

Chapter 13

Methodological Issues in the Application of the Latent Growth Curve Model

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13.1 Introduction

Latent growth curve analysis (McArdle, 1986, 1988; Meredith & Tisak, 1990; Willett & Sayer, 1994) is well suited to analyze systematic change in longitudinal data collected from a panel design. It represents outcome variables explicitly as a function of time and other measures. Specifically, latent growth curve analysis is a statistical technique to estimate the parameters that represent the growth curves that are assumed to have given rise to the structure of the repeatedly measured outcome variable over time. Growth curve analysis can be applied just to get a (unconditional) description of the mean growth over a certain period of time. However, the emphasis of this technique lies in explanation of differences between subjects in the parameters describing the growth curves; in other words, in the systematic inter-individual differences in intra-individual change.

As a special case of the general structural equation model, the latent growth curve model can benefit from the advantages of structural equation

modeling approach. As such, the latent growth curve model can be extended in several ways. Two of such extensions have been proven to be of special value: (1) fixed factor loadings can be estimated to account for nonlinear growth, and (2) the factorial structure of the repeatedly measured variable can be incorporated explicitly in the latent growth curve model (see for instance McArdle, 1988; McArdle & Hamagami, 1996; Hancock, Kuo & Lawrence, 2001). This chapter provides a demonstration of these two extensions, together with a discussion and clarification of two methodological issues that may hinder the interpretation of the results of these extended latent growth curve models. Both issues are related scaling problems concerning the latent variable structure, and will be introduced in the next paragraphs.

Regarding the first issue, the linear latent growth curve model is often too restricted to fit the data. A possible way out of this type of situation is to include one or more higher-order polynomial terms into the growth model to account for the nonlinear growth or development present in the data or theory, for instance, via a quadratic or cubic term in a polynomial growth curve model. An alternative approach to the inclusion of higher-order polynomial terms is the "latent basis" approach, originating from the work of Rao (1958) and Tucker (1958), and introduced in SEM by McArdle (1986) and Meredith and Tisak (1990). In contrast to higher-order polynomial growth curve models, in which all coefficients of the basis functions (i.e. the factor loadings) are fixed to known values, the "latent basis" approach describes nonlinearity in the growth curves by estimating the basis function coefficients for the growth factor, instead of including higher-order polynomials. While this model presents a challenging and elegant way of modeling growth, it contains some inherent pitfalls, which are only partially known in the literature. The problems are recognized in common factor analysis. It is important to recognize these problems also in latent growth curve modeling, for they may play an essential role. The pitfalls concern the apparent differences of the growth parameter estimates and of the standard errors due to the scaling of the latent growth factor. De Pijper and Saris (1982) reported already, for the general confirmatory factor model, that standard errors might change as the model incorporates a different scaling of the latent variable structure. The differences in these statistics can be so large that on the same data one set of restrictions can lead to the conclusions that a correlation between two factors is not significantly different from zero, while another set of restrictions can lead to the opposite conclusion (de Pijper and Saris, p.182; cf. Mellenbergh, Kelderman, Stijlen & Zondag, 1979; Saris, 1978). The aim of this part of the chapter is to clarify by demonstration that this scaling issue is also present in the latent growth curve model since it can

be regarded as a special case of the general common factor model¹. It is illustrated that standard errors and test statistics (i.e. the Wald statistic) of some of the parameters of the latent growth curve model change as a consequence of a different scaling of the basis function of growth rate. Furthermore, regarding this scaling issue, the question is explored whether a two-stage approach (cf. Jöreskog & Sörbom, 1988) can overcome these differences. This is important because the Wald statistic is used to test model parameters in many tutorials and applications of the latent growth curve model.

The second issue this chapter addresses concerns the fact that often the same set of indicators is assessed at each measurement occasion of a longitudinal study. A possible approach to such multiple indicators is to create summated scale-scores at each time point for each individual, also known as 'item partialling'. Subsequently, a growth model, or some other type of longitudinal model (e.g. an autoregressive model), may be built for this new variable. Although item partialling appears a natural way to deal with multiple indicators, it has some important drawbacks (Bandalos, 2002; Hall, Snell & Singer Foust, 1999). It may lead to biased estimates of the model parameters if the indicators within each measurement occasion violate the assumption of unidimensionality (Bandalos, 2002). In addition, item partialling will not yield as stringent a test of structural equation models because the reduction of data points relative to the original items may be too influential (Bandalos, 2002).

A better approach to multiple indicators may be to model them, explicitly, as indicators of a latent construct or factor at each measurement occasion. A growth model may be constructed then to explain the variance and covariance among the first-order latent factors. This approach has been termed second-order growth modeling by Hancock, Kuo and Lawrence (2001) in contrast to first-order growth modeling on the observed indicators. Different names for the same model are 'curve-of-factors model' (McArdle, 1988), and 'multiple indicator latent growth model' (Chan, 1998). We refer to Hancock et al. (2001) for a recent illustration of this approach. Modeling multiple indicators in a longitudinal setting requires a test on the structure of the measurements, i.e. a test of measurement invariance. Complications concerning the test of measurement invariance will be highlighted, more specific, it will be demonstrated that the choice of a specific indicator as reference for the scaling of the latent variable may influence the fit of the latent growth curve model as well as the parameter estimates. Measurement invariance has been the topic of discussion and investigation before, see for instance (Bechger, 1997; Byrne, Shavelson & Muthén, 1989; Dolan &

¹ Most applications of the common factor model concern latent psychometric variables; the latent growth curve model, on the other hand, concerns latent chronometric variables (McArdle, 1989)

Molenaar, 1994; Horn & McArdle, 1992; McArdle & Cattell, 1994; Meredith, 1964; 1993).

The topics to be discussed concern a relatively advanced use of latent growth curve modeling. A detailed enunciation of latent growth curve modeling is, therefore, beyond the scope of this chapter. Good introductions are provided by Willett and Sayer (1994), Stoolmiller (1995), MacCallum et al. (1997), McArdle & Epstein (1987), Muthén and Khoo (1998) and Duncan, Duncan, Strycker, Li and Alpert (1999). In the next section the latent growth curve model with an estimated basis function will be discussed; Section 3 will be devoted to the issue of measurement invariance in second-order latent growth curve models.

13.2 The Latent Growth Curve Model with an Estimated Basis Function

Consider the simple latent growth curve model in Equation 1 for an outcome y_{it} of individual i at time point t :

$$\begin{aligned} y_{it} &= \eta_{0i} + \eta_{1i} b_t + \varepsilon_{it} \\ \eta_{0i} &= \nu_0 + \zeta_{0i} \\ \eta_{1i} &= \nu_1 + \zeta_{1i} \end{aligned} \quad (1)$$

where η_{0i} represents the (initial) level factor, and η_{1i} represents the growth rate factor, while the basis function for the growth rate factor, b_t , represents an elementary function of time (e.g. measurement occasion or age), and ε_{it} is a time-specific residual; ν_0 and ζ_{0i} , and ν_1 and ζ_{1i} are the mean and deviation of respectively the level factor and the growth rate factor. The variances and covariance of the level and growth factor are respectively ψ_{00} , ψ_{11} , and ψ_{01} . Conceptually, growth of the individuals in the population is characterized by growth curves in which the growth parameters (level and growth rate) may vary. Equation 1 provides a general form of the latent growth model that can be used to describe nonlinear growth.

Although the unobserved growth rate score is an individual characteristic which remains the same across time, the observed impact of this growth rate score changes as a function of time b_t (McArdle & Hamagami, 1992). To represent linear growth, the coefficients in the basis function for the growth rate factor (Meredith & Tisak, 1990, p. 108), b_t of Equation 1, are fixed to specific values; for instance, to values corresponding to the measurement occasion (e.g. $b_t = [0, 1, 2, 3]$) or some related scaling. With a fixed basis function, the level and growth rate factors have a straightforward interpretation: the level factor represents the status at the time point defined as zero, and the growth rate factor represents the amount

of change per time unit. Often, just the first measurement occasion is defined as the zero time point, and is loosely phrased by many authors as the 'initial status' (but see Stoel & Van den Wittenboer, 2003).

The nonlinear growth model of McArdle (1986) and Meredith and Tisak (1990) is obtained by estimating the values of the basis coefficient in Equation 1 (i.e. b_t) from the data. For purposes of identification at least two basis function coefficients need to be fixed, the remaining coefficients are estimated. The basis function coefficients reflect a common factor of individual differences in the pattern of change over time (McArdle & Hamagami, 1996, p.109). Muthén and Khoo (1997) explain the estimation of the basis function coefficients as the estimation of the time scores. The essence is captured effectively with the following citation of Garst (2000, p.259): "Statistically, a linear model is still estimated, but the nonlinear interpretation emerges by relating the estimated time scores to the real time frame...Therefore, a new time frame is estimated and the transformation to the real time frame gives the nonlinear interpretation". In other words, what would be the value of the time scores (i.e. basis function coefficients) if true growth were linear? Thus, although the model in Equation 1 keeps being linear, it can be given a nonlinear interpretation.

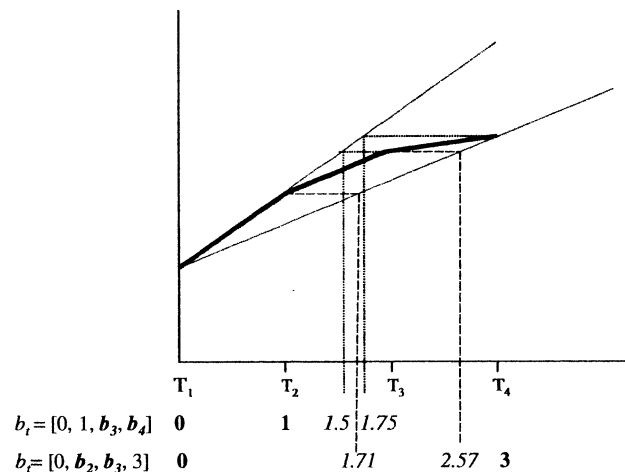
To elucidate this in more detail, suppose the following basis function in Equation 1, with the first two values fixed: $b_t = [0, 1, b_3, b_4]$. Assume this basis function is used to analyze data from a time-structured design (Bock, 1979) with equal spacing of the four occasions of measurement of, say, one year. The level factor still represents the status at the first measurement occasion since it is invariant to this type of transformations of the basis function. The growth rate factor, however, now represents the (linear) growth between the first and the second measurement occasion. For example, if the time scores are estimated as $b_t = [0, 1, b_3 = 1.5, b_4 = 1.75]$, this implies that the growth between the third and second occasion equals $1.5 - 1 = .5$ times the growth rate factor; and the growth between the fourth and third occasion equals $1.75 - 1.5 = .25$ times the growth rate factor. In other words, the increments of the elements of the basis function decrease. Thus, growth is the strongest between the second and the first occasion, and gets less strong during the subsequent occasions. If true growth were linear, then the mean score reach at the third measurement occasion ($t=2$), would have been reached at $t=1.5$.

The basis function described above, with the first two values fixed, represents the usual scaling. This scaling is, however, somewhat arbitrary. McArdle & Bell (2000) demonstrate that any scaling, with two basis function values fixed, could be used instead. With a different scaling of the basis function the overall model fit is identical, as well as the estimated mean growth curve.

Now, let a different scaling of the basis function be $b_t' = [0, b_2, b_3, 3]$ (cf. McArdle & Bell, 2000). Growth is now interpreted in terms of the full

period. Since the ratio of b_t/b_{t+1} remains the same for a different scaling of the model with the same data (McArdle & Bell), the values of an alternative basis function can be computed easily. Thus, for example, b_3/b_4 of basis function b_t equals b'_3/b'_4 of basis function b'_t . This implies that the basis function from the previous example, $b_t = [0, 1, b_3 = 1.5, b_4 = 1.75]$, corresponds to $b'_t = [0, b'_2 = 3*(1/1.75) = 1.71, b'_3 = 3*(1.5/1.75) = 2.57, 3]$. In other words, the elements of the basis function are multiplied by a factor $3/1.75 = 1.71$. Figure 13.1 illustrates this graphically. From the figure it can be seen that both basis functions lead to the same mean growth curve.

Figure 13.1. Two basis functions procedure the same mean growth curve



As an illustration, the analysis of an empirical data set will now be presented. The data were taken from the Dutch longitudinal PRIMA cohort study (Driessen, Van Langen & Vierke, 2000). The measurements in the present example are from a subsample, presenting the complete data of 679 children on four consecutive measures of children's academic motivation measured with two indicators (x_t and y_t). The covariance matrix and mean vector are given in Table 13.1. In this section only the measures of x_t are used; simultaneous analysis of both x_t and y_t will be the topic of the next section. The goal of these example analyses is to clarify by demonstration the problem of arbitrariness of the basis function's scaling, and to illustrate the consequences for the parameter estimates and model fit.

Table 13.1. Sample means and covariance matrix of academic motivation

	Language							
	x_1	y_1	x_2	y_2	x_3	y_3	x_4	y_4
Mean vector	3.604	3.640	3.518	3.248	3.443	3.209	3.446	3.241
Covariance matrix								
x_1	.881							
y_1	.531	.763						
x_2	.213	.228	.850					
y_2	.260	.282	.551	1.069				
x_3	.166	.239	.268	.337	1.059			
y_3	.259	.315	.359	.453	.631	1.106		
x_4	.204	.231	.262	.336	.403	.427	1.121	
y_4	.216	.275	.287	.399	.367	.514	.720	1.101

Note: x_t and y_t refer to the measurement of x respectively y at time point t .

Growth curve models with a different scaling of the basis functions are fit to the covariance matrix and mean vector of Table 13.1 using *Mplus* 1.04 (Muthén & Muthén, 1998). The basic model to be fit was the model expressed in Equation 1. Table 13.2 shows the relevant parameter estimates and fit measures of the model with different basis functions.

Models are fit to the data with varying basis functions: Model 1.1 with $b'_t = [0, 1, b_3, b_4]$, and Model 1.2 with $b'_t = [0, b_2, b_3, 3]$. The zero time point did not change, however, and is placed at the first measurement occasion for both models. As may be expected, the overall fit of Model 1.1 and 1.2 is the same. Thus, Models 1.1 and 1.2 can be regarded as statistically equivalent, and graphs with a similar curve will emerge. Both basis functions, in combination with the means of the level factor and the growth factor, lead to a curve characterized by a decreasing growth rate as time passes by. However, Table 13.2 shows that the mean and variance of the growth rate factor (respectively $-.07$ vs. $-.05$; and $.05$ vs. $.03$), as well as their test statistics (respectively -2.34 vs. -3.12 and 1.32 vs. 2.42) differ across the two models. The mean and variance of the level factor are the same, as well as their test statistics. This is as expected since the zero time point did not change. The correlation between the level and growth factor ($-.25$) is the same across these two models², but not its test statistic ($-.84$ vs. $-.96$).

² If, however, the origin of the time scale would also have been changed between the two models, then the correlation between the level and growth factor could have changed as

Table 13. 2. Maximum Likelihood estimates of the parameters of the fitted growth models with different basis functions

Parameter	Model 1.1: $b_t^1=[0, 1, b_3, b_4]$			Model 1.2: $b_t^2=[0, b_2, b_3, 3]$		
	estimate	s.e.	est./s.e	estimate	s.e.	est./s.e
Mean (level)	3.60	.034	106.65	3.60	.034	106.65
Mean (growth)	-.07	.028	-2.34	-.05	.015	-3.12
Var (level)	.24	.058	4.18	.24	.058	4.18
Var (growth)	.05	.040	1.32	.03	.012	2.42
Correlation (level, growth)	-.25	.034	-.84	-.25	.022	-.96
b_t	[0, 1, 2.49, 2.19]			[0, 1.37, 3.40, 3]		
CHISQ	$\chi^2(3)=1.08, p=.78$			$\chi^2(3)=1.08, p=.78$		
RMSEA	.000			.000		

Note: The test statistic (Wald test) = Estimate/ standard error. The test statistic of the correlation is computed using the covariance as the parameter estimate.

Since the basis function coefficients are different and therefore the unit of the time scale, it is obvious that there must be differences in the estimates and standard errors. In the next section it is demonstrated that the differences in the parameter estimates can, indeed, be explained by a (linear) transformation of the basis function (cf. McArdle & Bell, 2000). However, some standard errors do not change according to this transformation, which led to different values of the test statistics (i.e. parameter estimate/ standard error) for the same parameter estimate. To put it in other words, the observed change in the test statistics implies an additional change in the standard errors above the linear transformation. In the next section it is also illustrated that the differences in the test statistics are a consequence of the fact that a different factor loading (i.e. basis function coefficient) is fixed to a nonzero value (cf. de Pijper & Saris, 1982).

13.2.1 Clarification of the problem

The consequences of a linear transformation of the basis function for the parameter estimates of the LGC model are well known (Mehta & West,

well. Several authors have shown that, depending on the origin of the time scale, this correlation may take any value between -1 to +1 (e.g. Mehta and West, 2000).

2000; Rogosa, Brandt and Zimowski, 1982; Rogosa and Willett, 1985; Stoel & Van den Wittenboer, 2003), and have even been studied for the LGC model with an estimated basis function (Rovine & Molenaar, 1998). However, the focus of these articles was primarily on the effect of changing the time point of initial level. In other words they focused on transformations of the basis function by adding or subtracting a constant. Nevertheless, the equations provided by Stoel and Van den Wittenboer, can be used to explain the effect of transformations by multiplying the basis function with a scaling factor.

A comparison of the estimated basis function coefficients in the previous section reveals that they are just linear transformations of each other. The basis function of Model 2 can be computed by hand by multiplying the elements with the scaling factor $\beta = 3/2.19 \approx 1.37$. Equations 2 to 4 give the effect of the scaling factor on the variance/ covariance estimates (see Equation 13 to 15 of Stoel & van den Wittenboer, 2003, p.28). The derivation of these equations is also given by Stoel & van den Wittenboer.

$$\psi_{00}^* = \psi_{00} - \frac{2\alpha}{\beta}\psi_{10} + \frac{\alpha^2}{\beta^2}\psi_{11} \quad (2)$$

$$\psi_{10}^* = \frac{1}{\beta}\psi_{10} - \frac{\alpha}{\beta^2}\psi_{11} \quad (3)$$

$$\psi_{11}^* = \frac{\psi_{11}}{\beta^2} \quad (4)$$

where ψ_{00}^* represents the transformed variance of the level factor, ψ_{11}^* the transformed variance of the growth rate factor, and ψ_{01}^* is the transformed covariance between these parameters. α is the shift on the basis function, and β is the scaling factor. To keep things simple, it is assumed in the following that α equals zero. Consequently, the basis functions of Model 1.1 and 1.2 both had the first coefficient constrained to zero. This section, thus, only focuses on transformation of the basis function by multiplying its coefficients with the scaling factor, and not on a shift of the basis function. Shifting the origin of the time scale does, however, have additional consequences for the estimates of the growth parameters, as well as for the effect of time-invariant covariates. These issues have recently been discussed by Mehta and West (2000), and Stoel and van den Wittenboer (2003).

Equation 2 to 4 clearly show that the variance of the level factor is not affected by the scaling factor, that the variance of the growth rate becomes a factor β^2 smaller, and that the covariance between the level and growth rate

becomes a factor β smaller. With these equations it can be shown that the correlation between the level and growth rate does not change as a function of β . Equation 5 shows that the scaling factor β plays no role in the computation of the correlation between the level and the growth factor.

$$\text{Correlation (level, growth)}^* = \frac{\frac{1}{\beta} \psi_{10}}{\sqrt{\psi_{00}} \sqrt{\frac{\psi_{11}}{\beta^2}}} = \frac{\psi_{10}}{\sqrt{\psi_{00}} \sqrt{\psi_{11}}} \quad (5)$$

It can be concluded that the correlation between the level and growth rate is scale invariant, and that the variance of the level, the variance of the growth rate and their covariance are scale free. That is, though these parameters are affected by a change in the basis function, they can be translated into each other by a linear transformation. If we know the parameter estimates with one type of scaling, we implicitly know the parameter estimates at another scaling of the basis function. It can be easily shown that the mean of the growth rate factor is also scale free, and that it becomes a factor β smaller. However, if the models were completely scale free, then the standard errors would also have to change proportionally resulting in equivalent test statistics. As noted before, the observed change in the test statistics implies an additional change in the standard errors above the linear transformation.

This change of the values of the test statistic does not only occur in growth curve models with an estimated basis function. It occurs in any confirmatory factor analysis with some, but not all, factor loadings fixed to a value unequal to zero (de Pijper & Saris, 1982; see also Bollen, 1989, and Browne, 1982). Since, in the past years, the focus was mainly on growth curve models with an entirely fixed basis function (e.g. linear or quadratic), the issue has not arisen previously. The solution is provided by Jöreskog and Sörbom (1988) for a confirmatory factor analysis, but seldom used. They implicitly state that invariant estimates of the standard errors can be obtained with a two-stage approach, by reanalyzing the model while fixing the factor loadings to the estimated values of the first analysis. For the growth curve model with an estimated basis function this would mean that in the second step the coefficients of the basis function are fixed to their estimated values of the first step. With this two-stage approach, the standard errors follow the same linear transformation as the parameter estimates, and consequently the values of the test statistic will be invariant across models.

Table 13.3. Maximum Likelihood estimates of the parameters of the fitted growth models with basis functions fixed to the estimated values of step 1.

Parameter	Model 2.1: $b_1=[0, 1, 2.489, 2.194]$			Model 2.2: $b_1=[0, 1.367, 3.403, 3]$		
	estimate	s.e.	est./s.e.	estimate	s.e.	est./s.e.
Mean (level)	3.596	.033	110.08	3.596	.033	110.08
Mean (growth)	-.065	.018	-3.66	-.048	.013	-3.66
Var (level)	.244	.054	4.55	.244	.054	4.55
Var (growth)	.052	.015	3.45	.028	.008	3.45
Corr. (level, growth)	-.252	.025	-1.14	-.252	.018	-1.14
CHISQ	$\chi^2(5)=1.076, p=.96$			$\chi^2(5)=1.076, p=.96$		
RMSEA	.00			.00		

Note. The test statistic (Wald test) = Estimate/ standard error. The test statistic of the correlation is computed using the covariance as the parameter estimate.

Table 13.3 provides the correct estimates of the standard error and test statistics for Model 2.1 and Model 2.2. All parameter estimates and their standard errors are now scale invariant or scale free. The parameter estimates of Model 2.2 can be computed by hand from the parameter estimates of Model 2.1 using Equations 2, 3 and 4 with $\alpha=0$, and $\beta=2.194/3=.731$. As expected the value of the χ^2 -test statistic is the same, however, the overall model fit seems to be better because the model has two degrees of freedom more caused by the fact that the basis function is now entirely fixed.

13.3 Multiple Indicators: The Issue of Longitudinal Measurement Invariance³

Longitudinal measurement invariance, or measurement invariance across time, means that the numerical values across measurement occasions are obtained from the same measurement scale (Drasgow, 1987; cf. Meredith, 1993). The general question of measurement invariance is one of whether or not, under different conditions of observing and studying phenomena, measurement operations yield measures of the same attribute (Horn & McArdle, 1992). In other words, invariance of measurements ensures an equal definition of a latent construct over time (Hancock et al. 2001). In other words, do indicators with the same face validity (i.e. identical scaling

³ Meredith (1993) uses the concept of weak measurement invariance to stress that only the first two moments of the probability distribution function are invariant (see also Lubke, Dolan, Kelderman and Mellenbergh (2001). Our concept of measurement invariance corresponds to weak measurement invariance as defined by Meredith.

and wording) relate to the underlying construct in the same fashion over time (Sayer & Cumsille, 2001)?

Within the context of the common factor model, measurement invariance is often investigated using the concept of factorial invariance (Meredith, 1964; 1993). Meredith describes factorial invariance as being composed of 3 hierarchical levels, respectively: weak, strong, and strict factorial invariance, with strict factorial invariance being the strongest form of factorial invariance. It will not be surprising that factorial invariance, in general, assumes configural invariance, being that the same indicators of the latent construct are measured at each occasion. Furthermore, weak factorial invariance requires the measurement parameters of each indicator (i.e. the factor loadings) to be invariant over time. Second, strong factorial invariance requires equal indicator intercepts. Third, strict factorial invariance requires also the residuals to be equal over time. Meredith (1993) has shown that strict factorial invariance almost certainly ensures measurement invariance⁴.

In practice, strict factorial invariance is likely to be violated, and researchers have been investigating whether the requirements of strict factorial variance can be relaxed (e.g. Byrne, Shavelson & Muthén, 1989; Horn & McArdle, 1992; McArdle & Cattell, 1994; Pentz & Chou, 1994; Lubke & Dolan, 2002). Some researchers state that invariance of the residuals is not required for testing hypotheses about common factors. Unequal residual variances are indicative only of differences in reliability of the observed variable (Little, 1997), and do not concern the common factors (Oort, 2001). In other words they suggest strong factorial invariance to be a sufficient test for measurement invariance. A recent simulation study by Lubke and Dolan (2002), however, casts some doubt on this latter suggestion. They show that under some conditions differences in residual variances can mask differences in indicator intercepts. More liberal scholars have even been suggesting relaxations of strong factorial invariance, termed 'partial measurement equivalence' (Byrne et al.). A measurement model possesses partial measurement invariance if some measurement parameters are free and others are constrained to be equal over time. Byrne et al., as well as Pentz and Chou, argue that partial measurement invariance is a sufficient requirement, and a more realistic goal compared to strong or even strict factorial invariance. Methodological research about partial measurement invariance is scarce, but, as already noted by Meredith (2001), it deserves more attention than it has received. In this section it will be demonstrated that partial measurement invariance may present some problems if the latent variable structure is scaled using a reference indicator. The term 'full

⁴ Lubke, et al. (2002) discuss instances in which strict factorial invariance between groups in the cross-sectional common factor model is not a sufficient condition for measurement invariance, hence the phrase "almost certainly". They conclude, however, that these exceptions do not represent a serious threat in practice.

measurement invariance' is used to indicate a model with strong factorial invariance, and 'partial measurement invariance' is used to indicate a model in which some of the requirements of strong factorial invariance have been relaxed.

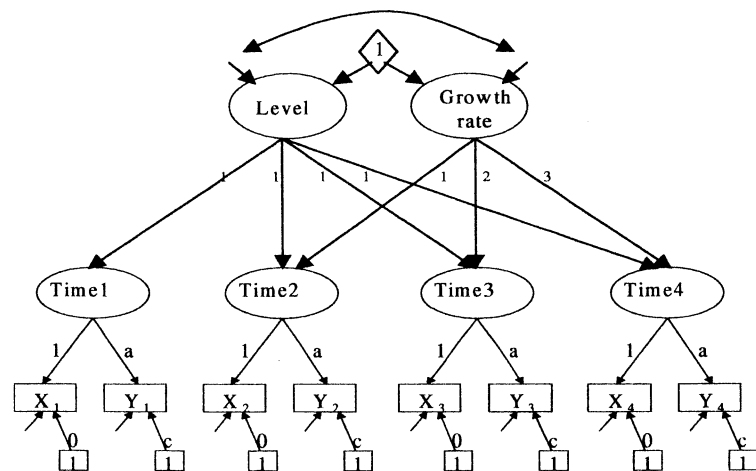
Consider the second-order growth model of Figure 13.2, in which we restrict ourselves to a growth model with two indicators (x_t and y_t) at four equally spaced measurement occasions t . Generalizations to more indicators at each measurement occasion are straightforward, but beyond the scope of this chapter. Figure 13.2 schematically represents full measurement invariance: (1) one factor loading of one indicator within each measurement occasion is fixed to the value of 1 for the purpose of scaling the covariance structure of the latent variable. (2) The factor loadings for the other indicator are estimated, but they are constrained to be equal across time. (3) One indicator intercept of one indicator within each measurement occasion is fixed to the value of zero for the purpose of scaling the mean structure of the latent variable⁵ (4). The intercepts of the remaining indicator is estimated, but are constrained to be equal across time. In other words, each repeatedly measured indicator (x_t and y_t) has the same factorloadings and intercepts across time. In Figure 13.2, indicator x_t is used to scale the latent variable; we will refer to this indicator as the reference indicator. However, y_t could also have been used as the reference indicator. Full measurement invariance has the attractive property of being invariant to the scaling of the latent variable. That is, if the indicator y_t were used to scale the latent variable instead of indicator x_t , the same parameter estimates and overall model fit would have emerged.

Partial measurement invariance is not as strict as full measurement invariance in that a few violations are tolerated. That is, the factorloadings (a), and or the indicator's intercepts (c) do not have to be of the same value, need not necessarily to be equal for the full time period. Following the arguments for full measurement invariance, it may be expected that using either x_t as the reference indicator and to estimate the factorloadings for y_t , should result in the same model fit as using y_t as the reference and to estimate the factor loading for x_t . Differences may arise, however, if the indicator's intercepts are not constrained to be equal; i.e. if parameter c is estimated uniquely at each occasion. Now, the model is no longer insensitive to the scaling of the latent variable; if y_t is used as the reference variable instead of x_t , different model parameters and a different overall model fit will emerge. In other words, the latent growth curve model under partial measurement invariance is not invariant under a different scaling of the

⁵ An alternative approach to scale the mean structure of the model might be fixing the mean of the level factor to zero instead of the intercept of one indicator each time point (see Chan, 1998; Dolan and Molenaar, 1994; Horn and McArdle, 1992). In many applications, however, the way of scaling the mean structure as presented here is implemented.

mean structure of the latent variable by using a reference indicator. We will illustrate this in the example of the next section.

Figure 13.2. Schematic presentation of full measurement invariance in a linear latent growth curve model



Note: Intercepts of indicators are conceptualized as regression on a constant equal to one (See Hancock et al, 2001). Only the relevant parameters are presented. Factorloadings for x_t are fixed to 1.00 prior to estimation; factorloadings of Y_t are constrained to be equal (a); intercepts of Y_t are fixed to zero; intercepts of Y_t are constrained to be equal (c). The curved double-headed arrow represents a correlation between the latent factors.

13.3.1 Example

The data to be analyzed are presented in Table 13.1 of Section 2. In this section both indicators of academic motivation are incorporated into a latent growth curve model. The model to be analyzed is equivalent to the model depicted in Figure 13.2. Thus, it is a linear growth curve model with the basis function for the growth rate factor fixed to $b_T=[0, 1, 2, 3]$. Table 13.4 presents the relevant parameter estimates.

Table 13.4. Parameter estimates of growth models without (Model 3.1), with full (Model 3.2 and 3.3), and with partial measurement invariance (Model 3.4 and 3.5)

Parameter	Model 3.2	Model 3.3	Model 3.4	Model 3.5
Mean (level)	4.86	3.54	4.36	3.54
	(15.75)	(113.96)	(14.31)	(113.83)

Mean (growth)	-.06	-.06	-.03	-.06
	(-8.87)	(-8.87)	(-3.29)	(-9.12)
Var (level)	.34	.34	.32	.30
	(6.42)	(6.42)	(6.44)	(6.13)
Var (growth)	.01	.01	.01	.01
	(2.95)	(2.95)	(2.86)	(2.53)
Cov	-.01	-.01	-.00	-.00
(Level,Growth)	(-.48)	(-.48)	(-.41)	(-.06)
CHISQ	$\chi^2(15)=80.01$	$\chi^2(15)=80.01$	$\chi^2(12)=12.70$	$\chi^2(12)=62.53$
	p=.00	p=.00	p=.39	p=.00
RMSEA	.08	.08	.01	.08
	(.06 -.10)	(.06 -.10)	(.00 -.04)	(.06 -.10)

Note: Estimate/ standard error in brackets for the growth parameters; for the RMSEA the 90% confidence interval is given in brackets.

Model 3.2 and Model 3.3 are latent growth curve models under full measurement invariance with, respectively x_t and y_t as reference indicator. Model 3.4 and 3.5 have partial measurement invariance with, respectively, x_t and y_t as reference indicator. Firstly, from Table 13.5 it can be seen that the latent growth curve model, under full measurement invariance, is insensitive to the choice of the reference indicator (Model 3.2 and 3.3). The overall fit measures of Model 3.2 and Model 3.3 are equivalent ($\chi^2(15)=80.09$ p=.00; RMSEA = .08). The only difference between the models is in the estimates of the mean of the level factor, 4.86 respectively 3.54. Apparently the two indicators, x_t and y_t , have different means over time, and using one instead of the other as a reference indicator changes the scale, and therefore the mean of the latent factors. This does, however, not change any substantive interpretation based on the model, it simply illustrates the arbitrariness of the mean of the level factor.

Inspection of Model 3.2 and Model 3.3 shows that full measurement invariance is not supported for these data. The significant chi-square $\chi^2(15)=80.01$, and the large RMSEA values clearly lead to a rejection of full measurement invariance. Inspection of the results suggests that the misfit might have been caused by the equality restrictions on the estimated indicator intercepts (see constraint 'c' in Figure 13.2). In other words, the requirement of full measurement invariance might be too strict for these data. Consequently, this requirement will be relaxed in Model 3.4 and Model 3.5 by removing the equality constraint for the intercepts. In other words, Model 3.4 estimates the intercepts of y_t , and Model 3.5 estimates the intercepts of x_t .

Table 13.4 shows clearly that Model 3.3 and Model 3.4 are quite different. Model 3.4 can be regarded as fitting excellently, whereas the fit of Model 3.5 is quite poor. In addition, the parameter estimates in both models differ. Thus, an apparently arbitrary choice of the reference indicator can have serious consequences for the model fit and the parameter estimates of the corresponding models.

The data of the example consist of two repeatedly measured indicators (x_t and y_t). Since Model 3.4 (with x_t as the reference indicator) provided a good model fit, while Model 3.5 (with y_t as the reference indicator) did not, it can be concluded that the intercepts of y_t cannot be constrained to be equal across time. In other words, the intercepts of x_t may be regarded as being invariant, but this does not hold for the intercepts of y_t , and thus choosing one of the two in an LGC model with partial measurement invariance affects the parameter estimates. Although it was not illustrated in this example, it is not difficult to see that this may also occur for the factor loadings. If a different reference indicator is chosen with partial measurement invariance of the factor loadings, parameter estimates and model fit may change.

13.4 Discussion

In the first section of this chapter, we clarified by demonstration that growth interpretations based on the growth curve model with an estimated basis function are indeed nonarbitrary, and a plot with a similar shape will be drawn from the growth estimates obtained under any rescaling (McArdle & Bell, 2000, p.107). More specifically, it is demonstrated that, given a two-stage approach, the parameter estimates and standard errors of the LGC model with an estimated basis function will be scale invariant or scale free.

The latent growth curve model with an estimated basis function has been applied relatively few times compared to its linear variant (but see for instance McArdle, 1989; McArdle & Anderson, 1990; McArdle & Hamagami, 1992; Muthén and Khoo, 1997; Raykov & Marcoulides, 2000; Rovine & Molenaar, 1998). The reason for this might have been the difficult interpretation and changes in the standard errors. However, in this chapter it is demonstrated that the differences could be easily overcome by adopting the two-stage approach advocated by Jöreskog and Sörbom (1988), in another context. Actually, if in any factor analysis model a different factor loading is fixed to a value unequal to zero for purposes of scaling, the estimates of the standard errors may be different. Although it is comforting that the two-stage approach advocated here leads to the same linear transformation in the standard errors as in the parameter estimates, an objection against the approach might be that it eliminates the sampling variability in the estimated coefficients. In what way the standard errors are underestimated requires further research, which will be the topic of future work.

The results of this chapter support the statement of McArdle & Bell (2000, p.82) that the LGC model with an estimated basis function "is certainly a viable optional basis in LGC models". However, this LGC model presents a somewhat exploratory approach to the modeling of nonlinear growth, and more rigorous hypotheses about nonlinear growth can be tested

as illustrated by McArdle & Hamagami (1996). A Gompertz-growth model, for example, can be specified by adding a third latent variable together with nonlinear restrictions on the basis functions coefficients (Browne & DuToit, 1991). In general, these are all alternative forms of latent growth curve models (McArdle & Hamagami, p.109).

In the second part, apparently similar multiple indicator latent growth curve models are compared under full and partial measurement invariance. While changing the reference indicator has no serious consequences for the model under full measurement invariance, model fit and parameter estimates may change drastically under partial measurement invariance. Full measurement invariance, i.e. strong factorial invariance may, therefore, be a necessary condition for a valid interpretation of change in latent constructs for models that scale the latent variable structure with using reference indicators. Freely estimating the intercepts may lead to completely different models depending on the choice of the reference indicator in this case. This argues against an unsophisticated relaxation of full measurement invariance to partial measurement invariance. In other words, though the choice of the reference indicator does not influence the model under full measurement invariance, it matters under partial measurement invariance.

The approach discussed here consists of identifying the covariance and mean structure by constraints on, respectively, the factorloadings and indicator intercepts, and constitutes the approach to second-order latent growth curve modeling as recently presented by, for example, Hancock, Kuo and Lawrence (2001), Oort (2001) and Sayer and Cumsille (2001; cf. Vandenberg & Lance, 2000). Under full measurement invariance, and no violation of other model assumptions, the approach leads, as was illustrated, to the correct parameter estimates and model fit. Under partial measurement invariance, however, the approach might lead to incorrect parameter estimates and model fit. An alternative approach might solve the problems discussed above for models that fail to satisfy full measurement invariance. That is, the covariance and mean structure of the model can also be identified by means of constraints on the latent variable structure (e.g. Chan, 1998; Dolan & Molenaar, 1994; Horn & McArdle, 1992) instead of constraints on the measurement part of the model (i.e. on the factorloadings and intercepts). Though this approach might prove an attractive alternative for the general longitudinal factor model under partial measurement invariance, it may lead to other complications if a latent growth structure is imposed on the first order factors. The mean structure, for example, can be identified by constraining the mean of the level factor to zero, while estimating indicator intercepts. However, in some instances the mean of the level factor can be interpreted as the initial status of the growth process, and thus may be a substantively interesting parameter. Constraining its mean to zero makes inferences regarding this parameter impossible. Nevertheless,

under partial measurement invariance, not being able to interpret some parameters should be preferred to getting an unwanted solution.

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