase transitions are permitted. This effect is to be expected (see discussion). For the other stripe widths we obtain a small but significant increase of the sensitivity with increasing coherence length. This increase occurs even for very long wave-trains.

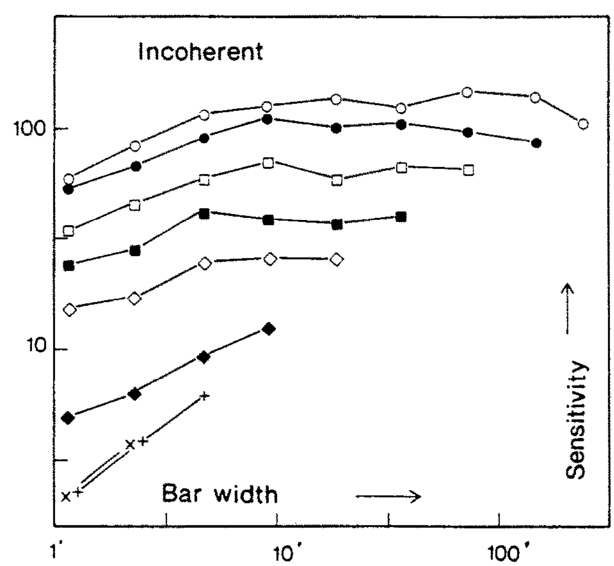

Fig. 7. The sensitivity (the reciprocal of the threshold modulation depth in percent) as a function of the bar width for randomly light or dark bars (the coherence length equals the bar width). Subject AD. The total field width (W) is: ○ $W = 240'$. ● $W = 147'$. □ $D = 73'$. ■ $W = 37'$. ◇ $W = 18'$. ◆ $W = 9'$. + $W = 5'$. × $W = 2'$.

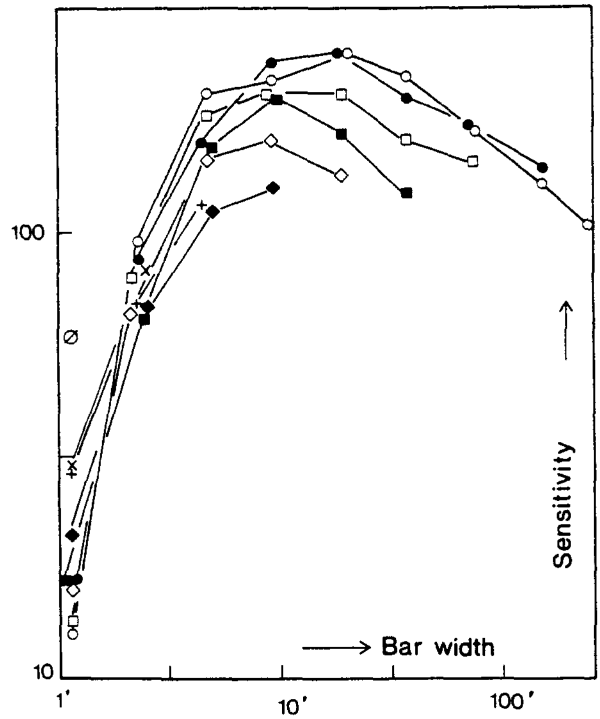

Fig. 8. The sensitivity (the reciprocal of the threshold modulation depth in percent) as a function of the bar width for partially coherent (coherence length less than the total field width) gratings with the coherence length as parameter. The target subtends 4 × 4°. Subject AD. The coherence length (*C*) is: ○ *C* = 240′, ● *C* = 147′, □ *C* = 73′, ■ *C* = 37′, ◇ *C* = 18′, ◆ *C* = 9′, + *C* = 5′, × *C* = 2′, ∅ *C* = 1′.

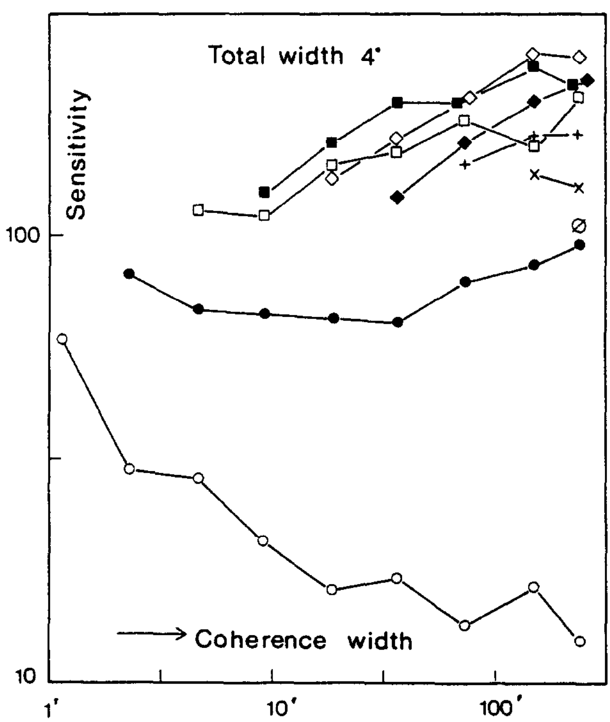

Fig. 9. The sensitivity (the reciprocal of the threshold modulation depth in percent) as a function of the coherence length with the bar width as a parameter. The target subtends 4 × 4°. Subject AD. The bar width (*D*) is: ∅ *D* = 240′, × *D* = 147′, + *D* = 73′, ◆ *D* = 37′, ◇ *D* = 18′, ■ *D* = 9′, □ *D* = 5′, ● *D* = 2′, ○ *D* = 1′.

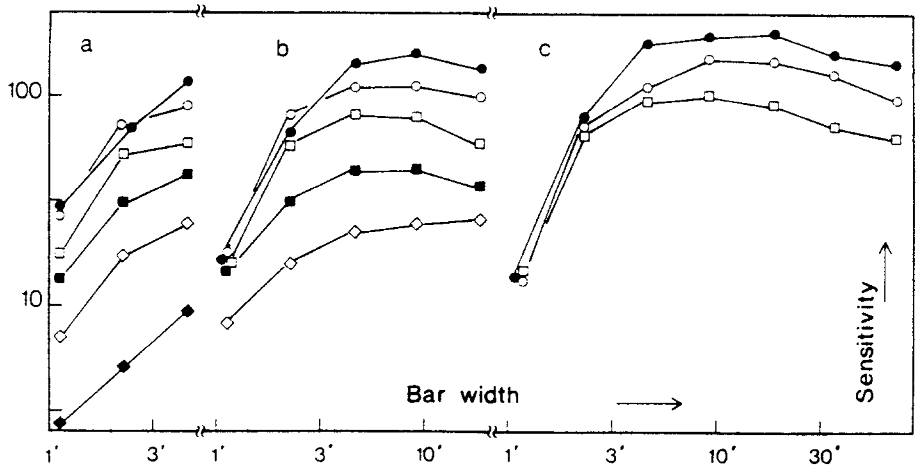

Fig. 10. The sensitivity (the reciprocal of the threshold modulation depth in percent) as a function of the bar width with the total field width as a parameter, for the coherence lengths (C) equal to: $C = 5'$ (a), $C = 18'$ (b), $C = 73'$ (c). Subject AD. The total field width (W) is: ● $W = 240'$, ○ $W = 147'$, □ $W = 73'$, ■ $W = 37'$, ◇ $W = 18'$, ◆ $W = 9'$.

DISCUSSION

Figures 2 and 3 show that the summation effect for regular grids extends over an area that depends on the structure of the stimulus; a fine regular grid is summated over an area that is more restricted than the summation area for coarser grids. This is entirely in accord with earlier experimental evidence (Hoekstra *et al.*, 1974; Howell and Hess, 1978) and theoretical predictions (Koenderink and van Doorn, 1978e). More interesting is the fact that the dependence of sensitivity on field width seems to depend little on stimulus structure if this structure is coarse enough. For both periodic patterns and one-dimensional spatial noise we find that the sensitivity is proportional to the field width. This strong result is illustrated in Figs 11 and 12. For a moderate stripe width ($D = 5'$), it is of no importance for the detectability whether the

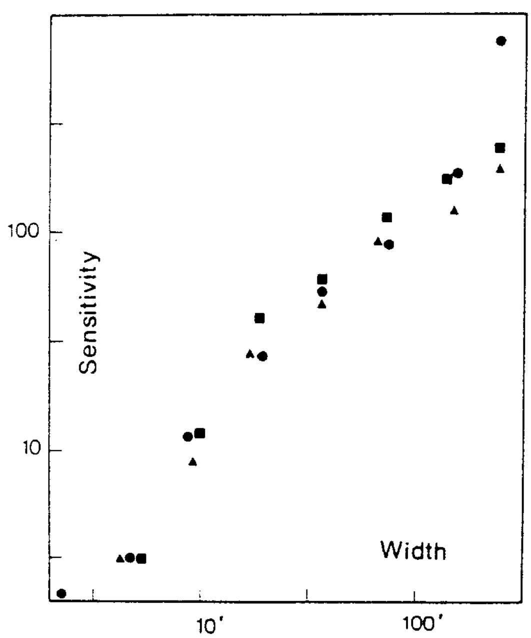

Fig. 11. The sensitivity (the reciprocal of the threshold modulation depth in percent) as a function of the total field width for three different patterns. The target field is divided into bars with a width of 5'. The bars are modulated either in phase (●, a homogeneous field), or in counterphase (■, a regular rectangular grid), or with random phase (▲, one-dimensional visual noise). The single point at a total field width equal to 2' has been added for reference. Subject JK.

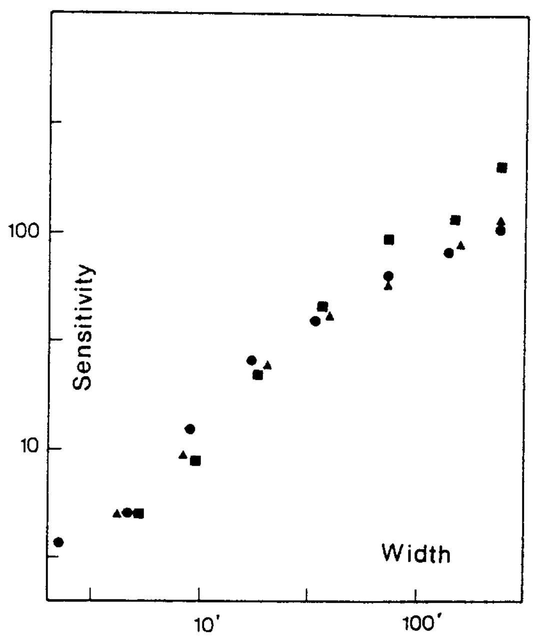

Fig. 12. The sensitivity (the reciprocal of the threshold modulation depth in percent) as a function of the total field width for three different patterns. The target field is divided into bars with a width of 5'. The bars are modulated either in phase (●, a homogeneous field), or in counterphase (■, a regular rectangular grid), or with random phase (▲, one-dimensional visual noise). The single point at a total field width equal to 2' has been added for reference. Subject AD.

stripes all flicker in phase, in counterphase, or in a random fashion. The proportionality to field width was also obtained for peripheral vision by Koenderink *et al.* (1978a, b, c). For the latter case it was shown that the sensitivity is proportional to the square root of the number of stimulated ganglion cells (Koenderink *et al.*, 1978c). In the present case we lack a direct proof but if the reasonable assumption is made that the number of stimulated perceptive fields is proportional to the target area (from ganglion cell counts we know that the number of illuminated ganglion cells increases almost linearly with the target area for our stimuli (Drasdo, 1977)), it also follows that the sensitivity is proportional to the square root of the number of fields (Figs 2–5). Such a dependence makes it likely that the threshold is determined by the *numerical energy* of the signal. Independent evidence for the latter hypothesis has been presented by Quick *et al.* (1976) and Mayhew and Frisby (1978). This hypothesis of power integration would imply incoherent summation and consequently we would predict that the stimulus structure is of minor importance for the summation effect. Thus, the functional dependence of the sensitivity on the field width can be related to the fact that the summation effect is equal for periodic and random patterns.

It seems probable that this summation is physiological, not probability summation, because application of the convenient formalism due to Quick (1974) to our results leads to the expectation that the sensitivity should be proportional with a much higher power of the number of fields than is actually measured. Data from Quick *et al.* (1976) and Mayhew and Frisby (1978) also point in this direction. They show that perceived contrast is proportional to the numerical energy of the signal. In addition Nachmias (1977) has shown that the "internal effect" in response to a grating is proportional to the square of the contrast in the neighbourhood of the threshold.

Whatever the summation mechanism may be, it follows that attempts to detect "frequency channels" with pure threshold measurements are doomed to fail if the experiments are performed at a single location. In the case that power integration holds the phase spectrum is immaterial, and knowledge of the contrast sensitivity function permits us to predict thresholds for *all* stimuli.

As a first approximation we can summarize our data as follows: if the patterns are coarse, then *only the field width, and not the number of coherent cycles in the pattern determines the threshold.* The deviations from this rule are small compared with the total sensitivity variation. However, these deviations are significant and are interesting in their own right; the power integration is not exact, although a good approximation, and visibility improves as the coherence length of the wave-train is increased even beyond many cycles. The amount of the effect is comparable to that reported by Mostafavi and Sakrison (1976) for the variation of sensitivity of a 2.5 c/deg grating as a function of radial bandwidth for constant small angular bandwidth. [Part of this effect (but not all of it) can be ascribed to the fact that the number of light-dark transitions increases somewhat as the coherence length is increased.]

For small stripe widths we encounter another effect: the visibility of a very fine grating pattern improves if the coherence length is *diminished.* The explanation of this effect seems obvious: its mechanism is similar to that used to explain the pseudo-flash effect of Levinson (1968), it has been worked out for spatial patterns by Campbell *et al.* (1969). In other words, the 1′ bars correspond to a spatial frequency of 30 c/deg to which we have a very low sensitivity; as the coherence length is *reduced,* the spectrum *spreads* to spatial frequencies to which we are more sensitive.

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REFERENCES

Campbell F. W., Carpenter R. H. S. and Levinson J. Z. (1969) Visibility of aperiodic patterns compared with that of sinusoidal gratings. *J. Physiol.* **204**, 283–298.

Drasdo N. (1977) The neural representation of visual space. *Nature* **266**, 554–556.

Estévez O. and Cavonius C. R. (1976) Low-frequency attenuation in detection of gratings: sorting out the artefacts. *Vision Res.* **16**, 497–500.

Graham N. (1977) Visual detection of aperiodic spatial stimuli by probability summation among narrowband channels. *Vision Res.* **17**, 637–652.

Hoekstra J., van der Goot D. P. J., van den Brink G. and Bilsen F. A. (1974) The influence of the number of cycles upon the visual contrast threshold for spatial sine wave patterns. *Vision Res.* **14**, 365–368.

Howell E. R. and Hess R. F. (1978) The functional area for summation to threshold for sinusoidal gratings. *Vision Res.* **18**, 369–374.

Kelly D. H. (1975) Spatial frequency selectivity in the retina. *Vision Res.* **15**, 665–672.

Koenderink J. J. (1977) Current models of contrast processing. In *Spatial Contrast, Report of a workshop held in Amsterdam, 1976*, (Edited by Spekreijse H. and van der Tweel L. H.). North-Holland, Amsterdam.

Koenderink J. J., Bouman M. A., Bueno de Mesquita A. E. and Slappendel S. (1978a) Perimetry of contrast detection thresholds of moving spatial sine wave patterns—I. The near peripheral visual field (eccentricity 0–8 degrees). *J. opt. Soc. Am.* **68**, 845–849.

Koenderink J. J., Bouman M. A., Bueno de Mesquita A. E. and Slappendel S. (1978b) Perimetry of contrast detection thresholds of moving spatial sine wave patterns—II. The far peripheral visual field (eccentricity 0–50 degrees). *J. opt. Soc. Am.* **68**, 850–854.

Koenderink J. J., Bouman M. A., Bueno de Mesquita A. E. and Slappendel S. (1978c) Perimetry of contrast detection thresholds of moving spatial sine wave patterns—III. The target extent as a sensitivity controlling parameter. *J. opt. Soc. Am.* **68**, 854–860.

Koenderink J. J. and van Doorn A. J. (1978d) Detectability of power fluctuations of temporal visual noise. *Vision Res.* **18**, 191–195.

Koenderink J. J. and van Doorn A. J. (1978e) Visual detection of spatial contrast: influence of location in the visual field, target extent and illuminance level. *Biol. Cybernet.* **30**, 157–167.

Levinson J. (1968) Flicker fusion phenomena. *Science, N.Y.* **160**, 21–28.

Mayhew J. E. W. and Frisby J. P. (1978) Suprathreshold contrast perception and complex random textures. *Vision Res.* **18**, 895–897.

McCann J. J., Savoy R. L. and Hall J. A. Jr (1978) Visibility of low-frequency sine-wave targets: dependence on number of cycles and surround parameters. *Vision Res.* **18**, 891–894.

Mostafavi H. and Sakrison D. J. (1976) Structure and properties of a single channel in the human visual system. *Vision Res.* **16**, 957–968.

Nachmias J. (1977) Theoretical approaches: method of analysis. In *Spatial Contrast. Report of a workshop held in Amsterdam, 1976*, (Edited by Spekreijse H. and van der Tweel L. H.). North-Holland, Amsterdam.

Quick R. F. Jr (1974) A vector-magnitude model of contrast detection. *Kybernetik* **16**, 65–67.

Quick R. F. Jr, Hamerly J. R. and Reichert T. A. (1976) The absence of a measurable "critical band" at low suprathreshold contrasts. *Vision Res.* **16**, 351–355.

Savoy R. L. and McCann J. J. (1975) Visibility of low-spatial-frequency sine-wave targets: dependence on number of cycles. *J. opt. Soc. Am.* **65**, 343–350.

Wilson H. R. and Giese S. C. (1977) Threshold visibility of frequency gradient patterns. *Vision Res.* **17**, 1177–1190.