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2011 New J. Phys. 13 045010

(http://iopscience.iop.org/1367-2630/13/4/045010)

New Journal of Physics
The open-access journal for physics

Spin drag in ultracold Fermi mixtures with repulsive interactions

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New Journal of Physics **13** (2011) 045010 (10pp)
Received 16 December 2010
Published 11 April 2011
Online at http://www.njp.org/
doi:10.1088/1367-2630/13/4/045010

Abstract. We calculate the spin-drag relaxation rate for a two-component ultracold atomic Fermi gas with positive scattering length between the two-spin components. In one dimension, we find that it vanishes linearly with temperature. In three dimensions, the spin-drag relaxation rate vanishes quadratically with temperature for sufficiently weak interactions. This quadratic temperature dependence is present, up to logarithmic corrections, in the two-dimensional (2D) case as well. For stronger interaction, the system exhibits a Stoner ferromagnetic phase transition in two and three dimensions. We show that the spin-drag relaxation rate is enhanced by spin fluctuations as the temperature approaches the critical temperature of this transition from above.

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Contents

1. Introduction

Interest in electronic transport ranges from everyday applications to fundamental physics. One of the most interesting phenomena that span this entire range is the influence of a thermodynamic phase transition on the electrical conductivity. The most direct example is the phase transition from a normal conductor to a superconductor characterized by a vanishing resistance. The applications of this phenomenon are ubiquitous and the basic physics that underlies the transition in superconductors, the Bose–Einstein condensation of fermionic pairs, has emerged in research fields from astroparticle physics [1] to cold-atom systems [2, 3].

A system in between the latter two temperature extremes, in which analogies of superconductivity have been predicted, is that of a two-dimensional (2D) electron–hole bilayer [4, 5]. In this case, the pairs that condense are excitons formed by electrons from one layer with holes in the other. The relevant transport probe is in this case the Coulomb drag measurement [6]: a current I is driven through one layer, known as the 'active' layer, causing a voltage drop V_{D} in the other. As the layers are separated by an essentially impenetrable tunnel barrier, the voltage drop is predominantly caused by Coulomb scattering, and the drag resistivity $\rho_{\mathrm{D}} = V_{\mathrm{D}}/I$ has, up to logarithmic corrections, the characteristic quadratic Fermi-liquid-like low-temperature dependence $\rho_{\mathrm{D}} \propto T^2$. When the excitons undergo Bose–Einstein condensation, however, the drag resistivity is predicted to jump from the relatively small value proportional to T^2 to a value equal to the ordinary resistivity of the active layer [7]. Although conclusive evidence of exciton condensation is still lacking, two experimental groups [8, 9] have recently reported the observation of an upturn in the drag resistivity as the temperature is lowered. This upturn is interpreted as being due to strong pairing fluctuations that precede exciton condensation [10], and thus serves as a precursor signal for the transition, similar to the enhancement of conductivity in superconductors due to superconducting fluctuations above but close to the critical temperature [11].

A closely related situation arises when the two layers of a 2D electron–electron bilayer placed in a strong perpendicular magnetic field are close enough to allow the establishment of interlayer coherence [12]. In this case, the two layers in the system can be labelled 'up' and 'down' along a 'z'-axis, so that the which-layer degree of freedom becomes a spin one-half pseudospin. Interlayer coherence in this language corresponds to pseudospin ferromagnetism with an easy x–y-plane, since this orientation of the pseudospin describes a particle that is neither in the left nor in the right layer, but in a coherent superposition of the two. Furthermore, Coulomb drag becomes pseudospin drag, the mutual friction between two pseudospin states due to Coulomb scattering. This analogy prompted studies of spin drag, the frictional drag between electrons with opposite spin projection, in a single semiconductor [13]. While the realization of separate electric contacts to the two spin states remains an experimentally challenging problem, the spin drag is observed indirectly, by measuring different diffusion constants for charge and spin [14].

Because of the presence of other relaxation mechanisms, spin-drag effects are usually not very large in semiconductors, and are even smaller in metals. This is completely different in cold atomic gases, where scattering between different hyperfine spins is the only mechanism to relax spin currents, and was considered both for fermionic atoms [15, 16] and for bosonic ones [17]. In this paper, we consider spin drag in a two-component Fermi gas, in one, two and three dimensions. We point out that a particularly interesting situation occurs when spin drag is considered in a two-component Fermi gas that is close to a ferromagnetic instability [18]–[22], as can occur for sufficiently strong and repulsive interactions in two and three dimensions. We show that the spin drag is strongly enhanced as the ferromagnetic state is approached from the normal side [23], as expected from the analogy between electron–hole bilayers and pseudospin ferromagnets. In one dimension, however, where the ferromagnetic phase transition is absent, the effects of spin drag vanish linearly with temperature. One of our motivations for considering this effect is the recent observation of ferromagnetic correlations in a two-component Fermi gas with strong repulsive interactions [24]. The fact that spin-polarized domains were not directly observed adds to the theoretical interest [25] in this experiment. Because atoms are neutral, the relevant experimental quantity is the spin-drag relaxation rate, which, for instance, determines the damping rate of the spin-dipole mode in trapped cold-atom systems [16] and is thus accessible experimentally. Interestingly, an electronic analogue of the spin-dipole mode also exists [26].

The remainder of this paper is organized as follows. We first derive an expression for the damping of the spin-dipole mode from the Boltzmann equation. As mentioned before, this damping is determined by the spin-drag relaxation rate, which is subsequently evaluated in one, two and three dimensions. We end the paper with our conclusions and a short discussion.

2. The spin-dipole mode and spin-drag relaxation rate

We consider a mixture of fermionic atoms of mass m in d dimensions, with two hyperfine states denoted by $|\uparrow\rangle$ and $|\downarrow\rangle$. The grand-canonical Hamiltonian with external trapping potential $V(\boldsymbol{x})$ and chemical potential μ is given by

$$\hat{H} = \int \mathrm{d}^d\boldsymbol{x} \sum_{\sigma\in\{\uparrow,\downarrow\}} \hat{\psi}^\dagger_\sigma(\boldsymbol{x}) \left(-\frac{\hbar^2\nabla^2_{\boldsymbol{x}}}{2m} + V(\boldsymbol{x}) - \mu\right) \hat{\psi}_\sigma(\boldsymbol{x}) + U \int \mathrm{d}^d\boldsymbol{x}\; \hat{\psi}^\dagger_\uparrow(\boldsymbol{x})\hat{\psi}^\dagger_\downarrow(\boldsymbol{x})\hat{\psi}_\downarrow(\boldsymbol{x})\hat{\psi}_\uparrow(\boldsymbol{x}), \tag{1}$$

in terms of fermionic creation and annihilation operators $\hat{\psi}^\dagger_\alpha(\boldsymbol{x})$ and $\hat{\psi}_\alpha(\boldsymbol{x})$, respectively. At low temperatures s-wave scattering, described by a pseudopotential $V(\boldsymbol{x}-\boldsymbol{x}') = U\delta(\boldsymbol{x}-\boldsymbol{x}')$, dominates, and we have therefore omitted other interaction terms from this Hamiltonian. We consider here only the balanced case in which there is an equal number of atoms N in each hyperfine state.

Following the discussion in [27], we now derive an expression for the damping of the spin-dipole mode from the Boltzmann equation for the distribution function $f_\sigma(\boldsymbol{x}, \boldsymbol{k}, t)$ for the atoms in spin state $|\sigma\rangle$, given by

$$\frac{\partial f_\uparrow}{\partial t} - \frac{1}{\hbar}\nabla V \cdot \frac{\partial f_\uparrow}{\partial \boldsymbol{k}} + \frac{\hbar \boldsymbol{k}}{m} \cdot \frac{\partial f_\uparrow}{\partial \boldsymbol{x}} = \Gamma_{\mathrm{coll}}[f_\uparrow, f_\downarrow] , \tag{2}$$

where we take the trapping potential to be harmonic $V(\boldsymbol{x}) = m\sum_{j=1}^{d} \omega_j^2 x_j^2/2$. The equation for $f_\downarrow$ is found by replacing $f_\uparrow \leftrightarrow f_\downarrow$ in the above. Below we give an explicit expression for the collision integral $\Gamma_{\mathrm{coll}}[f_\uparrow, f_\downarrow]$.

We solve this inhomogeneous Boltzmann equation by making the ansatz $f_\uparrow(\boldsymbol{x}, \boldsymbol{k}, t) = N_{\mathrm{F}}(\epsilon_{\boldsymbol{k}-m\boldsymbol{v}_\uparrow(t)/\hbar} + V(\boldsymbol{x} - \boldsymbol{x}_\uparrow(t)))$, with a similar expression for $f_\downarrow(\boldsymbol{x}, \boldsymbol{k}, t)$. Here, $\epsilon_{\boldsymbol{k}} = \hbar^2\boldsymbol{k}^2/2m$ is the single-particle dispersion and $N_{\mathrm{F}}(\epsilon) = [\mathrm{e}^{\beta(\epsilon-\mu)} + 1]^{-1}$ is the Fermi–Dirac distribution function with $\beta = (k_{\mathrm{B}}T)^{-1}$ the inverse temperature. This ansatz is parameterized by the centre-of-mass velocity $\boldsymbol{v}_\sigma(t)$ and position $\boldsymbol{x}_\sigma(t)$ of the atomic cloud of atoms in the spin state $|\sigma\rangle$. From this, we obtain the equations of motion

$$\begin{aligned} Nm\frac{\mathrm{d}\boldsymbol{v}_\uparrow}{\mathrm{d}t} &= -N\frac{\mathrm{d}V\left(\boldsymbol{x}_\uparrow\right)}{\mathrm{d}\boldsymbol{x}_\uparrow} + \boldsymbol{\Gamma}(\boldsymbol{v}_\downarrow - \boldsymbol{v}_\uparrow, \boldsymbol{x}_\downarrow - \boldsymbol{x}_\uparrow), \\ Nm\frac{\mathrm{d}\boldsymbol{v}_\downarrow}{\mathrm{d}t} &= -N\frac{\mathrm{d}V\left(\boldsymbol{x}_\downarrow\right)}{\mathrm{d}\boldsymbol{x}_\downarrow} - \boldsymbol{\Gamma}(\boldsymbol{v}_\downarrow - \boldsymbol{v}_\uparrow, \boldsymbol{x}_\downarrow - \boldsymbol{x}_\uparrow), \end{aligned} \tag{3}$$

with the function $\boldsymbol{\Gamma}(\boldsymbol{v}_\downarrow - \boldsymbol{v}_\uparrow, \boldsymbol{x}_\downarrow - \boldsymbol{x}_\uparrow)$ given by

$$\begin{aligned} \boldsymbol{\Gamma}(\boldsymbol{v}_\downarrow - \boldsymbol{v}_\uparrow, \boldsymbol{x}_\downarrow - \boldsymbol{x}_\uparrow) = &\int \mathrm{d}^d\boldsymbol{x} \int \frac{\mathrm{d}^d\boldsymbol{k}}{(2\pi)^d}\hbar\boldsymbol{k} \\ &\times \Gamma_{\mathrm{coll}}\left[N_{\mathrm{F}}(\epsilon_{\boldsymbol{k}-m\boldsymbol{v}_\downarrow(t)/\hbar} + V(\boldsymbol{x}-\boldsymbol{x}_\downarrow(t))), N_{\mathrm{F}}(\epsilon_{\boldsymbol{k}-m\boldsymbol{v}_\uparrow(t)/\hbar} + V(\boldsymbol{x}-\boldsymbol{x}_\uparrow(t)))\right], \end{aligned} \tag{4}$$

where

$$\begin{aligned} \Gamma_{\mathrm{coll}}[f_\downarrow, f_\uparrow] = \frac{(2\pi)^{d+1}}{\hbar}U^2 &\int \frac{\mathrm{d}^d\boldsymbol{k}_2}{(2\pi)^d} \int \frac{\mathrm{d}^d\boldsymbol{k}_3}{(2\pi)^d} \int \frac{\mathrm{d}^d\boldsymbol{k}_4}{(2\pi)^d} \delta^d(\boldsymbol{k} + \boldsymbol{k}_2 - \boldsymbol{k}_3 - \boldsymbol{k}_4)\delta(\epsilon_{\boldsymbol{k}} + \epsilon_{\boldsymbol{k}_2} - \epsilon_{\boldsymbol{k}_3} - \epsilon_{\boldsymbol{k}_4}) \\ &\times \left\{[1 - f_\uparrow(\boldsymbol{x}, \boldsymbol{k}, t)][1 - f_\downarrow(\boldsymbol{x}, \boldsymbol{k}_2, t)]f_\uparrow(\boldsymbol{x}, \boldsymbol{k}_3, t) f_\downarrow(\boldsymbol{x}, \boldsymbol{k}_4, t)\right. \\ &\left. - f_\uparrow(\boldsymbol{x}, \boldsymbol{k}, t) f_\downarrow(\boldsymbol{x}, \boldsymbol{k}_2, t)[1 - f_\uparrow(\boldsymbol{x}, \boldsymbol{k}_3, t)][1 - f_\downarrow(\boldsymbol{x}, \boldsymbol{k}_4, t)]\right\}. \end{aligned}$$

We linearize the above equations using that $\boldsymbol{\Gamma}(\boldsymbol{v}, \boldsymbol{x}) \simeq \Gamma'\boldsymbol{v}$ due to the isotropy of the collision integral. The linearized equations then yield a collective-mode spectrum with $2d$ modes, corresponding to two types of oscillation in the d-dimensional trap. One set of modes is undamped and has frequencies ω_j, $j \in \{1, \ldots, d\}$, and corresponds to an in-phase oscillation of the two clouds in the harmonic trap. The other mode corresponds to the out-of-phase spin-dipole oscillation of the two-spin states. This mode is damped as a result of the friction, i.e. the spin drag between the two spin states during the oscillation. This friction is due to collisions between particles of opposite spin and results in the transfer of momentum between the two clouds, leading to spin drag and damping of these modes. These modes have the frequencies

$$\omega_j^{\mathrm{dip}} = -\mathrm{i}\gamma_{\mathrm{sd}} + \sqrt{\omega_j^2 - \gamma_{\mathrm{sd}}^2}. \tag{5}$$

The imaginary part of the above frequencies gives the damping rate of the modes, and is given by $\gamma_{\mathrm{sd}} \equiv (2\tau_{\mathrm{sd}})^{-1} = \Gamma'/Nm$ with τ_{sd} being the spin-drag relaxation time. We conclude that the spin-drag relaxation time determines the damping of the spin-dipole mode of the atomic clouds.

Instead of performing the appropriate trap average shown in equation (4), we proceed by giving an expression for $\tau_{\rm sd}$ for a homogeneous system with density n per spin state for which we, in the first approximation, take the central density in the trap to make a connection with the inhomogeneous case. Since the density is highest in the centre of the trap, this somewhat underestimates the spin-drag relaxation rate in one and two dimensions. (This is because in this case the spin-drag relaxation rate becomes larger for smaller densities, cf equations (8) and (12), at low temperatures.) In three dimensions, the spin-drag relaxation rate turns out to be approximately independent of density (see equation (14)) in the low temperature limit and so the results for the homogeneous and trapped case should be comparable. In terms of the noninteracting (Lindhard) response function at nonzero temperature,

$$\chi_0(q,\omega) = 2\int \frac{{\rm d}^d \boldsymbol{k}}{(2\pi)^d}\frac{N_{\rm F}(\epsilon_{\boldsymbol{q}+\boldsymbol{k}}) - N_{\rm F}(\epsilon_{\boldsymbol{k}})}{\epsilon_{\boldsymbol{q}+\boldsymbol{k}} - \epsilon_{\boldsymbol{k}} - \hbar\omega - i0}\,, \tag{6}$$

the expression for Γ' can be worked out to yield

$$\frac{1}{\tau_{\rm sd}(T)} = \frac{\hbar^2}{4\,mnk_{\rm B}T}\int\frac{{\rm d}^d\boldsymbol{q}}{(2\pi)^d}\frac{q^2}{d}\int_{-\infty}^{+\infty}\frac{{\rm d}\omega}{\pi}U^2\frac{[\Im m\ \chi_0(q,\omega)]^2}{\sinh^2[\hbar\omega/(2k_{\rm B}T)]}. \tag{7}$$

In the next section, we present the results obtained by evaluating this expression. We end this section by noting that the above expression for the spin-drag relaxation rate is similar to the expression for the drag resistivity in electron–hole bilayers [6]. This also implies that the spin-drag relaxation rate defines a genuine dissipative transport coefficient for an impurity-free cold-atom system and thus represents a natural starting point for studying transport phenomena in these systems.

3. Results for the spin-drag relaxation rate

In this section, we present results for the spin-drag relaxation rate $1/\tau_{\rm sd}$. These results are, among other parameters, characterized by the Fermi wave number $k_{\rm F} = [d\Gamma(d/2)n/4\pi^{d/2}]^{1/d}$, where $\Gamma(x)$ is the Euler Γ function. We also introduce the Fermi energy $\varepsilon_{\rm F} = k_{\rm B}T_{\rm F} = \hbar^2k_{\rm F}^2/2m$. The results for the 1D (3D) case are also discussed in [15] ([23]).

3.1. One dimension

A 1D trapped gas can be experimentally realized by tightly confining two directions in the harmonic trap. We therefore take $\omega_1 = \omega_2 \equiv \omega_\perp$ to be much larger than ω_3. In the limit $a \ll a_\perp$, where $a_\perp = \sqrt{\hbar^2/m\omega_\perp}$ and a is the 3D s-wave scattering length, one has for the effective 1D coupling constant that $U_{\rm 1D} = 2\hbar^2a/ma_\perp^2$ [28]. It is also customary to introduce the dimensionless Yang parameter $\gamma = mU_{\rm 1D}/\hbar^2 n$.

In figure 1, we show the results that follow from equation (7) by taking $d = 1$ and $U = U_{\rm 1D}$. From this plot it is seen that the spin-drag relaxation rate vanishes linearly with temperature. It can be shown [15] from equation (7) that

$$\frac{1}{\tau_{\rm sd}(T)} \overset{T\to 0}{\to} \left[\frac{8}{9\pi}\gamma^2\frac{k_{\rm B}T}{2\varepsilon_{\rm F}} + \frac{8}{3\pi}\gamma^2\left(\frac{k_{\rm B}T}{2\varepsilon_{\rm F}}\right)^2\right]\frac{\varepsilon_{\rm F}}{\hbar} \quad \text{(one dimension)}, \tag{8}$$

in agreement with the numerical results for $T/T_{\rm F} \ll 1$ (see the inset of figure 1). This leading order behaviour is in agreement with results from bosonization [29]. A quantitative comparison

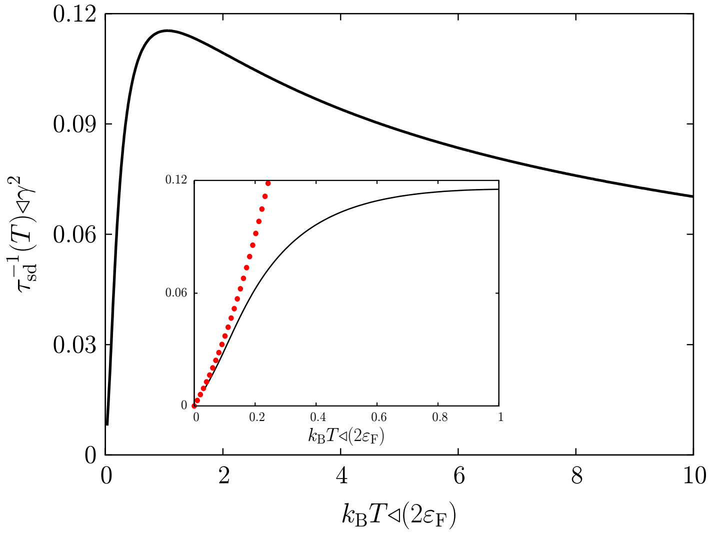

Figure 1. Spin-drag relaxation rate $\tau_{\rm sd}^{-1}$ (in units of $\varepsilon_{\rm F}/\hbar$) as a function of the temperature for a one-dimensional (1D) Fermi gas. The inset shows a zoom of the low temperature region $0 \leqslant k_{\rm B}T/2\varepsilon_{\rm F} \leqslant 1$, with the filled circles representing the analytical result in equation (8).

is not possible as the results presented in [29] have an undetermined numerical prefactor. The linear-in-T term in square brackets in equation (8) originates from contributions to the spin-drag relaxation rate that are controlled by momenta q of the order of $2k_{\rm F}$, while the quadratic-in-T term comes from momenta q near 0. Calculations beyond second-order perturbation theory have been carried out by Pustilnik *et al* [30] in the context of Coulomb drag between quantum wires: these authors have considered only contributions to the drag transresistance $\rho_{\rm D}$ coming from momenta q near 0 and found $\rho_{\rm D} \propto T^2$ for $T \to 0$. Neglecting the $2k_{\rm F}$ contributions to $\rho_{\rm D}$ is fully justified in their case since the inter-wire Coulomb interaction at wave vectors of the order of $2k_{\rm F}$ is suppressed by the exponential factor $\exp(-2k_{\rm F}\ell)$, where ℓ is the inter-wire distance.

3.2. Two and three dimensions

In two and three dimensions, the spin one-half Fermi gas is predicted to undergo a ferromagnetic phase transition [18]–[22], which motivated the experiments by Jo *et al* [24]. Assuming a second-order phase transition (note, however, that the transition is predicted to become first order at very low temperatures [22]), the transition is signalled by a diverging spin susceptibility $\chi_{S_zS_z}(q,\omega)$ at zero wave vector ($q=0$) and frequency ($\omega=0$). Within Stoner mean-field theory, this spin susceptibility is calculated by summing all random-phase approximation (RPA) bubble diagrams, which yields $\chi_{S_zS_z}(q,\omega) = \chi_0(q,\omega)/[1+U\chi_0(q,\omega)/2]$. Hence, the critical temperature, both in two and three dimensions, is determined by the condition $1+U\chi_0(0,0)/2=0$. This equation gives, together with the equation $n=\int {\rm d}^d\boldsymbol{q}\, N_{\rm F}(\epsilon_{\boldsymbol{q}})/(2\pi)^d$ that determines the chemical potential, the critical temperature $T_{\rm c}$ as a function of U. In two dimensions, this yields

$$\frac{T_{\rm F}}{T_{\rm c}} + \log\left(1-{\rm e}^{-T_{\rm F}/T_{\rm c}}\right) = -\log\left[\frac{U\nu(\epsilon_{\rm F})}{2} - 1\right], \tag{9}$$

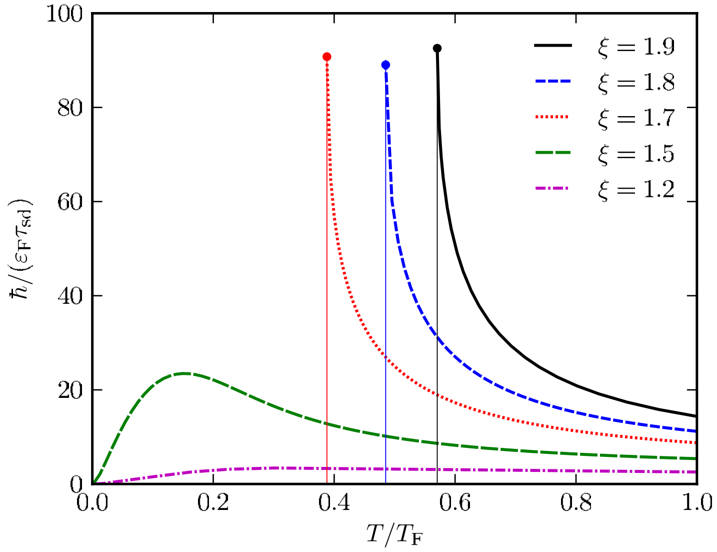

Figure 2. Spin-drag relaxation rate in two dimensions as a function of temperature, for various strengths of the interactions determined by $\xi \equiv \pi a/a_z$. The vertical lines indicate the critical temperature for the ferromagnetic phase transition.

with $\nu(\epsilon_F)$ being the total density of states at the Fermi level. Note that only when $U\nu(\epsilon_F)/2 > 1$, the Stoner criterion, there exists a ferromagnetic phase transition.

Experimentally, the 2D situation can be achieved by tightly confining the system in one direction by making one (say ω_z) of the three trapping frequencies much larger than the other two. The effective 2D interaction is then determined by $U_{2D} = 4\pi a\hbar^2/ma_z$, where $a_z = \sqrt{\hbar/m\omega_z}$. This result for the effective interaction, and its counterpart for the 1D case, are obtained by assuming that the atoms occupy the lowest-lying mode of the trap in the tightly confined direction that is well described by a Gaussian wave function of width a_z.

To account for the effect of ferromagnetic fluctuations, we evaluate the effective scattering amplitude between atoms by summing all RPA bubble diagrams. This gives

$$A_{\uparrow\downarrow}(q,\omega) = U_{2D} + \frac{U_{2D}^2}{4}\chi_{nn}(q,\omega) - \frac{3U_{2D}^2}{4}\chi_{S_zS_z}(q,\omega), \tag{10}$$

with $\chi_{nn}(q,\omega) = \chi_0(q,\omega)/[1 - U\chi_0(q,\omega)/2]$ being the RPA density response function. In what follows, we numerically evaluate the result in equation (7) with the above effective interaction, i.e. after making the replacement $U \to |A_{\uparrow\downarrow}(q,\omega)|$ in equation (7). In figure 2, we show the results for the spin-drag relaxation rate in two dimensions, as a function of temperature and for various values of the dimensionless parameter $\xi = \pi a/a_z$. Clearly, for sufficiently strong interactions, i.e. sufficiently large ξ, this rate is enhanced upon approaching the ferromagnetic phase transition, as discussed in the introduction. For interactions that do not fulfil the Stoner criterion, the spin-drag relaxation rate vanishes quadratically with temperature, as expected for a Fermi liquid. We note that, in two dimensions, there is a logarithmic correction to this quadratic temperature dependence [31], although this is hard to discern from the numerical results in figure 2. This logarithmic correction is due to the fact that the imaginary part of the Lindhard

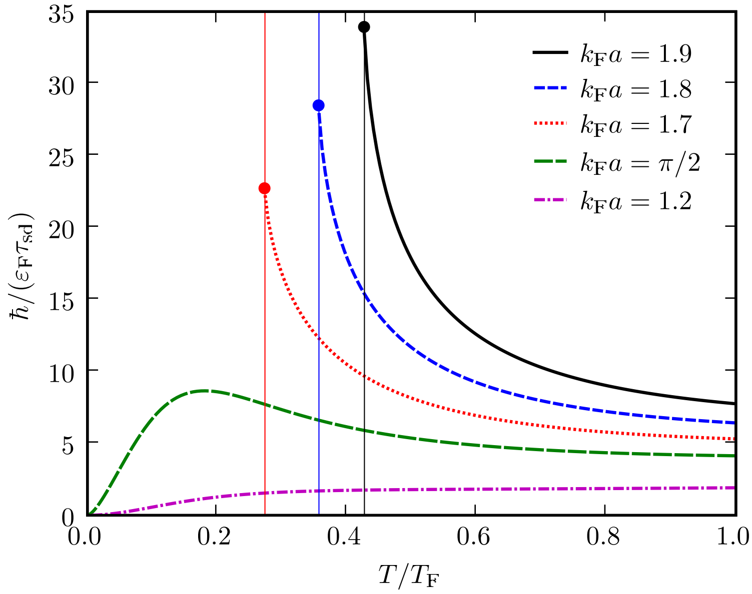

Figure 3. The spin-drag relaxation rate of a 3D Fermi gas as a function of temperature T for various values of the interaction parameter $k_F a$. The vertical lines indicate the critical temperature for the ferromagnetic phase transition.

function in two dimensions at zero temperature behaves as

$$\Im m\ \chi_0(q,\omega) \simeq -\frac{m\omega}{\hbar q k_F}\frac{\nu(\varepsilon_F)}{\sqrt{1-(q/2k_F)^2}}, \tag{11}$$

for $\omega/q \to 0$ and for $q_{\rm min} < q < q_{\rm max}$. Here, $q_{\rm min} = k_F(1-\sqrt{1-2m\omega/\hbar k_F^2})$ and $q_{\rm max} = k_F(1+\sqrt{1-2m\omega/\hbar k_F^2})$ [31]. Using this result in equation (7), and ignoring the effects of fluctuations on the effective interaction that are not important for the low-temperature behaviour if the Stoner criterion is not fulfilled, ultimately gives

$$\frac{1}{\tau_{sd}(T)} \stackrel{T\to 0}{\to} \left[-\frac{4}{3\pi^3}\xi^2\left(\frac{T}{T_F}\right)^2 \log\left(\frac{T}{T_F}\right)\right]\frac{\epsilon_F}{\hbar}. \quad \text{(two dimensions)} \tag{12}$$

The 3D results are obtained by replacing U_{2D} with the 3D two-body T-matrix $4\pi a\hbar^2/m$ [23]. The results for the spin-drag relaxation rate are shown in figure 3 and are qualitatively similar to the 2D case. For weak interactions, such that there is no ferromagnetic phase transition, the spin-drag relaxation rate vanishes quadratically with temperature. This quadratic temperature dependence is understood by using that in three dimensions and at zero temperature we have

$$\Im m\, \chi_0(q,\omega) \simeq -\frac{\pi\nu(\varepsilon_F)m\omega}{2\hbar k_F q}, \tag{13}$$

for $q < 2k_F$ and $\omega/q \to 0$. This yields for the spin-drag relaxation rate

$$\frac{1}{\tau_{sd}(T)} \stackrel{T\to 0}{\to} \left[\frac{32\pi}{9}(k_F a)^2\left(\frac{T}{T_F}\right)^2\right]\frac{\epsilon_F}{\hbar} \quad \text{(three dimensions)}. \tag{14}$$

4. Discussion and conclusions

We have presented the results for the spin-drag relaxation rate for a 1D, 2D and 3D two-component Fermi gas of ultracold atoms. In two and three dimensions and for sufficiently strong interactions, such a system may undergo a ferromagnetic phase transition. The spin-drag relaxation rate is strongly enhanced as this transition is approached from above, which could be observed experimentally as an increased damping of the spin-dipole mode. This enhancement is determined by including all bubble-diagram contributions to the effective interaction between different spin components of the gas. This essentially treats the ferromagnetic phase transition within Stoner mean-field theory. In three dimensions, this is most likely qualitatively correct, although the transition occurs at strong coupling. In two dimensions, the mean-field results for the critical temperature are an upper bound. This is because in this case, and in particular in the experimentally relevant case that an external field is present to trap two low-field seeking hyperfine species, the phase transition is of the Kosterlitz–Thouless type. It is known that the Stoner mean-field theory overestimates the Kosterlitz–Thouless transition temperature.

In one dimension, the spin-drag relaxation rate vanishes linearly with temperature. In principle, we could also have included an effective interaction that included all bubble-like diagrams in the 1D. This would result in an enhancement, upon approaching some temperature $T_{\rm SDW}$, of the spin-drag relaxation rate due to the divergence of the spin-density response function at zero frequency and $q = 2k_{\rm F}$ that signals the onset of spin-density wave antiferromagnetism. This mean-field treatment, however, is not accurate in one dimension, not even qualitatively. Instead, it is known from bosonization theory that the linear dependence at small temperatures is the correct one. Since this behaviour is reproduced by our expression in equation (7) without including additional contributions to the effective interactions, we do not include such fluctuation corrections in one dimension. In principle, it would be interesting to explore further the connection between bosonization and our Boltzmann methods; this, however, is beyond the scope of the present paper.

In future work, we will investigate the behaviour of the spin drag in the spontaneously spin-polarized phase, i.e. for temperatures $T < T_{\rm c}$. Further studies will also investigate the role of critical fluctuations close to the critical temperature, and the situation of negative scattering length.

Acknowledgments

This work was supported by the Stichting voor Fundamenteel Onderzoek der Materie (FOM), Netherlands Organisation for Scientific Research (NWO) and European Research Council (ERC). GV acknowledges support from NSF grant no. 0705460. MP acknowledges very useful conversations with Rosario Fazio and Andrea Tomadin.

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