THE USE OF SPONTANEOUS VOLTAGE FLUCTUATIONS FOR THE MEASUREMENT OF LOW TEMPERATURES

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Summary

A discussion is given of the method proposed by Lawson and Long to use the noise across a resistance as a measure of its absolute temperature. The statistical fluctuations in the measured noise are calculated, taking into account the bandwidth of the amplifier. This calculation shows that is not necessary to use a first tube with lowered voltages on plate, screengrid and heater and it shows that a quartz crystal is essentially worse than a pure resistance or a L—C circuit as a thermal noise generator for thermometrical purposes.

§ 1. In 1946 Lawson and Long[1]) made the suggestion to use the spontaneous voltage fluctuations across a resistance as a measure of its absolute temperature. Instead of a pure resistance in principle any circuit can be used, that has a resistive part e.g. an $L - C$ circuit or a quartz crystal. In the discussions [2]) [3]) [4]) [5]) on this subject the important influence of the statistical fluctuations in the measured noise and its relation to the bandwidth of the amplifier have not been worked out. It is our intention to do so in this paper.

§ 2. Let us first consider a pure resistance R_g cooled to temperature T, as the grid leak of an amplifier tube. The thermal voltage fluctuations contained in the frequency band $\Delta\nu$ are then given by:

$$\overline{V_{th}^2} = 4R_g\, kT\, \Delta\nu \quad (k = \text{Boltzmann's constant}). \tag{1}$$

Besides this "signal noise" there is a "zero noise", the shoteffect of the first tube, which we write in the convenient but unfamiliar form:

$$\overline{V_{sh}^2} = 4(R' + R_g^2/R'')\, kT_0\, \Delta\nu, \tag{2}$$

where R' is the aequivalent noise (plate-) resistance and T_0 roomtemperature. The second term describes the grid current noise; the resistance R'' defined in this way is equal to $4\ kT_0/2\ e\ I_g$, where e is the electron charge and Ig the grid-current [6] [7]). The grid leak can be chosen so as to make the ratio of signal noise to zero noise a maximum (with fixed temperature T). This optimal grid leak is equal to:

$$R_g = \sqrt{R'R''}, \tag{3}$$

giving a signal noise to zero noise ratio:

$$\overline{V^2_{th}}/\overline{V^2_{sh}} = (T/2T_0)\sqrt{R''/R'}. \tag{4}$$

So far is was not necessary to make any assumption on the bandwidth $\Delta\nu$. In fact the thermal noise and grid current noise are contained in a frequency band extending from zero up to the frequency, where the grid leak is shorted by the grid capacity Cg:

$$\Delta\nu = 1/2\pi\, R_g C_g = 1/2\pi\, C_g\, \sqrt{R'R''}. \tag{5}$$

We give the amplifier this same high-frequency cut-off so as to limit the plate current noise to the same frequency band. Equations (1) to (4) hold only exactly, when the input impedance and the amplifier response are constant up to the frequency cut-off given by (5) and then drop sharply to zero, but results are not changed substantially when we drop these assumptions.

The noise passed by the amplifier is squared by some quadratic element and measured (mean current $\bar{i}$). The fluctuations of the indication will be determined by the characteristic averaging time τ of the meter and by the bandwidth passed by the amplifier [8]) [9]):

$$\sqrt{\overline{\left\{\frac{1}{\tau}\int_0^\tau (i - \bar{i})\, \mathrm{d}t\right\}^2}}\bigg/\bar{i} = 1/\sqrt{\tau\Delta\nu}. \tag{6}$$

The lowest temperature T_{min} that can be measured by this method is reached when the thermal noise indication of the meter is equal to the fluctuations in the shot noise indication. With (4), (5) and (6) we get in this way:

$$T_{min} = (1/\sqrt{\tau\Delta\nu})\, 2T_0\sqrt{R'/R''} = 2T_0\sqrt{2\pi C_g/\tau}\ \sqrt[4]{R'^3/R''}. \tag{7}$$

Lawson and Long [1]) suggested the Western Electric D-96475 electrometer tube which has a grid current as low as 10^{-15} A as a first tube. We do not know if the grid current noise of this tube corresponds to this low figure (which is not necessarily true [6])), and so we prefer to calculate the lowest measurable temperature using the data of the best tube (Philips AF 7) investigated by Keller [6]) [7]). The tube was operated with lowered plate-, screen grid-, and heater voltages. It has an aequivalent plate resistance of: $R' = 125$ kΩ and a grid current of: $Ig = 1{,}2.10^{-12}$ A, which gives $R'' = 4{,}3.10^{10}\ \Omega$. With $T_0 = 300$°K, $Cg = 10$ pF and $\tau = 10$ sec. we get: $T_{min} = 2{,}2.10^{-2}$ °K.

The optimal grid leak given by (3) is: $Rg = 7.10^7\ \Omega$, and the bandwidth given by (5) is: $\Delta\nu = 230$ c/s. It is interesting to note that the thermometrical usefulness of a tube is expressed by the product R''/R'^3 or $1/Ig.R'^3$. When e.g. the tube cited above is operated (Keller [7]) p. 72) at nearly normal voltages (plate 120 V, screen grid 70 V, heater 4 V), the equivalent plate resistance is 10 times lower, the grid current 1000 times higher, so that accidentally $I_g R'^3$ and thus T_{min} remains the same. The optimal grid leak is then: $Rg \doteq 7.10^5\ \Omega$, and the bandwith: $\Delta\nu = 23$ kc/s.

It must be noted, that flicker effect was absent in this tube at least down to a frequency of 10 c/s. Essential is also that the mean grid current and plate current of the first tube must be constant over long times, because the zero noise can only be subtracted by reading the meter at two known temperatures.

A constancy of the zero noise of 2% would be necessary for the AF 7 with lowered voltages, and to profit fully of the extended bandwidth the constancy would have to be better than 2‰ for the AF7 with normal voltages.

If we should wish to measure higher temperatures, where the thermal noise is large compared to the shot noise, the relative accuracy would be given by (6). For the AF7 with lowered voltages this would be again 2%, with normal voltages 2‰.

§ 3. Instead of cooling a resistance we can make use of a L—C circuit with cooled coil. The thermal and grid current noise is now contained in a band around the resonance frequency. The amplifier must contain a similar circuit to limit the plate current noise to the same frequency band. For this case all equations of § 2 remain unchanged, when only Rg is replaced by Zg, the (real) impedance of the L—C circuit at resonance.

From (7) we can draw the conclusion that again the grid capacity must be as small as possible, but that the values of the resonance frequency and of the Q-factor of the coil are quite immaterial, when only $Zg = \sqrt{R'R''}$ is fulfilled, which will not be very difficult if the first tube is operated under normal conditions. Also it is not advisable to make the resonance frequency very high because then the losses in the input capacity might become prohibitive. Apart from technical reasons the L—C circuit has no advantages or disadvantage compared to a pure resistance.

§ 4. By Lawson and Long [1]) a quartz crystal was also suggested as a thermal noise generator. We will show however that a quartz crystal gives essentially worse results than a pure resistance or a L—C circuit.

Electrically the crystal can be replaced by the circuit of fig. 1 [10]).

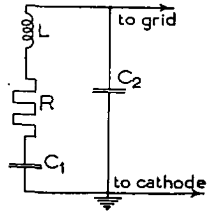

Fig. 1

The capacity C_1 is of the order of 0,1 pF, C_2 is about 10–30 pF. Because $C_2 \gg C_1$ the bandwidth at parallel resonance is given in good approximation by:

$$\Delta\nu = C_1/2\pi\, Z_g C_2^2, \qquad (8)$$

where Z_g is the impedance at resonance of the circuit between grid and cathode. Replacing R_g by Z_g all formulae of § 2 again hold true except (5) which must be replaced by (8), and as a consequence we must change C_g in (7) into C_2^2/C_1. Putting the ratio C_2/C_1 equal to 100, then the lowest measurable temperature is now 10 times higher than we calculated in § 2.

The only advantage of a quartz crystal would then be the easy way to obtain high impedances, because the Q-factor is so large. However, as we saw in § 2, these high grid impedances are not necessary when the first tube is operated under normal conditions, a grid impedance of ≈ 1 MΩ being enough.

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