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THE CONDUCTANCE FOR THE GAS FLOW OF AN ACCELERATING TUBE

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Measurements of the conductance in the molecular flow region of accelerating tubes of an EN tandem are given and discussed. These measurements have been carried out on both straight and inclined field tubes. A theoretical model is constructed by means of which the conductance of an accelerating tube can be calculated. The calculated values of the conductance turn out to be in good agreement with the measured ones.

1. Introduction

Accelerating tubes, as used in Van de Graaff accelerators, consitute rather a complicated vacuum system. The tubes are made up of a large number of metal electrodes separated by pyrex or ceramic insulators. The apertures in the electrodes, through which the beam passes, act as a series of diaphragms in the vacuum system.

For stable functioning there must be a good vacuum inside the tube; otherwise various disturbing collision processes may occur, such as charge exchange of the ions, scattering of the beam and the production of secondary electrons, photons and X-rays. The vacuum inside an accelerating tube of a Van de Graaff accelerator is practically always produced by a pumping system outside the pressure tank. This means that with a single-ended machine the source gas of the ion source, and with a tandem accelerator the stripper gas, are pumped away through the long accelerating tubes.

If the vacuum system is to have the right dimensions we first have to have detailed information about the conductance of the gas flow in an accelerating tube. In order to obtain this information we performed some measurements on both straight and inclined field tubes. These measurements will be discussed in section 2. In section 3 it is shown that by using and expanding the existing theory the conductance can be calculated for a model case. The values for the conductance of an accelerating tube calculated with this model are in good agreement with the measurements. Although the observations apply to the HVEC * tubes of an EN tandem, the results will also hold for other accelerating tubes.

2. Conductance measurements

In this section a description is given of measurements which have been performed on the HVEC acceleration tubes of an EN Tandem Van de Graaff accelerator. These tubes fall into two categories: straight and inclined field tubes [1]. For a straight tube, the apertures in the electrodes, which are perpendicular to the tube axis, are circular. For beam optical reasons the diameters of the apertures are not the same. It is only the apertures of the electrodes in one section of the tube which have the same diameter. In an inclined field tube the apertures in the electrodes are slot-shaped. The dimensions of all the slots in the electrodes are the same; only the angle of inclination θ of the electrodes alters alternately from $+12°$ to $-12°$ in succeeding sections. In figs. 1(a) and (b) a straight and an inclined field tube are depicted. The electrodes are bent near the pyrex insulating rings in order to protect these rings from material that is sputtered from the aperture edges when the beam hits the electrodes.

The conductance measurements were performed with the equipment drawn in fig. 2. One end of the accelerating tube is connected to a turbomolecular pumping system and the other end to an apparatus

* High Voltage Engineering Corporation, Burlington, Mass. USA.

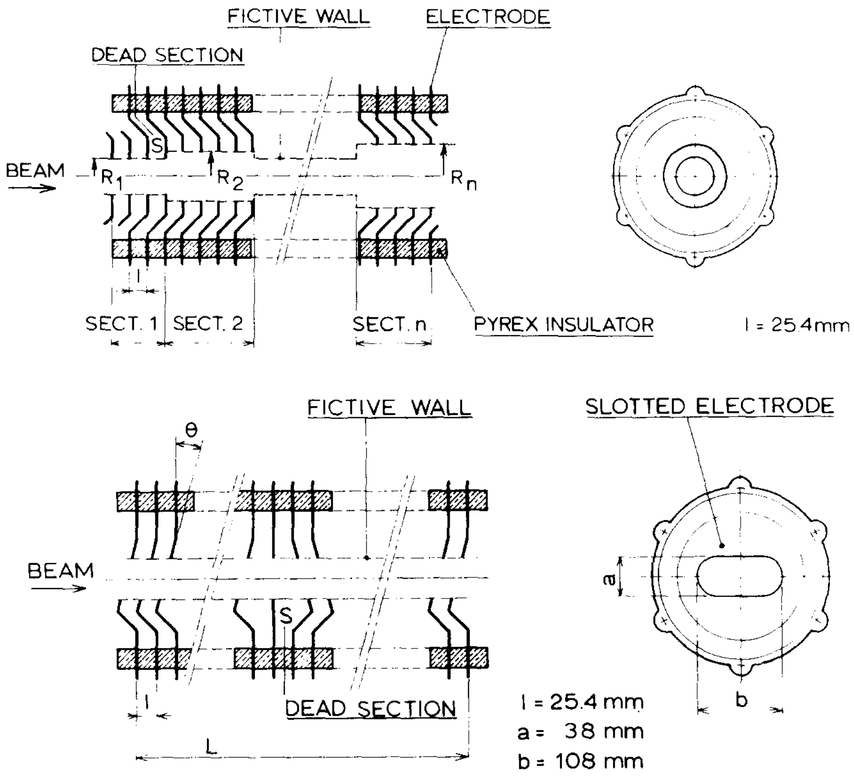


Fig. 1. (a) Straight tube. (b) Inclined field tube.

which lets a given amount of gas into the system. This apparatus is connected to the tube via a regular leak valve. The pressures of both sides of the accelerating tube are measured by two identical ionization gauges, IG1 and IG2. The conductance of the tube is given by the relation $Q = C\Delta p$, where Q is the throughput of the gas and Δp is the pressure difference over the tube. The amount of pumped gas Q can be determined at a given pressure by means of the measuring device drawn in fig. 2; the following procedure must be followed. Let the gas flow from the cylinder into the accelerating tube via the valves V_1, V_2 and the needle valve. By means of the precision manometer M the gas pressure P_a is set to atmospheric pressure and with the needle valve the pressure in the tube is adjusted to the value required during the measure-

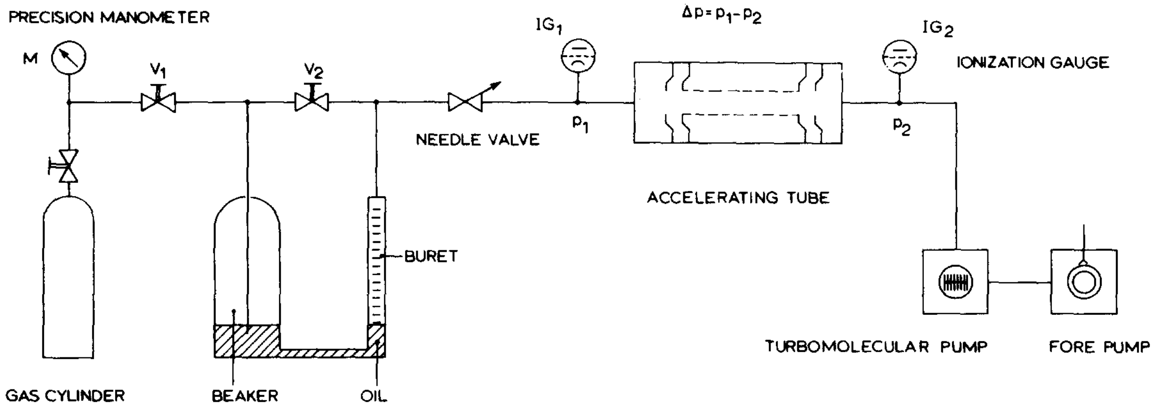


Fig. 2. Method for measuring the conductance of a tube.

ment. After closing valve V_2 the oil level in the burette will rise and the pumped volume V in the time t can be determined. The conductance of the accelerating tube can now be calculated with the formula

$$C = VP_a/(t\ \Delta p)\,. \qquad (1)$$

If V is expressed in litres, the pressure difference Δp and P_a in mbar and t in seconds, the conductance C will be expressed in l/s. The pressure difference is $\Delta p = p_1 - p_2$, the pressures p_1 and p_2 being indicated in fig. 2. The values of the pressures p_1 and p_2 are corrected for the residual gas pressure at the beginning of the experiment.

The measurements are performed in the pressure range from about 4×10^{-6} to 6×10^{-3} mbar. The accuracy of the measurements is within 15% and decreases at lower pressures. In fig. 3 the measurements shown were done on a straight accelerating tube. In the EN tandem this HE tube is the fourth as counted from the ion source. For this tube the conductance have been determined for the gases nitrogen, helium and xenon. For the second and third tube the conductance has been determined for nitrogen only (see fig. 4). The pressure along the abscissa of the graph is the average pressure inside the accelerating tube, which is given by $(p_1 + p_2)/2$. The graph shows that the conductance is constant over a large part of the pressure range but increases at higher pressures. The picture is the same for a cylindrical tube [2]. At lower pressures the flow of gas through the tube is called molecular flow. In that case the

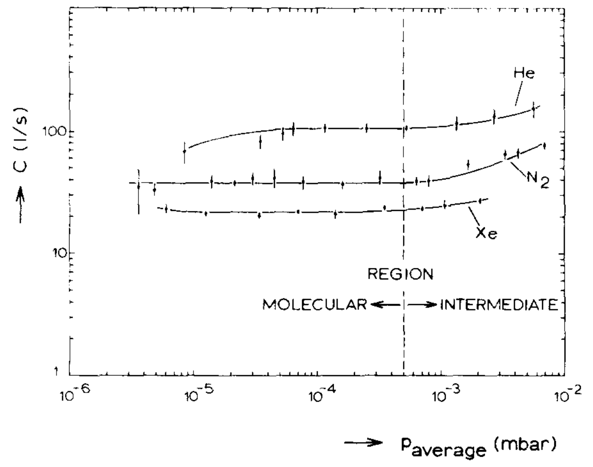


Fig. 3. Conductance of a straight tube. (No. 4 of an EN tandem) The measurements are done for the gases nitrogen, helium and xenon.

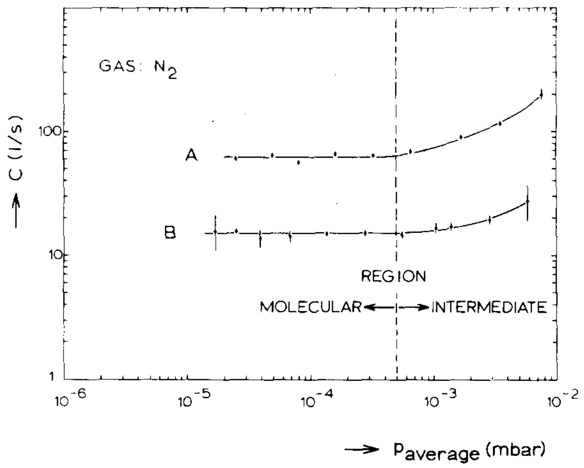


Fig. 4. Conductance of a straight and an inclined field tube for nitrogen. A. Straight tube (No. 3 of an EN tandem). B. Inclined field tube (No. 2 of an EN tandem).

mean free path λ of the molecules is longer than the diameter D of the tube, so $\lambda > D$. With molecular flow, collisions between the molecules are rare and collisions with the wall predominate. In this molecular region the conductance of a vacuum component is determined only by its geometrical dimensions. At higher pressures the collisions between the molecules start to play a role, and for $\lambda \ll D$ the flow will become viscous. The region between molecular and viscous flow is called the intermediate flow region (see ref. [2]). In figs. 3 and 4 the beginning of this flow region is shown. In practice all accelerating tubes are used in the molecular flow region for the reasons mentioned above. The kinetic gas theory shows that for molecular flow the conductance of a tube depends on the type of gas. The correction factor respective to nitrogen is proportional to the root of the ratio of the molecular masses. Here the correction factors are $[M(\mathrm{He})/M(\mathrm{N_2})]^{1/2} = 0.38$ and $[M(\mathrm{Xe})/M(\mathrm{N_2})]^{1/2} = 2.2$. The ratio of the conductances in fig. 3 is in good agreement with these factors, within the accuracy of the measurements. The ionization gauges used are calibrated for nitrogen. For helium the meter indication must be multiplied by a factor 6 and for xenon by a factor 0.36, because of the different sensitivity of the gauge for various gases. As figs. 3 and 4 show the accuracy of the measurement decreases at lower pressures. This is due to the fact that in eq. (1) the error in $\Delta p = p_1 - p_2$ increases at smaller values. Next to that the influence of the residual gas pressure on Δp increases.

In fig. 4 the conductance of an inclined field tube is also plotted. The curve for the conductance, which is given for the second LE tube, shows the same behaviour as a straight tube, as we might expect. For this tube the measurement has only been performed for nitrogen. In the next section it will be shown that the conductance of an accelerating tube can also be calculated with the help of a theoretical model.

3. Model calculation

3.1

As mentioned earlier, an accelerating tube constitutes a complicated vacuum system. We can try to estimate the conductance by considering the tube as a series of diaphragms. If we calculate the conductance C with the resistance method

$$1/C = 1/C_1 + 1/C_2 + \dots 1/C_n \quad (2)$$

in which C_1, C_2 ... C_n are the conductances of the different diaphragms, we find much too low a value. In this formula we have

$$C_n = 3.64(T/M)^{1/2}A \text{ l/s} \quad (3)$$

(ref. [2]) for molecular flow, A being the surface of the aperture in cm^2, M the molecular weight and T the temperature (K). For the fourth tube (straight) for instance, we calculate with this formula the conductance for nitrogen to be $C = 13$ l/s, whereas according to the measurement it is 39 l/s *. This big discrepancy can be attributed to the fact that in the calculation of the conductance of a diaphragm we use the fact that the diaphragm links two volumes the dimensions of which are large compared with the cross section of the aperture. After the gas molecules have passed through the diaphragm, complete randomization of the molecules occurs. Inside an accelerating tube, the distance between the diaphragms is short and the volumes between them are too small to produce complete randomization. The chance that a molecule which has passed through one diaphragm will also pass through the next is greater than when there is complete randomization of the gas flow between the diaphragms. This means that the measured values for the conductance are higher than the values calculated with the resistance method mentioned above.

* The authors thank HVEC (Amersfoort) for putting the necessary geometrical data at their disposal.

3.2

In the following we shall show that it is possible to construct a theoretical model for the tube, which gives results in good agreement with the measurements. For this purpose use is made of the existing theory, which will be expanded for our purposes. Firstly we shall give a short summary of the theory of conduction, as developed in the past. We restrict ourselves here to molecular flow, since this is the only type of flow of importance in a running accelerator. At the beginning of this century Knudsen [3] derived a formula to describe the flow of a rarified gas through a long tube, ($L/r \gg 1$, with L = the length and r = the radius of the tube). From this theory the conductance of a long uniform tube can be written as [2]

$$C = \frac{3.44 \times 10^4}{\pi^{1/2}} \left(\frac{T}{M}\right)^{1/2} \left(\frac{A^2}{BL}\right) \text{ l/s} \quad (4)$$

in which A is the surface and B is the periphery of the cross section of the tube. We shall show that this formula can also be used for an accelerating tube, in which the electrode apertures are the same size. This is the case with an inclined field tube. We shall now replace the accelerating tube by a fictive tube, which has the same length and of which the cross section is the same as the apertures of the electrodes. In figs. 1(a) and 1(b) this fictive tube is indicated by dotted lines. The justification for this model is that the parts of the tube, marked in fig. 1 by S, are dead sections as far as the gas flow is concerned. In these dead sections there is no overall gas flow in the direction of the pump. Hence there seems to be a fictive wall inside the tube. On the one hand molecules from the central part of the tube will pass through this wall and enter the dead sections; on the other hand molecules from the dead sections will be emitted continuously into the tube. This process resembles that occurring on the wall of a vacuum tube, where molecules are continuously absorbed and re-emitted. For molecular flow the number of collisions between the molecules is small and the collisions with the wall predominate.

3.3.

The tube model with the fictive wall can also be used for a straight tube, which is built up of short sections each with the same circular cross section. However, now we must add the conductances of the different short tube sections. This means that we cannot use the long tube eq. (4). For short tubes we have to use the formalism given by Clausing [4,5]. Consider a tube with a certain length; the molecules arriving at the opening of the tube strike the wall after flowing for a certain distance. At this point reflection occurs which is random in all directions. Some of the molecules are reflected back and can return to the vessel from which they originate, directly or via another reflection. There is also a chance that a back-reflected molecule via a following reflection continues in the original direction. Hence, a molecule entering the tube has only a certain probability of being transmitted to the other end. This probability is called the transmission probability or the Clausing factor. In fig. 5 the Clausing factor α is plotted for a round cylindrical tube as a function of L/r. For molecular flow the value of α is dependent only on the geometry of the tube. For an orifice ($L = 0$), α is equal to unity, since no molecules are reflected by an infinitely short tube. For a tube with molecular flow the conductance can be expressed by the product of the conductance of the orifice of the tube and the Clausing factor of the tube, so

$$C_{\text{tube}} = \alpha C_{\text{orifice}} \,. \tag{5}$$

It follows from eqs. (3) and (5) that for a tube with unequal cross sections at the two ends we have

$$\alpha' A' = \alpha'' A'' \tag{6}$$

because the conductance of a tube is independent of the flow direction. In eq. (6) α' and A$'$, α'' and A$''$ are the probability factor and the surface of the entrance opening, respectively, at the two ends of the tube. For a cylindrical tube the factor α is tabulated [6]. If two tubes of equal diameter and with transmission probabilities α_1 and α_2 are connected in series (see fig. 6), the transmission factor α for the combined

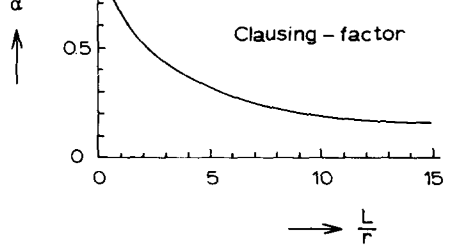
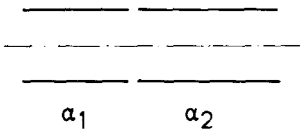


Fig. 5. The Clausing factor α as a function of L/r.



$$\frac{1}{\alpha} = \frac{1}{\alpha_1} + \frac{1}{\alpha_2} - 1$$

Fig. 6. Two tubes of the same radius in series.

tubes is not given exactly by $1/\alpha = 1/\alpha_1 + 1/\alpha_2$. This can be seen from fig. 5. The decrease of α is not as sharp as it would be if this equation were valid. The reason for this is that the flow conditions at the beginning of the second and the end of the first tube have changed. The molecules entering the second tube, for example, no longer come from all directions but from a beam formed by the first tube. A formula has been derived by Oatley [7] for the transmission probability of two tubes connected in series. This equation

$$1/\alpha = 1/\alpha_1 + 1/\alpha_2 - 1 \tag{7}$$

takes the beam effect into account.

By connecting a very short piece of tube to both sides of the narrowest tube with the diameter of the largest tube Oatley succeeded in applying eq. (7) to the combination of two tubes of unequal diameter in series (see fig. 7a), of the largest tube. This piece of tube is so short in fact that it does not really influence the conductance (see fig. 7b). In this way we have two tubes of the same diameter. He should however have given the tube with an enlarged diameter a transmission probability α_1' which is smaller than α_1 by a factor $1/f$ ($f = R^2/r^2$); R is the radius of the largest and r is the radius of the smallest tube. [See eq. (5).] By using eq. (7) for the transformed situation of fig. 7b he could write

$$1/\alpha_R = 1/\alpha_1' + 1/\alpha_2 - 1 \,, \tag{8}$$

in which

$$\alpha_1' = \alpha_1/f \,. \tag{9}$$

Substituting of eq. (9) into eq. (8) gives

$$1/\alpha_R = f/\alpha_1 + 1/\alpha_2 - 1 \,. \tag{10}$$

If we revert to fig. 7a and considers the inlet to have radius r, then we find with the aid of eq. (6)

$$1/\alpha_r = 1/\alpha_1 + 1/f\alpha_2 - 1/f \,. \tag{11}$$

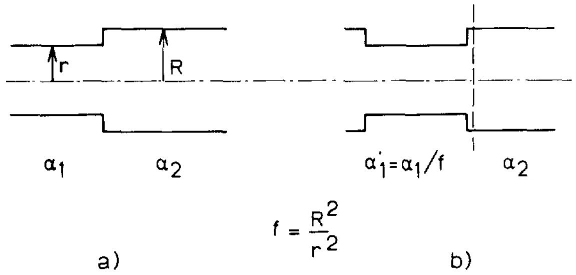


Fig. 7a,b. Two tubes of different radii in series.

The conductance follows from: $C = \alpha_r c\pi r^2$, or $C = \alpha_R c\pi R^2$, $C_1 = \alpha_1 c\pi r^2$ and $C_2 = \alpha_2 c\pi R^2$, in which c is a constant which for a round tube can be calculated with eq. (3). For nitrogen gas flow at room temperature c has the value 11.8 l/s cm^2, if the radius is given in cm. Substituting the above conductances in eqs. (10) or (11) gives

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} - \frac{1}{c\pi R^2} \tag{12}$$

because the conductance C is independent of the flow direction. In eq. (12) the radius occurs of the largest of the two tubes

3.4.

The above formalism can be expanded so that it is applicable to a straight tube. An HVEC straight tube consists of a number of sections (sometimes more than twenty), each with the same circular aperture. Following the procedure described above a straight tube can be represented by a series of cylindrical tubes of different radii R_1 to R_n, as sketched in fig. 8. By combining continually two adjoining tube sections and by making use of eq. (12), we obtain the following expression for the conductance of a tube of n sections and $n-1$ junctions

$$\frac{1}{C} = \sum_{n=1}^{n} \frac{1}{C_n} - \sum_{n=1}^{n-1} \frac{1}{c\pi R_n^2}, \tag{13}$$

where C_1, C_2 ... C_n and R_1, R_2 ... R_n are the conductances and the radii for the different tubes as given in fig. 8. It must be pointed out that in the second summation of eq. (13), starting with tube 1, only the largest radius at every junction must be taken into account [see also eq. (12)]. This means that for $R_n/R_{n-1} > 1$ only the radius R_n and for $R_n/R_{n-1} < 1$ only the radius R_{n-1} appears in eq. (13). This also implies that in eq. (13) depending on

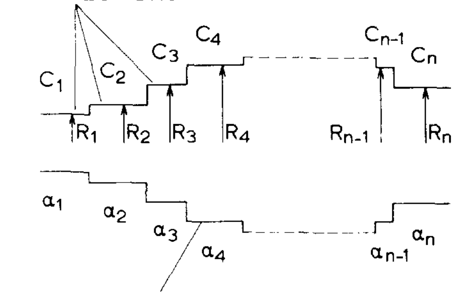


Fig. 8. A cascade of tubes of unequal diameter, representing a straight tube.

the increase and decrease of succeeding tube radii, it is possible that some radii do not appear, while other radii appear twice.

4. Application

4.1

We shall apply eqs. (4) and (13) discussed in section 3 to the tubes shown in figs. 3 and 4. The second tube of an EN tandem is an inclined field tube with 75 electrodes, 70 of which have an inclination angle of 12°. The dimensions of the slots in the electrodes taken from the HVEC drawing are: area of the aperture 3794 mm^2, circumference of the aperture 259 mm. The length of the tube is 1876 mm. We find the area of the cross section of the fictive tube by multiplying the surface of the slot by cos 12°. Because 5 electrodes are not inclined there is a small error, which, however, is negligible. Substituting these values in eq. (4) gives $C = 17.8$ l/s for the conductance of the inclined field tube for nitrogen at room temperature. This result is in good agreement with the measured value $C = 15$ l/s (fig. 4).

4.2.

We shall now apply the derived eq. (13) to the two HE tubes of the EN Tandem. The third tube consists of 13 sections, the diameters of which alter gradually from 5″ to 3.5″. The fourth tube has 21 sections, of which the two end sections have diameters of 3″ and 3.4″. The conductance of a section expressed as $C_n = \alpha c\pi R_n^2$ can be calculated by determining the L/R value of the section and by looking up the Clausing factor α in the literature. By substituting the values for C_n and R_n we find, with the aid of eq. (13), for the third tube a conductance of 56.6 l/s and for the fourth tube 35 l/s. Here too the calculated values are in good agreement with the measurements.

5. Conclusion

In the foregoing we have shown that it is possible to calculate the conductance of a hybrid vacuum system, such as is formed by an accelerating tube. The good agreement between theory and experiment shows that the model used is useful. We would point out that the method described can also be used for calculating the conductance of sections of a linear accelerator (Linac), the structure of which resembles the configurations described here.

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