Willebrord Snellius (1580–1626)
a Humanist Reshaping the Mathematical Sciences

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a Humanist Reshaping the Mathematical Sciences

Willebrord Snellius (1580–1626)
een humanist hervormt de wiskundige wetenschappen
(met een samenvatting in het Nederlands)

Proefschrift

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Preface

Although the choice for one of the most eminent Dutch early-modern mathematicians as the topic for my thesis seemed to be a natural one after my master's degrees in mathematics and classics, the right approach, relevant questions and most telling sources turned out to be rather difficult to identify. The aim of this book is to strike a new balance between an internalist, technical approach of history of mathematics, and an externalist approach that focuses on the cultural and institutional context, in one specific case. It tries to explain early modern mathematics from within the questions, motivations, problems and scientific excitement of its own period.

I am very grateful to a large number of people who contributed in some way to the shaping of this book. In the first place, I want to thank my supervisor prof. dr. Henk Bos for teaching me to become a researcher, through sharing some of his enormous knowledge with me in many long discussions, in which he showed me the importance of questions which other people had ignored. Moreover, he gave me large amounts of comments on my written work, on all aspects, from the general structure to grammatical details. I also thank him for giving me the freedom to explore a side of the history of mathematics which was less familiar to himself.

When my contract in Utrecht was finished, prof. dr. Jan Hogendijk reconnected me to the Leiden Mathematical Institute. Moreover, his knowledge of the classical tradition and his genuine interest in my work have been a great help in the later phases of my research. I am very grateful to my closest academic sibling, Steven Wepster, for large amounts of comments on my chapters, discussions of the life of PhD-students, computer advice, help in the design of the cover of this thesis and strong tea, all equally sharp and agreeable. I thank prof. dr. Klaas van Berkel for suggesting the topic of this thesis and for regularly stressing the importance of studying the historical embedding of mathematics. I thank dr. Helen Hattab and dr. Kim Plofker for correcting the English of parts of this work.

I have much enjoyed and benefited from the graduate seminars and meetings of the Huizinga Instituut (the research school for cultural history in the

Netherlands) and the lively discussions in graduate student gatherings in various fields, in particular the GWAD (a group of young historians of mathematics and astronomy), and the discussion societies on humanism and cultural history. Of the cultural historians, I want to mention in particular Juliette Groenland, whom I thank for her combination of genuine interest, sharp wit and friendship. I am very happy that Juliette and Steven are my *paranimfen*, as they will thus represent the unity of arts and sciences in my work at my thesis defense.

I also thank other colleagues in the Netherlands and abroad for their various contributions, among whom dr. Kirsti Andersen, dr. Danny Beckers, dr. Jeroen Blaak, dr. Rob van Gent, dr. Michiel van Groesen, prof. dr. Chris Heesakkers, prof. dr. Kees Meerhoff, dr. Henrik Kragh Sørensen, Sébastien Maronne, Wijnand Rekers, prof. dr. Volker Remmert, dr. Rienk Vermij, and dr. Jan Waszink, Sr.

Further, several institutes have enabled me to start and finish my work, for which I thank them. The Mathematical Institute in Utrecht provided a good working environment and allowed me to continue my work after my appointment was finished. The Mathematical Institute in Leiden also offered hospitality. In the Institute of Medical Statistics and Bioinformatics in Leiden (LUMC), I have found an interesting field and pleasant colleagues, who gave me the freedom to finish my thesis. Moreover, I thoroughly enjoyed my enriching stay in the KNIR (Royal Dutch Institute) in Rome, which the Dr Ted Meijer Stipend offered to me. The mini-sabbaticals which I spent in the Classics Department in Nijmegen in 2006 helped me to make considerable progress. The staff of many libraries and archives have assisted me, of which I want to thank in particular the helpful staff of the Rare Books Collection of Leiden University Library.

Finally, but especially, I want to thank my family: my parents for teaching me the importance of books and perseverance and for their support in more recent times, in particular by taking care of Hanneke, and my father by translating large sections of Swedish scholarship; my husband Jan Waszink for his unconditional and constant moral, intellectual and practical support and encouragement, and for giving his views on the translations of the Latin quotations; and my daughter Hanneke for her joyful presence and her earnest endeavours to grasp the meaning of a *promotie.*

Chapter 1

General introduction

> We should not lose courage when we encounter topics that we do not understand immediately. Geometry cannot be as easily understood as a story.[1]

1.1 Images

What do we see if we consider Willebrord Snellius, professor of the mathematical sciences in Leiden in the first quarter of the seventeenth century, as a living person? Although our view is clouded, some images can be discerned.

On a day in 1606, we see Snellius, somewhat pensive, walking home through the centre of Leiden after a day of hard work. In the morning, he had discussed a geometrical problem with his former mathematics teacher Ludolph van Ceulen, and together they had tried to find a more elegant solution than the existing ones. Van Ceulen had proposed to make the problem easier by assigning numerical values to the sides of the triangle under discussion and then experimenting with the 'rule of cos', which involved calculating the value of unknowns. Once again Snellius had lost track when Van Ceulen started to manipulate the numbers and symbols very rapidly, and he was somewhat suspicious whether this method could really yield a better solution than the traditional one with ruler and compass. In the afternoon, Snellius had gone to the house of Josephus Justus Scaliger, the eminent philologist whose private tuition he had enjoyed as a student and who still supervised his studies. They had looked at some textual difficulties in the Greek manuscript of Pappus's *Collection* of which Scaliger was

[1]'Men moet ook de moet niet verloren geven, indien ons saaken voorkomen, die men terstont niet verstaat. De Geometrie is so ligt niet te begrypen als een Historie.' [Pardies, 1690, First page *Instructie des Autheurs*].

the proud owner. At the end of their session, Scaliger had suggested that Snellius make a more profound study of the mathematical contents of the manuscript. He had singled Snellius out for praise declaring that, among all of Scaliger's brilliant students, he was the obvious person for this noble task. Now Snellius is pondering whether he should spend more time trying to master Van Ceulen's methods, or instead show Scaliger that he is indeed able to cope with his assignment.

In December 1610, we are witness to a joyful family scene. Willebrord Snellius and his wife Maria are sitting near the fireplace, together with Willebrord's father Rudolph and his mother Machtelt, who live in the same house. Maria is recovering from the birth of their first daughter. The adults all look tenderly at the toddler Jacob, who is stroking the baby as carefully as he can. Willebrord tells his family that he has just received a letter from their much appreciated and respected relative Amelis van Rosendael, in which he accepted Snellius's invitation to be present at the baptism of the newborn. Rudolph remarks that when Amelis visits them, they should not forget to show him their telescope, because he is always so keen to hear about the latest scientific developments. They discuss whether they should have a telescope made for their relative as well, or whether they had better refrain from such an expensive present, because Amelis could feel too much pressured to do something in return.

In the summer of 1615, we see Snellius on his way to Mechlin with the Barons Sterrenberg, two of his students, and their tutor. Some servants, who are in charge of Snellius's enormous astronomical quadrant, lag behind. Snellius's feelings are mixed. Their long travels through the Northern and Southern Netherlands have exhausted him, and he is worried about the hazards of travelling in this region which could still be considered as the domain of the Spanish enemy, and about the damage which his precious instrument could incur. Yet he enjoys the enthusiastic assistance of the brothers Sterrenberg. Although they sometimes complain about their strains, they are very helpful in taking measurements. Snellius, who is used to students who follow his courses without genuine interest, because they do not see their relevance either to current pleasures or to a future career, is happy to supervise the brothers and impart some of his knowledge to them. He looks forward to educating his son in the same way. Moreover, he is determined to bring his project to a good conclusion, ambitious to improve earlier attempts to calculate the circumference of the earth and confident that his method will actually yield a better result.

Although some elements of these scenes, in particular Snellius's emotions, originate from my imagination, most of these reconstructions is firmly source-based. It is the aim of this thesis to construct a more complete, accurate and enlightening picture of Snellius than has henceforth been available, with a number of new insights into his life and work and his position within and outside mathematics in his own time. The purpose and plan of the book will be further explained in the final part of this general introduction. Before this, a section will be devoted to a brief survey of Snellius's life and work, and another to previous studies on Snellius.

1.2 The star of the story

Willebrord Snellius lived from 1580 to 1626. After extended educational travels, he became first a teacher and then a full professor of mathematics at the University of Leiden, which was becoming one of the major centres of scholarship in the world in this period. The university was a stronghold of humanist learning, which was, in a nutshell, the dominant form of European scholarship between c. 1470 and 1640. In humanist learning the texts of classical authors formed the basis and point of departure for new works.

Snellius's father Rudolph (1546–1613) had been the first professor of mathematics in Leiden, but his knowledge of mathematics was not very deep. Willebrord, by contrast, wrote, translated and edited books that covered almost the whole field of the mathematical sciences. It must be noted that the Latin words *scientiae mathematicae* ('mathematical sciences') or the equivalent term in various languages indicated a larger field in the seventeenth century than 'mathematics' does today. The mathematical sciences consisted of pure mathematics (the abstract sciences of magnitude and number, geometry and arithmetic respectively) and 'mixed mathematics' (disciplines in which objects from reality were measured or counted, e.g. astronomy, optics, navigation and surveying), encompassing an ever-growing number of subfields in Snellius's time. Mixed mathematics is *not* a synonym for practical mathematics, nor for applied mathematics (a term which suggests a difference between 'real' mathematics and its derivatives outside mathematics, a difference which would have been conceptually inconceivable in Snellius's time). If confusion about the modern and old meaning of 'mathematics' is possible and if the distinction is relevant, I use the term 'mathematical sciences' when referring to the broad seventeenth-century meaning.[2]

All Snellius's books except one were written in Latin. This exception was a Dutch translation of Petrus Ramus's *Geometria*, of which Snellius was the initiator and supervisor. His Latin works were mainly meant for a university-educated audience. The achievement that makes Snellius's name still sound familiar to many nowadays, the discovery of the Law of Refraction of light, was made public posthumously. Snellius was not only a prolific writer, but also active in teaching (which was in fact the core of his work as a professor) and as a scientific consultant, giving expert advice to, among others, colleagues in Leiden and abroad and to the government of the Dutch Republic. In his own day,

[2][Imhausen and Remmert, 2006, p. 73]; see for a contemporary explanation e.g. Martinus Hortensius, quoted in id., p. 92. Cp. [Mulder, 1990].

Snellius was greatly respected and admired as a scholar, partly for work that later received little attention, such as his edition of astronomical observations. His position among contemporary mathematicians is indicated by the great Kepler's charming description of him as 'the ornament of the geometers of our age.'[3]

1.3 Earlier work on Snellius

Modern research on Snellius started in the nineteenth century, when he caught the attention of several Dutch historians of science. Their publications were primarily intended to show the richness of the largely unknown material to those interested in the mathematical past in their country. The main authors of this group are P. van Geer and D. Bierens de Haan.[4] Somewhat later, the Law of Refraction attracted most of the scholarly attention. D.J. Korteweg and C. de Waard discovered and discussed some sources.[5] Another source relevant for Snellius's optics, his annotations to Risnerus's *Optica*, was studied and partly edited by J.A. Vollgraff, who also published two of Snellius's letters.[6] C. de Waard's entry from the *Nieuw Nederlandsch Biografisch Woordenboek* remains a very good survey of Snellius-related sources and the older literature.[7] Johannes Tropfke and Rudolf Wolf gave good overviews of Snellius's contributions to different parts of mathematics.[8] All this literature, which testifies to the thorough scholarship of its authors, is still indispensable for a modern Snellius-scholar, yet its general focus on those parts of Snellius's works that were relevant for the later development of mathematics and its lack of concern for non-scientific influences make most of it outdated.

During the twentieth century, other valuable studies on Snellius were published. Henri Bosmans and N.D. Haasbroek did substantial research on the *Eratosthenes Batavus*, Snellius's work on the measurement of the earth. Haasbroek made many calculations to analyse the accuracy of Snellius's work. Snellius's editions of Ramus's work were discussed in the very thorough study of J.J. Verdonk on Ramus's mathematics, and D.J. Struik wrote an article on him for the *Dictionary of Scientific Biography*.[9] Recently, the Law of Refraction has received fresh assessments by F. Hallyn and Klaus Hentschel.[10]

In recent decades, more attention has been paid to Snellius in different

[3]'Geometrarum nostri saeculi decus', [Kepler, 1960, p. 71].

[4][van Geer, 1883] and [van Geer, 1884b]; [Bierens de Haan, 1878b] and [Bierens de Haan, 1878a].

[5][Korteweg, 1898]; [de Waard, 1935].

[6][Vollgraff, 1918], [Vollgraff, 1913b], [Vollgraff, 1936] and [Vollgraff, 1914].

[7][de Waard, 1927b].

[8][Tropfke, 1940], [Tropfke, 1923]; esp. [Wolf, 1973a] and [Wolf, 1973b].

[9][Bosmans, 1900]; [Haasbroek, 1968]; [Verdonk, 1966]; [Struik, 1975].

[10][Hallyn, 1994]; [Hentschel, 2001]. This latter article includes a German translation of the manuscript that contains the Law of Refraction.

seventeenth-century contexts. Klaas van Berkel has written about the relationship between the Snellii and Leiden University and formulated a plausible hypothesis about the connections between Rudolph Snellius's Ramism, the engineering school in Leiden and Rudolph's appointment as a professor.[11] Both Van Berkel and Rienk Vermij have seen Snellius as a representative of humanist mathematics.[12] Their insights into the connections between the Snellii and the world of scholarship around them are valuable, yet they left much work to be done, because they did not give more than a few illustrations from the works of the Snellii to substantiate their statements.

Some parts of Snellius's work have been discussed in books with a wider focus: Henk Bos has studied several of Snellius's contributions to the tradition of geometrical problem solving, Rienk Vermij has investigated to what extent Snellius was a Copernican and Tabitta van Nouhuys has related Snellius's and others' work on comets to the decline of the Aristotelian worldview.[13] I know of two cases in which (rather) recent mathematicians found inspiration in Snellius's work for their own mathematics: A. Haerpfer elaborated on the resection problem from *Eratosthenes Batavus*, and Frits Beukers and Weia Reinboud further developed Snellius's work on the quadrature of the circle.[14]

Although Snellius is one of the better-known Dutch mathematicians from the past, only the relatively few in-depth studies mentioned above have been devoted to him. Many aspects of his work have not been researched at all, and almost none of his own texts have been edited or translated into a modern language. No book or large article has attempted to analyse the relations between all of Snellius's activities, and to place his mathematics in both its mathematical and extra-mathematical contexts, as will be done here.

1.4 Text in context: goals and methods

1.4.1 Sources, questions and themes

In this book, I analyse Snellius as a mathematician, which means that I will describe the content of part of his mathematical work and investigate his views on mathematics and alongside the relationship between his work, his professional activities and the outside world. I focus on what Snellius himself deemed important and on his position in his own time, paying less attention to what later turned out to have been his contributions to the development of mathematics.

[11] E.g. [van Berkel, 1985b], [van Berkel, 1998], [van Berkel, 1988a].

[12] [van Berkel, 1998, p. 45]; [Vermij, 1996], [Vermij, 2002, pp. 21–22]: 'The happy marriage between the world of mathematics and that of classical philology is nowhere better illustrated than by the career of Rudolf Snellius' son and successor, Willebrord Snel van Royen.'

[13] [Bos, 2001a]; [Vermij, 2002]; [van Nouhuys, 1998].

[14] [Haerpfer, 1910], [Beukers and Reinboud, 2002].

My main source material consists of Snellius's published works, all of which except one appeared during his lifetime. Naturally, his own presentation of his mathematical results to his readership gives the best information about his scientific ideas and the form in which they were intended to reach his contemporaries. Other sources which have been used for this thesis are printed works of others and manuscript material like letters and archival sources. Most of the sources that I have consulted have scarcely been studied, and the existence of some archival material, notably a number of letters written by and sent to father and son Snellius from libraries in Marburg and Kassel, was unknown.

In this book, the stories told by these sources are embedded in several contexts. Without choosing a position in the debates within the history of science community about the relative merits of the internalist and the externalist tradition, I want to stress the necessity of the study of both content and context for a good understanding of Snellius's mathematics. A study of Snellius cannot succeed without proper attention to the content of his works, in which he purposefully reacted to and contributed to an old mathematical tradition. Yet several non-mathematical factors were so influential that these works cannot be properly assessed without taking these factors into account. My choice of contexts has been led by their explanatory value on the one hand, and on the other hand by the availability of sources. Contexts that I found significant in most cases were fashionable topics and discussions in mathematics, Snellius's intellectual network, the Leiden academic environment, Ramism and the role of mathematics in society—though they did not equally matter in all cases. All these different contexts made an interdisciplinary approach to the topic of this book necessary. Less significant are economical and political factors. Examples of contexts that had to be largely ignored due to lack of sources are Snellius's teaching and his faith.

Given the number of books written or edited by Snellius, their difficulty,[15] and the scarcity of literature on them, it was not possible to analyse all of his works and all relevant connections between them and various contexts in this thesis. Therefore, the thesis is organized as follows: the first part is a biography of Snellius, in which attention is also paid to most of the relevant contexts; the second part consists of case studies. Before giving a more thorough motivation for this approach and more information on the content of the chapters, some general themes that play a role in a large part of the book will briefly be introduced.

[15]Cp. [van Geer, 1884b, pp. 4–5]: 'Is aldus het leven van Willebrord Snellius in korte trekken weer te geven, minder is dit het geval met zijne werken. Wel zijn deze niet vele, doch zij bevatten onderzoekingen en leerstellingen, die eene blijvende plaats in de wis- en natuurkundige wetenschappen hebben verkregen. Daar zijne werken alle in het Latijn zijn geschreven en de klassieke vorm streng wordt gehandhaafd, zijn aan de beoefening daarvan voor den tegenwoordigen tijd eigenaardige moeilijkheden verbonden.'

The first theme is the absence of a role model for Snellius. Although it is clear what 'mathematics' meant in his time, it is difficult to define 'mathematicians'.[16] This term referred to members of a heterogeneous group of individuals, such as professors in universities, teachers, *Rechenmeister* (teachers of calculation), engineers and court astronomers. This diversity of the group of practitioners was a common phenomenon in all sciences:

> Before the development of societies and institutions devoted exclusively to scientific pursuits in Europe from the 1660s onwards, and the subsequent emergence of codified and tacit forms of professional ethics specific to such institutions, natural philosophers and mathematicians attempting to make novel claims about the natural world were obliged to look outside science for models of acceptable conduct in the prosecution and presentation of their work. Rather than being obliged to acquiesce into a single model of personhood, scientific practitioners were free to make their own creative synthesis from a smørgasbord of religious and courtly models, to name just two of the more obvious options.[17]

Thus, a professional group of mathematicians did not yet exist, which means that Snellius could not merely adjust to the norms of a peer group, but had to make conscious choices in deciding what it would mean to be a professor of mathematics for him. He could be described as a 'research mathematician', because part of his work consisted of creating new mathematics. This word has to be used with caution for persons of his period, because it does not have the usual connotation of a mathematician paid to do research in an institution specially designed for that purpose, such as the nineteenth-century university. Still, it is a useful term to designate the rare species of mathematicians interested in and working on new mathematics, both for its own sake and for its applications.[18]

The second theme is Snellius's motivation. If we want to learn more about Snellius's role in his own time, instead of focusing on his contributions to later science, it is important to try to understand what drove him, what kind of mathematics excited him, why he studied certain mathematical problems in an uncommon way, which audiences he wanted to reach and which purposes he had in mind. Some explanation is needed for Snellius's choice to pursue the study of mathematics with so much zeal, because other career possibilities, for instance becoming a lawyer, offered easier paths to success. Although the sources are

[16]Cp. [Andersen and Bos, 2006, pp. 697–702].

[17][Gorman, 2003, p. 2]. Note that in the Dutch situation, the church and the court were less obvious producers of role models than the regents' class and the universities.

[18]Cp. [Andersen and Bos, 2006, p. 722]: 'As a historical category, pure mathematics in the early modern period is primarily defined by its subject matter, not by the social cohesion of the group who taught or developed it. Pure mathematics did not constitute a profession, and few if any scholars restricted themselves to it.'

rather taciturn on this point, the tentative answers to these questions help in understanding Snellius's significance.

In the third place, Snellius's method for achieving his results will receive ample attention. A humanist approach is at the centre of it, and this is combined with several other ways of acquiring knowledge. This method is characteristic for Snellius and helped him reach his achievements.

The fourth theme is Snellius's preference for exactness, most visible in his pure mathematics, but also in his mixed mathematics. He strove for well-founded knowledge, precisely-defined concepts, statements based on what he called 'reason and observation', preferring these to speculation or the exploration of new tools.

1.4.2 Outline of the book

The core of this book consists of a biographical part (chapter two), two chapters with case studies on mixed mathematics (chapters three and four), an introductory chapter on geometry including two smaller cases (chapter five), two chapters on geometrical cases (chapters six and seven) and a chapter in which the characteristics of Snellius's mathematics are analysed, followed by a general conclusion (chapter eight). Neither in the biographical part, nor in the first two case studies, is mathematics discussed in great technical detail in more than a few isolated places. This is different in the three geometrical chapters. I have striven to explain all concepts and background needed for a good understanding of my points in all chapters, taking into account the different backgrounds of potential readers, which could include historians of mathematics, of science, and of intellectual history as well as mathematicians. For those readers who prefer to skip the technical details, the core of my arguments should still be accessible.

A **biography** is the most natural form to present a survey of the life and work of a person and his or her connections to several contexts. Such a survey did not previously exist for Snellius, because my thesis is the first substantial volume that is completely devoted to him. This part also serves to give an overview of the relevant sources and secondary literature. My aim is to present a solid base for the rest of the book. Therefore, it includes many different story lines and pieces of evidence, most of which fit into the larger picture of Snellius's life. Yet some fragments may seem to be isolated. I have presented them anyway, not only because a life is not a novel and therefore some parts of it can be disconnected from the rest, but also because fragments which may seem to be without further meaning now could acquire meaning when more sources are found or studied, or when they are used by researchers with different questions than my own. The case studies do not contain similar 'loose ends'; each of them presents one story, in which my own taste, selection and interpretation play a larger role than in the biography.

The biographical part contains discussions of hitherto (almost) neglected sources (e.g., the Apollonius reconstructions, Snellius's correspondence, his auction catalogue), some corrections of earlier work (e.g., the number of his children was seven—or some more—rather than 18, Snellius was not to blame for the sloppy printing of the *Fundamenta*, the extant quadrant is not the one described in *Eratosthenes Batavus*), some shadings of earlier viewpoints (e.g., about Rudolph Snellius's position in Leiden, and the idea that *Eratosthenes Batavus* was Snellius's principal work) and many additions to what was known previously. All conclusions are firmly based on sources; if they are merely interpretations, this is indicated explicitly. The interpretation of the meaning of the events of his life and of his work is my own, and hence my story as a whole is new.

The term 'biography' may give rise to some expectations that cannot be satisfied. Information about Snellius's character and private life is almost completely absent. The sources are biased towards his contacts with highly educated adult males, although his daily life must have been coloured by his contacts with less privileged groups as well. Snellius is no exception in being unfit as a leading figure for a standard biography. All potential biographies of early modern persons are hampered by the problem of the sources: not only have many of them disappeared in the long period that has elapsed since, but the surviving sources are also often less telling than the modern researcher would desire. The reason for this relative lack of information about the inner life in older sources is that only since Romanticism has the individual been perceived as unique, which has led to a tendency to reflect on the inner part of oneself and other individuals. For instance, when Barlaeus wrote about his grief caused by the death of Snellius, he may sincerely have felt sad. Yet the language in which he expressed his loss is so artificial and full of *topoi* or rhetorical figures, that the modern reader who is not trained in unravelling these expressions finds it difficult to ascertain what Barlaeus really felt when he wrote: 'Which Hercules will succeed this Atlas?'[19] (see p. 109).

The solution to this potential problem of unsatisfied expectations is to call this part of the thesis an *intellectual* biography. This name expresses the emphasis on the scholarly side of Snellius's pursuits—this restriction is not problematic, because the primary reason why we are interested in Snellius is his mathematics and not his private life. And after all, humanist scholars themselves felt that the achievements of the mind are central to a human being.[20]

[19]'Quis Atlanti isti succedet Hercules?', [Barlaeus, 1667, p. 173].

[20]Cp. Juliette Groenland in her explanation of why an intellectual biography is suitable for the analysis of the humanist pedagogue Joannes Murmellius: 'De gedachte dat het geestesleven, en niet het gevoelsleven, het belangrijkste en beste deel van een persoon vertegenwoordigt is bij uitstek humanistisch. Een intellectuele biografie is derhalve het geschikte genre bij uitstek om een humanist te portretteren.' [Groenland, 2006, p. 17].

Besides presenting Snellius's life and work, the biography also discusses his scholarly contacts and the persons and environments that influenced him. Special attention is paid to the impact of Willebrord's father Rudolph, who stimulated the interest of his son in Ramus and by his scholarly network facilitated his entrance into the world of learning. As Rudolph Snellius was no specialized mathematician himself, his son did not just copy his example, but incorporated Rudolph's influence in his own work alongside other influences. Some other persons who contributed to Snellius's development were Scaliger, the symbol of the Leiden humanist-philological tradition, the virtuoso mathematician-engineers Van Ceulen and Stevin, Kepler (another scientist with humanist interests) and Snellius's prominent relative Rosendalius. The currents of Ramism and (mathematical) humanism will also be discussed, and the place of mathematics at Leiden University and elsewhere in the Dutch Republic will be reviewed.

In the **case studies** (chapters three to seven), depth instead of breadth is aimed for. The first two discuss topics from **mixed mathematics**. In the first of these (chapter three), Snellius's determination of the circumference of the earth as described in *Eratosthenes Batavus* is analysed. Although Haasbroek devoted some accurate and detailed studies to this project, several important issues have not been considered by him or others. My aim is to give a clear overview of all steps of Snellius's method, in their correct chronology, with particular attention to his motivation and his endeavours to inform his readers about the exact length of the unit of measure that he had used. Snellius's description of his effort to determine the relative density of water shows him as a practical and ingenious scientist-engineer, a surprising, hitherto unknown side of him.

In the second mixed mathematics case (chapter four), Snellius's astronomical treatises *Observationes Hassiacae* and *Descriptio Cometae* are central. They were both dedicated to Landgrave Maurice of Hessen, the son of William of Hessen, who had strong astronomical interests himself and was praised for this by Ramus. Maurice felt much sympathy for Rudolph Snellius, and he became a patron of Willebrord through this family connection. The connection between both Snellii and Maurice, and the relation of this to Snellius's astronomical work are discussed in this chapter, largely on the basis of hitherto (virtually) unknown letters, of which large parts are published here for the first time.

The next three chapters deal with topics from **geometry**. It has to be noted that geometry was still very 'traditional' in this period: in general, new results were solutions to problems found by old methods, rather than new methods or the exploration of new kinds of questions. Almost all these old methods were found in works from classical Greek authors, especially Euclid. They gave rules for the manipulation of points, lines, surfaces and solids with ruler and compass, without attaching numerical values to line segments, surfaces and volumes; however, this last feature was subject to change in the early modern period.

The most important source of inspiration for the geometry part of this thesis has been Henk Bos's book *Redefining Geometrical Exactness: Descartes' Transformation of the Early Modern Concept of Construction.* Not only does it give a new interpretation of Descartes's *Geometry*, it also offers a very effective and valuable method for exploring pre-Cartesian geometry. Bos summarized his plan as follows:

> The present study, then, is about the opinions and arguments of mathematicians concerning the acceptability of geometrical procedures, in particular procedures of construction. These procedures served to *solve* problems and to make geometrical objects *known*, and so the issue of their acceptability hinged on the questions: When is a problem solved? When is an object known? Which procedures are acceptable in mathematics to solve problems and represent objects?[21]

His central concept is 'exactness', defined as 'the complex of qualities that were (and are) invoked with respect to propriety and demarcation in mathematics'.[22] Elsewhere, Bos remarked: 'The interpretation of exactness is extra- or meta-mathematical, in the sense that no answer can be derived from the pertaining formalized mathematical theory itself'.[23]

I found the framework offered by Bos particularly useful for catching the characteristics of Snellius's geometry. Whereas a traditional look on his work, in which the focus would be on new mathematical methods or solutions to problems hitherto unsolved, would not yield more than some isolated results to distinguish Snellius from other mathematicians of the period, the exactness perspective does help to find such a distinction. Some of the topics discussed here in this framework have previously been mentioned in Bos's book, but they have received far less attention there. The main topic which caused Snellius's concern about exactness was the introduction of numbers and arithmetical procedures into pure geometry. A study of his anxiety helps to explain why it took analytical geometry a long time to be conceived and accepted as useful. Criteria for exactness can often be discerned more easily in pure mathematics than in applied or mixed mathematics, because in the latter case exactness is often sacrificed for applicability in some extra-mathematical domain.

The first geometry chapter (chapter five) gives a short introduction to the field of early modern geometry. Different from most introductions to the topic, it does not hurry from Viète to Descartes, but stops to sketch a broader overview of the topics at stake in 'research geometry' in the period. The topic of arithmetic in geometry is first touched upon in the second part of this chapter. It figures prominently in Snellius's dedicatory letter in the *Fundamenta*, his Latin

[21] [Bos, 2001a, p. 6].
[22] [Bos, 2001a, p. vi].
[23] [Bos, 1993a, p. 39].

translation of a book by Van Ceulen. An elaborate analysis of this letter will unveil its rhetorical structure, Snellius's goals with it and the mathematical discussion in which he participated. This very rich letter has received only scanty attention so far. One of its subjects is the troublesome character of Book X of Euclid's *Elements*. In order to understand Snellius's viewpoint, a comparison of his ideas to earlier and later reactions on this much debated book is undertaken. The rest of the chapter is devoted to a small case study: namely, Van Ceulen's introduction of the four elementary arithmetical operations on line segments of which the length is given in numbers, and Snellius's commentary thereon. This illustrates clearly their different approaches to the use of arithmetic in geometry.

Comparisons of Snellius's statements to those of his contemporaries and predecessors are also undertaken in the next two cases. Because Snellius did not venture far from the tradition in general, a very close study is often needed to track the marginal differences between him and other mathematicians and to be able to interpret their significance. This is a normal phenomenon for authors of the period: all humanist writers referred frequently to the traditional framework, which is based on the work of their classical predecessors, for which reason the overlap between their works is often considerable. Moreover, humanist scholars were not used to discussing method in general terms, as e.g. Anthony Grafton remarks in his intellectual biography of Scaliger. Therefore, Grafton recommends deducing their principles from what they actually did and what practices they attacked. An 'archaeological approach' is often necessary for this:

> One must sink many shafts through the rubble of sixteenth-century technical literature to locate the few firm strata where personal allegiance and joint intellectual aims combined to produce schools.[24]

The first geometry case (chapter six) deals with the problem of the division of a triangle by a line through a given point. This topic illustrates the lively activity of geometrical problem solving in the period. Detailed attention is given to a number of solutions by different mathematicians. No new techniques were developed, but a close look teaches us much about the interests of Snellius and his contemporaries. This and similar topics have hardly been studied in modern scholarship. Snellius came back to the problem several times in his career, which shows his fascination with it. Moreover, his proceedings illustrate some of his viewpoints on good solutions of geometrical problems and the importance of structure in geometry.

In the second geometry case (chapter seven), the use of arithmetic in geometry is again at stake. Snellius followed Ramus in his rejection of the traditional proof of Heron's Theorem for the determination of the area of a triangle. This theorem was situated somewhere on the borderline between pure and applied

[24][Grafton, 1983, p. 7]. Grafton mainly talks about classical scholars and chronologists, but his analysis applies to a broader group.

geometry, which made a part of its proof ambiguous. Snellius was the first to state and prove a variant of the theorem by exclusively Euclidean means. Further, a study of Snellius's discussion of the construction and area of a cyclic quadrilateral enables us to refine the view of Snellius as a rigorous Euclidean.

Finally, the results of the biographical part and the case studies are combined in a chapter with an overview of the **characteristics** of Snellius's mathematics (chapter eight). In this place, some seeming inconsistencies will also be explained. Thus, an image of Snellius's mathematics and the forces behind it is revealed, which would probably not be changed essentially if the number of case studies were increased.

1.5 How to read this book

A short explanation of my approach to some technical points is given here to facilitate the reading of this book:

- This thesis has been written for several different audiences, with different backgrounds. In particular, I expect the knowledge of (old) mathematics to vary widely. Therefore, I will sometimes have to explain topics familiar to some readers to make this thesis as much as possible self-contained. Technical terms are explained at their first appearance in the text.

- When paraphrasing old mathematics, I have tried to find an equilibrium between a verbatim translation (which would be unnecessarily troublesome to read) and a streamlined modernization (which could cause false associations with modern mathematics). In particular, in geometrical problems I refer to the rectangle determined by line segments a and b as 'rect (a, b)' instead of 'ab' to distinguish explicitly between the operations of rectangle formation and (arithmetical) multiplication. Similarly, by 'sq (a)', I mean 'the square with side a'.[25]

- I quote from modern editions if available; if not, I quote from the original sources unless otherwise stated. As is customary, the orthography is maintained in citations from vernacular sources, and normalized in Latin quotes ('j' is written as 'i', 'u' and 'v' refer to the vowel and the consonant respectively, and accents are omitted). Neo-Latin spellings that diverge from their classical counterparts (e.g. 'relligio') have been retained. Obvious

[25]Notation borrowed from [Bos, 2001a, p. 19]. Cp. id., pp. 17–18: 'It is impossible to render mathematical arguments from earlier times exactly as they were. The modern reader has [...] both too much and too little mathematical knowledge. [...] In accordance with these general remarks I have not hesitated to modify the presentation of mathematical argument with respect to notation, symbols, and figures, in such cases where this makes the essence more clear and does not change the meaning.'

printing errors or otherwise trivial mistakes have been silently corrected, less obvious ones or otherwise strange or incorrect wordings have been either marked by '[sic]' or emended. Emendations are written in *italics* in the original text and explained after the quotation in square brackets.

- Wherever I refer to a generic 'he' (for instance referring to the reader), I mean 'he or she'. Obviously, a reader, student or professor was generally a 'he' in the seventeenth century.
- All quotations in the main text from Dutch, Latin, Greek, French, Italian and German have been translated into English. The translations are my own, unless otherwise stated. Some quotations are given in the footnotes in a more elaborate form than in the main text because the sources are often difficult to find and consult. Book titles have only been translated if they are informative; an abbreviated title in Latin or English is used in subsequent references.
- Persons that play a key role in the story are introduced by (very) short biographies. I use both Latin, vernacular and translated names, most often maintaining the Latin version, because that was the commonly used one in the discourse of Snellius and his colleagues, but sometimes preferring the name with which they are indicated in newer literature. In the spirit of the sources, I have aimed for readability and clarity, not for a consistent but rigid system. The same holds for place names.

  'Snellius' in general refers to Willebrord, but the reader should be aware that his father could also be intended. In paragraphs about both, I use their first names.

Chapter 2

Biography and background

2.1 Introduction: Snellius embedded

Although Willebrord Snellius speaks to us eloquently and elaborately through his published works, large parts of his life and work are rather obscure. In this chapter I will bring together the extant material, including some hitherto unknown or little used sources, to sketch a coherent picture of Snellius's activities. The main question that will be explored is which people and other factors helped create Snellius's own mathematical style and achievements. Of special importance are Snellius's relationship with his father and through him with Ramism, and Snellius's place in the learned community of the university of Leiden. The background information necessary to answer this question is discussed in some digressions. The content of Snellius's mathematics will only be discussed briefly in the current chapter.

My focus will be on the learned aspects of Snellius, not only because this is the most interesting side of Snellius's personality and therefore the focus of this thesis, but also because the sources do not permit otherwise. These are predominantly bookish, which makes the picture of Snellius biased towards his activities as a writing scholar. Snellius left the world a pile of published works, which contain very diverse and rich material with which he presented himself to his contemporaries and their descendants, yet how he brought about the results enclosed in these volumes must often remain the subject of speculation. Moreover, we have little information on what he did when he was not writing his books, but still professionally active, notably in interaction with mathematical practitioners, students and members of the elite. About his private life not much is known. Due to the imbalance of the sources, it is difficult to make out the character of Snellius.[1]

The material in this chapter is mostly organized chronologically, but this is interrupted in some instances for thematic sections. A detailed discussion of a number of mathematical themes, announced in this chapter, is postponed till the case studies in the later chapters.

[1]Some older biographical articles of Snellius exist, of which especially De Waard's is very useful, bringing together a wealth of material, [de Waard, 1927b]; other articles include [Struik, 1975], [van Geer, 1883], [van Geer, 1884b], [Haasbroek, 1960], [Gerrits, 1948], [Beek, 1983], [van Berkel et al., 1999, pp. 561–563] (by Van Berkel). All of them must be read with some caution, as they contain (minor) mistakes or unfounded conclusions. Cp. my own biographical article [de Wreede, 2003b].

2.2 1580–1600: education and preparation

Willebrord Snellius was probably born on 13 June 1580.[2] His last name was Snel van Royen in Dutch, but like his father he usually used the Latin version of their name, both in and outside academia. His parents were Rudolph Snellius and Machtelt Cornelisdochter; they had named their son after Rudolph's father.[3] The first name was usually spelled as Willebrordus, but sometimes, also

[2]The question of Snellius's date of birth deserves an elaborate footnote. Most older biographical surveys give 1591 as year of birth (e.g. [Universallexicon, 1743, c. 132], [Poggendorff, 1863, c. 948]), for which there is no indication in the sources and which, moreover, would only be possible if Snellius was a prodigy, taking into account his activities in the first decade of the seventeenth century. This persistent error still arises frequently, although some nineteenth-century authors had tried to correct it to 1580 on the basis of his funeral oration (for this source see below; [van Geer, 1884a]; Van der Schaaff blamed Siegenbeek, who wrote about the history of Leiden University, for the mistake and pointed out that Dodt van Flensburg had already identified the error in 1842, [van der Schaaff, 1884, p. 1]).

No archival record documenting Snellius's date of birth exists. His funeral oration gives 1580, [Iachaeus, 1626, p. 21]. However, its author, the professor of physics Gilbertus Jachaeus, had to write it in a hurry (Snellius died on 30 October 1626 and his funeral, where Jachaeus delivered this speech, took place on 4 November) and therefore he did not have the time to check many details. For the greater part, he derived the facts from Snellius's biography in *Athenae Batavae* by Joannes Meursius, adorning them with some rhetorical flourishes.

Meursius's book is the main source for the lives of both Snellii. It is trustworthy because the biographies of father and son were in all probability written by Willebrord himself (in his preface, Meursius explains that the living subjects wrote their own biographies, whereas those of the deceased were written by persons close to them, [Meursius, 1625, fol. **4^r]). The political purpose of this book has to be noted: Meursius meant to convey the message that the scholars of Leiden University had given their university a central place in the world of learning through their works. Therefore, he emphasized their research. Anthony Grafton remarks about these (auto)biographies: 'The patterns by which Leiden professors evoked their past were thus every bit as stereotyped as those now familiar to us [...]', [Grafton, 2003, p. 24], cp. pp. 14–18 of the same.

No date of birth is given in Meursius's account, but it says that Snellius was nineteen years old when he started his *peregrinatio* (educational journey), [Meursius, 1625, p. 297], which cannot have been earlier than 1600, from which 1581 as earliest possible year of birth follows. The earliest archival evidence of his existence dates from September 1581; according to the register of the inhabitants of Leiden, Rudolphus Snellius, Machteld Cornelisdr and their son Willebrord then lived in a house on the Pieterskerkhof, [van Geer, 1884a, p. 122].

Willebrord's birthday can be deduced from a letter written by Rudolph Snellius, which was written on 13 June of an unnamed year. The references to current affairs make 1597 most likely. Rudolph tells his correspondent Magnus that he is writing the letter on the seventeenth birthday of his oldest son, whom he remarkably calls Philibertus—this must have been a nickname, for no one else than Willebrord can have been intended, [Snellius, 1597, fol. 1^v]. This letter has not received any attention so far. Rudolph gave Willebrord his own copy of Scaliger's *Opus de Emendatione Temporum* on 17 June 1607, probably as a somewhat belated birthday present, [Scaliger, 1598, last page].

Weighing these sources, I conclude that it is most likely that Snellius was born in 1580, and that he made a mistake when he wrote down his age at the beginning of his journey some 25 years later in his account for *Athenae Batavae*.

[3][Meursius, 1625, p. 117].

by Snellius himself (especially before 1615), as Wilebrordus or Wi(l)lebordus. Willebrord was born in Leiden,[4] where he would spend most of his life. His father was a university teacher then (for Rudolph's biography see section 2.6). Willebrord later described his mother, who came from the local elite of Oudewater, as a 'most excellent woman'.[5] The family lived in a house on the Pieterskerkhof, very close to the university, in which they lodged a number of students.[6]

As a child, Willebrord must have roamed the streets and canals of Leiden, a city full of university buildings, markets, students and professors. Much of this environment has been preserved to the present day. When the professor of physics Jachaeus pondered the connection between Snellius and his home town in Snellius's funeral oration, he wrote (not without rhetorical exaggeration):

> Leiden, apple of the eye of the Dutch towns, if the gatherings of so many very learned men and the crowds of the most brilliant of minds that were once assembled in you—Scaliger, Lipsius, Heurnius, Junius, Donellus—would not make you illustrious, if the terrible siege would not make you famous enough, and if the loveliness of the region, the fertility of the fields, the ease of traffic would not recommend you enough, still one pupil and citizen would suffice to secure your fame. For in the regions in which mathematics thrives (and it thrives everywhere), the memory of Willebrord Snellius thrives, and with him that of the town of Leiden. And therefore I am forced to doubt whether he received more glory from his paternal town, or gave it to the town.[7]

Some foreign visitors were less impressed by Leiden than this quotation would suggest: they found the canals smelly and the university buildings modest. The town, which mainly thrived on the booming cloth industry, was densely populated. It contained a mixture of magnificent houses and humble shacks, poor and wealthy inhabitants, powerful Dutch Reformed Protestants and more or less tolerated Roman Catholics, Lutherans, Jews and (later in Snellius's life) Remonstrants (Arminians), old Leiden families and immigrants from the rest of the Dutch Republic and abroad.[8]

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[4] [Meursius, 1625, p. 297].

[5] 'lectissima foemina', [Meursius, 1625, p. 297]; [Iachaeus, 1626, p. 4].

[6] [Witkam, 1973, pp. 82–83].

[7] 'Lugdunum Batavorum, ocelle urbium Belgicarum, si concursus tot doctissimorum virorum, si tot olim praestantissimarum animarum, Scaligeri, Lipsi, Heurni, Iuni, Donelli frequentia te non illustraret, si gravissima obsidio parum nobilitaret, si regionis amoenitas, agri ubertas, vecturae opportunitas parum commendaret, tamen vel ex uno alumno et cive satis inclaresceres. Quibus enim in oris vigent Mathemata (vigent autem ubique) viget memoria Willebrordi Snelli, et cum eo urbis Lugdunensis. unde dubitare cogor, plus ne gloriae a patria acceperit, an eidem reddiderit.' [Iachaeus, 1626, p. 6].

[8] [Otterspeer, 2000, pp. 51–58, 436–437].

Although the Dutch Republic was at war with Spain, Leiden was a safe place, which Rudolph in particular must have appreciated, as his place of birth, Oudewater, had been destroyed and its inhabitants massacred by the Spanish in 1575.[9] Leiden had also been besieged by the Spanish in 1574, and was only liberated after much hardship for its population, as alluded to by Jachaeus. Willebrord was aware of the danger of the times, as a later recollection by him shows. He wrote that as a boy the adults had told him about the hazards and difficulties of the earlier phases of the Dutch Revolt, during which period prognostic phenomena had appeared in the heavens. He also remembered how the people around him had described a sea-fight to him which had been seen in the air above Amsterdam, shortly before the Spanish Armada had actually appeared on the North Sea (cp. section 4.4.3 for Snellius's interpretation of those phenomena).[10]

Apart from the continuous threat to the Dutch Republic by the Spanish, Willebrord's youth was also burdened by more personal grief: he had to mourn the loss of his two younger brothers, Jacob and Hendrik. Hendrik died in childhood, Jacob when he was sixteen (in 1599). In Jacob, the family lost a son 'of a lively character, capable of studying all sciences, and not even untalented as a poet'.[11]

Willebrord received his first education from his father, who taught him Latin and Greek, and made him read philosophical authors, perhaps along with the pupils who attended Rudolph Snellius's private school. As soon as he had acquired enough knowledge to be able to attend the courses at the university, he started to study law at the instigation of his father.[12] However, as a typical adolescent, Willebrord chose another track, about which he wrote later:

[9] [Bangs, 1971, pp. 41–43].

[10] 'Memini me puerum ab aetate provectioribus audivisse, antequam diuturnum et difficile hoc bellum Belgicum, quo vix aliud partium studiis vehementius, rei magnitudine difficilius, eventu felicius gestum est cum primum adeo difficilis belli initia quodammodo apparerent, tunc illas aërias Φαντασίας praelusisse secuturis eventibus. Visas nocturno tempore easdem rerum facies, et auditas quoque, quales paulo post obsessis et captis urbibus eveniebant. Tormenta, tubas, sclopetorum displosiones, castrorum metationes, atque alia id genus quam plurima. Quin adeo priusquam classis Hispanica nostro et Brittannico Oceano, instar Xerxis cuiusdam, vincla et virgas iniectura videretur anno supra sequimillesimum octavo et octogesimo, memini me admodum puerum a notis et familiaribus, qui etiamnum in vivis sunt, audivisse, visas Amstelodami in ipso aëre paulo ante Solis occasum navium conflictus maritimos, impressiones hostiles, hos victos illos victores abire, tanquam in puerili naumachia.' [Snellius, 1619, pp. 61–62], see [van Nouhuys, 1998, pp. 531–532].

[11] '[...] vivido ingenio, et ad omnes disciplinas percipiendas idoneo, nec poeta infelix [...]', [Meursius, 1625, p. 119].

[12] [Meursius, 1625, p. 297]; his registration at the university as *Litterarum Studiosus* ('student of humanities') dates from 1 September 1590, [Album, 1875, c. 28], but that does not mean he was actually following courses at the tender age of ten. For the students living in the house see the register of inhabitants of Leiden of 1581 and the Album Studiosorum [van Geer, 1884a, pp. 122–123].

> because he himself preferred to follow the impulse of his heart, and could hardly suffer to be confined within such narrow boundaries, and because he loved the study of mathematics more eagerly, he stuck to that for some time.[13]

This decision to turn to mathematics was not obvious, although Rudolph was a professor of mathematics. In the early modern period, there were no clear-cut ways to become a mathematical specialist, nor were there more than a few job opportunities. The group of 'professional' mathematicians was small and heterogeneous and its members often worked in isolation, although they knew each other's works through personal contacts and publications.[14]

In 1595, Tycho Brahe proposed to Rudolph Snellius, through the agency of Johannes Pontanus, that he (Tycho) send his own son to Leiden to have him educated by this 'excellent mathematician' Snellius, because he did not doubt his 'conscientiousness and integrity as a teacher'.[15] Rudolph then suggested that he send Willebrord, who was the same age as young Tycho, to Brahe in exchange. This plan appealed to Tycho Brahe, not least because he had been told that Willebrord had already learned arithmetic and geometry. This proposal was not immediately carried out, but the idea of sending his son abroad for his education certainly appealed to Rudolph Snellius, who had himself studied and worked abroad for many years. Two years later, in 1597, he announced that he would conduct Willebrord to Heidelberg the following year, but this plan does not seem to have been put into effect either.[16]

While staying at home for the present, Willebrord developed his mathematical skills. He was a pupil of Ludolph van Ceulen (see p. 29), together with Nathaniel Claeszoon.[17] Van Ceulen included several of Snellius's mathematical results in his *Arithmetische en Geometrische Fondamenten*. Already in 1599, Snellius assisted the much more experienced Ludolph van Ceulen in solving the problem of dividing a triangle (see section 6.3)[18] and Van Ceulen made him solve

[13]'cum ipse animi impetum sequi mallet, et aegre ita arctis limitibus circumscribi se pateretur, impensius mathematum studio delectatus, istis aliquantisper adhaesit.' [Meursius, 1625, p. 297]. The quotation is in the third person because it was incorporated in the biography in *Athenae Batavae* (see footnote 2).

[14]Cp. [Andersen and Bos, 2006, pp. 697–702]; cp. section 2.4.

[15]'R. SNELLIUM, eximium illum Lugdunensium Mathematicum [...] nihil de ipsius fide et integritate addubitans, quin seduli et fidelis praeceptoris ac moderatoris munus sit obiturus.' Tycho Brahe to Johannes Isacius Pontanus, 5 September 1595, [Dreyer, 1972a, p. 372]; cp. [Meursius, 1625, pp. 297–298].

[16]Letter to Joannes Magnus, 13 June 1597, [Snellius, 1597, fol. 1^v]. For the date of the letter see footnote 2.

[17][van Ceulen, 1615a, p. 242]. Note Snellius's small emendation of Van Ceulen's text, where no distinction is made between the two students, in his Latin translation where he stressed his own contribution: 'Hoc zetema solvit Willebrordus Snellius RF. et postea quoque Nathanael Classonius' [van Ceulen, 1615b, p. 233].

[18][van Ceulen, 1615a, p. 212].

another triangle problem which Snellius handled, in Van Ceulen's words, 'more ingeniously than I'[19] (see section 5.3 for more examples of collaboration between Van Ceulen and Snellius).

In 1600 Willebrord was himself teaching at the university on 'extraordinary days' (Wednesday and Saturday, when the regular professors did not teach), treating the first three books of Ptolemy's *Almagest*. He did this as an 'exercise', not of his own volition, but 'he obeyed his father in this matter, because he did not dare to resist him'. This shows that Snellius senior considered Snellius junior as a future professor, not pushing his son too much into becoming a lawyer.[20]

Willebrord Snellius was destined and prepared to become a scholar and a teacher from his earliest youth, almost from the cradle. As the only child of his parents to reach adulthood, he probably had a close relationship with them. His father educated him both as a son and as a pupil. For this reason, the background and scholarly views of his father and the intellectual environment in which the Snellii functioned have to be scrutinized more closely, which will be done in the following four sections.

2.3 *Leiden University:* Praesidium Studiorum Humaniorum

The story of the foundation of Leiden University is well known. After Leiden had heroically survived the siege by the Spanish, it was rewarded with a university, the first in the Northern Netherlands, of which the chief aim was to educate Protestant ministers. Its set-up was traditional: students were supposed to start their studies in the *artes* faculty, which took care of the propaedeutic or preparatory teaching, after which they proceeded to one of the higher faculties: theology, medicine and law. Many subjects belonged to the *artes*, such as Latin and Greek language and literature, natural philosophy, ethics, rhetoric, history, and different branches of mathematics.[21]

In the first decades, the number of professors and students was rather small, and sometimes there were vacancies for longer periods of time. There was no fixed curriculum, and not much about the contents of the lessons is known. It is clear, however, that the texts of classical authors were the core of the discussed material. All teaching was done in Latin, and almost all writings of the professors

[19]'[...] constiger als ick [...]', [van Ceulen, 1615a, p. 230].

[20]'Concessum filio Snellii ut diebus extraordinariis possit exercitii causa praelegere Mathematicen, hora 8^a vel 11^a.' [Molhuysen, 1913, p. 126].

'[...] parenti, cui refragari non audebat, in eo obsecutus.' [Meursius, 1625, p. 297].

For ordinary and extraordinary days and hours, see [Otterspeer, 2000, p. 228].

[21]Because much good literature is available on this topic, I only give a very brief summary here. See e.g. [Otterspeer, 2000], [Dibon, 1954], [Berkvens-Stevelinck, 2001], [Lunsingh Scheurleer et al., 1975], [Grafton, 1988], [Vermij, 2002, pp. 15–25].

were in the same language. Unlike the situation in traditional universities, the character of the education was predominantly humanist and the arts received ample attention in the curriculum.

As this term is used in many different meanings, I will make explicit here what it means in the framework of this book. It must first be noted that the terms 'humanism' and 'humanist' have been used in a meaning close to that given here only since the nineteenth century. Snellius's epoch did not have a specific term to denote what we now call 'humanism',[22] which makes it sometimes complicated to distinguish humanist elements in works of learning. Humanism was the current in the Renaissance that was concerned with scholarship and education. Its main characteristic is the renewed interest in Greek and Roman Antiquity, and mainly in its authors, which started in Italy in the fourteenth century and determined the dominant form of scholarship in Northern Europe between c. 1470 and 1640. This interest manifested itself in an intensive study of a wide variety of classical texts and material remnants and by the application of the wisdom contained in them to different parts of life, such as politics and art. The major aim of humanism was to educate adolescents into good citizens on the basis of classical (mainly rhetorical and philosophical) texts. Because the education of the elite was predominantly humanist (no longer scholastic) for several centuries, humanism had an enormous impact on society.[23]

Language, especially the Latin language, was central to humanist scholars. They aimed at a perfect command of Latin, because that was necessary to become a complete human being, and therefore searched for the purest form of classical Latin. The language and the content which it expressed were inseparable from each other. For this reason, humanist scholars considered the writings of Greek and Latin authors as a never-empty *Fundgrube* or mine of inspiration, both for the content and for the form of their works. They used this as the basis for their own work in the renowned model of *translatio-imitatio-aemulatio* ('translation, imitation, emulation'). Although modern readers are sometimes disappointed in humanist writings because they consider them second-hand and unoriginal, a closer scrutiny and reconsideration of the concept of 'originality' often helps to see the additions and innovations within the classical material made by the humanists. Many humanists themselves liked to stress how much they differed from the Scholastics of the 'Dark Ages', yet, in fact, the approaches to education and scholarship of the two groups are often strikingly similar: in both cases, the reading and study of classical texts occupied centre stage, while

[22] The terms 'studia humanitatis' and 'studia humaniora' were sometimes used, but they were somewhat more restricted in meaning than the modern term 'humanism'. The 'studia humanitatis' were grammar, rhetoric, poetry, ethics and history, [Groenland, 2006, p. 13].

[23] [Kristeller, 1980]. This paragraph, and some other parts of this section, are based on a research paper written in 2000 by Juliette Groenland, Marleen van der Weij and Liesbeth de Wreede, for which see www.murmellius.com.

theology sat on the throne of the arts and sciences.

The humanists distinguished themselves from their predecessors by the profundity of their studies in a large range of sometimes nearly lost ancient texts. They made these texts available through editions and translations (mainly from Greek into Latin) and critically scrutinized the language, content and context of these texts with ever-increasing philological skill. They also taught their students 'good' classical (mainly Ciceronian) Latin, instead of the 'barbarian' Medieval Latin, and developed their own thoughts by building their own edifices on classical foundations. Their works bear witness to their mastery of rhetoric in communicating their message. The study of rhetoric was a key part of humanism, for which authors like Cicero and Quintilianus had furnished both case material and theoretical expositions. The language used was mainly Latin, although many humanists felt some interest in the vernacular as a means of reaching larger audiences. This emancipation of the vernacular became ever more important, first in Italy and then, from the sixteenth century onwards, also in Northern Europe.

An important aspect of the emulation was the reconciliation of pagan and Christian wisdom. All humanists took for granted the truth and superiority of the Christian faith, in one or another of its denominations. Some humanists introduced a third source of wisdom: the pre-antique *prisca scientia*, the enormous knowledge that early civilizations, such as the Egyptian and the Babylonian, were thought to have possessed. This could only be accessed through hints in later sources and some writings at first believed to be primaeval but much later unmasked as more recent, such as those of Hermes Trismegistus. The *Wijsentijt* ('Age of the Sages') invoked by Simon Stevin was such a pre-classical period of wisdom, regarded by him as almost lost and only recoverable by huge efforts in his time.

Although scholars and humanists often worked in geographical isolation, they felt themselves to be members of the same *Respublica Litterarum* ('Republic of Letters'), in which they learnt about each other's works through the art of printing. Moreover, they stayed in contact with each other through extended correspondences and the tendency (somewhat surprising given the practical obstacles) of some members to make extended educational trips. The letters they exchanged were an important way of spreading knowledge before the appearance of scientific journals, and they were often circulated among a number of interested individuals. This Republic always retained a personal character due to its small scale.[24] To belong to a network of kindred spirits was essential to humanists and much humanist literature is devoted to the well-phrased expression of feelings of friendship. Although the aims of the humanists may have been very elevated, they were often involved in polemics in which they insulted each

[24] See e.g. [Dibon, 1990] or [Stegeman, 1996] for a case study.

other in rather vulgar ways. These calumniations were not always completely serious: they also contained an element of playfulness and showed how witty their composers were.

Since the Middle Ages, the arts were divided into two groups, the *trivium*, consisting of rhetoric, grammar and dialectic,[25] and the *quadrivium*. The latter, also known as 'mathematical sciences', consisted of pure mathematics (arithmetic and geometry) and mixed mathematics (in the Middle Ages only music and astronomy were considered as such, but later an ever increasing list of topics was studied under this name). The focus of humanism was on language and moral philosophy more than on dialectic, which received most attention in Scholasticism. Both in the Middle Ages and in the early modern period, the mathematical sciences were generally treated in stepmotherly fashion in the university curricula. The students were not very interested, and the teaching was often done by junior staff. Only in some exceptional cases was more advanced tuition offered, usually on the fringe of or outside academia. The primary task of mathematics in academic education was to teach future physicians astrology.[26]

In the first part of the sixteenth century, many Dutch secondary schools had been converted to institutions founded on humanist pedagogical principles. This meant that the teaching of linguistic competence was the principal aim, and that—much more than in the Middle Ages—the general education of the pupil was central, rather than his preparation for specific professions. This went together with an increasing interest in pedagogical aspects and a more practical approach to language learning, which is shown by, e.g., the teaching of the Latin equivalent of everyday language. Especially here, (Latin) language learning dominated all other pursuits, and non-language disciplines served just as auxiliary sciences. Sometimes, some attention was paid to them in the last classes to prepare the pupils for university studies. The mathematical sciences were hardly ever taught. This development formed a context for the new university.[27]

Since mathematics only had a marginal role in the humanist curriculum, the study of the influence of humanism on mathematics is somewhat problematic. Moreover, much mathematical activity sprang from specific practical problems studied far from the world of humanist learning. Yet a part of the mathematical work from the Renaissance and early modern period certainly had humanist characteristics. This part was, like other parts of scholarship, thoroughly 'textual': oriented towards the problems and techniques of classical, that is, Greek, math-

[25]'[...] humanists always refer to their study of ratiocination as 'dialectic' (reasoning conducted between two interlocutors), rather than as 'logic', to emphasise the active, pragmatic nature of the argumentation which captures their interest.' [Jardine, 1988, p. 176].

[26][Schöner, 1999]. Cp. [Brockliss, 1996, pp. 589–593], who is somewhat more optimistic about the mathematical tuition on offer in the universities. [Bot, 1955, p. 76]: 'De uitgesproken, en meestal exclusieve aandacht voor de taalfunctie, voor het 'lingueren', maakt het meest wezenlijke element uit in het Humanisme'.

[27][Groenland, 2006], [Bot, 1955, pp. 73–101].

ematics and elaborating on claims and arguments with the help of references to classical authors. Predominantly Italian philologists had unearthed many manuscripts with classical mathematical texts and had made them accessible by editions and translations. Archimedes, Euclid, Apollonius and Diophantus all received their fair share of attention. While struggling with manuscripts that were sometimes corrupt or otherwise incomprehensible, these humanists had recourse to their own mastery of mathematics to come to grips with the ancient texts, thus often creating new mathematics while trying to restore the old. They also tried to discover how mathematics had developed.

Moreover, humanistically inclined authors wrote textbooks about the divisions, method, function and concepts of the mathematical sciences. The study of astronomy, which could unveil part of God's plan in the creation of the world, was considered to be an elevated activity. Due to the exertions of the humanists, mathematics gained in 'cultural prestige'[28] in the sixteenth century, becoming a science of its own. Temporarily, in the first half of the sixteenth century, it conquered a stronger position within the university, especially the astronomical part.[29]

As in other fields, humanism influenced not only the content, but also the (literary) style of mathematical works. This can be seen most clearly in the passages which do not deal with mathematics proper, such as dedicatory letters: these are written in an eloquent Latin style, in which the erudition of the author is shown.

Leiden University had attracted some of the luminaries of Northern European scholarship, for which reason it boasted the reputation of 'stronghold of the humanist sciences'[30]. The most notable examples are Justus Lipsius, who had left Leiden when Snellius was still a child, and Josephus Justus Scaliger, who was to play a vital role in the humanist education of a whole series of talented young men, including Snellius. Scaliger (1540–1609) had been attracted to the university by most generous conditions: he would earn a very high salary, but not be obliged to do any teaching—just to be present and radiate wisdom would be enough. Scaliger's fame was spread everywhere in Europe, and he earned 'a position as the arbiter of erudition for the whole world of learning', according to his biographer Grafton.[31]

Scaliger published an enormous amount of work, and he pretended to expertise in almost the whole universe of scholarship, including mathematics, but sadly betrayed himself when he published his erroneous quadrature of the circle.

[28] [Porter, 1996, p. 550].

[29] A very good survey of the activities of the Italian humanists in mathematics is [Rose, 1975]. See [Hoppe, 1996] for a listing of early modern mathematical works with a humanist flavour. [Schöner, 1999, p. 104].

[30] 'Praesidium Studiorum Humaniorum'.

[31] [Grafton, 2003, p. 23].

Insensible to the warning of Viète, one of the ablest mathematicians of his time, he made his 'discoveries' public in a series of eye-catching works (1594), in which he extensively condemned Archimedes and his methods, among which notably reduction to absurdity. One of his key statements was that the circumference of an inscribed polygon could exceed the circumference of the circle in which it was inscribed—at least in calculations, not in geometrical constructions, a 'noble paradox'. According to Scaliger, the ratio between the circumference and the diameter of a circle was $\sqrt{10} : 1$. This was in fact a very imprecise approximation. His results were immediately attacked by a number of mathematicians. Of course, Scaliger was not able to give a sensible answer and had to have recourse to other means. One of them was to deprecate mathematicians: 'A very bright character cannot be a great mathematician.'

However poor his mathematical explorations may have been, Scaliger's knowledge of Greek was unparalleled and his work on chronology, which also involved mathematics, in particular knowledge of ancient astronomy, much appreciated. Scaliger did not teach regular courses, but he did offer private tuition at home to a small group of students, whom he stimulated to carry out real research and publish it as well. Most of these students were the sons of members of the elite. Among them were Hugo Grotius, and the future professors Daniel Heinsius, Johannes Meursius and Petrus Cunaeus.[32]

Apart from Scaliger, Leiden had other attractive features for eager students and scholars: a whole collection of humanistically inclined professors and a wealth of books and other materials. Although the education of students for a career in the church or government was the core business of the university—as in all other universities and academies across Europe—the climate at Leiden University was more favourable towards research by its scholars than was customary elsewhere. Although the traditional view of Leiden as a research-driven university has been somewhat modified by Anthony Grafton and others, and although the courses may not have been influenced much by the professors' new works of scholarship, still Leiden was one of the foremost centres of learning, maybe even the most eminent, in Snellius's time. Philology especially, in all its forms, had reached a very high level.[33]

Rudolph Snellius was professor of mathematics while his son grew up. In the early days of the university, some lessons in this field had been taught by other people, but in Willebrord's lifetime, the Snellii alone covered the mathematical sciences. Gilbertus Jachaeus, who became the professor of physics in 1612, could

[32][Grafton, 2003, pp. 18–27], [Grafton, 1983], [Grafton, 1993a], notably pp. 378–384 on the quadrature, and p. 488, quote from *Scaligerana* about Clavius: 'praeclarum ingenium non potest esse magnus Mathematicus'. See [Bierens de Haan, 1878c], [Hoftijzer, 2005], and in this collection notably [de Jonge, 2005] on Scaliger's private teaching.

[33][Grafton, 2003].

be considered as Willebrord's closest colleague.[34]

Although the story of the founding of the university was heroic, its teaching programme idealistic, and the state of its scholarship elevated, its academic community was far from harmonious. The University was plagued by troubles in its early decades. The amount of influence granted to the Calvinist church was an issue since the beginning, which culminated in the conflict between the adherents of the professors of theology Arminius (the Remonstrants) and Gomarus (the Contra-Remonstrants) that nearly led to civil war in the Republic during the 1609–1621 Twelve Years Truce with Spain. In 1619, when the Contra-Remonstrants had triumphed, mainly through the protection of the Stadholder Maurice of Nassau, the university was purged of Remonstrant elements. The professors Coddaeus (who delivered Rudolph Snellius's funeral oration) and Jachaeus were reprimanded and temporarily suspended.

Moreover, the university administration was forever looking for good professors, a rare commodity, made even rarer by political and religious restrictions. The students were sometimes involved in violent incidents or misbehaved otherwise.[35] Although all this must have influenced Snellius's professorial performance, it is not reflected in the sources concerning him.

2.4 Mathematics in the Dutch Republic

Much mathematics of the period was developed and used in a practical context. In the young Dutch Republic, people educated in practical mathematics were in demand, e.g. as military and civil engineers or surveyors, in the expanding merchandise sector or in architecture. Engineering science became of growing importance.[36]

The stadholder Maurice of Nassau, also commander-in-chief of the Dutch army, was very interested in the use of scientific insights in warfare and therefore decided to establish a school for the purpose of the education of engineers. Thus, in 1600 the Leiden engineering school was founded. It was called 'Duytsche

[34] [Vollgraff, 1913a, pp. 167–171]; see [van Berkel, 1981] for the position of the physical sciences in Leiden.

See [Vermij, 1996] for an overview of mathematics in Leiden in the period. His stress, however, is too much on the use of mathematics as an applied science, and he does not seem to appreciate contributions to pure mathematics—e.g. he considers the endeavours to solve the quadrature of the circle as 'the riding of a hobby-horse' (p. 78). He does pay attention to the connections between (practical) mathematics and humanism, particularly in the case of Willebrord Snellius.

[35] [Otterspeer, 2000, passim].

[36] Cp. [van Maanen, 1987, pp. 5–15]. See [van Berkel, 1985b] for a survey of Dutch mathematics education in the seventeenth century, focused on the situation in Franeker. See also [van Berkel, 1985c] or [van Berkel, 1999] (the revised English version of the same work) for a general survey of the history of mathematics and science in the Netherlands from 1580 onward; cp. [van Berkel, 1998, pp. 27–84].

Mathematique' (Dutch mathematics) because the teaching was done in Dutch.[37] The proposal for the curriculum was probably written by Simon Stevin, although he did not become a teacher at the school.[38]

This polymath was probably born in 1548, in Bruges. Since 1593/4, he was the scientific advisor and mathematics teacher of Count Maurice of Nassau, and later also quartermaster of the Dutch army. He published on many topics from the extended field of the mathematical sciences, including arithmetic, geometry, perspective, navigation, mechanics and military science. In many of these fields, he had innovative ideas, often originating from his inclination to take both 'spiegheling' (reflection, theoretical investigation) and 'daet' (act, practical activity) into account. His enormous *Wisconstige Gedachtenissen* gives an account of his tutoring of Maurice, also containing contributions by the pupil. One of his achievements is his propagation of Dutch as the ideal language for scientific discourse, for which he developed neologisms when necessary. Some of them have been very successful: the most famous example is 'wisconst'[39] for mathematics. Although no explicit adherent of Ramism, he shared a number of insights with Ramus (see the next section), which he may have borrowed from him, maybe learning them from Rudolph Snellius. Stevin died in 1620.[40]

The classes were taught in the vernacular, because they were meant for pupils from a humbler social class than university students; typically, only the latter would have attended a Latin school. Yet the programme that was offered was broad and solid, containing both the foundations of mathematics and their application in surveying and fortification. This latter had recently been developed into a real science, in which Stevin was an expert. The students also did fieldwork. One of their textbooks was Sems's and Dou's *Practijck des landmetens* ('Practice of surveying').[41] Their knowledge could not only be applied to warfare, but also to more peaceful occupations like building and land reclamation.

The first teachers of the engineering school were Symon Fransz van Merwen and Ludolph van Ceulen. The latter would have an important role in Snellius's life. Van Ceulen was his teacher, and Snellius translated several of his works into Latin. Van Ceulen was born in 1540 in Hildesheim. He became a fencing master in Delft and later in Leiden, soon teaching mathematics besides. In 1599, he became a teacher of practical mathematics at the engineering school.

[37] [van Maanen, 1987, pp. 5–8, 16–17]; cp. [van Winter, 1988, pp. 14–27] and [van Berkel, 2005].

[38] See for this programme [Molhuysen, 1913, pp. 389*–391*].

[39] Meaning 'the certain art'. Mathematics in modern Dutch still has almost the same name: wiskunde.

[40] [Koninklijke Bibliotheek van België, 2004], in particular the article [van den Heuvel, 2004]; [Dijksterhuis, 1955]; [Dijksterhuis, 1943, pp. 65–111] for a summary of his mathematics (in the modern sense of the word) and pp. 298–320 for Stevin and the Dutch language. See [Verdonk, 1969] for Ramus's alleged influence on Stevin.

[41] [Sems and Dou, s a].

He taught his fencing classes in the former Faliede Bagijnen-Church, where this school also had its seat. In 1600, he was elevated to the rank of professor at the school. Some of his teaching material found a place in the *Arithmetische en Geometrische Fondamenten*.[42]

The mathematical level of Van Ceulen exceeded by far that of an average mathematics teacher. He was an able and ingenious calculator, in which capacity he calculated π in 35 decimals (all correct), and developed some mathematics concerning chords of regular polygons. He was also involved in several polemics on geometrical problems with other practitioners. The large book on algebra which he announced on several occasions did not appear.

Although Van Ceulen had a good mathematical reputation, his social position made him vulnerable. Adrianus Romanus (see p. 47) published a telling example of this. When Scaliger had published his erroneous quadrature of the circle, Van Ceulen read it, discovered the mistakes and had Scaliger warned through intermediaries, urging him to withdraw the book. Scaliger ridiculed him and said that not even a man of learning, let alone a *pugil* (a boxer, referring disdainfully to Van Ceulen's job as a fencing master), would be able to check the contents of the book so promptly, thus denying Van Ceulen the title of 'mathematician'.[43] After the deaths of Van Merwen and Van Ceulen, both in 1610, Frans van Schooten Senior became the professor of mathematics at the engineering school.[44]

Outside Leiden, some other Dutch mathematicians were active, but they were geographically more isolated. Some of them will be mentioned in the rest of this chapter as Snellius had personal contacts with them. *Rechenmeister* (or reckoning masters) were active in many Dutch towns. They taught practical arithmetic and often held the limelight by solving difficult mathematical problems or claiming that others were not able to do so. The proverbial representative of this category is Willem Bartjens, who wrote the very popular *De Cijfferinghe* ('The Art of Calculation'; the first edition of many appeared in 1604).[45] A reasonably large group of teachers of navigation could also earn their living in the seafaring Dutch Republic.

When Snellius was a child, the only other university of the Dutch Republic was in Franeker, in Friesland (founded in 1585). The professor of mathematics there was Adriaan Metius (1571–1635), a former student of Rudolph Snellius and Ludolph van Ceulen. His teaching was oriented towards practical applica-

[42][van Ceulen, 1596], [van Ceulen, 1615a]; [Katscher, 1979], [Bierens de Haan, 1878b], [Bos, 2000a], [Struik, 1971], [Oomes and Breugelmans, 2000].

[43][Katscher, 1979, p. 110]; see [Bierens de Haan, 1878b, p. 153] for the quote in Latin.

[44][van Maanen, 1987, pp. 5, 16]. Van Maanen states that Van Schooten also taught courses in elementary mathematics in the university, in Latin, but I have not found any indication for that, for which reason it seems to be unlikely. See in particular [Molhuysen, 1916].

[45][Bartjens (introduced by Danny Beckers and Marjolein Kool), 2004]; see its introduction for the historical-mathematical background.

tions: he taught for instance on surveying, navigation and military engineering, giving both Latin and Dutch lessons, for different audiences. Later in Snellius's life, universities were also founded in Harderwijk (1600) and Groningen (1614). The professors of mathematics in Snellius's lifetime in these institutions were Johannes Isacius Pontanus and Nicolaas Mulerius, respectively. They had to teach other subjects as well. Mulerius (1564–1630) had been a student of Rudolph Snellius. He published a new edition of Copernicus's *De Revolutionibus* and was involved in endeavours to solve the problem of the determination of longitude at sea. In all these universities, the teaching of mathematics was aimed either at practical applications or for the general development of the students in the propaedeutical phase. There was no room for more advanced mathematics in the curriculum. It is not known if and how Snellius was in touch with these fellow-professors.[46]

The amount of contact between the Latin-dominated world of learning and of the Dutch-speaking practitioners is still subject to scholarly debate.[47] The sources do show that contacts between the two groups took place, and that the expectations of the applicability of the (mathematical) sciences were high. Ramism is a good candidate to explain this faith and interest in science functioning outside schools and universities (see the next section). However, it is impossible to delineate its precise influence. Not all cases in which the words 'use' or 'practice' are mentioned are proofs of the influence of Ramism, or humanism in general.[48]

In the rest of this chapter, it will become clear that although the fragmentary evidence of Snellius's role in the world of practical mathematics does not permit a very elaborate analysis, several examples show that he certainly was involved in this world.

2.5 Petrus Ramus

Because the thought of Petrus Ramus had considerable influence on Willebrord Snellius and dominated his father's complete oeuvre, the works of this sixteenth-century French philosopher will be introduced in this section. Much debate was provoked by Ramus's philosophy, both during his life and afterwards, and many scholars were fiercely pro or contra him. Rudolph Snellius was his most fervent Dutch advocate. Ramus had a special interest in mathematics and published books on geometry and arithmetic, on the basis of which father and son Snellius wrote some works.

[46] [van Berkel et al., 1999, p. 525], [van Maanen, 1987, p. 15], [van Berkel, 1985b, pp. 215–216, 222, 228, 234], [Jorink, 2003], [Vermij, 2002, pp. 45–52].

[47] E.g. Davids argues against Frijhoff that the contact between these two worlds was at its summit between c. 1730 and 1800, [Davids, 1990].

[48] [Davids, 1990, pp. 23–24] thinks that the role of Ramism has been overestimated.

Although controversial, Ramus's philosophy broadly overlapped with humanistic thought in general, for example in its attention to didactics. Moreover, his popularity was partly based not on his ideas, but on his role as a Protestant 'martyr'. For these reasons, the influence of Ramus and general humanistic traits of Willebrord Snellius's work can not always be distinguished completely, yet in some cases Ramus is unmistakably present in his work.

Although all aspects of Ramus's person and work seem to have been (and partly still are) subject to controversy, it is fair to see him as a humanist, who developed his own thoughts in dialogue with the ancients and in forceful anti-Aristotelian polemic. Much of his thought was not original, yet influential in his version. Over the last decade, a huge amount of studies on almost all aspects of Ramus's work have appeared, of which only a small part can be discussed in this sketch, which aims to introduce those aspects most relevant to the Snellii. The reader interested in Ramus is referred to the relevant sources and secondary literature.[49]

Pierre de la Ramée was born in 1515 in Cuts in France. After completing his studies, he taught in several colleges in Paris. When his vehement anti-Aristotelianism caused trouble, he diverted his attention from philosophy to rhetoric and mathematics. His appointment to the Collège Royal in 1551 gave his method royal acknowledgement. This resulted in large numbers of students and publications. He devoted much time to his major task in life: the restoration of the liberal arts according to his own method. After some years, the interest of students dwindled and he lost the esteem of his patron by his professed sympathy with the Huguenots. The rest of his life was eventful: he had to flee Paris several times, was popular and despised at the same time, and kept publishing his books. His life ended dramatically: he was murdered in the anti-Protestant violence known as the St. Bartholomew's Day Massacre (1572).[50]

2.5.1 Ramus as a philosopher and a pedagogue

'Philosophy' has a somewhat restricted meaning for Ramus and his followers. It is the *cognitio* or *doctrina artium liberalium*, the knowledge of the liberal arts. Ramus was mainly interested in their teaching at the university, which makes his philosophy predominantly pedagogical.[51] Mathematics formed a part of philosophy for him, so Ramus's statements about mathematics have to be considered in the framework of his general philosophy. Of crucial importance is the union between philosophy and eloquence: together, they served to illuminate

[49]See for recent bibliographies on Ramus and Ramism [Sharratt, 2000] and [Magnien and Michel, 2004, pp. 195–207].

[50][Verdonk, 1966, pp. 5-47], cf. [Mahoney, 1975], [Magnien and Michel, 2004, pp. 209–210].

[51]Cp. [Mahoney, 1975, p. 287]: 'Thus "method" was for Ramus primarily a pedagogical concept; accordingly, his contributions to the sciences were essentially pedagogical and propagandistic in nature.'

the meaning of texts and made it possible to produce new ones. Usefulness in society, in the shape of practical reading, writing and speaking of texts, was the goal of education; a focus which Ramus shared with mainstream humanism.[52]

Central to Ramus's philosophy is his method, which was the same for all disciplines, including mathematics. It meant that he used three laws, borrowed from Aristotle's *Posterior Analytics*, to organize the contents of every *ars* or liberal art. These are:

1. The *lex veritatis* or *lex κατὰ παντός* (the law of truth): all precepts of a discipline must be generally valid and necessarily true. E.g. a precept stating that the angle of a triangle is right, is against this law, because it is only true for some triangles.

2. The *lex iustitiae* or *lex καθ' αὑτό* (law of justice): precepts must be included in the discipline to which they naturally belong and not in any other one. E.g. precepts about areas belong to geometry and should not be included in arithmetic, because that deals solely with numbers.[53]

3. The *lex sapientiae* or *lex καθ' ὅλου* (law of wisdom): more general precepts always have to precede more specific ones in the presentation of a discipline. E.g. the rule that the three angles of a triangle add up to 180^0 has to be presented in the section about all triangles. Thus, students would learn 'the most general things generally, but specific things specifically'.[54]

The purpose of this method was to present the material of the topics that pupils had to study in an ordered, short and clear way, thus permitting them to work as efficiently as possible. All arts were purified by the method: they had to be reduced to their essence, for which purpose Ramus dissected the material offered by the classical authors, reading it hypercritically, and confronting the opinions of various authors with each other, after which he did not replace the old material by something new, but abbreviated, modified and rearranged it. His most favoured method of presentation was the dichotomy, the division of a concept into two categories. Once the student had gone through the essential material of all disciplines separately, these disciplines were again combined in their actual use.

[52][Meerhoff, 2001a, pp. 201, 214], [Meerhoff, 2001b, pp. 355–359]. Cp. [Hooykaas, 1958] for an overview of Ramus's philosophy, in particular of the mathematical sciences.

[53]See e.g. [Ramus and Talaeus, 1969, p. 385], [Pantin, 2004, pp. 385–386] about this separation of the disciplines of arithmetic and geometry.

[54]'[...] generalissima [...] generaliter, quae vero specialia, specialiter.' [Snellius, 1596i, p. 19], quoted from [Sellberg, 1979, p. 65]. The rest of this paragraph is borrowed from [Sellberg, 1979, pp. 43–44, 65]. I thank my father, W.J. de Wreede, for his meticulous translation of large sections from the Swedish. Some points from the book are repeated in English in [Sellberg, 2001]. Cp. [Bruyère, 1984, pp. 267–275], [Verdonk, 1966, pp. 322–324].

The method did not always yield convincing results—not even in the analysis of a rhetorical piece, its ‘core business’. This is exemplified by Marc van der Poel’s analysis of Ramus’s discussion of one of Cicero’s orations on the Agrarian Law and the reaction of Ramus’s contemporary Turnebus. Van der Poel shows that although Ramus succeeded in making visible the logical structure of the oration, his analysis is not entirely satisfactory and his terminology is complicated. Moreover, Turnebus’s criticism that Ramus ultimately did not manage to explain the success of the oration in convincing the public makes sense; Turnebus pointed out that the counting of arguments and figures of style, central to Ramus’s method, was not useful for explaining the effect of the speech.[55]

Ramus was a fervent defender of the use of the vernacular. He planned to translate all his textbooks into French, and argued that the Gaulish culture was the basis of European civilisation.[56]

2.5.2 Ramus as a mathematician

Ramus had ambitious plans in the field of the mathematical sciences, which he considered as ‘the most certain and accurate of all disciplines’.[57] He aimed to publish a *corpus matheseos*, editions of a number of Greek mathematical texts with Latin translations. Moreover, he planned several textbooks and commentaries, both in Latin and in the vernacular. Although he was not able to achieve all this, his message about the importance of the Greek source texts for contemporary mathematics was picked up. His defence of the role of mathematics in the liberal arts curriculum was so forceful that it had a lasting influence.[58]

Ramus did publish a number of works on mathematics: several editions of a textbook on arithmetic (gradually further removing himself from the Euclidean material), one on geometry, a book on algebra, *Prooemium Mathematicum* and *Scholae Mathematicae*, which contained the *Prooemium*. The *Prooemium* discussed the history and use of mathematics, defending it against the charge of obscurity, and addressed the question whether mathematics could be made simpler and more systematic. Ramus paid particular attention to the ancient use of mathematics, both theoretical, as a basis for natural philosophy, and practical, for example as applied in mechanics and astronomy. He blamed Euclid for

[55][van der Poel, 2005, pp. 336–337].

[56][Meerhoff, 1986, pp. 34–40], [Meerhoff, 2001b, pp. 370–371].

[57]‘[...] hae disciplinae omnium certissimae essent et accuratissimae [...]’, [Ramus, 1569, p. 113].

[58]For this section I have gratefully drawn from the standard work on Ramus’s mathematics, [Verdonk, 1966]. The argument of the book is summarized in German in [Verdonk, 1968]. See also [Loget, 2004] and [Pantin, 2004]. Cp. [Bruyère, 1984, pp. 354–364] and [Robinet, 1996, pp. 153–170] for Ramus’s philosophy of mathematics; [Robinet, 1996, pp. 171–176] on the Snellii contains some interesting remarks, but his interpretation of Willebrord’s expression *universa mathesis* as *mathesis universalis* seems to be unfounded (p. 171).

having written the *Elements* in an obscure form, without paying attention to applicability.[59]

In the *Scholae*, much attention was paid to pointing out the shortcomings of Euclidean geometry, and again, usefulness took centre stage:

> We consider him a mathematician who has not just demonstrated arithmetical and geometrical propositions by means of some syllogism, but who has also, and primarily, made clear the profit of arithmetic and the usefulness of geometry by means of examples and works.[60]

Usefulness also guided Ramus in selecting his favourite authors, as is e.g. shown in this quote on Heron:

> Therefore this author appeals to me first of all because he has connected Plato's geometry with the mechanics of Archimedes, and science with the application of science so ingeniously and diligently.[61]

Pure mathematics only had to be studied because it laid the foundation for applications.

Ramus tried to advance the mathematical sciences by delivering a number of orations in their favour, in which he pointed out to his French audience how some German princes, among whom was William of Hessen, had been helpful in the rebirth of the mathematical sciences, setting examples which should be followed in France.[62] Mathematics should, in particular, be fostered in the universities, a viewpoint not generally supported in Ramus's period, when mathematics mainly flourished outside the universities. He tried to have a specialized professorship in mathematics founded in France and actually paid for one through an inheritance. His significance for the history of mathematics is in the first place found in this unremitting defence and support of the mathematical sciences, which must certainly have stimulated the Snellii.[63]

Ramus's chief complaint against Euclid was that the *Elements* were not arranged according to his method, which he considered the natural one. He blamed him for not having followed the rules of logic (for the explanation and examples of a typical Ramist objection, that of 'hysterology', see p. 263). In his view, proof was a less desirable aspect of a mathematical exposition, which should

[59] [Mahoney, 1975, p. 288].

[60] '[...] mathematicum iudicemus, qui non solum propositiones arithmeticae et geometriae syllogismo aliquo concluserit, sed multo magis qui exemplo atque opere fructum arithmeticae, utilitatem geometriae praestiterit.' [Ramus, 1569, p. 111].

[61] 'Quamobrem iste mihi imprimis placet author, qui Platonis Geometriam cum Archimedis mechanica, qui artem cum artis usu tam solerter atque industrie coniunxerit.' [Ramus, 1569, p. 35].

[62] All relevant prefaces and orations have been reprinted in [Ramus and Talaeus, 1969].

[63] Cp. [Pantin, 2004, pp. 82–84].

only be deployed in very obscure cases. He replaced the deductive method by his own, which was meant to give insight into the truths of propositions by their ordering and by giving illustrative examples. Euclid especially annoyed him by not obeying Ramus's third law. Apparently, Ramus had not understood the structure of the *Elements*, in particular the way in which some general propositions were deduced from more special ones, and merely considered the proofs as (inadequate) explanations of the propositions. However, although Ramus strove after enhanced clarity by presenting the material according to his method, his definitions are sometimes rather obscure and the frequent absence of proof does not help in following the argument. Moreover, his plan to select only those parts of the *Elements* that would lay a foundation for applications in practice was not carried out consistently.[64]

For Ramus's purposes, mathematics was a field of finite measure, the contents of which could be rearranged or even rediscovered, but in principle not extended. He did not consider the possibility that some propositions of which the practical use was unknown in the present might be applied in the future. Ramus himself was not an advanced mathematician, for which reason he had to be assisted by several collaborators, who translated Greek manuscripts for him and helped him to learn mathematics. One of them was the German Fridericus Risnerus, who wrote a book on optics, which Willebrord Snellius would study closely (see section 2.9.6). Ramus's publications did not reach beyond elementary mathematics, which made his philosophy of mathematics less relevant for specialized mathematicians.

A somewhat intriguing part of Ramus's mathematics is his astronomy. Ramus advocated an 'astronomy without hypotheses': he claimed that the ancient Babylonians, Egyptians and early Greeks had been able to predict celestial phenomena without having recourse to hypotheses. This astronomy, which had been spoiled by the classical Greeks and later astronomers, had to be recovered, by means of logical derivations from experience. Although his contemporaries did not exactly understand what he meant, a number of them reacted to it or even took up the challenge, among whom were Tycho Brahe, Rothmannus and Kepler.[65]

In the above, only a few characteristics of Ramus's mathematics have been discussed. On many great and small topics, he had very outspoken opinions, some of which will be discussed in detail elsewhere in this book (see section 5.4.2 for Ramus on book X of the *Elements* and section 7.3 for Ramus on Heron's Theorem).

[64] See e.g. *Scholae Mathematicae* p. 100, quoted in [Oldrini, 1997, p. 82]. [Verdonk, 1966, pp. 327–328, 332–341, 349–350].
[65] [Jardine and Segonds, 2001].

2.6 Rudolph Snellius: Ramism embodied

Rudolph Snellius, who educated Willebrord as a son and a pupil, had a major influence on his son's career and a less strong, but still perceptible influence on his scholarly activities and ideas. Willebrord Snellius often presented himself as 'R.F.', *Rudolphi Filius*. Because of this influence, Rudolph's life and works will be discussed here to some extent.

2.6.1 Rudolph Snellius's life

Roeloff Snel van Royen, who would later latinize his name to Rudolphus or Rodolphus Snellius, was born on 8 October 1546[66] in Oudewater (between Gouda and Utrecht).[67] His father, who was called Willebrord Snel van Royen, lived on his private means; his mother was called Gloria Henrici.[68] After having attended school in Utrecht, Snellius came back to Oudewater because his father had died. He studied poetry at home, and when he was fifteen, he continued his education abroad, in Cologne, where he studied Hebrew, mathematics, and Aristotle's works.

After three years, he came home again, but he left again soon because of the death of his mother. He visited Hamburg, Jena, Wittenberg and other Saxon towns and subsequently studied Hebrew in Heidelberg, which he left because of the plague. In 1565, he registered as a student at the Calvinist University of Marburg, where he became familiar with and began to admire the philosophy of Petrus Ramus. For some years he taught Greek, Latin, Hebrew and the liberal arts at the Marburg *Paedagogium*, a preparatory school for the university. In 1572, he received the degree of *magister artium*. The influential theologian Jacobus Arminius was his special pupil, and again followed his courses in Leiden for a very short time. Undoubtedly, Snellius was the source for Arminius's enthusiastic Ramism.[69]

With his younger brother Rudolph then left for Italy, where he studied medicine in Pisa and Florence and visited Rome. He returned to Marburg to resume his teaching. Although contemporary sources assert his popularity, in one instance the university professors requested his removal. But he was protected

[66]See e.g. Snellius's own confirmation of this date in [Snellius, 1603].

[67]This section is largely based on [de Wreede, 2003a]. [de Waard, 1927a] brings together most of the biographical information. Important sources for Snellius are [Witkam, 1973, pp. 82–89], where a number of archival sources is quoted, [Molhuysen, 1913] and [Molhuysen, 1916]. [Alma, 1614, pp. 175–177] is a contemporary short biography; another slightly later biographical sketch was probably written by Willebrord, see [Meursius, 1625, pp. 117–121]. Rudolph Snellius's funeral oration by Coddaeus contains more or less the same information, [Coddaeus, 1613].

[68]This is the latinized form of Meursius's biography, [Meursius, 1625, p. 117].

[69][Bangs, 1971, pp. 37–38, 55].

by Lazarus Schonerus, the director of the Paedagogium and himself a professed Ramist. Schonerus also supported Snellius's work, as is shown for instance by a letter from Schonerus to Snellius included in one of Snellius's works.[70] Snellius returned to Oudewater in 1578, which in the meantime had been cruelly destroyed and its population massacred by the Spanish.[71] He married Machtelt Cornelisdochter there. Three sons were born to them of whom only the eldest, Willebrord, reached adulthood.

At the request of some students and the curators of the new University of Leiden, Rudolph went there to give some lectures (1578), although at first the curators did not give him an official position. In 1581, he was appointed as extraordinary professor of mathematics, under the condition, however, that this would last only until someone more experienced in mathematics became available.[72] Count Maurice of Nassau was one of his pupils. The students were not completely satisfied by his teaching, as is shown by a petition of 1582 directed against the *via compendiosa* (teaching by means of abridged texts instead of original sources) of the professors, in which the *Physica* of Valerius, which was used by Snellius in his courses, was explicitly mentioned.[73]

A small part of Rudolph Snellius's correspondence of this period, mainly with his German acquaintances, is still extant, giving some useful information about his life and work. Six of the seven letters which I have found seem to have been unknown to scholarship before I started my research, and their contents are presented here for the first time. An overview of the letters is given in table 2.1, quotations from the letters are especially given in section 4.2.[74]

In 1585–86, Snellius taught Hebrew in addition to mathematics. This extension of his classes was temporary, as his knowledge of Hebrew was only sufficient to explain the basics. His lectures were based on the Ramist grammar of Pierre Martinez; Snellius greatly appreciated its clear presentation.[75] Some of the mathematical writers which he taught were Ramus, Euclid (*Elements* and *Optics*), Aratus, Proclus, Dionysius Alexandrinus (*De Situ Orbis*), Manilius, Maestlin (*Planetarum Theorica*), Plinius and Pomponius Mela (both for geography). Probably, Snellius was responsible for the edition of Stadius's commentary of Ramus's *Arithmetica* that appeared in Leiden in 1584 (he may have used it in

[70] [Meursius, 1625, pp. 118–19], [Coddaeus, 1613, pp. 12–13], [Friedrich, 2000a, pp. 717, 731], [Snellius, 1587, fol. Av^r–Avi^r]. Lazarus Schonerus (1543–1607) made an edition of Ramus's *Arithmetica*, *Geometria*, [Ramus, 1599], and *Scholae Mathematicae*, [Ramus (L. Schonerus ed.), 1627]. Rudolph Snellius had once lived in his house, [Kalckhoff, 1750, fol. 313^v].

[71] [Bangs, 1971, pp. 37–43].

[72] [Molhuysen, 1913, p. 26].

[73] [Verbeek, 2001, p. 43].

[74] I thank prof. dr. Chris Heesakkers for his transcription of [Snellius, 1603] and [Snellius, 1609].

[75] [Molhuysen, 1913, p. 124*], [Meerhoff, 2003b, pp. 174–175], [Snellius, 1608].

Year	Correspondent	Source	Modern edition
1596	to Hatzfeld	[Kalckhoff, 1750, fol. 312^r–314^r]	
1597?	to Magnus	[Snellius, 1597], [Kalckhoff, 1750, 2, fol. 321^r–323^v]	
1603	to Hatzfeld	[Snellius, 1603], [Kalckhoff, 1750, fol. 314^v–316^r]	
1603	from Maurice of Hessen	[Kalckhoff, s a, 1, fol. 45^r–47^v]	
1608	to Rosendalius	[Snellius, 1608]	[Meerhoff, 2003b], [Vollgraff, 1914], facsimile in [van Berkel, 1987, p. 34]
1609	to Magnus	[Snellius, 1609], [Kalckhoff, 1750, 2, fol. 316^v–318^v]	
1612	from Magnus	[Kalckhoff, 1750, 2, fol. 319^r–320^v]	

Table 2.1: Rudolph Snellius's correspondence

his lessons before he had his own commentary printed) and of Pena's translation of Euclid's optical and catoptrical propositions (Leiden 1599).[76]

The teaching of Ramus, however, met with opposition: in 1592, Snellius received permission to diminish the number of his weekly lectures on the condition that he stopped lecturing on Ramus and discussed Euclid, Aratus, Proclus or another ancient author instead.[77] Although Snellius favoured Ramus, he was certainly also receptive to the use of other authors. Proof of this is seen in a large schedule with authors both ancient and modern on all parts of pure and mixed mathematics that Snellius drew up for Isaac Beeckman. Beeckman (1588–1637) would start his career as a producer of candles and water conduits, but later became a deputy headmaster and headmaster in several schools. He was an active mathematician and natural philosopher, with a special interest in experiments and the application of mathematics to physics, developing fruitful and original mechanistic theories.[78]

Snellius also kept a private school, until c. 1598, in which other teachers were employed as well.[79] Moreover, he received a large number of students as boarders in his home in the same period. This cannot always have been easy: in 1594, two of his lodgers were caught running amok, one of them with a sword and the other with stones.[80] Snellius may have quitted these activities when finances permitted. In 1598, he was also a candidate for the position of professor of physics, but without success, because the curators decided that two positions

76[Ramus (I. Stadius ed.), 1584], [Euclid, 1599]. See [Verdonk, 1966, pp. 112, 431–432].

77[Witkam, 1973, p. 88].

78[de Waard, 1953, pp. 17–19]. The list is probably from 1607, [van Berkel, 1983b, p. 25]. [Hooykaas, 1970].

79[Coddaeus, 1613, p. 16], [Witkam, 1973, p. 83].

80[Molhuysen, 1913, p. 81].

involved too much work for one person.[81]

Not until 1601 did he become a regular professor. Klaas van Berkel proposes an explanation for this late appointment. He asserts that

> Ramism as taught by Snellius was a plea for breaking down the social and intellectual barriers between the theoretical science inside the university and the practical arts outside.[82]

Snellius's final promotion to *ordinarius* may be connected with the foundation of the Leiden School for Engineers in 1600. The curators of the university may have thought that now that two lecturers in mathematics had been appointed in the School and the university of Franeker had a full professorship in mathematics, they could not lag behind and had to raise Snellius to the same level.[83] Van Berkel concludes:

> The early history of mathematics in Leiden does not show us how mathematics was *welcomed* in Leiden, but how it was forced upon a reluctant university.[84]

Although the sources are too scarce or too little studied to draw an accurate picture of the attitude of the members of Leiden University towards mathematics, or to distinguish that from their opinion of the person of Rudolph Snellius, Van Berkel's statements are certainly plausible.

In 1603, Snellius visited Hessen on the invitation of Landgrave Maurice, himself an admirer of Ramus (cp. section 4.2). Other contacts with Germany also continued during his Leiden period. Snellius was rector of Leiden University in 1607 and 1610, and served as an inspector and mathematics teacher in the 'Staten-college' where state-supported theology students lived. He was involved in the examinations of the engineers from the Leiden School, and was a member of several patent committees of the States General. After a period of deteriorating health (he had asthma), he died in Leiden in 1613 (see also p. 74). Coddaeus, the professor of Hebrew, delivered his funeral oration, in which he described Snellius as a defender of mathematics and the encyclopaedic arts.[85]

Some books from Rudolph's private library have been preserved, e.g. a Bible in Hebrew with his own annotations, Philippe de Mornay's *Tractatus de Ecclesia*, Scaliger's *Opus de Emendatione Temporum*, Cleomedes's *Meteora* (bought from Scaliger's library after his death), Aratus's *Phaenomena* to which he added many notes. They testify to Snellius's broad range of scholarship.[86]

[81] [Molhuysen, 1913, pp. 107, 113].
[82] [van Berkel, 1988a, p. 158].
[83] [Molhuysen, 1913, pp. 132, 136], [van Berkel, 1988a, pp. 157–161].
[84] [van Berkel, 1988a, p. 161].
[85] [Coddaeus, 1613, p. 10]. See [Meerhoff, 2003b, pp. 163–165] for an overview of the Ramist characteristics of the oration.
[86] The Bible is in the Municipal Archive in Amsterdam;

2.6.2 Works: education and dissemination

Rudolph Snellius's voluminous oeuvre has not been studied extensively by modern scholarship and it is therefore not possible now to give a general assessment of his contributions and opinions. Some of his work has been opened up by good articles by Klaas van Berkel and Kees Meerhoff. Van Berkel's bibliography of Rudolph Snellius's works is especially worth mentioning. It is a valuable survey, notably because Snellius's works are not well represented in Dutch libraries and therefore their titles and abodes had not been well known before this article appeared.[87]

Snellius was the most outspoken Dutch Ramist of his time. Although the work of some other Dutch scholars of the period shows Ramist influences—notably that of William Ames (1576–1633), an army preacher and later professor of theology in Franeker—Ramism was not a strong movement within the universities. Snellius's Ramism originates from his German period and it fits very well with the popularity which the works of Ramus and Ramist authors enjoyed as teaching material in Central Europe around 1600. These authors were mainly used in Calvinist, pre-university schools, like the Paedagogium in Marburg.[88] According to Willebrord, Ramus was Rudolph's 'Apollo'.[89]

The aim of Rudolph Snellius's works was rather to make accessible the views of Ramus and some other, mainly related, authors than to spread his own original thought. They cover more or less the entire field of the propaedeutic arts (he wrote on mathematics, dialectic, rhetoric, physics, ethics[90] and psychology),[91] and consist of commentaries by himself and other scholars and of his lectures. He was also preparing notes on Ramus's *Theologia*, a topic outside propaedeutics, but these did not appear in print. The reason for this seems to have been that his friend Magnus, who intended to have them printed, lost a copy of them, after which Snellius may not have had the opportunity to make a new copy before his

[de Mornay, seigneur DuPlessis-Marly, 1585], [Scaliger, 1598], [Cleomedes, 1605], [Aratus, s a].

[87] [van Berkel, 1983a, pp. 189–194]. Through modern catalogues, I was able to trace some more copies and reprints. Some small extensions are found in the bibliography by Kees Meerhoff: [Meerhoff, 2003a, pp. 34–36]. [van Berkel, 1983b, pp. 271–279], [van Berkel, 1987]; [Meerhoff, 2003a], [Meerhoff, 2003b]. Cp. [Vollgraff, 1913b, pp. 610–612], [Verbeek, 2001, pp. 38–40, 43].

[88] [Verbeek, 2001], [Freedman, 1999]. Cp. [Dibon, 1953].

[89] [Snellius, 1607b, p. 3].

[90] Ramus had not lived long enough to write an ethics textbook, although ethics was an essential part of the curriculum. Several of his followers filled the gap after his death, [Meerhoff, 2001b, p. 367].

[91] [Snellius, 1600], a contribution is included in a larger collection in [Cramerus and Goclenius, 1600], [Snellius, 1587], [Snellius, 1592], [Snellius, 1597a]; [Snellius, 1595], [Snellius, 1596a]; [Snellius, 1607a]; [Snellius, 1594], [Snellius, 1597b]; [Snellius, 1596b]. *Psychologia* was a neologism, only coined in 1575, [Meerhoff, 2003a, p. 17].

death.[92] Snellius also disseminated the Ramist legacy by propagating the work of his followers, as is for instance shown in his letter to his relative Rosendalius.[93]

During Rudolph's lifetime, many books appeared which bear his name. However, in some cases the publication was not authorized by him. According to his funeral oration, he could not be persuaded to publish his works by himself, and he took the initiative for the publication of only one of his books, the commentary to the rhetoric of Talaeus. Other works were published by friends or (former) students, who brought their lecture notes to the publishing companies, which apparently expected that Snellius's work would attract buyers. This is confirmed by, e.g., the note of the typographer in *Commentarius doctissimus*: Snellius had discussed this material in Marburg, and it had now been offered for print by his friends. Thus, it is likely that he was not involved in the publication of the books which appeared under his name in Germany in the period when he was already in Leiden. They may reflect courses that he had given in his German period, which were meant for pre-university students, a different audience than he had as a Leiden professor.

According to his friend Hatzfeld, some books allegedly written by Snellius were not actually his. However, it is not clear which these would be: all titles now known as Snellius's (and mentioned in this section) are given in his biography in *Athenae Batavae*. As Willebrord is presumably the author of this biography, it would mean that these are actually Rudolph's own works.[94] Hatzfeld's statement could mean that around 1600, some books were sold as Snellian productions that are now either unknown or attributed to other authors, because Rudolph Snellius's name was not mentioned on the title page.

Due to his Ramism, Snellius seems to have been more appreciated by his students than by learned colleagues like Lipsius and Scaliger, but the sources are not completely consistent. Scaliger, for example, scorned Snellius's vanity and when Snellius came to him to point out a mistake in a calculation, he told him: 'Ass, why should I reckon according to your method?' (for Scaliger and the Snellii, cp. section 2.8.1).[95]

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[92][Kalckhoff, 1750, fol. 319^r].

[93][Snellius, 1608].

[94]See Snellius to Hatzfeld, 1 June 1596, [Kalckhoff, 1750, fol. 312^v–313^r], and in reaction to this Hatzfeld to Magnus, 10 June 1596: 'Indignatur ut vides Commentaria sua promulgari. Et sane non immerito, cum multa pro Snellianis venduntur, quae ipsi nunquam in mentem venerunt.' [Kalckhoff, 1750, fol. 311^r]. Cp. [van Berkel, 1983a, p. 189]. [Coddaeus, 1613, p. 13]; the edition meant must be the one that appeared in Leiden in 1600, [Snellius, 1600]. [Meursius, 1625, pp. 118, 120], [Snellius, 1587, fol. *Aii*^{r}].

[95]'Qui demandera à luy [Gomarus] & à Snellius, si ce siecle portera de plus grands hommes que les precedens, il [sic] respondront sans doute qu'ouy, parce qu'ils pensent estre les plus sçavans.' [Scaliger, 1669, I, p. 95].

'Snellius me vint dire vne fois que ie m'estois trompé, parce que ie ne contois pas selon sa methode: ie le renuoyai & luy dis, Asine cur numerarem secundum tuam methodum?' [Scaliger, 1669, I, p. 228].

Moreover, Scaliger was at least doubtful about Ramus's merits. A letter of 1598 written by Scaliger to the curator Janus Dousa shows the former's concern about the intended admittance of the teaching of Ramus's doctrine at Leiden University (the exact meaning of this is not explained). Scaliger feared that a combat between Ramists and Aristotelians would ensue, to great harm of the university, and he asked Dousa to prevent this admittance. Although this source can be interpreted as a sign of Snellius's difficult position, it also serves to show that the university administration was positively inclined towards Ramism. Most professors seem not to have cared so much, and some were also in favour of Ramism. Rudolph Snellius's nomination for the professorship of physics by the senate also is a sign of the esteem in which he was held.[96]

In his commentary on Manilius, Scaliger spoke out against Ramus's astronomy without hypotheses and his belief in a superior pre-ancient astronomy, without mentioning Ramus's name. He seems to have inherited his anti-Ramism from his father, who had polemized against Ramus's mathematics. Yet Scaliger also respected Ramus's learning.[97] Lipsius expressed himself scornfully on Ramus and his teachings on several occasions: 'Never will he be great for whom Ramus is great'.[98] It is very well possible that the Ramist methodology, which served to make complex topics perspicuous, mainly appealed to the junior students in Leiden and that the likes of Scaliger propagated more advanced study material instead. Yet all taken together, Snellius's professed Ramism does not seem to have been an impediment for a respectful treatment by fellow professors and the university administration.[99]

Snellius's works clearly reflect his teaching. Much effort is spent to defining concepts, explaining them and (in true Ramist style) dichotomising them in order to make their meaning clearer. Use and usefulness of the subject matter are important themes. Ramus is both implicitly and explicitly present, also in

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[96][Heinsius, 1627, pp. 130–131], quoted in [Lebram, 1976, p. 348].

'Précisons encore que les rares indications que nous pouvons glaner sur l'attitude des professeurs de Leyde à l'égard du ramisme—tant dans les manuels que dans les disputes—ne nous permettent nullement de supposer que leur fidélité à Aristote se soit jamais traduite par une hostilité systématique et militante à l'égard de Ramus et de sa pédagogie.' [Dibon, 1954, p. 29].

[97]For Scaliger Senior's critique on Ramus's mathematics, see [Carpentarius, 1564, fol. 8^v–9^r]. Scaliger on Ramus: 'Bonus Orator, qui facultatem dicendi sibi comparaverat. ἀμέθοδος plane, ut pote qui aliud in disciplinis tradendis iter novamque viam inquirere voluit. Flumen verborum, guttula mentis. Religionis purioris quam profitebatur ergo, mortuus est anno 1572. [...] Thalaeus Ramo suo longe doctior.' [Scaliger, 1669, I, pp. 127–128].

'Aujourd'huy on ne fait estat que des Ramistes. Ramus estoit homme docte, mais on en fait trop grand estat. [...] Ramus estoit grand personnage; Ramus magnus fuit vir, sed magni nimis fit; ipsius Mathematica sola bona, sed ipse non est Autor. Ramus male scribebat.' [Scaliger, 1669, II, p. 201]. Cp. [Grafton, 1983, pp. 215–217].

[98]See e.g. [Sellberg, 1979, p. 22]: 'Numquam ille magnus erit, cui Ramus est magnus.' Cp. [Sellberg, 1979, p. 133], [van Berkel, 1983b, pp. 278–279].

[99]Klaas van Berkel speaks of a Ramist 'sub-culture' among younger students in Leiden, [van Berkel, 1983b, p. 279]. Not enough evidence exists for this statement.

books on other authors: e.g. some opinions on his philosophy are quoted, and his definitions for straight and parallel lines are used in the commentary on Valerius's *Sphaera*.[100] Due references are made to the classical authors. Large parts of Snellius's work can only be understood if accompanied by the commented texts, because they give a running commentary in which only enough of the original text is quoted to identify the relevant phrase.

Rudolph Snellius gave the most extended overview of his philosophical position in his *Snellio-Ramaeum philosophiae syntagma*, although the book does not cover as much material as its elaborate title promises, where a general treatment of Ramist philosophy, and discussions of dialectics, rhetoric, arithmetic, geometry, astronomy, physics, psychology and ethics were announced. Only the first part did actually appear. As to mathematics, Snellius discussed its position in the system of sciences briefly, not treating other topics. This book has been studied by Erland Sellberg, who actually used a large number of quotes to illustrate Ramus's and Ramist statements.

In this book, Snellius followed Ramus closely, yet he was no epigone of Ramus as other Ramists, that is, at least he claimed that he did not follow Ramus without using his own judgment as the 'pseudo-Ramists' did. As a good student of Ramus, he stated that ultimately 'truth, nature and reason' were the foundations of Ramus's philosophy, and not 'authority, opinion and belief'.[101] He would even exchange Ramus for some better author if someone would prove that the former's ideas were wrong. Paradoxically, Ramus's advocacy of a Socratic attitude led his students to questioning his own authority. They blamed the Aristotelians for preferring authority to reason. Ramus himself had emphasized that Aristotle distinguished himself favourably from his epigones as a lover of truth.

In his explanation of Ramus's first law, Snellius remarked that precepts had to be eternal and unchangeable.[102] The goal of this principle was educational, not directed to the discovery of new science, but to the presentation of what was already known. Even so, Ramus was aware that knowledge was in reality not obtained in the fixed order of his method. Yet, once it was collected, it should be arranged in the prescribed manner, because this corresponded to the order of

[100][Snellius, 1587, fol. *Aii*^{v}–*Aiiii*^{r}], [Snellius, 1596g, pp. 16–17].

I have not been able to find evidence that supports Klaas van Berkel's claim that Snellius assigned even greater importance to the work of artisans and engineers than Ramus, and that he made practice into the basis for theory, thus (together with other Ramists) transforming Ramus's educational ideal into a scientific ideal, [van Berkel, 1983b, p. 278].

[101]'Hoc igitur Petri Rami fundamentum est, quo Dialectica eius et tota philosophia nititur, nempe non autoritas, non opinio, non persuasio, sed veritas, sed natura ipsa, sed ratio.' [Snellius, 1596i, p. 115], quoted in [Sellberg, 1979, p. 103]. I have corrected Sellberg's copying mistakes in this and the next quotes. See also [Sellberg, 1979, pp. 106, 110, 131].

[102]'Lex κατὰ παντός ita dolat et polit artium praecepta, ut non patiatur illa esse aut falsa, aut contingentia, mutabiliaque, sed efficit vera, necessaria, aeterna, et immutabilia [...]', [Snellius, 1596i, p. 13], quoted in [Sellberg, 1979, p. 43]. Cp. [Sellberg, 1979, pp. 68–69].

nature. Of course, Snellius also stressed the role of the use of science in human life, which was the origin and final purpose of science.[103] In their actual use, the various disciplines were no longer kept separated.

Snellius pointed out some shortcomings of other Ramists: their preoccupation with method instead of content, their lack of knowledge of Aristotle's own and other classical works and their inclination to lump together Aristotle and his commentators. He wanted to have the philosophies of both Ramus and Aristotle explained and connected in education.[104] Thus, Snellius had a moderate point of view on all the hobbyhorses of other Ramists.

Ramus's advocacy of mathematics is reflected in the prominent position granted to the mathematical sciences by most Ramist authors in the philosophical system. It is noteworthy that their application of the Ramist method led to a number of different classifications of the arts. Rudolph Snellius classified the arts in those that give more, respectively less certain knowledge. The mathematical arts are counted among the former, physics among the latter.[105] Snellius's books on pure mathematics are again commentaries on Ramus's books, one on his geometrical and one on his arithmetical textbook.[106]

Snellius's commentary on the *Arithmetica* remained very close to the source text, and could not be consulted without a copy of Ramus's text at hand. Both the content of Ramus's text and the commentary are elementary. Snellius's annotation to the sieve of Eratosthenes, a procedure that Ramus had mentioned without further explanation, shows that he did not completely understand all the relevant mathematics.[107] He did add a rather long preface in which he explained what mathematics was, from where it had derived its name and what its purpose was. Mathematics was 'the art of quantifying well',[108] the word 'quantify' being chosen to apply both to arithmetic and to geometry. He contradicted the usual opinion that music and astronomy were part of the mathematical sciences with reference to the Ramist laws, considering them as physical sciences instead. Although all these ideas were borrowed from Ramus, his name is not mentioned. Some less contested authors are referred to, probably to give the text a more

[103]'Origo et initium philosophiae est ab usu: finis philosophiae est in usu: philosophia ipsa tendit ad vitae humanae usum ac fructum.' [Snellius, 1596i, p. 44], quoted in [Sellberg, 1979, p. 89].

[104]'Tam necessaria hoc tempore est sedula Rameae et Aristoteleae Philosophiae collatio et coniunctio, ut sine ea nemo perfectus hodie Philosophus evadere possit.' [Snellius, 1596i, p. 149], quoted in [Sellberg, 1979, p. 131] (erroneously as [Snellius, 1596i, p. 153]); cp. [van Berkel, 1987, p. 35].

[105][Freedman, 1999, pp. 102–105], [Sellberg, 1979, p. 42].

[106][Snellius, 1596e], [Snellius, 1596d]. The commentary on the *Sphaera* is sooner of a natural-philosophical than of a mathematical character, as is also indicated by Snellius in his introduction (see p. 144).

[107][Verdonk, 1966, p. 167]. See the index of the same book for references to more detailed and often critical commentaries on Rudolph's mathematical works.

[108]'Est autem Mathesis ars bene [...] quantitandi.' [Snellius, 1596d, pp. 3–4].

neutral dress. Ramus himself had also excluded astronomy from mathematics. Yet in the *Syntagma*, Snellius included some parts of mixed mathematics, such as mechanics, architecture and cosmography, in the mathematical sciences, whereas Ramus ranged all of these under physics.[109]

Snellius's commentary on the *Geometria* covers mainly elementary material. According to Verdonk, it contains some errors.[110] In his preface, Snellius paid much attention to the historical background of the mathematical disciplines, their name, content and use. All this is probably borrowed from Ramus's *Scholae Mathematicae*. Especially striking is the large amount of verbal explanation and the lacking of technical geometrical parts in the book. Some simple figures are included, yet—as in Ramus's own book—there is no room for intricate constructions and figures. Snellius tried to explain the stereometrical part with hardly any figures. He circumvented some more complicated issues, e.g. in the case of Heron's Theorem (see section 7.4).

Snellius wrote commentaries on works of Cornelius Valerius, Philippus Melanchthon and Audomarus Talaeus, too. Valerius and Melanchthon were certainly no full-blooded Ramists, which again shows that Snellius was not a dogmatic Ramist. The ideas of Melanchthon had dominated Marburg University. In Germany, the reformations of the curriculum by Melanchthon and Ramus often merged in the so-called Philippo-Ramism.[111]

After Rudolph Snellius's death, his popularity seems to have dwindled quickly. Two of his books were reprinted. One of them was printed in Hanau,[112] the other one in Leiden. In this second case, Willebrord Snellius was probably somehow involved, although his name is not mentioned in the book. The book was called *Commentarius in Rhetoricam Audomari Talaei*, a reprint of *Dialogismus rhetoricus, qui commentarius sit in Audomari Talaei Rhetoricam e praelectionibus Petri Rami observatam*, and is again a unambiguously Ramist work. The reprint was dedicated to Nicolaus Zeistius, a Leiden regent.[113] It is written in the shape of a dialogue between Rudolph Snellius and Joannes Magnus, who once was, according to Snellius, his favourite pupil, and later (in 1600, when the book was written) counsellor of Maurice of Hessen.[114]

That Willebrord was the initiator, or at least supported the reprint, can be inferred from the *Eratosthenes Batavus*. In a passage about the downfall of Oudewater, Snellius quoted his father's description of this catastrophe from his 'very accomplished' book. In the new edition of *Eratosthenes Batavus* which Willebrord was preparing, he wanted to add a sentence about Magnus, his fa-

[109] [Sellberg, 1979, p. 41].
[110] [Snellius, 1596e], [Verdonk, 1966, p. 229].
[111] [Snellius, 1596h], [Snellius, 1596g], [Snellius, 1596f], [Snellius, 1596b], [Snellius, 1596a], [van Berkel, 1983b, p. 277].
[112] [Snellius, 1614].
[113] [Snellius, 1617a], [Snellius, 1600].
[114] [Snellius, 1617a, p. 1].

ther's old friend and the Landgrave's counsellor. Willebrord may have republished this book as a sort of gift for Magnus, with whom he was also acquainted, and Maurice of Hessen, whose grandfather Philip was praised in the book.[115] In that case, it would be the prelude to a patronage relationship of his own with Rudolph's old patron Maurice of Hessen (for which see further section 4.3).

Judging Rudolph's mathematical abilities from his publications, we assume that he could teach his son and his students the foundations of mathematics, but that they had to look for more specialized knowledge elsewhere. He seems to have been more interested in the role of mathematics in the whole spectrum of sciences than in (advanced) mathematics itself. Although his Ramism was not shared by the other professors, his position in the university seems to have been unproblematic in general, yet not very distinguished. His position enabled him to further his son's career, but Willebrord's own exertions were also indispensable.

2.7 1600–1603: the travelling mathematician

As soon as Willebrord Snellius had reached adulthood, he decided not to stay in Leiden but to travel abroad for some years. Apparently, his teaching duty oppressed him:

> However, as soon as he noticed that people were trying to secure that he, bound in this way, would never be allowed to set a foot outside his country, or to go away from home anywhere, he obtained permission of his parents to leave, pointing at his age as a reason (for he had only just turned nineteen).[116]

According to Jachaeus, he went abroad because he felt confined by the downcast and insignificant Dutch spirit.[117] Emotional motives do not have to offer the complete explanation of his decision: such an educational travel was very common for young men of the elite. However, a difference between him and the majority of travelling students must have been his interest in what he could learn as a future scholar instead of in merely gaining life experience.

In 1600, Willebrord left for Germany, where he became acquainted with many men of consequence, who remained in his network after he had returned home.[118]

[115][Snellius, 1617c, p. 179], [Snellius, 1617a, pp. 160–162].

[116]'Cum vero iam id agi sentiret, ut hac ratione astricto nusquam e patria pedem efferre, aut quoquam peregre discedere liceret, aetatem causatus, (vix enim decimum nonum aetatis annum expleverat) a parentibus missionem impetravit.' [Meursius, 1625, p. 297]. [Meursius, 1625, pp. 297–298] is again the main source for this part of Snellius's biography; it was probably written by Snellius (see footnote 2). Cp. [Iachaeus, 1626, pp. 7–13].

[117]'abiecti siquidem et pusilli animi esse credens angusto hoc Bataviae theatro circumscribi [...]', [Iachaeus, 1626, p. 7].

[118]'[...] multis et claris viris ibi tum innotuit, et charus vixit, cum quibus etiam absens postmodum arctam amicitiam coluit.' [Meursius, 1625, p. 297].

We know the names of some of them. First, Snellius visited Adrianus Romanus in Würzburg. Romanus (1561–1615), or Adriaan van Roomen in his native tongue, was an able mathematician and physician, originating from Louvain. His broad interests and knowledge characterised him as a humanist scholar. His main contributions to mathematics are in the fields of trigonometry and *mathesis universalis*. When Snellius visited him, he was professor of medicine at the university of Würzburg. Among his correspondents were Ludolph van Ceulen, François Viète and Christopher Clavius. He stimulated Van Ceulen to solve certain problems,[119] and he wrote to Clavius about Van Ceulen's and Stevin's achievements. Thus, it is not unlikely that he already proposed to his guest to make their Dutch works accessible to foreign scholars by means of a Latin translation.

Snellius then went to Prague, where his patron Romanus, who was summoned by Emperor Rudolph II, introduced him to Tycho Brahe. Romanus was very impressed by the precious collection of scientific instruments of the famous astronomer. Snellius's visit to Tycho was the postponed fulfilment of half of the exchange plan of some years earlier. He stayed with Tycho for some time, assisting him with his observations and other astronomical work. Tycho's environment was an excellent place to master astronomy further, and Snellius must have acquired many useful skills and knowledge there. In this way he prevented that he would become, in Jachaeus's words, one of 'those that had to be ridiculed because they thought that they could become judicious astronomers on the basis of literature alone without observing the stars'.[120] Snellius's later work in astronomy and surveying shows Tycho's influence. The internship was probably ended by Tycho's death on 24 October 1601, to which Snellius may have been a witness. In Prague Snellius met also Johannes Kepler and Otho Valentinus. The German Kepler (1571–1630) was a very ingenious mathematician and astronomer, with a special inclination to astrology, and moreover 'one of the most distinguished humanist scholars of his time'.[121] He was Tycho's research assistant at the time. The connection with Kepler would remain intact the rest of Snellius's life. Kepler mentioned Snellius several times in his works, both with approval and disapproval (see sections 2.9.1, 2.9.4 and 5.4).

Snellius would later publish some of Tycho Brahe's observations of 1599–1601, of which he probably made an extract during his visit, in his *Observationes Hassiacae* (cp. section 4.3.2).[122] He added a description of Tycho's last illness and death to these observations, which description was a copy from the account

[119] [Bockstaele, 1976, pp. 85–89], [Bosmans, 1910, pp. 114 ff.].
[120] '[...] ridendi, qui sine observatione stellarum cordatum Astronomum ex sola lectione fieri autumant.' [Iachaeus, 1626, pp. 9–10].
[121] [Grafton, 1991b, p. 182]; [North, 1994, pp. 312–326].
[122] [Bockstaele, 1976, p. 267]. Cp. Caspar in [Kepler (M. Caspar ed.), 1959, p. 548].

by Kepler, in the observation ledger.[123]

Kepler read and used the *Observationes Hassiacae*, as can be inferred from several references in his correspondence and in his *Tabulae Rudolphinae*.[124] He was occupied by the question whether Snellius had possessed the original manuscripts of the observations that he had edited, or just a copy. In 1628 Kepler asked Georgius Brahe, Tycho's son, whether he could try and obtain Tycho's own notebook from 1600 and 1601 from Snellius's widow. He wrongly supposed that someone had brought the book to Leiden during the international political troubles of 1618. Fearing that part of Tycho's own observations were lost, he wrote that 'it would be a shame that we should then content ourselves with just the excerpts of Snellius and trust his edition'.[125] Kepler's worries were unnecessary: he received the requested volume from Tycho's heirs.[126] It is therefore most likely that Snellius copied the notes while he was in Bohemia and that he was present when Tycho died.

After this episode, Snellius continued his *peregrinatio* to other German towns and universities, still accompanied by Adrianus Romanus. He befriended other scholars, with whom he would later exchange letters. Among them was Joannes Praetorius in Altdorf, whom Romanus described as an 'eminent mathematician'. Praetorius (1537–1616), after having been an instrument-maker, and a mathematics teacher in Wittenberg, was appointed to teach mathematics at the university of Altdorf. He invited Snellius to study quadrilaterals inscribed in circles, the results of which Snellius would later publish as an excursus in Van Ceulen's *Fundamenta* (see section 7.6 for the mathematics).[127] In Tübingen he encountered Maestlin. Michael Maestlin (1550–1631) was a mathematician with a special interest in astronomy, who was professor in these fields at the University of Tübingen. He counted Johannes Kepler and Wilhelm Schickard among his students.[128]

Snellius also stayed in Hersfeld (now Bad Hersfeld, in Hessen), where he was received by his father's friend and former student Wilhelm Hatzfeld and by Christophorus Vulteius.[129] There he could make a 'pilgrimage' to the tomb

[123] [Snellius, 1618, pp. 83–84], edited in [Dreyer, 1963, pp. 386–387]. Dreyer mentions no other publication of the text than Snellius's, [Dreyer, 1963, p. 310].

[124] Quietanus to Kepler, 20 October 1618, [Kepler (M. Caspar ed.), 1955, p. 277], Kepler to Guldin, 10 December 1625, [Kepler (M. Caspar ed.), 1959, p. 254] and 7 February 1626, [Kepler (M. Caspar ed.), 1959, p. 258], [Kepler (F. Hammer ed.), 1969, p. 44].

[125] '[...] das wär eine Schand das wir uns alsodan der blössigen Excerptorum Snellii betragen und seiner Edition trauen müesten.' [Kepler (M. Caspar ed.), 1959, p. 362]. Letter of 17 August 1628, both published in [Kepler (M. Caspar ed.), 1959, pp. 361–363] and (in abridged form) in [Dreyer, 1972b, pp. 296–297].

[126] This can be deduced from its location in Kepler's legacy, see Caspar in [Kepler (M. Caspar ed.), 1959, p. 548].

[127] [Bockstaele, 1976, p. 267], [van Ceulen, 1615b, p. 190].

[128] [Bockstaele, 1976, pp. 106–107], [Betsch, 1996, p. 121].

[129] [Snellius, 1603].

of Ramus's student Risnerus, whose optical treatise Snellius would later study profoundly (see section 2.9.6).[130] It was probably during this same visit that he became acquainted with another former student of his father, Joannes Magnus.[131]

In the meantime Rudolph had bought a new house in Leiden, on the Koepoortsgracht, now Doezastraat, on 21 July 1601. He paid three thousand guilders for this big house with five fireplaces, which was a large amount when compared to his salary of 400 guilders per year. He could not afford to pay this sum at once and therefore arranged to redeem it in terms between 1601 and 1609. After 1603, he stopped taking in boarders, and his son would never have them.

Rudolph did not live in complete peace with his neighbours, with several of whom he had quarrels. Already on 21 August 1601, he had an argument with his neighbour about the ownership of their boundary wall. In 1606, another neighbour complained twice that Rudolph had clogged the water outlet.[132] In 1612 Rudolph bought an adjacent open yard for 600 guilders. Willebrord would live in this house until his own death. The house is no longer extant, because it was demolished by the explosion of a gunpowder ship in 1807.[133]

Willebrord came back to Leiden in the early spring of 1602, because his father called him back, no longer able to bear his absence.[134] He stayed at home for a year, in which time he prepared his first publication. This did not yet contain his own work, but a summary of Ramus's *Geometria*, entitled *Petri Rami Geometriae Libri XXVII* ('Twenty-seven books of Geometry by Petrus Ramus'). The booklet was published in 1604 and again in 1612.[135]

Snellius dedicated it to his recent tutor Adrianus Romanus, whom he addressed as the 'apple of the eye of the mathematicians'. In the short dedicatory letter, three typically Ramist themes are touched upon: method, usefulness and an inimical reception of the work.[136] The booklet itself only contains a sum-

[130] Snellius's note on the tomb in his own copy of the *Optica* suggests that he had seen it himself. See [Risnerus, 1606, title page] or the reproduction in [Vollgraff, 1918].

[131] On 7 February 1612 Magnus wrote to Rudolph Snellius: 'Filium tuum Wildebordum doctissimum, et mihi charum meo nomine quaeso peramanter salutes, meque apud illum commendes.' [Kalckhoff, 1750, fol. 320^r]. For Hatzfeld and Magnus as former students of Rudolph, see the letter of Hatzfeld to Magnus, 10 June 1596, [Kalckhoff, 1750, vol. 2, fol. 309^r].

[132] [Rechterlijk Archief, 1601], [Rechterlijk Archief, 1606b], [Rechterlijk Archief, 1606a].

[133] [van Geer, 1884a, p. 123], [Haasbroek, 1968, p. 60].

[134] '[...] pius parens desiderium filii, et tantae quidem indolis amplius non ferens, domum ad se revocat [...]', [Iachaeus, 1626, p. 10].

[135] The dedication letter is subscribed Leiden, 9 November 1602, [Ramus (W. Snellius), 1612, fol. $A3^r$]. The first edition is [Ramus (W. Snellius), 1604], and there is a reprint in J.H. Alsted, *Compendium Philosophicum*, Herborn 1626, [Verdonk, 1966, pp. 226–227].

[136] 'mathematicorum ocellus'; '[...] doctissimus quisque in id nervos intendat suos, ut veterum dogmata qua veritati consentanea sunt methodo, eaque quam brevissima, sic comprehendat, ne qua cuiusquam artis particulae umbra illis confusa fuisset [...]', '[...] methodum istam quae rem a summo capite arcessit et in sua membra diducit [...]'; '[...] cuius usus in terrae, maris, et orbium coelestium dimensione iisdem quibus solis radii terminis circum-

mary in which Ramus's propositions, definitions and corollaries are given, but no proofs, explanations or figures. Ramus's geometry textbook was especially suitable for being summarized because of the subordinate value that he attached to proofs. The right disposition of the material gave more insight than anything else and exactly this characteristic was maintained in a summary. The booklet must have been used in education. One example of a pupil who learnt geometry from it is Joannes Broscius (cp. section 5.4.3 for Broscius).[137]

Snellius felt still too young and restless to settle down permanently, 'almost wasting away at home',[138] and he persuaded his parents to let him go to France. In 1603 he went to Paris to study law again, which was the normal study for a future magistrate. His background and family connections made this career path a realistic option.[139] Here, he extended his network as well,[140] although he stayed for only a few months. One of his new acquaintances must have been Jacobus Alealmus, whom Snellius later publicly claimed as his friend.[141] He must have arrived just too late to meet the greatest pure mathematician of the era, François Viète, to whose work he would later appreciatively refer in many of his publications (see p. 302).[142] Alealmus had been a fellow student of Viète, and became prefect of military engineering under King Henry IV, according to Snellius.

At home Rudolph Snellius was preparing a trip to his patron Landgrave Maurice of Hessen and some of his (Rudolph's) old German friends (see section 4.2). Once he was at the court in Kassel, in the summer of 1603, he made Willebrord come there as well, realizing the value of the patronage of the Landgrave for the

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scribitur.'; '[...] tibi hoc munus licet ab autore et a te adversus invidorum malevolentiam bene munitum sit [...]'. [Ramus (W. Snellius), 1612, fols. $A2^r$–$A3^r$].

[137]For Broscius, see [Broscius, 1652, pp. 41, 63] and a letter of Broscius to Romanus, [Bockstaele, 1976, pp. 292–294]. The function in education is also concluded by [Verdonk, 1966, p. 226], who writes that 'de uitgave geen grote betekenis heeft', apparently meaning 'significance' in a strictly mathematical way, that is, ignoring its significance for the spread of Ramus's mathematics.

Both copies that I have been able to consult (University Library Amsterdam, and Lincoln Cathedral Library) were preceded by *Petri Rami ... Arithmetices Libri duo*, Hanoviae 1611. This work has the same organization, but no dedication. Its editor is unknown; they contain the propositions in the shape which Lazarus Schonerus had given to them, [Verdonk, 1966, p. 114]. The addition on the title page 'nunc primum hac manuali forma in gratiam studiosae iuventutis in lucem editi' proves that this compendium was certainly meant for students.

[138]'[...] domi quasi contabesceret [...]', [Iachaeus, 1626, p. 10].

[139]'Nam cum ex antiqua et opulenta familia originem duceret, haberetque in Republica summos viros sanguine iunctos, difficile non fuit illi ad honores grassari.' [Iachaeus, 1626, p. 11].

[140]'[...] multum renitentibus ibi amicis, et claris viris, et Professoribus Regis, quique eum postea per literas saepe revocare conati sunt [...]', [Meursius, 1625, p. 298].

[141][Snellius, 1617b, p. 103].

[142]Viète died 23 February 1603. De Waard uses an inscription in the *album amicorum* of Rivetus to show that Willebrord was in Paris in 1602, [de Waard, 1927b, c. 1155], but he is mistaken: the inscription to which he refers is by Rudolph, and was written in Leiden, [Rivet, 1896, p. 334].

career of his son. Willebrord then gave up his studies, probably not regretting his second, and this time final, farewell to a judicial or administrative career, because he neither returned to Paris nor to his study of law afterwards. For some reason, however, Rudolph did not wait for his son, but went home before his arrival. Nevertheless, it is very likely that Willebrord visited the court in Kassel, and in particular its observatory, and continued his astronomical studies by making an excerpt of the observations of William of Hessen (Maurice's father) and his collaborators (cp. section 4.3.2).[143] He then travelled to the Swiss Alps via the Rhine and visited Basel, Strasburg, Cologne and Worms.

After this double *Grand Tour*, Willebrord Snellius had seen enough of the world to last a lifetime. From then on, he would spend almost all his time in Leiden, making only occasional trips, mostly to other parts of Holland. He must have considered his period abroad as crucial for his development, which is shown by the relative length of the account of these travels in Snellius's autobiography: more than half of it is devoted to this part of his life, which only lasted for three years. Crucial it was indeed: Willebrord extended both his mathematical and astronomical knowledge, and his network of scientists, and decided to abolish a career as a lawyer or an administrator.

Rudolph Snellius was both the instigator of Willebrord's study of the law and its preventer, allowing him to travel to men of mathematics abroad, probably preparing his son's contacts with them, and even asking him to interrupt his stay in Paris. Both father and son must have realized at some moment that Willebrord's main talents were in mathematics, and they must have relied on his being able to make a living out of that—by becoming a professor at some university—after having acquired enough skills, friends and patrons.

Whether he would have made a good magistrate and whether his network of members of the local elite in Leiden, Oudewater or Schoonhoven was powerful enough for such an alternative career cannot be said. It is irrelevant, because he eventually managed to become a successful professor of mathematics. It will be explained in the next section that his path towards that success was not strewn with roses.

2.8 1603–1613: preparing the professorship

2.8.1 First steps in teaching and publishing

Willebrord dedicated the next decade to a multitude of activities, which almost all show his growing familiarity with large parts of the mathematical sciences. He gradually became an appreciated member of the Leiden academic community in his own right, no longer only acting as his father's son, but also as his colleague.

[143] [Snellius, 1618, p. 85].

However, he had to wait for his father's retirement until he could procure a decent official position.[144]

When Willebrord was back in Leiden from his *peregrinatio*, he assisted his father, whose health was deteriorating, by teaching mathematical courses at the university and private lessons.[145] Rudolph taught geometry, geography and optics, whereas Willebrord gave the courses in arithmetic and astronomy, thus putting to use the astronomical expertise that he had obtained abroad. He continued to develop this expertise, for instance by observing the comet which appeared in 1607.[146] According to Willebrord's funeral oration, father and son operated so well as a team that crowds of students rushed together from all corners of Europe to listen to them.[147]

Willebrord's teaching tasks acquired a more official status in 1609, when the University Senate decided that he would teach at 8 o'clock every morning on Saturdays.[148] In July 1610, the Senate made young Snellius teach the fourth hour of the afternoon on weekdays henceforth. It may not have been a coincidence that this happened while Rudolph Snellius was rector of the university. The curators, realizing that Willebrord was working hard for little money, gave him an emolument of 150 guilders in 1612, yet they did not want to appoint a second professor of mathematics. To keep him satisfied, they promised him that if Rudolph were to retire in the future, they would recommend Willebrord as his successor.[149]

His teaching load left Willebrord some time for other work, and he began his career as a prolific producer of books (see table 2.2 for all works written, edited or translated by him; see the bibliography for details). He translated Simon Stevin's enormous *Wisconstige Gedachtenissen* into Latin as *Hypomnemata Mathematica*. The publication is dated partly 1605, partly 1608; thus Snellius's translation activities must have occupied him during several years.[150] Stevin had written down his lessons to Maurice of Nassau in this book, also including some questions and solutions to mathematical problems by his student. The Latin translation made the book available to an international audience, which Stevin and Maurice must have thought desirable.[151]

[144] From this period of his life onward, the autobiography and the funeral oration do not give much information. The main source of Snellius's activities are now his own books. These are supplemented by some letters, some archival material and references in the work of others.

[145] [Iachaeus, 1626, p. 13].

[146] [Iachaeus, 1626, p. 14]; for Rudolph Snellius's lessons in optics, see p. 68. [Snellius, 1619, pp. 19–20].

[147] 'certatimque ex omnibus Europae angulis huc ad tantos viros audiendos concurritur.' [Iachaeus, 1626, p. 14].

[148] [Molhuysen, 1913, p. 182]; cp. [Meursius, 1625, p. 298].

[149] [Molhuysen, 1916, pp. 2, 41, 42].

[150] [Stevin, 1608b], [Stevin, 1608a].

[151] In his *Parallelon*, Hugo Grotius suggested that Maurice had insisted on a translation, but Dijksterhuis doubts this, [Dijksterhuis, 1943, p. 330].

Maurice was one of the two most powerful men of the Dutch republic (the other being Johan van Oldenbarnevelt, the Grand Pensionary, 'raadpensionaris' of Holland). Maurice and his cousin Willem Lodewijk had managed to reform the army of the Republic into a winning one, stimulating discipline, developing better strategies and exploring and applying technical means of warfare, such as building good works of fortification and digging trenches. Willem Lodewijk found his inspiration mainly in the classical sources, Maurice in mathematics, by which he was passioned. He was most of all interested in its practical applications, but also practised it for its own sake. He even took the *Wisconstige Gedachtenissen* with him on his campaigns, found new solutions for some old astronomical problems and discovered an improvement of the compass.[152]

Snellius mastered Latin so well that he was very suitable for this translation job. In general, the translation is a faithful rendition of Stevin's text. Snellius dedicated the book to its obvious patron, Count Maurice. In the long (six pages) letter of dedication, he gave an overview of the applications of different parts of mathematics—arithmetic, geometry, optics, astronomy, geography, mechanics—to warfare, supported by mainly classical examples. The explicit statement that the typographer had paid for the publication himself was certainly meant to stimulate Maurice to give a grant.[153] This dedicatory letter is the first proof of Snellius's command of the humanist discourse.

Snellius also worked on his first own mathematical publications in this period. These were reconstructions of three lost treatises of the Greek mathematician Apollonius of Perga (c. 200 BC). These treatises belong to the domain of 'analysis', a part of pure geometry unconnected to practical mathematics. The method of analysis aroused much early modern curiosity, which could not be satisfied because many of its applications either had never been written down, or else the relevant manuscripts were lost (see section 5.2.2). Snellius described Apollonius as the mathematician who had made analysis perfect.[154]

The full titles of Snellius's treatises are *Περὶ λόγου ἀποτομῆς, καὶ περι χορίου ἀποτομῆς resuscitata geometria* ('The Revived Geometry of *Cutting off of a Ratio* and *Cutting off of an Area*'), published in 1607, and *Apollonius Batavus, seu, Exsuscitata Apollonii Pergaei Περὶ διωρισμένης τομῆς Geometria* ('The Dutch Apollonius, or the re-awakened Geometry of Apollonius of Perga of the *Determinate Section*') of 1608. These titles are abbreviated by me to *Cutting off* and *Apollonius Batavus*. The former book contains the reconstructions of the two Apollonian treatises *Cutting off of a Ratio* and *Cutting off of an Area*, which titles I also use to refer to the two parts of Snellius's text. *Apollonius Batavus* contains the reconstruction of the *Determinate Section*. According to Meursius,

[152] [van Deursen, 2000, p. 77], [Dijksterhuis, 1943, pp. 321–332].
[153] [Stevin, 1608a, fol. $+2^r$–$+4^v$].
[154] [Snellius, 1607b, p. 4].

Year	Title	Remarks	Bibliographical key
1604	*Geometriae Libri XXVII*	summary of book by Ramus	[Ramus (W. Snellius), 1604]
1605/08	*Hypomnemata Mathematica*	translation of work by Stevin	[Stevin, 1608a]
1607	*Περὶ λόγου ἀποτομῆς, καὶ περι χορίου ἀποτομῆς*		[Snellius, 1607b]
1608	*Theses Philosophicae*		[Snellius, 1608b]
1608	*Apollonius Batavus*		[Snellius, 1608a]
1612	*De Maculis in Sole Animadversis*	anonymous (by Snellius)	[Snellius (attr.), 1612]
1613	*De Re Nummaria Liber Singularis*		[Snellius, 1613]
1613	*Arithmeticae Libri Duo*	edition of and commentary to book of Ramus	[Ramus (W. Snellius ed.), 1613]
1615	*Fundamenta Arithmetica et Geometrica*	translation of work by Van Ceulen, commentaries added by Snellius	[van Ceulen, 1615b]
1616	*De Re Nummaria Dissertatio*	edition of book by Scaliger	[Scaliger (W. Snellius), 1616]
1617	*Eratosthenes Batavus*		[Snellius, 1617b]
1618	*Coeli et Siderum … Observationes Hassiacae* *Et Spicilegium … Tychonis Brahe*	edited by Snellius	[Snellius, 1618]
1619	*Descriptio Cometae* *Chr. Rhotmanni … Descriptio … Cometae*	first part written by Snellius, second part edited by Snellius	[Snellius, 1619]
1619	*De Circulo et Adscriptis Liber*	translation of work by Van Ceulen, appendix added	[van Ceulen, 1619]
1621	*Cyclometricus*		[Snellius, 1621]
1622	*Meetkonst, in XXVII boecken vervat*	translation by Houtman of book by Ramus, supervised by Snellius, appendix added	[Ramus, 1622]
1624	*Tiphys Batavus*		[Snellius, 1624]
1626	*Canon Triangulorum*		[Snellius, 1626]
1627	*Doctrina Triangulorum Canonica*	written by Snellius, edited by Hortensius	[Snellius, 1627]

Table 2.2: Snellius's published works (the works without remarks are completely his)

Snellius had also prepared a work on Apollonius's *Vergings*. This was, however, never published and no trace remains of it.[155]

Although Apollonius's main work, *Conics*, does not belong to plane geometry, the treatises studied by Snellius do, and therefore, all their problems can be solved by ruler and compass (cp. section 5.2). As an example of the kind of problem that Snellius solved in these treatises, the general problem of *Cutting off of a Ratio* will be given here. In his own words, this is:

> To draw a straight line through a given point, which from two straight lines given in position cuts off segments, which end in points given on them, in a required ratio.[156]

In modern language, the problem can be formulated as follows:

Problem 2.1 (*Cutting off of a Ratio*) *Given a point A, two lines in position l_1 and l_2, two points on these U_1 and U_2 and a ratio $\rho = \alpha : \beta$. It is required to determine points A_1 on l_1 and A_2 on l_2 such that A, A_1 and A_2 are on one straight line and $A_1U_1 : A_2U_2 = \rho$ (see figure 2.1).*

As most analytical treatises were lost, Snellius and his contemporaries had to take recourse to the late classical work of Pappus, who had become a well-known author among mathematicians since Commandino had published a Latin translation of his main mathematical work, the *Collection*, in 1588.[157] Pappus gave short summaries of most Greek analytical treatises and explained some of the lemmas that should be used for the solution of their problems.

Remarkably, Snellius did not use this text, but a Greek manuscript. No printed edition was yet available of the Greek text. Scaliger, a very eminent Greek scholar, was the person who provided Snellius with his own manuscript of Pappus and may also have taken the initiative for this enterprise. Snellius expressed his gratitude for this generosity in *Cutting off*.[158] A small mystery is

[155]'[...] quibus et quartum, iam diu perfectum, περὶ νεύσεων de lineis inclinatis propediem prima quaque occasione adiiciet.' [Alma, 1614, p. 225].

Struik may have had this reference in mind when he wrote: 'Snel's work [sc. on Apollonius] was in three parts: the first remained in manuscript and is preserved at the library of the University of Leiden [...]', [Struik, 1975, p. 500]. Such a manuscript, however, is not to be found in this library. De Waard's remark that a copy of *Cutting off of a Ratio* with notes from Snellius himself is kept in the same library cannot be substantiated either, [de Waard, 1927b, 1156]. I thank dr. Anton van der Lem from the Leiden University library for his assistance in the search for these sources.

[156]'Per datum punctum rectam educere, quae a duabus rectis positione datis ad data in illis puncta absumat segmenta in ratione imperata.' [Snellius, 1607b, p. 8].

[157][Pappus of Alexandria, 1588]; [Pappus of Alexandria, 1986] is a modern edition and translation of the relevant part.

[158]'Horum igitur argumentum, et primum quidem περὶ λόγου αποτομῆς, ex codice manuscripto (quem liberalitate et munificentia Ill. viri Iosephi Scaligeri usurpavimus) descriptum ante oculos ponimus.' [Snellius, 1607b, p. 7]. This manuscript is still present in the Leiden University library, [Pappus of Alexandria, s a].

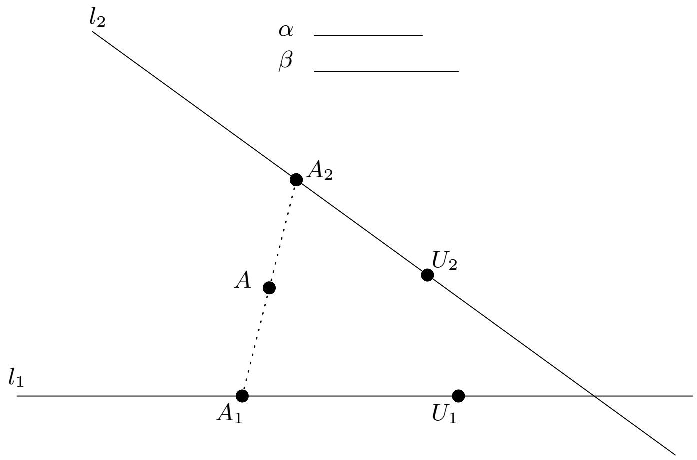

Figure 2.1: The general problem of *Cutting off of a Ratio*

connected to the manuscript. It is identical, in text and layout, to the manuscript copy of Pappus that Ramus's collaborator Nancel had prepared and translated for Ramus. However, when Nancel asked Scaliger in a letter of 1594 to publish this Pappus manuscript and some other manuscripts, which, as far as he knew, were in the hands of Scaliger's friend De Thou by then, Scaliger flatly denied that he owned the Nancel copy of Pappus, asserting that his manuscript of Pappus originated from the same place whence Ramus had his manuscripts copied (without specifying this place). This could be true, but it is more likely that Scaliger had the Nancel copy copied, yet wanted to have the free disposal of his own manuscript without obligations towards Nancel.[159]

Snellius was the first to publish parts of the Greek text of the *Collectio*.[160] He did not just copy the text that he found, but also made some emendations, thus showing some Greek scholarship himself. These philological skills were remarkable in a mathematician.

Scaliger also wrote a preliminary poem for the *Apollonius Batavus*, in which he made Apollonius praise Snellius as the person who gave him eternal life.[161]

[159] [Verdonk, 1966, pp. 53–54, 400, 410–411], [Heinsius, 1627, pp. 424–425].

[160] Based on the list in [Pappus of Alexandria, 1986, pp. 63–64].

[161]

> Clara Mathematici fax Luminis, ortus ad illa
> Litora, Pamphyli quae ferit ira sali,
> Evocor in lucem tenebris *excitus* ab imis,
> Dum me Snelliaci luminis aura fovet.
> Vita fuit brevior, mihi quam, mea Perga, dedisti.
> Aeterna est, quam nunc Snellius ipse dedit.

The fact that Scaliger did this effort for the young Snellius shows his appreciation for him. In all likeliness, Snellius belonged to the informal group of talented young people surrounding Scaliger.[162] Scaliger's dislike of Rudolph Snellius makes his sympathy for Willebrord the more noteworthy. His regard for the son was certainly higher than for the father:

> The son of Snellius is a nice boy, completely different from his father. Snellius thinks that he is a learned mathematician, but his son knows ten times more about it than he does.[163]

Yet this negative image must be modified: Scaliger presented 'the most learned Rudolph Snellius' with a copy of his precious book *Opus de Emendatione Temporum*.[164] It seems that Scaliger had a reputation for his inconstancy of opinions, as is for instance shown by what Raphelengius wrote to Lipsius in 1595:

> Scaliger is vehement in both praise and abuse, and often of the same man or thing. [...] Many would be offended if he had not become more a figure of fun than a cause of hatred.[165]

Scaliger's poem for Willebrord Snellius was later included in his biography in *Athenae Batavae*, probably at Snellius's own instigation. His positive judgement of Willebrord was also commemorated in the latter's funeral oration.[166]

The learning of Scaliger continued to influence Snellius after the former's death in 1609. This is for example testified by the Cleomedes text which Rudolph bought from Scaliger's legacy. It contains some marginal notes and calculations which must be Willebrord's.[167] In his *Eratosthenes Batavus*, Snellius complimented Scaliger as the 'Phoenix of this time'.[168] However in some other works, Snellius did not hesitate to show that he had learned to assess all authorities scrupulously by referring critically to Scaliger's work several times (see section 7.6.1 and p. 73).

Snellius's reconstructions could be considered as specimina of research mathematics, because new, advanced mathematics was discussed in them. They can be seen as a 'PhD thesis' (in the modern sense of the word): Snellius proved

[*excitus*: *Apollonius Batavus* reads *excitur*; I corrected to *excitus*.] [Snellius, 1608a, p. 2].

162 H.J. de Jonge also includes Snellius in his enumeration of Scaliger's students: [de Jonge, 2005, p. 136]. Cp. [Grafton, 1993a, pp. 492 ff.] for Scaliger's tuition.

163 'Le fils de Snellius est gentil garçon, c'est bien autre chose que son Pere. Snellius pense estre sçauant Mathematicien: son fils en sçait dix fois plus que luy.' [Scaliger, 1669, I, p. 228].

164 'Doctissimo viro Rudolpho Snellio [...]', [Scaliger, 1598, title page].

165 Letter of Raphelengius Junior to Lipsius (1595), quoted in [Grafton, 2003, pp. 20–21].

166 [Meursius, 1625, p. 299], [Iachaeus, 1626, pp. 16–17].

167 [Cleomedes, 1605].

168 'Phoenix huius seculi Iosephus Scaliger', [Snellius, 1617b, p. 85]; 'vir doctissimus et phoenix nostri seculi', [Snellius, 1617b, p. 144].

by them that he could contribute to a scientific discipline that also attracted other bright mathematicians. Indeed, more reconstructions of lost Apollonian works appeared in the same period: Viète had published a reconstruction of *On contacts*, called *Apollonius Gallus* (1600); Marinus Ghetaldus devoted several studies to Apollonius's *Vergings* and also wrote a supplement to Viète's *Apollonius Gallus*; and Alexander Anderson also studied the *Vergings*.[169]

Snellius considered Viète's booklet as a source of inspiration[170] and the title of his work clearly mirrored Viète's. He may have become acquainted with this work during his stay in Paris, where the recently deceased Viète must have been often spoken of in learned circles. Anderson visited Snellius,[171] which makes a direct, personal influence of Snellius on him very likely. However, the books of Ghetaldi and Anderson do not show any influence of Snellius, nor vice versa.

Because Pappus's summary was very succinct, Snellius had to take recourse to his own mathematical ingenuity for the reconstruction of the Apollonian treatises. He used traditional, Euclidean means to solve the problems, hardly using analyses—or at least not writing down more than a few (see section 5.2 for an explanation of the geometrical background). The central problems of the three treatises all had to be divided into a number of sub-problems, dictated by the different possibilities of arranging the givens. These sub-problems entailed different cases. All these cases had to be studied separately before the general problem could be considered solved. In some cases, a solution only existed if extra restrictions applied. The determination of all the cases and the special conditions under which they existed was a non-trivial part of the solution of the problem, made more difficult by the fact that a figure always represents one particular case and not the general problem. Sometimes Snellius left some details, such as discussing special cases, for his readers, who were supposed to be rather advanced in geometry.

An remarkable feature of Snellius's reconstruction technique is that he reduced some sub-problems of *Cutting off of a Ratio* and *Cutting off of an Area* to sub-problems of *Apollonius Batavus*, in this way working very efficiently. He thus made Apollonius's *Cutting off*-treatises dependent on the *Determinate Section*, which is inconsistent with the order in which Pappus had described them and the years of publication of Snellius's reconstructions: *Cutting off* in 1607, *Apollonius Batavus* in 1608. Probably, he worked on both booklets in the same

169 [Vieta, 1646a]. [Ghetaldus, 1607a], which was completed by [Ghetaldus, 1613]; [Ghetaldus, 1607c]; all his works have been reprinted in [Ghetaldus (Ž. Dadić ed.), 1968]. [Andersonus, 1612], supplemented by [Andersonus, 1615]. Modern discussions are found in [Bos, 2001a, pp. 83–84] and [Brigagli and Nastasi, 1986].

170 'et quae ab Apollonio [...] propositionibus plus quam 90 erant discerpta, [sc. Fr. Vieta] in paucissima problemata ita contraxit, ut dubium sit utrum illa brevius, an clarius demonstrarit. Cuius studium nos quoque aemulati libros Apollonii [...] nostra opera resuscitatos in usum φιλομαθῶν vulgamus.' [Snellius, 1607b, p. 7].

171 [Iachaeus, 1626, p. 16], probably between 1608 and 1613.

period and gradually discovered the relations between their contents. They may not have appeared far apart, or even at the same time.[172]

Snellius's two treatises contain three very informative dedicatory letters. *Cutting off*'s first part was dedicated to Rudolph Snellius, its second to the *Archimedean man* Simon Stevin. They were both part of Willebrord's inner circle, which may have been the major audience for this small publication.[173] The first dedication is even somewhat unusual, because Willebrord did not have to beg for his father's support. In the first place, his purpose must have been to express his recognition of the education received from his father, thus simultaneously confirming Rudolph's status as a scholar. Willebrord asserted, in a Greek quote of Theodoretus, 'that I understood both being and non-being from you'.[174]

In the second place, Willebrord supported his father's Ramist cause in public:

> Because you had stimulated me from my youth onwards to apply myself truly and fully to scholarship, and spurred me on when I hesitated, nothing was more important to me than to examine more attentively the exactness, clearness and brevity in the proofs of the ancients by means of your precepts, as rules and norms, because your Apollo resolutely urged to do so in his *Prooemium Mathematicum.*[175]

The 'Apollo' of the last part of the sentence is no one else than Petrus Ramus, the author of a *Prooemium Mathematicum.*

The Ramist theme reappeared later in the letter, but not before Willebrord had given a lively sketch of the seemingly insurmountable obstacles that a researcher meets on his path. He told his father how his strife for simplicity, and his abhorrence of piles of details had determined his first experiences with the reconstruction of the treatises:

> And while I was rereading the traces in Pappus's work more carefully,

[172]Cp. also 'id namque in Apollonio Batavo [...] a nobis demonstratum est.' [Snellius, 1607b, p. 13].

[173]This restricted audience is also suggested—not proven—by the current rarity of the booklet: I have traced two copies in the Netherlands, both in Leiden. However, the examples of reception given at the end of this section show that a number of mathematicians knew or had at least heard about Snellius's work. The evidence suggests that *Apollonius Batavus* was diffused more than *Cutting off.*

[174]'Cur hic libellus ad te eat, pater optime, pater venerande, quia balbuties et infantia mea me prohibet dicere, loquar per interpretem, et alienis verbis explicabo, quod meis non queo. ὅτι παρὰ σοῦ καὶ τὸ εἶναι, καὶ τὸ οὖ εἶναι ἔλαβον. Theodoretus tuus me facit eloquentem.' [Snellius, 1607b, p. 3].

[175]'Cum me ab ineunte aetate ad vere solideque philosophandum invitasses, et haesitantem impulisses, nihil mihi fuit antiquius quam ut praeceptis tuis, tanquam regula et norma, veterum in demonstrando subtilitatem, claritatem, brevitatem diligentius examinarem: quia id Apollo tuus in prooemio suo mathematico obnixe suaderet.' [Snellius, 1607b, p. 3].

> I was frightened by the enormous diversity of so many problems, so that I took a loathing for the work that I had already began, because of the excessive hair-splitting in the individual problems (although I had understood that this had also been Apollonius's own practice). What should I do? I had finished much work, but more still awaited me. [...] Then I saw that the problems that I had already examined and investigated also contributed very much material that helped acquire knowledge of hidden things. For this reason I returned to the task. And behold, no sooner had I heaped up an enormous pile of different theorems and problems than I regretted having returned to a work so shapeless.[176]

Fortunately, the education which he had received from his father had finally given Snellius the means to organize the material properly:

> Only then did I appreciate your tools, honourable father, and have I reduced the countless repetitions of identical things, which had escaped me because of the variety of conditions and cases, to a single heading, by applying induction and the other instruments of purer logic. With their help, I have contracted eight, ten, twenty problems to one problem each.[177]

Why did Snellius tell his father and the reader this story which could be interpreted as his showing a lack of interest in the material or of perseverance? The reasons for this may be several. By stressing the difficulty of the enterprise, Snellius showed that he had achieved something praiseworthy by finishing it. His reluctance to go into hair-splitting was an indication that as a scholar he was firmly rooted in the normal world, as opposed to those who shunned all practical aspects of science—a theme treated more often by Snellius—, and that he was careful not to loose time on worthless studies. These viewpoints had a clear Ramist ring.

Moreover, Snellius could thus attract the attention to the importance of working according to a method to organize the material, which made a crucial

[176]'dumque vestigia apud Pappum posita diligentius relego, magna multorum problematum πολυσχιδία animum meum terruit: ut iam coepti operis, propter nimiam in singulis problematis λεπτολεσχίαν (quanquam idem ab Apollonio factitatum animadverterem) taederet. Quid facerem? multum operis post terga erat relictum: ante oculos autem plus adhuc restabat. [...] deinde videbam quoque explorata iam et investigata problemata maximam materiam ad rerum occultarum cognitionem afferre. quamobrem ad eundem laborem me recipio. Et iam denuo maximum variorum theorematum problematumque cumulum congesseram, cum ecce rursum tam informis operis me poeniteret.' [Snellius, 1607b, p. 4].

[177]'ibi demum tua mihi placuere instrumenta pater honorande, et innumerabiles tautologias, quae me ob ἐπιταγμάτων πτῶσεώντε varietatem latuerant, adhibita ἐπαγωγῇ, reliquisque purioris logicae instrumentis ad unum caput revocavi, illorumque ope problemata octo, decem, viginti in singula contraxi.' [Snellius, 1607b, p. 5]. Cp. [Snellius, 1608a, p. 10] for Snellian complaints about the obscurity of the Pappian rendering of the Apollonius treatises.

difference between just solving a pile of problems and understanding their structure. The organization also made a shorter presentation possible. The method alluded to as his father's 'tools' was Ramus's method for all sciences, to which Rudolph Snellius must certainly have devoted some private teaching to his son. The most relevant part of the method here is the necessity to discuss everything on its own level: 'the most general things generally, but specific things specifically' (see section 2.5.1). In fact, the method probably helped Snellius to keep a cool head while facing the difficulty of his job and stimulated him to look for a hierarchy in the problems, but Ramus's work offered no example or guiding lines of how such a structure should be built in the case of advanced mathematics.

In his letter of dedication to Stevin, Snellius pleaded for a mixture of, or at least a collaboration between, theoretical and practical science. Stevin, who stressed that 'spiegeling' and 'daet' should go together, was just the right person for such an approach, which can again be connected to the Ramist sympathies of father and son Snellius.[178] The dedicatee of *Apollonius Batavus* was again Count Maurice of Nassau, another lover of both the theoretical and the practical side of mathematics. Like in his dedication of the *Hypomnemata*, Snellius stressed Maurice's interest in applied mathematics, especially in warfare, although this context could hardly be called relevant for the Apollonius reconstructions.[179] Maurice found his gift copy interesting enough to include it in his library.[180]

This letter gives more insight into Snellius's opinions about the best way to handle mathematical problems. He wrote that he had recovered the lost treatise on the *Determinate Section*

> not, however, with that abundance of proofs or that over-exactness, which they[181] applied, because that would have been both unpleasant and useless; no, I have rather of this science within my grasp drawn the outlines and the usefulness in one go, and the chief content in its own terms. For all that is valuable in mathematical demonstrations is judged by its suitability and its skilful joining of things; and neither are more preferred to less, nor longer to shorter [proofs].[182]

[178][Snellius, 1607b, pp. 15–16]; cp. p. 311.

[179]'Neque enim post homines natos cuiusquam memoria extat, qui artes mathematicas cum ipsarum usu tam felici eventu copularit unquam, aut coniunxerit.' [Snellius, 1608a, p. 3].

'Te igitur, Princeps Illustrissime, qui etiam in acie, et media inter praelia harum artium dignitatem utilitatemque, vel solus, vel maxime recognoscis, neque earum curam ex animo *deponis* unquam, rogo ut libellum hunc, mole exiguum, utilitate (ut spero) permagnum, inspicere non graveris.' [*deponis*: *Apollonius Batavus* reads *depuis*; I corrected to *deponis*.] [Snellius, 1608a, pp. 6–7].

[180][Wiekart, 1998, p. 49].

[181]This refers to the first inventors of the material of the *Determinate Section*, who were mentioned earlier on the same page.

[182]'[...] non quidem illa demonstrationum copia aut περιεργείαι, quam illi usurparunt, id enim et insuave et inutile foret: quin potius artis quam concepi et summam, et utilitatem eodem

Snellius explained here that for him, the reconstruction of a lost treatise did not mean that the original text had to be recovered literally, but that the mathematical essence had to be represented in a concise way. Although the ancient mathematicians had developed the standards for the type of mathematics under discussion, this did not mean that their work could not be improved.

In the following passage, Snellius stressed the non-rhetorical character of mathematics and the need for a method. The reader is probably meant to think of Ramus again when coming across this word, but the context is not completely Ramist, as Snellius made a distinction between mathematics and other sciences, instead of pointing to the unicity of the method for all sciences:

> Indeed, though the mathematician is, when proving, well provided with a variety of facts and cases, he is more sparing with words. For just as in general all sciences comprise many, but homogeneous[183] topics in their sphere, this is true especially for mathematics; and therefore [mathematics] must be constructed according to reason and method. And for this reason it is childish to long to want to your genius in such diversity and labyrinths of demonstrations; the ability to give clear and perspicuous explanations is the mark of a learned and intelligent man. And for this reason, I am delighted by short and sharp corollaries when they are needed, which must not be so long in order to be clear.[184]

Snellius kept to his word and in general gave clear mathematical constructions expressed in standard terms. He wound up his argument by showing signs of disapproval of the work of most other contemporary mathematicians:

> This view must certainly be defended by all mathematicians, for those who exert themselves in such a copiousness of proofs prove too wretchedly to my opinion: we must state everything briefly and without ornament. However, because this approach requires very much study and exercise, we see that among the more recent [mathematicians] a few strive for this way; and that the others either do not

cursu, resque ipsas suis verbis dimensus sum. Demonstrationum enim mathematicarum omne bonum convenientia et concinnitate iudicatur: nec plura paucioribus, nec longiora brevioribus anteponuntur: [...]', [Snellius, 1608a, p. 5].

[183]Probably another Ramist reference.

[184]'Mathematicus enim in demonstrando rerum et casuum varietate locuples, oratione sit licet ieiunior. Nam cum artes omnes plurima, eaque homogenea suo ambitu complectantur, ita maxime Mathematicae; quaeque ideo ratione constituendae sunt ac via: quare in istis demonstrationum varietatibus, et labyrinthis ingenium suum ostentare velle, puerile est; plane autem, et perspicue expedire posse, docti et intelligentis viri. atque ideo, cum opus erit, consectaria me brevia et acuta delectant; quae, ut perspicua sint, ita longa esse non debent.' [Snellius, 1608a, pp. 5–6].

know it, or are not able to do it, but have in any case given it up.[185]

This last remark is so non-specific that it cannot be said whether Snellius had any specific authors or books in mind; its function is merely rhetorical, highlighting his own achievements.

Snellius's Apollonius reconstructions have undeservedly been left to their fate by modern scholarship. Although it must be admitted that they do not contain much surprising or innovative mathematics, they were a worthy proof of his capacity to be a member of the Republic of Learning. He must have intended them as exhibits: both books showed his familiarity with ancient sources and their core part proved to his fellow mathematicians that he could contribute to a difficult topic that occupied several of the more able of them. Moreover, in the preliminary matter he showed off his rhetorical skills, professed his scholarly allegiances, tried to win Maurice as a patron and developed some thoughts on good mathematics. All these elements play a role in most of his later works, although not always so explicit as here.

The importance of these works is also shown by the fact that they were used by a number of later mathematicians: Pierre Herigone gave the content of Snellius's treatises in a new, short notation; Fermat referred to them in his own study on Apollonius's *Plane Loci* (around 1637); Mersenne published a summary of Snellius's three Apollonius-reconstructions in his mathematical encyclopaedia. Between 1622 and 1671, three German mathematicians worked on a *Apollonius Saxonicus* ('Saxon Apollonius'), which remained in manuscript. The name is a conscious reference to the earlier Apollonii of different nationalities. Frans van Schooten mentioned Snellius's work in the preface to his own Apollonius reconstruction (1657). Benjamin Bramer also followed Viète's and Snellius's habit of giving Apollonius different nationalities, calling his book, in which conic sections are an important topic, *Apollonius Cattus* ('Hessian Apollonius').[186]

Even in the eighteenth century, Snellius's texts were the basis for further research. An English translation of *Apollonius Batavus*, by John Lawson, appeared in 1772, accompanied by a new reconstruction by William Wales. In the same period, the Glasgow professor Robert Simson worked on his own reconstruction of Apollonius's *Determinate Section*. He reviewed the *Apollonius Batavus* in his introduction. The depreciative words with which he started this part make clear that both the rhetorical conventions and the mathematical state of the art had

[185]'ista enim omnibus mathematicis tuenda sententia est. Nam qui tanta demonstrationum ubertate laborant, squalidius mihi demonstrare videntur: breviter et enucleate omnia nobis dicenda sunt. verum quia haec res plurimae commentationis et exercitationis indiget, videmus inter recentiores unum aut alterum hoc genus sequi; caeteri aut noluerunt, aut non potuerunt, certe reliquerunt.' [Snellius, 1608a, p. 6].

[186][Herigone, 1634, pp. 890–904], [de Fermat, 1679, p. 12], [Mersennus, 1644, pp. 382–383]. [Elsner, 1988]; see p. 13 for Van Schooten. [Bramer and Burgi, 1684]; Snellius is explicitly mentioned in the preface from 1646.

changed enormously since Snellius's time.[187]

2.8.2 Entering adult life

Several signs show that Snellius gradually became an authority in his field in this period, whose recommendations were appreciated by those interested in mathematics. Already in 1606, he wrote a complimentary letter to Philips Lansbergen, although he considered himself as too young for this task. Nevertheless, Lansbergen included the letter in his *Progymnasmata Astronomiae*, in which he overtly sustained the Copernican system.[188] Lansbergen, or Lansbergius (1561–1632), a clergyman from Zeeland, was an avid mathematician and astronomer in his spare time. Snellius later stimulated Lansbergen to publish this book (cp. section 4.5).[189] Lansbergen included another letter by Snellius, dated 11 October 1607, in his *Cyclometria Nova*. This letter discussed the approximate quadrature of the circle. Snellius would later use Lansbergen's book for his own calculation of the ratio of the circumference and the diameter of a circle.[190]

Snellius also assisted the navigator Jan vanden Brouck, who developed a geometrical 'construction' for the determination of cubic roots (in fact an approximation of their numerical value) by means of compass, multiplication and division. In his *Instructie der Zee-Vaert* ('Teaching of Navigation', 1609), Vanden Brouck tells us that he had had his 'invention' studied and approved by Snellius. He was a distant relative and a former teacher of Isaac Beeckman, through whom he may have become acquainted with Snellius.[191]

Outside Leiden, Snellius's star as a scholar was also rising. He was visited by scholars from abroad, among whom was Alexander Anderson, to whom he later referred as his friend. He corresponded with others and also gave advice to Count Maurice of Nassau.[192]

After having worked in the field of mathematics for some years already and having gained some renown, Snellius must have felt the need to obtain an official degree to consolidate his status and income, which was especially urgent since he was about to marry and thus to become responsible for a family. He defended a number of theses to become a *Magister Artium* on 12 July 1608. His *Philosophical Theses* cover an extended part of the *artes liberales*: *grammatica, rhetorica, logica, arithmetica, geometria, analysis/algebra, physica, optica, astronomia, geographia, gnonomica, statica* and *ethica*. Although the name of

[187][Snellius, 1772, pp. 1–14].
'[...] in quo multa quidem magnifice de se iactat, nihil autem praestitit quod immodicae sui ipsius laudationi convenit.' [Simson (J. Clow ed.), 1776, p. ii].
[188]Letter of 10 December 1606, [Lansbergius, 1628, fol. $*4^v$].
[189][Lansbergius, 1628, fol. $*3^v$], [de Waard, 1912a].
[190][Lansbergius, 1616, fol. $A4^{r-v}$].
[191][vanden Brouck, 1610, pp. 98 ff.], [van Berkel, 1983b, p. 25].
[192][Iachaeus, 1626, pp. 15–17], [van Ceulen, 1619, 2, p. 50].

Ramus is nowhere mentioned, his spirit is manifest in many statements. The name did not have to be mentioned, because in Ramist spirit, Snellius let truth and a critical lecture of the ancients prevail.

He expressed for example his agreement with the Roman author on rhetoric Quintilianus that the goal of rhetoric is not to persuade, because whether the orator was successful in this depended partly on chance. This strictness of definition was linked to the discussion in Ramus's *Scholae Rhetoricae*. The thesis that no elements of one art could belong to another art as well is directly derived from one of Ramus's laws, the *lex iustitiae*. In the mathematical part, the most outspoken examples of Ramist echoes are a thesis on book X of the *Elements* (see p. 197) and the use of the term 'hysterology' (see p. 274).[193] Snellius never acquired the doctoral degree, which was not available in the mathematical sciences.

In the month after the defence of his theses, Snellius married Maria de Langhe, 'a virtuous young woman, more sensible than one would expect of her age' in the standard phraseology of Snellius's obituary.[194] She was the daughter of Laurens Adriaens de Langhe, a burgomaster of Schoonhoven, and Janneke Symons. According to Jachaeus she originated from an old and respectable family; her brothers Simon and Adriaen were lawyers.[195] It is not known how the couple became acquainted. The families must have known each other before (Oudewater is not far from Schoonhoven and they were related, see below) or the brothers met the Snellii in an academic context. The intended marriage was given notice of ('ondertrouw') on 1 August 1608.[196] Bride and groom received the honourable present of a printed booklet with two nuptial songs, written by Bonaventura Vulcanius, the professor of Greek, and Jacob Letting, a lawyer, in which their families and achievements were highly praised.[197]

According to Snellius's funeral oration, Snellius and his wife had eighteen children.[198] This number, which is not supported by archival evidence, seems to be improbably high, given the duration of the marriage of eighteen years. It can be reconstructed that they had at least seven children, but some young children may have died without leaving a trace of their existence.[199] As far as extant sources show, their baptisms and funerals were registered by the Reformed Church.

On 4 August 1609, a son Jacob was baptized in Schoonhoven.[200] He must

[193][Snellius, 1608b], [Molhuysen, 1913, p. 177]. I thank prof. dr. Kees Meerhoff for pointing out to me the Ramist content of all theses on the trivium and on ethics.
[194]'virgo casta et supra aetatem prudens [...]', [Iachaeus, 1626, p. 15].
[195][Iachaeus, 1626, p. 15], [de Waard, 1927b, c. 1156].
[196][Kerkelijk Ondertrouwboek G, 1614, fol. 15^v].
[197][Vulcanius and Letting, 1608].
[198][Iachaeus, 1626, p. 19].
[199]The registers of baptisms of the Leiden Reformed Church have only survived since 1621.
[200][Dopen Schoonhoven, 1650]. I thank Mrs. W.H.Th. Damen for this information.

have been the eldest son (assuming no children had been born before the wedding), yet he was not named after Willebrord's father, but after his deceased uncle. In February 1618, a Jacobus Snellius was inscribed in the *Album Studiosorum* of the university. Although the age does not correspond to that of the boy, it was probably the same son, inscribed at the same age as his younger brother in 1625.[201] Later, probably in November 1610, Maria gave birth to a daughter. Snellius wrote to his relative Rosendalius (for whom see below) a letter asking him to be a witness to the baptism two days later. In the name of his wife, he entreated Rosendalius's wife to be present as well. He hoped that the baby's uncles would be the 'governors of her faith and her ecclesiastical tuition'.[202] At the end of December, he sent some more news about the health of his recovering wife and about the 'very sweet baby'.[203]

On 16 December 1614, a child of the couple, probably this girl, was buried.[204] In c. 1616, a son Rudolph was born, called after his paternal grandfather. When he was nine years old, he was inscribed in the *Album Studiosorum*.[205] On 25 November 1618, another child (Jacob?) was buried.[206] A daughter Jannetgen was baptized on 26 June 1622; she was named after her maternal grandmother. Her grandmother and her uncle Simon were among the witnesses.[207]

When the Leiden population was counted in 1623, Snellius and his wife had only two children at home, Rudolf and Jannetje (a variant of Jannetgen).[208] In the same year, on 13 October, another son, named Laurens, was born, called after his maternal grandfather. His uncle Adriaen was among the witnesses.[209] These three children were still alive when Snellius died three years later. In the meantime, two other babies were born and died young. They were buried on 15 November 1624 and on 18 October 1626.[210]

The birth of seven (or more) children and the loss of four (or more) of them was a normal part of the fate of early modern couples, and they must have given Willebrord and Maria a fair amount of joy and sadness. The hazardous enterprise of child bearing certainly distressed Snellius. He once expressed his worries in a letter to Rosendalius:

> And finally, the delivery of my wife has absorbed the greater part of

[201] The age given is 20, [Album, 1875, c. 134].

[202] '[...] relligionis et ecclesiasticae disciplinae moderatores [...]'. This birth is named in the same letter as the auction of Vulcanius's library, which took place in 1610, [Snellius, 1610b].

[203] 'το γλυκύτατον τεκνίδιον', [Snellius, 1610a].

[204] [Begraafboek 3, 1617].

[205] Year of birth reconstructed from this inscription, which was 'Hon.[oris] ergo' and dated 14 June 1625, [Album, 1875, c. 185].

[206] [Begraafboek 4, 1624].

[207] [Doopboek Hooglandse Kerk 12, 1628].

[208] [van Geer, 1884a, p. 123].

[209] [Doopboek Pieterskerk 1, 1644].

[210] [Begraafboek 5, 1627].

> my person from that moment onwards. I will inform you about this, although somewhat later than I had intended: which did not happen out of negligence, but out of anxiety and concern, because the health of the new mother has easily driven all other considerations from our mind, and in this way it has truly held us all equally alarmed.[211]

Willebrord may have regretted having relatively young children, probably waiting anxiously for them to grow up so that he could educate and help them with their further development just as his father had done with him. Yet he did not live long enough to see any of his children beyond the age of ten. It was maybe partly due to this loss that neither of his sons became a mathematician. The increasing size of his family must have entailed increasing expenses, and his responsibilities as a *pater familias* must have forced Snellius to raise his income accordingly.

Of special interest is Snellius's relationship to Amelis van Rosendael (1557–1620), or Aemilius Rosendalius in Latin, the language in which Snellius and he corresponded. He was married to Aleijda de Langhe, the daughter of Adriaen de Langhe and thus the aunt of Maria de Langhe, and he descended from Rudolph Snellius's uncles. Through him, Willebrord and his wife were distant relatives. He was an influential person, a lawyer at the Hof of Holland, the provincial high court. His father Jan Jacobsz van Rosendael had been a burgomaster of Gouda, and a person of some importance in the Dutch Revolt.[212] A number of letters from Snellius to Rosendalius are still in existence. Not all letters are dated, and the topics of some of them are difficult to understand without background information, yet others give valuable information about some scientific pursuits of Snellius, and also a look behind the scenes of academic policy. Rosendalius acted as a patron of Snellius several times, and they shared some scholarly interests. The letters have received only scanty attention in scholarship. An overview of the extant letters is given in table 2.3; see the bibliography for details (some of the dates are explicitly given in the letters, while others are deduced from internal evidence). The content of the letters are discussed in the relevant sections.

The base of their relationship was the family connection. Snellius always greeted his 'very dear aunt'[213] as long as she was alive. When Rosendalius's daughter stayed with the Snellius family, Snellius wrote to her father to announce

[211]'Tandem etiam uxoris partitudo ex illo tempore non pessimam partem nobis praecidit. Cuius etsi serius quam putaram certiorem te faciam: nulla id factum est negligentia; sed sollicitudine et cura; Puerperae enim valetudo reliquas cogitationes omnes facile ex animo nostro recussit, ita enim omnes nos aeque habuit sollicitos.' [Snellius, 1610b].

[212][Kuyk, 1914], [Wittert van Hoogland, 1906, pp. 343–351]; Snellius calls Rosendalius's wife 'amita' (paternal aunt), e.g. [Snellius, 1612a]; descent of uncles Hugo and Jacobus: see [Coddaeus, 1613, p. 11].

[213]'et simul salutem amitae charissimae [...] dicito.' [Snellius, s ab].

Year	Source	Remarks
1610	[Snellius, 1610c]	cp. [Vollgraff, 1914]
1610	[Snellius, 1610a]	cp. [Vollgraff, 1914]
1610?	[Snellius, 1610b]	
1612	[Snellius, 1612a]	cp. [Vermij, 2002, pp. 44-45]
1612?	[Snellius, 1612b]	
1612?	[Snellius, 1612c]	
before 1613	[Snellius, s ac]	
1612/13?	[Snellius, s ab]	
after 1613	[Snellius, s ad]	
1615?	[Snellius, 1615]	
1616	[Snellius, 1616]	

Table 2.3: Willebrord Snellius's correspondence with Aemilius Rosendalius

that she could not go home on the day she had decided on previously because of the bad weather.[214] Snellius and Rosendalius also corresponded about illness and health of members of the family.[215] On one occasion Snellius communicated some news about the family and his greetings in Greek, as a playful style exercise.[216] Other correspondents of Rosendalius also mixed Greek expressions into their Latin and Simon de Langhe even wrote a complete letter in Greek to his uncle.[217]

2.8.3 Mathematics mainly mixed

Snellius continued his astronomical work in this period and soon became acquainted with the major new invention of the telescope. Although some forerunners were known—tubes without lenses, magnifying lenses—, the telescope started to play its spectacular role in astronomy only after Galileo had learned about its design, had improved its performance and published his findings in the *Sidereus Nuncius* (1610). With the telescope, he made several discoveries that were hard or even impossible to reconcile with Aristotelian cosmology.[218]

Father and son Snellius possessed a telescope almost as soon as its success story began. This early date is not very strange given the Dutch origins of the instrument.[219] Rudolph had already shown a telescope to one of his students after a class in optics in 1609,[220] and Willebrord wrote two letters to Rosendalius about telescopes in 1610. Snellius's telescope had probably caught Rosendalius's attention when he had visited the Snellius family on the occasion of the baptism

[214] [Snellius, s ad].
[215] E.g. [Snellius, 1612c], [Snellius, s ac].
[216] [Snellius, 1610a].
[217] [de Langhe, s a, fol. 263].
[218] [North, 1994, pp. 326–337].
[219] See [Van Helden, 1977] for an extensive study of the invention of the telescope.
[220] [de Waard, 1939, p. 12].

of their daughter. In the first letter, Snellius wrote that he had had lenses ground for Rosendalius, but that unfortunately, neither his own nor these new lenses gave very clear images: 'because this alone would be of great advantage to this light of optics'.[221]

Snellius continued to tell Rosendalius that the correct distance of the two glasses had to be found by sliding the two tubes and letting his eyes decide what this distance should be.[222] Apparently, Rosendalius could not operate the telescope well enough, because in the second letter, Snellius again stressed that the right interval of the lenses should be found by selecting the best observations.[223]

Neither Snellius nor Rosendalius were completely satisfied about their telescopes. Snellius had the impression that his telescope did not bring the objects as near as when he first acquired the instrument, but he realized that this could not be true.[224] It seems that Rosendalius and he used the instruments in the first place to magnify terrestrial objects; no astronomical observations are alluded at.

In 1612, Snellius published his first astronomical work, in which he discussed the spots of the sun. It was called *De maculis in sole animadversis, et, tanquam ab Apelle, in tabula spectandum* [sic] *in publica luce expositis, Batavi Dissertatiuncula* ('A small treatise of a Dutchman about the spots that have been observed on the sun and that have been publicly exhibited, on view in a painting like one made by Apelles'). The sunspots were a hot topic that engaged Christopher Scheiner, Galileo Galilei and Marcus Welser in a polemic.[225] The dedicatee of Snellius's book was Cornelius van der Mijlen, a curator of the university, who had more or less ordered this book, as Snellius explained to Rosendalius:

> Following the orders[226] of the most honourable lord, the distinguished gentleman Van der Mijlen, whose will I could not oppose, nor whose command I could refuse to carry out, I have found it necessary to write this down about material and observations plainly unheard of until this age.[227]

[221]'[...] unum enim illud ad illam optices lucem maximo esset emolumento.' [Snellius, 1610c].
[222]'Oculorum enim iudicio hoc facilius, quam multis verborum ambagibus diffinietur.' [Snellius, 1610c].
[223]'In diffinienda conspiciliorum διαστάσει nullus industriae est locus, oculorum indicium et iudicium consulendum, et ex αὐτοψίᾳ τῆς διοπτρίας ratio, et intervalli modulus constituendus.' [Snellius, 1610a].
[224][Snellius, 1610a]. These two letters have been almost completely edited and translated into Dutch by Vollgraff. He gives the later of the two letters as the first, [Vollgraff, 1914, pp. 108–110].
[225][Snellius (attr.), 1612]. Cp. [Galilei, 1895], [Matteoli, 1959]. Apelles, the most famous Greek painter, had been mentioned in Scheiner's title.
[226]Snellius had replaced 'voluntati' by this word ('imperio') to stress Van der Mijlen's authority.
[227]'Amplissimi viri D.C. vander Milii imperio obsecutus, cuius neque voluntati obluctari, neque auctoritatem detrectare poteram, necesse habui ista exarare super materia et observationibus plane ad hoc aevi [sic] inauditis.' [Snellius, 1612a].

The book was published anonymously, but in fact it had been written by Snellius. This is proved by this same letter: not only do the dedicatee and the subject matter mentioned in the letter suit the book, Snellius also asked Rosendalius to forward his booklet to Van der Mijlen, who should send it to Welser, and he called it a *dissertatiuncula*.[228]

Snellius argued that sunspots could be used to determine whether the earth moved or was at rest. He did not mention any observations done by himself, explaining this as follows:

> Preferring in this matter to lean on reasonings rather than to be informed by observations, I consider it as necessary to declare what I think.[229]

In fact, it is more likely that his instruments were not good enough to take such observations.

The reason for the anonymity of the publication may have been Snellius's wish to be careful to attach his name to a book with polemical contents (he reacted critically to Scheiner's work), thus protecting his own and his father's name. Moreover, he had not taken the initiative for this book and may not have seen it as a great achievement. His later work shows the importance which he attached to observations, which means that in fact, he may have considered it as unsatisfactory that he did not have good own observations at his disposal.

The book reached both Kepler, who was not very impressed by its contents, and Galileo.[230] Neither of them seems to have been aware of Snellius's authorship, yet it cannot have been difficult for someone knowing the field to guess the name of the astronomer connected to the university of Leiden. They may not have bothered to research the question of the authorship because the contents of the book did not add much to the learned discussion.

In the same period,[231] Snellius told Rosendalius that he was also preparing another work with polemical contents, directed against an English author who wrote 'a very silly book' 'on the computation of longitude', as he told

[228]'hanc nostram dissertatiunculam', [Snellius, 1612a]. Snellius's authorship and its proof were discovered independently by Rienk Vermij, [Vermij, 2002, pp. 44-45], and myself. Ludendorff also thought that Snellius was the author, but his arguments were less conclusive, [Ludendorff, 1937, pp. 291–292]. See for a summary of the contents [Ludendorff, 1937, pp. 292–294].

[229]'Qua in re rationibus potius nixi, quam observationibus instructi, quid sentiremus, enuntiare necesse habuimus.' [Snellius, 1612a].

[230]Letter of Kepler to Matthias Bernegger, 4/14 September 1612, [Kepler (M. Caspar ed.), 1955, p. 26]. A manuscript copy of the work, probably ordered by Federico Cesi for Galileo, belongs to the collection of the Accademia dei Lincei, [Snellius, s aa].

[231]Snellius wrote about it in a letter to Rosendalius, in which his wife was greeted; she died in 1613, which is therefore *terminus ante quem* of the letter.

Rosendalius.[232] It is most likely that Snellius referred to the famous problem of the determination of geographical longitude at sea. A polemical treatise as announced by Snellius did not appear, but this letter is a strong indication that Snellius was already studying navigational matters long before the publication of *Tiphys Batavus* in 1624.

Another letter to Rosendalius from around the same period gives some food for speculation about this longitude project. Snellius reported on 'our Nonius' who was delayed. Pedro Nuñez, Nonius in Latin, had acquired fame as an author on navigation. Snellius could have borrowed his name for a work of his own on the same topic, or he could have prepared a new edition of a work by Nuñez. He told Rosendalius that he was worried about the work: some months ago he had sent it (this must be the manuscript) to a publisher in Cologne who would forward it to Albinus in Mainz to have it published, but by now he had completely lost track of it.[233] It seems that the treatise was lost forever, or maybe published with another name and title.

More certainty can be obtained about another occupation: Snellius published another edition of a work by Ramus in 1613, this time of the *Arithmetica* and including his own commentary. He probably needed this edition for his teaching in arithmetic. His father had written a commentary on Ramus's *Arithmetica* earlier, but Willebrord preferred to make his own rather than to have his father's reprinted. Indeed, the character of the two commentaries is very different. One reason for this may have been that Willebrord's was meant for a more advanced audience. It contains Ramus's own text and is thus readable independently of a copy of the source text, as opposed to Rudolph's. Although Verdonk, the specialist on Ramus's mathematics, judged the annotations to be of little importance in general, Willebrord's book has some interesting features, which become more distinct when it is compared to Rudolph's edition. In total only a very limited number of commentaries on the *Arithmetica* appeared.[234]

Willebrord dedicated the book to Joannes Karolus a Rechlingen,[235] who had stimulated him to make this edition because of his exceptional love for pure mathematics, according to Snellius in his dedicatory letter. He remarked that he remained closer to Ramus's text than earlier editions. Even so, he had been compelled to make some changes. He also told that he had restored the changes

[232]'[...] de longitudinis ratione ab Anglo [...] insulsissimi libri [...]', [Snellius, s ab]. The work which he meant might be *Descriptio ac delineatio geographica detectionis freti : sive, Transitus ad Occasum, supra terras Americanas, in Chinam atque Iaponem ducturi, recens investigati ab M. Henrico Hudsono Anglo*, published in Amsterdam in 1612.

[233][Snellius, s ac].

[234][Verdonk, 1966, pp. 116–117]; see the index of this book for references to more remarks about the Snellii commentaries. Verdonk mentions only one commentary besides those of the Snellii, by Jacobus Martinus.

[235]I have not been able to trace the dedicatee, who was addressed as a 'Patritius Augustanus', probably a patrician from Augsburg, [Ramus (W. Snellius ed.), 1613, fol. $*2^r$].

made by his father, 'a man of most penetrating discernment'.[236]

This letter demonstrates that Snellius was an adherent of Ramus, and willing to express so explicitly. Yet his commentary does not follow Ramus's text so slavishly as the letter suggests. Whereas his father's commentary was attached to single words or parts of sentences, young Snellius first gave the text of Ramus's paragraph and then his own comment, which gave him more liberty to digress from the original text. Sometimes Ramus's topics stimulated Snellius to write long excursions, for instance on Greek words for division or on the notation of large numbers in Antiquity. In that last case, he showed his concern about the didactical side of the matter. Similar topics were lacking in Rudolph's commentary, who on the other hand had paid more attention to the philosophical dimension.

Willebrord also left out passages that he considered less relevant or he gave them without comment, e.g. Ramus's paragraph on the sieve of Eratosthenes. He corrected an error by Ramus which his father had overlooked and changed the way in which Ramus had explained division. He added some examples from astronomy, which could be considered as un-Ramist, because astronomy was a part of physics, not of mathematics, according to Ramus and Rudolph. In true Ramist mind, Willebrord often used schedules to explain calculations—more than his father. Typographical restrictions may explain why Rudolph did not have them. Both father and son corrected an error in the solution of an exercise about arithmetical series.

Willebrord took position in the debate about the right name for the operation of compounding ratios (see p. 258 for an explanation of this technique). Euclid had called this addition, but some sixteenth-century writers preferred the name multiplication because of the analogy with this operation in numbers. Ramus had been in two minds and used both terms. Snellius advocated the classical term, because if it were changed, it would make the works of the ancients less accessible. Moreover, he argued, this was not only the viewpoint taken by most mathematicians, but also corresponded to popular usage, which point he illustrated by an example in which different monetary units had to be compared.[237]

Another publication of Snellius from this period lies somewhat beyond the boundaries of his normal interests, showing his versatility as a scholar to his readers. It is called *De Re Nummaria Liber singularis* ('On money, in one book') and discusses ancient money.[238] In a letter to Rosendalius, Snellius announced this book about Roman, Greek and Jewish money. Although he was aware of

[236] '[...] parens meus vir acerrimi iudicii [...]', [Ramus (W. Snellius ed.), 1613, fol. $2**^v$]. Cp. [Snellius, 1596d].

[237] 'Omnis mathematicorum schola iam inde antiquitus istam phrasin et loquendi modum secuta est, ut non temere mutari debuerit, nisi ad veterum scripta aditum nobis intercludi postulemus [...]', [Ramus (W. Snellius ed.), 1613, p. 54], discussed in [Verdonk, 1966, p. 182]. Cp. also Ramus's own commentary on his *Arithmetica* in [Ramus, 1569, pp. 116–145].

[238] [Snellius, 1613].

the existence of earlier publication on the topic, he thought that there was still room for his new book, because the others 'seem to prefer diverting the reader by a diversity of topics rather than to instructing him'.[239]

Although the topic was non-mathematical, Snellius proposed a mathematical approach to Rosendalius:

> Therefore, I have presented the entire topic summarily as though in the form of its own propositions, which I have then proved by the authority of the ancients. Thus it has been achieved that the topic, which was scattered over the works of various authors, was reduced to one or two quires.[240]

This organisation of material again reflected the Ramist adage to discuss 'general things generally'.

Snellius asked Rosendalius also for help: Rosendalius had a good collection of ancient coins, of which Snellius wanted to know some properties.[241] Snellius was allowed to borrow the coins from Rosendalius for the preparation of this book for some time. He asked him whether he could consult the coins of a regent from Antwerp as well.[242]

De Re Nummaria was dedicated to Hugo Grotius, the most clever and famous member of the Scaliger school, and like Rosendalius a lawyer and a member of the elite of regents. In the dedicatory letter, Snellius referred to his predecessors in his field, and explained how he had studied both the ancient authoritative texts and the coins themselves, and that he had sometimes been forced to deviate from the former because the overwhelming evidence of the latter had forced him to do so.[243] His own study of ancient numismatics entitled Snellius to criticize Scaliger, whose treatise on ancient coins he would later edit. Notwithstanding this critique, Scaliger was probably the person who had made Snellius familiar with numismatics. In the *Eratosthenes Batavus* he called this treatise very learned, yet he noted that Scaliger had distorted a quotation by Plinius in it.[244]

[239]'Habeo iam perfectum caput, an libellum dicam, de vetere moneta Romana, Graeca, Iudaica: idem argumentum diligentissime ab aliis tractum non sum nescius (neque eam laudem mihi tribuo) sed ita tamen ut aliorum industriae locum aliquem reliquerint, cum lectorem magis distinere rerum varietate, quam docere velle videantur.' [Snellius, 1612c].

[240]'Rem igitur universam summatim tanquam per sua theoremata proposui, quae deinde veterum autoritate probarem. ita factum est, ut quae res apud varios sparsim diffusa legebatur, in unum aut alterum caternionem contracta recideret.' [Snellius, 1612c].

[241][Snellius, 1612c].

[242][Snellius, 1612b].

[243]'Et certe, in materia tam ubere, ista non scribentium solum autoritate examinanda, sed etiam nummorum veterum momentis fuere comprobanda; quorum consensu mutuo, tanquam incorruptissimo testimonio, omnis veritas certissime confirmetur. Atque ideo si quando ab illorum sententia abeo diversus—abeo autem non semel—, maxime hoc praedictum velim, evidentissimis argumentis et ipsa veritate victum me in illam sententiam descivisse.' The letter has been republished in [Nellen, 2001, pp. 99–101].

[244][Snellius, 1617b, p. 144].

Snellius also studied optics. He never published anything in this field, but his marginal notes to his copy of the *Optica* of Ramus and Risnerus have survived, which allow us to learn some of his considerations. They were partly edited by Vollgraff, who found 1611 as the earliest date given (see further section 2.9.6). Optics became even more relevant to astronomy once observations were carried out by means of telescopes, the lenses of which caused distortions.

Finally, after having taught at the university for many years without an official position, Snellius became an extraordinary professor on 8 February 1613, replacing his father who was too old and ill to teach any longer. It was decided that Willebrord would earn 300 guilders a year and that he would teach every day from 9 till 10 o'clock. He taught in the *Auditorium Medicum*, in the Academy Building. If his father were to recover, Willebrord had to yield the position to him again.[245] This kind of 'descendance' of professorial chairs from father to son was rather common in Leiden. Four other examples can be found in the first four decades of the university, and in the first century of the university, about 20 % of the sons of the professors became a professor themselves, mainly in Leiden as well. These sons did not rise in the hierarchy more quickly than others.[246]

Rudolph could not enjoy his retirement for long: he died either 1 or 2 March 1613.[247] Everardus Bronchorstius, professor of law, was present during his last hours. He had been an old friend of Rudolph ever since they had been housemates in Marburg. At Rudolph's deathbed, Bronchorstius recited a passage from a psalm to him, to which Rudolph replied with another psalm quote, spoken with distinct voice. According to Bronchorstius, he then died piously and calmly.

Although Rudolph had bought a family tomb in the Pieterskerk in Leiden in 1583,[248] he was buried in Oudewater. On 6 March, the professors and burgomasters saw him off to the boat that would take him to his native town. Gulielmus Coddaeus, professor of Hebrew, delivered his funeral oration in the Academy Building. Aemilius and Jacobus Rosendalius were among the audience.[249]

His feats were commemorated on a plaque in the church in Oudewater. This epitaph had been composed by Willebrord, and was also established in the name of his son Rudolph. It did not only mention the deceased's teaching duties, but also the esteem in which he was held by the two Maurices, the Count of Nassau and the Landgrave of Hessen. The Latin text was followed by a short record in Dutch.[250] Rudolph's widow did not continue to live together with her son

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[245][Molhuysen, 1916, p. 45], [Meursius, 1625, p. 298], [Orlers, 1641, p. 186], [van Slee, 1898, p. 130].

[246][Clotz, 1998, pp. 122–123], [Otterspeer, 2000, p. 305].

[247]1 March according to Bronchorstius, see [van Slee, 1898, p. 131], 2 March according to the copied text of his funeral stone (see below) and according to Willebrord in his manuscript notes to the *Eratosthenes Batavus*, [Snellius, 1617c, p. 177].

[248][VIII Grafboeken Pieterskerk en Engelse Kerk, 1610, fol. 69].

[249][van Slee, 1898, p. 131], [Coddaeus, 1613, p. 11].

[250][Meerhoff, 2003a, pp. 7–8], [Kalckhoff, 1730, fol. 1^r–1^v]. [Knöll, 2003, pp. 418–419] de-

and his family, but moved to Oudewater. According to Willebrord, she did so to be near the tombs of her beloved, but the fact that she owned some property there may also have played a role.[251] Willebrord intended to publish his father's uncompleted works, but he did not find the leisure to do so.[252]

Now that Rudolph Snellius had died and the results of his career could be assessed, it was clear that his merits had mainly been in his prolific teaching and the resulting publications. His core task, the teaching of mathematics, had yielded commentaries on the *Arithmetica* and *Geometria* of Ramus, which discussed mathematics on a elementary level, in the service of the explanation of Ramus's works. Willebrord had already surpassed his father in the field of mathematics by publishing original work, notably the Apollonius reconstructions. Moreover, he had made himself familiar with several parts of mixed mathematics in which he would penetrate further later on. This had probably happened with the full assent and support of Rudolph, who had given his son the opportunity to develop this side and had even sent him to some excellent mathematicians abroad. Would it be too bold to surmise that Rudolph had noticed that his own somewhat shallow knowledge of the field of his professorship had hindered his career prospects and that he hoped that his son would do better? From a financial point of view, the results of the Snellii firm were satisfactory. Rudolph Snellius had abolished his private school and his boarders at some moment, apparently earning enough as a professor, and his son did not have either a school or students living with him.

2.9 1613–1626: professor of the mathematical sciences

2.9.1 Sowing and reaping

Snellius developed a broad range of activities in the thirteen year period in which he was a professor, continuing his previous work and extending his interests. He published much work and was an active member of the Leiden academic community, and also had contacts with the Dutch elite and with scientists abroad.

Snellius's struggle to acquire a decent position at the university continued. On 8 February 1614 his salary was raised from 300 to 400 guilders a year. This was not much when compared to the salaries of other professors. Two of the theologians, for instance, earned as much as 1200 guilders each. The payment of the other regular professors ranged between 575 and 987.50 guilders, whereas

scribes a design for the plaque that is now in the Regional Archive in Leiden. Her transcription is not completely accurate.

[251] [Snellius, 1617b, p. 177], [Witkam, 1973, pp. 83–85].

[252] [Meursius, 1625, p. 299].

the *extraordinarii* received less: between 300 and 600 guilders. The teachers from the philosophical faculty were remunerated less generously than those of the higher faculties. Apart from these salaries, the professors had some other resources and they were exempted from certain taxes. Their revenues were comparable to those of clergymen and teachers. The large majority of professors originated from the middle class, and their position, income and influence gave them a prominent position within the middle class.[253]

Because he considered his new salary as insufficient, Snellius entreated Rosendalius to help him by asking the curators of the university whether he could become a regular professor (*ordinarius*) and earn as much as his father had at the end of his life, 600 guilders (see section 2.9.3). Rosendalius's plea was partly successful: Snellius was ordained a regular professor in February 1615, on the same salary as before however.[254]

Understandably, Snellius was not yet satisfied, and in a letter of 6 January 1616 he asked Rosendalius again to help him. He addressed the issue straightforwardly, refreshing Rosendalius's memory that be should plead Snellius's case with the curators Van Mathenesse and Van der Mijlen, which Rosendalius might have forgotten due to the amount of his daily occupations.[255] He then made clear that he wanted to have his salary augmented and remarked that because the Leiden burgomaster Seistius stayed with Rosendalius, and because they were judges in the same lawsuit, Rosendalius could easily urge the burgomaster about this issue.

Snellius also had another trump card with which he could win the sympathy of the university administration. He had prepared an edition of a work by Scaliger on ancient money, named (almost identical to his own book on the same topic) *De Re Nummaria Dissertatio, Liber posthumus* ('A Treatise on Money, published posthumously'), which he dedicated to all regents involved in his promotion: the curators of the university and the magistrates of the town.[256]

The combined offensive of Snellius and Rosendalius was successful: Snellius's salary was raised to 500 guilders. Only from 1618 onward he received the 600 guilders he had asked for three years earlier. He earned this amount till his death, but as an extra remuneration he received 200 guilders each year for instruments

[253] [Molhuysen, 1916, p. 51]; see [Clotz, 1998, p. 101] for an overview of salaries of Leiden professors in 1585, 1600 and 1615, and her pp. 100–106 for a further analysis of the salaries. See also [Otterspeer, 2000, pp. 304–310].

[254] [Molhuysen, 1916, pp. 56, 59].

[255] 'Etsi haud dubitem causam meam tibi curae esse, atque eam de meliore nota Dominis Curatoribus Mathenesio Vander-Milioque commendatam: cum tamen quotidianarum occupationum tantus sit cumulus, ut ea sollicitudo hanc curam non quidem omnino expectorare, at leviter obliterare possit. Omnino mihi faciendum putavi, ut eius memoriam tibi refricarem.' [Snellius, 1616].

[256] [Scaliger (W. Snellius), 1616]. It has been reprinted as [Scaliger (W. Snellius ed.), 1737].

from 1620 onward.[257]

Although Snellius was certainly not among the best paid professors, his salary provoked the jealousy of the professor of medicine Heurnius, who wrote a letter to the university administration in 1623, complaining that he only earned 700 guilders, the same amount as in the past ten years, whereas his job had become ever more demanding. He also pointed out that Meursius and Snellius received the same salary as he, although they had been appointed as professors more recently, and moreover, although they were professors in the propaedeutical sciences 'which were valued much less than medicine in all universities'.[258] If these sentiments were shared by other university members, Snellius must have had a difficult time to explain the value of mathematics.

Snellius only had to look at the example of his father to know that some perseverance was needed to become a full professor. Rudolph had started as a *Privatdozent*, and had then become an extraordinary professor of mathematics in 1581, on a salary of 200 guilders a year. He was meant to be replaced by someone more experienced in mathematics, but this never happened. His salary was raised several times and in 1601 he was finally installed as a regular professor. In the next year he received another increase in salary, earning 500 guilders. From 1604 till his death he received 600 guilders a year. This was not considered as much by the curators and therefore he received a silver cup worth 120 guilders in 1607, and a silver dish in 1608. In this same period, Scaliger stood lonely at the summit with an income of 2000 guilders.[259]

When we compare this to the salary earned by the professor of mathematics in Franeker, Metius, we see that the Snellii had some reason to complain. Metius, who was nine years older than Willebrord, already earned 600 guilders a year in 1605. His salary sometimes increased, and sometimes decreased in later years: it was in the range between 500 and 700 guilders in Snellius's lifetime, and was even raised to 1000 guilders in 1634.[260]

Snellius continued his teaching in this period. Hardly anything is known about the content of his classes. Probably he lectured on a range of classical authors from the whole field of the mathematical sciences, adding a few moderns such as Ramus. Unlike his father in his last decade, he had to do all teaching alone. A few of his own notes at the back of his copy of Risnerus's *Optica* tell us that he indeed discussed mixed mathematics: he taught *sphaera* (motion of heavenly bodies) and optics on alternate days from December 1617 till July 1618 and he taught optics in 1619. He again started his courses on *sphaera* and

[257][Molhuysen, 1916, pp. 65, 81, 92, 106, 117].

[258]'niet en bedienen officium in Facultate maer in Philosophia en Literis, welcke scientien van veel minder waerden en gewichte in alle academien geestimeert werden als de Medicine.' [Molhuysen, 1916, p. 199*].

[259][Molhuysen, 1913, pp. 26, 37, 66, 82, 111, 132, 136, 142, 156, 175, 179], [Witkam, 1973, pp. 85–89].

[260][van Berkel, 1985b, p. 233].

optics in October 1621. At least this year, he used near-contemporaneous books for these topics: by Maestlin and Risnerus respectively. He also gave practical surveying classes outside the town in this period (see section 3.4). Moreover, he may have taken over the physics classes of Jachaeus, who was suspended from 1619 to 1623 because of the religious troubles of the Twelve Years Truce period. If he indeed did so, we have a testimony of such a course in the shape of the notes of Cornelis Booth added to his copy of Cornelius Valerius's *Physicae, seu de Naturae Philosophia Institutio* from around 1622–1623.[261]

Snellius was a candidate for the rectorship several times, but never was appointed as such. In 1623, he received five or six votes, whereas the winners, Walaeus and Cunaeus, had seven or eight votes. In 1625, Antonius Walaeus and Snellius were nominated by the Senate, of whom Maurice of Nassau, who had the final decision power, choose Walaeus. A year later, Snellius was nominated again, this time together with Polyander. Maurice, however, disregarded this proposal and continued Walaeus's rectorship. The reason for this was probably not any distrust of Snellius's capacities, but the Stadholder's wish to keep a firm grip on the religious policy of the university: Antonius Walaeus was one of those who represented the orthodox side in the Truce conflicts. He had been appointed in 1619, when Maurice's side (the orthodox) had won. His father's old friend Bronchorstius did not give up Willebrord's case and in October 1626, he proposed to make Snellius a candidate for next year's rectorship again, yet then Snellius's death interfered.[262]

One of Snellius's key publications appeared in this period: the *Eratosthenes Batavus*, his book on his endeavours to determine the length of the circumference of the globe (this part of Snellius's work is discussed at length in chapter 3). It was published in 1617, and in the years both before and after that Snellius spent much time on this project. In contrast to his habitual residence in Leiden, much travelling had to be done for the measurements, not only in the Dutch Republic, but also in the Southern Netherlands. Snellius claimed that he left his house and family unwillingly for these expeditions.[263]

Snellius dedicated this book to the States General, perhaps inspired by Adriana Simons's successful dedication of the *Fundamenta* to them (see p. 90). He received 200 guilders, 40 % of his annual salary. This is the earliest known instance of a direct financial remuneration received by Snellius for the dedication of a book; in earlier cases, his dedications may have yielded him other profits, such as support in his career. Two years later Snellius dedicated *De Circulo*, again a Van Ceulen translation, to the States General as well, for which he received 100

[261][Risnerus, 1606, last page], cp. [Vollgraff, 1918, p. 30b]; [Vermij, 2002, p. 41].

[262][van Slee, 1898, pp. 177, 188, 194, 198], [Otterspeer, 2000, p. 283].

[263]'[...] sed invitum pene domo me, et a meis abduxere.' [Snellius, 1617b, p. 177]. Snellius speaks about the stimulation of the Sterrenberg brothers and their tutor to actually perform his project, see p. 124.

guilders (see p. 83).[264]

Dedicating books to both private persons and local, regional or general authorities was a common practice, and books were often dedicated to several patrons at the same time. The authorities frequently gave financial rewards in return. In Snellius's time, amounts between 10 and 300 guilders were common. The official (incomplete) resolutions of the States of Holland show that their average remuneration for a book was 274 guilders between 1610 and 1619.[265]

Snellius was also in demand as a mathematical adviser, of which the most telling example is his membership of a committee that had to judge a new solution for the problem of determining longitude at sea. The method under consideration was developed by Jan Hendrik Jarichs, the collector-general of the College of the Admiralty in Friesland, who had filed a patent application. A solution of this difficult problem would be most welcome to the government of the Dutch Republic as a major maritime nation. Jarichs proposed a way to correct the roughly estimated longitude by using known data, in particular the difference between estimated and observed latitude. He had devised some aids to facilitate its application: auxiliary charts and transparent paper with loxodromes.[266]

Some representatives of the States General were selected to study Jarichs's method. To assist them, a committee of experts was established, consisting of Snellius, Simon Stevin, Jan Pietersz Dou, Melchior van Kerckhove, Jan Cornelis Kunst and Jooris Joosten. On 3 November 1617 they all gathered to study the matter. As a final conclusion was not immediately reached, it was decided to test the method in practice. A ship was prepared and some navigators were asked to experiment with the method during a long voyage. When they had returned, Snellius was again asked to come to The Hague to listen to the report with the results. Snellius and his fellow committee members, this time Stevin, Dou and Jan Pietersz Marlois, also questioned the navigators. On 5 January 1619 the committee presented their report on Jarich's invention to the States General.

The committee concluded that the first part of Jarichs's invention was not very practical, mainly because a precise measurement of the altitude of the Pole Star (necessary for the determination of the latitude) was impeded by the movements of the sea. Yet they did appreciate Jarichs's auxiliary devices. Moreover, they saw another positive consequence of the experiment: since some experienced navigators had now seen the different outcomes of the calculations of the position of their ship in practice, they and others would now be more motivated

264[Smit, 1975, p. 241]; [Smit and Roelevink, 1981, p. 263].

265[Verkruijsse, 1991, p. 232]; see for more data cf.hum.uva.nl/bookmaster/boekenmecenaat/-index.htm, a website devoted to maecenatism of books in the seventeenth century, edited by dr. P.J. Verkruijsse.

266Loxodromes are curves on the globe that cut all meridians under the same angle.

to seek for the best method for determining their position instead of just doing what they were used to. Snellius did not have to do this consultancy for free. He, Marlois and Dou (Stevin is not mentioned) were paid 165 guilders and 1 stiver fee attendance money for 8 days (Snellius's part of that is not specified in the sources).

A patent was granted to Jarichs for twelve years for the publication of his book *'t Gesicht des Grooten Zeevaerts* (1619). This book made the value of the Snellius mile, as based on the results of *Eratosthenes Batavus*, known to navigators (see section 3.6). Jarichs had personally discussed the matter with Snellius.[267] Naturally, Jarichs dedicated his book to the States General and this time Snellius was asked by them to study the book, together with Pieter Nannincx, a surveyor, and Lambert Palmeto, a teacher of navigation and mathematics (April 1620). This new committee judged Jarichs's method more favourably. They stated that his method was almost always valid and also practicable. This outcome yielded Jarichs a huge amount of money: from 1625 onward he received 600 guilders per annum of the Admiralty of Friesland, which annuity was inherited by his children. Nevertheless, the longitude problem was not really solved by Jarichs's rule, which in fact gave no more than an—often but not always—reasonable approximation. Snellius's fee was four guilders a day. Although this was much more than the other two committee members received—2 guilders and 10 stivers—,Snellius asked for an increment, just as Nannincx, but this request was not granted. Apparently Snellius thought that he in his position was entitled to a higher reward, even though he already was paid more than his fellows.[268]

Later, Snellius was again asked to be a navigational expert. When in 1621 Claes Jacobsz claimed to the States General to have found the solution to the problem of determining longitude at sea, he was asked to report to Snellius, at his own expenses. Snellius could then advise the States General.[269] He also wrote a letter of recommendation for the textbook *Schat-kamer des grooten seevaerts-kunst* by C.J. Lastman of 1621.[270]

Finally, Snellius also wrote a book of his own on navigation, the *Tiphys Batavus*, published in 1624. Tiphys had been the mythological pilot of the Argonauts. The book was dedicated to the States of Holland, which discussed

[267] [Davids, 1990, p. 8].

[268] [Smit, 1975, index 'Jarichs'], [Smit and Roelevink, 1981, index 'Jarichs']. See [Dodt van Flensburg, 1848, pp. 17–19, 49–52] for the text of some of the resolutions. The report of 5 January 1619 is [Stevin et al., 1619]; Jarichs's request to the States General of the same day is [Jarichs, 1619] (both now in the National Archive in The Hague). Cp. the document of the Notary Van Banchem, now in the Municipal Archive Amsterdam: [Van Banchem, 1619]. See [Davids, 1986, pp. 80–85, 285–287] for an explanation of the method and its further history.

[269] [Roelevink, 1983, p. 113].

[270] [Davids, 1986, pp. 115–116]. Davids claims that there is no extant copy of the first edition of this book, but the University of Aberdeen has it in its online catalogue. I have not been able to trace a copy of the 1653 edition mentioned by Davids.

it in their meeting. They granted Snellius 300 Pound (guilders) for it because they considered him as 'an extraordinarily experienced man who has rendered a good office to the country and could do more in the future by his skills'.[271]

In the dedicatory letter, Snellius expressed the wish to contribute to the solution of the problem of determining position at sea, like many others. In his preface to the reader, he added that he wanted to give a thorough exposition, in which the assertions would be proved from their foundations, and that he wanted his work to be useful for daily practice.[272] He also explored the use of the magnetic compass for navigational purposes. This was no trivial matter, because the directions of the magnetic North and the geographical North were not the same, and their difference varied between different places on earth. Snellius and his contemporaries were well aware of this fact. Snellius followed Stevin with his approach. He used his results from *Eratosthenes Batavus* to relate the angular and linear distances at sea.[273] In particular, he paid much attention to the mathematical properties of the loxodrome, a curve on the surface of the earth that cuts all meridians under the same angle. He had introduced the term 'loxodromia' as the translation of Stevin's neologism 'kromstreeck'. The *Tiphys* consists of two parts, one theoretical and one more practical. In the second, practical, part, some examples were included and Snellius stressed the importance of keeping a logbook.[274]

The summit of Snellius's astronomical activities was in the years 1618–1619. He published two books: the first, *Observationes Hassiacae* ('Hessian Observations'), contained observations by other astronomers, the second, *Descriptio Cometae* ('Description of the Comet'), Snellius's own discussion of the characteristics of the large comet of November 1618, to which an older treatise by Rothmann about the comet of 1585 was added. Both books were dedicated to Maurice Landgrave of Hessen and the second was even an answer to a question which he had asked Snellius (see further chapter 4).[275] Snellius may also have been preparing a manuscript on the influences of comets on human destiny.[276]

Some of the readers of these books are known: e.g. in the preface of his *Tabulae Rudolphinae*, Kepler gave Snellius a compliment on an aspect of his edition of observations conducted by Tycho and Maurice's father in the *Observationes Has-*

[271]'Waerop geconsidereert dat hij is een extraordinare ervaren man die de landen veel dienst gedaen heeft ende noch door sijn const soude connen doen, is 't boeck met danckbaerheyt aengenomen ende hem daervoor vereert de somme van 300 £.' [Veenendaal-Barth et al., 1987, p. 357].

[272][Snellius, 1624, fol. $7 * 3^{r-v}$].

[273][Snellius, 1624, p. 6].

[274][Davids, 1986, pp. 115, 386].

[275][Snellius, 1618], [Snellius, 1619].

[276]Beeckman mentions a manuscript by Snellius called *Cometarum apotelesmata* ('Influences of comets upon human destiny'), found in Simon Stevin's inheritance in 1624, [de Waard, 1942, p. 291]. Rudolph Snellius may also have been the author, or it might not have been a new work, but an extract from the *Descriptio Cometae*.

siacae. Kepler's correspondent Johannes Remus Quietanus mentioned the *Observationes Hassiacae* to Kepler, and informed after the *Descriptio Cometae*.[277]

2.9.2 Geometry in Latin and Dutch

Snellius also continued his work on pure mathematics in this period. His rendering of Van Ceulen's *Fondamenten* contains in fact much material of himself (see section 2.9.3 and references there). He also exerted himself for one of the standard geometrical problems of his time, the quadrature of the circle. The problem of squaring the circle had been famous and notorious since Antiquity. It was required to construct a square with the same size as a given circle. This is equivalent to determining the value of π (this symbol was not yet in use in the seventeenth century; instead, mathematicians referred to the ratio between the circumference and diameter of a circle). Even though many mathematicians had tried their hand, none had been able to solve it in the only way truly satisfactory to those educated with the norms of Euclidean geometry, that is, with ruler and compass alone. Therefore, the problem kept challenging mathematicians, some of which proposed very strange solutions, while others, although not truly able to solve it, developed fruitful mathematics. The deadlock was overcome in two ways: more geometrical means were allowed to solve the problem, and numerical approximations of π were given instead of an exact solution.[278]

Van Ceulen and Snellius chose this second approach. They both used the method developed by Archimedes, who had approximated the value of π ever more precisely by inscribing and circumscribing the circle with polygons of an increasing number of sides. Ludolph van Ceulen had written down his results in *Vanden Circkel* (1596). Snellius translated a small part of this book into Latin, adding some annotations. He may not have translated all of it because he found his own method more useful; in one place, he complained that Van Ceulen's proof was 'thorny' and 'entangled'.[279] This translation was published together with a reprint of most of the *Fundamenta*, as *De circulo et adscriptis liber* ('Book on the Circle and its Circumscribed and Inscribed Figures') in 1619. Snellius added an appendix on triangle division to it (see section 6.4.3 for the appendix).[280]

Snellius also published his own work on the approximate quadrature of the circle, in the *Cyclometricus* ('Circle Measurer', 1621). He improved the method of Archimedes and Van Ceulen in such a way that he could determine more decimals of π than they on the basis of a polygon with the same number of

[277][Kepler (F. Hammer ed.), 1969, p. 44], [Kepler (M. Caspar ed.), 1955, pp. 277, 395].

[278]See e.g. [Jesseph, 1999, pp. 22–28], Herman C. Schepler in [Berggren et al., 1997, pp. 282–305]. In fact, it is impossible to solve the problem by ruler and compass.

[279]'Autoris demonstratio spinosa est et variis linearum ductibus intricata.' [van Ceulen, 1619, 2, p. 3].

[280][van Ceulen, 1596], [Bosmans, 1910].

sides. Even so, he referred to the work of Van Ceulen respectfully. With enormous time-consuming calculations, the latter had calculated one more decimal than Snellius's 34. Van Ceulen's result was printed for the first time in the *Cyclometricus*. It was also engraved on his memorial stone. Even though Snellius's method was faster, he did not feel incited to surpass the precision of Van Ceulen.[281] Snellius explained, but did not prove, the central propositions on which his method was based. Christian Huygens later supplied the proofs in his *De Circuli Magnitudine Inventa*.[282]

Apart from their geometrical contents, Snellius's circle publications contain other interesting features. Snellius dedicated *De Circulo* to the States General. He wrote a dedicatory letter in which he sketched the historical background of the squaring of the circle and announced his own *Cyclometricus*. He offered the book to the States General together with a hand-written letter in Dutch, in which he again stressed the difficulty and weight of the enterprise of squaring the circle, and pointed out how many capable mathematicians from abroad had failed. The States General accepted Snellius's present and he received a recompense of 100 guilders in return.[283] *Cyclometricus* was again dedicated to Maurice of Nassau. The usefulness of the work was contended once more. The Leiden curator Mathenes had had some influence on its contents: he had induced Snellius to write an appendix.[284]

Slightly later, Snellius's only vernacular publication appeared. This was the *Meetkonst* (1622), the Dutch translation of Ramus's *Geometria*.[285] It had been translated by Dirck Houtman, a clergyman in Loosdrecht,[286] under supervision of Snellius. Although the title page claims that Snellius had 'enriched and explained'[287] the book, the differences between the original text and the translation are minimal and the statement merely seems to have served advertising purposes. Nevertheless, three features make this book worthy of some further deliberations:

281 [Snellius, 1621, pp. 54–55].

282 Propositions 28 and 29 in the *Cyclometricus*, 12 in Huygens's book. See [Jesseph, 1999, p. 284] or [Bierens de Haan, 1878a, pp. 175, 180]. A modern rendering of Huygens's method can be found in [Heilbron, 1998, pp. 271–277]. See for a recent follow-up of Snellius's method [Beukers and Reinboud, 2002].

283 The original letter is still extant in the National Archive in The Hague, [Snellius, 1619]. For a summary of the resolution of 8 October 1619, see [Smit and Roelevink, 1981, p. 263]; for its text, see [Dodt van Flensburg, 1848, p. 87].

284 [Snellius, 1621, p. 91].

285 The book is discussed in [Verdonk, 1966] and briefly in [van Wijk, 1941]. This last article contains some mistakes; e.g. the author thinks that the *Meetkonst* is an independent adaptation of Ramus's *Via regia ad geometriam*, whereas in fact it is an almost literal translation of the *Geometria*, [van Wijk, 1941, p. 148].

286 [Ramus, 1622, fol. * 3^v]. Theodorus Houtmannus came from Amersfoort and was registered as a student of theology in Leiden in 1587 and 1589. From 1611 until his death in 1629 he was a minister in Nieuw-Loosdrecht. Snellius also mentioned him in his notes to Risner's *Optica*, [Verdonk, 1966, pp. 227–228].

287 'verrijckt, en verklaert', [Ramus, 1622, title page].

the elaborate preliminary matter, the translation of technical terms into Dutch and the appendix in which Snellius discussed Heron's Theorem, which expresses the area of a triangle in terms of its sides (see section 7.5 for this appendix). Moreover, it is telling that Snellius wanted precisely this book to become available to a vernacular audience and not a more recent work on the same topic. This shows that he considered it as a good textbook, still endorsing Ramus's work publicly long after his father had died. Yet he did not make a new Latin edition, which suggests that the level was too elementary for his students.

The preliminary matter of the book consists of the dedication of Houtman to Adriaen Pauw, a magistrate from Amsterdam and curator of Leiden University; Snellius's preface to the reader; and two poems. The first of these was by Petrus Scriverius, who did not write his full name, but abbreviated it to 'P.S.'. That he is the author behind these initials, is proven by two manuscript letters of Snellius to Scriverius, in the first of which Snellius urged him to send him his poetry for the edition of Ramus's *Geometria*, because the publisher was already 'pulling his ear', i.e. admonishing him to hurry. Scriverius did not react very quickly and in the second letter Snellius almost had to beg him to finish the poem. Scriverius (1576–1660) was an antiquarian scholar, who lived in Leiden. Although he was not officially connected to the university, he was a prominent member of the Dutch world of learning.[288] The second poem was by Joost van den Vondel. It is not likely that Snellius and this famous Dutch poet knew each other personally. The publisher Willem Jansz Blaeu probably arranged the poem to be written himself.

The division of the tasks between Snellius and Houtman is explained by them in the beginning. Snellius was the mathematician and therefore the intellectual leader of the two, taking the initiative, overseeing Houtman's work, translating the technical Latin terms into Dutch and giving some extra clarifications.[289]

The two laudatory poems highlight Snellius's role in this undertaking much more than Houtman's. Scriverius praised Houtman as 'Goudman' (Goldman)[290]

[288] [Snellius, 1622], [Grotius (edited and translated by Jan Waszink et al.), 2000, p. 28].

[289] Houtman writes: 'tot welck stout bestaen niet alleen de overvloedighe rijckdom van onse spraeck my oorsaecke ghegeven, maer oock veel meer het aenporren en raedt van den Hooghgeleerden Heere Willebrordus Snellius gedreven, heeft.' [Ramus, 1622, fol. * 3^r].

Snellius writes: 'Hebbe daerom den welgeleerden en verstandighen Dirck Hendricxsz Houtman daer toe gheport om dien arbeyt op hem te nemen, en daer toe mijn hulpighe handt gheboden, om hem tot dien eynde willigher te maecken, ghelijck als ick mede, daer toe versocht zijnde, gedaen hebbe. En diens volgende syne oversettinghe ernstlijck doorsien, verrijckt, en hier en daer nae den eysch verklaert en verlicht hebbe, en ghearbeyt de Latijnsche kunstwoorden [in margin: vocabula technica] in plat duyts uyt te drucken, ende in alle manieren den duytschen Leser nae mijn vermoghen behulpich te zijn.' [Ramus, 1622, fol. * 4^r].

[290]

> Ontkendt, o Snelli, niet; cieraet van onse' Athenen,
> U weldaedt en u lof: te recht ginght ghy verleenen
> U dierbaer tijd en'raed aen Houtman uwen vriendt,

and complimented Ramus posthumously 'with his excellent and large intellect' for bringing together all material of the classical mathematicians, in that way even surpassing Euclid.[291] Even though this book must have been meant for a diverse audience of not very far advanced 'lovers of art'[292], as its level and language shows, the humanistic way of thinking is present. Snellius defended the invention of new words with a reference to the Roman practice to borrow from the Greek, and Scriverius developed a complicated argument about the relative values of old and new knowledge. Vondel's poem contains some puns on the names of Ramus ('branch') and Snellius (in Dutch 'snel' means 'quick'), whom he addressed as 'quick Snellius, Euclid's most beloved fellow'.[293]

The decision to make a translation into the vernacular was very close to Ramus's own intentions. Ramus himself had not only wanted to publish a French translation of the *Arithmetica* and *Geometria*, in which he did not succeed, but had also expressed his interest in spreading his thoughts in French in other contexts.[294] The *Meetkonst* was one of the rare instances of translations of

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> Daer Rami heil'ge asch, en'hy was me'e gediendt.
> En' menigh Idioot, die dese vruchten smaecken,
> En'aen soo swaeren werck soo lichtelick gheraecken.
> O Houtman! veelen sult ghy Goutman zijn gesegt
> Om dat ghy haer de Konst soo rondelick uytlegt,
> In hare moeders tael: om dat ghy gaet toe meeten
> 'T geen vande Meet-konst ons heeft Rami pen doen weeten: [...]

[Ramus, 1622, fol. $**^r$].

[291]'Die Ramus van verstandt uytnemend' en' soo groot, [...]', Scriverius in [Ramus, 1622, fol. $**^r$].

[292]'Voor den kunst-lievenden', as Houtman has it, [Ramus, 1622, fol. $* 3^r$].

[293]

> Klinkert
> Wat snelgewiekte bood brengt ons den gulden Tak?
> 'T is Snellius, die snel van geest, van sinnen wakker,
> Dien snellijk plukken liet op Ramus vetten akker,
> En snel dees Spruyte gaf een kracht die haer ontbrak.
> Veel sneller salmen nu gaen meten 's werelds dak,
> O snellen Snellius, Euclides weerdste makker!
> Ghy vliegt de kunst voorby in snelheid: want men sprakker
> Noyt sneller af met re'en eer Snel het hoofd opstak.
> Ghy snelle Geesten volgt, en sneller op wilt merken,
> Vermids u Snel gaet voor met snelle en lichte vlerken:
> Of giert hy u te snel, so trekt een snelle schacht
> Uyt sijn geswinde wiek, soo spoedy langs hoe sneller,
> En hoe ghy sneller stygt, hoe haer de Meetkunst heller
> En sneller op sal doen tot in haer volle kracht.

The poem is subscribed by 'I. v. Vondelen', [Ramus, 1622, p. $** 6^v$]. The editors of Vondel's *Collected Works* have included it, [Sterck et al., 1929, p. 428], which identification I follow. I do not presume to be able to translate Vondel's highly literate Dutch into adequate English—the non-Dutch reader is kindly requested to skip this poem.

[294][Verdonk, 1966, pp. 9–10, 47], [Meerhoff, 2001b, pp. 370–371].

Ramus's mathematical works. Snellius suggested that Houtman and he would also translate the *Arithmetica*, but no trace of this project has remained.[295]

Snellius was especially attentive to the translation of technical terms into Dutch. In general, the Latin terms, the meaning of which was more widely known, were put in the margin next to their translation. Snellius and Houtman borrowed some technical terms from Stevin, who had made a conscious effort to develop a clear and consistent scientific terminology in Dutch, thus enabling Dutch mathematicians to write about mathematics in the vernacular on the same level as in Latin. In this way, the *Meetkonst* helped these terms to become familiar to the Dutch audience. Examples of these are 'wisconstenaer/-konstenaer' for 'mathematicus', 'platcloot/platkloot' for 'astrolabium', 'bran(d)tsne' for 'parabola' and 'pael' for 'terminus'. Sometimes, Snellius and Houtman chose other Dutch words than Stevin, showing that they had their own ideas about the development of a Dutch mathematical language, e.g. they translated 'axioma' as 'ghemeene kennissen' ('common knowledge'), Stevin as 'ghemeene regel' ('common rule'), and 'ellipsis' as 'langkloot', Stevin as 'lanckrondt'.[296]

2.9.3 *Behind the scenes of the publication of the* Fundamenta *(1615)*

In a letter which probably dates from early 1615, Snellius promised Rosendalius a very special New Year's gift, fit for a lover of books, as a token of his friendship.[297] Before he could actually offer this present, he had to give some explanation to his correspondent. A few years after Ludolph van Ceulen's death in 1610, his widow and some other heirs had decided to publish part of his manuscripts, which were written in Dutch. They would appear in 1615 under the title *Arithmetische en Geometrische Fondamenten* ('Arithmetical and Geometrical Foundations'). Snellius told Rosendalius that these heirs had asked him to translate this work into Latin in order to make it accessible for an international

[295] 'Sult derhalven, beminde Leser D. Holtmanni arbeydt, en myne ghenegentheydt ten besten duyden, en soo voor lief nemen, op dat ghy meer uyt de selve koker mueght ontfanghen.' [Ramus, 1622, fol. * 4^v].

[Verdonk, 1966, pp. 117–118, 227–228] mentions the following translations of both works: the *Geometria* into English (1636); the *Arithmetica* into German (1569, on the authority of Ramus himself), into English (1592) and into Dutch (1636, by Bernard Lampe).

[296] See [Dijksterhuis, 1943, pp. 308–310] for a list of (purely) mathematical terms in Dutch that have either been coined or been popularized by Stevin.

[297] 'Constitueram autem haud nudum votum afferre, sed simul xenium veluti tesseram animi et affectus indicem. non illud quidem splendidum aut de divite censu depromptum: sed qualis hoc hominum genus decet qui curam suam libris addixerunt. Id ipsum tamen te inconsulto haud facere ausus fui, neque prius quam consilium meum tibi exposuissem.' [Snellius, 1615, fol. 224^r].

learned audience,[298] to which request he had yielded reluctantly:

> although I considered myself more burdened than honoured, I have nevertheless accepted to do it, in order to show that the memory of my deceased friend is very dear to me, and in order to extend Van Ceulen's fame abroad, which we have already acknowledged in the Netherlands in these sciences.[299]

Although there is no reason to doubt the sincerity of the wish of Adriana Simons, Van Ceulen's widow, to enhance her deceased husband's reputation by making his work known, she was certainly driven by financial motives as well. She even had the *Fondamenten* printed with three different dedicatory letters: to Count Ernest of Nassau and the States of Gelderland, to Count Maurice of Nassau and the States of Holland and West-Friesland, and to the Admiralties of Holland and West-Friesland,[300] apparently determined to gain as much as possible from the book by addressing different groups of potential patrons.

Snellius wrote to Rosendalius that he had to put much effort into the *Fundamenta*, and that yet Simons demanded the right to enclose a dedicatory letter at the beginning of the volume, to either the States[301] or the Stadholder. Snellius yielded this to her, although he thought that she was quibbling, on the condition that he was allowed to include his own dedicatory letter in the middle of the book. The part starting there, the 'best and richest part of the whole work', was destined for Rosendalius.[302]

Snellius insisted on this right because he wanted to use the book as a tool for his own career. He did not keep his motives for the dedication to Rosendalius hidden in his letter to his patron, explicitly asking him for a favour: Rosendalius should talk to the curators to arrange that Snellius would obtain 'without sweat'

[298]'Evenit superioribus diebus ut haeredes Ludolphi a Ceulen (cuius industria in Logisticis et Algebricis satis laudata atque adeo doctis et iis qui in hac arte celebres sunt commendata) ut haeredes inquam et vidua quaedam postuma eius monumenta publicare constituerent, captoque iam opere obnixe me rogarent, ut et amici famam ne desererem, et eiusdem viduae hac in re gratificarer, atque idem illud opus etiam Latine edoctis communicarem.' [Snellius, 1615, fol. 224^r].

[299]'hic quamvis plus oneris quam honoris mihi imponi cernerem: tamen ut defuncti quondam amici recordationem mihi non ingratam *ostenderem*, et nomen atque famam, quam in his artibus in Belgio iam assecuti sumus irem amplificatum, facturum recepi.' [*ostenderem*: the original is damaged; I have completed *osten*[...]*em* to *ostenderem*.] [Snellius, 1615, fol. 224^r].

[300][Bierens de Haan, 1878b, p. 148].

[301]The States General are meant, see below.

[302]'Ibi vidua cuius illud erat aucupium, sibi operis dedicationem ut concederem rogavit, quod pari facilitate ipsi concessi, qua operis versionem in me receperam: assensus itaque ei sum ea lege ut saltem secundae partes meae essent, hoc est, quod illa in totum opus sibi sumebat, id ego in parte aliqua pro meo iure usurparem. Cum itaque illa vel ad ordines vel ad Principem ire constituerit, ego partem optimam et totius operis sumen Tibi destinavi, nisi ipse secus sentias et aliud malis e nostris merum.' [Snellius, 1615, fol. 224^r].

that for which others had to exert themselves much,[303] to be more precise: Snellius wanted to become a regular professor and receive an increase in salary to the level of his father's.[304] Rudolph Snellius had received 600 guilders at the end of his life, whereas Willebrord earned 400 when he wrote this letter. This bold request was somewhat softened by its closing remark:

> This is what I felt that I had better explain in a letter than in person, because a letter does not blush.[305]

The last part of this sentence was a phrase borrowed from Cicero.[306]

The Latin version of the *Fondamenten*, entitled *Fundamenta Arithmetica et Geometrica*, was also published in 1615.[307] Snellius did much more than merely translating: he corrected mistakes, changed the formulation of problems, added his comments and a number of his own mathematical inventions. Apparently, Rosendalius granted him his permission for the dedication, because Snellius did indeed write a flattering dedicatory letter to him, which was printed on pages 83 and 84 of the *Fundamenta*. This letter is a complex and rich text with some programmatic and polemical statements (it is discussed in detail in section 5.4).

It seems that the Latin book was printed in a great hurry to have it ready in time for the big Frankfurt book fair. Snellius expressed his irritation about the impossibility to finish his work at leisure repeatedly, for instance writing openly in the book that lack of time had hindered him from making an addition.[308] There was no time to wait for new figures to be engraved, about which Snellius expressed his annoyance a number of times.[309] He wrote for example that he had developed an instrument to construct roots of degree 2^n, which he would like very much to explain to the readers,

> if the publishing company did not press me for this translation too rudely, because they try to pay the Dutch edition with the Latin

[303]'Quamobrem hoc illud est vir Amplissime quod nunc obnixe rogatum Te [?] velim, cum ipsa oportunitas nos huc quasi invitare videatur, ut si mihi tua facilitate et opera frui liceat, num [?] D. Curatores pro me interpelles, et mihi tuo benificio, quo etiam alienissimi gaudent, hoc impetrare liceat ἀνιδρωτί, quod alias cum sollicitudine et cura esset adnitendum.' [Snellius, 1615, fol. 224^r].

[304]'Summa petitionis haec est, ut in Professorum ordinariorum numerum allegerer, et stipendio doceam eodem quo Parens meus p.[iae] m.[emoriae] olim docuit, hactenus enim ducentis florenis annuis ab illa summa absum.' [Snellius, 1615, fol. 224^r].

[305]'Haec erant quae per literas potius quam coram explicare me posse putavi, cum litera non erubescat.' [Snellius, 1615, fol. 224^r].

[306]Cicero to L. Lucceius, *Epp. ad Fam.* V.12.1.3; cp. [Vollgraff, 1948].

[307]See chapters 5, 6 and 7 for a discussion of the mathematical contents of this book.

[308]Snellius to Rosendalius: 'praela enim hic festent et operae typographicae nos urgent: id enim summis viribus contendimus ut proximis nundinis lucem videat [...]', [Snellius, 1615, fol. 224^r]. Cp. [van Ceulen, 1615b, p. 135].

[309][van Ceulen, 1615b, pp. 135, 221, 230, 233].

one.[310]

Elsewhere in the book, he remarked that the publishing company did not give him the freedom to write an extension, that he was forced to postpone a demonstration to the next edition and that he was prompted to make haste by the printers. Although he was restricted, he seized the occasion to claim certain inventions as his own, which he could expand in a next edition or another volume.[311]

The Dutch text must have been prepared for printing by someone else than Snellius, who paid much less attention to a careful exhibition than he, as is testified for example by a wrong presentation of a proposition by Van Ceulen on triangle division (see section 6.3). The Dutch version ends very abruptly: no answer follows after the enunciation of the last problem. Snellius explained that Van Ceulen's death had prevented him from finishing this problem and he gave his own solution.[312]

The printing of the *Fundamenta* was done sloppily[313] and it seems that there was so little time that the printer started before Adriana Simons had finished her dedicatory letter, which is suggested by its lacking in at least one of the extant copies.[314] Simons dedicated the book to the States General in a letter with the usual rhetorical flourish about the splendour and usefulness of mathematics, much more standard than its Snellian counterpart. Much less usual was the fact that such a letter was written by a woman. It is unlikely that she knew the discourse well enough to be able to produce such a letter herself, and even more so that she was proficient in Latin. Snellius would have been the obvious person to assist her, but his annoyance about her may have prevented his interference. It is not known whether he did help her or not.

[310]'[...] nisi operae typographicae versionem istam, dum cum belgica editione paria facere conantur, nimis importune urgerent.' [van Ceulen, 1615b, p. 109].

[311][van Ceulen, 1615b, pp. 210, 235]. Cp. p. 121: 'sed et hoc, et alia huius generis complura nostra data occasione in lucem et utilitatem philomathōn aliquando proferemus.' Cp. p. 188 for some more examples.

[312][van Ceulen, 1615a, p. 271], [van Ceulen, 1615b, pp. 267–269].

Pages 91–94 are lacking in the *Fondamenten* (at least in the two copies with I have seen), whereas the missing propositions do figure in the Latin text, in [van Ceulen, 1615b, pp. 54–58], which suggests that Snellius prepared his translation on the basis of a manuscript.

[313]Bierens de Haan's remark: 'men zoude bijna meenen, dat hij [sc. Snellius] [...] zich aan den nauwkeurigheid van den druk [...] niet veel liet gelegen liggen.' is not fair for Snellius, who most likely did not get the chance to do any proof reading. [Bierens de Haan, 1878b, p. 149].

[314]It lacks in the copy of Leiden University Library, shelf mark 2360 C 18; there are no signs of its removal after printing. This book was printed 'Apud Iacobum Marcum Bibliopolam'. I own a digital version of another copy (of which I do not know the location), which includes the letter. It was printed 'Apud Iustum a Colster Bibliopolam' according to its title page, and 'Apud Iustum a Colster, et Iacobum Marci. Bibliopolas.' according to the next page. This last addition is absent in the copy of Utrecht University Library (P. q. 1032), which does also contain Adriana Simons's letter.

Simons mentioned the benevolence which she had experienced previously from the States General, of course hoping to stimulate their generosity. Oddly, she did not refer to Snellius's share in the present work at all, although the book so clearly bears his marks, maybe fearing that she had to share the revenues of the book with him.[315] It is not known how much success she had with the dedications of the Dutch version, but she received 72 guilders from the States General for the Latin edition.[316]

This brief history of the genesis of the *Fundamenta* shows that both Van Ceulen's widow and Snellius wished to pursue their own interests by its publication, next to serving Van Ceulen's memory. The letter to Rosendalius is a very rich source, not only because of the information it contains about the publication history of the *Fundamenta*, but also because it allows us to see how Snellius used a patron to further his career, and was used by Van Ceulen's widow for her own benefit. Two reprints of the *Fundamenta* appeared a few years later (see section 5.3). It is unknown whether Snellius played a role in these.

2.9.4 A network of learned friends

Although only a small sample of Snellius's undoubtedly large correspondence has survived the ravages of time, these letters give us some telling examples of the relationship between Snellius and other scholars on the one hand and some patrons on the other hand. These letters have not been studied in any detail before. An overview is given in table 2.4 (details are given in the bibliography); the letters to Rosendalius are given in table 2.3.

An example of the way in which Snellius functioned in the context of the Leiden Academia is found in his correspondence with Cunaeus from around 1616. Petrus Cunaeus (1586–1638) was a great scholar, professor of Latin and law at the university. He asked Snellius's opinion about at least two issues. In December 1616, they corresponded about the exact meaning of the term *Jubilaeum*. Snellius argued that the Jubilee year covered the second half of every forty-ninth year (beginning in the autumn) and the first half of every fiftieth year; if one counted inclusively, this was the fiftieth year since the last Jubilee. He based himself on a number of Biblical quotations for this interpretation, and he asked Cunaeus not to disdain his opinion because it was new, but to judge the arguments themselves.[317] Cunaeus replied that he did not agree with Snellius's

[315] Fol. 3* of the second copy mentioned in the previous footnote.

[316] [van Deursen, 1984, p. 462]. Cp. [Bierens de Haan, 1878b, pp. 148, 166]; Bierens de Haan mistakenly assumed that she received this amount for her dedication of the Dutch version to Count Maurice of Nassau and the States of Holland.

[317] 'Neque vero nostra sententia ideo erit repudianda, quia nupera, quia nunc demum nata. quin potius quam vim ipsa argumenta habeant expendendum et iudicandum censeo [...]', [Burmannus, 1725, p. 138]. [Otterspeer, 2000, pp. 252–253]

Year	Correspondent	Source	Remarks
1616[1]	from Cunaeus	[Cunaeus, 1616]	
1616	to Cunaeus	[Snellius and Cunaeus, 1616]	edited in [Burmannus, 1725, pp. 136–138]
1616[2]	from Cunaeus	[Snellius and Cunaeus, 1616]	edited in [Burmannus, 1725, pp. 138–139]
?[1]	to Cunaeus	[Snellius and Cunaeus, 1616]	edited in [Burmannus, 1725, pp. 139–140]
?[2]	to Cunaeus	[Snellius and Cunaeus, 1616]	edited in [Burmannus, 1725, pp. 140–141]
1618	to Maurice of Hessen	[Snellius, 1618]	see section 4.3.2
1619	from Maurice of Hessen	[Maurice of Hessen, 1619]	see section 4.3.3
1619	to States General	[Snellius, 1619]	in Dutch
1622	from Bainbridge	[Bainbridge, 1622]	
1622[1]	to Scriverius	[Snellius, 1622]	
1622[2]	to Scriverius	[Snellius, 1622]	
1625[1]	from Gassendi	[Gassendi, 1964, pp. 2–4]	
1625[1]	to Gassendi	[Gassendi, 1964, pp. 391–393]	
1625[2]	to Gassendi	[Gassendi, 1964, p. 393]	
1625[2]	from Gassendi	[Gassendi, 1964, pp. 6–10]	

Table 2.4: Willebrord Snellius's correspondence (except with Rosendalius)

interpretation, and that he followed Scaliger's opinion.[318]

These two letters are antedated by a letter of thanks of 11 December 1616 from Cunaeus to Snellius, in which the former stated that he was glad that he had consulted Snellius's 'oracle' on a case of which he had understood nothing and that he was now certain to have received Snellius's sensible opinion.[319] On another occasion, Snellius wrote two letters to Cunaeus about the classical unit of measure *arura*, again referring to classical authors, this time Herodotus, Varro and Plinius, and reducing the *arura* to several other units of measure. In his second letter, he proposed an emendation of the *Suida* on the ground of Herodotus's authority and of the absurd consequences of the uncorrected statement, changing the reading of Budaeus and Agricola. He also explained a quote of Josephus.[320]

In 1617, Cunaeus asked again for Snellius's assistance, this time on behalf of his correspondent Apollonius Scottus. Cunaeus had tried to solve a problem of chronology proposed to him by Scottus, but it turned out to be too troublesome for him. Therefore, Snellius had to supply his mathematical expertise. When Cunaeus was writing to Scottus, he was anxiously waiting for the results of the industrious Snellius.[321]

[318][Snellius and Cunaeus, 1616], edited in [Burmannus, 1725, pp. 138–139].
[319][Cunaeus, 1616].
[320][Snellius and Cunaeus, 1616], edited in [Burmannus, 1725, pp. 139–141].
[321][Burmannus, 1725, pp. 19–20].

Not only did Snellius support Cunaeus, the latter was also involved in Snellius's work. He served as a kind of private poet for Snellius, writing poems to four of his books: he adorned Snellius's astronomical works with a poem on the observations of William of Hessen, which Snellius had edited in *Observationes Hassiacae* (1618), and another praising Snellius and discussing comets as omens in *Descriptio Cometae* (1619), and he extolled Snellius's achievements in the preliminary matter of *Eratosthenes Batavus* and *Cyclometricus*.[322]

It is very likely that Snellius was used as a quantitative expert by his fellow professors. Long after Snellius's death, the humanist scholar Gerardus Joannes Vossius remembered how he had appreciated Snellius as a colleague, not only because of his ingenuity and scholarship, but most of all because of his friendship. He also quoted laudatory phrases of Mersenne and Boulliau. Vossius and Snellius had had frequent discussions, mainly on astrology.[323] The example of Snellius and Cunaeus also shows that while they both had their own specialization, Snellius could discuss and explain topics relevant to Cunaeus. This was made easier by the fact that they shared their methods and a broad knowledge of classical texts, both being humanist scholars.

The relationship with colleagues was sometimes more practical. On one occasion, Snellius lent his house to Bronchorstius for the delivery of a present and later he was a guest at Bronchorstius's house when the latter gave a dinner on the occasion of the doctoral defence of his son.[324] Snellius's relationship with Stevin was so close, that the latter asked him to become the guardian of his children after his death. For some reason Snellius and the other candidate guardian, Bartholomeus Panhuysen, refused this when Stevin had actually died in 1620. Stevin's widow tried to compel them through a lawsuit at the Hof of Holland, to no avail.[325]

As an example of a scholarly exchange outside Leiden we can consider how fifteen years after Snellius's death, D. de Wilhem remembered in a letter to Constantin Huygens that Snellius had once sent him a letter in which he had explained his opinion on a certain change of the Poles and the different determinations of several locations by different astronomers; and he had also enquired about the reason for the heavy rainfall in De Wilhem's place of residence.[326]

Snellius was in touch with scientists from abroad as well. The evidence is scanty, yet the cases discussed here may again be considered as exemplary. John Bainbridge wrote a letter to Snellius in 1622, in which he expressed his admiration for Snellius's astronomical publications and gave some news about his own

[322] [Snellius, 1618, fol. (...)(...)2^v], [Snellius, 1619, pp. *5–*6], [Snellius, 1617b, fol.)?(v^v], [Snellius, 1621, *1^v].
[323] [Vossius, 1650, pp. 70–71, 202, 419]. For Vossius see [Rademaker, 1999].
[324] 31 July 1613, [van Slee, 1898, p. 135]; 22 November 1618, [van Slee, 1898, pp. 142–143].
[325] [Dijksterhuis, 1943, p. 20].
[326] [Worp, 1914, pp. 195–196].

endeavours in this field. Bainbridge also corresponded with Christopher Heydon on the comet of November 1618. They referred in their letters to Snellius's work on the comet.[327] Snellius was also the person who stimulated his friend Jacques de Valois to start astronomical observations.[328]

Late in his life, Snellius also had a scholarly exchange of letters with Pierre Gassend(i) (1592–1655). The main interests of this French clergyman, named Petrus Gassendus in Latin, were philosophy and astronomy.[329] Two letters of either correspondent have been preserved.[330] It was Gassendi who wrote the first letter, in February 1625. He praised Snellius for his erudite *Eratosthenes*, compared him favourably to those dusty scholars who only studied books and not real matters, and explained that he appreciated him highly

> because to the unremitting perusal of books, because of which you deserve to be called 'the Reader', like the Stagirite himself, you wanted that excellent observations of the heaven and the earth were added, which you have yourself performed with admirable skill.[331]

To introduce himself, Gassendi sent Snellius one of his own works, probably the anti-scholastic *Exercitationes Paradoxicae*. He also offered him the data of the latitudes of three towns, Digne, Aix and Grenoble, adding meticulously with whom he had observed each of them, in case Snellius wanted to include them in an extended list of latitudes (such a list was contained in *Eratosthenes Batavus*).

These different kinds of gifts were a long introduction to a request that Gassendi wanted to make. He realized that the friendship between Snellius and him had just started and that therefore it was rather early to put it to use already, but since Gassendi himself was used to doing everything for his friends, he trusted that he would be treated with the same kind sentiment. He complained first how in Paris he was surrounded by people interested in astrology, yet not in the stars themselves, let alone in observing them with their own eyes. Only one man was an exception, and Gassendi wanted to have a proper measuring instrument, a quadrant of steel with a radius of at least two Parisian feet, made for this Monsfortius. The Parisian artisans were not able to make an instrument with the desired degree of precision.

[327] [Bainbridge, 1622], [Feingold, 1984, p. 144].

[328] Communication in a letter of Peiresc to Dupuy, [Tamizey de Larroque, 1888, p. 384].

[329] [Rochot, 1972].

[330] They are known through their publication in Gassendi's *Opera Omnia* (for exact references see the next footnotes). The letters written by Gassendi have recently been translated into French by Sylvie Taussig, who added an elaborate commentary. Her remarks on Snellius are not always correct and have to be used with caution, [Gassendi (S. Taussig ed.), 2004, 1, pp. 4–6, 9–16; 2, pp. 7–10, 14–20].

[331] '[...] cum tu ad assiduam librorum evolutionem, ex qua, ut Stagirites ille, Anagnostes dici promereris, accedere volueris exquisitas rerum coelestium, terrenarumque observationes, quas ipsemet admiranda quadam solertia peregeris [...]', [Gassendi, 1964, p. 3]. The Stagirite is Aristotle.

Gassendi had understood that what he wanted for Monsfortius was close to one of Snellius's quadrants, probably thinking of the references to his instruments and their observations which Snellius had made in *Descriptio Cometae* and *Eratosthenes Batavus*. Therefore, Gassendi asked Snellius to help him and his friend by tellling them the exact measures of the instrument, how to make it as accurate as possible, how to move it and some more of its qualities. Preferably, Snellius would communicate directly to a good artisan, have the quadrant made and keep an eye on the proceedings. As a reward for his efforts, Snellius would receive the data of the observations which Monsfortius would conduct with the quadrant.[332]

Snellius answered in May of the same year. He thanked Gassendi for his gift, expressing his agreement with Gassendi's anti-Aristotelianism and anti-Scholasticism in virulent terms, and told him that he had ordered an artisan to make a quadrant similar to his own. He was very grateful for the latitude data which he had received, and explained how he used them and the distances from itineraries to establish a more precise difference of longitude between Leiden and Rome. Now it was Snellius's turn to ask for a favour: he liked to know more latitudes, and also the exact measures of the foot and ell in a number of French places, and preferably also in Italy. This knowledge would benefit Snellius's preparation of the extended edition of *Eratosthenes Batavus* (see section 3.4; see section 3.3 for Snellius's preoccupation with units of length). He also announced that he wanted to write a letter to Jacobus Valesius.[333]

When Snellius did not receive an answer from Gassendi, he wrote a second letter to him, in which he referred to the contents of his first letter. This letter is dated July 1625. The instrument for Monsfortius was now ready and was in Snellius's house. He had corresponded with Monsfortius about its price. Snellius ended his letter by apologizing for not writing in French to Gassendi.[334]

Just after Gassendi had received this second letter, he wrote to Galileo and asked him, among other things, to send Snellius a Florentine foot, indicated on a wooden ruler or in some other way. Gassendi presented himself as Snellius's patron for the occasion, as he wrote.[335]

Later in August 1625, Gassendi wrote a response to Snellius's letters, which he had both received. He was very grateful for them, because they showed how most intelligent and kind Snellius was. The reason for his late answer was that he had written to his friends to ask them for the lengths of their local feet, after which he had had to wait for their answers. Although he was not very satisfied

[332][Gassendi, 1964, pp. 2–4].
[333][Gassendi, 1964, pp. 391–393].
[334][Gassendi, 1964, p. 393].
[335][Gassendi, 1964, pp. 4–6]. Gassendi wrote: '[...] non poenitebit me egisse apud te illius Patronum.', which Taussig translates as though Galileo was Snellius's patron, not Gassendi, which seems to be a wrong interpretation of the Latin sentence both because of the Latin and because of the meaning, [Gassendi (S. Taussig ed.), 2004, 1, p. 9].

with the ones which he had received so far, which he blamed to warfare in several regions, he no longer wanted to postpone his own letter to Snellius, to whom he told the results of his endeavours in the neighbourhood. Together with this letter, he sent Snellius a wooden ruler on which were engraved several feet and some other units of length. Gassendi explained how and by whom the different data had been collected and compared them to the length of the Rhenish foot (the unit used by Snellius). He had also found a stick with the length of a Rhenish foot indicated on it, which length he reported to Snellius to have it corrected 'in order that some day, we may also have the opportunity to find this out with a bit more certainty'.[336]

Gassendi also told Snellius that Joannes Lombardus had done an experiment to determine the weight of a cubic palm of several liquids, and considered and calculated how Lombardus's unit of weight was related to Snellius's (see section 3.3 for Snellius's experiment to determine the weight of a cubic Rhenish foot of water). He then again discussed several longitude matters, and praised Snellius's 'invention' of calculating longitude differences as explained in the *Tiphys Batavus* as most useful for geography. He was eager to know what job Snellius wanted him to do—did it involve climbing mountains and determining angular distances? Gassendi consulted Snellius about a good observational instrument for this purpose, explaining that what he had was either too small to give reliable results or too heavy for transport. He asked him to have a good new one made—it was up to Snellius to decide what exactly Gassendi needed. Gassendi also sent more astronomical observations, among which were those of solar and lunar eclipses. After mentioning some critique which Snellius had ventured on himself in the *Descriptio Cometae* for not paying enough attention to the observation of the tail of the comet, Gassendi offered his own observations of this phenomenon.[337]

Either no more letters were exchanged between the two by now cordial friends after this one, or they were not preserved. Snellius died fourteen months after Gassendi's last letter. In later letters, Gassendi referred a few times to his deceased friend. In 1630 he disclosed to Wilhelm Schickard his plan to travel to the Orient to determine the latitude and longitude of a number of localities. The instrument that he wanted to use for this plan was a brass quadrant, over two feet long. Willebrord Snellius, his 'singular friend', had had this instrument made for Gassendi. In the same year, Gassendi asked Snellius's successor Golius to send him a stick with the exact length of the foot which Snellius had employed as the base for the further study of distances. This suggests that Snellius had an instrument made for Gassendi, as he had asked in his last letter, but had not reacted to the latter's enquiry about the correct length of the Rhenish foot.[338]

[336]'[...] ut et nobis aliquid certius experiri aliquando liceat.' [Gassendi, 1964, p. 7].
[337][Gassendi, 1964, pp. 6–10].
[338][Seck, 2002, 1, p. 571], [Gassendi (S. Taussig ed.), 2004, 1, p. 69].

The correspondence between Gassendi and Snellius is very telling, although the number of letters is restricted. Not only does it give much information about the impression which Snellius made on the world of learning around him as a man of books and observations, it also adds information about his work on the second edition of *Eratosthenes Batavus*. Moreover, it is a good example of the functioning of Snellius's scholarly network: it tells us about the exchange of information and other goods between scientists with shared interests and thus the significance of their 'friendship'. Gassendi provided Snellius with astronomical and geographical observations and information about units of measure, and Snellius paid him back by ordering two observational instruments.

Although no letters between Kepler and Snellius are known, some indications show that Kepler followed the progress of his old acquaintance. In 1615 Kepler published his exploration of the volumes of wine casks and other three-dimensional magnitudes, the *Stereometria*. After the exposition of an unsolved problem, he spurred Snellius, 'the ornament of the geometers of our age',[339] to find a valid solution to this and other problems. He hoped that such a demonstration would further Snellius's career: a Maecenas should reward him for his ingenuity.[340] This was a very open plea for a patron for Snellius.

Elsewhere in his book, he incited Snellius and Adrianus Romanus, 'the Netherlandish Apollonii',[341] to procure a demonstration or refutation of a certain conjecture. The great Kepler's apparent incapability to solve these problems explains Snellius's silence in answer to this challenge: he was not able to improve on Kepler's work. After Kepler had read Snellius's dedicatory letter to the *Fundamenta*, he seems to have lost some of his esteem for his old friend, who had in his view desecrated Euclid's *Elements* by abusing book X (see section 5.4.3). Nevertheless, he was still willing to send Snellius's astronomical works to Paul Guldin.[342]

Snellius's network did not only consist of other scholars, but also included his students. The names of some of them are known. He conferred the degree of *Magister Artium* to Franco Petri Burgersdyck on 30 March 1620, who then became professor of logic and ethics. He also educated other future professors: his successor Jacobus Golius and the Amsterdam professor of mathematics Martinus Hortensius. The latter (Maarten van den Hove, 1605–1639) had been a pupil of Isaac Beeckman, who may have kindled his love for mathematics. He arrived in Leiden only in 1625. In 1634, he started his teaching at the Amsterdam *Athenaeum Illustre*, to become full professor the next year. When Hortensius

[339]'Geometrarum nostri saeculi decus', [Kepler, 1960, p. 71].

[340]'reservatur, ni fallor, haec inventio Tibi, ut existat Maecenatum aliquis, qui tuae fortunae splendorem reputans, et verecundia instigatus, dignum aliquid hac sollertia, quo scilicet notabilis aliqua tuae rei fiat accessio, remuneretur [...]', [Kepler, 1960, p. 71].

[341]'Apollonii Belgae', [Kepler, 1960, p. 71].

[342]Letters of 10 December 1625 and 7 February 1626, [Kepler (M. Caspar ed.), 1959, pp. 253–254, 257–258].

wrote a short guide for the study of mathematics, he included a number of Snellius's works. Snellius made an educational travel with the Sterrenberg brothers (see p. 124). Florentius Schoonhovius, who had probably followed some of Snellius's courses, wrote and published an ode for Snellius in a collection of his own poems, in which he praised his astronomy and descent. He was related to Aemilius Rosendalius, to whom he dedicated a volume of poetry.[343] The names of some other students of Snellius are Jacob Spoors, Godefridus van Haestrecht, Matthias Pasor and Antonius Aemilius.[344]

Traces of Snellius's network of students and men of learning are also found in a number of entries in *alba amicorum*. When the German student Lucas Holstenius (the later librarian of the Vatican library) studied in Leiden, he collected the autographs of many of the professors, among whom was Snellius (1619). Snellius adorned the album of Ernst Brinck with a quotation from Sophocles's *Antigone*. In 1621, Johannes ab Heemskerck received a pious text by Ambrosius as an album contribution from Snellius. In 1624, Snellius wrote a quote of a satire by Juvenal together with some Greek phrases in the album of Cornelis Montigny de Glarges, who according to Snellius was not only ennobled by birth, but also by his longing for science.[345]

All this evidence shows that Snellius was firmly rooted in a Dutch and international world of scholars and of (former) students, and that he was generally held in esteem by them. Negative assessments are much rarer than positive ones, which shows that Snellius acquired himself a respectable position as a mathematician.

2.9.5 In the workshop: books and instruments

Snellius could not have developed his mathematics if he had not had a good collection of tools at his disposal. I divide them into two main categories: his books and his instruments designed for observations and measurements.

Snellius's library was sold on an auction in 1629, of which the catalogue is still in existence.[346] The advantage of this source is that we have many titles of books from Snellius's property, yet unfortunately, it is not possible to declare with certainty which books were Snellius's, for two reasons. Firstly, his books were sold together with those of Thomas Segeth.[347] As Segeth's name figures

[343][Schoonhovius, 1975, pp. 235–236], [Kuyk, 1914, c. 1096].
[344][van Slee, 1898, p. 150], [Worp, 1911, p. 372]. On Hortensius, see [Imhausen and Remmert, 2006, pp. 75–77], [van Berkel, 1998, pp. 63–84], [van Berkel, 1983b, p. 143], [Snellius, 1627], [Hortensius, 1637, pp. 116, 120, 124, 128]. [Vermij, 2002, p. 105], [Barlaeus, 1667, p. 173], [Feingold, 1997, p. 479], [van Otegem, 2003, p. 4].
[345][Blom, 1984, p. 37]; [Brinck, s a, fol. 76^r], edited in [Ridder van Rappard, 1856, p. 57]; [ab Heemskerck, s a, fol. 31^r], [Montigny de Glarges, s a, fol. 150^r].
[346][Catalogus Snellii, 1629].
[347]Hardly any biographical data of Segeth are known. Two inscriptions in the Leiden *album*

less prominently on the title page, it can be assumed that Snellius's library had a larger share in the books for sale. By selling the two collections together, the auctioneer made it more attractive for buyers to come from outside Leiden and attend the auction.

Secondly, the catalogue must have become diluted by additions from the stock of the auctioneer. Some books in the main part were only published after Snellius's death, and sometimes more copies wit the same title appear. There are also two Appendices, which do not contain much mathematical fare. Moreover, the main list contains over 2,000 titles, which is a very large number for two private libraries. According to the book historian Van Selm, a private library with over 500 books can be considered as sizeable in the first decade of the seventeenth century. It was indeed customary for auctioneers to seize the occasion of an auction to reach a large audience of interested buyers to try to get rid of part of their own fixtures, which would then be found in Appendices (in this case also in the main part).[348]

The catalogue did not only contain more, but also less than Snellius's complete library, as is evident from the absence of books of which he or his father was the author. They (and others) may have been preserved by his descendants and friends. In addition, the absence of more pedestrian (vernacular) literature in the catalogue does not show that Snellius only owned scholarly works, but that only books of this last category were precious enough to be auctioned.

Although it is not possible to reconstruct Snellius's library exactly, the catalogue still gives a very good picture of the books available to Snellius—either at home or else somewhere in his environment.[349] It covers extended fields of learning: classical authors of all kinds, philosophy, mathematics, history, theology, medicine and law. By far the biggest part was in Latin, but there were also Greek (some bilingual Latin-Greek editions), French, Dutch, English, Italian and Spanish works included.

A wide range of mathematical works figure in the catalogue, both classical and modern, mainly covering pure mathematics and astronomy, but also other parts of the mathematical sciences such as optics, mechanics and architecture. Among the Greek mathematicians we find Euclid (several editions, both Latin and Greek), Archimedes (two editions), Diophantus (several editions of the *Arithmetica*), Pappus, Hipparchus's commentary of the *Phaenomena* of Aratus and Eudoxos, Apollonius, Ptolemy, Aristarchus, Heron, Pythagoras and Proclus. The moderns included near-contemporaries and some earlier

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studiosorum mention this name: one as a 'Magister Artium' in 1589, which also gives his Scottish descent, [Album, 1875, c. 26], and one of someone with the same name, probably his son, in 1625 (also Scottish; 35 years), [Album, 1875, c. 187].

348 [van Selm, 1987, p. 92]. For auction catalogues as a source of ownership of books, see especially pp. 75–144 of the same book.

349 Cp. also the first printed catalogue of the Leiden university library. The description of the mathematical section only filled two pages in 1595, [Bertius, 1995, fol. $I1^{r-v}$, $L4^{r-v}$].

authors: Viète (*Canon mathematicus ad triangula*, *Universalium inspectionum liber*), Clavius, Maurolicus, Ghetaldus, Commandinus (*De superficierum divisionibus*), Cardanus, Regiomontanus, Benedictus, Galileo (among other things *De maculis in sole*), Gemma Frisius, Metius and Purbachius.

Some books must have occupied a special place in Snellius's library because he had known their authors in person: Kepler, Tycho Brahe, Lansbergen, Adrianus Romanus and Maestlin. He also seems to have owned many books from both living and dead members of the Leiden humanist school, a number by Scaliger in the first place, but also by Lipsius, Merula, Scriverius, Heinsius, Vulcanius, Dousa, Meursius, Grotius, Cunaeus and Vossius. The catalogue also contains a number of Ramus's books (*Scholae in Artes Liberales*, *Dialectica*, *Rudimenta Grammaticae Latinae*, *Rhetorica*, *Scholae metaphysicae*) and his followers, such as Audomarus Talaeus, Nancellus, Rennemannus and Schonerus.

Apart from the auction catalogue, some other sources inform us about Snellius's library. A significant part must have originated from his father's property. This is for example shown by two books formerly owned by Rudolph Snellius, an edition of Cleomedes's *Meteora* and one of Aratus's *Phaenomena*, which contain marginalia which are in all probability Willebrord's. He used Cleomedes for the *Eratosthenes Batavus*. His father had given him his own complimentary copy of Scaliger's *De Emendatione Temporum* (see p. 57).[350] These three books are now in the University Library of Leiden.

Books were not only bought from booksellers, but also exchanged between friends. Snellius for instance sent some books with Bible texts to Rosendalius.[351] At another occasion, he bought a number of books from the auction of Vulcanius's library, some of which he sent to Rosendalius. If Rosendalius did not want them, Snellius wrote to him, he would like to have them himself. Snellius had not bought some books because he considered them too expensive, for example a Koran which had to cost 17 guilders.[352] Cunaeus restituted the copy of a book by Calvisius which he had borrowed from Snellius.[353] A comparable example of the help of friends in purchasing books is that of Daniel Mögling, who asked Schickard, who had some Dutch friends, to find out whether the *Hypomnemata Mathematica* were still for sale. If not all parts were available in Latin, he was willing to read them in Dutch or French.[354]

Snellius was also the owner of some manuscripts, for instance Heron's *Stereometria* in Greek, and a copy of several texts by Euclid in Greek, which he had probably bought at the auction of the books of Hieronymus Commelinus in 1607.[355] He could also borrow some manuscripts, for instance Scaliger's copy of

[350] [Cleomedes, 1605], [Aratus, s a], [Snellius, 1617b, pp. 56, 61, 62, 88].
[351] [Snellius, 1612b].
[352] [Snellius, 1610b].
[353] [Burmannus, 1725, p. 139].
[354] Letter of 1633, [Seck, 2002, 2, p. 136].
[355] Snellius mentioned the *Stereometria* in [Ramus (W. Snellius ed.), 1613, p. 40]; the

Pappus.[356] Moreover, Snellius had some ancient coins.[357]

The diversity and sheer length of this enumeration shows that Snellius had access to a wealth of material, covering all his interests. The abundance of the books available through his father, the circle of university scholars and his own library must have stimulated his curiosity and his humanist style of mathematics.

Sometimes Snellius wrote comments in the margins of his books. He provided his copy of Risnerus's *Optica* with many annotations. His copy of *Eratosthenes Batavus* contains corrections, remarks and extensions which were clearly meant for a new edition.[358] Some other books with Snellian marginal notes were mentioned in the auction catalogue of Golius, but are lost now: Viète's *Responsio ad problema Adr. Romani*, Ramus's *Arithmetica* and *Arithmetica et Geometria*, (Euclid's ?) *Elementa geometrica* in Greek, and Snellius's own *De Re Nummaria*. At the same auction, *T. Locri de mundi anima et natura liber singularis* was also on offer, with notes of (Willebrord?) Snellius. Some other unspecified mathematical writings from Snellius could also be purchased.[359] Hence Golius must have acquired a substantial part of Snellius's library.

Snellius was not only a keen user of books, but also of scientific instruments. His early interest in telescopes has already been discussed (see p. 69). Snellius also mentioned a telescope as an auxiliary tool when measuring the directions towards other towers from a number of towers in *Eratosthenes Batavus*, which was not unproblematic because of optical effects. Snellius also informed the reader about the use of other measuring devices in *Eratosthenes Batavus* (see p. 125).[360]

Although Snellius already owned some large observational instruments in 1614/1615, they could not fulfil his needs when the comet of 1618 appeared. In his *Descriptio Cometae*, he explained that his copper quadrant of $2\frac{1}{2}$ feet could not be used because of troubles with its pivots and because moreover two persons were needed to handle it (apparently, Snellius did not have an assistant at his disposal).[361] This is the same quadrant as that mentioned in the *Eratosthenes Batavus* with a radius of $2\frac{1}{5}$ feet: in this book, Snellius had defined a foot as one tenth of a Rhenish rod, whereas he used the conventional division of one rod in 12 feet in the *Descriptio Cometae*.

Apparently Snellius was able to convince the university administration that he needed more and better instruments to conduct his research: he received a yearly allowance of 200 guilders for buying instruments from 1620 onward.

Euclid manuscript is now in the Leiden university library (shelf mark BPG 7), see [de Meyier and Hulshoff Pol, 1965, pp. 11-12].

356 [Snellius, 1607b, p. 7].

357 [Snellius, 1612c].

358 [Risnerus, 1606], [Snellius, 1617c].

359 [Catalogus Golii, 1668, pp. 28, 39, 134, 136, 137]; [de Waard, 1927b, c. 1162].

360 [Snellius, 1617b, p. 170].

361 [Snellius, 1619, p. 7]; cp. section 4.4.2.

It is not completely clear what he had made and with what purpose, yet we know that when he died he had a good collection of instruments, a number of which were included in the auction of his library. Among them were five quadrants: a very large iron one (the one used for some of the measurements for the *Eratosthenes Batavus*), three wooden ones and one from (or designed by) Lansbergen. Other items for sale were more surveyors' gear, a terrestrial and two celestial globes, an astrolabe, a nautical instrument, several time measuring devices and 'Euclidean bodies' (maybe models of regular solids). No telescopes were on offer in the auction.[362] When Golius's library was auctioned in 1668, at least one of Snellius's instruments was again for sale: one of the quadrants which he had used for his measurements in the *Eratosthenes Batavus*.[363]

All these instruments are lost now. However, one impressive instrument of Snellius is still alive and even on display: a huge quadrant (with a radius of over 2 m) of wood with a brass mounting. It is now on view in Museum Boerhaave in Leiden. Willem Jansz Blaeu made it, after the example of Tycho Brahe's huge quadrant.[364] The lack of references to its use in Snellius's works makes it likely that it was made late in his life. It had (almost) the same size as his large iron quadrant and was very accurate. He may have intended to use it or actually used it for new measurements for the calculation of the circumference of the earth or for astronomical observations at home. The university acquired it from the heirs of Snellius, on the request of Golius, in 1632. They paid 125 guilders for it and had a cabin made on the roof of the Academy Building for the preservation of the instrument and the demonstration of the course of celestial bodies to students. This was the third observatory that was founded in Europe, after those of Tycho Brahe in Denmark and of William of Hessen in Kassel.[365]

Although Snellius was a capable user of instruments, a note by Isaac Beeckman shows that Snellius understood the principles of mechanics less clearly than he. Beeckman wrote that practitioners often claimed that they had invented instruments that were much more powerful than the existing ones. Snellius had told him about such a case, in which he believed, whereas Beeckman considered these claims as impossible, because they would yield a *perpetuum mobile*.[366]

One of Snellius's roles in the world of learning was that of an intermediary between the producers and the customers of instruments, as the examples of Rosendalius and Gassendi show (see p. 69 and p. 95). He had instruments of good quality made which is e.g. shown by the extended later use of the university

[362][Catalogus Snellii, 1629, pp. 68–69].
[363][Catalogus Golii, 1668, p. 138].
[364]www.museumboerhaave.nl/collectie/hoogtepunten.html. Inv. V06500. [Vossius, 1650, p. 200].
[365][Molhuysen, 1916, p. 177], [de Waard, 1927b, c. 1162]. Recently, a copy of the instrument has been made to adorn the Snellius building, the current home of the Mathematical Institute of Leiden University.
[366][de Waard, 1945, pp. 306–307].

quadrant.

2.9.6 The Law of Refraction

Most people who nowadays know Snellius's name do so because they learned 'Snell's Law' at school, the optical Law of Refraction. Therefore, this law gave Snellius the major part of his posthumous fame. Yet it played no role whatsoever in his contemporary reputation, because it was only published after his death—and there are no indications that he spread it otherwise. It was published later in the seventeenth century by other scientists (Isaac Vossius and Christian Huygens) who possessed a manuscript by Snellius that has disappeared since. This has made Snellius's discovery of the Law of Refraction the subject of much speculation. The topic will not be treated in much detail here, as the major focus of this book is on Snellius's position in his own time. Moreover, some good articles have been written on the issue, to which the interested reader is referred.[367] Only a short summary of the relevant sources and secondary literature is given here.

Although there does not exist a book or finished treatise on optics by Snellius, we do have two sources that give first-hand information about his optical activities, both handwritten by Snellius. The first are his annotations to the *Optica* of Risnerus, the second an outline for a treatise on optics.

Risnerus had prepared his *Optica* in collaboration with his master Ramus. It was only printed in 1606, long after their deaths. Rudolph Snellius had been a zealous advocate of this publication (see section 4.2). The annotations to the *Optica* give us information about Snellius's interest in optics for a long period of his life. The oldest dated one is from 1611, the youngest from 1622; most are undated.[368] These annotations do not contain the Law of Refraction, which is one of the indications that Snellius discovered it late. Snellius may have become interested in the behaviour of (breaking) light through its relevance for astronomy and (indirectly, through astronomy) navigation.

Snellius annotated his copy heavily, in the margins and sometimes on interspersed leaves. It is evident from the handwriting and the contents that he was indeed the author of the notes: e.g. the author had a son Rudolphus, lectured on optics and made a journey from Oudewater (his father's natal town) to Schoonhoven (from where his wife originated).[369] Snellius used the book

[367]Recently a long article by Klaus Hentschel, which contains a German translation of most relevant sources, has been published: [Hentschel, 2001]. See for more references the notes to this section.

[368][Vollgraff, 1936, pp. 723, 725].

[369]Snellius's own copy is in the Leiden University library now: [Risnerus, 1606]. See [Vollgraff, 1918, pp. 24b–25b] for these and more proofs of Snellius's authorship. Vollgraff's introductory chapter on Ramus and Snellius was also published as a separate article in French as [Vollgraff, 1913b].

as teaching material and the marginalia must either have been meant for his classes, or for a book on optics that he was preparing. There are different kinds of notes: quotations of authors, both classical and modern, and thoughts about them; his own observations, and geometrical clarifications of the behaviour of light, including figures. The annotations to the first book have been edited by Vollgraff, who also discussed some of the annotations to the other books.

The quotations and considerations mainly concern the character and behaviour of light. Snellius's conceptions are predominantly Aristotelian and animistic. The classical authors to whom he referred were, among others, Aristotle, Euclid, Philoponus and Cicero. He also used the work of the Arab Alhazen and of more modern writers like Peckham, Vitello, Aguilon, Cardanus and Kepler. This abundant use of older authorities was true to the character of the book: Ramus and Risnerus described the nature of light on the basis of the works of ancient authors and of Alhazen and Vitello, not having done any experiments themselves. Risnerus had translated Alhazen from Arabic into Latin.[370]

Snellius's observations mainly describe celestial phenomena. He made a number of travels in the country, continuously looking around and above him. He observed rainbows for many years, as is shown by the notes on them in the manuscript that range between 1611 and 1622. E.g. in December 1617, he noted a rainbow on a trip to The Hague. He described this same observation in *Descriptio Cometae*, adding his wish that he could write about those issues on a later occasion more elaborately.[371] His family was also infected by his observational enthusiasm: in 1621 his little son Rudolph warned him that he had seen a double rainbow. Snellius also mentioned an observation of the setting sun (12 September 1622), and of a circle around the moon (23 December 1619).[372] Other people were also involved, such as Houtman, the translator of the *Meetkonst*, who reported that he had seen the tower of (the cathedral of) Utrecht from a great distance (Loosdrecht); knowing its height, he had been able to calculate this distance.[373] These observations show that Snellius was much interested in the physical world around him. He included some remarks on the behaviour of magnets in his notes.[374]

Snellius mentioned some experiments that he had done to study the reflection of light in concave and convex mirrors in December 1621, but no experiments with refraction. Since experiments were not yet commonly used as means of discovery in his time, it is worth noting that Snellius conducted them. He also

370[Vollgraff, 1918, pp. 10a–17a, 23b, 29b, 311-312], [Vollgraff, 1936, pp. 720–721]. Snellius only refers to Kepler's *Paralipomena ad Vitellionem*, not to his *Dioptrice*, [Vollgraff, 1918, p. 30b]. See also [Hallyn, 1994, pp. 132–134] for a Dutch translation and discussion of some of Snellius's notes.

371[Risnerus, 1606, p. 242], [Vollgraff, 1918, p. 24b]; [Snellius, 1619, p. 33].

372[Risnerus, 1606, fol. 155^v, p. 229], [Vollgraff, 1918, p. 25b], [Vollgraff, 1936, p. 725].

373[Risnerus, 1606, p. 216].

374[Vollgraff, 1936, p. 721].

mentioned an experiment in which he had studied the reflection of the flame of a candle in a semi-cylinder of ice, performed on 2 January 1622. A somewhat obscure note written by Snellius in the context of the experiments on reflection could be an early allusion to his law.[375]

The second autograph source on optics is a manuscript that contains the outline of a treatise on optics. It is now kept in the University Library of Amsterdam and both the handwriting and internal indications prove that it was also written by Snellius, although his name does not figure on the manuscript. C. de Waard has drawn the attention to this manuscript and argued convincingly Snellius's authorship.[376] The manuscript is not dated; *terminus post quem* is 1611 because of the reference to a book by Maurolycus published in that year. Its unfinished state makes it likely to have been written shortly before Snellius died. He may have interrupted his work on the improved version of *Eratosthenes Batavus* (see section 3.4) for this treatise and then died before he had finished either.[377]

In the outline, only topics are mentioned related to the refraction of light in different bodies, including some applications in the heavens. The maximum angles of refraction in water and glass are given in degrees and minutes. The treatise was again based on the work of (between others) Witelo and Alhazen (both in Risnerus's edition). Conspicuously absent this time was Kepler: would Kepler's annoyance with Snellius in the Book X issue be the cause for Snellius's neglect?[378]

This manuscript contains Snellius's formulation of the Law of Refraction. I will first give the modern formulation, and then Snellius's, and show that they are (nearly) equivalent. In modern language, this optical law is: if rays of light travel from one medium to another, then (see figure 2.2):

$$\frac{\sin(\text{angle of incidence})}{\sin(\text{angle of refraction})} = \frac{\rho_1}{\rho_2}, \tag{2.1}$$

in which expression the ρ_1 depends only on the rarer medium and ρ_2 only on the denser medium.

Snellius formulated this law as follows:

> The real radius has to the apparent radius the same proportion in one and the same different medium. The secant of the complementary

[375][Risnerus, 1606, fol. 145–146 (pencil)], [Vollgraff, 1936, pp. 722–724].

[376][Snellius (attr.), 1625]. On its cover, a calculation has been made that seems to belong to *Eratosthenes Batavus*, [de Waard, 1935, p. 52]. This article also contains an edition of the text of the manuscript, see [de Waard, 1935, pp. 60–73].

[377]See [Snellius (attr.), 1625], [Weinrich, 1998, p. 25] for the *terminus post quem*. Hentschel dates the manuscript between 1622 and 1626, [Hentschel, 2001, p. 313].

[378][Weinrich, 1998, p. 26]. For the polemic about book X of Euclid's *Elements* see section 5.4.

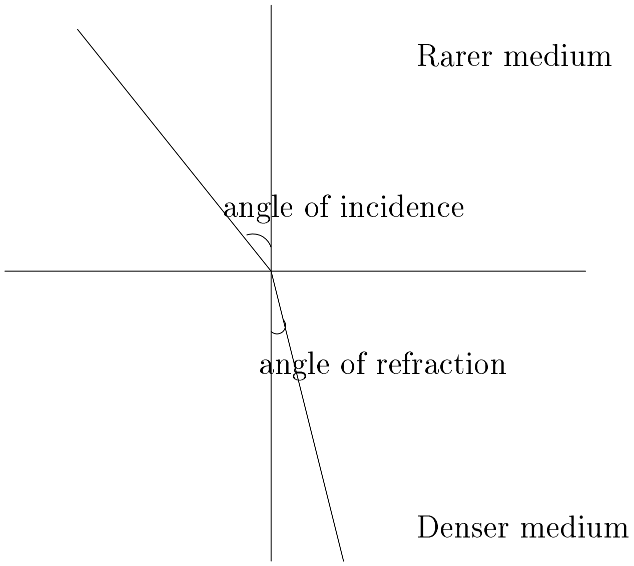

Figure 2.2: The Law of Refraction in modern terminology

> angle of the inclination in the rarer medium has the same ratio to the secant [of the complementary angle] of the broken [radius] in the denser medium, as the apparent radius has to the true or incident radius.[379]

As usual, Snellius formulated his results in terms of proportions. By 'one and the same different medium' he meant that refraction between two different given media, with varying angles, is studied. This can be translated into symbolic language as follows (see also figure 2.3 for the meaning of the symbols). $p : q$ is a ratio, r_v is the true radius (the ray of incidence seen as a line segment), r_a the apparent radius (the ray of light after refraction as a line segment), O the point of intersection of the ray of light and the interface, A the point of intersection of the interface and a perpendicular through the starting point of r_v, A' on the interface to the other side of O such that $OA = OA'$, α' the angle between the true radius and the interface between the two media, β' the angle between the apparent radius and this interface. All the relevant terms had been

[379]'Radius verus ad apparentem in uno eodemque medio diverso eandem habet inter se rationem. Ut secans complementi inclinationis in raro ad secantem *complementi* refracti in denso, ita radius apparens ad verum seu incidentiae radium.' [Snellius (attr.), 1625, fol. 5^v–6^r]. [de Waard, 1935, pp. 64–65] deletes 'inter se' and reads 'habent' in the first sentence, which I do not follow. *complementi* is an addition by De Waard, which I follow. I thank Wijnand Rekers for taking photographs of the manuscript.

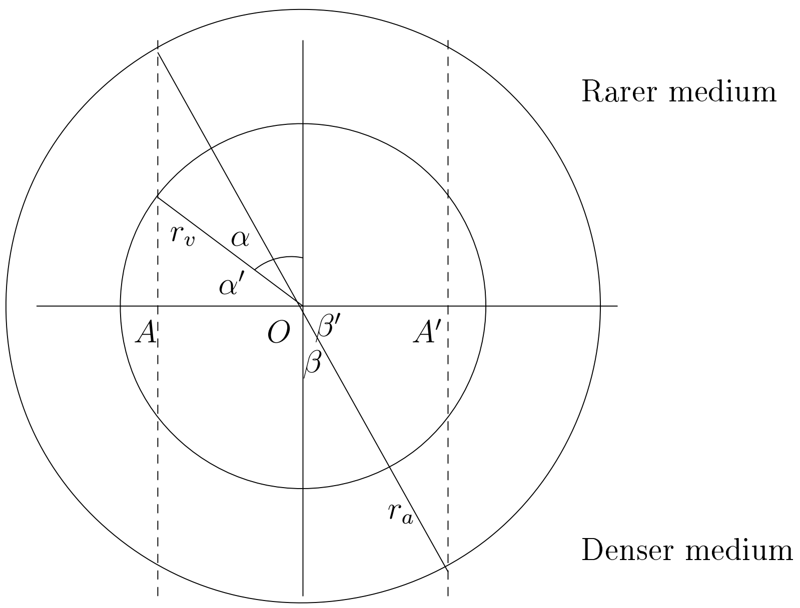

Figure 2.3: The Law of Refraction in Snellius's terminology

introduced by Snellius in the first part of the manuscript (without an explicit definition).[380] The first sentence of his formulation states that there is a ratio $p : q$, independent of the angle of incidence, such that

$$r_v : r_a = p : q.$$

This is indeed near-equivalent to the modern expression, which can be seen by rewriting the left-hand side of this equality as (take $\alpha = \frac{\pi}{2} - \alpha'$, $\beta = \frac{\pi}{2} - \beta'$):[381]

$$r_v : r_a = \frac{OA}{\cos(\alpha')} : \frac{OA'}{\cos(\beta')} = \cos(\beta') : \cos(\alpha') = \sin(\beta) : \sin(\alpha). \tag{2.2}$$

Expression (2.1) can be derived from (2.2) by substituting $\rho_1 : \rho_2$ for $q : p$ and interpreting this expression as a fraction. Note however that $q : p$ depends on the two media together and ρ_1 only on medium 1, ρ_2 on medium 2.

Snellius made a mistake in the second part of his statement, in which he tried to express the radii mentioned in the first part in standard trigonometrical

[380] Cp. [Hentschel, 2001, p. 309].

[381] Surprisingly, none of the modern authors have bothered to show explicitly this equivalence in their publications. Hentschel is the only exception, but one of the equalities in his formula does not apply, which compensates Snellius's mistake in the second part, [Hentschel, 2001, p. 303].

magnitudes. His rule says:

$$\sec(\alpha') : \sec(\beta') = r_a : r_v.$$

This is not true: the left-hand side is equal to

$$\frac{1}{\cos(\alpha')} : \frac{1}{\cos(\beta')} \text{ and therefore to } r_v : r_a.$$

Thus, the order of the last two terms in the second part of the statement must be reversed. This mistake in the formulation of the explanation has not been noticed thus far. It does not affect the statement of the law itself, because that was contained in the first part.

The Amsterdam manuscript is not the only source that proves Snellius's discovery of the law that is named after him. In the decades after his death, a manuscript by his hand containing it circulated among Dutch scientists. Although this manuscript has disappeared, some of its contents have been preserved in the work of its readers. In 1628, Mersenne told the publisher Maire that he was willing to complete Snellius's optical treatise, so that it could be printed, but he received no reply. Golius was able to consult the manuscript in 1632. He communicated his findings in a long letter to Constantin Huygens.[382] This letter, in which the Law was included, is a strong indication that nobody in Snellius's surroundings was aware of his discovery before his death, which makes it probable that he invented it late in his life. Golius compared the methods of Descartes, who had discovered the law independently, and Snellius for finding the law in the following words:

> the Frenchman on the basis of principles and causes, and the Dutchman on the basis of effects and observations [...] have drawn exactly the same conclusion.[383]

Golius also told Huygens that Snellius had used the calculations and tables of Witelo and his own observations, which he had repeated frequently and in different forms, to create his optical theorem. In 1629, he apparently had not yet seen the manuscript, because then he had written to Huygens that he sincerely hoped that Snellius's optical manuscript would make its appearance again so that he could make it public 'to the glory of their inventor, my teacher and friend'.[384] In 1661, Isaac Vossius was able to consult an incomplete work on

[382][Tannery et al., 1937, pp. 157–158], [Worp, 1911, pp. 371–375]. The formulation of the law is here: 'Si duplex fuerit medium, densitate et raritate differens, radius quivis incidentiae verus ad suum apparentem in eius generis medio eandem servat rationem', [Worp, 1911, p. 372]. The ensuing elucidation of the Amsterdam manuscript lacks.

[383]'[...] per principia et causas Gallus, per effectus et observata Batavus [...] concluserunt prorsus idem', [Worp, 1911, p. 372].

[384]'[...] ad gloriam inventoris, praeceptoris et amici [...]', [Worp, 1911, p. 263].

optics by Snellius in three books, which Snellius's son had lent to him. The Amsterdam manuscript must contain the table of contents of this unpublished work.[385]

Snellius may have discovered his law through a combination of experiments, thinking and calculating. According to Christian Huygens, who may have heard about it from his father Constantin and had also been able to consult Snellius's optical treatise himself, Snellius had done many experiments. Snellius had learned of the experiments done on the refraction of rays of light by Ptolemy and Alhazen from Risnerus's book.[386] He mentioned Alhazen's *experimentum elegans*. His familiarity with trigonometrical tables may also have guided his thoughts.

The first to publish the Law of Refraction was Descartes, who did so in the *Dioptrique*, one of the essays to the *Discours de la Méthode*. The Cartesian version of the Law of Refraction became known in Holland in 1629 at the latest. Beeckman mentioned it in his notes, apparently at that moment not yet aware of the Snellian discovery.[387] In later scholarship, some discussion arose whether Descartes had stolen the law from Snellius, but this is unlikely. As has happened more often in history, the law seems to have been 'in the air' and was discovered almost simultaneously by a number of people. Harriot had discovered it in 1602, yet without making it public.[388] This certainly does not mean that it was a simple step to find the law. Illustrious scholars had been looking for it in earlier ages without finding it. Only very few laws of nature had been formulated so far, which also made it more difficult to find new ones.

2.10 Death and beyond

Snellius was caught by death on 30 October 1626 after a short illness, less than two weeks after the funeral of one of his children. He died from colic (a paroxysm of the internal organs), together with fever and paralysis of the arms and legs. According to the chronicler Bronchorstius, Snellius died calmly; two hours before his death, he said to Andreas Rivetus

> that he was prepared to serve the university and his calling if God granted him life; if however God had rather decided to call him out of this life, he would willingly follow the summoning God.[389]

[385][de Waard, 1935, pp. 55–58].

[386][Vollgraff, 1918, p. 20b], [Vollgraff, 1936, p. 719].

[387][Korteweg, 1898, pp. 66–68]; [de Waard, 1945, p. 97]; [Weinrich, 1998, p. 53].

[388][Korteweg, 1898], [Weinrich, 1998].

[389]'[...] se paratum esse, si Deus illi vitam concesserit, Academiae vocationique suae operam dare, vel si Deo placuerit eum ex hac vita evocari, lubenter Deum vocantem sequi.' [van Slee, 1898, p. 199]; cp. [Iachaeus, 1626, pp. 21–22].

The professor of physics Gilbertus Jachaeus gave a few more details about Snellius's death in his funeral oration. When Snellius had fallen ill, the medical doctors Heurnius and Screvelius had been consulted, but they had not been able to prevent the deterioration of his situation. In the evening of 30 October, Heurnius and Jachaeus went to visit Snellius to see the effects of a new medicine. This had not helped at all and after giving him a suppository for some relief, they left. Snellius had dinner with his wife. Because he was not able to walk, his servants had to lift him up. He then suddenly lost consciousness and died, 46 years old.[390]

He was buried on 4 November in the Pieterskerk in Leiden. Twenty students carried his coffin. After the funeral, a (in Bronchorstius's words) 'elegant and learned'[391] obituary was delivered by Jachaeus. The oration was printed together with funeral poems by Daniel Heinsius, Gerardus Joannes Vossius, and Caspar Barlaeus, who spread his grief over two poems, filling seven pages. He wrote to Snellius's former student Godefridus van Haestrecht that he had written these humble poems very quickly, because the printer had urged him to make haste.[392]

Barlaeus also corresponded about this loss and his poem with Arnoldus Buchelius and Huygens.[393] The day before Snellius's funeral, Barlaeus wrote to Van Haestrecht that Snellius's death was a 'common loss for our university', because mathematicians were rare, especially those that mastered the field so well that they were capable teachers. He succinctly commemorated Snellius's contributions to arithmetic, optics, astronomy, surveying and navigation:

> This great master and judge of numbers has withdrawn himself from the number of the living. The light in the eyes that have illuminated the science of light and eyes to the Dutch has been extinguished, and soon the earth will cover him whom the starry heavens have so often held captured by directing his eyes to it. He who has measured the whole earth with his compass marks the outlines of his narrow grave with his dear little body. He has gone away to his Eratosthenes, and Tiphys will not bring him back. Which Hercules will succeed this Atlas?[394]

[390][van Slee, 1898, p. 199], [Iachaeus, 1626, pp. 19–22]; cp. [Orlers, 1641, 348–349]. The municipal book of funerals puts him under 29 October, [Begraafboek 5, 1627]. The age is deduced from Snellius's reconstructed day of birth.

[391][van Slee, 1898, p. 199].

[392]'Crede, versus extemporanei sunt, nec ter scribenti effluxit clepsydra. Festinabat enim typographus, qui vacuas aliquot paginas sordibus istis conspergi volebat.' [Barlaeus, 1667, p. 176].

[393][Barlaeus, 1667, pp. 167], [Worp, 1911, p. 209].

[394]'At doleo publicam Academiae nostrae iacturam, quae tanto viro orbata frustra parem sperabit. Minus enim parabiles hoc seculo sunt Mathematici, et rarius haec studia penitus addiscuntur, ut docere ea possint, qui didicerunt. Excessit e vivorum numero magnus ille numerorum arbiter et iudex. Extincta sunt lumina, quae Batavis luminum oculorumque doc-

Snellius had already made provisions a few years earlier. His last will, dated 28 January 1623, was drawn up by a notary from Oudewater, named Jongsten. Snellius appointed his surviving children as his universal heirs, and his brother-in-law Dirck Gijsbrechtsen van Praet, a former burgomaster of Oudewater, as their guardian and the executor of the will. Other members of the family of Snellius's wife also had to play a role.[395] At the end of his life, he had also decided that if none of his sons was to follow in his mathematical footsteps, all his mathematical books and manuscripts would be bequeathed to the university.[396] This request was not granted: the university library now has no Snellius manuscripts, apart from a few letters, and only a few books of which he has been the previous owner. Its key item, Snellius's annotated copy of Risnerus's *Optica*, only came to the library after Golius's death.[397]

Shortly after Snellius's death, the university administration decided to grant his widow Maria de Langhe an extra year of salary payment. She had stayed behind with three young children, Rudolph, Laurens and Jannetgen. It was discussed whether all widows of deceased professors would enjoy this privilege from now on, but the curators decided against it.[398] Maria only survived Willebrord somewhat more than a year: she died 11 November 1627.[399] Their children placed a memorial stone for their parents in the Pieterskerk, in Latin with (in the spirit of their father) some Greek words. The actual ledger stone for the couple gives the information about the buried in Dutch, but Snellius is still called by his latinized name. This is a last proof of the close ties between his professional and private identities.[400]

Snellius's death was a cause for grief and appreciative memories both in the Netherlands and abroad, as is for instance shown by letters of Peiresc and Huygens. Vossius wrote to Joannes Meursius about the loss of their common friend, 'an outstanding light of the university'.[401]

In 1627, Snellius's last work was published, *Doctrina Triangulorum* ('The Doctrine of Triangles'). It had been written by Snellius, but completed by Hortensius, and it cannot always be decided what the latter added to or changed

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trinam illustraverunt: operiet iam illum tellus, quem toties conversis in se oculis detinuit stellifer aether. Sepulchri angustias corpusculo suo metitur, qui radio totum descripsit gentibus Orbem. Ad Eratosthenem suum abiit, nec reducet eum Tiphys. Quis Atlanti isti succedet Hercules?' [Barlaeus, 1667, p. 173].

395[Jongsten, 1623]. I thank dr. Jeroen Blaak for his transcription of this text.

396Letter of Golius to Constantin Huygens, [Worp, 1911, p. 263].

397[Catalogus Golii, 1668, p. 37].

398[Molhuysen, 1916, pp. 130, 134]. Note that according to Bronchorstius, all widows would receive such a payment, [van Slee, 1898, p. 200].

399This is the date on the memorial stone; the municipal book of funerals files her under 16 November, [Begraafboek 5, 1627].

400[Knöll, 2003, pp. 419–421] gives transcriptions.

401'egregium eius [sc. Academiae] lumen', [Colomesius, 1691, p. 111]. Peiresc to Dupuy, [Tamizey de Larroque, 1888, p. 139], Constantin Huygens to Golius, [Worp, 1911, p. 260].

in the original text. Snellius had already planned its publication some years earlier, which is shown by his reference to 'our Menelaos or Science of Triangles'.[402] Apparently he had wanted to give the volume a title that fitted in his series of Batavian brothers of classical figures: after dealing with Apollonius, Eratosthenes and Tiphys, he would show how he had improved the trigonometrical work of Menelaos of Alexandria. *Canon Triangulorum* , which served as an appendix to the *Doctrina* with trigonometrical tables, was published still in 1626.[403] Hortensius received 10 Flemish pounds (60 guilders) from the States of Holland for his dedication to them.[404] He had also dedicated the book to the regents of Dordrecht, Delft and Alkmaar.

The book discussed propositions and examples from plane and spherical trigonometry, with some astronomical applications. Snellius's philological interests induced him to include a note on the etymology of the word *sinus* ('sine'). He used some algebra. A proposition from plane trigonometry (if one side and two angles of a triangle are given, the other sides are given) was illustrated by an example from the *Eratosthenes Batavus*-measurements.[405]

No more works of Snellius were available for publication, to the regret of at least one of his friends. Hugo Grotius told his brother in a letter of 1628 that a friend of Snellius had come to him to ask whether Grotius knew if Snellius's heirs had still unpublished manuscripts of him, because the friend hoped to publish them. Grotius did not mention the name of the friend, but only wrote that he came from the province. While he was in Paris at that moment, the editor of his correspondence suggests Gassendi, or maybe Mersenne.[406]

In 1629, Snellius's books were auctioned. The Senate of the university decided to keep the day of the auction free of lessons to commemorate Snellius. In the same year, a successor of Snellius was finally appointed, namely Jacobus Golius. His initial salary was 600 guilders. In 1632, the university bought Snellius's large wooden quadrant of his heirs on the request of Golius (cp. p. 101).[407]

When Maria de Langhe had also died, their children were orphans. Her brothers Simon and Adriaen became their guardians.[408] As far as we know, the surviving children did not develop any scientific activity. The two sons

[402]'quemadmodum in Menelao nostro sive doctrina triangulorum Canonica lib. 1 propos. .. ostendimus' [Snellius, 1617c, ad p. 222-2]. The proposition number had not yet been filled in.
[403][Snellius, 1627], [Snellius, 1626].
[404][Huysman et al., 1989, p. 407].
[405]Some of its results are given in modern notation in [Tropfke, 1923, pp. 65–66, 75, 77, 83]; see [Tropfke, 1923, p. 98] for a reference to Snellius's solution of the so-called problem of Hansen, [Snellius, 1627, pp. 97 ff.]. See p. 4 for the etymology of sine, p. 67 for the *Eratosthenes Batavus*-example.
[406]Hugo Grotius to Willem de Groot, 24 May 1628, [Molhuysen and Meulenbroek, 1961, pp. 311–312].
[407][Catalogus Snellii, 1629], [Molhuysen, 1916, pp. 141, 146–147].
[408][Guardianship of Snellius's children, 1628]. The Regional Archive in Leiden has several later documents with information about the guardianship of the children.

were (re)inscribed in the *Album Studiosorum*, but left no other traces at the university.[409] They died unmarried, the daughter married to Adriaen Adriaense Vroesen, who was several times burgomaster of Rotterdam.[410] She remained in the same social circles as her maternal grandfather, who had also been a burgomaster.

2.11 Conclusion

The overview of Snellius's life and work in this chapter teaches us which roles he had in his own time. These roles manifested themselves in his teaching and publications, and also in his professional and private contacts. Of this last category, too little information is extant to satisfy the curiosity. Yet the sources show clearly enough that he was known and respected by fellow mathematicians and other scholars in the Dutch Republic and abroad and that his work was discussed, sometimes critically, but most of the time with approval. He played a role as a mathematical adviser, e.g. discussing an ancient unit of measure with Cunaeus, taking care of the production of a quadrant for Gassendi and a telescope for Rosendalius and advising the States General on a new method for the determination of longitude at sea. He also spent much money and energy on the acquisition and use of his own scientific tools, among which were not only measuring devices, but also books, manuscripts and ancient coins.

Snellius was a hard worker, which is evidenced by the amount of works he published, which are of good quality. The 'publish or perish'-pressure must mainly have been self-imposed, as his professorship did not oblige him to do more than teaching. The huge production shows his ambition, and at the same time the necessity, to show what he was worth, and in this way to improve the position of mathematics and of himself. This is most clearly exemplified by his dedication of the *Fundamenta*, which is at the same time a short philosophy of mathematics, as well as a tool both to flatter his patron Rosendalius, and to convince the university administration of the need for his promotion to regular professor.

What other traits of character can be gleaned besides ambition and determination? They are mostly hidden in humanist flourish or mathematical constructions. We can see a man of broad interests, generally moderate in his opinions but not afraid of polemics. His inclination towards mathematics was firm from his youth onward, even although he knew that it would not give him easy success in society. Conspicuously invisible in Snellius's life is his faith, maybe kept to

[409] Laurens in 1640, as a student of law; his age was erroneously given as 20, [Album, 1875, c. 316]; Rudolph for the second time in 1642, again 'Hon. ergo'. He then lived in The Hague, [Album, 1875, c. 337].

[410] [Haasbroek, 1968, p. 62].

himself for reasons of safety in the religious conflicts of the period. There are no indications that his religious position influenced his mathematical work in any way (apart from his belief in the prognostic value of comets), and therefore we can safely keep it in the confinement of his unknown private life. His family life remains in the shade as well. Although it is generally assumed that Snellius had eighteen children, of which he lost fifteen during his lifetime, I believe that the number of his children was seven (or some more). Even so, the facts that he had no siblings when he had grown up, lost most of his children, and did not see his children old enough to educate them like his father had done with him, must have pained him. That he was not insensitive to these issues, is testified by his correspondence to Rosendalius, in which he reported about his sweet baby and expressed his worry about his wife's health.

Snellius's prolific production consisted of his own works, and editions and translations of works of others. He covered a large part of the mathematical sciences in his publications: pure mathematics (geometry, arithmetic) and mixed mathematics (astronomy, surveying, navigation), and even ventured just outside this domain in his works on ancient money. Although the *Eratosthenes Batavus* is widely considered as his main work, this does not say that Snellius judged it so himself. True, he put much effort in its preparation and conception and was even working on a second edition, but this does not diminish the significance of his other publications to him. It is best to see these works as a whole, supplementing each other, and each highlighting other aspects of the mathematical sciences. Moreover, it has to be realized that Snellius was rather young when death crept up on him. He was still full of plans and projects: a book on optics in an advanced stage, another Apollonius reconstruction, a book on (pure?) geometry, an edition of his father's unpublished works; maybe a polemical navigational treatise and another work on the astrology of comets, and perhaps still more. After his death, Snellius lived on through his publications and more directly through his students, of which at least two became mathematicians of some distinction: Hortensius and Golius.

Almost Snellius's whole oeuvre was destined for an international, well-educated audience and therefore written in Latin. Although Snellius's main task was teaching, this is not reflected in his books. None of the books of which he himself was the author was appropriate as teaching material, only his editions of the *Geometria* and *Arithmetica* of Ramus were textbooks for students. He may have used his own Latin translation of Van Ceulen's *Fondamenten*, a textbook for future engineers, for his university students. He also supervised the translation of Ramus's *Geometria* (*Meetkonst*) into Dutch, for pupils outside the university system, maybe of the engineering school. His research for *Eratosthenes Batavus* stimulated him to give practical surveying classes outside Leiden.

All his books discuss problems that were fashionable, and his contributions were useful, both in a pure and in an applied sense. An echo of Ramus's plea for the *usus* and against the obscurity of the mathematical sciences can easily be heard in this. This may also have been the reason why he did not venture very far into advanced pure mathematics. Some of his books treated age-old problems, that were still relevant (or at least of interest) in his period: the quadrature of the circle, the reconstructions of lost treatises of Apollonius, the determination of the circumference of the earth and the refraction of light. Others were even closer to everyday problems: his navigational treatise, and related to that his book on trigonometry, and his analysis of the comet of 1618 (written on the specific request of Maurice of Hessen). He also reacted to the new discovery of sunspots, but the fact that he published this work anonymously shows that in this case he did not want to be identified with his own statements publicly.

Snellius would not have managed to achieve all these results without the fruitful influences of a number of persons. His father must have been of prime importance in stimulating his son's intellectual development and have made him aware of some of the merits of Ramism: the defence of the importance of mathematics in a humanist curriculum and the attention to useful, non-obscure mathematics are leading themes in Willebrord's oeuvre. Once it became clear that Willebrord had a special interest in mathematics, however, Rudolph's teaching did no longer suffice, although he was professor of mathematics himself. Rudolph then used his network to provide better teachers, of which Van Ceulen, Romanus and Tycho Brahe were the most important. Willebrord also became part of Scaliger's privileged circle, the best place to learn philological skills.

Snellius's network shows that he was connected both with persons from higher as from lower social standing. The latter category consisted for a large part of mathematical practitioners. Snellius must have had an ambiguous relationship with them. On the one hand, they applied mathematics and thus showed its usefulness to the world at large, which message was very welcome to Snellius. On the other hand, association with them and their work could harm the prestige of mathematics in the world of learning. The low standing of someone like Van Ceulen is best exemplified by Scaliger's arrogant reaction when a mere 'boxer' came to correct him. Snellius's position was precarious: to fit in the humanist framework of Leiden University and thus to be recognized himself as a good scholar, he had to elevate mathematics to the level of humanist scholarship, yet (loyal to Ramus) without obscuring its applicability. This struggle for the right place for himself and his mathematics, for finding the right balance between following his own interests and pleasing his different audiences, must have been a leading force in his life.

Chapter 3

Scholarship and rainwater: Eratosthenes Batavus

3.1 Introduction

In the years between circa 1614 and 1625, Snellius exerted himself to determine the length of the circumference of the earth. He published the major part of his results in his *Eratosthenes Batavus* of 1617. This is one of the key works on which Snellius's posthumous fame is based. One of the striking aspects of the book is its diversity. It offers a survey of old knowledge, a methodological improvement of surveying, practical considerations about taking measurements and a load of data. Snellius was the first to measure the circumference of the earth on the basis of a triangulation, that is, the plotting of a net of triangles between two towns far apart.[1]

This interdisciplinary, time-consuming operation must have demanded much of Snellius's attention. Both this fact and the rich contents of the *Eratosthenes Batavus* make it a central work in Snellius's oeuvre. The book will be discussed in this chapter, together with Snellius's work in the same field done after the publication of the book.

The aim of this chapter is threefold. In the first place, a concise overview of Snellius's method will be given, with special attention to those aspects to which he himself drew the attention. This will show, among other things, a side of Snellius unknown in secondary literature: he was an experimenter with some knowledge of chemistry. In the second place, an attempt has been made to carefully establish the chronology of Snellius's activities relating to *Eratosthenes*

[1][Haasbroek, 1968, p. 63]. For a summary of the book, see [Wolf, 1973b, pp. 170–174] and for a more elaborate survey the first chapter of [van der Plaats, 1889].

Batavus, which adds to his biography. In the third place, the results of this chapter will be used for the general analysis of Snellius's mathematics in chapter 8.

3.2 Purpose and method of Eratosthenes Batavus: general use versus endless efforts

The task of measuring the earth was a huge one, and it is evident that Snellius would not have ventured on this enterprise if he had not perceived it as very useful. He gave several motivations in his dedicatory letter to the States General. According to Snellius, the question of the size of the earth was a very old one, which had occupied many scientists. Hence the person who answered it would make a good addition to scientific knowledge and could make a claim to eternal fame.

Besides, the problem of determining one's longitude was most urgent, especially for the Dutch ships which ventured far away from home to formerly unknown regions. Snellius proudly proclaimed his contribution to the solution of this problem:

> I have tackled a problem the solution of which has always been desired by everyone, which has been tried very often, and which has also been made famous by the endeavours of great men. I present here an accurate assessment of the size of the globe [...].[2]

Indeed, the circumference of the earth was a relevant parameter in some methods for determining one's position at sea. These estimated the distance sailed by the ship on the basis of its velocity, and used it to calculate its new position when its original position was known.[3] Linear distance (travelled miles) was transformed into angular distance (the angle between a referential meridian and the present meridian) in this way. Other methods only used this angular distance, for instance those based on astronomical observations or time differences between the point of departure and the meridian under consideration.

In fact, the *Eratosthenes Batavus* offers no direct link between the meridian measurement and the problem of finding longitude at sea. Snellius only seems to have mentioned the problem to explain the relevance of the book to the dedicatees. And these indeed invited him a few years later to become a member of a committee installed by them to judge an alleged solution to the problem. The

[2]'Rem aggressi sumus ab omnibus semper desideratam, saepius tentatam, et magnorum quoque virorum industria nobilitatam [...] Orbis terrae quantitatem accurate definitam hic exhibeo, ut inde omnis longitudinis et latitudinis mensura ex itinerum intercapedine tanto minus erroribus sit obnoxia.' [Snellius, 1617b, fol.)?(*iii*^{v}–)?(*iiii*^{r}].

[3]Cp. Jarich's method in section 2.9.1.

problem again occupied him in his book *Tiphys Batavus*. The States General had offered an opulent financial recompense for the solution of this problem.

Rudolph Snellius also served as an examiner of several proposed solutions. The inventor of one of them, Thomas Leamer, was so disappointed when his method was not accepted that he devoted a long pamphlet to the explanation of his method and the refutation of the objections of Rudolph Snellius and Robbert Robbertsz. One of the key elements of Leamer's method consisted of assigning numerical values to Hebrew words in order to discover their hidden meanings. Snellius and Robbertsz also made Leamer calculate some examplary problems, but they found the results unsatisfying and concluded that he did not even master the foundations of astronomy.[4] This polemic may have induced father and son Snellius to discuss more sensible approaches to the longitude problem, and it may have led Willebrord Snellius to examine one aspect profoundly.

Furthermore, the work had a more local interest: Snellius surveyed a large part of Holland and the surrounding provinces which enabled the States General 'beyond doubt, to register their home-country more accurately' than the Greeks, Romans or any other rulers.[5] Altogether, the problem was challenging for a scholar and its solution well-suited to serve the public good. Moreover, it was useful for astronomy, which was probably not mentioned in the dedicatory letter because this application was less relevant for the dedicatees. The size of the earth was an important parameter in some astronomical calculations, e.g. of the solar distance. This consideration must also have stimulated Snellius.

Snellius's project can be divided into a number of steps, which will first be mentioned briefly and then explained more elaborately. The main source is the *Eratosthenes Batavus.* Yet after its publication, Snellius continued his measurements, because he was not satisfied with his results. Part of his corrections and additions, some of them several pages long, can be found in his own copy of the *Eratosthenes Batavus*, in which he prepared a second edition that has never been published. This copy is now in the Royal Library in Brussels. Most of these changes were published in 1729 by Petrus van Musschenbroek in his *Physicae experimentales, et geometricae, ... de magnitudine terrae ... dissertationes* ('Physical-experimental and geometrical discourses on the size of the earth'). The manuscript which Van Musschenbroek had at his disposal was slightly different from the Brussels copy, containing more changes in some places and fewer in other.

Snellius's measurements and calculations, including those from after the publication of *Eratosthenes Batavus*, have been studied thoroughly by N.D. Haasbroek, lecturer at the department of surveying of the Technological University of Delft. He also compared Snellius's results with modern data, made his own

[4][Leamer, 1612], cp. [de Waard, 1912b].

[5]'Patriam autem hanc iam accuratius et certius consignari posse ex ipso opere facile constabit.' [Snellius, 1617b, fol.)?(*iiii*^{r}].

calculations to check those of Snellius and scrutinized Van Musschenbroek's additions. Part of what is written below is founded on Haasbroek's excellent work, published in four Dutch articles and in a book in English. For technical details I refer to these publications, where a number of relevant maps can be found as well. The focus of the current chapter is somewhat different: it is less technical, and more methodological and historical than Haasbroek's.[6]

Snellius applied a *triangulation*, that is, a method to survey land by dividing it into triangles. He improved earlier efforts by Gemma Frisius and Tycho Brahe. Gemma Frisius explained the principles of triangulation for the first time in an appendix to his *Cosmographicus liber Petri Apiani*, published in 1533. The surveyor was to collect the data of the directions of different places from one place by means of a magnetic compass and a large circle, then travel to the next place and repeat the procedures. The distances between these places could be determined by walking and counting the steps. Later in the book, Gemma Frisius proposed to take the angles of the network instead of the directions, draw them on a map and calculate the required distances by using proportions. Thus no trigonometrical functions were used. Snellius would improve the precision of the method by calculating the sides of the triangles in the network by means of trigonometrical functions instead of measuring them on a map. There are no indications that Frisius actually carried out a substantial triangulation.[7]

The direct inspiration for Snellius's endeavours was probably Tycho Brahe, who performed a triangulation in Denmark. Snellius knew Tycho personally. The latter used a combination of astronomical observations (azimuths, see below) and angle measurements to interrelate the positions of a number of Danish localities. However, he did not actually calculate these positions. If he had, he would have noticed that his results were not very accurate.[8]

Snellius's programme for the determination of the circumference of the earth consisted of the following steps, in the order of his own presentation:[9]

1. He studied the works of classical and early modern authors on the same issue,

2. and he defined his unit of length carefully.

3. He measured several base lines in the fields around Leiden.

[6]Cp. the introduction to this chapter. [Haasbroek, 1960], [Haasbroek, 1965], [Haasbroek, 1966], [Haasbroek, 1967]; the part on Snellius of [Haasbroek, 1968] has almost the same content as the Dutch articles.

[7][Haasbroek, 1968, p. 7, 10–14].

[8]For Snellius's visit to Tycho see section 2.7. An elaborate study of Tycho's triangulation is in [Haasbroek, 1968, pp. 29–58].

[9]Cp. [Haasbroek, 1968, p. 66]. Because Snellius did not publish his later emendations himself, his own 'presentation' does not apply there; that part is based on chronology as much as possible.

4. He calculated the distance between Leiden and The Hague.
5. From a number of towers in Dutch towns, he measured the directions towards other towers. He also measured another base line.
6. He calculated the distances between these towns and their relative position.
7. As a result, he could determine the distance between Alkmaar and Bergen op Zoom, two places with a small difference in longitude.
8. He determined the latitude of Leiden, Alkmaar and Bergen op Zoom, and the azimuth from Leiden to The Hague.
9. He used all these to calculate the circumference of the earth and published his work in *Eratosthenes Batavus* (1617).
10. After the publication of the book, he repeated many of the measurements and calculations.
11. In 1622 he measured a new base line, but he did not make his calculations anew,
12. and he extended his triangulation network to the Southern Netherlands.
13. He collected new material through (among others) Gassendi.

3.3 Before publication: a scholar dirties his hands

Snellius did most of his work before the publication of the *Eratosthenes Batavus*. Afterwards, he did not make any fundamental changes. His work will be discussed by elaborating on the steps given above.

Ad 1. The *Eratosthenes Batavus* consists of two books. Its first book is devoted to a historical survey. Snellius shows himself a true humanist scholar here, knowing an extended range of sources and able to use them. He addressed a number of relevant issues, such as the shape of the earth, its location in the universe,[10] and earlier endeavours to measure the earth.

Snellius's most famous predecessor was Eratosthenes of Cyrene (276–194 BC), who had computed the circumference of the earth on the basis of the distance and the difference in latitude between two localities in Egypt almost on the same meridian. Snellius devoted many pages to a precise explanation of this work, including many figures and calculations. He also had some critique—understandably: if Eratosthenes's work had been perfect, there would have been

[10] Cp. section 4.5.

no need for his own enterprise. Snellius also mentioned Hipparchus and Ptolemy and discussed the work of some Arab scholars and of Jean Fernel, a sixteenth-century French author, as well.

Ad 2. Snellius was aware that it only made sense to talk about distances if his readers knew exactly what unit of measurement had been used. As there existed no standard units of measurement, this was no trivial problem. The first part of the second book was devoted to its solution. He developed no less than four means to convey his information. The first of them was to relate the unit used by him, the Rhenish foot ('pes Rijnlandicus', 'Rijnlandse voet'), to Roman measures, for which the ancient sources were scrutinized again. These were not only bookish sources, but also archeological evidence: he considered the dimensions of the Brittenburg, a Roman fortress near Katwijk (and Leiden). His conclusion was that the Roman and the Rhenish foot were equal.[11]

The most direct way to inform his readers about the used unit of measurement was to show it to them. This was somewhat problematic, however, because the size of the paper changed in the process of printing, as Snellius explained to the reader. Even so, he included a picture of half a Rhenish foot in the book. After printing, this turned out to have the wrong measure, which necessitated a correction on the last page of the book.[12]

Snellius also expressed the length of the Rhenish foot by relating it to other Dutch and foreign standards. Firstly, Snellius gave this information for different feet. He had investigated this himself in Dordrecht, Den Briel, Middelburg, Goes, Zierikzee, Antwerp, Leuven and Mechlin, and the information had been sent to him from other places. He had also borrowed some information from books. One of them had been sent to him by the Bohemian Joannes Smil a Michalovicz, someone he probably knew from his stay with Tycho Brahe.[13] This shows Snellius's methods of acquiring knowledge in a nutshell: he used printed sources, acquaintances and his own observations. It also tells us that he had travelled to even more places than those in his triangulation network, which did not include Zeeland, Den Briel or Leuven.

Secondly, Snellius related the Rhenish foot to ells. Snellius explained that the correct (local) measure of the ell was made known publicly in towns to prevent tradesmen from fraud, because this unit of measurement was used for cloth. He had gained information in the same three ways as for the foot, now giving first hand information for Oudewater, Leiden, Amsterdam and Antwerp.[14]

Snellius then devoted an interesting excursus to his highly original fourth endeavour to fix the value of the Rhenish foot. He argued that although units of length were not well established in general, this problem did not hold for weights,

[11] [Snellius, 1617b, p. 132].
[12] [Snellius, 1617b, pp. 124, 194, 264].
[13] [Snellius, 1617b, pp. 124–126].
[14] [Snellius, 1617b, pp. 140–143].

because these were connected directly to the value of coins and therefore were supervised continuously. He first gave an overview of the weights and relative values of a number of coins. Then he announced that he would determine the weight of a cubic Rhenish foot of water accurately, aiming posterity to reap the fruits of his labour.

A careful description of his experiment was necessary,

> because this is a topic that has kept men of learning fully engaged, and in which some have too boldly dared to determine something just on the basis of their own thoughts.[15]

He needed to measure the volume and the mass of an amount of water exactly. While the determination of the mass was not difficult, the determination of volume was more troublesome, and Snellius had to design a special measuring instrument to cope with some of the complications. First, he had a hollow cylinder of copper or bronze made with a diameter of half a foot and equal height. He made a hole in its upper covering to which he fitted a smaller cylinder (diameter and height 0.1 foot). The insides of the two cylinders were now connected. He also made a small outlet in the covering of the larger one to let the air escape, to enable it to be filled with water completely. After having done so, he closed this opening with a screw and filled the small cylinder.

Snellius wrote that he had constructed the instrument in this way to master the 'rising' or 'swelling' (*tumor*) of the water, which had troubled him at first. He probably referred to the phenomenon that the surface of water in a vessel is not flat due to the force of adhesion. The construction with the two cylinders seems to have been designed to ensure that the instrument both had a rather large volume and that it could be filled completely, so that he could determine the volume of the water exactly. This also explains why he did not take a ready-made instrument for measuring the volume, e.g. a jug: its contents could never be determined with the required degree of precision. After filling, he connected three screws to the bottom to make the vessel stand horizontally.

Now that the instrument was practicable to measure precisely the volume of water, he had to consider what sort of water he could employ. In the beginning he had decided to use rainwater, assuming that rainwater would be equally heavy to all experimentators everywhere because it came straight from the heavens. He compared the weight of rainwater and well water on 10 May 1617. After discovering to his dismay that the former was heavier than the latter, he understood that this could happen because it had rained very recently, and thus the water had arrived in the water reservoir carrying along dregs which had not had the time to settle down.

[15]'cum ista quoque materia sit quae viros doctos admodum habuit sollicitos, et in qua non nulli nimis audacter ex suo tantum conceptu quidquam definire ausi fuerint.' [Snellius, 1617b, p. 150].

Snellius then decided to use a chemical process as an auxiliary means to solve this problem. He distilled the rainwater to purify it, for which he employed a bain-marie (*balneum Mariae*), because he considered that to be the least violent method of all. To do so, he filled a cooking-pot with water, put a vessel of glass half-full of the rainwater in it and put another vessel on top. When he had brought the water in the cooking-pot to the boil, the rainwater also started to evaporate. Through the upper vessel the steam was directed to a tube from which it emerged as purified water. Next, he weighed the double cylinder twice: once empty and once full of distilled water. On 31 May, he filled it again with rainwater from the cistern, after a period of 10 or 12 days during which the sediments had settled down. He reported its weight, and also the weight of the instrument filled with well water.

He subsequently found the weights of the three kinds of water by subtracting the weight of the vessel from the totals. He used his knowledge of the volumes of bodies to calculate the weights of the different kinds of water in the large cylinder, which was easy because of the similarity of the large and small cylinder. He remarked that the weights were in the same proportion as the volumes and therefore the weight of the water in the total vessel was to the weight of the water in the large cylinder as $0.5^3 + 0.1^3$ to 0.5^3, that is $126 : 125$. The weight of the water in a cylinder with diameter and height 1 foot would be 8 times that of the water in the big cylinder.

The calculation of the weight of a cubic Rhenish foot of water was equally straightforward: the proportion between the weight of the water in a cylinder with diameter and height 1 foot and a cubic foot of water was $\pi : 4$. Snellius gave the final values and concluded optimistically that his data made it easy for his readers to determine the size of the Rhenish foot expressed in their own unit of length, thus ignoring all his own troubles.[16]

Snellius's experiment has gone unnoticed in modern Snellius scholarship,[17] perhaps because it does not fit in the modern picture of the activities which a mathematician should pursue. Yet it is full of information, stirs the imagination and raises many intriguing questions. Although his description is very detailed, some problems were glossed over. Did he consider taking other material than water, with a weight that would depend less on the circumstances? Did he consider the change of volume of the water due to temperature shifts? Was the vessel not deformed when filled with water? Snellius's description of his instrument and experiment are so detailed that in principle other scientists could repeat and check it if they wanted, but the many complications with which he was confronted make one wonder whether this was actually possible. Hence although Snellius described many practical details, the experiment seems to be mainly relevant as a thought experiment: how a unit of length can in principle

[16] [Snellius, 1617b, pp. 143–156].

[17] I have only found a short summary in [van der Plaats, 1889, p. 7].

be expressed in a unit of weight. Snellius could also use his description of the experiment as a showcase, showing his aptness in solving practical problems and some knowledge of chemistry. It is unknown whether he ever did similar experiments and where he had acquired the necessary knowledge.

One reaction to the experiment is known. In a letter to Snellius, Gassendi discussed the experiments which Joannes Lombardus had done to establish the weight of solid palms of water, wine and oil. Gassendi used Snellius's relative density of water to compare Lombardus's palm to Snellius's foot, unfortunately having to conclude that the result did not match the relation between those two units as known from elsewhere. He did not know where this difference came from. Snellius shared this interest in the precise determination of standard measures along with some other contemporary scholars.[18]

Even though Snellius gave all this information, it cannot be made out exactly how long the Rhenish rod ('Rijnlandse roede') used by Snellius was. Haasbroek used 1 rod $= 3.766$ m. Although traditionally the rod was divided into 12 feet and a foot into 12 inches, Snellius used a decimal division to facilitate calculations (thus, 1 foot $= 0.3766$ m).[19] This choice may well have been influenced by Stevin's plea for decimalization.

Ad 3. Snellius used a surveyor's chain to measure three base lines, two in the fields between Leiden and Zoeterwoude (one on the straight line between the towers of the two places and one perpendicular to it) and one between Wassenaar and Voorschoten (all three villages in the neighbourhood of Leiden). The first one is the shortest: 87.05 rods (327.8 m). By measuring angles between the lines connecting the end points of the base lines and points in the two places nearby, he could calculate the distance between these points in Leiden and Zoeterwoude, and between those in Wassenaar and Voorschoten.[20]

Ad 4. Although all the localities under consideration were located on a sphere (the earth), Snellius did not use spherical trigonometry, but calculated as though all places were in one plane. He was aware that this was a simplification, but because the distances were not very large, the differences in his results were negligible. This means that he adjusted his degree of exactness to his aims in practical geometry.[21]

After having done his first series of measurements in the fields, he conducted the rest of his observations from towers, in this way having a higher viewpoint, which enabled him to oversee larger distances. He did some measurements from the Town Hall in Leiden and from a church in The Hague and used them, together with his earlier results, to calculate the distance Leiden–The Hague in

[18] [Gassendi, 1964, p. 7], [Gassendi (S. Taussig ed.), 2004, 2, pp. 13–15].
[19] [Snellius, 1617b, p. 157], [Haasbroek, 1968, pp. 63–65].
[20] [Snellius, 1617b, pp. 156–160, 163–164], [Haasbroek, 1968, p. 70].
[21] [Snellius, 1617b, p. 198].

two different ways.[22]

Ad 5. Snellius then established a triangulation network connecting Alkmaar and Bergen op Zoom. This was a schematic representation of a number of Dutch towns, interconnected by straight lines representing their distances. Snellius did not plot them on a map, but only made a sketch. This did not have to be to scale because it was not meant for measuring: the distances were calculated on the basis of the measurements of the angular distances each time between two places, starting from the base lines.

The actual determination of the distances in the triangulation network was a huge work, which involved much travelling. Snellius did observations in Leiden, Alkmaar, Haarlem, Amsterdam, Utrecht, Gouda, Oudewater, The Hague, Zaltbommel, Breda, Willemstad, Dordrecht and Bergen op Zoom. He took his measuring instruments to all these places and climbed towers, from where he measured the angular distances between the towers in other places of his network. Rotterdam was also included in the network, but no measurements were taken from there.[23]

He did not have to do all these observations by himself. In 1615, he travelled around with Erasmus and Casparus Sterrenberg, two young barons. Snellius told the readers of the *Eratosthenes Batavus* that they had already learned arithmetic, geometry and (spherical) trigonometry and now liked to exercise their abilities in a useful matter. When they longed to relax their minds somewhat during the summer holidays, their tutor Joannes Philemon proposed to them to travel to the adjacent regions to prevent his pupils from dissipating their time in too much leisure. They asked Snellius to come along, stimulating him to try his hand at determining the circumference of the earth, 'which I had once said in passing but which they took in earnest'.[24] Their role must have been somewhat exaggerated here.

The Sterrenbergs did part of the observations and calculations. The group first travelled to Oudewater, where Snellius's widowed mother lived. Snellius described his father's place of birth elaborately in the *Eratosthenes Batavus*, dwelling on the horrible siege of 1575, the cruelties done by the Spaniards after their victory and the demolition which it had suffered some decades earlier. Near the town, Snellius and his companions measured a new base line, which they used to determine the distance between Oudewater and Montfoort.[25] In August, they travelled hence to Amsterdam. In the same year, Snellius did a stellar observation in Mechlin together with the Sterrenbergs and Philemon, in order to determine the geographical latitude. He dedicated the second book of

[22] [Snellius, 1617b, pp. 161–167].

[23] [Haasbroek, 1968, p. 88] gives a table and a schematic map of all the 54 measured angles.

[24] 'Ecce quid facerem illa quae olim tantum obiter a me dicta, serio ab ipsis accepta.' [Snellius, 1617b, p. 177].

[25] [Snellius, 1617b, pp. 176–179].

the *Eratosthenes Batavus* to them.[26]

The taking of the measurements was a time-consuming business, which took over a year. An indication of this is that Snellius determined the distance between Wassenaar and Voorschoten twice, based on two different configurations, the measurements for which were done more than a year apart, according to Snellius's own testimony. He added that they had both been witnessed by knowledgeable people, and in one of the measurements he had also been helped.[27]

Snellius was hampered by all sorts of practical difficulties, some of which he disclosed to his readers. When standing on towers, he was bothered by the wind, which was stronger than on the ground. The observations were also made less accurate by the fact that it was often not possible to stand at the centre of the tower. In addition, it could be difficult to identify the correct towers from a large distance.[28]

He stressed these difficulties to impress the reader by his Herculean task, which he would have abandoned

> if the general profit, and the illustrious energy invested on this topic in so many centuries before me, had not spurred me on and forced me to take my pen in hand again, and to raise my body and the sharpness of my eyes to the peaks of towers.[29]

And he had told the reader only the tip of the iceberg:

> What I report here is hardly the hundredth part of the exertion, trouble and expenses which I have endured.[30]

To emphasize this rhetorical exaggeration, he even proposed that they replace 'one hundredth' by 'one thousandth' in the second edition of *Eratosthenes Batavus*. He clearly wanted to ascertain that the reader would appreciate his *magnum opus* sufficiently.

[26]The data about the year of the travel are not consistent. Observation in Mechlin in 1615: [Snellius, 1617c, ad p. 208-1] (cp. nr. 12 of Snellius's programme). In the dedicatory letter to the Sterrenbergs, Snellius refers to their summer holiday trip two years before, which should be 1615—unless the letter was written some months before the publication of the book, then it could be 1614, [Snellius, 1617b, pp. 119–120]. In a note to Risnerus's *Optica*, Snellius wrote that he travelled from Oudewater to Amsterdam in 1615 with the Sterrenbergs. However, in *Eratosthenes Batavus* he wrote that they were in Oudewater in the year after the death of Rudolph Snellius (1613) ('[...] charissimi parentis mei obitum anno superiore [...]', [Snellius, 1617b, p. 177]).

[27][Snellius, 1617b, p. 164].

[28][Snellius, 1617b, p. 171].

[29]'nisi publica utilitas, et tot iam seculis fatigata tam nobilis cura stimulos mihi addidisset, et rursum calamum in manum, corpus et oculorum aciem in turrium fastigia attollere coëgisset.' [Snellius, 1617b, p. 171].

[30]'Haec enim ipsa quae hic affero vix centesima pars sunt laboris, molestiae, impensarum quas exantlavimus.' [Snellius, 1617b, p. 171].

When Snellius made his measurements for the *Eratosthenes Batavus*, he needed instruments that were both accurate and could survive transport. On his expedition with the Sterrenbergs, he used a semi-circle (diameter $3\frac{1}{2}$ feet, about 1.33 m) for measuring angular distance between towers (in the horizontal plane) and a quadrant of iron with a radius of over $5\frac{1}{2}$ feet (a. 2.09 m) with an edge of copper for determining the polar altitudes. He used a smaller quadrant ($2\frac{1}{5}$ feet, a. 84 cm) when determining the position of his base line between Leiden and Zoeterwoude. When he was in Gouda, he measured the angle Leiden-Gouda-Den Haag several times, both with this last quadrant and with his semi-circle.[31]

Snellius received some data from other scholars. Beeckman for instance measured the angular distances between a number of places from the tower of Zierikzee on Snellius's request, yet Snellius did not include them in the *Eratosthenes Batavus*.[32]

Haasbroek pointed out some deficiencies in the triangulation network, e.g. the insufficient determination of the positions of Haarlem and Amsterdam in relation to the other points in the network.[33]

Ad 6. On the basis of the measured angles and the length of one side of the network (Leiden–The Hague), the directions and the lengths of all other sides could be calculated by simple trigonometry. Snellius did not manage to make all these calculations without making errors, as has been remarked by Van Musschenbroek and Haasbroek. The latter called him 'a shoddy calculator' and noticed that Snellius did not check all his calculations, although he sometimes could have done so by computing the same distance in different triangles.[34] Of course, he did not have a method like that of least squares at his disposal to minimize the influence of measuring errors, but he corrected the outcomes of his calculations in such a way that the sum of the angles of every triangle was 180^0.[35]

Ad 7. Snellius used his previous work to calculate the distance between

[31]'[...] semicirculum diametri pedum Rhijnlandicorum trium et semis, ad gaeodaesias distantiarum et angulorum quantitatem e turribus observandam. Quadrantem etiam amplissimum ferreum, aere incrustatum, amplius quinque et semis pedum, ad poli altitudinem explorandam.' [Snellius, 1617b, p. 177], '[...] maximo quadrante ferreo cuius radius propemodum erat sexpedalis, limbus autem ad commodiorem sectionem aere incrustatus [...]', [Snellius, 1617c, ad p. 208-1] (edited in [Bosmans, 1900, p. 121]); cp. [Haasbroek, 1968, p. 65] for an English translation of p. 177. 'Aes' meant any crude metal except gold and silver, esp. copper, or an alloy, mainly bronze (probably not brass, which is 'orichalcum', a word indeed used elsewhere by Snellius, [Snellius, 1617b, p. 150]). The quadrant of $2\frac{1}{5}$ feet was 'aereus', which can mean made of copper, or furnished/covered with copper/bronze, [Snellius, 1617b, p. 156].

Measurement from Gouda: [Snellius, 1617b, pp. 167–169].

[32][de Waard, 1939, p. 105]. Of the places mentioned, only Bergen op Zoom was in Snellius's network.

[33][Haasbroek, 1968, pp. 99-100].

[34][van Musschenbroek, 1729, pp. 359–380], [Haasbroek, 1968, pp. 72–73].

[35][Haasbroek, 1968, p. 89].

Alkmaar and Bergen op Zoom. His final value is 34,710.6 rods (130.7201 km), which differs about 172.5 m from the same value according to modern measures.[36] Haasbroek concluded more mildly than in the previous section:

> It is the best obtainable result in those days, also thanks to the eminent determination of the base line LHg [sc. Leiden–The Hague] and its excellent checks and in spite of the rather poor construction of the northern part of the meridian chain and the many errors in the calculation.[37]

Ad 8. Snellius then had to find the geographical latitudes of Alkmaar, Bergen op Zoom and Leiden, which he did by measuring the height of the Pole Star. His instruments did not allow him to determine these latitudes with the same degree of precision as the distance between Alkmaar and Bergen op Zoom. He also determined the azimuth (the direction in relation to the meridian) from his own house to the Leiden town hall and to The Hague and used it to calculate the azimuth Leiden–The Hague. This was necessary to orient his triangle network and thus to determine the difference in longitude between Alkmaar and Bergen op Zoom. He had selected these last two places because they were located almost on the same meridian.[38]

In connection to this, Snellius solved a geometrical problem, which gave him some lasting fame. It is called the Resection Problem. Snellius had to determine the distance of his house to three points in Leiden (two churches, the Pieterskerk and Hooglandse Kerk, and the Town Hall), the mutual positions of which were known. He considered his solution of the problem to be of no little importance himself: he devoted a separate, rounded-off chapter to it and proudly announced his useful invention for surveying:

> I have invented an elegant theorem for that problem, which can have a widespread application in our country from now on, because the distances between so many illustrious places have been registered with such precision.[39]

From a geometrical point of view, the problem is no more complicated than many other construction problems. Its practical application, however, made it stand out. Snellius's solution consisted of five steps:[40]

[36][Snellius, 1617b, p. 194], [Haasbroek, 1968, pp. 104–105].
[37][Haasbroek, 1968, p. 105].
[38]Cp. [Haasbroek, 1968, pp. 105–107].
[39]'Et ad eam rem theorema scitum excogitavi, cuius usus iam in patria nostra deinceps permagnus esse possit, cum tot illustrium locorum intervalla tam accurate sint consignata.' [Snellius, 1617b, p. 199].
[40][Snellius, 1617b, pp. 203–206].

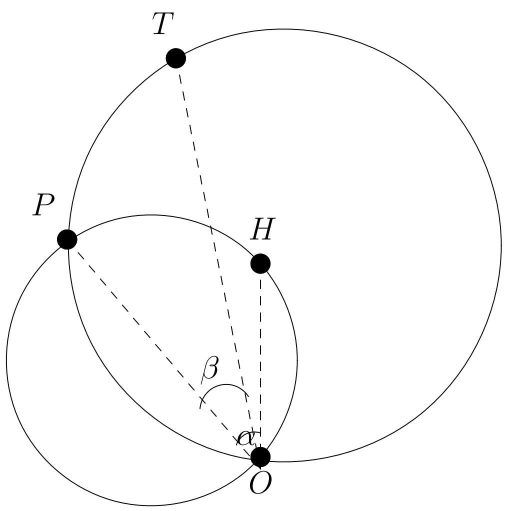

Figure 3.1: The Resection Problem

1. Two more data were needed to determine the problem. A number of possibilities existed for this, but as this is a problem from applied geometry, Snellius chose for a feasible solution: he measured the two angles between Pieterskerk-house-Hooglandse Kerk and Pieterskerk-house-Town Hall. Taking the angle Town Hall-house-Hooglandse Kerk instead of one of these two would have yielded a similar solution.

   In geometrical terms, the problem is now:

   Problem 3.1 (Resection Problem) *Given three points in position, P, T and H, $\angle POH = \alpha$ and $\angle POT = \beta$. It is required to determine OP, OT and OH (see figure 3.1).*

2. Snellius first gave the general idea of the solution. He argued that O lies on the intersection of two given circles, one drawn on the chord PH with circumference angle α and a second drawn on the chord PT with circumference angle β (cp. *Elements* III.33).

3. Next, Snellius gave an exact geometrical construction of the point O, by constructing these two circles. He remarked that there was no other solution than O, because two circles cut each other in at most two points (of which P also had to be one). If the problem had originated from pure geometry, Snellius would have been ready.

4. Now that it was a practical problem, two extra steps were necessary in order to find a solution in numbers. One by one, Snellius considered a number of triangles in the figure, of which three properties (angles or sides) were known, from which the other three could be deduced. He was still dealing with the general case, not with a numerical example. The sought distances were now 'given'.

5. Snellius reworked all the steps of his argument with the numerical values which he had measured and calculated previously.

   He could have considered to rework his calculations after replacing α or β by $\angle TOH$ and then to take the average value of his results to minimize the influence of measurement errors, but this line of thought seems not to have occurred to him.

Before and after Snellius, other mathematicians solved this or similar problems independently. The Greek astronomer Hipparchus solved a problem from astronomy that was mathematically equivalent to problem 3.1. It is found in Ptolemy's work and could have been known to Snellius, but he may not have thought of it while working on the resection problem. A later contribution was made by Laurent Pothenot, who published a solution in 1692. He was incorrectly assumed to have solved the problem for the first time and therefore the problem was often called after him in later times. One of the more recent approaches was to describe the problem in a Cartesian coordinate system, and to generalize it to more given points.[41]

In 1960, the Delft surveyors' society *Snellius* placed a memorial stone in the building currently occupying the space where Snellius's house formerly stood, to commemorate the first resection (the building, in the Doezastraat, is now a medical centre).[42] The present whereabouts of this stone are unknown, but a new plaquette has been fixed to the building by the same society in 2005.

Ad 9. Snellius now knew the distance between Alkmaar and Bergen op Zoom and their relative positions (latitude and difference in longitude). One of the other data needed to reach the desired result was the value of π. Snellius gave the approximations of Viète, Romanus and Van Ceulen, in many more digits than his measuring accuracy necessitated. Through a long series of calculations he arrived at the end of his quest: the length of one degree on the meridian of Alkmaar was 28,500 rods (a. 107.33 km), and therefore the length of a meridian 10,260,000 rods, about 38,639 km. This is about 3.65 % less than the modern value.[43]

[41] [Haasbroek, 1968, pp. 110–111], [van der Plaats, 1889, pp. 31–38], [Wolf, 1973a, pp. 181–182], [Tropfke, 1923, pp. 97–98], [Haerpfer, 1910, pp. 14–20].

[42] [Haasbroek, 1968, pp. 112–113].

[43] [Snellius, 1617b, pp. 198, 212–217], [Haasbroek, 1968, p. 108].

He published his work in the *Eratosthenes Batavus* of 1617. The full title is *Eratosthenes Batavus De Terrae ambitus vera quantitate, a Willebrordo Snellio, διὰ τῶν ἐξ ἀποστημάτων μετρουσῶν διοπτρῶν, suscitatus*, which means 'The Dutch Eratosthenes, on the true size of the circumference of the earth, recalled from the grave by means of optical instruments according to measured distances'. Snellius announced clearly in whose footsteps he followed by the explicit reference to his Greek forerunner.

3.4 After publication: improvements

Ad 10. Van Musschenbroek tells us that Snellius discovered some mistakes in his book after its publication when teaching his students practical geometry. Snellius showed them how he had taken measurements in the fields around Leiden to calculate the distances between the villages in the neighbourhood. After his discovery, he decided to start all over again and redo all the measurements and calculations, maintaining the same base line.[44]

The new results are found in the Brussels copy of the *Eratosthenes Batavus*, written in Snellius's own hand, and they were published by Van Musschenbroek in the first part of his treatise. Pages 185–192 of *Eratosthenes Batavus*, which contain an important part of the observations and calculations of the network, are lacking in the Brussels copy, and consequently we do not have a direct source of Snellius's corrections there. However, Van Musschenbroek had at his disposal these changes and included them in his book.[45] Not all Snellius's changes were improvements: some new errors were introduced.[46]

Ad 11. According to Van Musschenbroek, Snellius decided to publish a corrected and extended version of the *Eratosthenes Batavus* when he found out that the book had shortcomings. The completion of this second edition was thwarted by a unique chance to measure a more accurate base line when in January 1622, during a very rough winter, the fields and meadows around Leiden were inundated and it started to freeze. In this way, Snellius had a large, perfectly smooth surface at his disposal. He measured a new base line with a chain near Voorschoten on 3 February. He repeated his measurement three times for extra precision, then used it to determine anew the distance between two towers in Leiden and Zoeterwoude, which turned out to be almost 5 rods (a. 18.8 m) more than in his former calculations.[47]

[44][van Musschenbroek, 1729, p. 358], French translation of pp. 358–361 in [Bosmans, 1900, pp. 114–116].

[45]There are more indications that Van Musschenbroek had a further reworked copy of the *Eratosthenes Batavus*; cp. [Bosmans, 1900, pp. 118–119]. However, the description of the Belgian extension of the triangulation network lacks in his book.

[46][Haasbroek, 1968, p. 66].

[47][van Musschenbroek, 1729, pp. 358–359, 401], Haasbroek discusses three new base lines of

No details about this new base line can be traced in Snellius's preliminary work for the new edition, but he does write that a few years after his first measurements, he seized the opportunity to determine the distance between Leiden and Noordwijk when it was freezing, and that this distance would be his line of reference from then on, instead of that between Leiden and The Hague.[48] However, no calculations connecting this new line to his other observations are known.

After having explained these extensions by Snellius, Van Musschenbroek wrote that at this point Snellius would have had to calculate all the triangles for the third time; worn out by all the labour, Snellius had just remarked that the new base line (near Voorschoten) had to be preferred to that in *Eratosthenes Batavus* and 'he did nothing further'.[49] It seems odd that Snellius would have ended his enormous project before reaching the finish, especially because adjusting his triangle network to this new base line did not involve as much travelling as his last round of observations. If Snellius had indeed decided to stop at this point, method must have mattered more for him than result. It seems more likely, however, that he did not have the time to finish the job immediately, therefore postponed it and did not have the chance to come back to it before he died only a few years later, in 1626.

In the second part of his treatise, Van Musschenbroek used this new base line and Snellius's observations (some of them checked by him) to calculate an improved approximation of the circumference of the earth. Haasbroek thought that Van Musschenbroek falsified some of Snellius's observations to make the computations more consistent.[50] He concluded too sternly:

> With this falsification Van Musschenbroek's work is fully condemned; it is entirely unreliable and it contrasts very badly with the faithful work carried out by Snellius a century earlier.[51]

Ad 12. Snellius wanted to improve his results in yet another way, extending his net of triangles to Mechlin in the Southern Netherlands in order to improve his final result by using two places further apart than Alkmaar and Bergen op Zoom. He included Antwerpen, Hoogstraten and Mechlin in his network, and he used the polar altitude determined in Mechlin in 1615. The other necessary

1622: [Haasbroek, 1968, pp. 70, 79–87].

[48]'Nam initio distantiam inter Leidam et Noortwicum non eramus ex ipso fundamento dimensi; quod tamen aliquot annis post ad certiorem operis rationem oblata oportunitate, hiberno tempore per glaciem commodissime absolvimus.' [Snellius, 1617c, p. 183].

[49]'adeoque iterum ab initio calculus omnium Triangulorum erat repetendus: pertaesus procul dubio laboris, quo iam bis perfunctus erat Auctor, tantum adnotavit, ultimae modo mensurae fidendum esse, non primae, quam in Eratosthene statuit, atque ulterius fecit nihil [...]', [van Musschenbroek, 1729, p. 359].

[50][Haasbroek, 1968, pp. 68, 79].

[51][Haasbroek, 1968, pp. 83–84].

measurements were probably done in 1625. Van Musschenbroek thought that this addition had been lost, but it is included in the Brussels copy of *Eratosthenes Batavus*.[52] Apparently, there were no serious obstacles related to travelling in hostile territory, nor even to climbing towers and surveying the country. This extension did not lead to a new final conclusion either.

Ad 13. The correspondence between Snellius and Gassendi from 1625 shows that Snellius was still acquiring new material then, notably on the sizes of feet and ells in different places, because, as Snellius wrote, 'my Eratosthenes has to be augmented and when it has been enriched, it must be published'.[53]

3.5 Conclusion

Snellius's efforts to measure the earth were huge. He had to make many travels, followed by numerous calculations. The transport of his instruments, especially of the largest quadrant, must have been a tremendous job, as they were heavy. Yet Snellius took the large quadrant with him al the way to Mechlin, trusting this instrument more than whichever he could have borrowed there. This shows his determination in making his observations as exact as possible and his willingness to solve practical problems if an important matter was at stake.

Moreover, this whole measuring expedition probably cost Snellius some money. Part of the expenses may have been paid by the Sterrenbergs and Snellius's dedication to the States General yielded him 200 guilders.[54] This was a considerable amount, 40 % of his annual salary in 1617. Yet it is quite likely that his costs exceeded this amount and that he had to supply it from his own not too full pocket. The remuneration for Snellius was not financial, but intellectual and social.

Snellius showed both signs of pride of his achievements and awareness of his shortcomings. The latter induced him to work on an improved version of *Eratosthenes Batavus* continually, after wrapping up his research and writing the book rather quickly at first. He shared many of his considerations and activities

[52][van Musschenbroek, 1729, pp. 358, 361]; extension in [Snellius, 1617c, ad p. 208-1–7], edited in [Bosmans, 1900, pp. 121–126].

The chronology is not completely clear. Van Musschenbroek dates the Belgian extension between the publication of *Eratosthenes Batavus* and 1622. In the new chapter which Snellius wrote about this extension, he mentioned an astronomical observation done in Mechlin in 1615 and the death of Coignet, who died 'in the previous year 1624', [Snellius, 1617c, ad p. 208-6], without making explicit which events belong to which year. In fact, Coignet died 24 December 1623, [Quetelet, 1873]. All taken together, 1625 is the most likely year for the triangulation in the Southern Netherlands; if Snellius had done all this work in 1615, he certainly would have included it in the first edition of his book.

[53]'Eratosthenes meus augendus, et locupletior est in lucem edendus.' [Gassendi, 1964, p. 393].

[54][Smit, 1975, p. 241], [Dodt van Flensburg, 1848, p. 16].

with his readers, which is best exemplified by the digression on the tragic fate of Oudewater and his lengthy explanation of the experiment to determine the relative density of water. This expansiveness might be a figure of speech: Snellius may have meant to incite the reader to empathize with him and to appreciate his immense project more.

Eratosthenes Batavus is indeed an epitome of Snellius's *oeuvre*, not only because of its contribution to surveying, but also because of its programmatic value: Snellius showed in it how fruitful a combination of bookish scholarship and practical research could be.

3.6 Reception

Some examples may suffice to show the reception of the *Eratosthenes Batavus*. A practical consequence of Snellius's project was the frequent later use of the Snellius mile for navigational purposes. This mile was defined as one fifteenth of the length of one degree on the meridian, that is 22,800 Rhenish feet (a. 85.864 km). The *Eratosthenes Batavus* was not directly accessible to sailors because of the language barrier, but a portion of the results could be learned through vernacular texts. Snellius's measure of the circumference gained wide acceptance.[55] In this way Snellius had contributed to the solution of the pressing problem of the absence of standard measures, to which he had paid much attention in the book. Just as he had stated in his dedicatory letter, the work was useful for navigation, yet it did not contribute substantially towards solving the longitude problem.

Snellius introduced a new development of the technique of triangulation in his book, yet this did not reach the Dutch surveyors for a long time because they did not master Latin either. As they could not read the book, they may have considered its topic as too esoteric for their purposes, evidently unaware of the relevance of Snellius's work for making local maps. The distances between places that he had calculated were not used either.

Abroad, a number of scientists read Snellius's work and although they had some criticisms, they used the method for their own degree measurements or they selected other results. For instance Kepler mentioned Snellius's reduction of longitude differences in Western Europe in his *Tabulae Rudolphinae*, and he also referred to *Eratosthenes Batavus* in the *Somnium*. Wilhelm Schickard asked Matthias Bernegger in Strasbourg on behalf of Kepler and himself to send them his (Bernegger's) local foot and ell expressed on a solid, not moistened piece of paper, so that they could use Snellius's comparison of these measures with Roman ones. Between 1624 and 1635, Schickard also surveyed the duchy of

[55][Davids, 1986, pp. 113–114, 126, 150, 212, 270–271, 310].

Württemberg using the principles of triangulation as developed by Snellius.[56]

Snellius's work also reached map makers abroad. For instance Tobias Mayer used Snellius's data in several maps of the Low Countries in the eighteenth century.[57]

[56][Pouls, 1997, pp. 261–264], [Kepler (F. Hammer ed.), 1969, pp. 95, 98], [Kepler (V. Bialas and H. Grössing eds.), 1993, p. 362]; [Seck, 2002, 1, p. 172], [Betsch, 1996, pp. 136–142].

[57][Mayer, 1747], [Mayer, 1748a], [Mayer, 1748b]. [Mayer, s a] contains some of Mayer's own notes on *Eratosthenes Batavus*.

Chapter 4

'In your father's footsteps': astronomy and patronage (1618–1619)

4.1 Introduction

In 1618 and 1619 two astronomical treatises by Snellius appeared: he edited the *Observationes Hassiacae* ('Hessian Observations') and wrote the *Descriptio Cometae* ('Description of the Comet'). Both of them were dedicated to Maurice, the Landgrave of Hessen. Willebrord Snellius had 'inherited' Maurice of Hessen as a patron from his father Rudolph. Maurice explained this double patronage in a letter to Willebrord from 1619 in the following way:

> Soon it became clear to us, Illustrious Sir, that there is no difference in learning or affection between you and your father of blessed memory, and therefore we now adorn you with the same favour and sympathy that we gave him in the past—for our inclination to good and learned men is such that we do not only give attention to these men themselves, but also to their progeny who strive after their paternal qualities, with as it were extended affection. Nor do you have to doubt whether you enjoy equal favour with us, which you do, because you are following in your father's footsteps. That you can be assured of this in every respect is the purpose of this letter and at the same time we want to give you an opportunity to display equal veneration for us.[1]

[1] 'Cito nobis persuasum est, vir clarissime, inter te et parentem tuum piae memoriae studiis

This chapter will examine the ties which join together Maurice of Hessen and father and son Snellius, and the related link between Maurice's father William and Willebrord Snellius. The younger Snellius came into touch with the late elder Landgrave's work in the field of astronomy through the shared interest in Ramism of the younger Landgrave and Snellius senior. Special attention will be paid to the question what influence William of Hessen's work and his son's ideas had on Willebrord Snellius's two works originating from the patronage. The chapter offers a well-documented case of the role of patronage in early modern science.[2] Several hitherto unknown or hardly known letters by the Snellii and Maurice of Hessen shed light on their roles, contributions and interests. Because no editions of these letters are as yet available,[3] large quotations will be given to allow their authors to express their opinions in their own voices. The different character of the relation between Rudolph and Maurice, and that between Willebrord and Maurice will illuminate the diversity of patronage relationships in general.

Furthermore, some of Willebrord Snellius's views on astronomy will be discussed, in particular those on observation, authority and astrology, as they can be inferred from the two treatises under consideration. These works offer good specimens of the tensions between the theories of much respected ancient authors and newer insights that was so typical for the work of humanists. Snellius expressed his debatable opinions forcefully in some places.

Snellius's astronomical treatises deserve a more thorough study than they receive in this chapter. I will examine here only those aspects that are related to his work on other parts of the mathematical sciences or to one of the central themes of this book. This does not mean that other aspects are uninteresting—notably, the accuracy of his measurements and his calculations, and his use of older data—, but they just fall outside the scope of this study.

aut affectione, nihil esse divisum; et inde, quo illum olim, eodem te quoque nunc prosequimur favore et clementia. Ita namque animati sumus in bonos et doctos viros, ut non solum ipsos, sed et ex iis genitos et in bona paterna nitentes propagato quasi amore contemplemur. Nec tu, quia paternis vestigiis insistis, dubitabis, quin in pari quoque apud nos sis gratia: De quo ut omnino possis esse certus, hisce literis efficere volumus, simulque ostendere occasionem parem nobis exhibendi cultum.' [Maurice of Hessen, 1619, fol. 1^r].

[2] Cp. the influential [Biagioli, 1993], in which the interwovenness of the contents and circumstances, in particular patronage, of the mathematical sciences are highlighted in the case of Galileo.

[3] With the exception of Rudolph Snellius's letter to Rosendalius. See tables 2.1, 2.3 and 2.4 for a survey of the extant correspondence of Rudolph and Willebrord Snellius.

4.2 Rudolph Snellius and Maurice of Hessen: Ramist ties

Rudolph Snellius and Maurice of Hessen must have become acquainted before Willebrord's birth. In the years 1565–1578 Rudolph had been a student and then a teacher in Marburg, which belonged to the realm of Hessen.[4] Maurice of Hessen (1572–1632) was a true Renaissance prince, who not only ruled his country, but also had wide cultural interests, and who was an able orator and sportsman. He had earned himself the epithet 'the learned' because of his interest and participation in many fields of art and learning, primarily in medicine and alchemy, and because of his contacts with an international circle of scholars. This interest in learning had been bequeathed to him by his father William of Hessen, a cultivator of astronomy and an astronomer himself.[5]

Several ties connected Maurice to the Netherlands, and in particular to the Stadholder Maurice of Nassau. The Landgrave's second wife was a cousin of the Stadholder. Besides, Maurice of Hessen was very keen to know about the Dutch army reforms instigated by Maurice of Nassau and sent two of his sons to The Hague to learn about them from the Dutch Maurice and his cousin William Frederick.[6] The two Maurices also shared scientific interests, as shown, for instance, by a letter of 1602 in which Maurice of Nassau asked Maurice of Hessen to send him a copy of the latter's father's observations of the stars.[7]

Maurice of Hessen founded a court school in 1592 that received the name *Collegium Mauritianum* in 1599, when it changed into a school for a wider audience than only the children of the nobility. He was a staunch Calvinist,[8] which the timetable of this school shows most clearly: about 32 hours had to be spent on services, catechism, religious music and other religious activities every week, whereas the mathematical sciences did not play an important role: the only part that was taught was arithmetic, for four hours a week (all in all, the time table covered no less than 112 hours a week).[9] Yet Maurice did not neglect mathematics. For some time, he even had a court mathematician, Nicolaus

[4]William and Maurice were Landgraves of Hessen-Kassel, which did not include Marburg until 1604. From 1567, when Hessen was split into four parts, until 1604, Hessen-Marburg was a separate realm. However, the university of Marburg remained common Hessian in this period, [Menk and Kümmel, 1997, p. 87].

[5][Borggrefe, 1997b, pp. 15–16], [Borggrefe, 1997a, p. 30], [Menk, 2000b, pp. 50–57]. Two excellent recent books on Maurice of Hessen with contributions by various experts are [Borggrefe et al., 1997] and [Menk, 2000c]. See [Moran, 1991] for an extensive study of the alchemical circle around Maurice; cp. [Moran, 2000]. For William see section 4.3.1.

[6][Löwenstein, 1997, pp. 47, 51, 74–75].

[7][Maurice of Nassau, 1602]. For these observations see section 4.3.1.

[8]For the complicated story of Maurice's confessional politics and his growing identification with Calvinism, see [Menk, 2000a].

[9]A guest of Maurice, John Louis of Nassau-Hadamar, also noticed a lack of mathematical education, [Menk, 2000b, p. 55].

Chesnecopherus, who defended theses about the usefulness of mathematics under Maurice's direction in his Collegium.[10]

Rudolph Snellius and Maurice of Hessen shared a profound interest in Ramism.[11] Already as a twelve year-old, Maurice had studied Ramus's works,[12] and he ordered that rhetoric and dialectic should be taught at the court school according to Ramist principles. The Landgrave was influenced by Ramism in other fields, too.[13] Ramism and Calvinism went very well together, Ramus himself having been a Huguenot and his murder in the Saint Bartholomew massacre having given him the role of a martyr of the Protestant faith. It has been argued that Maurice of Hessen founded the Mauritianum to root Ramism and Calvinism in his territory.[14]

It is not known what conversations, if any, took place between Rudolph Snellius and Landgrave Maurice when they both lived in Hessen. What we do know, is that Maurice wanted to see him again when Rudolph had been living in Leiden for a long time. In 1600 Rudolph was invited by Maurice to come over to his court in Kassel 'because of the old friendship'. Although Rudolph gladly accepted this invitation, he only managed to get away from home in 1603. He was then most generously treated by Maurice:

> and soon he [the Landgrave] let him [Rudolph Snellius] go again with honour, with a golden necklace and a portrait of the Prince as gifts, and he was [taken] to Frankfurt in the Landgrave's chariot drawn by four horses, and at his expense, as a sign of honour, after which he went home.[15]

This display of honours bestowed by Maurice on Snellius must have been one of the major events of his life, giving him recognition as a scholar. They even made the great Scaliger jealous, who complained that the golden chain should have been his because of his noble descent—he claimed kinship with Maurice's

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[10][Löwenstein, 1997, pp. 72–73], [Borggrefe, 1997a, pp. 28–29]. These theses are summarized in [Stegmann, 1757, pp. 9–14]. Stegmann tried to argue that Maurice was highly interested in the mathematical sciences, considering Maurice's direction of the defence as a proof of his thorough understanding. Needless to say, Stegmann's argumentation is explained by its context; his panegyric of Maurice must be seen in the light of his goal to offer an agreeable lecture to his Landgrave on the occasion of the latter's birthday.

[11]For Ramus's own ideas, see section 2.5. For Rudolph Snellius's Ramism and his German period, see section 2.6.

[12][Borggrefe, 1997b, p. 15].

[13]Borggrefe writes: '[...] Moritz [strebte] gestützt auf die Lehren des Petrus Ramus eine umfassende Reform in allen weltanschaulichen und gesellschaftlichen Bereichen an.' [Borggrefe, 1997a, p. 28]; see further [Borggrefe, 1997a, pp. 28–31], [Löwenstein, 1997, p. 70].

[14][Friedrich, 2000b, p. 170].

[15]'[...] et mox honeste iterum dimissus, aureo torque, et effigie Principis donatus, et quadrigis denique eius atque impensis honoris ergo Francofurtum, domum rediit.' [Alma, 1614, pp. 176–177].

wife.[16]

Public as this honourful treatment may have been, the precise reason of the visit and the contents of the discussions were kept more private. A newly discovered source shows that Rudolph and Maurice discussed a religious problem. In 1599, Maurice was confronted by a request from the Count of Nassau to have his sons, who were being educated at the court of Kassel, receive Communion according to Calvinist rules. This issue was also of great importance to Maurice himself, who had already made clear in 1587 that he did not agree with the Lutheran concept of the Lord's Supper.[17] In Hessen-Kassel, this was the beginning of the Second Reformation, the period in which the Calvinist faith began to play a more dominant role in everyday life in a number of German states. Maurice consulted several scholars, all of them Calvinist lawyers and theologians, about the right form of Holy Supper. He also asked and received Rudolph Snellius's opinion, and devoted a text to the problem himself.[18]

The problematic question, which was full of religious and political pitfalls, was not solved for some years, and when Snellius was in Kassel in 1603, the Landgrave discussed it with him. When he had just left Kassel, Maurice wrote a letter to him about the liturgical breaking of the bread to recapitulate the contents of their conversations. The letter can therefore be used to accurately date Rudolph's visit to the Landgrave in July 1603.[19] Maurice told Rudolph that he had chosen him for consultation because he was one of the few scholars of his age who 'wants to apply himself to philosophy with his own philosophy, which means to put an effort into the pure and clear truth' and who was 'an ardent, learned and true philosopher, which means someone who loves the truth, not pretence'.[20] They had been talking about the breaking of the bread and

> about the fact which, as I said, troubles me most, i.e. that many Reformed declare our ceremonies hypocritical, but that on the other hand others—no fewer than they—revere the same with us.[21]

[16]These honours are not only mentioned in the biography quoted in the previous footnote, but also implicitly referred to in another contemporary biography, [Meursius, 1625, p. 120], and in Rudolph Snellius's funeral oration, [Coddaeus, 1613, pp. 17–18]. Scaliger's remark was recorded in the *Scaligerana*, [Scaliger, 1669, II, p. 109]: 'Le Lan[t]grave de Hesse a renvoyé à Snellius vne chaisne d'or plûtost qu'à vn honneste homme comme moy, qui suis parent de sa femme selon mes ancestres.'

[17][Borggrefe, 1997b, p. 15].

[18][Menk, 1993, pp. 176–178].

[19]The letter was written 'ex Musaeo nostro' ('from our Museum', probably a room in the castle in Kassel) on 31 July 1603. Maurice writes: 'Redeo itaque ad nostram quam ante paucas horas habebamus collationem de ritibus Ecclesiasticis, tibique in memoriam revoco omnia ea, quae tum inter nos agebantur [...]', [Kalckhoff, s a, fol. 45^r]. Only a copy of the letter is extant, see [Kalckhoff, s a, fols. 45^r–47^v].

[20]'[...] qui cum philosophia sua philophari, id est candidae et apertae veritati operam dare velit. Cum a iam Deo, sic dante, in te vivum doctum et vere philosophum, id est, qui veritatem amet, non ostentationem [...]', [Kalckhoff, s a, fol. 45^r].

[21]'[...] de eo, quod me maxime perturbare inquiebam, quod nimirum multi ex reforma-

Maurice argued that he was not opposed to those wanting to break the bread in the Lord's Supper, but that the ritual was not supported by any Biblical prescription. He used several arguments based on the Bible, the Church Fathers and the Reformed theologians Jean Calvin and Theodorus Beza to underpin his viewpoint. As a prince, he was not only religiously motivated to solve the problem, but also politically, as he wished to prevent the turmoil that would arise from a schism:

> so long as these ceremonies of the Reformed have not been arranged by a national council, that is, in a meeting of all the churches that embrace the purer religion, and so long as they have not formally been described, I consider no church bound to these, nor do I judge that any other church has sufficient reason to split itself off because of this.[22]

After these considerations Maurice appealed to Snellius's loyalty:

> My last point is a shorter one, because I have no doubt at that point at all that you suffer together with us, and that you very often deplore the proud attempts of that kind of people. In you, I have a faithful, ardent and sincere witness, whom I can invoke and ask to give evidence that the pious among my ancestors in embracing, fostering and advancing the purer religion have determined, organized and arranged all their good-will, accomplishments, efforts and successes to this pious intention of theirs.[23]

The letter shows Maurice's understandable worries about the ecclesiastical developments in his realm and his need for good advice. Maurice and Rudolph Snellius seem to have shared a rather detailed knowledge of the matter at hand. We do not know Snellius's private opinion about this problem, nor about the faith and its significance relative to education and scholarship in general, but the appreciation that he received from Maurice indicates that the latter had no reason to doubt his Calvinist orthodoxy, and the same must have been true of Willebrord. Moreover, the position of the Snellii in Leiden would have been

tis nostras ceremonias pro hypocriticis exclamarent, tum tamen alii non pauciores easdem nobiscum colerent.' [Kalckhoff, s a, fol. 45^r].

[22] '[...] quandiu istae ceremoniae reformatorum, non nationali concilio; et ex conventu omnium Ecclesiarum puriorem doctrinam amplectentium manduntur, et observandae praecipiantur, nullam Ecclesiam, ad eas astrictam esse existimo, neque ullas alias Ecclesias propter eam causam separandi sese habere sufficientem iudico.' [Kalckhoff, s a, fol. 46^v].

[23] 'Ad postremum brevior ero, nullum enim mihi ibi est dubium, quin et nobiscum doleas, et saepius super eiusmodo hominum elatis conatibus ingemiscas; habeo te certum, vivum et syncerum testem, quem invocare possum et testari, pios maiorum meorum in religione puriore amplectenda, fovenda et promovenda affectus et effectus, conatus, eventus omnes ad piam ipsorum intentionem terminantes, correspondentes et quadrantes.' [Kalckhoff, s a, fol. 47^r].

much less comfortable if they had not shown their adherence to the Reformed Church.

Rudolph, however, wanted to gain more from the visit to Kassel than honours and religious conversations, also desiring to have the *Optica* of Fridericus Risnerus printed. Risnerus had been Ramus's German assistant and Ramus himself had also contributed to the book. It was an uncompleted work and had remained in manuscript since Risnerus's death in 1580 or 1581. Risnerus had been in contact with Maurice's father William, for whom he had solved an optical problem and to whom he had sent a concave mirror. After his death, Maurice had bought his library, which probably made him the owner of the manuscript.[24]

In 1597 Rudolph Snellius had already tried to persuade his friend Joannes Magnus, counsellor to Maurice, to assist him in his endeavours:

> You had promised that the *Optica* of Ramus and Risnerus would appear within sight and reach of the public, and you had inflamed with great joy the followers of the Ramist school, and me in the first place, in an amazing way so that all waiting seemed too long to me and that I have since extolled your zeal in advancing optics to everyone [...]. But poor me, I was mistaken, and I have been deprived of the hope that had enlightened my life. For however often I examine the catalogues of the book fairs with the greatest accuracy, I find no optical treatise whatsoever—not only nothing by Ramus or Risnerus, but not even by a mean or common author on optics.
>
> But come on, my Magnus, stimulate the very illustrious hero and Prince of the Hessians, Maurice, to publish the *Optica*, to make it available to all and in this way obtain the same praise as that given to his father. For in this way your Hessen will be rendered famous by a double triumph: on the one hand an astrological prize from the father, on the other hand an optical one from the son. And in my opinion, this will not be difficult for you, because the Prince himself is inclined and disposed to serve the scholarly cause. [...] Address him and arrange it, and I shall have no reason to envy the Arabians their treasures.[25]

[24] For Risnerus (Reisner), see [Verdonk, 1966, pp. 66–73]. For Ramus's involvement, see [Waddington, 1855, pp. 436–437, 475–476]. For the connection with the Princes of Hessen, see [von Rommel, 1835, pp. 776–777].

[25] 'Opticam Rami et Resneri promiseras in hominum ὄψιν et manus exituram, et magna laetitia Rameae αἱρέσεως asseclas, me in primis mirum in modum ita inflammaras, ut omnis mora mihi longa esset, tuam praeterea in Opticis promovendis diligentiam apud omnes praedicaverim, et, tanquam Marathonium quendam campum nactus, declamitaverim. Sed, me miserum, falsus sum, et spe, qua fovebar, excidi. Nam quamvis Catalogos Librorum Nundinales quam accuratissime excutio nihil Opticum non modo non Rami aut Resneri, sed ne plebei aut gregarii Optici invenio. [...]

Sed age mi Magne Illustriss[imem] Heroem et Cattorum principem Mauritium exsuscitato, Snellius's plea did not have the desired effect and he probably realized that he should seize the opportunity to draw attention to the *Optica* when he would meet Maurice of Hessen in person. On 22 July 1603, when he was about to leave Leiden for Kassel, he sent a letter to his friend Guilielmus Hatzfeldius. Snellius wrote that he had to do a job of great importance in Kassel, in which Magnus was also involved, and that every delay would be harmful. This probably referred to the same project of having the *Optica* published.[26] It is not known what was said between Maurice and Rudolph Snellius, but Risnerus's *Optica* did indeed appear in print in Kassel in 1606, and on its title-page Maurice of Hessen's role in its publication was mentioned, which must mean that he contributed largely in its costs.[27]

Rudolph was indeed very interested in optics. He lectured on the subject several times, and described it in the same letter to Hatzfeld as the best topic one could teach, playing with the almost equal words 'optimum' and 'optica'.[28] In 1608 he sent a copy of Risnerus's *Optica* to his relative Rosendalius, praising its worth:

> Indeed, the famous statue of Minerva by Phidias misses the finishing touch, but even as it is, it is of such quality that it leaves all other works by all optical writers very far behind; being a work of undoubtedly Daedalian quality, both in the arrangement of the theorems and in the clearness of the method.[29]

After the meeting in 1603, Maurice of Hessen and Rudolph Snellius seem to have been in touch only occasionally. In 1609 Rudolph told Magnus he had not been writing to Maurice or anyone else for a long time, yet now he sent the Landgrave an alarm clock to show his gratitude. This fitted very well in the collection of instruments and curiosities that Maurice owned. He had inherited a large number of clocks of excellent quality from his father, which had been

Optica edat, publici iuris eam faciat, et paternae laudis aemulus sit. Hoc enim pacto Hassia vestra gemino triumpho nobilitabitur Astrologico quidem a patre, a filio autem Optico. Nec difficile hoc tibi fore arbitror, ipso principe ad bene de re literaria merendum prono et propenso. [...] Aggredere et efficies, et ego Arabibus suas gazas non invidebo.' [Snellius, 1597, fol. 1^r].

[26] [Snellius, 1603].

[27] [Risnerus, 1606, titlepage]: 'Nunc demum auspiciis Illustriss[imi] et Potentiss[imi] Principis ac Domini, D[omi]n[i] Mauritii Hassiae Landgravii, etc[etera] e situ et tenebris in usum et lucem publicam producti.'

[28] 'Tempus enim publicae praelectionis iam nunc urget. Quale autem publicae lectionis argumentum sit exquiris? Optimum est. Elementa enim Optica, studiosis dictito.' [Snellius, 1603]. He had also taught Euclid's optics in 1599, [Molhuysen, 1913, p. 384*], and Ramus's (and Risnerus's) optics in 1609, [de Waard, 1939, p. 12].

[29] '[...] ultimam enim manum illa Phidiae Minerva desiderat, sed tamen quale quale est, eiusmodi est, ut omnes omnium Opticorum Labores post se longissime relinquat, opus, cum theorematum conformatione, tum Methodi claritate plane Daedalum [...]', [Snellius, 1608]. Phidias was a famous sculptor; by 'Phidiae Minerva', Snellius means a work of art of outstanding value. Daedalus was the mythical builder of the Cretan labyrinth.

made by the talented instrument maker and mathematician Jost Bürgi. The collection also contained a merely playful mechanical maybug, able to move its legs, wings and antennas. It had been the property of Maurice's first wife and it expressed the interest, common in the period, in a mechanical view of nature.[30]

In his letter to Magnus, however, Snellius did not refer to this collection, but motivated his choice for this present with a Ramist argument:

> Since I see that the arts that Petrus Ramus recommends to a professor, in no lesser place than his Mathematical Testament, were treated so carelessly, yes even neglected, I have resolved to draw them out of the darkness as well as I could, refining them and making them public. The first of those is the art of machines, which is deduced from the nature of the circle.[31]

This argument must have appealed to Maurice as an adherent of Ramism.

We have seen that Maurice of Hessen showed an interest in Rudolph Snellius over the years, which the latter returned on several occasions. None of Snellius's books, however, seems to have been written on Maurice's request. In that respect, the patronage-relationship that the Landgrave and Willebrord were to entertain was different: two of Willebrord's astronomical works were dedicated to Maurice and at least one was written on Maurice's request, which makes Maurice the most important motivator of Willebrord's astronomical work.

This raises the question whether Rudolph could not have written such works for Maurice. Certainly, Rudolph Snellius was not ignorant of astronomy. As a professor of the mathematical sciences, he taught astronomy and cosmography, yet his choice of authors shows that he was mainly interested in the astrological and natural philosophical side of the discipline, and much less in the observational, mathematical side of it. In his courses he treated among others Manilius, Dionysius Alexandrinus, Aratus and Maestlin. The latter was one of the few modern authors discussed in Snellius's lessons.[32]

[30]See for this collection of clocks, quadrants, armillary spheres, astrolabes, globes, [Mackensen (mit Beiträgen von Hans von Bertele und John H Leopold), 1979, pp. 118–151], [von Mackensen, 1997a, pp. 385–389] and [von Mackensen, 1997b, pp. 391–402]. The maybug was made before 1603 ([von Mackensen, 1997a, p. 388]; earlier, Mackensen had dated it around 1620 in [Mackensen (mit Beiträgen von Hans von Bertele und John H Leopold), 1979, p. 148]).

[31]'Cum enim videam artes a Petro Ramo etiam Testamento Mathematico professori commendatas, tam supine tractari imo negligi, animo obfirmavi meo, illas, quantum in me, e tenebris eruere, expolire et in publicum ponere. Ex quibus Mechanica prima est, e circuli Natura deducta.' [Snellius, 1609]. I thank prof. dr. Chris Heesakkers for his assistance with the transcription of this letter.

[32][Molhuysen, 1913, pp. 158*, 192*], [Coddaeus, 1613, p. 15], [Witkam, 1973, p. 88]. Snellius's copy of Aratus's *Phaenomena*, filled with extensive marginal notes, is now in Leiden University Library, [Aratus, s a].

Rudolph Snellius's interest in the astrological side of astronomy is shown by his published lecture notes on the 'sphere' (which means astrology) of Cornelius Valerius. Snellius explained that astrology was not even a mathematical science, because 'it does not teach the science of quantity in lines, surfaces and volumes, but only the nature and the influence of the heavenly bodies'. It was a part of physics instead, he wrote.[33] When father and son Snellius shared their teaching duties, it was Willebrord who taught astronomy.[34] Rudolph seems never to have made any observations either. Thus it would have been difficult for Snellius senior to write books for Maurice, because he was not an expert in astronomy, nor, in fact, in any of the fields of Maurice's particular interest.

The question can now be answered what advantage Rudolph and Maurice gained from their relationship. Rudolph must have considered himself a lucky person to have acquired the support of a person as high-ranking and influential as Maurice of Hessen, who was a partisan of the Ramist cause, like himself. The mere awareness of their relationship may have induced those Dutch scholars otherwise not too keen on Ramism to think somewhat more favourably of it and of Rudolph Snellius. Maurice's funding of the *Optica* of Risnerus was important to Snellius, and Snellius himself may have gained financially from the relationship as well. The Landgrave enjoyed the company of scholars, and he must have valued the discussion of learned and religious issues with Rudolph Snellius, the 'learned and true philosopher'. In this way, he could involve himself in the world of scholarship that he valued without having to spend too much of his rare free moments to extensive studies of his own.

That Rudolph had a German patron was not a conscious choice. When he was young, no Dutch Reformed university existed yet. For this reason he went abroad for his studies. His long stay in Marburg had provided a connection to the most learned Hessian prince, and he managed to continue this relationship when he was back in the Netherlands. In this way, he prepared the contact between his son and Maurice. Of course Rudolph knew about William's observatory and he must have realized that Maurice might one day continue his father's work there. Though Rudolph Snellius himself neither had the inclination, nor the knowledge to be of use in this mathematical and observational side of astronomy, his son was becoming an able mathematician and astronomer. Rudolph may even have stimulated him to develop this side because of the Kasselian job opportunities that he saw.

[33]'[...] neque Astrologia docet quantitatis doctrinam in linea, superficie, et corpore, sed tantum naturam et affectionem corporum coelestium. [...] Est autem Astrologia Physicae pars.' [Snellius, 1596g, p. 4].

[34][Iachaeus, 1626, p. 14].

4.3 Willebrord Snellius and Maurice of Hessen: astronomical services

4.3.1 William the Wise

Ironically, in the case of the Landgraves of Hessen, it was the father who was most interested in astronomy, as opposed to the case of the Snellii. William of Hessen (1532–1592), who was surnamed 'the Wise', had been an observer of the skies himself. He had founded the first permanent astronomical observatory in Europe in his palace in Kassel in 1560, where he had surrounded himself by able collaborators such as Bürgi and the court astronomer Christoph Rothmann. The first-hand testimony of Andreas Christianus expresses to what an exceptional extent the Landgrave was attracted by astronomy:

> Mathematical devices pleased our Prince in the highest degree, and he felt more affection for students of mathematics than for other students. In his private room, or wherever else he went, even abroad, he always had celestial and terrestrial globes and other mathematical instruments, and he took them along. As a result, I have hardly ever been with him, even while handling and settling often very serious business, without catching him observing and examining the motions of the stars.[35]

An anecdote told by Tycho Brahe and quoted by Snellius reinforces the impression that William's interest was really profound. William had already been carrying out many observations of the newly discovered star of 1572, and he did this so earnestly that once, when some servants came up to his observatory to tell him that part of the building was on fire, just as he was measuring the star's altitude, he was not at all unnerved by the news, but judging that there was no big danger in the fire, he continued his observations.[36]

William particularly focused his attention on the measurement and calculation of the positions of the fixed stars and the production of a catalogue on the

[35]'Noster Princeps machinis mathematicis ut plurimum delectabatur, et mathematum studiosos prae aliis amabat, et in suo conclavi, seu quocunque etiam peregre abiret, semper globos coelestes et terrestres, et alia instrumenta mathematica habebat, et secum circumferebat, ita ut nullo unquam fere tempore cum ipso fuerim, quo non tractando et expediundo etiam varia et saepe gravissima negotia, simul motus siderum contemplantem et considerantem deprehenderim.' Quoted from [Stegmann, 1756, p. 5].

Stegmann writes that Christianus was 'Nassauischer und Hanauischer Rath' and quotes from page 40 of his *Oratio de vita et morte Guilielmi Hassiae Landgravii*, [Stegmann, 1756, p. 3]. Stegmann's account of William's achievements is highly laudatory and thus cannot count as an adequate historical account for modern readers, but it contains many quotations of relevant sources.

[36][Snellius, 1619, p. 68]. For the 'stella nova', which would nowadays be recognized as a supernova, see [North, 1994, pp. 299–300].

basis of these data, which was to contain 1032 stars. This task was so large that it was not finished when William died, and it was continued by Bürgi until 1596. In that year, the observations were ended and when Rothmann offered his services to Maurice in 1597, he received no positive answer. William's observations were the most accurate of his time.[37]

Tycho Brahe visited the observatory in 1575 to work together with William. After he had departed, the Landgrave sent a letter of recommendation to the Danish King Frederick II, asking him to support Brahe financially. The monarch indeed decided to do so, giving Brahe the means to build his own observatory. Brahe corresponded with William and Rothmann about several astronomical topics, such as instruments, the Copernican system and comets, from 1585 until 1591.[38]

The exceptional endeavours of William on behalf of astronomy did not go by unnoticed by his contemporaries. Ramus, for instance, in his *Scholae Mathematicae* eulogized him:

> It seems that William Landgrave of Hessen has transferred Alexandria to Kassel: he has instructed the makers of instruments for the observation of the stars, and he finds pleasure in the daily observations by means of the constructed instruments, all in such a way that Ptolemy seems to have come from Egypt to Germany with his armillary spheres and measuring-rods.[39]

By telling a French audience about the good position of the mathematical sciences in Germany, Ramus wanted to stimulate them to follow the German example and to develop the mathematical sciences (including astronomy) in France as well. The Snellii implicitly subscribed to Ramus's programme by associating with Wilhelm and Maurice of Hessen. In his introduction to the catalogue of fixed stars, Rothmann in his turn praised Ramus, yet he contradicted Ramus's conviction that astronomy should be done without hypotheses.[40]

Maurice of Hessen's interest in astronomy was much smaller than his father's.[41] Ludolf von Mackensen suggests that Maurice was too egocentric to

[37] [Mackensen (mit Beiträgen von Hans von Bertele und John H Leopold), 1979, pp. 9, 13–21, 38], [von Mackensen, 1997b, p. 392], [von Mackensen, 1997a, p. 386], [Hamel,]. For Bürgi, see [Mackensen (mit Beiträgen von Hans von Bertele und John H Leopold), 1979], [Zinner, 1967, pp. 268–276]; for Rothmann, see [Multhauf, 1975].

[38] [Mackensen (mit Beiträgen von Hans von Bertele und John H Leopold), 1979, pp. 16, 20], [Multhauf, 1975, p. 561].

[39] 'Guilielmus Landgravius Hessiae videtur Cassellas Alexandriam transtulisse: Sic Cassellis artifices organorum observandis syderibus necessariorum instruxit, sic quotidianis per instructa organa observationibus oblectatur, ut Ptolemaeus ex Aegypto in Germaniam cum armillis et regulis venisse videatur.' [Ramus, 1569, p. 67].

[40] [Granada et al., 2003, pp. 171–177].

[41] [Borggrefe et al., 1997, p. 11]: 'Moritz haben Astronomie und Botanik, die wissenschaftlichen Interessensfelder seines Vaters, weniger berührt, doch hat er das innovative

operate in his father's shadows and therefore preferred to acquire his own fame instead of continuing his father's work, yet maintained the observatory 'more out of reverence and respect for his father's fame as an astronomer than out of his own scholarly interest'.[42] Maurice, however, did not completely abandon his father's project. In contrast to von Mackensen, Gerhard Menk considers Maurice's correspondence with Tycho Brahe and his extensive observation of a star as tokens of Maurice's genuine interest in astronomy.[43] It is in this field that Maurice's and Willebrord Snellius's paths crossed.

Willebrord Snellius had already shown his keen interest in astronomy both inside and outside his teaching. The first course that he gave was devoted to Ptolemy's *Almagest* and he visited Tycho Brahe in Prague, assisting him with his observations. Back in Leiden he again taught astronomy, observed the comet of 1607 and anonymously published a booklet on sunspots (1612).[44]

William of Hessen would have been an excellent patron for Willebrord Snellius with his astronomical inclinations, being a very high placed person and an astronomer himself, a highly exceptional combination. Regrettably, William had died when Willebrord was only a child and once the latter had grown up, the Kassel observatory was not even in use anymore. Willebrord could only hope to take advantage of his father's association with William's son Maurice, even though the younger Landgrave showed no outspoken interest in astronomy.

4.3.2 First contacts: services for a prince

In 1618 the first astronomical work bearing Snellius's name appeared. The book had the name *Coeli et siderum in eo errantium Observationes Hassiacae, Illustrissimi Principis Wilhelmi Hassiae Lantgravii auspiciis quondam institutae* ('Hessian Observations of the heaven and the heavenly bodies that wander upon it, which have once been begun under the command of the most distinguished Prince William Landgrave of Hessen'). Its first part contained Snellius's edition of some of the observations carried out in Kassel by Bürgi and Rothmann under William's direction. Because Maurice of Hessen was the heir of their notes, Snellius certainly could not have published them without Maurice's consent and it was only natural that Snellius dedicated the book to the Landgrave.

In his nine-page long dedicatory letter, Snellius did not tell the reader who took the initiative for the publication. This silence shows that Maurice was not overactive in this business, otherwise Snellius would certainly not have missed

Moment des Kasseler Hofes auf anderen Gebieten stärker ausgeprägt.'

[42] [von Mackensen, 1997a, p. 387];
[Mackensen (mit Beiträgen von Hans von Bertele und John H Leopold), 1979, p. 38]: '[...] mehr aus Pietät und Achtung vor dem astronomischen Ruhm des Vaters als aus eigenem Forscherinteresse'.

[43] [Menk, 2000b, p. 52].

[44] See sections 2.2, 2.7 and 2.8.3; [Snellius, 1619, pp. 19–20].

an opportunity to bestow praise on him.[45] Maurice was merely expected to be a passive supporter of the enterprise:

> Illustrious Prince, I beg you to allow that these small brooks will flow out from your sources to the general use, so that everyone will extol and make publicly known the inborn kindness and generosity of Your Highness.[46]

In one of the last sentences of the letter, Snellius alluded to his own reward from Maurice:

> the sun would sooner lose its warmth, than Your Highness his mildness, kindness or generosity: our family has experienced this privately and we frequently call it to mind with gratitude.[47]

Snellius meant the patronage received by him and his father, which probably had a considerable financial component, about which he could not be too specific in this place. Although the scholarly interests of father and son were somewhat different, Willebrord Snellius never objected to being associated with his father. He even underlined their family ties by frequently calling himself R.F., *Rudolphi Filius*, 'son of Rudolph', an addition not necessary to distinguish Willebrord from others in their small family.

Most of the letter, however, was dedicated to topics less intimately connected with the Prince: the laudation of astronomy and a survey of its history. Plato was quoted, who had written that 'the true astronomer must necessarily be the wisest of all men.'[48] Astronomy was supposed to show the 'foundations of truth'. Snellius also gave the argument common in the Book of Nature-tradition that God's wisdom and plans could not only be learnt from study of the Bible, but also from Nature, His creation:

> For that reason, God has raised man from the ground, and created him high and erect, in order to enable him to grasp knowledge of Him when he contemplated the sky.[49]

[45] Von Mackensen's statement that 'Snellius zeigte höchstes Interesse an dem Sternenkatalog Wilhelms IV. und so gestattete Moritz ihm, diesen zu veröffentlichen.' can, however, not be proven; it may reduce the Landgrave's role too much, [von Mackensen, 1997b, p. 393].

[46] 'Patere quaeso Illustrissime Princeps hos rivulos e tuis fontibus ad publicam utilitatem emanare, [...] ut liberalitatem et munificentiam Tuae Celsitudini ingenitam publice etiam omnes laudent et depraedicent.' [Snellius, 1618, fol. $(\cdots)(\cdots)2^r$].

[47] 'Etiamsi citius Soli calor, quam Celsitudini Tuae clementia, liberalitas, aut munificentia sit defutura: quod et privatim nostra familia sensit, et grato animo saepius secum recolit.' [Snellius, 1618, fol. $(\cdots)(\cdots)2^r$].

[48] '[...] verum astronomum necessario omnium hominum esse sapientissimum.' [Snellius, 1618, fol. $(\cdots)2^r$].

[49] 'Ideo Deus homines humo excitatos celsos et erectos constituit, ut eius cognitionem coelum intuentes capere possent.' [Snellius, 1618, fol. $(\cdots)2^v$].

Snellius did not elaborate on the connection between the observations edited in this book and the higher vocation of astronomy, nor did this theme play any role in the core of the book. The defence of the usefulness of a certain field was generally a very becoming topic for a dedicatory letter.[50]

In the historical part of the letter, pre-classical antiquity was first discussed, mainly the Babylonian and Egyptian contributions to astronomy. Snellius explained that it was important to go back far into the past because the secrets of nature were so difficult to fathom that observations and investigations made over long stretches of time were necessary to make any progress. In this survey, Snellius remarked that 'this interest and the majesty of such an honourable subject took possession even of the minds of Kings and Princes'.[51] The relevance of this sentence becomes evident in the last part of the historical survey, when we have almost arrived in Snellius's own era. William of Hessen received a generous amount of praise there for his astronomical activities, his own observations and the employment of others in the observation of the sun, the planets and the stars, and their work on the catalogue of fixed stars. A long list of Greek, and a very short one of Roman and post-classical achievements were given in summary. The shortness of the second list let William of Hessen's deeds stand out even more. The preliminary matter also contained a poem by Petrus Cunaeus[52] honouring William of Hessen and Snellius.

The book did not contain the observations of the fixed stars that William had been so keen on, but the observations of planets and the sun done by William and his collaborators between 1561 and 1597. Snellius supplemented these Hessian observations with some of Tycho Brahe (from Bohemia, 1599–1601) and a technical section by himself. In the second part of the book, some observations by Johannes Regiomontanus and Bernhard Walterus were edited (from Nuremberg, 1462–1504).[53] Snellius added a text by Regiomontanus on comets, some fragments by Johannes Schonerus about astronomical instruments, an excerpt of a text by Kepler on an optical problem and a small text by himself in which he corrected an error made by Schonerus.

Snellius had to collect much material for this publication. Part of it had never been published before, so he had to lay his hands on notebooks from these astronomers. Those by William of Hessen and Bürgi were in the observatory in Kassel, and he made an abstract from them there, as Snellius tells us in the

[50] For Snellius's defence of other parts of the mathematical sciences, see section 8.6.1.

[51] 'Sed ea cura, et tam illustris rei splendor etiam Regum ac Principum animos invasit.' [Snellius, 1618, fol. $(\cdots)4^r$].

[52] On this Leiden professor of Latin and law, who was in contact with Snellius on several occasions, see section 2.9.4.

[53] In the book itself, their names are written as 'de Monteregio' and 'Waltherus', see e.g. [Snellius, 1618, fol. 1^r]. The part of the book devoted to their work has its own pagination, which can be distinguished from that of the first part by the fact that in the first part normal page numbers are used, and in the second part folio numbers.

book itself:

> At the entrance of this book, I have presented the observations of the sun, conducted for twenty-one years without interruption. I once made a transcript of them in Kassel in Hessen, when this wealth of material was made available to me, as if it were only a pile of leftovers from the very dense work of this excellent Prince.[54]

This statement is important, because it is the only indication that Snellius had actually visited the court at Kassel, probably in the summer of 1603, and had had access to the collections of the observatory. It is likely that Snellius had made an extract of Tycho's notes during his visit to him in 1600–01.[55] The Nurembergian observations probably all originated from printed sources. Regiomontanus's works were published in 1544 under the title of *Scripta clarissimi mathematici M. Ioannis Regiomontani*; this edition was probably Snellius's main source.[56]

Snellius made a very succinct edition of these various observations, in general just adding short notes on locations and instruments. Halfway through the book, he addressed the reader more extensively in a lengthy piece about the technical difficulties connected to the determinations of parallax and refraction. He based his considerations on a critical assessment of many observations done by different astronomers from Eratosthenes to Copernicus and Brahe (but not by Snellius himself), and made calculations involving spherical trigonometry.[57]

Such a publication of observations was very appropriate within a Ramist programme: Ramus had advocated an 'astronomy without hypotheses', founded on empirical observations instead of on theoretical considerations.[58] The presentation of material from different astronomers enabled the reader to make his own calculations over longer stretches of time and to compare the utility of the observations. The reader was helped by Snellius's technical exposition, the short remarks throughout and the sections by Schonerus on instruments.

After the death of his father in 1613, Willebrord Snellius obtained his own patronage relationship with Maurice of Hessen. Important and active in scholarship as Maurice still was, political developments had made him less influential than before. After the murder of the French King Henry IV in 1610, the power of the German Catholics increased. The relations between the Dutch Republic and Maurice were strained since the negotiations for the wedding of Maurice's

[54]'In vestibulo huius libri exhibuimus Solares observationes annorum unum et viginti continuorum, quas Cassellis Hessorum, cum nobis esset facta istarum copia, tanquam spicilegium duntaxat ex spississimo opere optimi illius Principis [...] olim descripsimus.' [Snellius, 1618, p. 85].

[55]Cp. Caspar in [Kepler (M. Caspar ed.), 1959, p. 548]; cp. section 2.7.

[56]For some parallels between the 1544 edition and Snellius's, see [Rosen, 1975, p. 352].

[57][Snellius, 1618, pp. 85–109].

[58]See section 2.5.2.

daughter and Frederick Henry, Maurice of Nassau's younger half-brother, had failed. In 1624, Maurice of Hessen's second wife Juliane even publicly accused her husband of making diplomatic, military and judicial errors and of assigning to her the task to reign in his absence yet without trusting her or granting her real power. She also tried to claim a large inheritance for her children. Meanwhile, the German emperor tried to subdue Hessen-Kassel.[59] As a politician, Maurice showed much less talent than as a scholar and a Maecenas.[60] Part of the Landgrave's troubles were financial: his sumptuous court and patronage activities cost too much money,[61] and in a poem written in 1617 he described himself as 'a Lord without either gold or money.'[62] This means that by the time Snellius solicited his attention, Maurice did not have the means to do much for him.

Although these troubles claimed a large part of Maurice's energy and attention, he was involved in Snellius's astronomical pursuits in 1618 and 1619. In these years, he travelled to the Netherlands to solve some of the political tensions. On 27 and 28 August and on 9 September 1619 he visited Leiden, where he probably met Snellius.[63] The word *coram*, 'in your presence', in the letter that Snellius wrote to the Landgrave of Hessen on 1 September 1618 indicates that they had also spoken to each other in that year. On that occasion the plan for the edition of the *Observationes Hassiacae* may have been born. This letter of 1 September was not intended for publication. It is probably the letter of presentation that accompanied the two copies of the *Observationes Hassiacae* that Snellius sent to Maurice through Caspar Meusch, a messenger of Maurice.[64]

In Snellius's long, panegyrical letter, he exerted himself to honour Maurice and his house. The first point of laudation was their role in the dissemination of the Protestant faith, the 'purer religion':

> Because we all acknowledge gratefully that the grandfather of Your Highness, Philip the Magnanimous, has installed and promoted the purer religion.[65]

[59] [Borggrefe, 1997b, p. 19], [Löwenstein, 1997, pp. 50, 55–56], [Gräf, 1997, p. 112], [Menk, 2000b, p. 14].

[60] [Menk, 2000b, pp. 60–78], especially the last two pages.

[61] [Menk, 2000b, p. 77], [Löwenstein, 2000].

[62] 'Ich bin ein Her ohn Goldt und Geldt [...]', [Löwenstein, 2000, pp. 93–94].

[63] [Gräf, 1997, p. 112].

[64] The autograph of Snellius's letter is now in Kassel, [Snellius, 1618]. A summary of it is given by Wolf, [Wolf, 1888, pp. 52–55], and some (large) quotations are given by Rommel, [von Rommel, 1837, p. 529], and Stegmann ([Stegmann, 1756, pp. 6–7] and [Stegmann, 1757, pp. 14–15]; Stegmann's programmes are now very rare). In the rest of the literature about Snellius, this rich manuscript is not discussed.

Meusch's letter is kept in Marburg, [Meusch, 1618]. It was written in Kassel on 2 October 1618. For Meusch's job as a messenger, compare another letter written by him to Maurice (in English!), Hessisches Staatsarchiv (Marburg) 4a, 39, 125.

[65] 'Nam et religionis purioris instaurationem simul ac propagationem a T[uae] C[elsitudinis]

The whole family of Maurice was then applauded for 'calling back this old and truly golden age in religion'[66] and in a deliberate confusion of pagan mythology and Christian religion Snellius equated the Princes of Hessen to gods because of their role in the defence and preservation of God's laws.

After this crucial matter had been dealt with sufficiently, Snellius could proceed to discuss topics closer to his own interests. William of Hessen was extolled to the stars for his role in the development of astronomy: he was both compared to the mythological figures Atlas, Hercules, Perseus and Cepheus and to the historical persons Ptolemy, Hipparchus and King Alfonso, which last three had also been mentioned in the dedicatory letter to the *Observationes Hassiacae*. And William would beat the trio, 'because he was not only their equal in learning, but also far superior in carefulness and zeal'.[67]

William's observations of the fixed stars, 'the most difficult topic of all'[68] according to Snellius, and his stimulation of Tycho's astronomical career made Snellius conclude:

> And therefore it must be [acknowledged as] the everlasting benefit of this Great Atlas, William Landgrave of Hessen, that astronomy carries round its head more magnificently in this century.[69]

It was then Maurice's own turn to be praised. Strikingly, Snellius did not mention his astronomical interests, even in a letter in which references to them would have been highly appropriate. In this way he implicitly suggested again that these interests were only small. After some general acclaim of the huge qualities of the Landgrave, phrased with the rhetorical adornment suitable to the genre of the dedicatory letter, Snellius arrived at a more concrete point: Maurice was an excellent patron of both living and dead authors, as he had demonstrated by the efforts that he had put into the publication of the *Optica* of Ramus and Risnerus.

Snellius thought that this last work was a very valuable acquisition for the field of optics, mainly because its subject material was ordered according to the Ramist method. He suggested that the publication of more works by Ramus and Risnerus would be beneficiary, by which wish he continued his father's quest for the publication of the *Optica*:

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avo Philippo magnanimo factam, omnes gratis animis agnoscimus: [...]', [Snellius, 1618, fol. 1^r].

[66]'[...] pristinum illud et vere aureum in relligione seculum revocando [...]', [Snellius, 1618, fol. 1^r].

[67]'[...] cum scientia illis non solum par, sed diligentia et sedulitate etiam longe fuerit superior.' [Snellius, 1618, fol. 1^r].

[68]'[...] rem unam omnium difficillimam [...]', [Snellius, 1618, fol. 1^r].

[69]'Quamobrem, quod sideralis scientia hoc seculo caput illustrius circumferat Magni illius Atlantis Wilhelmi Hassiae Lantgravii sempiternum beneficium esto.' [Snellius, 1618, fol. 1^r–1^v].

> And although these are merely the outlines of a future work, they have nevertheless already given so much light to scholars and students that the study of optics, which has almost completely been neglected due to the obscurity of the writers, now flourishes more than ever before and is advanced eagerly. Then the mathematical sciences will also own this to Your Highness.
>
> Please do not suffer that the other works of the same authors are held in confinement in their dungeons and behind bars for any longer; no, let them rather be brought out in the open air. Truly, this right method places the outlines of the whole structure so clearly before the eyes that unfinished treatises by men of such greatness are far to be preferred to dense volumes of others—because in the former everything intricate is unfolded in an easy way, as though with Ariadne's thread, and guidance is given to wandering footsteps.[70]

Snellius then suggested that more research should be done in the field of optics, because much was still unknown, and part of the transmitted knowledge was incorrect. He wrote that it is a task assigned by God to uncover nature's secrets.[71] His own intensive reading of the Ramist *Optica* and his subsequent discovery of the law of refraction show that his eagerness on this point was real.[72]

All these commendations were the introduction to a request that Snellius wanted to make, which is in fact the core of the letter. He phrased his question in a very indirect way, indicating only after many preparatory words that he hoped that the interrupted series of observations of William of Hessen would be continued again:

> This is what Urania herself resolutely entreats from Your Highness, this is what the ghost of the Very Heroic William Landgrave of Hessen demands as a very agreeable sacrifice in his honour, and this gift from Your Highness will be celebrated by posterity with eternal praise:

[70]'Et quamvis ea tantum sint lineamenta futuri operis, tantam tamen lucem doctis et studiosis attulerunt, ut opticum studium, quod iam propemodum obscuritate scriptorum neglectum iacebat, nunc magis quam ante hac unquam vigeat, et certatim excolatur. Hoc igitur Tuae Celsitudini mathemata quoque debebunt. Ne patere quaeso reliqua eorundem autorum, diutius carceribus et seris coërceri, quin potius in apertam proferantur lucem. Enimvero ista εὐμεϑοδία [sic] tam clare totius corporis lineamenta ante oculos ponit, ut opera a tantis viris inchoata aliorum spissis voluminibus longe sint anteferenda. Istis enim, tanquam Ariadneo filo, involuta omnia facile evolvuntur, et errabunda vestigia diriguntur.' [Snellius, 1618, fol. 1^v].

[71]'Multa sunt in opticis hactenus ignorata, a multis etiam veterum et recentiorum vel non tentata vel perperam tradita, amplius tertia huius parte altissimis tenebris sepulta iacebant, in qua amplissimus opticae utilitatis campus versatur. Certe Dei munus est in naturae arcana penitius introspicere; eam salivam curiosis movit editio Resneriana, quibus inventis vix quidquam in opticis amplius latere possit.' [Snellius, 1618, fol. 1^v].

[72]See section 2.9.6.

> that the observations of the sun and the planets will be undertaken carefully from now on.[73]

A few sentences later, Snellius finally revealed his plan: he implied that he would be a good candidate to continue the work of William and his collaborators:

> If something could be done in these fields, I would willingly and with great pleasure offer all my services to Your Highness.[74]

This could be interpreted as meaning that Snellius would stay in Leiden and fulfil any tasks for Maurice there, but it could also mean that he was applying for the position of court astronomer. The last interpretation is not unlikely, because if Maurice seriously wanted to emulate his father's achievements in astronomy, he could not do that without his own court astronomer. If this interpretation is correct, it means that Snellius ultimately was not satisfied with his position as a professor in Leiden and wanted to improve it by becoming Maurice's employee. If we continue on this track, we could explain this ambition by assuming that Snellius wanted to have a higher salary, without teaching duties, or that he felt that he was not held in enough respect in the Republic.[75]

If Snellius's goal was indeed to obtain a job from Maurice, it also becomes easier to understand why he presented his edition of the Kassel observations as just a *spicilegium*, a 'pile of leftovers'. It was a topos of modesty, but it also implied that the work had not been finished, that much more work had to be done, that old observations had to be edited and new ones carried out—and Snellius was the right person to do all that.

Snellius ended his letter by mentioning Rudolph's gratitude for Maurice's lavish reception of him, and informed him that Rudolph had actually planned to dedicate a book to Maurice himself as a memorial to the Landgrave's well-doing, but that old age and illness had prevented that. Willebrord now acted in his place.[76] The lack of any reference to a specific book that Rudolph would

[73]'Hoc illud est quod ipsa Urania obnixe a T[ua] C[elsitudine] contendit, hoc Magni Heroïs Wilhelmi Hassiae Lantgravii manes, tanquam inferias gratissimas, deposcunt: Hoc posteritas a T[ua] C[elsitudine] acceptum aeternis laudibus celebrabit, si solis et Planetarum observationes deinceps porro accurate instituantur.' [Snellius, 1618, fol. 1^v].

[74]'Ego si in his oris res geri possit, sponte omnem meam operam T[uae] C[elsitudini] libentissime offerrem.' [Snellius, 1618, fol. 2^r].

[75]It is even conceivable that Snellius wanted to flee Leiden because of the religious and political dissensions between Remonstrants and Contra-Remonstrants, which were reaching their summit just then—in August 1618, Oldenbarnevelt, Snellius's acquaintance Grotius and some others had been imprisoned, [Jansen, 1988, p. 103]. Lack of sources prevents us to know Snellius's thoughts about and reactions to the troubles in the Dutch Republic. It is, however, safe to conclude that the undisturbed continuation of his professorial activities shows that he was perceived as orthodox, or at least as having not very strong Remonstrant sympathies.

[76]'Saepe publice aliquo monumento idem testari instituit: sed aetate in senium inclinata, et nonunquam morbis gravi impeditus, quod animo destinaverat re ipsa praestare haud potuit. neque ideo tamen eam curam ex animo unquam deposuit suo, ut etiam moribundus

have been working on makes it impossible to determine whether Rudolph was really planning to do so; in any case, Willebrord wanted to gratify Maurice by underlining his father's recognition and thought it prudent to stress the relationship between his father and himself, trying in this way to succeed him as a client.

In these two letters by Snellius, both addressed to Maurice of Hessen, one of them available for every reader, the other only for Maurice himself, Snellius showed that he knew how to write laudatory letters, how to build long Latin sentences, how to use classical quotations in the right places and how to refer to a revered past and the almighty God. He exerted himself to explain both the usefulness of astronomy and the greatness of the house of Hessen to the general reader. The private letter is much more specific about the deeds of Maurice and his family than the public letter.

Conspicuously absent in the two long texts is a reference to Maurice's interest in astronomy. Snellius implied by his silence that Maurice would have to develop more interest and activity in this field if he wanted to enjoy the honours bestowed on his father. Perhaps the Landgrave's involvement in the publication of the *Optica* could serve as an appetizer to continue his patronage of the mathematical sciences, to which astronomy also belonged?

We cannot determine whether Snellius planned the *Observationes Hassiacae* mainly as an advertisement campaign for himself, for the accomplishments of observational astronomy of the past one and a half century or just as a useful compendium of knowledge, yet we do know that he had some success with his readers: Kepler used the book, and Maurice of Hessen gave Snellius a new assignment. Daniel Mögling described it in a letter to Wilhelm Schickard of 1631 and he sent an excerpt of the book a year later. Later in 1632, Pierre Gassendi asked François Luillier on behalf of Schickard first to send Schickard a copy of the *Observationes Hassiacae* and then to ask Ismaël Boulliau to make an excerpt of the observations of Mercurius. Boulliau did as requested, sending his notes directly to Schickard.[77]

4.3.3 Maurice's assignment

Maurice wrote a letter to Snellius in 1619, which may be considered an answer to his request to receive a position at court, albeit an indirect answer. He asked Snellius for the observation and explanation of several aspects of the comet that had appeared at the end of the previous year:

> we wish for an observation of the comet that can teach us what we want to know about its first origins, its size, place, motion, disap-

hoc debitum mihi, tanquam in tabulis accepti et expensi relatum reliquerit.' [Snellius, 1618, fol. 2^r].

[77] [Seck, 2002, 1, pp. 605, 680–683, 692; 2, pp. 11–17].

> pearance and its prognostic by means of mathematical foundations and physical reasonings.[78]

Three comets appeared in 1618, of which the comet of November was the most brilliant.[79] The Landgrave gave this assignment to Snellius, because he could not find any astronomers who were as good as Snellius in his entourage:

> Indeed, our men[80] have tried something in this matter, and they seem to have observed something, but because they did not strive after precision—at least not as much as the matter requires, especially with respect to the parallax[81]—they have not felt confident to conclude anything with certainty. However, since we have understood that it is the method both of your own researches and of tasks committed to you, that you leave absolutely nothing out of those things which might to affect the exactness and the carefulness of the observation of the aforementioned phenomenon, and that you have an excellent mastery of what either the astrological rules, or the conjectures established by experience announce in the case of such phenomena, we singularly and mildly wish you to communicate all that to us as soon as possible.[82]

[78] '[...] cuius observationem desideramus talem, quae nos de primo eius exortu, quantitate, loco, motu, evanescentia et praesagio per fundamenta mathematica et rationes physicas satis instruere queat.' [Maurice of Hessen, 1619, fol. 1^r].

The letter must be dated in 1619, because Maurice refers to the 'appearance' of November and December of the previous year, and this must be the comet of 1618 to which Snellius devoted a treatise (see below). The quote from the beginning of this chapter is from the same letter.

[79] [van Nouhuys, 1998, p. 159].

[80] It is not clear to whom Maurice refers, maybe to scholars from the University of Marburg.

[81] 'Parallax' in general is the angle through which an object seems to be displaced when viewed from two different positions. The geocentric or diurnal parallax is meant here: the positions of planets and other celestial bodies are expressed relative to the earth's center, but in reality, an observer is standing somewhere on its surface. This difference needs to be taken into account when calculating the positions of heavenly bodies. It takes observations separated from each other by twelve hours to calculate this effect.

Stellar or annual parallax refers to the phenomenon that because the earth moves around the sun, a star that is relatively close to the earth seems to describe an ellipse when seen against the background of the stars further away (the period of the ellipse is a year; it takes observations six months apart to calculate this parallax). This effect cannot be measured in the case of a comet, because it moves and disappears too quickly. See [North, 1994, p. 103].

[82] 'Nostri quidem homines hac in parte aliquid tentarunt, et nonnihil observasse visi sunt, verum quia non spectarunt ἀκρίβειαν, quantam quidem hoc negotium, et praecipue circa parallaxim, sibi deposcit, nec certi quidpiam pronunciare ausi sunt. Quando vero intelligimus eam esse et studiorum tuorum, et commissi muneris rationem, ut procul dubio nihil omiseris eorum, quae ad exactam et accuratam dicti phaenomeni observationem pertinere videntur, probe etiam teneas, quid vel regulae Astrologicae vel coniecturae experientia comprobatae, per eiusmodi phaenomena denuncient, ea omnia, ut prima quoque occasione per literas nobis communices, unice et clementer cupimus.' [Maurice of Hessen, 1619, fol. 1^r–1^v].

However, Maurice did not propose that Snellius should do this job as a court astronomer anywhere in the letter, he just promised that Snellius would be rewarded generously.

Although Maurice had only asked Snellius to send him the results of his work, Snellius made a book out of it, entitled *Descriptio Cometae qui anno 1618 mense Novembri primum effulsit* ('A Description of the Comet that shone for the first time in November 1618'). He added a treatise by the late Rothmann[83] about the comet of 1585 to his own work.

The book was dedicated to Maurice of Hessen. In the dedicatory letter, Snellius intimated that previously, he had deemed it sufficient to study only the positions of the comets since he knew that the heaven was subject to the same alterability as earthly matter, and for this reason the properties of heavenly bodies would be changing as well. The Landgrave's request had now made him investigate the other characteristics of the comet more thoroughly:

> However, as soon as Your Highness had proposed this very honourable inquiry to me, everything thenceforth has seemed less important to me, and I have decided that I should pursue the investigation of the causes themselves zealously and very eagerly. [...] So that a nod of Your Highness will make or break my judgement.[84]

From the outset all readers knew that Snellius had written this book in commission, and that it had been Maurice who had induced its focus on the qualitative characteristics of the comet (Snellius's views will be discussed in detail in the next section).

A remarkable feature of Snellius's dedicatory letter is his criticism of all his predecessors. This makes the letter strikingly different from the two letters related to the *Observationes Hassiacae*, in which the topics discussed are both very general—the connection between them and the content of the book is only superficial—and non-controversial, whereas Snellius made some polemical statements that specifically apply to comets and their investigators in the dedicatory letter to the *Descriptio Cometae*.

Snellius considered the writings of the ancient authors to be of hardly any assistance. The more recent investigators were satisfactory on the point of the parallax (Maurice had specifically drawn Snellius's attention to that point in his letter), yet 'the other topics, about its origin, material, tail and prognostics

[83] His exact year of death is not known, it must be somewhere between 1599 and 1608, [Multhauf, 1975, p. 561]. Von Mackensen gives 1597 as his year of death, [Mackensen (mit Beiträgen von Hans von Bertele und John H Leopold), 1979, p. 20].

[84] 'Simul ac vero a T[ua] C[elsitudine] tam nobile zetema mihi propositum vidi, omnia ex eo tempore mihi minora visa sunt, atque ad ipsarum causarum investigationem summa alacritate contendendum existimavi. [...] Ut T[uae] C[elsitudinis] nutu ea sententia stet aut cadat.' [Snellius, 1619, pp. *–*2].

have been described only meagrely, not to say unhappily'.[85] Now, stimulated by the Landgrave to express his views on the nature of the comet, he summarized his thoughts on its characteristics and its purposes, part of which went squarely against those of other students of comets.

Snellius showed the reader that he was aware of his controversial stances, which to some would make him look like someone who 'has lately glided down from the heaven to recite a pleasant, yet fantastic song before the mortals.'[86] Self-consciously, he claimed to have good reasons to digress from the opinions of other authors and to trust that Maurice would appreciate them.

In the main part of his treatise, Snellius first examined several ancient and new opinions about the distance of comets from the earth, then he described and calculated the path of the comet of November 1618, considered its parallax and concluded that it was located far beyond the moon. Subsequently, he discussed several aspects of the tails of comets and the matter of which they consisted, and he ended with their astrological significance.

Further on in the book, William of Hessen was glorified again, this time in the words of Ramus and Tycho Brahe.[87] These quotations form the transition to Rothmann's treatise about the comet of October and November 1585. No introduction by Snellius explains its appearance in the book, but its connection to his own work is clear. Just like Snellius had received a commission from Maurice of Hessen to describe the comet, Rothmann had received a similar task from William, and just like Maurice had induced Snellius to pay attention not only to the position of the comet, but also to its causes, William had asked Rothmann, after they had both observed the comet of 1585:

> to put it [sc. the comet] in writing for the profit of posterity, and for the benefit for the search for Truth (because a comparison of opinions always contributes much to the uncovering of the truth).[88]

The arrangement of the chapters of Rothmann and Snellius is highly similar as well, which probably means that Rothmann's treatise served as a model for Snellius's, and that both found inspiration in Tycho's work on the comet of 1577. Rothmann first discussed his observations and the motion of the comet. Then he considered some aspects that conflicted with the traditional Aristotelian worldview (see next section): the comet had no parallax, it belonged to the sphere of Saturn, and the planets did not move in impenetrable spheres. He

[85] '[...] reliqua autem de natalibus, materia, cauda, prognosticis admodum ieiune, ne dicam infeliciter expressa sint [...]', [Snellius, 1619, p. *].

[86] '[...] tanquam ἐχθὲς καὶ πρώην e coelo delapsus mortalibus iucundum quidem, sed tamen fabulosum ἀκρόαμα referre.' [Snellius, 1619, p. *3].

[87] [Snellius, 1619, p. 68]. Ramus's quote and Tycho's anecdote are given in section 4.3.1.

[88] '[...] ut in posteritatis utilitatem, et veritatis inveniendae causa (collatio enim opinionum ad inventionem veritatis plurimum conducere solet) eum scripto comprehenderem [...]', Rothmann in [Snellius, 1619, p. 71].

proceeded to discuss the material of which comets were made and the (religious) use of their study. At the end of Rothmann's treatise, its editor Snellius remarked that the autograph was incomplete and he added a short notice by Rothmann about the comet of 1596.[89] By showing that he was able to equal, and better still, to surpass Rothmann, Snellius may have hoped to improve his chances of obtaining a job from Maurice of Hessen.

Not much came out of it. There are no indications of any further relations between Snellius and the Landgrave after the publication of the *Descriptio Cometae*. Maurice must have been too preoccupied by statehood affairs, too poor and too little interested in astronomy to be able to fulfil all wishes that Willebrord could have of his patron. Nevertheless, two worthwhile books came out of the association.[90]

4.4 Snellius and the comet of 1618: 'arguments derived from geometry'

4.4.1 Against Aristotle

When Snellius decided to comply with Maurice's request and devote a book to the comet of November 1618, he was touching a potentially controversial subject. A number of authors from the Northern and Southern Netherlands had devoted treatises to comets between 1577 and 1620. Among them especially the comets of 1577 and of November 1618 (the third and most brilliant of that year) received much attention.[91] There had not been much agreement between them, neither about the characteristics of the comets, nor about their meaning. Snellius plunged himself into this debate and professed some outspoken opinions.

Tabitta van Nouhuys has made an extensive study of the *Descriptio Cometae* in the framework of her excellent study on Dutch treatises on the comets of 1577 and 1618 and the changes in the Aristotelian worldview which they reflect. The following discussion of Snellius's work is largely based on Van Nouhuys's book, entitled *The Age of Two-Faced Janus* (1998). Van Nouhuys's analysis of the *Descriptio Cometae* is very thorough and its value is enhanced by the comparison to other Dutch cometary treatises. As an extension to her work, some neglected aspects that help to understand Snellius's book are discussed here, the most important of which is to what extent Snellius was influenced by

[89] [Snellius, 1619, pp. 69–156].

[90] Moreover, Willebrord was not forgotten in Hessen, as *Apollonius Cattus* shows, a book written in the 1640's by Bürgi's brother-in-law Benjamin Bramer. Bramer explained in the preface that the title, 'the Hessian Apollonius', was based on Viète's *Apollonius Gallus* and Snellius's *Apollonius Batavus*, [Bramer and Burgi, 1684, fol. **4^v]. See section 2.8.1 for the Apollonius reconstructions.

[91] [van Nouhuys, 1998, p. 159].

Maurice and adapted his work to the alleged taste of his patron. The letter quoted above, in which Maurice entreated Snellius to study the comet, shows that Snellius did not take the initiative for this book himself. This suggests that Snellius took the taste of his patron into account when writing his book. Maurice would have appreciated Snellius's special attention to the accomplishments of William of Hessen and his circle, and his emphasis on God's power. Further, the connection between the *Descriptio Cometae* and Rothmann's cometary treatise is worth some attention, as is the relation of the book to Snellius's other works, notably those on astronomy. The inclusion of Rothmann's work makes that of Snellius more Kassel oriented, and less towards a Dutch audience.

As was the case in almost all natural philosophical issues, Aristotle had developed a theory about comets that had been authoritative for many centuries. He had described comets as sublunary entities, consisting of burning terrestrial vapours which were set on fire in the atmosphere. In 1577, however, Tycho Brahe had managed to calculate that the comet of that year was located beyond the moon. This appearance of a new body in that region challenged Aristotle's explanation of the nature of the comets and even the fundamental Aristotelian axiom of the unchangeability of the (superlunar) heavens, and led several authors to a modification of their Aristotelian views. In the dedicatory letter to the book, Snellius indeed explicitly stated that matter on earth and in the heavens had a similar changing nature. Only later the Aristotelian view of a geocentric universe, with its division into a super- and sublunary realm which both had their own laws, and with its solid spheres around which the planets circled, would be completely abolished.[92]

Snellius's treatise was one of the ten publications devoted to the comet of 1618 in the Low Countries. In the Northern Netherlands, six pamphlets in the vernacular appeared, apart from Snellius's Latin treatise. According to Van Nouhuys, Snellius's was 'the most self-assured work to have been occasioned by the comet of 1618 in the Netherlands'. In the Southern Netherlands, three professors from Louvain University wrote Latin treatises as well.[93]

Traditionally, natural philosophers were interested in the nature of comets in general, whereas the study of specific comets was left to physicians, for whom the astrological aspect was relevant, or to mathematicians, who would be taking measurements. Indeed, the 1618 comet did not receive any special attention from the natural philosophers of Leiden University, but it was studied by the mathematician Snellius. At the University of Groningen, it was the professor of medicine Nicolaus Mulerius who wrote a treatise about it.[94]

One of the striking aspects of Snellius's treatise is its anti-Aristotelianism, which pervades the *Descriptio Cometae* and makes it 'the most consistently anti-

[92] [van Nouhuys, 1998, pp. 3 ff.].
[93] [van Nouhuys, 1998, pp. 159, 337].
[94] [van Nouhuys, 1998, p. 328].

Aristotelian of all the Dutch cometary treatises', which 'was pervaded with biting anti-Peripatetic rhetoric,' according to Van Nouhuys.[95] No mistake is possible on the position of Snellius, who accused the Aristotelians of 'unwittingly allowing the truth to be violated by the concoctions of one single philosopher'.[96] He wrote that even more recently, scholars had not dared to contradict the general opinion,

> preferring to make themselves less trustworthy rather than the trivial conclusions of the philosophers, and rather doubting their own experiences than other people's conjectures.[97]

This abhorrence of the domination of natural science by the Aristotelians, in which authority took the place of reason, had been a key aspect of Ramus's programme, and Snellius showed in this book that he agreed with it.

Snellius's professed loathing for the Aristotelian explanations of comets did not let him distance himself altogether from the concepts of this school. His description of the nature of the comet in the dedicatory letter makes use of the Aristotelian concept of cause, denoting material, form, starting-point and end (or goal) of a thing[98]:

> However, our comet was at least four times as far away as the moon; I ascribe to its motion the path of the comet (in a great circle, the node of which falls upon the $13\frac{1}{2}$ degree of Scorpion), and its velocity, which is different in the beginning, middle and end, to its motion. I conclude that in its essence, the matter consists of exhalations from the burning body of the sun blown out through its craters, which happens in such a way that sometimes a certain burning fragment of the sun is cast forth at the same time, as I think happened in this particular case as well. I conclude that its appearance is that of something burning, since the above-mentioned substance is kindled into a visible burning by the force that it carries with it from its very origins, or by the rays of the stars. For if the head of the comet was not consumed by its own fire, then hardly any comet would be visible here on earth, or at least would show its tail for us to look at, with such a varied and uncertain undense state and length. For this tail is also an exhalation of the burning comet, or a flame, which by

[95][van Nouhuys, 1998, pp. 339, 358].

[96]'[...] dum unius philosophi figmentis veritati vim fieri tam secure patiuntur.' [Snellius, 1619, p. 1], translation slightly adapted from Van Nouhuys's, [van Nouhuys, 1998, p. 338].

[97]'[...] ut sibi potius, quam levibus philosophorum conclusiunculis fidem derogare; et suae potius experientiae, quam alienis coniecturis diffidere maluerint.' [Snellius, 1619, p. 3], translation slightly adapted from Van Nouhuys's, [van Nouhuys, 1998, p. 338].

[98]See Aristotle's *Metaphysics*, Δ, 2.

> some hidden force is pointed in the direction of the space away from the sun.
>
> Finally, I conclude that its purpose is the glory of God the highest, who spurs on the obstinate minds of men to His veneration by such an unusual display and punishes the stubborn.[99]

Snellius elaborated on the nature of the comet in the core of the book, basing himself on older texts, his own observations, reasonings and calculations.

4.4.2 'Reason and observation'

Snellius's rejection of these Aristotelian 'conjectures' did not imply that he thought that all previous comet students had worked in vain. Some of them, such as Regiomontanus, Maestlin, Tycho Brahe and of course William of Hessen, had produced valuable work. Snellius had met two of these, Maestlin and Tycho, during his travels.[100] Even though he was convinced of the quality of their work, Snellius preferred to make his own observations, which had to be as accurate as possible, and were processed with the aid of mathematics.[101] In the dedicatory letter, he phrased the primacy of the observations as follows:

> Now surely if the causes of the phenomena that are born and perish in the high air were evident and known to us, then it would not be very difficult to determine their positions and motions on the basis of these as well. However in reality we know about their location from observations, and only from those we explore and examine their causes, and we investigate their essences on the basis of their accidents. I count place and motion as accidents.[102]

[99]'Abfuit autem cometa hic noster, ut minimum, quadruplo longius quam Luna: ad motum refero cometae tramitem in maximo suo quodam circulo, cuius nodus incidit in $13\frac{1}{2}$ scorpii; et velocitatem variam in principio, medio, fine. In essentia materiam statuo flagrantis solaris corporis per suos crateras exhalationem, ita ut nonnunquam solis quoddam ἀποσπασμάτιον flagrans simul etiam eructetur, quemadmodum in isto factum existimo. Formam, conflagrationem: dum vi illa, quam ab ipso ortu secum trahit, aut siderum radiis ea materia *exstimulata* in *manifestum* ignem prorumpit. nisi enim caput cometae ab igne suo depasceretur, vix ullus nobis hic in terris aut videretur, aut saltem caudam adeo varia et incerta raritate ac longitudine nobis spectandam exhiberet. Haec enim quoque flagrantis cometae exhalatio est, aut flamma, occulta quadam vi in plagam a sole aversam directa. [...] Finem denique constituo, summi DEI gloriam, qui adeo insolito ostento obstipas hominum mentes ad sui venerationem excitet, et refractarios castiget.' [*exstimulata*, *manifestum*: *Descriptio Cometae* reads *existimulata* and *manifectum*; I corrected to *exstimulata* and *manifestum*.] [Snellius, 1619, pp. *2–*3].

[100]See section 2.7.

[101][van Nouhuys, 1998, pp. 338–340].

[102]'Certe equidem si istorum causae, quae in summo aethere nascuntur et denascuntur nobis essent perspectae et cognitae, haud foret admodum difficile illorum quoque locum et motum ex iisdem definire. Nunc contra ex observationibus situm istorum edocti, inde demum causas scrutamur et inquirimus, atque ex affectionibus ipsorum essentiam indagamus. In affectionibus locum et motum reputo.' [Snellius, 1619, p. *2].

Here Snellius protests against the Aristotelian deductive approach, yet he takes recourse to the Aristotelian concept 'accident', an attribute which could either belong to a subject or not, without affecting its essence.[103]

Snellius was aware that he needed the best instruments to be able to make these observations, and he distinguished himself in this way from the Louvain comet students who did not even have a Jacob's staff[104] at their disposal. He gave the reader several details of his proceedings in this field. When the comet first appeared, he had had to use a Jacob's staff, which did not yield very accurate measurements. The other instruments that he could have used either had just been disconnected to be sent to the blacksmith, or were too heavy to determine the distance between celestial bodies (those apparently had another prime function, probably for surveying). His quadrant could only be handled by two men at the same time, which could mean that it was based on Tycho Brahe's design of the sextant, which required two observers to watch two stars simultaneously.[105] This enumeration shows that Snellius had a very decent collection of scientific instruments, but that all of them had drawbacks for his current purpose.

According to Snellius's own words, he then asked Maurice Prince of Orange to send him a sextant made by Bürgi from his own collection. Although the Dutch Maurice was an amateur of the mathematical sciences, his appearance here is unexpected and it seems not too bold to consider 'Prince of Orange' as an error of a composer who, being Dutch, thought of Maurice of Nassau, Prince of Orange, in the first place if the name Maurice was mentioned. The Maurice intended would then again be Maurice of Hessen, who did indeed own a collection of instruments made by Bürgi. Sending a letter to Kassel and waiting for the sextant must have cost some time, but Snellius probably judged that the good quality of the sextant and the opportunity to attract Maurice's attention would compensate for that. This loan preceded the letter sent to Snellius by Maurice of Hessen and Snellius's appeal probably taught the Landgrave that he was studying the new phenomenon in the heavens. Snellius promptly received what he had asked for, which allowed him to make more accurate observations from 11 December onwards, although even this instrument had a flaw that made it less accurate.[106]

Rothmann had boasted of this or a similar sextant in his description of the comet of 1585:

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[103][Guthrie, s a, p. 148].

[104]The Jacob's staff or cross-staff (*baculus Jacob* in Latin) was a nautical and astronomical instrument consisting of a wooden four sided staff and one or more vanes perpendicular to it. Its purpose was to determine the altitude of celestial bodies above the horizon or more generally the angle between two celestial objects, [Mörzer Bruyns, 1994, p. 25].

[105][Gingerich, 2001, p. 25].

[106][Snellius, 1619, p. 7]. Cp. [van Nouhuys, 1998, p. 341], who silently assumes that the Maurice intended is Maurice of Hessen.

> However, in order to determine the distances we have used neither a wooden staff, nor a Jacob's staff, but a new instrument, made with a new method. It is customarily called a sextant, because it is an instrument made in a certain manner from the sixth part of a circle enclosed in steel, and shaped in such a way that it can very easily be turned in any plane you like, and remains in its turned position on its own, without any support of the hands. And by an ingenious invention, it has been divided into its parts in such a way that it clearly shows us not sixth, not twelfth, not twenty-fourth parts of degrees, and not even so small as single minutes, but even (which seems hardly possible) some parts of minutes.[107]

The sextants made by Bürgi indeed made possible very accurate measurements. They were constructed in such a way that angles in different planes could be measured.[108]

Snellius's observations formed the basis of calculations of the position of the comet on several days, and then of its orbit and some other characteristics (the parallax, the point of intersection of the orbit with the ecliptic and the angle of inclination). His stress on the importance of observations was certainly not just rhetorical. The treatise contains 'more, and more accurate, data for the comet than those of his Louvain colleagues'.[109]

In his consideration of the distance of the comet from the earth, Snellius used both qualitative and quantitative arguments, but attached greater force to the second category:[110]

> These [sc. qualitative considerations] cannot force such an unwilling and reluctant person to approve them, as something near the truth, or take away every doubt. Therefore, we will have to derive our arguments with respect to this matter from Geometry and we will have to summon observations to support us so that we can explain

[107]'Usi autem sumus ad distantias capiendas non ligno, non baculo: sed instrumento novo nova ratione confecto: quod Sextantem appellare consuevimus. Est enim e sexta circuli parte certo modo chalybi inclusa ita formatum instrumentum, ut in quodvis planum facillime flecti possit, flexumque per sese absque manuum ope firmiter subsistat: estque ingeniosa inventione ita in partes suas distinctum, ut in observationibus non Sextantes, non uncias, non semuncias graduum, nec etiam singula tantum minuta, sed et (quod vix posse fieri videtur) minutorum aliquot partes distincte nobis exhibeat.' [Snellius, 1619, pp. 73–74].

See [Mackensen (mit Beiträgen von Hans von Bertele und John H Leopold), 1979, p. 66] for the ingenious division which Rothmann mentions.

[108]For the reconstruction of a similar (wooden) sextant, and for pictures of metal sextants ascribed to Bürgi, see [Mackensen (mit Beiträgen von Hans von Bertele und John H Leopold), 1979, pp. 61–69]. See [Zinner, 1967, pp. 268–276] for a survey of Bürgi's instruments.

[109][van Nouhuys, 1998, pp. 340–341, 358]; [Snellius, 1619, p. 9].

[110][van Nouhuys, 1998, pp. 344–345].

> what they tell us us by drawing it with a stick in the dust.[111]

And later he said that he had studied the position of the tail of the comet himself 'so that I would not need to rely on other people's testimonies'.[112]

Snellius did not merely observe the comet, but also used auxiliary sciences, notably optics. In the dedicatory letter he warned the reader:

> Because I see that the explanation with which the most distinguished people have agreed (namely that [the tail] is formed by the rays of the sun out of the very bright ball itself of the comet), can in no way be reconciled with the truth of optical theories.[113]

In his further study of the location and essence of the tails of comets, he used arguments from the science of optics to convince the reader of their real, superlunary existence, instead of their being optical illusions in the atmosphere.[114]

Optics became especially relevant as an auxiliary science for astronomy when the telescope was introduced to take observations, because the lenses caused distortions. Rudolph and Willebrord Snellius possessed a telescope,[115] yet Willebrord gave no hint of having used a telescope for observing the comet in *Descriptio Cometae*. It indicates that he was not able to buy or make a telescope of high enough quality to improve his observations, and that for the moment, he considered the observations made by the sextant of Bürgi as accurate enough. Remarkably, the Groningen professor Mulerius did use a telescope to observe the same comet.[116]

The telescope does play a role in the section of *Descriptio Cometae* on the matter of which the comet was made. This part shows a specimen of Snellius's method of acquiring new knowledge by combining ancient sources and modern discoveries: first he told the reader about the theory of Anaxagoras, a classical Athenian philosopher, who had argued that the sun was a glowing mass, then he announced:

> Recently, reason and observation have convinced us that this is really true, because we have started to grasp it completely since the

[111]'Sed ista ut probari possint, tanquam vero assideant, ita invitum et reluctantem cogere non possunt, aut scrupulum omnem eximere: Quamobrem a Geometria nobis ad hanc rem argumenta erunt mutuanda, et observationes huc nobis advocandae, ut quid illae nobis addicant radio et pulvere explicemus.' [Snellius, 1619, p. 21].

[112]'[...] ut non haberem necesse alienis testibus credere [...]', [Snellius, 1619, p. 30], translation by Van Nouhuys, [van Nouhuys, 1998, p. 346].

[113]'nam quod summis viris placuisse video, eam a solaribus radiis per ipsum cometae pellucidum globum deformari, id ab opticorum theorematum veritate omnino est alienum.' [Snellius, 1619, p. *3].

[114][van Nouhuys, 1998, p. 347].

[115]See p. 68.

[116][Jorink, 1994, p. 73].

> Batavian telescope shows us the truth.[117]

This 'information' probably did not come from his own observations, otherwise he would certainly have said so. He then explained how sunspots, one of the recent discoveries made thanks to the telescope, indicated that the sun was a fiery ball, because they could be interpreted as exhalations of the burning sun.[118]

Snellius took recourse to other parts of the mathematical sciences than optics to prove his hypotheses in other places. For example, he used calculations to demonstrate that the comet's head could not consist of terrestrial exhalations, nor of sublunary matter in general, by explaining that it was too big to allow for that possibility, thus using the mathematician's tools to refute a natural philosophical idea. In these calculations, he used the value of the radius of the earth that he had calculated himself in the *Eratosthenes Batavus*. Because the matter of the comet could not be sublunary, the comet had to be composed of superlunary matter, and because the sun was an enormous fiery sphere, this matter was formed of exhalations of the sun and it was similar to its counterpart under the moon. Snellius contradicted Aristotle again with this opinion.[119]

4.4.3 Ancient wisdom: shadows of knowledge

Even though Snellius greatly valued the powers of the mathematical sciences in describing and explaining the characteristics of the comet, this appreciation certainly did not entail a refusal to take into account the opinions of the classical authors, whom he generally considered as authorities. However, the knowledge that he could extract from them was not his final purpose. This goal was the pre-classical pristine knowledge, which, Snellius believed, had been of higher quality than the knowledge of his own age and that sadly had been lost in the meantime. Tabitta van Nouhuys suggests:

> His preoccupation with mathematics and experience can even be said to have been inspired to a certain extent by the desire to recover, through calculation, the lost wisdom that had been almost intuitively known to the pristine sages.[120]

In particular, Snellius was impressed by the astronomical achievements of the Babylonians, a theme also discussed in the dedicatory letter to his *Observationes Hassiacae*. He thought that they did not discover all their wisdom themselves, but inherited it from much earlier generations. The reason for the loss of a great part of this treasure was that it had been guarded throughout the centuries by a

[117]'Hoc adeo verum esse apud nos nuper ratio et observatio ad certum perduxit. nam id omnino coepimus scire veritatem monstrante Batavica dioptra.' [Snellius, 1619, p. 40].
[118][Snellius, 1619, pp. 39–42]; cp. [van Nouhuys, 1998, pp. 355–356] and section 2.8.3.
[119][van Nouhuys, 1998, pp. 354–355].
[120][van Nouhuys, 1998, p. 349].

small group of initiates, who were fearful that common people would abuse the knowledge. Therefore, the experts were very reluctant to share their 'mysteries', 'so that we now possess only shadows of the highest things'[121] known to the ancients.

Snellius shared the belief in a *prisca scientia* with many other humanists and other contemporaries. The idea of pre-ancient knowledge that had to be restored was an important part of Neo-Platonism, and, closer to Snellius, of Ramus's and Stevin's thoughts. Although so much had been lost, Snellius saw reason for optimism, considering the recent appearance of a number of stars and comets as signs of hope, sent by God to encourage scholars to recover the lost science.[122]

Snellius was well aware that future generations might then (again) possess more knowledge than he had now. In some cases, he ventured a theory that had the status of a mere hypothesis, for instance when he tried to explain whence comets originated that were born far away from the sun. He wrote:

> I for me consider it to be a thing of great importance that if in a new and hitherto unknown field something close to truth is invented, until this history of man, and frequent observations either prove or refute it; for to persist in one's opinion even if experience goes against it, amounts to trusting no one but one's own witnesses, and wrongfully relying on them.[123]

Snellius devoted his last chapter to the purpose of the comet. In his dedicatory letter he had already explicitly made clear that God was its creator, and that He had a clear plan with it, and he repeated this opinion in his chapter: 'God does not send vast bodies to move in the immense ether or to glide under the stars, in vain, or without any effect.'[124] According to Snellius, their significance could be understood by a combination of one's own study of the comet and a scrutiny of reports of the experiences of earlier comet researchers. He approached these accounts critically, for instance, rejecting Ptolemy's rules such as 'when comets appear in the morning, their effects will manifest themselves soon, and when they appear in the evening, late'.[125]

[121]'[...] ut maximarum rerum solas umbras possideamus.' [Snellius, 1619, p. 48].
[122][van Nouhuys, 1998, pp. 348–351].
[123]'Ego utique magni existimo referre in re nova et hactenus incerta aliquid vero affine excogitasse, donec ista crebra et diutina observationum historia aut approbet, aut refellat: pertinaciter enim suam sententiam invita et reluctante experientia tueri, est suis testibus tantum fidem habere, et iisdem importune insistere.' [Snellius, 1619, p. 54], translation partly by Van Nouhuys, [van Nouhuys, 1998, p. 357].
[124]'A Deo vastas moles non frustra, aut nullo effectu per immensum aethera cieri, et sidera subterlabi [...]', [Snellius, 1619, p. 57], translation partly by Van Nouhuys, [van Nouhuys, 1998, p. 529].
[125][van Nouhuys, 1998, pp. 529–530].

Because of the unsatisfactory nature of existing scholarship, he considered it necessary to try and discover the principles of astrology himself, the science that explained the meaning of heavenly bodies. In order to achieve this, he had to rely on observations:

> I would like to have the principles of this science [sc. astrology] somewhat more accurately determined. For if it is indeed a science, it should, in any case, like all the other sciences, have as foundations experience, observation, sense perception and induction. In all true and systematic theories, those things are put first from the very beginning. I wish for the same principles here, and I search for them—because no one with a healthy brain will allow old wives' tales and the gibberish of lunatics to be forced upon him, as if they were certain and undisputed truths.[126]

The Aristotelian series listing 'experience' to 'induction' often occurs in Ramus's work as well.

According to Snellius, the powers of the comets were equal to those of the sun, since comets were solar exhalations. They could actually affect the earth with these powers: for instance the comet of 1618 had caused a storm when it was in conjunction with a certain star. The effects of the comets on the earth obeyed certain laws, which were as yet largely unknown, but could be discovered by observation and study.

Comets could sometimes also be considered divine signs; in this respect they were similar to several phenomena in the air with predictive powers. Snellius for instance told the reader that before the outbreak of the Dutch Revolt, people had seen apparitions of besieged cities, explosions and military camps in the sky, which would soon afterwards all appear in reality. Yet Snellius stated that unlike the comets, these phenomena had no physical influences, because they were in fact only optical illusions, sent by God to show His anger.[127]

The modern reader should realize that, in principle, the astrological and religious interpretations of celestial phenomena were incompatible, but that, in practice, they were often combined. They conflict because the astrological meaning of heavenly bodies is much more concrete than the more general religious message that a comet sent by God would spread. Calvin had protested against

[126]'Vellem equidem huius artis principia paulo accuratius constituta. Nam si ars sit, habebit utique natales et initia ad instar artium reliquarum ἐμπειρίαν, ἱστορίαν, αἴσθησιν, ἐπαγωγὴν, experientiam, observationem, sensum et inductionem. in omnibus theorematis veris καὶ ἐντέχνοις istis locus primus ab ipsa origine est attributus. Eadem hic ego principia desidero, et quaero. neque enim aniles fabulas, aut commenta delirantium pro ἀναμφισβητήτοις sibi obtrudi quisquam patietur cui sinciput erit sanum.' [Snellius, 1619, p. 61], translation partly based on [van Nouhuys, 1998, p. 531].

[127][van Nouhuys, 1998, pp. 531–533].

predictions about the fate of individual human beings on the basis of the observation of the heavens, which was superstition according to him. For his later followers, this did not imply that astrology as a whole should be abandoned. Mulerius, for instance, was both a fervent combatant for the Protestant faith and a practitioner of predictive astrology.[128]

Because astrology had been so poorly developed, Snellius was sceptical about the prognostications made by his contemporaries on the occasion of the appearance of the comet. However, he expressed his optimism in hoping that, in the future, the necessary rules would become known, and although he did not have much confidence in his understanding of the laws that governed the effects of the comet, he ended his treatise with one prophesy. He noticed that the comet which he had observed closely resembled the comet described by the Medieval Arab scholar ʻAlī ibn Ridwān, and the appearance of this comet had been followed by wars, murders and other catastrophes. Snellius feared that the same would happen now and he prayed God to avert these evils from His church and mankind, 'because He alone is able to do so.'[129] This short prayer was finished by *amen*, the last word of the treatise.

The importance of God's role was thus stressed, even if Snellius was not very specific about how God operated the comets and their consequences. Believing that the almighty God would certainly be capable of creating comets and catastrophes, he did not need to consider that problem. Because of the general nature of his prediction, he did not have to fear the disapproval of Calvinist ministers. His predictions could even make his readers repent and improve their ways, which was a welcome message for the Reformed church. Especially during the internal religious strife of the Twelve Years Truce, the orthodox Calvinists were a powerful group that one better not antagonize.[130]

In short, for Snellius astronomy and astrology were both real sciences, with the same method. For this reason, it would not do to call his astrology superstition, even though until recently it was a common practice among historians of science to call the astrological part of cometology by that name. Eric Jorink has shown that physical explanations of characteristics of the comet and speculations about its astrological and religious meaning went very well together in all Dutch treatises about the comet of 1618.[131]

Overseeing the treatise as a whole, we can answer the question to what extent Snellius took into account the wishes and interests of his prime reader, Maurice of Hessen. Although we find no specific clues as to Maurice's influence, we can see that Snellius treated all topics for which Maurice had asked in his letter of request: observations, origins, orbit, disappearance and aim of the comet.

[128] [Jorink, 1994, pp. 72, 75].
[129] '[...] quia solus id potest.' [Snellius, 1619, p. 67].
[130] [van Nouhuys, 1998, pp. 534–537], [Jorink, 1994, pp. 76–78].
[131] [Jorink, 1994, pp. 69–70].

This last part of cometology was probably closest to Maurice's own alchemical interests. All in all, the attention paid to these issues makes the treatise different from Snellius's usual work, in which mathematical (quantitative) considerations play a much larger role, and this is most likely due to Maurice's assignment.

Maurice of Hessen was a very religious man and he certainly would not have allowed Snellius to stray far from the path of Calvinism in his astrological ventures. Snellius may have stressed God's capacity to please his patron, yet there is no reason to doubt his sincerity, and ultimately we cannot decide whether we read Snellius's opinion or statements calculated to be liked by Maurice.

4.5 The choice between geocentricity and heliocentricity

One other question must be raised here: can we deduce from the *Descriptio Cometae* whether Snellius was a Copernican, or, more specifically, whether he believed in heliocentricity, the central position of the sun in the universe? This issue is treated not because it is very prominent in Snellius's work, but because it has great appeal to modern students of the history of science. Although Snellius clearly spoke his mind in the *Descriptio Cometae* several times, he did not do so on this topic. Rather, he referred to the opinion of other people, both ancient ('that sun, which the earliest natural philosophers always called the Heart of the heavens'[132]) and modern:

> Venus and Mercury go round it [sc. the sun] closest as satellites, although Tycho could have seen right that the other, superior, planets do the same, but with very large circumvolutions, which also include the earth.[133]

The system in which Venus and Mercury circled the sun, and the sun and the other planets circled the earth, was the Capellan system, based on the work of the late-antique writer Martianus Capella. This was the mainstream conception among Dutch scholars. In the Tychonic system, the immobile earth was in the centre of the universe, the sun moved around the earth and the planets around the sun. Thus, Snellius subscribed to the Capellan system, and only tentatively endorsed the system of Tycho.[134]

His other works forbid a definite conclusion about his viewpoint as well. In the *Theses Philosophicae*, defended to obtain his master's degree in 1608, he

[132]'Sol ille, quem physici Veterrimi Cor caeli vocitabant [...]', [Snellius, 1619, p. 48].

[133]'Venus et Mercurius tanquam laterones eum proximi ambiunt. Etsi reliquos superiores idem quoque facere, sed amplissimis conversionibus quibus terram quoque includant, non inepte Tychoni sit visum.' [Snellius, 1619, p. 49], translation adapted from Vermij's, [Vermij, 2002, p. 43].

[134]See [van Nouhuys, 1998, pp. 351–352, 359–360] and e.g. [North, 1994, pp. 303–304].

stated: 'It seems to be more likely that the earth moves in an annual orbit than that it is at rest',[135] which shows a small, very carefully phrased preference of heliocentricity. Because the theses are given without any arguments, we do not know how Snellius defended this. Four years later, in *De maculis in sole*, Snellius suggested that sunspots could be used to determine whether the earth was moving.[136] Around the same period he used a model to explain the trepidation of the earth. Isaac Beeckman wrote down how it functioned, starting his description as follows:

> Snellius held a wooden globe in his left hand for the sun at rest, and in his right hand another globe for the movable earth, which was connected to the first one by a rather long piece of wood.[137]

The sun at rest points to heliocentricity.

On the other hand, in his *Eratosthenes Batavus* (1617) Snellius wrote that 'the earth is in the middle of the whole universe, and as it were its centre'.[138] This statement is chosen for practical rather than for philosophical reasons, as is shown by the end of the chapter:

> If some have other opinions on its [sc. the earth's] place, that does not affect my argument. [...] However, here we prefer to follow this line of thought [that the earth is immobile], as it is simpler and less complicated for proving what we contend.[139]

What Snellius means here is that the earth is so small in comparison to the distance between it and the stars, that the effect of parallax can be disregarded when the stars are observed. Around 1622–23, Snellius probably taught the Capellan system, as the notes of a student suggest.[140]

Snellius certainly knew some adherents or at least readers of Copernicus's work. William of Hessen and Rothmann were Copernicans, Rudolph Snellius had advised his student Beeckman to read Copernicus, and Simon Stevin and Philips Lansbergen were in favour of Copernicanism, and even wrote defences of the

[135]'Probabilius videtur terram moveri in orbe annuo, quam quiescere.' [Snellius, 1608b, fol. A 5^r].

[136]For a—not completely clear—discussion of the argument, see [Vermij, 2002, p. 45].

[137]'[...] sinistra tenuit globum ligneum loco Solis quiescentis, dextra autem alterum globum, oblongo ligno priori annexum, loco Terrae mobilis.' [de Waard, 1939, p. 21].

[138]'Terram totius mundi esse mediam, et tamquam centrum.' [Snellius, 1617b, p. 11].

[139]'[...] si qui tamen aliter de eius loco censeant, iis mecum hic quidem nihil negotii [...] Verum hanc rationem tanquam planiorem, et ad demonstrandum id quo contendimus minus irricatam hic sequi maluimus.' [Snellius, 1617b, pp. 13–14], translation adapted from Vermij's, [Vermij, 2002, p. 44].

[140][Vermij, 2002, pp. 41–43]. Vermij based his argument on manuscript notes written by Cornelis Booth, probably made during a course in physics when he was a student in Leiden. This course was probably given by Snellius, because the normal professor of physics, Jacchaeus, was suspended for some time as a result of the religious troubles.

Copernican system in the vernacular. According to Lansbergen, Snellius was the prime instigator of the Latin book that he devoted to the Copernican system, the *Progymnasmatum astronomiae restitutae Liber I. De motu solis* ('Preparatory Exercises of the Restored Astronomy, Book One, on the Motion of the Sun').[141]

The question of Snellius's view on Copernicus has attracted some attention in the last few decades. N.D. Haasbroek, who made a thorough study of the *Eratosthenes Batavus*, wrote:

> In our opinion it may be queer that during his life Snellius remained a supporter of this [sc. Ptolemy's] geocentric structure of the universe. Apparently Copernicus' theories could not convince him.[142]

This summary does not do Snellius justice, as the arguments mentioned above show.

Rienk Vermij, who in his fine study of the reception of the new astronomy in the Dutch Republic gave a good overview of most of Snellius's arguments related to Copernicanism, grasped the elusive nature of Snellius's opinion much better by concluding:

> This shows sympathy for the Copernican system, but not real conviction. [...] As a geocentrist, Snellius does not sound very convinced either.[143]

All in all, Snellius showed an interest in the problem of geo- or heliocentricity, but did not reach a single conclusion—or at least not one that remained unchanged during his life. One could say that Snellius was old-fashioned, but in truth, none of the humanisticly inclined Leiden scholars defended heliocentricity in public,[144] nor were many other knowledgeable persons convinced of its truth. One could also argue that Snellius did not dare to publicly admit his adherence to such a contested viewpoint, but given his outspokenness in other matters (the *Descriptio Cometae* contains many of them), this explanation is not very convincing either. It is more likely that he found himself lacking arguments that decisively showed the truth of one of the systems of the world, and that he did not feel the need to select one of them. To his learned contemporaries, the Copernican model was attractive mainly because of the advantages which it offered in the simplification of many astronomical calculations, but this consideration was not of much relevance to Snellius. When he did make calculations or studied other astronomical problems, he worked with the model that served his needs best on the occasion.

[141] [von Mackensen, 1997a, p. 387]; [de Waard, 1953, p. 17]; [Vermij, 2002, pp. 65–68, 82–88]; [Lansbergius, 1628, fol. $*3^v$], from the dedication written in 1619.
[142] [Haasbroek, 1968, p. 62], probably based on [de Waard, 1927b, c. 1159].
[143] [Vermij, 2002, pp. 43–44].
[144] [Vermij, 2002, p. 52].

4.6 *Concluding remarks on* Descriptio Cometae

A comparison of Snellius's *Descriptio Cometae* with other Dutch cometary treatises shows that Snellius attached more importance to observations and calculations than his colleagues, but that his natural philosophical assumptions were not very different. They all agreed that the heavens were subject to change and that the paths of heavenly bodies were not determined by solid orbs, thus they all denied two fundamental points from classical Aristotelian cosmology. To substantiate their viewpoints, they could refer to other classical authorities, notably Seneca as an important representative of Stoicism. Only Snellius judged it superfluous to give any arguments for these statements, because they had been conclusively demonstrated by Tycho Brahe, Maestlin and William of Hessen.

The anti-Aristotelian rhetoric of Snellius is very strong; its force distinguishes Snellius's book from the other Dutch comet treatises. Yet it should not mislead the reader into thinking that Snellius's own theory was largely at variance with Aristotle's. In fact, they were largely similar: Snellius, disapproving of Aristotle's theory that comets consisted of exhalations from the earth, replaced it with a theory that considered comets as solar exhalations with large lumps of the sun's material that would catch fire.[145]

All Dutch cometary authors also agreed in their support of geocentricity. Some of them, however, preferred Tycho's system to the standard geocentric system. Snellius's viewpoint—he tentatively subscribed to Tycho's model—thus fits with that of the other scholars, yet his utterances in other places prove that he was not constant in his professed opinion on this question.[146]

The world views of the Dutch authors on comets are highly similar, yet one important difference can be found: their reaction to the problem to what extent the human mind could truly understand nature and its laws. Some authors hardly saw any possibility for such an achievement, but Snellius had more confidence in the enterprise—in fact more than any other of the Dutch authors—if only accurate observations and mathematical calculations were used. Thus, although Snellius was sceptical about the achievements of astrology so far, he was optimistic about its future developments. Mulerius by contrast, who wrote in Dutch about the comet, showed his approval of traditional astrology. This difference may have been caused by the difference in intended audience: while Snellius aimed at a learned public, Mulerius wrote for the less educated, who might not have been interested in philosophical debates on new developments in science.[147]

Abroad, cometary treatises appeared as well, written among others by Galileo,

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[145] [van Nouhuys, 1998, p. 359]. Cp. [Jorink, 2006, pp. 137–138] for a discussion of *Descriptio Cometae* in the framework of other Dutch cometary treatises.
[146] [van Nouhuys, 1998, pp. 11, 358–359, 368, 372].
[147] [van Nouhuys, 1998, pp. 373–374, 536–537].

Kepler and the Jesuit Horatio Grassi. Snellius's *Descriptio Cometae* appeared somewhat earlier (1619) than Galileo's *Il Saggiatore* (1623) or Kepler's *Hyperaspistes* (1625), which makes it possible that the latter two scholars knew Snellius's work, but no trace of influence appears from their books.

The editor of these foreign books, Stillman Drake, saw their importance thus:

> The controversy which sprang from the comets of 1618 enables us to see in strong contrast the spirit of traditional philosophy and the spirit of modern science—the former with its emphasis upon authority, dogma, and verbal exegesis; the latter with its stress on observation, skepticism, and mathematical analysis.[148]

This stark contrast is not found in Snellius's treatise. Although he himself stressed time and again the importance of observations and mathematics, which makes him look 'modern', his reverence for the knowledge of (pre-)ancient authors and his careful reading of mostly ancient texts for gathering ideas show his 'traditional' side. Snellius evidently made an effort to combine the two approaches, between which no boundaries or opposition existed for him.

Snellius made a thorough study of the characteristics of the comet, following the humanist method. This meant that most of his ideas had a classical origin. Because the ancient sources were so diverse, Snellius had enough room to develop and explain his own ideas while referring to those sources. However, he did not consider ancient astronomy as the summit of knowledge. The pre-classical accomplishments had been larger, but were sadly lost. Snellius thought that by working hard in the right way, they could be recovered.

As a mathematician he stressed the importance of his own experience and calculations in the search for truth. He preferred an inductive to a deductive approach. His description of the instruments that he used, in which the sextant from Kassel played the leading part, shows the value that he attached to good observations. By mentioning this sextant, he again showed his attachment to the Kasselian observatory both directly and indirectly, since Rothmann had mentioned such a sextant as well.

Snellius used optics and spherical trigonometry as auxiliary sciences for astronomy, thus merging three of the mathematical sciences in this work. The arguments based on optics were merely qualitative, and were meant to explain why the tail of the comet had to be a real, material entity. He used calculations to explain why the head of the comet did not consist of sublunary matter. In this way he combined qualitative and quantitative methods to acquire knowledge about the comet.

Snellius's paragraph about the purpose of the comet is not mathematical at all and therefore more at variance with his other works than the rest of the

[148] [Galilei et al., 1960, p. viii]. Drake wrote this sentence in 1960; a modern historian of science would probably phrase it more subtly, not as the opposition of two groups of concepts.

Descriptio Cometae. However within the context of the treatise, it is not out of place: the question of the purpose of the comet was asked very generally, and Maurice of Hessen in particular had demanded its reply. By refusing to give specific astrological rules himself, Snellius could avoid the tricky problem of the potential irreconcilability of God's omnipotence and the predetermination of human fate by the stars.

The *Descriptio Cometae* is of a very different nature than the *Observationes Hassiacae*, which had also been dedicated to Maurice of Hessen. The latter is mainly a collection of material, whereas the former also contains an analysis of the material, allowing Snellius's own voice to be heard more clearly. His discussion of sunspots implicitly referred to the anonymous work that he had published on the topic. The comparison of Snellius's viewpoint on geocentricity in the *Descriptio Cometae* to that in his other works shows that he either was not able or did not want to claim a singular world system to be true, which is remarkable when compared to the self-assured way in which he framed his thoughts at other moments.

4.7 Conclusion

In this chapter, most of Snellius's astronomical work and his relationship with one of his main patrons have been discussed. These two topics are connected: of the three books on astronomical topics which Snellius published, two were dedicated to Maurice of Hessen. Astronomy was certainly a lifelong interest for him, which must have been greatly stimulated by his sojourn with the already famous Tycho Brahe and his brilliant assistant Kepler in his youth.

In his astronomical pursuits, Snellius could combine several of his skills: of calculating, using instruments, critically scrutinizing classical sources, and writing about his results and conclusions. It is impossible to determine precisely in what proportion heartfelt interest in astronomy, the wish to do useful work and the need to receive financial remunerations from a patron were found in Snellius's mind, but all of these factors must have been relevant. When studying his astronomical work, it is useful to realize that Snellius did not take the initiative for either the work on sunspots, or that on the comet of 1618.

The background of Snellius's familiarity with Maurice of Hessen had nothing do to with astronomy. Maurice had made the acquaintance of Rudolph Snellius much earlier, probably when the latter was a teacher in Marburg, which belonged to Hessen. He seems to have valued Rudolph as a philosopher, in particular because of their shared enthusiasm for Ramism, and as an adviser in the Second Reformation. Rudolph, for his part, saw in Maurice a powerful stimulator of the publication of the *Optica* of Ramus and his student Risnerus. Their relationship was clearly that of a patron and his client: both gained from it, and there was a clear hierarchy between the two. The patronage relation of the Landgrave and Rudolph could be called private: it was not celebrated in publications, but kept within a small circle, for which reason not much of it is known. Once, when Rudolph had been back in Leiden for a long time, he was entertained lavishly by Maurice. The patronage relation was then publicly recognized, causing gritting teeth from Scaliger in Leiden.

The patronage relationship between Maurice and Willebrord Snellius, on the other hand, was of a more public character. Everyone could read the two books dedicated to Maurice, and private conversations do not seem to have played a significant role in their relationship. It is very likely that Maurice paid for the two books that Snellius dedicated to him.[149] Snellius seems to have hoped for more, namely the position of court astronomer, but he did not obtain this. Not only was Maurice too distracted by several serious political troubles when Snellius asked him for this favour, he also lacked money, and astronomy was not his prime interest anyway. His support of Snellius's endeavour to publish some of the observations carried out in the observatory of his father William must be seen as an obligation to his father, not as a new venture into astronomy.

And yet Maurice seems to have been so pleased by this work that he singled Snellius out to study the comet that appeared at the end of 1618. The comet received much attention, and certainly not only from scholars, but also from common people who usually saw it as a sign of divine wrath. It is not unusual that Maurice with his devout disposition wanted to know more about it. Since the letter in which Maurice asked Snellius to make this study is still extant, we can be certain that the Landgrave took the initiative for this book, whereas in the case of the *Observationes Hassiacae* the lack of any reference to a guiding role of Maurice makes it more likely that Snellius was the instigator. Maurice specifically asked Snellius to inform him about a number of aspects of the comet; in this way, he not only influenced the coming into existence, but also the contents of the *Descriptio Cometae*. These cases show, therefore, that different shapes of patronage can be discerned in the case of the Snellii.

Rudolph Snellius may have stimulated his son to develop his astronomical expertise and render service to the Landgrave, who was the heir to an observatory and observations. The older Snellius must have been aware that he was not a suitable candidate to continue the work of the previous generation himself. A few years after Rudolph's death, Willebrord indeed did his utmost to please the Landgrave, applying his knowledge of Latin rhetoric in flattering letters and his astronomical scholarship in the contents of the books, and trying to use the credit that his father had built up with Maurice.

It remains the question whether the investments in the patronage made by

[149]Cp. the relationships of Maurice with other clients: one of them wrote a long treatise with the aim of asking Maurice for money to buy a house and another received the magistracy of a Hessian town, [Moran, 2000, pp. 222–223, 225].

the son were really worth the effort. The relationship might have ended because the interests of Maurice and Willebrord remained too far apart. Would not much more have come out of the association had Maurice had the same commitment as his father? Then Willebrord would have had the perfect patron: the prince-astronomer, who showed the value of mathematics, even for those of high descent, by his activities. And this association would have strengthened Snellius's programme of enhancing the status of the mathematical sciences.[150] In this way, he would have followed in the footsteps of Ramus, the most outspoken promoter of mathematics of the sixteenth century, who had specifically presented William of Hessen as a laudable example of a patron-scientist to his French royal audience.

Descriptio Cometae also shows that Snellius was not afraid to stick out his neck if he thought that the matter was worth it, and that he could then condemn other scholars forcefully and ruthlessly. In this way, he could show his own position and merits. The problem of the truth of Copernicanism was not such an important matter for him, but the further downfall of the old scholastic domination of natural science clearly was.

[150] Cp. Bruce Moran's analysis of the role of the scientific patronage by the Hessian princes: 'The patronage of Wilhelm IV and his son Moritz did not directly inspire revolutionary movements in science. Nevertheless, the style of patronage associated with each of their courts is, I think, emblematic of both social and intellectual changes important to the Scientific Revolution. The active involvement in projects by princes like Moritz of Hessen, and his father, Wilhelm, not only helped bridge the gulf between scholar and craftsman, but helped also to elevate the status of technical and empirical skills, as opposed to strictly textual learning, in the pursuit of the understanding and control of nature.' [Moran, 2000, p. 228].

Chapter 5

Snellius's geometry: number as the 'exact translator of every measure'

5.1 Introduction

The aim of this chapter is to introduce Snellius's activities in the field of geometry, the liveliest part of pure mathematics of his day. Firstly, a short survey of the state of the art in this domain in Snellius's time will be given. Only topics necessary for understanding the following sections and chapters will be discussed. Most general books on history of mathematics treat early modern geometry (before Descartes) in stepmotherly fashion. The most complete book on the topic is Henk Bos's *Redefining Geometrical Exactness*. Although it is not meant as a survey, but aims to answer some specific questions, it certainly fills the gap left by the general books.[1] Some of the issues raised in this book have guided a crucial part of my study of Snellius's geometry and the following survey is borrowed from it for a large part.

Secondly, a major source for Snellius's geometry will be further introduced. This is the *Fundamenta Arithmetica et Geometrica*, Snellius's translation of Ludolph van Ceulen's *De Arithmetische en Geometrische Fondamenten*. It may be somewhat surprising that a book of which Snellius is merely the translator is crucial for understanding his geometry. The reason for this is twofold: he included many comments of his own on topics from pure geometry in the book

[1] Its central subject is 'the concept of construction and the changes it underwent in the early modern period', [Bos, 2001a, p. 13]. Recently a good survey article on pure mathematics by Kirsti Andersen and Henk Bos has appeared, [Andersen and Bos, 2006].

and he reacted to Van Ceulen's work, in this way adding elements of a dialogue between two representatives of different currents in mathematics. Snellius planned to write a book in which many of the topics touched upon would be further developed, but he never published such a book.

The plan of the present chapter has the shape of a funnel: in the beginning its focus is wide and then it narrows down from a general account of the *Fundamenta* to a more detailed analysis of two of its most interesting parts. These are Snellius's dedicatory letter, which contains his mathematical philosophy in a nutshell, and Van Ceulen's introduction of the four elementary arithmetical operations performed on line segments of which the length is expressed by a number, with Snellius's reaction.

5.2 Some characteristics of early modern geometry

5.2.1 Problems and tools

In Snellius's time, pure geometry was generally perceived as the science offering the most certain knowledge and it was, as such, much appreciated. It found many students on different levels, ranging from school pupils to specialized mathematicians. They often studied old problems, of which solutions were already known, which were repeated, or, if they were not to their satisfaction, substituted by new ones. The focus was on solving problems, not so much on proving theorems. In the words of Bos:

> The debates primarily concerned the solution of point construction problems, that is, problems that admitted one or a finite number of solutions only. Solving such problems was indeed seen as a major, if not the main, aim of geometry.[2]

To solve a problem meant to find a construction that yielded the result on the basis of given (that is, known) elements. Questions about the existence or the number of solutions were only rarely raised.

Although some of these problems were motivated by practical questions, they were mainly theoretical, belonging exclusively to the field of pure geometry. This meant that they had to be solved by pure means, that is, by exact constructions and not by approximative procedures. In traditional pure geometry, no numbers were used to express the size of magnitudes and ratios.

Most problems and the tools to solve them originated from Greek geometry. The three main representatives of this geometry were Euclid, Apollonius of Perga

[2] [Bos, 2001a, p. 59]. The period under consideration is c. 1590–c. 1650.

and Archimedes. The knowledge of this tradition had grown considerably in the century before Snellius's time. The original sources were published several times and translations and commentaries appeared. Euclid's *Elements* were available in printed editions in Latin, Greek and some other languages (including Dutch, but only books I–VI), and their contents, mainly book I to VI, were widely taught. The axiomatic-deductive structure of the work was generally accepted as correct and appropriate to the subject (an exception was Ramus, see section 2.5.2), yet not many works appeared repeating the structure of the *Elements*. Other works by Euclid were also known by direct or indirect transmission, such as his work *On Divisions*.

The works of Apollonius and Archimedes were more advanced and therefore they were less widely studied than the *Elements* in the early modern period. Mathematicians tried to reconstruct some of their treatises that had not survived the 'dark ages'. Such was the case for Apollonius's minor treatises, three of which were reconstructed by Snellius. Apollonius's major work, the *Conics*, was the standard work on conic sections. Although the name may give a different impression, conic sections were generally used for problems in the plane. Archimedes's work, which represented a different style from that of Euclid and Apollonius, also found early modern followers, who developed the so-called 'precalculus'.[3] Snellius did not study this topic.

Some discussion developed among Snellius's contemporaries about the constructions that could be allowed in pure geometry. The cornerstone was a passage in Pappus's *Collection*; this book became more widely known among students of geometry after the publication of its Latin translation by Commandino in 1588. Pappus classified geometrical problems as plane, solid or line-like.[4] These terms did not indicate whether the problems were located in the plane or in space (most were in the plane), but by which type of curves they could be solved.

The first category consisted of plane problems. These were problems that could be solved by only (ideal) ruler and compass, or, in other words, straight lines and circles. It was evident to early modern mathematicians that Euclid's *Elements* could be used as a treasury of standard constructions when plane problems had to be solved. One of the problems under consideration in the next chapters is triangle division. This is an example of a traditional problem, discussed many times since Euclid's to Snellius's time, and solvable by straight lines and circles alone. Two reasons for solving such problems over and over again is that mathematicians of all levels wanted to train their skills and to demonstrate their virtuosity by giving a different turn to an old problem.

[3] So called by most historians, who see forerunners of the calculus of Newton and Leibniz in the infinitesimal methods developed in the early modern period.

[4] I use Bos's translation of the Latin word 'linearis' to avoid confusion with the modern 'linear', [Bos, 2001a, p. 38].

The next category of problems was called solid, which does not (necessarily) mean that the magnitudes under consideration were three-dimensional, but that the problems could be solved by means of the conic sections, combined with straight lines and circles. The problems of the last category, which were called 'line-like', had to be solved by more complicated curves than lines, circles and conic sections. These curves were traced by the points of intersection of other (simpler) moving curves or straight lines. Sometimes (ideal) instruments consisting of moving straight lines were developed for the construction of new curves. An important difference between pure and practical geometry was that in practical geometry more instruments were accepted as tools for problem solving.[5]

It has to be noted that this classification was based on the *solution* of the problem and not on its formulation. Thus, it was often not clear from the outset to which species a certain problem belonged. A geometer might solve a problem by means of a conic section, whereas circles and straight lines would have sufficed. If he did so, he would act against Pappus's 'precept', which stated that geometrical problems had to be solved by the simplest possible curves, or, in other words: 'Problems should be constructed with the means appropriate to their class'.[6] Circles and straight lines counted as the simplest in the hierarchy of curves. This rule was taken very seriously in Snellius's time. However, there was some ambiguity in Pappus's discussion of the class of line-like problems. He seemed to suggest that the construction of curves needed for them was not legitimate in general, but elsewhere in his book his attitude towards construction means such as instruments and complicated curves was more positive. Because his opinion could not be pinned down precisely, his statements left ample space for discussion.[7]

Sometimes, solving a problem was made complicated by the fact that the different possible configurations of the givens dictated a number of sub-problems, which in their turn entailed different cases. This aspect of problem solving was particularly relevant in the Apollonius reconstructions. All these cases had to be studied separately before the general problem could be considered solved. In some cases, a solution only existed if extra restrictions applied. The determination of these different cases and conditions was often a non-trivial part of the solution of the problem, made more difficult by the fact that a figure, even if the draughtsman tried to make it generic, always represented one particular case. Only a careful consideration of all possible combinations of lines and points could yield all cases and criteria for their distinction. Often, only some of the cases were discussed and no rules about the conditions of validity of their solutions were given.

[5] [Bos, 2001a, pp. 37–38].
[6] [Bos, 2001a, p. 49].
[7] [Bos, 2001a, pp. 48–50].

Some of the classical problems had not yet been solved to general satisfaction in Snellius's time, and mathematicians kept proposing new solutions for them. The three most famous ones were: the quadrature of the circle (to construct a square equal in size to a given circle), the trisection of the angle (to divide a given angle into three equal parts) and the duplication of the cube (to construct a cube double in volume to a given cube; equivalent to finding the two mean proportionals between 1 and 2). Of this last problem, a generalization was often studied: to find two mean proportionals between two magnitudes.[8] These new solutions sometimes provoked discussions about acceptable means of construction. The trisection and duplication problems could be solved by means of conic sections. The most famous example of a problem not solvable through the help of either of the standard curves is the quadrature of the circle. Many ingenious solutions had been proposed, and equally many arguments had been used to deny their validity.[9]

Theorems had a minor role in geometry, and their legitimation did not cause so much debate as the correct means of problem solving.[10] Yet remarkably, Snellius's most outspoken contribution to an exactness debate is in the context of the proof of a theorem, Heron's Theorem relating the area of a triangle to the lengths of its sides (see section 7.5).

Snellius's work in the field of pure geometry is spread out over a number of books: the *Fundamenta*, the Apollonius reconstructions, *Cyclometricus* and the *Meetkonst*. He did not devote a single volume to geometrical problem solving, but he was preparing one. Some of the issues raised above are not relevant to his work, for instance conic sections and line-like problems hardly played a role. Much of Snellius's geometrical work is found in his books on topics from mixed mathematics. Geometry was not only an aim in itself, but was also applied in many other fields. Trigonometry, for instance, played a major role in astronomy. This mixed face of geometry is visible in e.g. *Doctrina Triangulorum*, *Eratosthenes Batavus* (see chapter 3) and *Observationes Hassiacae* (see section 4.3.2).

5.2.2 Innovations

In the previous section, it was argued that although geometry had a rich history, uncertainty existed about the proper solution method for a large category of problems. In Snellius's time, change was in the air, and some mathematicians experimented with new tools, hoping to increase the power of solving methods.

[8][Bos, 2001a, p. 27], cp. [Jesseph, 1999, p. 22]. The two mean proportionals of magnitudes a and b are x and y such that $a : x = x : y = y : b$.

[9]For an overview of popular early modern problems, see [Bos, 2001a, pp. 59–94], especially pp. 60–61.

[10]'It should be noted that in the early modern period there was much less concern about the rigor of proofs than there was about the legitimacy of constructions.' [Bos, 2001a, p. 8].

One of the stimuli behind this was the rediscovery of analysis, mainly found in the work of Apollonius. Analysis was a method for finding the solution of a problem, after which it was actually constructed by means of a synthesis using elementary geometrical operations. In a classical analysis, the problem was first assumed to be solved. In principle, the mathematician then argued backwards to the givens of the problem. However, practice must often have been at variance with this theory: mathematicians would start at any point between problem and solution where they saw a fruitful way to connect the two.

Early modern mathematicians were very interested in this Greek method. However, it was surrounded by many questions, because sources explaining its exact meaning and containing examples were rare. Analysis could also be used to prove theorems, but this did not receive much attention in Snellius's time. Snellius sometimes used a classical analysis in his solution of geometrical problems, e.g. when explaining Van Ceulen's construction of a cyclic quadrilateral and in one of the problems in his Apollonius treatises.[11]

It was in the context of the rediscovery of analysis that algebra became a tool in geometrical problem solving. Algebra also had old roots: it mainly stemmed from the medieval Arabic tradition. According to Bos

> from c. 1590 the development of this analytical use of algebra can be identified as the principal dynamics within the early modern tradition of geometrical problem solving.[12]

The key figure in this development was Viète, who invented a whole algebraic framework that could be applied in solving problems. For this purpose, the relations in the problem had to be described by equations involving known magnitudes and one (or more) unknowns. These equations had to be reduced to a standard form which could be handled by a standard geometrical construction.[13] For some time, classical and algebraic analysis were both used. Although algebraic analysis was often a powerful alternative, it also had problematic aspects: for instance the results of algebraic manipulations could not always be interpreted geometrically in a straightforward manner. Moreover, algebra often used numbers, which were traditionally banished from geometry (see the next section).[14] Snellius's work shows some hints of familiarity with the Vietean framework. It has to be noted that in Snellius's time, and for Snellius, 'algebra' sometimes meant calculation with integers, fractions and nested roots, without unknowns.[15]

[11][Bos, 2001a, pp. 95–102]. See section 7.6.1 and [Snellius, 1607b, pp. 12–14].

[12][Bos, 2001a, p. 97].

[13]See also [Bos and Reich, 1990, pp. 185–199] and [Bos, 1996, pp. 187–192].

[14][Bos, 2001a, pp. 95–98, 117]; see pp. 99–117 for examples both of classical and of algebraic analyses of problems.

[15][Andersen and Bos, 2006, p. 702]. During the sixteenth century, 'algebra' came to mean

Some geometrical problems could be tackled by either a more geometrical or a more algebraic approach. A good example of this is Van Ceulen's solution to a problem posed by Goudaen in 1580. It was required to determine the height of a quadrilateral.[16] In the more geometrical approach, Van Ceulen cleverly used some similar triangles to shorten calculations. In the more algebraic approach, he took the length of the required line segment as the unknown, deduced a second degree equation in this unknown and solved it. Of course the two approaches are not completely different: calculations had to be made in the geometrical approach as well, and the algebraic approach needed geometrical results, notably Pythagoras's Theorem. The former asked more creativity of the problem solver in hunting after adequate similar triangles, the latter in calculating with and simplifying complicated root forms.

Another change during this period is the growing enthusiasm of a number of mathematicians for problems involving enormously laborious calculations. Ludolph van Ceulen, who calculated 35 decimals of π and a table of chords of regular polygons, was a representative of this current. This large-scale reckoning, which had its roots in astronomy, made a solid mastering of clever ways of calculating necessary. In Snellius's time, the focus of the calculations was on trigonometrical functions. This field was innovated by the use of irrational numbers and of algebra, which both gradually became more accepted in this part of geometry.[17]

However, these innovations gave some mathematicians, who feared that the proverbial certainty of geometry would be affected, uneasy feelings. Snellius's reaction to the traditional proof of Heron's Theorem must be seen in this light.

5.2.3 The use of numbers in geometry

A central obstacle to the adjustment of algebra for use in geometry was the difficulty which Snellius and his contemporaries experienced in assigning a number to a line segment or to an area.[18] The numbers considered are integers, fractions, square roots of these and nested square roots, no negative or transcendental numbers. Scalar multiplication was not thought of as problematic. Some of the difficulties that were relevant in this period were:

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the theory of equations, and also the theory of the algebraic numbers (roots) necessary to solve them, [Dijksterhuis, 1943, p. 85].

[16] The problem is treated in Van Ceulen's *Solutie en Werkinghe*; my discussion is based on [van Loon, 2002, pp. 84–89].

The problem is as follows: given a quadrilateral ABCD, $\angle C = 90^0$, $AB = a, BC = b, CD = c, DA = d$. Its basis is CD, A is the top. Calculate its height (the distance between A and the basis).

[17] [Bos, 2000a, pp. 260–262].

[18] For a careful analysis of the terms under consideration, many relevant questions and contemporary answers to them, see [Bos, 2001a, pp. 119–158].

1. Numbers were traditionally absent from geometry, which meant that criteria for the exactness of operations involving them had not yet been developed.

2. The absence of a natural candidate for the unit in geometry, that is, the line segment with the same function as the number 1. Multiplying any number a (unequal to zero) by 1 yields a. The nearest operation to multiplication in geometry is rectangle formation, yet it is not possible to form a rectangle which is equal to a given line segment.

3. Connected to this is the next obstacle: the absence of dimensions in arithmetic. For this reason as many numbers as one wishes can be multiplied, and the product is still a number. On the other hand, all geometrical magnitudes have a dimension. Because an object with more than three dimensions was unthinkable in classical geometry, geometrical magnitudes could not be multiplied without problems of interpretation. Moreover, in order to be interpretable, an equation involving geometrical magnitudes had to be homogeneous, which means that all its terms had to have the same dimension.

4. The incommensurability problem. Only if two line segments are commensurable, which means that a magnitude exists of which both are multiples, can they both be expressed by a rational number. It was not clear how the relationship between incommensurable geometrical magnitudes could be described by numbers.

5. The lack of good proof techniques in arithmetic, partly caused by the absence of the concept of an indeterminate number. E.g. when equations involving parameters and an unknown were solved by means of algebra, it was not possible to explore in a general way in which way the necessary manipulations depended on the value of the parameters.

Although Snellius was not a member of the *avant garde* of the innovative mathematicians, he was certainly involved in the issues at stake. The most illustrative examples are his discussion of Heron's Theorem and the role of numbers in its proof, and his discussion of the construction and area of a quadrilateral (see sections 7.5 and 7.6).

5.3 *The* Fundamenta*: a Van Ceulen–Snellius dialogue*

Ludolph van Ceulen's *De Arithmetische en Geometrische Fondamenten* ('Arithmetical and Geometrical Foundations') is a rich work, collecting much standard fare of the period in the fields of arithmetic and geometry, but also containing some innovations. Among these is an original introduction of the use of numbers in geometry (see section 5.5). Other topics include the arithmetic of roots, a summary of results from the *Elements* and problems involving (regular) polygons and circles, some of which are solved with the aid of numbers, trigonometry or algebra. Van Ceulen often informed his readers about the genesis of a problem and its solution, which gives us a better insight into the mathematical practice of the period.

Snellius translated the book of his deceased teacher and friend into Latin as *Fundamenta Arithmetica et Geometrica*. This edition has a special extra feature: Snellius corrected mistakes and added elaborate commentaries, which make it function as a dialogue between Van Ceulen and Snellius. Van Ceulen's work triggered Snellius to study and then comment on his solutions and inventions. He sometimes expressed his approval of Van Ceulen, but on occasion he was in doubt about the value of his former teacher's ideas and every now and then he changed the presentation and added some ideas or inventions of his own. This makes the book a rare source for a historian of mathematics: Snellius does not only do mathematics, but he also talks *about* mathematics, making thus a direct comparison of his and Van Ceulen's approach of the same problems possible. Snellius also reported on some work in progress in his notes and announced that he would make several of his inventions public at a later occasion.[19]

The dialogue between the dead Van Ceulen and the living Snellius in the *Fundamenta* was the follow-up of their discussions and collaboration while both were still alive.[20] Snellius revealed for instance that a problem which Van Ceulen had included without explaining its origins had in fact been proposed by him. He compared his own and Van Ceulen's calculations and remarked that they had found different expressions for the same numbers. Van Ceulen had also included another complicated geometrical problem that had been proposed and solved by Snellius.[21]

Several examples of confrontations of Van Ceulen's and Snellius's approaches to topics will be discussed in the rest of this thesis. Snellius commented on Van Ceulen's rules for calculating with pairs of a line segment and a number and proposed some alternatives, he criticised and improved the traditional proof of Heron's Theorem as embodied in Van Ceulen's version of it, proved the validity of Van Ceulen's constructions of cyclic quadrilaterals, and extended this with a theorem similar to Heron's for the area of such a quadrilateral (see chapter

[19]E.g. [van Ceulen, 1615b, p. 121]. See for the publication history of the book and some of Snellius's purposes with it section 2.9.3. The *Fondamenten* are hardly mentioned in modern literature. See [Bos, 2001a, index] and [Katscher, 1979, pp. 118–120] for exceptions. Most students of Van Ceulen have concentrated on his circle quadrature. See www.wiskonst.nl for some general information on Van Ceulen and commentaries on most of his works.

[20]See section 2.2.

[21][van Ceulen, 1615a, pp. 227, 232], [van Ceulen, 1615b, pp. 215–216, 223].

7). Van Ceulen and Snellius collaborated when solving triangle division problems, for which they also developed their own solutions (see chapter 6). Snellius restructured large sections of the book to make the material fit better into a Euclidean mould (see the example in section 6.4.2).

The *Fundamenta* was republished under a different name:[22] it forms a major part of *De Circulo*, which also contains Snellius's translation of a part of Van Ceulen's other chef d'oeuvre. Snellius made some minor changes and added an appendix.[23] For neither Latin edition, did Snellius have the opportunity to have new figures cut. This is shown both by the fact that all figures were the same as in the *Fondamenten*, including those that have Dutch words written in them,[24] and the regret expressed by Snellius several times about not being able to give extensions to Van Ceulen's work due to lack of proper figures.[25]

5.4 The dedicatory letter: rhetoric and polemic

5.4.1 'Exceptional use'

Snellius's dedicatory letter to the *Fundamenta* is particularly relevant for revealing his opinions on some mathematical issues. This may be somewhat surprising, because the letter does not belong to the mathematical core of the book. However, this core contains predominantly mathematical results and does not digress on mathematical method, which is also the case for other mathematical works by Snellius and others—and a similar statement even holds for humanist scholarship in general. Thus, according to Anthony Grafton, 'In so far as there was a natural place for discussing method, it was the prefatory letter'.[26] The last part of Snellius's letter has a very polemical tone, as was often the case in the genre. His polemic about a seemingly irrelevant matter helps us a great deal in establishing his position in relation to that of other mathematicians. Indeed, as Grafton remarks, 'taking a position on a technical point about which earlier scholars had fallen out was one of the most forceful ways of declaring one's

[22]Some more editions entitled *Fundamenta* are mentioned in library catalogues; I have not seen them. The catalogue of the Bibliothèque Nationale de France has an edition 'Amstelodami, apud H. Laurentium' (1617), the Tresoar in Leeuwarden has 'Lugduni Batavorum, excudebat Georgius Abrahami A Marsse' (1618).

[23]See e.g. [van Ceulen, 1619, p. 188], where the wrong position of a part of a sentence has been corrected, but a new error was introduced. See section 6.4.3 for the appendix on triangle division.

[24]See e.g. [van Ceulen, 1615b, pp. 36, 39, 40].

[25]E.g. [A situation] 'quem ideo quia diagrammate destituimur explicare nunc non possum.' [van Ceulen, 1615b, p. 98].

'Potui aliam etiam concinniorem et parabiliorem factionem afferre, si per operas typographicas et diagrammatum sculptorem liberum fuisset: quamobrem in tempus magis oportunum et hanc, et alias differre cogimur.' [van Ceulen, 1615b, p. 127].

[26][Grafton, 1983, p. 6].

intellectual allegiance'.[27]

Snellius discussed three topics in his letter: the usefulness of mathematics, the use of numbers in geometry and Book X of Euclid's *Elements*. These three are connected in a complex and skilful rhetorical piece. As in other specimina of the genre, the methodological statements are somewhat elusive: not only because a detailed technical exposition did not fit in the genre of a letter, but also because of the two very different purposes which Snellius had with his letter.

In the first place, Snellius wanted to introduce Van Ceulen's book into an international learned circle by his translation from Dutch into Latin. This audience would not see the value of this work as evident: it had been written by a typical *Rechenmeister*, someone from outside their circle, who did not master Latin and Greek and could only access the wealth of Greek mathematics indirectly. A major purpose of Snellius's dedicatory letter was to explain the merits of the *Fundamenta* and thus to elevate the status of Van Ceulen's work and of his own translation at the same time.

In the second place, Snellius had an extra-mathematical purpose with his letter: he wanted the dedicatee to act as his patron and give him a leg up in his career as a professor. The dedicatee was Aemilius Rosendalius, who shared many scientific interests with Snellius and his father. Before addressing him publicly, Snellius had privately asked him for permission to dedicate the book to him and had revealed his expectations of Rosendalius's assistance at the same time.[28]

Snellius started the dedicatory letter with a defence of the usefulness of mathematics, including both pure and mixed mathematics. As examples of practical applications he mentioned weapons, ships, building machines, drawings, optical illusions and the study of heavenly bodies. He further stated the role of the mathematical sciences in elevating the mind:

> for apart from the conspicuous and wide-spread usefulness [of the mathematical sciences] in all parts of life, they also turn the mind and reasoning power away from the senses and direct them 'to the contemplation of being'—because the human soul, which is blinded and buried by barbarian filth, 'is purified and rekindled' by them.[29]

The role of mathematics as an instrument to help the mind rise to the higher world of the 'ideas' was a topos in philosophy, which was most vigorously advocated by Plato, who saw mathematics as an essential part of the education of the statesman-philosopher. Snellius included several quotations of Plato's *Republic*

[27] [Grafton, 1983, p. 7].

[28] See sections 2.8 and 2.9.3.

[29] 'Namque praeter usum, quem per omnes vitae partes habent singularem longe lateque diffusum, mentem quoque et cogitationem a sensibus avocant et convertunt ἐπὶ τὴν τοῦ ὄντος θέαν, his enim animus humanus barbarico caeno occaecatus et infossus ἐκκαθαίρεται καὶ ἀναζωπυρεῖται repurgatur et resuscitatur.' [van Ceulen, 1615b, p. 83].

in this paragraph to accentuate this point.[30]

As the dedicatee was a lawyer, Snellius purposefully gave some examples from the use of mathematics in law. He referred to the rules for the division of inheritances, and of properties in the case of a divorce, harvest and inundations in Roman law and scornfully remarked that some commentators had not been able to explain these rules properly. Rosendalius, who mastered both law and mathematics according to Snellius, could now be presented as a suitable person to dedicate this volume to.

The function of this first part of the letter was threefold. In the first place, it served as part of a general advertisement campaign for mathematics, a derivative of which was the advertising of the *Fundamenta*. The humanist terminology in which this was framed did indeed help to raise the book from a practitioner's work to a specimen of scholarship. In the second place, it put the dedicatee in the limelight. In the third place, this part was so undisputed, not advantaging or excluding any part of mathematics or approach, that the reader could only assent to it. This would make his mind more susceptible to the rest of the letter, which contained more controversial material.

The next topic raised in the letter was the use of numbers in geometry. Snellius defended Van Ceulen's experiments with this use in the *Fundamenta*. He claimed that numbers could accurately describe geometrical magnitudes and their relations:

> [...] that we have sometimes also admitted numbers in the company of this exactness [sc. of geometry], because number is the exact translator of every common measure, ratio and proportion.[31]

Snellius invoked Aristotle's support for this point of view, quoting Aristotle's phrase that 'arithmetic is more exact than geometry'. It seems that Snellius read too much into this passage when he wrote that:

> I fully agree with him if he means to say that the parts of whichever magnitude and the small parts of parts are more accurately expressed in numbers because of the infinite divisibility, which the geometer cannot obtain in his actual construction.[32]

Snellius seems to convey the message that two almost equal numbers can still be distinguished by expressing them in (decimal) fractions that only differ at the

[30]The two quotations above stem from Plato, *Republic* VII 525 a ('to the contemplation [...]') and VII 527 d ('purified and rekindled').

[31]'[...] quandoque numeros quoque in huius subtilitatis societatem admiserimus. Est enim numerus omnis commensus, rationis et proportionis accuratus interpres.' [van Ceulen, 1615b, p. 84].

[32]'[...] cui, si id dicat numeris ob infinitam sectionem, quam Geometra actu non assequatur, cuiuslibet magnitudinis partes et partium particulas accuratius exprimere, plane assentior.' [van Ceulen, 1615b, p. 84].

end, whereas the difference between two line segments of almost equal lengths can neither be observed nor constructed. The assertion is somewhat obscure, however, mainly because Snellius did not clarify his meaning by an example here or elsewhere. If the interpretation is correct, the statement is true, but hardly relevant to traditional geometers.

Snellius's interpretation of Aristotle is remarkably far removed from the source text. The Greek philosopher had in fact written:

> Of the sciences the most exact are those which are most concerned with first principles; those sciences which are based on fewer principles are more exact than those which are more conditioned: thus arithmetic is more exact than geometry.[33]

What Aristotle meant by this, is explained in a passage in the *Posterior Analytics*: geometry requires additional elements if compared to arithmetic, because numbers, which are only substance, are the subject of arithmetic, whereas in geometry points are studied, which are substance which have position.[34]

The discrepancy between Snellius's interpretation and the meaning conveyed by Aristotle himself can be explained by assuming that Snellius wanted to make the reader, who would not doubt Aristotle's authority, consent more easily with Snellius's potentially disputable statements on the use of numbers in geometry. The attentive reader could see that he was not cheated by Snellius, who had carefully written: '*if* he means'. This creative use of classical quotations, to be understood sometimes with, sometimes without their original context, was common among humanist scholars. In this way, they could show their erudition and express their own thoughts with classical quotations as their building material.

The ingenuity of this passage lies in the fact that it—again—elevates Van Ceulen's work. His assignment of numerical values to geometrical objects could be interpreted as just a normal action stemming from applied mathematics, but Snellius tried to give a theoretical justification for it, thus making it into a methodology founded in Aristotelian principles. In this place, he gave his unqualified assent to Van Ceulen's approach. If he wanted to captivate and convince the readers, he could not immediately raise objections. Yet the main text shows that Snellius was no unconditional supporter of Van Ceulen's way of employing numbers in geometry (see section 5.5 and the next two chapters).

Snellius did not elaborate on the concept of 'numbers' either in the dedicatory letter or in the main text. In his *Theses*, he had stated that irrational numbers

[33] *Metaphysics*, A, 2.982a 25–28, translation from [Heath, 1949, pp. 4–5].

[34] *Posterior Analytics* A, 27. 87a 31–37. This and related passages are given in English translation in [Heath, 1949, p. 5]; cp. [Heath, 1949, pp. 64–67].

Snellius may have found the quotation in Proclus's *Commentary on the First Book of Euclid's Elements*, but the interpretation cannot have been Proclus's, because his rendering of Aristotle's text was much closer to the original. [Proclus, 1970, pp. 47–48].

are not really numbers,[35] but here he was willing to accept Van Ceulen's root expressions, which were irrational, as numbers. He did not explain if and how irrational numbers should be approximated by fractions—apparently, once the principal step to express geometrical magnitudes in numbers was taken, the practical calculation rules for roots, which were discussed elaborately in the *Fundamenta*, were permissible.[36]

5.4.2 The 'cross for excellent minds'

After his free interpretation of the Aristotle quotation, Snellius connected Euclid's *Elements* X to the topic of the use of numbers in geometry:

> And therefore, we need not object to the use of numbers by the lovers of learning, and especially that of irrational and surd numbers, as is shown in these books. And it is even more important [to not do so], in order to make clear to everyone how useless this Pythagorean distribution of irrationality into thirteen species is for application. Euclid devoted the whole of the tenth book of the *Elements* to this distribution, although these general laws of writing numbers pay no attention to the question to which species the various numbers should be confined. For in fact, there exists one general rule for this writing of numbers.[37]

Before analyzing this quotation further, it is necessary to explain briefly the contents and reception of *Elements* X. Snellius was certainly not the first mathematician who had an awkward relationship with this book. It is devoted to the theory of the commensurability and incommensurability of magnitudes (often, but not necessarily, interpreted as line segments). Two magnitudes are called commensurable if a magnitude can be found of which they are both multiples. This concept was extended to commensurability 'in square': two straight lines are commensurable in square when their squares are 'measured by the same

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[35] 'ἄλογοι et ἄρρητοι qui vocantur numeri, vere numeri non sunt, cum per illos nihil diserte et explicite numeretur; sed facta resolutione tantum proxime.' [Snellius, 1608b, fol. $A3^r$].

[36] See e.g. [van Ceulen, 1615b, p. 149], where the area of the triangle with sides $30-30-30$ is determined. It is not stated explicitly that the rational expression of the value of the root involved in the calculation is an approximation.

[37] 'Eam ob causam numerorum, maxime irrationalium et surdorum usum istis libris illustratum philomatis invidere non debuimus: idque adeo tanto magis, ut clarum cuilibet sit, quantopere ad usum inutilis sit Pythagorea illa ἀλογίας in tredecim species distributio, in qua Euclides, totum 10 Elementorum librum occupavit, cum generales istae numerationis leges nihil pensi habeant ad quamnam speciem hic vel ille numerus sit referendus. Una enim et catholica huius numerationis regula est.' [van Ceulen, 1615b, p. 84].

For 'numeratio' as the writing and reading of integers in Hindu-Arabic notation, and sometimes some number theory, see [Kool, 1999, pp. 61–67]. Snellius extended the meaning, including irrational numbers as well.

area', which means that an area can be found of which they are both multiples. Another key idea in the book is that of rationality: straight lines are called rational if they are commensurable in length or in square with some reference line (which is also called rational).[38]

In the main part of Book X, the largest of the *Elements*, a classification of certain types of irrational magnitudes is unfolded. Ever since its appearance, the book has baffled its readers, especially because of its difficulty and unclear purpose (its application in Book XIII did not convince everyone of its use). After Antiquity, problems of interpretation of *Elements* X arose due to its unclear relationship to the fields of geometry and arithmetic. One of the methods used to get a grip on the subject matter was to describe it in algebraic-arithmetical terms, traces of which can already be found in the Arab world in the ninth and tenth century.[39] Although a description of the material by means of fractions and nested roots tempted later mathematicians, because it seemed to order the mass of material, this is not true to the character of the book in at least one respect: its foundations are geometrical, not arithmetical. This is proved by, among other things, the privileged position of square roots in the arithmetical translation: roots of higher degree are lacking because they do not correspond to line segments constructible by ruler and compass alone. Modern researchers still find it difficult to understand the purpose of Euclid's book, in which 'the study of the theory is transformed into unbearable tedium, while its few central ideas are overwhelmed by the mass of repetitious detail'[40] and which is 'a pedagogical disaster',[41] as writes Wilbur Knorr, who tried to uncover those central ideas.

A violent attack on the problematic tenth book was launched by Petrus Ramus in 1569. He devoted the 21st to 25th books of his *Scholae Mathematicae* to the subject matter of Euclid's Book X. After having given some general negative remarks, he criticised its contents in more detail and finally gave his own arithmetical-algebraic interpretation of the material contained in the book.

In the beginning of his assessment of Book X, Ramus made his negative judgement immediately clear to the reader. He considered Book X as useless and obscure, not because of the difficulty of its contents, but because of the absence of any indication of its underlying structure or its relevance:

> This subject[42] then is proposed in Book X and set out in such a way

[38] *Elements* X, defs. 2 and 3, [Euclid, s a, 3, p. 10].

[39] [Euclid, 1998, p. 15]. Vitrac also devotes some attention to the early modern reception of Book X: see [Euclid, 1998, pp. 13–15] and several places in the commentary.

[40] [Knorr, 1985, p. 34].

[41] [Knorr, 1983, p. 59]. See especially [Knorr, 1985, pp. 18–19] for the central ideas. Cp. [Fowler, 1987, pp. 190–192] for a modern interpretation, stressing its geometrical character.

[42] Before, Ramus had introduced the concept of (in)commensurability and explained that no 'smallest measure' existed in the realm of magnitudes; and that on the contrary in the realm of numbers, there is such as smallest measure, namely the unit.

> that nowhere in the arts and sciences have I uncovered an obscurity like this one: I do not mean that what Euclid is trying to teach is obscure to understand (because that which is present, and which is easily accessible, can be clear even for the unlearned and illiterate when they look at it), but to see through it completely and to find out what purpose and use are intended in the book, and [to find out] which are the kinds, species and differences[43] of the topics under discussion, because I have never read or heard anything so confused or intricate.[44]

Ramus proceeded to blame the 'superstition of the Pythagoreans' for the obscure exposition of the book and even contemplated excluding its subject matter from geometry, fixing his mind on the part of geometry that could be applied in some way:

> No part of geometry (if at least these subtleties should have any place in the real practice of geometry) is less useful, yet none more overloaded with precepts and theorems.[45]

He expanded this viewpoint in the next part of his exposition, where he argued how Book X gave both too little and too much: too little, because it did not actually help the reader understand the cause of the fact that certain magnitudes are irrational, and too much, because it would be enough to know if two magnitudes are rational or irrational; the manipulations of different categories of irrationality were inane.[46] He concluded that the essence from Book X should be liberated from its 'cross', that is, the complexity of the subject and its obscure exposition, a metaphor probably selected because of the similar forms of a cross and the capital X.[47]

In that way the book could finally be studied properly:

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[43]Here we recognize Ramus's predilection for divisions, from the general to the special.

[44]'Haec igitur materies est decimo libro proposita et eo modo tradita, ut in humanis literis atque artibus similem obscuritatem nusquam deprehenderim: obscuritatem dico non ad intelligendum quid praecipiat Euclides (id enim vel indoctis et illiteratis id solum quod adest, quodque praesens est intuentibus possit esse perspicuum) sed ad perspiciendum penitus et explorandum quis finis et usus sit operi propositus, quae genera, species, differentiae sint rerum subiectarum: nihil enim unquam tam confusum vel involutum legi vel audivi.' [Ramus, 1569, pp. 257–258].

[45]'[...] nulla pars geometriae (si tamen in vero geometriae usu locum ullum acumina ista habitura sint) inutilior, nulla tamen praeceptis et theorematis cumulatior.' [Ramus, 1569, p. 258].

[46]'[...] nullum tamen verbum est in Euclide ad demonstrandum quamobrem aut quomodo sint haec irrationalia [...] Et tamen si quid irrationale esset, sciri oportuit, satis fuit generaliter sciri, quia hoc uno argumento tenearis tales lineas numero datae mensurae inexplicabiles esse, qua generis specie quave differentia exquirere, vanus et inanis labor fuerit.' [Ramus, 1569, p. 258].

[47]*Crux* in classical Latin had the literal meaning of 'frame of execution, cross' and the transferred meaning of 'trouble, misery', [Lewis and Short, 1879, pp. 485–486].

> I for my part, having examined the whole of Book X earnestly and accurately, have not been able to reach any other conclusion than that a cross has been fixed to it in order to torture excellent minds on it. Therefore we must struggle with all our endeavour and zeal to unravel these topics most clearly, and to overturn and destroy the miserable and dismal cross and cast it down forever.[48]

After this devastating criticism, which was continued in more detail in the next two books (books 22 and 23), Ramus announced that he would 'deduce the whole material of the book on irrational lines with our method and in our way as if it were a scientific discipline',[49] to show that if it was exposed as clearly as possible and finally could be judged properly, it would appear that it really was useless. As an introduction to this exposition he gave an overview of calculations with integers, fractions, square roots and nested roots (book 24) and he ended his discussion of Book X with a very short rearrangement of its propositions, without proofs, but with references to Euclid's proposition numbers, some calculations and numerical examples (book 25).

Ramus's critical attitude can better be understood if it is seen against the background of his programme to reform the school curriculum and to give mathematics a central place in it. He wanted to attain this central position by showing how relevant and useful mathematics was. In his view, Book X contradicted these advantages of mathematics and it could frighten away those people who were needed for the development of the new curriculum: teachers, university officials and patrons. Moreover, he analysed and criticised the *Elements* in the first place for the benefit of students, more than for professional mathematicians, therefore attaching much value to accessibility. In fact, in Ramus's time not many students would have studied the *Elements* as far as Book X. In its Euclidean form, the complexity of Book X would certainly have baffled them. The absence of proofs in Ramus's own exposition fitted in his system, where in general insight in the truth of propositions was not gained by the study of their formal proof, but where the place of a proposition in a deductive scheme should convince the student of its truth.[50]

To conclude, Book X seems to have been a scapegoat for Ramus, containing every reproachable aspect of Euclidean geometry in a very explicit form: there were no connections with practice, it was badly structured, and its purpose and exposition were unclear. By distancing himself from the presumed shortcomings

[48] 'Equidem toto decimo libro studiose et accurate considerato nihil aliud iudicare potui quam crucem in eo fixam esse, qua generosae mentes cruciarentur. Quare omni studio diligentiaque connitendum nobis est, ut ista clarissime evolvantur, miseraque et funesta crux evertatur et prosternatur, atque in perpetuum affligatur.' [Ramus, 1569, p. 258].

[49] '[...] totam libri materiam de lineis irrationalibus nostra ratione atque via tanquam artem aliquam deducere.' [Ramus, 1569, p. 274].

[50] For Ramus and mathematics, see further section 2.5.2.

of Book X, Ramus must have hoped to gain more support for his strive to acquire a central role for mathematics in education.

Although not everyone shared Ramus's opinion of *Elements* X, his animadversion was influential. It even went so far that, according to Henk Bos,

> the uselessness of *Elements* X became a kind of partisan slogan of those who favored the use of irrational numbers to simplify matters in geometry.[51]

An example of such an advocate of numbers was Simon Stevin, who was also censorious, but less than Ramus. He devoted a separate part of his book *La pratique d'Arithmétique* (1585) to incommensurable quantities. In an appendix to the book, he gave his own version of Book X in which numbers were central, leaving out some of Euclid's categories to avoid a 'useless loss of time'[52]. In his preface to the reader of this *Traicté des incommensurables grandeurs*, he repeated Ramus's metaphor:

> the other commentators judged these propositions too obscure and the cross of the mathematicians.[53]

When reconsidering Snellius's statement about Book X in his dedicatory letter (see p. 192), we see that he considered its subject matter as (irrational) numbers, not as geometrical magnitudes. He argued that it was senseless to categorize these numbers, because the same (writing) rules applied to all categories. This attitude echoes Stevin's inclusive conception, who had argued that 'number is that by which the quantity of everything is expressed.'[54]

Snellius professed that he was not very impressed by the application of the theory of incommensurability to the study of the five regular solids, although Proclus had stated that the construction of those solids was one of the principal aims of the *Elements*:

> According to Proclus, Euclid however (a philosopher of the Pythagorean sect), turned his mind most of all to the Pythagorean construction

[51][Bos, 2001a, p. 138].

[52]'inutile perdition de temps'. [Stevin, 1585, p. 186].

[53]'les autres [commentateurs iugeoient] que ce sont propositions trop obscures, et la croix des Mathematiciens.' [Stevin, 1585, p. 162]. The preface is reprinted, with an English summary, in [Stevin (D.J. Struik ed.), 1958, B, pp. 713–721]. See [Stevin, 1585, pp. 161–201] for the appendix.

See for a summary and analysis of Stevin's point of view [Bos, 2001a, pp. 138–141]. Bos also indicates the weakness of Stevin's approach, which is the lack of a proper definition of number such that the existence of all Stevin's 'numbers' could be proved. 'However, such precision came to mathematics only in the late nineteenth century, so Stevin's defence of numbers may well have seemed, although perhaps not ultimately convincing, yet strong and legitimate enough.' [Bos, 2001a, p. 141].

[54]'Definition II. Nombre est cela, par lequel s'explique la quantité de chascune chose.' [Stevin, 1585, p. 1].

> of the solid bodies, as though that was the highest reward and final purpose of geometry.[55]

Just like Ramus, Snellius assumed that the Pythagoreans were responsible for the impractical side of the *Elements*.

Snellius then boldly stated that the contents of Book X appear nowhere in the work of Archimedes, Apollonius, Serenus, Theodosius, Menelaus, Ptolemy, Eutocius, Diophantus or even Euclid outside the *Elements*. Therefore, Book X should be disposed of, at least in education:

> therefore [we can say that] the cross has only been stuck on top of it, and that it can be removed very easily just by calculations on the abacus. And although these [discussions] could be stored in the mathematical library as mere subtleties, they should still be separated from the elementary instruction, as being less useful. Because in case they should turn out to be useful, then this entire subspecies (of which that book [Elements X] undertakes to explain merely a part), would no doubt include more faraway hidden learning and wisdom.[56]

Clearly, Snellius was not very patient with mathematical knowledge for its own sake here. He wanted mathematical learning to be useful, which means that it should be relevant outside mathematics, or that it could be applied in some other parts of mathematics. He acknowledged that more learning such as that in Book X could exist: not completely without value, but too far away from common experience to be worth the effort of understanding for anyone but an extremely small group.

Already in 1608, Snellius had defended the view that Book X should be removed from the *Elements* in his *Theses Philosophicae*:

> Since Book X of Euclid's *Elements* discusses only a special theme, and has for the larger part been derived from the general source of algebra, it must not be counted among the elements of geometry.[57]

[55]'Verum Euclides, Pythagoreae sectae philosophus, potissimum se ad Pythagoream solidorum corporum adscriptionem composuit, inquit Proclus, tanquam illud esset Geometriae summum bonum et finis extremus.' [van Ceulen, 1615b, p. 84].

Proclus had written: 'This, then, is its [sc. from the *Elements*] aim: both to furnish the learner with an introduction to the science as a whole and to present the construction of the several cosmic figures.' [Proclus, 1970, p. 59].

[56]'[...] crux igitur quaedam istic tantum defixa est, quae solo calculo in abaco facillime tollatur: et quamvis ista tanquam subtilia in Mathematica bibliotheca conservari possint: attamen ut minus utilia a στοιχειώσει segregari debent. nam si ista usum habeant, totum hoc genus, cuius ille liber particulam duntaxat aliquam explicandam sibi sumit haud dubie plus longe reconditae eruditionis et scientiae complectetur.' [van Ceulen, 1615b, p. 84].

[57]'Liber X. Elementorum Euclidis cum specialis tantum sit, ex generali Algebrae fonte maximam partem derivatus, inter elementa Geometrica connumerari non debet.' [Snellius, 1608b, fol. A4^r].

Book X could be described as algebraic because it discussed irrational magnitudes, and numbers expressing them often turned up in the solution of equations, which was the domain of algebra.[58] The emphasis on the right hierarchy of general and specific was contained in one of Ramus's laws. The division of a science in a general and a special part was also, though not uniquely, Ramist.[59]

Snellius continued his dedicatory letter in a slightly more moderate tone:

> But anyhow, since it is certain that Euclid has intimately connected the study of these [irrationalities] with bare [i.e. without numbers] lines and magnitudes (very certain proofs of which can be found everywhere in Antiquity, in the work of Archimedes, Eutocius, Ptolemy and others), it is no wonder that logicians of a better insight have removed this unfruitful exactness and have directed their discussion back to calculations with irrational and surd numbers, just like the most celebrated mathematicians of this age do. For do they in fact take their hands off this work, those who are the most fervent students of this kind of calculation, and who waste all their time doing this only, when the numbers grow exceedingly, and as it were collapse by their own weight?[60]

In this last part, Snellius propagated the algebraic-arithmetical approach to Book X, substantiating his argument by an appeal to the authority of some logicians and modern mathematicians, of whom he did not give the names, but among whom Ramus, who had always stressed the importance of the right (logical) method, and Stevin are certainly meant. The reference to the *cross* reflects the words of both Ramus and Stevin.

When Snellius entered upon the Book X-issue, he took a position on a controversial topic. Van Ceulen's *Fondamenten* did not make a discussion of Euclid's Book X necessary, because he usually calculated with well-chosen numerical examples, containing only integers and (nested) square roots, and he did not refer to it himself, perhaps unaware of it. Moreover, Book X did not forbid the use

[58] Snellius usually meant calculating with integers, fractions and (nested) roots when using the word 'algebra'.

[59] The *lex sapientiae*, for which see section 2.5.1.

[60] 'atqui cum certum sit Euclidem istarum contemplationem nudis lineis et magnitudinibus astrinxisse (eius enim rei documenta in omni antiquitate exstant certissima, apud Archimedem Eutocium, Ptolemaeum, alios) non mirum est purioris iudicii logicos sublata illa sterili ἀκριβολογία *earum* tractationem ad surdorum et irrationalium logisticam cum celeberrimis huius aevi Mathematicis reiecisse. Nam et illi ipsi qui huius numerationis sunt studiosissimi, quique omnem aetatem in ea sola triverunt, cum numeri ultra modum excrescunt, et quasi mole sua ruunt, ecquid manum de tabula tollunt?' [*earum*: *Fundamenta* reads *carum*; I corrected to *earum*.] [van Ceulen, 1615b, p. 84].

For the last expression, cp. C. Plinius Secundus, *Naturalis Historia* 35.80.5: '[...] quod manum de tabula sciret tollere, memorabili praecepto nocere saepe nimiam diligentiam.'

of numbers in geometry anyway. This seeming irrelevance makes Snellius's vehement statement all the more remarkable, because he must have chosen it very consciously, taking the risk that it could jeopardize the acceptance of the *Fundamenta*. He may have selected it to show that he was capable of judging a theoretical question, but objected to mathematics concentrating on arcane theoretical subtleties, in other words: to show that he knew the right middle way between pedestrian everyday applications and ethereal futilities. Probably, he assumed that well educated persons like Rosendalius had heard of the notoriously inaccessible Book X.

Moreover, there may be an explanation for this choice that goes back to Snellius's youth. Although this motive is uncertain, it is worth pondering. The explanation is based on a letter written by Adrianus Romanus to Clavius in 1595. In this period, Scaliger and Van Ceulen were involved in a dispute about the former's erroneous quadrature of the circle; Romanus kept his correspondent informed about this affair. He told him that Van Ceulen had written a refutation of Scaliger's *Appendix ad Cyclometrica*. The letter in which Van Ceulen's manuscript was contained was taken to Scaliger by a certain young man, who was in Scaliger's company on a daily basis and who was taught arithmetic by Van Ceulen. Continuing with Romanus's rendering of Van Ceulen's letter to him, we read that Scaliger stubbornly persevered in his error and that the young man had reported to Van Ceulen that Scaliger had begun to study Book X of Euclid and arithmetic diligently. Romanus supposed that Scaliger did so to enable him to understand Van Ceulen's answer, in which undoubtedly incommensurable or irrational numbers would have been included.[61]

The identity of this young man is not given, yet Snellius is a very probable candidate. He was a young man, fifteen years old, when the letter was written, was in touch with Scaliger and a pupil of Van Ceulen, as has been shown in the biography chapter. The scale of the university was very small, which makes the number of candidates for the 'young man' limited, especially because an average university student would neither attend Scaliger's *privatissima*, nor follow lessons in a subject as practical as arithmetic with a teacher from outside the university.

If we do indeed suppose that the reference is to Snellius, we can imagine that this episode was most unpleasant for him. Two of his teachers disagreed fundamentally on the solution of a hallmark mathematical problem. Scaliger tried to defeat Van Ceulen by disdainfully questioning his social status, whereas Van Ceulen was superior in terms of the value of his mathematical arguments. As the messenger between the two, Snellius may have felt pressed to choose sides. Although his mathematical education must have enabled him to understand the

[61][Bockstaele, 1976, pp. 120–121]. For Van Ceulen and Scaliger see chapter 2, especially section 2.4.

truth of Van Ceulen's objections, it cannot have been easy to take sides with him against Scaliger. The latter did not only embody humanist scholarship at the highest level, but he could also develop Snellius's philological skills and help him enter a more prestigious world than that in which Van Ceulen functioned.

This loyalty conflict, so symbolic for the quest for the true nature of and best position for his own mathematics, must have made a big impression on Snellius. Even if he was not able to understand all mathematical arguments and judge the value of the positions of the two fighting cocks as a boy, he would have been able to judge them as an adult. When he wrote the dedicatory letter to the *Fundamenta*, both his teachers had been dead for several years. By doing away with Book X, which had apparently played a role in the conflict, he may symbolically have tried to remove a bone of contention twenty years later.

Returning to firmer ground, we can certainly conclude that the connection between the approval of the use of numbers in geometry and the rejection of Book X shows the influence of Ramus and Stevin. Both of them had tried to make this book easier or even superfluous by a description of its contents by means of numbers. Neither they, nor Snellius could see any function of the book in solving geometrical problems, nor could any of them think of another useful application of its material. Snellius, instead of trying to grasp its hidden purpose, welcomed the innovation of the expression of ratios as numbers, in this case following the more recent authority of Ramus and Stevin. Assumptions about the aim of geometry also play a role. Snellius was focused on geometrical problem solving, in which (irrational) numbers can be used as an auxiliary device to increase efficiency, whereas Euclid built an axiomatic-deductive geometrical structure, in which the study of (in)commensurable magnitudes has a significant (though hard to grasp) part.

Nowhere in the letter did Snellius explicitly acknowledge his indebtedness to either Ramus or Stevin, but this need not surprise us, because he mainly used the letter as an advertisement for himself.[62] Nonetheless, his references, most notably to Ramus, are so clear that a large part of the expert audience must have recognized them. Implicitly, Snellius thus endorsed the Ramist programme for the propagation of lucid and practical mathematics.

After the denunciation of Book X, Snellius referred the reader to book V of the *Fundamenta* to see what Van Ceulen and he had actually done. This book deals with all sorts of geometrical problems that are solved by geometry or 'algebra' (calculations with both roots and unknowns). Some of the topics are the construction of a cyclic quadrilateral (see section 7.6) and triangle division (see sections 6.3.1 and 6.4.2).[63]

[62] Cp. [Grafton, 1983, p. 6].

[63] 'Problematum miscellaneorum liber *quintus*, quae hic vel Geometrice per solas lineas, vel per canonem triangulorum, aut denique per Algebricas positiones solvuntur.' [*quintus*: *Fundamenta* reads *quartus*; I corrected to *quintus*.] [van Ceulen, 1615b, p. 185].

At the end of the dedicatory letter, Snellius repeated the topical reference to *use* once more:

> To conclude: this handling of numbers must be approved in so far as profit flows forth from thence to other fields as well.[64]

To summarize, the dedicatory letter to the *Fundamenta* shows that Snellius was a competent humanist, who mastered rhetoric well enough to be able to write a showpiece, starting with some commonplaces, then addressing more controversial issues and showing the sharpness of his wit, and finally mitigating his tone again to show his reasonableness. He promoted the style of Van Ceulen, that of geometrical problem solving made more efficient by means of numbers, by developing a theoretical framework for it in a nutshell, leaning on earlier work by Ramus and Stevin. This style had a different goal than that of the *Elements*, yet that work was preserved as a treasury of geometrical tools.

Thus, this two page dedicatory letter, hidden in the middle of a book that Snellius had only translated, in fact contains a splendid summary of Snellius's mathematical style: he strove after a good balance of theoretical and practical mathematics, welcomed usefulness, hated obscurity and was open to moderate innovations of classical geometry. As a humanist, he found valuable material in the ancient authors, to whom he added Ramus as a new authority, but he was perfectly capable of voicing his own opinions and constructing his own text on the basis of these. And not unimportantly: the letter had a function in the extra-mathematical world, where it was an instrument to receive promotion in the academic hierarchy.

Snellius showed in this dedicatory letter that he dared to expose himself, following Ramus, who earlier had made Euclid's *Elements* X into a target for those who saw the advantages of a more relaxed attitude towards the use of numbers in geometry. Because the plan that Euclid had had with the book was difficult to discover, it did not receive much attention from those who considered its material as redundant, yet it was sought for and defended by others; some examples of this will now be discussed.

5.4.3 'The faithful disciple of Ramus'

Snellius's hard judgement caused hard reactions from some of these devotees of Euclid. They show that at least some of his contemporaries were provoked by the dedicatory letter. Kepler, who complimented Snellius in other places, was not willing to let this condemnation pass.[65] In the preface to the *Harmony of*

[64] 'Est itaque numerorum ista tractatio eatenus probanda, quatenus ad alia etiam aliqua utilitas inde redundet.' [van Ceulen, 1615b, p. 84].

[65] Cp. [Bos, 2001a, pp. 142, 187] for a short discussion of Kepler's reaction.

the World (1619), he criticised Ramus and his followers Lazarus Schonerus and Snellius for their lack of appreciation of *Elements* X.

The main underlying question is about the use of mathematics: is mathematics meant for everyday life or for deep insight into elevated truths?

> Nor is this yet the end of the damage which Ramus has inflicted on us. Consider the most ingenious of today's geometers, Snellius, clearly a supporter of Ramus, in his preface to the *Problems*[66] of Ludolph van Ceulen. First he says, 'That division of the inexpressibles into thirteen species is useless for application.' I concede that, if he is to recognize no application unless it is in everyday life, and if there is to be no application of the study of nature to life. But why does he not follow Proclus, whom he mentions, and who recognizes that there is some greater good in geometry than just being an art necessary for living? If he had accepted that, the application of the tenth Book in deciding the kinds of figures would have been evident.[67]

Snellius's use of numbers to facilitate the handling of irrationals is also condemned:

> 'Is it only a cross fastened to our talents?' I say, to those who molest the inexpressibles with numbers, that is by expressing them. But I deal with those kinds not with numbers, not by algebra, but by mental processes of reasoning, because of course I do not need them in order to draw up accounts of merchandise, but to explain the causes of things.[68]

Kepler's last remark about Snellius's preface deals with the real character of the *Elements*: is it just a textbook for students loaded with valuable material or does it have a plan of its own? Kepler is convinced of the second option:

> [Snellius] plays in every way the part of the faithful disciple of Ramus, and this builder does not fail to do his job properly: Ramus removed

[66]Kepler means the dedicatory letter to the *Fundamenta*.

[67]'Nec dum finis est damni, quod Ramus nobis dedit, ecce sollertissimum Geometrarum hodiernorum Snellium, plane suffragantem Ramo, praefatione in Ludolphi a Coellen Problemata: primum ait, *ad usum inutilem esse divisionem illam ineffabilium in tredecim species*. Concedo, si nullum ille usum agnoscat, nisi in vita communi, et si nullus contemplationum physicarum sit usus ad vitam. At cur non Proclum sequitur, quem allegat, qui agnoscit aliquod maius Geometriae bonum, quam sunt artes ad vitam necessariae? tunc equidem et decimi libri usus apparuisset in aestimandis figurarum speciebus.' [Kepler (M. Caspar ed.), 1940, p. 18], translation with small adaptations from [Kepler, 1997, pp. 12-13]. Kepler does not quote Snellius literally.

[68]'*Crux tantum defixa est ingeniis?* Equidem iis, qui numeris, hoc est effando vexant Ineffabilia. At ego has species tracto non numeris, non per Algebram, sed ratiocinatione Mentis; sane quia iis mihi non est opus ad subducendas Rationes mercatuum, sed ad explicandas rerum causas.' [Kepler (M. Caspar ed.), 1940, pp. 18-19], translation from [Kepler, 1997, p. 13].

> the form from Euclid's edifice, and tore down the coping stone, the five solids. By their removal every joint was loosened, the walls stand split, the arches threatening to collapse. Snellius therefore takes away the stonework as well, seeing that there is no application for it except for the stability of the house which was joined together under the five solids. How fortunate is the disciple's understanding, and how dexterously did he learn from Ramus to understand Euclid: that is, they think that the *Elements* is so called because there is found in Euclid a wealth of every kind of propositions and problems and theorems, for every kind of quantities and of the arts concerned with them, whereas the book is called *Elementary Primer* from its form, because the following proposition always depends on the preceding one right up to the last one of the last Book (and partly also that of the ninth Book), which cannot do without any of the previous ones. Instead of an architect they make Euclid a builders' merchant or a bailiff, thinking that he wrote his book in order to accommodate everybody else, and that he was the only one without a home of his own.[69]

We see that Kepler considered Proclus's explanation of the purpose of the *Elements* as valid. He showed that he was an orthodox Euclidean, not receptive to the alternative visions on geometry of Ramus, Van Ceulen and Snellius. In fact, Kepler's critique did not do justice to the thoughtfulness of Snellius's views as it is in particular manifested in the rest of the *Fundamenta*. Both in Snellius's and in Kepler's case, the dedicatory letter was no place for subtleties.

In 1638, the Polish mathematician Joannes Broscius expressed his disapproval of Ramus and Snellius in his *Aristoteles et Euclides defensus contra Petrum Ramum*, although just like Kepler, he showed respect for Snellius: 'I would not like to see Euclid condemned by the very learned Snellius in such a quick prejudice.'[70] He thought, however, that Snellius and Viète were mistaken

[69]'Omnino fidum Rami discipulum agit, nec ineptam locat operam: Ramus Aedificio Euclideo formam ademit, culmen proruit, quinque corpora; quibus ablatis, compages omnis dissoluta fuit, stant muri fissi, fornices in ruinam minaces: Snellius igitur etiam Caementum aufert, ut cuius nisi ad soliditatem domus sub quinque figuris coagmentatae nullus est usus. O foelicem captum discipuli, quam ille dextre Euclidem intelligere didicit a Ramo: sc. ideo putant Στοιχεῖα dicta, quod inveniatur in Euclide propositionum et problematum et Theorematum omnivaria copia, ad omne genus Quantitatum artiumque circa illas occupatarum: cum liber Στοιχείωσις sit dictus a forma, quod semper sequens propositio innitatur praecedenti, usque ad ultimam libri ultimi (partim et libri noni) quae nulla priorum carere potest. Ex Architecto saltuarium faciunt aut materiarium, existimantes Euclidem ideo librum suum scripsisse, ut omnibus aliis commodaret, solus ipse propriam domum nullam haberet.' [Kepler (M. Caspar ed.), 1940, p. 19], translation adaptated from [Kepler, 1997, pp. 13–14].

[70]Quoted from the 1652 edition, entitled *Apologia pro Aristotele et Euclide, contra Petrum Ramum, et alios*. Jan Brożek (Kurzelow, 1585 – Krakow, 1652) was influenced by his contacts with Adrianus Romanus. He had different chairs in the University of Krakow. He published

in preferring Ramus to Aristotle and Euclid.[71]

Although Broscius shared Kepler's admiration of Euclid, his arguments for the defence of Euclid against the charge of the futility of Book X are different and he may not have known Kepler's preface to the *Harmony of the World*. Broscius first said that only in the future would the use of many parts of science become clear, thus showing a more modest view of human knowledge than Snellius and Kepler:

> Time, father of all, will teach either us or our descendants many things which are now frowned upon as useless.[72]

He repudiated Snellius's proposal to use 'normal calculations' to clarify the obscurity of the book, in favour of an approach in which the figure is central. This is in fact also a departure from pure Euclidean geometry, probably motivated by Broscius's activities as a teacher.

> That calculation without a geometrical description will be senseless. The causes of the calculation are better understood from a figure. This same [approach] must be advised in the calculation of irrational [magnitudes].[73]

Broscius also countered Snellius's argument that the results of Book X were not applied anywhere else in Greek mathematics by pointing to the fact that it is not necessary to explicitly spell out their use. He explained his meaning by an attractive metaphor that was borrowed from Epictetus's *Manual*:

> Sheep do not show their herdsmen how much grass or hay they have eaten, they consider it sufficient to show the milk, that is, the fruit of the food.[74]

on Copernicus's work and also wrote on pure geometry, [Knaster, 1973]. In *Aristoteles et Euclides*, he discussed Ramus's work on the study of the area of isoperimetric figures and the efforts of Romanus, Giovanni Camillo Gloriosi and himself to improve on that, but he '[did] not make any substantial contribution to the problem', [Bockstaele, 1974, p. 106]. Verdonk is of the opinion that the book does not criticise the method of the *Geometria* fundamentally, [Verdonk, 1966, pp. 230–231].

'[...] nollem tam propero praeiudicio damnatum videre Euclidem a doctissimo Snellio [...]', [Broscius, 1652, p. 37].

[71]'Etsi enim summis in Geometria viris Vietae ac Snellio videatur [Ramus] λογικώτατος, aliis tamen rem diligenter considerantibus non tantus videtur ut propter illum Aristotelis et Euclidis via ac ratio deserenda sit.' [Broscius, 1652, pp. 3–4].

[72]'Tempus omnium pater aut nos aut posteros docebit multa, quae nunc tanquam inutilia contemnuntur.' [Broscius, 1652, p. 37].

[73]'Calculus ille sine designatione Geometrica caecus erit. [...] Causae enim calculi melius ex diagrammate cognoscentur. Hoc idem censeto de irrationalium calculo.' [Broscius, 1652, p. 37].

[74]'Oves non monstrant pastoribus, quantum herbarum aut faeni comederint, satis habent lac hoc est fructum cibi ostendisse.' [Broscius, 1652, p. 37]. The original Greek text is from Epictetus, *Encheiridion*, c. 46, 2, [Epictetus, 1959, pp. 530–531].

He added a reference to Book XIII of the *Elements* where some of the contents of Book X were used.[75] Although Broscius tried to refute Snellius's statements systematically, the arguments with which he explained why Snellius was really wrong in dismissing Book X and what its true merits were are not convincing, because too much depends on his trust in the value of the book.

These last two texts show that at least in two cases Snellius's dedicatory letter was read and taken seriously—that is, Kepler and Broscius thought it worthwhile to object to Snellius's views. His opinions were not really refuted; instead, Kepler and Broscius seemed to argue from a different set of assumptions, taking for granted the elevated value of the study of incommensurable magnitudes and their relevance for the study of the regular solids, whereas Ramus and Snellius had stressed the usefulness of mathematics for practical applications. In their world, mathematics was so central that it should not be spoiled by obscure subtleties. Because so much time elapsed between the different contributions to the early modern debate on *Elements* X (ranging from 1569 to 1638), there was no dialogue between its participants and their opinions lived on next to each other.

5.5 Van Ceulen's and Snellius's calculations with segment-number pairs

A fine example of Van Ceulen's and Snellius's actual use of numbers in geometry is found in the section of the *Fundamenta* in which they introduce the four elementary operations applied to line segments of which the length is expressible in numbers.[76] They do not calculate with indeterminate numbers, but always with exemplary values; the reader has to apply the method offered in the examples to his own problem, with other numbers, which is not always easy. Numbers are, as always in the *Fundamenta*, integers, fractions, square roots and nested roots, all positive.[77] If a unit line segment is given, line segments of all these lengths are constructible with ruler and compass alone. Van Ceulen explained the reverse; that is, how to construct a unit length on the basis of a line segment with its length given as a number.

Van Ceulen actually developed a new method in geometry here by linking line segments and numbers expressing their measure. This extension demanded some creativity, because the properties of numbers (e.g. being square) were involved.

[75] [Broscius, 1652, p. 37]. He had written eleventh book instead of thirteenth book by mistake; the proposition he referred to is XIII.11.

[76] Cp. the discussion in [Bos, 2001a, pp. 154–157].

[77] Elsewhere in the book, these categories are also used to approximate the solutions of equations of degree three and higher, which arise when the sides of regular polygons are calculated.

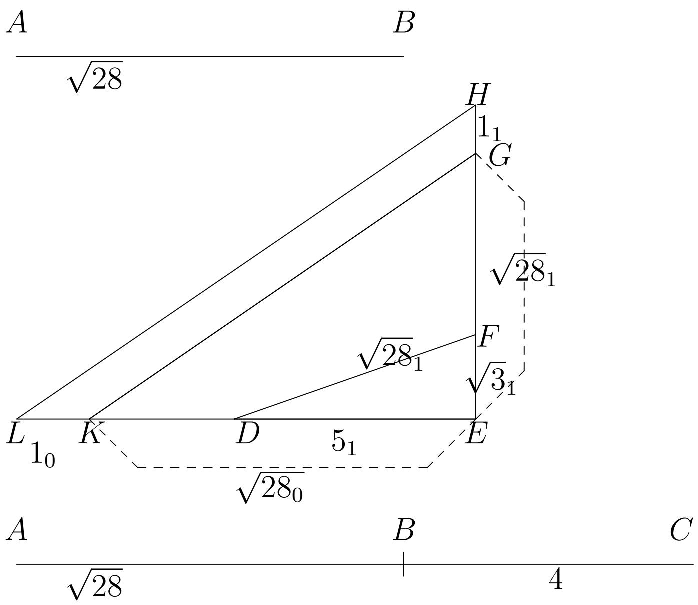

Figure 5.1: Van Ceulen: adding line segments

Some examples will clarify this. He first explained how two line segments had to be added.[78]

Problem 5.1 *Given a line segment AB of which it is given that the length is $\sqrt{28}$, it is required to construct a line segment AC of length $\sqrt{28}+4$.*

It has to be noted that this problem is not trivial. It would have been if it had been an arithmetical problem; then the sum of the two numbers $\sqrt{28}$ and 4 would have been $\sqrt{28}+4$. In a practical context, this sum could have been approximated by a fraction by means of a standard algorithm. If two line segments had been given, it would also have been easy to add them geometrically. However, now the solution to the problem is a constructed line segment, yet it falls beyond the reach of traditional Euclidean geometry. A line segment of length 1 has to be constructed on the basis of the given line segment. This unit is defined locally, which means that its length can be different in every problem.

Van Ceulen proposed the following solution (see figure 5.1):

Construction:

1. Draw a line segment DE of arbitrary length; call this 5 units. [We now have to distinguish the unit in which $\sqrt{28}$ is expressed, and this auxiliary

[78] [van Ceulen, 1615a, p. 133].

unit. They will be called 1_0 and 1_1 respectively. This is modern notation; Van Ceulen does not make this explicit distinction.] Now construct a line segment EF of length $\sqrt{3}_1$ perpendicular to DE through E.

[Van Ceulen does not explain how to find EF. It can be done by constructing another auxiliary right-angled triangle with sides 1_1 and $\sqrt{2}_1$, the hypotenuse of which is $\sqrt{3}_1$ (Theorem of Pythagoras). 1_1 can be determined easily because 5_1 is known (use *Elements* VI.9 to divide DE in 5 equal parts). $\sqrt{2}_1$ in its turn is the hypotenuse of a right-angled triangle with the other two sides 1_1.

Another possibility is to construct a right-angled triangle with hypotenuse 2_1 and base 1_1; its other side is $\sqrt{3}_1$.]

2. Connect D and F; this hypotenuse has length $\sqrt{28}_1$.
3. Prolong EF to G such that $EG = \sqrt{28}_1$. Prolong EG to H such that $GH = 1_1$.
4. Prolong ED to K such that $EK = AB = \sqrt{28}_0$. Construct the triangle GEK.
5. Construct a line parallel to GK through H; call its intersection point with EK prolonged L. Now $KL = 1_0$.
6. Prolong AB with four times KL to C. AC solves the problem.

The proof follows easily from the similarity of the triangles KEG and LEH, which implies $EG : GH = EK : KL = \sqrt{28}_1 : 1_1 = \sqrt{28}_0 : 1_0$. A similar construction can be made for any pair of numbers. Although the construction is straightforward, it contains one difficulty: it is not obvious how to 'resolve' a given number (in this case $\sqrt{28}$) into preferably two integers or else roots easier to construct (in this case Van Ceulen chose 5 and $\sqrt{3}$) in the most efficient way. Any root can, in principle, be reduced to integers in a finite number of steps by means of the method explained under step 1 and the unit can be constructed with ruler and compass by means of a number of auxiliary triangles and the Theorem of Pythagoras. However, the number of steps depends on the given number and on how well versed the mathematician is in calculating with roots and squares.

Snellius proposed a different method to solve this category of problems, which did not suffer from this last difficulty and for which he did not have to introduce an auxiliary unit.[79] For the solution of problem 5.1, he determined the mean proportional between $\sqrt{28}$ and $\frac{1}{28}\sqrt{28}$. This mean proportional is the unit, because $\sqrt{28} : 1 = 1 : \frac{1}{28}\sqrt{28}$. The mean proportional of two magnitudes could

[79][van Ceulen, 1615b, p. 107].

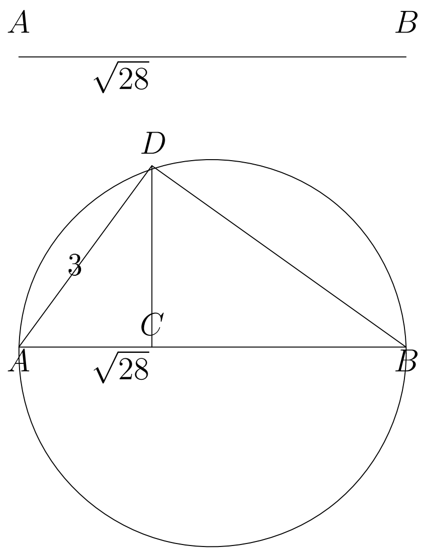

Figure 5.2: Snellius: adding line segments

be found by *Elements* VI.13. Once the unit was known, it was easy to construct $\sqrt{28}+4$.

Snellius then discussed a slight variation of Euclid's method of VI.13.[80] Again $AB = \sqrt{28}$ is given; it is now required to construct $\sqrt{28}+3$ (see figure 5.2).

Construction:

1. Divide AB in C such that $AB : AC = (\sqrt{28})^2 : 3^2 = 28 : 9$ [again use *Elements* VI.9 to divide AB into 28 equal parts; take 9 of them, starting from A, to determine C].

2. Construct a semicircle with diameter AB; draw a line through C perpendicular to AB. Call its intersection point with the semicircle D.

3. AD has the required length 3. Add AB and AD.

Proof:

Because of the similarity of $\triangle ACD$ and $\triangle ADB$, $AC : AD = AD : AB$ (AD is the mean proportional between AC and AB). Therefore, $AD = \sqrt{AB \cdot AC} = \sqrt{\sqrt{28} \cdot \frac{9}{28}\sqrt{28}} = 3$).

[80]The only difference from Euclid's method for the determination of the mean proportional is that Euclid determines CD as the mean proportional between AC and CB.

Snellius recommended his method as 'both very elegant and extremely easy to perform'.[81] Indeed, his method was conceptually simpler than Van Ceulen's because there was no danger of confusing different units. He came back to it after Van Ceulen's next example (given a line segment of length $\sqrt{19}$, to add a segment of $\sqrt{14}$ to it), claiming that his 'little theorem' could help to avoid Van Ceulen's troublesome method.[82] After Van Ceulen's last example (given a line segment of $\sqrt{\sqrt{15}}$, to add a line segment of 3 to it), Snellius commented that his own method could also be modified for this case. More modestly, he wrote that he did not know which method was more convenient and that the reader should judge for himself.[83]

Snellius did not intend to replace Van Ceulen's method by his own. Because they both only gave examples, he could not show which method was faster or more convenient. This depends partly on the numerical values in the problem. In order to solve problem 5.1, Snellius had to compose a segment of length 28, a tedious task, which Van Ceulen did not have to perform. This practical aspect was not what interested them most, they were both fascinated by methodological considerations, not by the fastest route to an answer. Moreover, it was not necessary to actually perform these operations in practical problems. Once it had been established that the constructions were sound, a short-cut could be taken, in which the number expressing the result of the operation was calculated; it was then sufficient to draw an approximation of it in the figure. Paradoxically, geometrical constructions yielded the most exact results in theory, whereas in practice calculations with numbers were more accurate. Snellius must have had this accuracy in mind when he described number as the 'exact translator of every measure'.

There is a telling difference in method: the reader must be a good calculator with squares, preferably a virtuoso like Van Ceulen himself, to be able to use Van Ceulen's method efficiently, whereas Snellius used an Euclidean construction. His method could be called more geometrical. It was also more general, and therefore indeed easier, because his algorithm did not depend on the actual numbers.

A problem in the next section, about subtraction, indicates more explicitly that Snellius was more geometrically minded than Van Ceulen. Van Ceulen gave a line segment AB with length $\sqrt{7}+\sqrt{3}$, and asked that a line segment equal to the root of AB be cut off.[84] Snellius remarked severely that Van Ceulen misap-

[81] '[...] modus iste elegantissimus iuxta ac parabilissimus sit.' [van Ceulen, 1615b, p. 107].

[82] 'Potuit vero per antecedens nostrum theoremation tam operosae factionis περιέργεια [sic] levissimo negotio declinari.' [van Ceulen, 1615b, p. 108].

[83] '[...] sed haud scio an vulgata illa autoris via in his medialibus $\sqrt{\sqrt{15}}$ caeterisque omnibus altiore gradu affectis expeditior sit, cuius tamen arbitrium penes lectorem relinquo.' [van Ceulen, 1615b, pp. 108–109].

[84] 'De hier ghestelde linie AB, doet $\sqrt{7}+\sqrt{3}$, daer van wilmen snijden een linie die den wortel

plied his own words, because he spoke about the side of a line, whereas he meant the side of a square equal to a rectangle contained by a line segment of the length of AB and by a line segment equal to the unit of the same measure.[85] Thus, Snellius re-interpreted Van Ceulen's problem in exact geometrical terms; in this context, a root is not a number, but the side of a square, and the dimensions of all magnitudes involved should be established unequivocally. This reformulation did not have any consequences for the actual solution of the problem.

An adherent of traditional Euclidean geometry would probably have his doubts about the non-systematic aspect of Van Ceulen's construction and the sloppy formulation, but he would really protest in the next case, where problems are worse. This is the multiplication of two line segments, where a similar difference in approach between Van Ceulen and Snellius is discernible. Van Ceulen's problem and solution are as follows:[86]

Problem 5.2 *Given a line segment a of which it is given that the length is $\sqrt{19}$ and a line segment b of which it is given that the length is 3, it is required to construct a line segment of length $3\sqrt{19}$ (see figure 5.3).*[87]

Construction:

1. Construct a line segment AB of $3+\sqrt{19}$; mark O on it such that $AO = 3$ and $BO = \sqrt{19}$.
2. Construct a line segment of length 1 from O in an arbitrary direction (only not along the line AB); its end point is C.
3. Construct a circle through A, B and C; call its intersection point with OC prolonged D.
4. OD solves the problem, because it has length $3\sqrt{19}$ (this follows from *Elements* III.35).

This construction contains no problematic aspects. It does not depend on the actual values of the numbers and is therefore more general than Van Ceulen's construction of addition. This example could even be solved in a simpler way, because 3 is an integer; just joining three copies of $\sqrt{19}$ would give the answer.

sy der selver AB.' [van Ceulen, 1615a, p. 136].

85 '[...] namque in ipso zetemate postulat latus lineae AB, cum non hoc vellet, sed latus quadrati aequalis rectangulo comprehenso sub longitudine AB $\sqrt{7}+3$, et latitudine aequante unam unitatem eiusdem mensurae.' [van Ceulen, 1615b, p. 111].

86 [van Ceulen, 1615a, p. 137], and see Bos's modern rendering in [Bos, 2001a, p. 156].

87 Note that there is a redundancy in the givens: either one of the line segments, or one of their lengths in numbers can be left out, because if the segment with length $\sqrt{19}$ is given, a segment with length 3 can be constructed and vice versa. Van Ceulen seems to intend this as an example of a general problem in which two line segments a and b are given, which are possibly irrational.

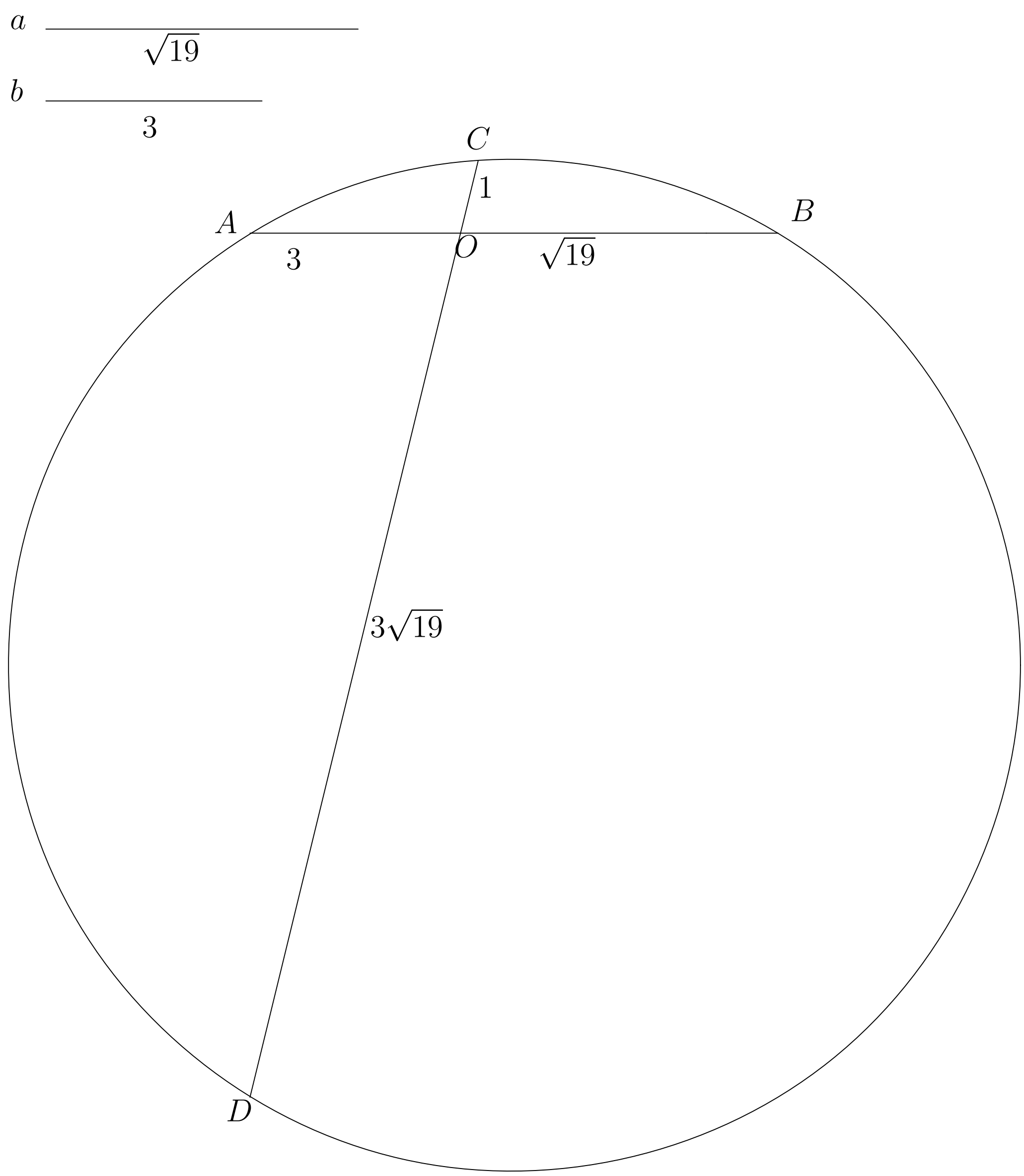

Figure 5.3: Van Ceulen: multiplying line segments

However, the formulation of the task deserves a closer scrutiny. Van Ceulen first asked which size a rectangle contained by the two given line segments would have, and then 'how long that product would be, that is, geometrically to find a line the number of which would be equal to the product.'[88] He did not *state* that the product of two line segments was a line segment, but through the detour of the measures of the line segments and areas, he seemed to imply that he meant this and Snellius indeed interpreted it in this way. If he really considered the product of two lines as a line, he would do away with a crucial feature of geometrical entities, which distinguishes them from numbers: that is their dimension.

Snellius was alarmed by the implication of these words by Van Ceulen and disassociated himself from them. In a long note following Van Ceulen's second example of a multiplication,[89] he wrote that the geometrical operation of forming a rectangle was analogous to the arithmetical operation of multiplication, and application of a parallelogram to a line equivalent to division.[90] To connect the two domains of arithmetic and geometry, Snellius borrowed a concept from Ramus. This is the *numerus figuratus*, a term used in Ramus's *Arithmetica*; this 'shaped number' was a number which corresponded with a geometrical figure.[91]

However, this analogy certainly did not mean that the domains of geometry and arithmetic overlapped. Snellius feared that a reader who was incautious or had not been very engaged in this business could be confused by Van Ceulen's improper phrasing, because

> what this author claims, i.e. that the result of the geometrical multiplication of two lines is a line, is not supported by any authority, just as that which follows, i.e. that a line would result from the mutual division of two lines.[92]

Thus Snellius indeed interpreted Van Ceulen's words as meaning that the product of two line segments is actually a line segment. He proceeded to say that although the problem was ridiculous, the solution was legitimate and therefore he rephrased the problem in terms that would make it geometrically valid according to his norms, returning to the old familiar terms of rectangle application and proportions, as follows:

> Two right lines are given according to the value of the same measure: A is $\sqrt{19}$, B is 3, and it is required to find the length which originates

[88]'[...] hoe lanck dat product zijn soude, dat is, Geometris een linie te vinden, welcke ghetal het product ghelijck zij.' [van Ceulen, 1615a, p. 137].

[89]Bos calls this a 'rather confusing note', [Bos, 2001a, p. 156].

[90]See p. 273 for an explanation of 'application'.

[91]'Figuratus autem numerus est, qui sua vi ac natura geometricae figurae respondet.' Quoted in [Verdonk, 1966, p. 134].

[92]'Namque quod hic autor postulat duarum linearum multiplicatione Geometrica lineam fieri, tam ἄκυρον est, quam id quod sequitur mutua duarum linearum divisione lineam existere.' [van Ceulen, 1615b, p. 113].

> from the application of the rectangle that is contained by them to the unit of the same measure. In other words: if the ratio 1 unit to B (which equals 3) is equal to the ratio A (which equals $\sqrt{19}$) to the unknown, [what is the unknown]?[93]

He then stressed that a line is essentially different from an area, that the outcome of a multiplication of two lines is a parallelogram and cannot be equal to a line, 'because no ratio or mathematical comparison exists between an area and a line'.[94]

To conclude: Snellius found himself in an awkward position when translating this part of the *Fondamenten*. Van Ceulen approached the topic of the four basic operations applied to line segments with numbers in a relaxed way with which Snellius fundamentally could not agree. Snellius's mathematical conscience could not allow him to translate a section that was too sloppy, in his view, without a critical commentary.

The introduction of numbers in geometry was a difficult matter, for which new kinds of problems had to be solved and this section shows how Van Ceulen and Snellius tried to deal with some of the key difficulties: the lack of a unit in geometry, the absence of dimensions and the need for proof methods in geometry with numbers. Snellius's cautious reactions show that his enthusiastic support of the admittance of numbers into the company of geometry of the dedicatory letter did not mean that this support was unconditional, and that he was too much attached to the classical geometrical concepts to exchange them altogether for attractive new methods.

[93]'Dantur duae rectae secundum eiusdem mensurae aestimationem A $\sqrt{19}$, B 3, quaeritur si rectangulum ab ipsis comprehensum ad eiusdem mensurae unitatem applicetur, quae num sit longitudo inde existens. Vel ut 1 mensura ad B 3, sic A $\sqrt{19}$ ad quem.' [van Ceulen, 1615b, p. 113].

[94]'[...] quoniam inter superficiem et lineam nulla ratio, aut mathematica comparatio intercedit [...].' [van Ceulen, 1615b, p. 113].

Chapter 6

A lifelong interest: the triangle division problem

6.1 Introduction

Snellius contributed several times to the solution of the triangle division problem, a traditional geometrical problem, in which a given triangle had to be divided by a line passing through a given point into two parts of which the areas had a given ratio. Snellius's contributions are of special interest because they allow us to cast a glance at his ideas on good geometry and at his different ways of approaching the same problem. Moreover, both the fact that he repeatedly paid attention to this problem and the stress put on it in his dedicatory letter to *Cutting off of an Area* show that it was one of his favourites. This special attention in the dedicatory letter is especially striking since it is only discussed in a corollary in this booklet, and thus it is not part of the main chain of reasoning.

Snellius's work cannot be assessed properly if the activities of his forerunners are not taken into account and therefore my analysis of his work will be preceded by a brief history of the solutions of the triangle division problem and its transmission. Several of Snellius's contemporaries also tried their hand at division problems and by comparing Snellius's work to theirs, we will obtain more insight into the lively activity of geometrical problem solving of the period: the mutual influences, collaboration and competition. Because this activity would become obsolete later, it has not received the amount of modern attention which its significance would justify.

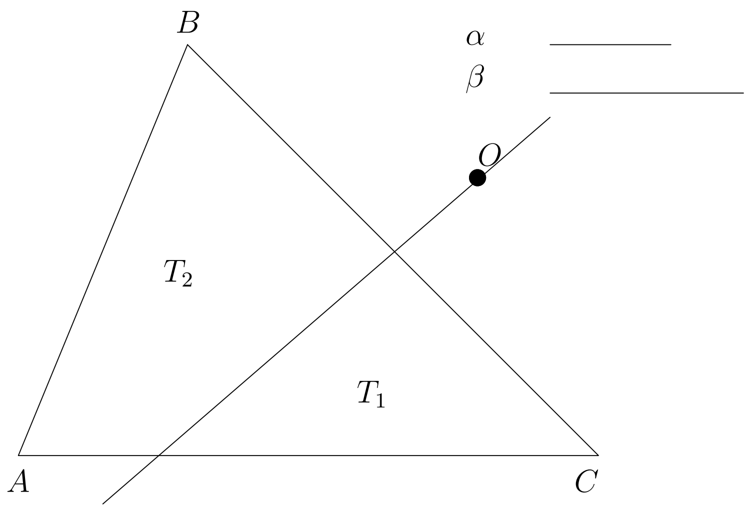

Figure 6.1: The general triangle division problem

6.2 Triangle division until Van Ceulen

Since the time of Euclid, mathematicians had studied several triangle division problems. In general terms, this family of problems could be stated thus:

Problem 6.1 *Given a triangle ABC, a point O and a ratio $\rho = \alpha : \beta$. Divide the triangle into two parts T_1 and T_2 by means of a line through O such that $T_1 : T_2 = \rho$ (see figure 6.1).*

The triangle, the point and the ratio are given in a geometrical sense. For the ratio, the reader is probably meant to think of the ratio between two given line segments, but generally this is not made explicit. The problem is plane, which means that it can be (and preferably was) solved by ruler and compass alone (see p. 181). All authors discussed here indeed only used plane means in their solutions. As we shall see, some authors used numbers as an auxiliary tool.

Several versions of the problem were studied. Some authors, for instance, solved the problem for different positions of O: on, outside or inside the triangle. Some of them only looked at special values of ρ, like 1 : 1. The problem was sometimes extended to the division of various kinds of other polygons, or the division of figures into more than two parts. A summary of the problems is given in table 6.1.

Type nrs	Problems	Remarks
I	Given a △, a point O and a ratio ρ. Divide the △ into 2 parts by means of a line through O so that the parts have ratio ρ to each other.	
Ia	Like I, for special value(s) of ρ.	
II	Like I, O on vertex.	
III	Like I, O on side.	Variant: divide the triangle into an arbitrary number of equal parts.
IIIa	Like III, for special value(s) of ρ.	
IV	Like I, O inside △.	
IVa	Like IV, for special value(s) of ρ.	
V	Like I, O outside △.	
Va	Like V, for special value(s) of ρ.	
VI	Given a △ and a ratio ρ. Divide the △ into 2 parts by means of a line $\parallel$ to a given side so that the parts have ratio ρ to each other.	Variant: divide the triangle into an arbitrary number of equal parts.
VIa	Like VI, for special value(s) of ρ.	
VI'	Like VI, trapezium instead of triangle.	
VII	Given a polygon, a point O and a ratio ρ. Divide the polygon into 2 parts by means of a line through O so that the parts have ratio ρ to each other.	Usually, less general variants of this problem are discussed, e.g. only points O on sides or vertices of the polygon are considered, or only certain categories of polygons.
VIIa	Like VII, for special value(s) of ρ.	
VIIb	Like VII, line $\parallel$ to a given side.	

Table 6.1: Types of triangle and polygon division problems

Author	Title of work (abbreviated)	Year of first publication	Type of problems	Predecessors mentioned	Numbers?
Euclid	*On Divisions*	ca. 300 BC	IIIa, IVa, IV, Va, V, VI, VI', VII	–	no[1]
Pisano	*Practica Geometriae*	1220[2]	II, III, IV, V, VI, VII		yes
Tartaglia	*General trattato* V	1560	IIIa, IVa, Va VIa, VIIa		yes
Commandino/ Dee/ Mohammed	*De superficierum divisionibus*	1570	II, III, VI, VII		
Clavius	*Euclidis Elementa*	1574	III	Commandino	no
Stevin	*Problemata Geometrica*	1583	II, III, VI, VI', VII, VIIb	Clavius	no
Benedictus	*Diversarum Speculationum*	1585	V	–	no
Van Ceulen	*Fundamenta*/ *Arith. en Geom. Fond.*	1599/1615[3]	II, III(a), IVa, V(a), VIa, VII, VII(a)(b)	Pouwelsz, Pietersz, Benedictus, Stevin	yes
Snellius	*Fundamenta*/ *Arith. en Geom. Fond.*	1599/1615[4]	V	Benedictus	[5]
Clavius	*Geometria Practica*	1604	III, V, Va, VI, VII, VIIb	Stevin, Commandino, Mohammed, Pisano, Tartaglia	no
Stevin	*Meetdaet*	1605	III, IV, V	Benedictus, Tartaglia, Cardano, Ludovicus Ferrarus	yes
Snellius	*Cutting off*	1607	I	Stevin, 'Bambalio'	no
Cardinael	*Hondert geometrische questien*	1612	IIIa, IVa, VIIa	–	yes
Snellius	Van Ceulen, *De Circulo*	1619	IV, V	–	no

Table 6.2: A survey of triangle dividers

A short survey of the transmission of the problem from Antiquity to 1600 will be given here to explain what prior results were available to Snellius. A list of the most relevant contributors up to and including Snellius is given in table 6.2.[6] The column 'Numbers?' indicates whether the authors discussed numerical examples, or only problems with general (geometrical) givens. As usual in plane geometry, Euclid formed the starting point of this tradition. Triangle division is not discussed in the *Elements*, but in a separate work, Περὶ διαιρέσεων βιβλίον/*Liber divisionum* ('On Divisions of Figures'). Many variants of division problems are found in this treatise: triangles, other polygons and circles are divided; those figures are divided into two or more parts, and these parts are either equal in size or in a given ratio; and the dividing line passes through a given point (at a vertex of a figure, on one of its sides, inside or outside it) or is parallel to the base of the figure.[7] The constructions are based on the material in the *Elements*, and some auxiliary propositions are developed.

Although this work was not directly accessible to Snellius, because it had been lost, it was not difficult to learn its contents. In 1570, John Dee and Federigo Commandino published a Latin translation of an Arabic adaptation of *On Divisions*. The author of the Arabic version was supposed to be one Mohammed Bagdedinus. Only the simpler triangle division problems were discussed in it, not those where O was situated inside or outside the triangle (IV/V). Snellius probably owned a copy of this book.[8]

Traces of Euclid's work could also be found in Jordanus de Nemore's *Liber Philotegni* ('The Book of the Lover of the Arts') and Leonardo Pisano's *Practica Geometriae* ('The Practice of Geometry', 1220). These works circulated in manuscript, but later other works based on Pisano's, e.g. those by Luca Pacioli, Niccolò Tartaglia and Joannes Baptista Benedictus (or Benedetti), appeared in

[1] This is not completely certain, as the original text has not been preserved.

[2] The book was written in 1220 and then circulated in manuscript copies. It only appeared in print in the nineteenth century.

[3] Both the Dutch and the Latin version of the book appeared in 1615, after Van Ceulen's death. In the text he explicitly remarked that he worked on the problem in 1599.

[4] See previous note.

[5] A numerical example is given in the treatment, but it cannot be determined whether this originated from Snellius or was added by Van Ceulen. Cp. section 6.3.1.

[6] The present list does not claim completeness. In particular, all Islamic contributions have been left out. Archibald gave a longer list of sources published after 1500, with other details than I give here, [Archibald, 1915, pp. 78–85]. He left out Stevin, and Van Ceulen's *Fondamenten* and *De Circulo*. The earlier sources are discussed in his historical survey, [Archibald, 1915, pp. 1–28].

[7] [Archibald, 1915, pp. 15–16]; see table 6.2 for a summary. Archibald made a reconstruction of Euclid's text based on two sources: an Arabic manuscript discovered by Woepcke in the nineteenth century and Pisano's *Practica Geometriae*, [Archibald, 1915, pp. 9–13]. I have used this reconstruction.

[8] [Bagdedinus, 1570]; see p. 99.

print.[9]

To summarize, the core of the triangle division activities was found in Italy during the Middle Ages and Renaissance. Tartaglia, for instance, discussed triangle division problems in his *La quinta parte del general trattato de' numeri et misure* ('The Fifth Part of the General Treatise on Numbers and Measures', 1560). The sub-problems connected to triangle division and the constructions which he used are Euclidean in style. He also calculated a numerical solution in some cases, and once warned the reader that the solution to a certain problem did not have to be unique.[10]

Somewhat later, Benedictus examined only the problem with O outside the triangle (V) in his *Diversarum Speculationum Mathematicarum, et Physicarum Liber* ('Book of Diverse Mathematical and Physical Explorations', 1585).[11] His solution consists of two parts, the first of which is not a construction, but a way to convince himself and the reader that the solution of the problem always existed, for which he implicitly used a continuity argument. In the second part, Benedictus gave an analysis of the problem that leads to a construction, to which I will come back in section 6.3.1.

Snellius's compatriots also began to work on division problems at the end of the sixteenth century. Simon Stevin considered these problems in the first book of his *Problemata Geometrica*.[12] He did not discuss the more difficult type of division problems where the point O is situated inside or outside the triangle (IV/V), but he did discuss the division of general polygons.

Stevin pointed out that his construction of the solution of one of the problems (type III) differed from those of other authors, among whom was Clavius in his edition of Euclid's *Elements*, because he had preferred to use his own 'general invention', which was also useful for the next set of problems (several problems of type VII; VI, VI', VIIb). Stevin's construction was less straightforward than Clavius's, but not essentially different. He claimed that this next set had never been described before, which shows that he did not know Dee's and Commandino's book.[13]

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[9] [Archibald, 1915, pp. 1–28] gives an elaborate overview of the history of the transmission of Euclid's work. See also [Bos, 2001a, pp. 84–86]. For Jordanus on polygon division, see [Clagett, 1984, pp. 161–164, 216–221, 269–271].

No manuscript of the original text of Euclid has resurfaced until now and our present knowledge is based on some Arabic texts and medieval Latin texts. See [Hogendijk, 1993] for an edition and English translation of the relevant Arabic texts, of which pp. 152–155 discuss triangle division.

[10] [Tartaglia, 1560b, fols. 23^v–32^r].

[11] [Benedictus, 1585, pp. 304–307].

[12] [Stevin, 1583], reprint edition with English translation in [Stevin (D.J. Struik ed.), 1958]; cp. [Dijksterhuis, 1943, pp. 97–98].

[13] [Stevin, 1583, p. 25]. See [Archibald, 1915, pp. 13–14] for a survey of the contents of the treatise by Mohammed, Dee and Commandino. Clavius's discussion of the problem of type III, borrowed from Commandino as he informs us, is found in [Clavius, 1589a, pp. 879–881].

6.3 Snellius enters the scene (1599)

6.3.1 The teacher and his student

Snellius's first work on the division of a triangle was done when he was still a young man. It is not found in a publication bearing his own name, but in Ludolph van Ceulen's *De Arithmetische en Geometrische Fondamenten* ('The Arithmetical and Geometrical Foundations', 1615). Van Ceulen discussed division problems in two different sections of the book, in the second of which a solution by Snellius played a role. In the first section, Van Ceulen discussed some relatively simple division problems, in which the point O was located on one of the sides of the triangle (type II, III in table 6.1) or on a side of some other rectilinear figure (VII). He also dealt with problems in which the cutting line had to be parallel to a side of the figure (VI, VIIb), and with the related category of problems in which a given area had to be subtracted from the figure.[14]

Van Ceulen started the second section on triangle division with some facts about his own and several other people's involvement in the solution of the problem. This passage is worth our attention because it tells us who was interested in this sort of problem from pure mathematics in the Dutch Republic and how these people collaborated. Van Ceulen wrote that Stevin had sent him a few problems in 1582 (that is, before the publication of the *Problemata Geometrica*); Stevin had not managed to find a solution to one of them, nor had anyone else, as far as Stevin was aware of. This was the problem of dividing a triangle by a line through a point outside it into two parts with a given ratio (type V). Although Stevin had promised fame to Van Ceulen, should he be able to solve it, Van Ceulen had not worked hard to find a solution, because he knew his own weaknesses, as he confessed himself.

A mathematical practitioner, Johan Pouwelsz, then sent Van Ceulen another problem: a triangle had to be divided into two equal parts by means of a line through a point inside it (type IVa). Pouwelsz asked Van Ceulen to solve this 'by numbers' instead of geometrically, and Van Ceulen managed to do so with the help of 'the rule of cos' (algebraically), after which he also found a solution 'with lines', which he was able to prove (purely geometrically), as he reported to the reader.[15] Van Ceulen had left the problem alone that Stevin had sent him until in the year 1599 another mathematician, Cornelis Pietersz, came to him to demonstrate that he had solved that problem.

Only some days later, Snellius showed Van Ceulen the way in which Benedic-

[14][van Ceulen, 1615a, pp. 119–132].

[15]See for Van Ceulen's actual solution [van Ceulen, 1615a, pp. 219–220] or [van Ceulen, 1615b, pp. 205–206]. In fact, as Snellius also remarked in his commentary, Van Ceulen gave a geometrical construction without a proof, instead working out a numerical example.

tus had solved the same problem in his *Diversarum Speculationum Liber*. Van Ceulen considered this solution skilful, but added that Snellius had even improved Benedictus's solution. Van Ceulen included Snellius's work in his book before his discussion of his own contributions to the solution of the triangle division problems of type IV and V.[16]

Before analysing Snellius's solution in detail, I will give a brief sketch of the section to which it belongs.[17] Van Ceulen discussed five problems in which a triangle must be divided by a line through a point outside the triangle, and three by a line through a point inside it. The first category received more attention than the second, because the different possible positions of the point O outside the triangle require (somewhat) different methods of solution. The formulation of the problems varies somewhat: a line must be drawn in some cases, the length of certain line segments must be calculated in other cases. One exemplary value is always selected for the ratio ρ. Every problem is concluded by an example, in which the sides of the triangle and sometimes other line segments are given in numbers. The presentation sometimes lacks in clarity: the givens are not always made explicit and some of the figures are confusing, which may (partly) have been caused by the fact that the publication was posthumous.

Snellius's solution as given by Van Ceulen is difficult to follow, because the letters used to indicate points in the text do not correspond to those in the accompanying figures. Moreover, one of the two figures contains an essential mistake (see below). Snellius changed the lettering of the text in his Latin translation of the *Fondamenten*, which appeared under the title of *Fundamenta Arithmetica et Geometrica*, also in 1615. He did not have new figures at his disposal for the Latin edition and therefore he had to maintain the wrong auxiliary figure in this case. In his explanation of the construction, he skipped the part where the figure should be adjusted. The missing step can be supplied from the rest of the construction and proof. The reason why this part is so sloppy, which is unusual in Snellius's work, may be that he was not able to amend it due to the pressure which the printers put on him to finish the book quickly.

The construction by Snellius as given by Van Ceulen is as follows:[18]

Problem 6.2 (Snellius, 1599) *Given a triangle ABC and a point O outside the triangle. Cut the triangle in two parts T_1 and T_2 by means of a line through O such that $T_1 : T_2 = 1 : 3$ (see figure 6.2).*

[16] [van Ceulen, 1615a, pp. 211–212]; Latin translation in [van Ceulen, 1615b, pp. 198–199].

[17] [van Ceulen, 1615a, pp. 212–222]. For the *Fondamenten* and *Fundamenta* in general, see section 5.3; for Snellius's translation of the part on triangle division, see section 6.4.2.

[18] Here and in the next construction, I have added some commentary, as well as the abbreviated notation of line segments by lower case letters and the algebraical interpretation of steps between square brackets to make the proof somewhat easier to follow for a modern reader. Standard constructions, such as dividing a line in equal parts, are not explained, less elementary constructions are either expounded or a reference to the relevant proposition of Euclid's *Elements* is given.

Construction:

1. Determine E on AC such that $CE = \frac{1}{4}AC$ and construct EB [call CE e]. Now $\triangle EBC = \frac{1}{4}\triangle ABC$.

2. Draw a line through O parallel to BC; call its intersection point with AC prolonged D. [Call DO d.]

3. Construct rect (BC, CE) in an auxiliary figure. [Call BC a, the rectangle is then equal to ae.] Make another rectangle, equal in area, with one side equal to DO (*Elements* I.43). [The other side is equal to $\frac{ae}{d}$; call it n.] Draw N on the base of the triangle such that CN is equal to the other side of this rectangle. $(*)$

4. Determine M on the base such that rect $(CM, MN) =$ rect (CN, CD). In order to achieve this, determine the mean proportional of $CN[= n]$ and CD [call this f] by *Elements* VI.13, and then apply *Elements* III.36 (see the second auxiliary figure). $(**)$[19]

   [Call $MN = x$. Now determining M is equivalent to solving the equation $(x+n)x = nf$, which has two solutions: $x_1 = -\frac{n}{2} + \sqrt{n(\frac{n}{4}+f)}, x_2 = -\frac{n}{2} - \sqrt{n(\frac{n}{4}+f)}$. Because $x_2 < 0$, it is not relevant for the present problem.]

5. Join M and O; call the intersection point of this joining line segment with BC P; now $\triangle MPC = \triangle EBC = T_1$, quadr.$(ABPM) = T_2$ and the problem is solved.

Proof:

$$\begin{aligned} \text{rect } (CM, MN) &= \text{rect } (CN, CD). \\ \text{Therefore [\textit{El.} VI.14]} : CD : CM &= MN : CN; \end{aligned}$$

Componendo[20] $DM : CM = CM : CN$.

$$\begin{aligned} \text{Because } \triangle MPC \sim \triangle MOD, DM : CM &= DO : CP. \\ \text{Combining the last two yields } CM : CN &= DO : CP. \\ \text{Then rect } (CM, CP) = \text{rect } (CN, DO) &= \text{[see}(*)\text{] rect } (BC, CE). \\ \text{Hence } CM : BC &= CE : CP \end{aligned}$$

[19] Snellius does not say that this mean proportional has to be constructed first, and does not explain how III.36 has to be applied. This application can be found in a scholium by Clavius in his edition of the *Elements*; see [Bos, 2001a, p. 65].

The line segments d and f seem to have merged into one in Van Ceulen's auxiliary figure (not reproduced here). The figure is used again in Van Ceulen's next example, where d and f are indeed equally long. This may explain the mistake. However, d and f cannot be considered as equal in the general solution.

[20] See *Elements* V.18: $A : B = C : D \Rightarrow (A+B) : B = (C+D) : D$.

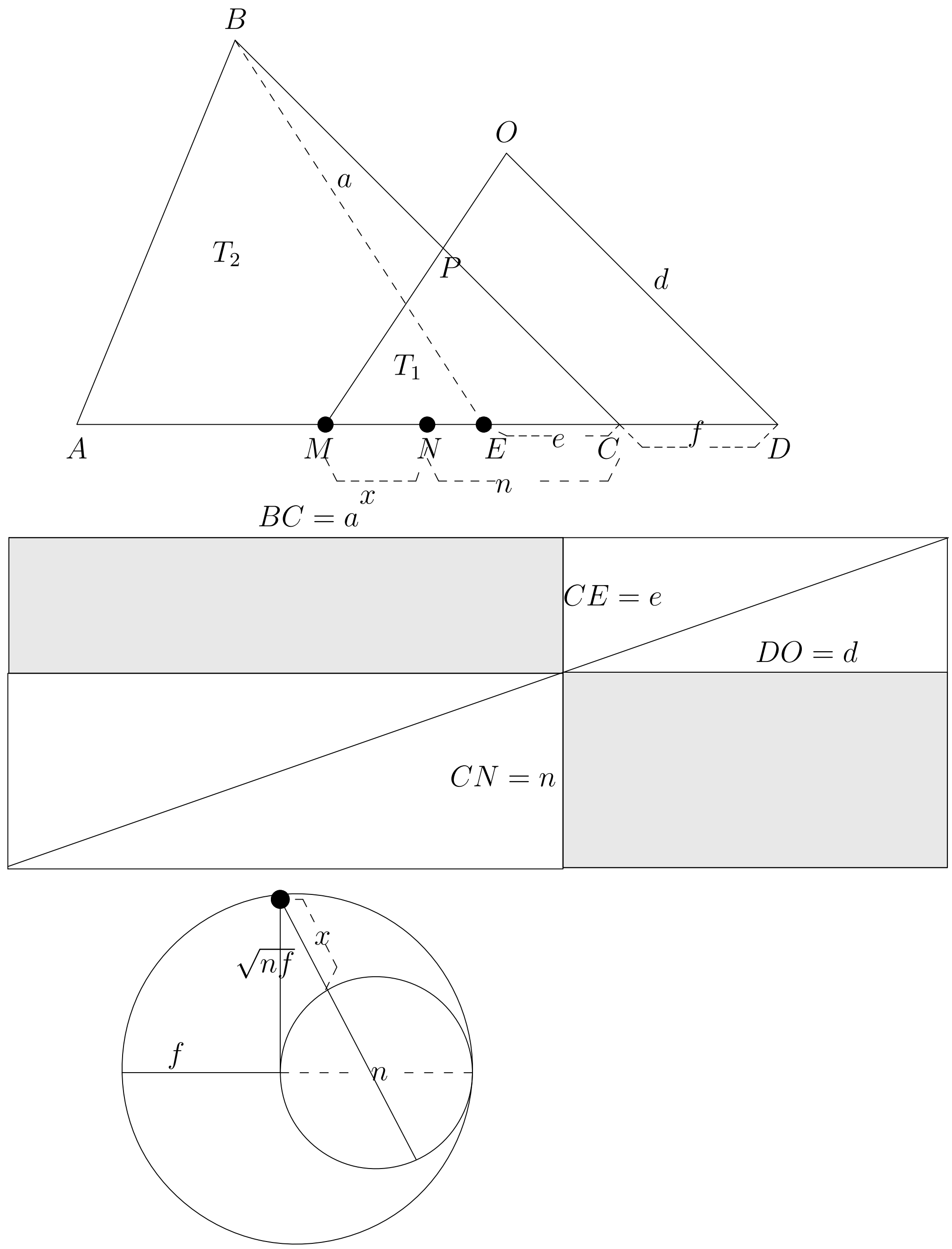

Figure 6.2: Triangle division problem, Snellius's solution (1599)

and (because they have one angle in common, *Elements* VI.4)

$$\triangle MPC = \triangle EBC = \frac{1}{4}\triangle ABC.$$

This construction can be generalized to an arbitrary ratio of the two resulting parts of the triangle without any difficulty.[21]

Van Ceulen had mentioned Benedictus's work as the source for Snellius's solution. Indeed, Snellius's and Benedictus's constructions are similar to a large extent. Instead of step (∗∗), Benedictus claimed to have used *Elements* VI.28 to determine the point on the base that solves the problem. If his procedure is translated into algebra, the same equation as in (∗∗) has to be solved. However, this equation cannot be solved by means of VI.28, which can be used to solve another type of second-degree equation. It seems likely that Benedictus did in fact mean VI.29.[22] The application of this proposition involves somewhat more work than Snellius's method. Benedictus's solution was probably borrowed from Leonardo Pisano's treatment of the problems of Euclid's *On Divisions*. Pisano discussed the problem in which the triangle had to be divided into two halves (Va). The step equivalent to (∗∗) was taken by the application to a given line of a rectangle equal to a given rectangle exceeding by a square, a special case of the problem of *Elements* VI.29.[23] Benedictus ended his exposition by pointing out what has to be done if M is located beyond A.

Van Ceulen solved the problem in which O is inside the triangle with a construction related to Snellius's. In this case the unknown x had to satisfy the equation $nf = x(n-x)$. Van Ceulen achieved this by considering $\sqrt{nf}$ as the mean proportional of x and $n-x$ in a construction similar to that in *Elements* VI.13 (see figure 6.3).[24]

6.3.2 Renewed attention: Clavius and Stevin

Before Snellius's first own publication on triangle division appeared, the field had been explored more extensively by other mathematicians as well. Two books, with different solutions, are of special relevance for comparison with Snellius's work. The first of these was Clavius's *Geometria Practica* from 1604. Clavius's discussed triangle division much more elaborately in this work than in his *Elements*-edition.

In the introduction to the sixth book, Clavius praised Stevin's work in the *Problemata Geometrica*, because Stevin had solved the division problem in an

[21] But notice step 1 of Stevin's construction, see p. 228.

[22] VI.28 can be translated into the equation $ax - \frac{b}{c}x^2 = S$, VI.29 into $ax + \frac{b}{c}x^2 = S$ with $a, b, c > 0$, [Euclid, s a, pp. 263, 266].

[23] See [Archibald, 1915, pp. 60–62]; the editor explains in a footnote that it is expedient to use *Elements* II.6 to tackle this sub-problem. Cp. p. 273 for a definition of application.

[24] [van Ceulen, 1615a, pp. 219–220].

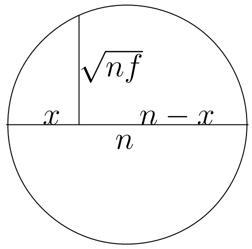

Figure 6.3: Van Ceulen's construction of the line segment x

easier and more general way than Commandino and Mohammed. Yet he also pointed out that Stevin's method could not be applied to all rectilinear figures. If he had wanted to achieve this, Stevin should have enunciated and proved two propositions first. Clavius wrote that he would follow in Stevin's tracks here, but if opportunity arose, he would develop more general propositions. Clavius considered as one of the weaker points of Mohammed's and Commandino's approach that the reader had to put much effort into dividing a multi-angular figure unless he remembered the division of all previous figures in their discussion.[25] He formulated criteria here which a good exposition of the division problem had to satisfy: rigour, generality and clarity.

Yet Clavius did not discuss all kinds of triangle division problems, but only those in which the points through which the line dividing the triangle has to pass are on one of the sides. The only exception to this is his problem VI.12, on the division of a triangle into two equal parts by means of a line through a point outside it (type Va). Only this ratio (1 : 1) is treated in full by Clavius, but he explained how problems in which parts have to be cut off according to other ratios can be solved analogously.[26]

Clavius proceeded in almost the same way as Snellius had done (see problem 6.2). Like Snellius, he had to solve a problem which can be expressed by the equation $(x+n)x = nf$, and like Snellius, he used *Elements* III.36 to do so. The precise way in which this proposition had to be applied was found in a scholium in Clavius's own edition of the *Elements*.[27]

Even though this similarity is great, no evidence can be found indicating that Snellius influenced Clavius or the other way around. According to Van Ceulen's report, Snellius's solution dates from 1599, and is therefore earlier than the publication of Clavius's book, but it was published much later. However, it

[25] [Clavius, 1604, pp. 264–265].

[26] [Clavius, 1604, pp. 294–295]. Bos gives a paraphrase of Clavius's construction in [Bos, 2001a, pp. 85–86].

[27] Unlike Snellius, Clavius determined the mean proportional of n and f explicitly.

is not unlikely that Snellius's work circulated among mathematicians in letter form, or that Snellius's and Clavius's solutions had a common source. They probably considered their own solution as simpler and therefore more elegant than that of Pisano and Benedictus.

Clavius remarked at the end of his discussion of triangle division problems that he did not include some other variants of V because these were 'more curious than useful'. He skipped the case with the point through which the line has to pass inside the triangle (IV) for (in his words) the same reason, 'because a line that cuts the triangle into two parts cannot always be drawn through an interior point, as we know from experience'.[28] This is only right if we understand 'two parts' as two unequal parts. The given ratio, shape of the triangle and location of the point O inside it determine whether or not the problem is solvable. Clavius paid no further attention to these factors, referring his readers to the work of Pisano and Tartaglia, where they could find problems of type IV and V. His own preference for general, clear and easy solutions may have been caused by the school context in which he was at home.

One year later, Stevin also published a second work in which he discussed triangle division: the *Meetdaet*, a part of the *Wisconstige Gedachtenissen* (1605). He had already written the *Meetdaet* in the 1580's, and therefore Snellius could have learned of its contents earlier.[29] Stevin had studied some more authors on division for this book than for the *Problemata*, and he told the reader that he had used the construction of Benedictus. Stevin presumed that Benedictus and Tartaglia had derived their knowledge from some manuscripts from the *Wijsentijt* (the period of pristine wisdom), and that the Greeks had not mastered this problem.[30] He also wrote that he and some other people had already been working on division problems before Benedictus's and Tartaglia's work had become known. In the *Meetdaet*, Stevin enlarged the group of division problems which he discussed relative to the *Problemata*. He now considered arbitrary points through which the dividing line had to pass, either on, outside or inside the polygon (type III, IV, V, VII).[31]

I discuss Stevin's solution of problem IV to be able to make a comparison to Snellius's approach in problem 6.2.[32] The notation is adapted to that of problem 6.2 (when the same letters are used in both constructions, the same points in

[28]'haec curiosa magis, quam utilia sunt [...] quia non semper per punctum interius duci potest linea, quae triangulum bifariam secet, ut experientia constat.' [Clavius, 1604, p. 295].

[29][Stevin, 1605b], [Stevin, 1608b]. See Struik in [Stevin (D.J. Struik ed.), 1958, p. 764], where only the first book of the *Meetdaet* has been reproduced.

[30][Stevin, 1605b, p. 144]. Snellius calls the *Wijsentijt* 'sapiens et eruditum usque seculum', [Stevin, 1605a, p. 132], and 'prudens et eruditum seculum', [Stevin, 1605a, p. 133], in his translation.

[31][Stevin, 1605b, pp. 138–156].

[32][Stevin, 1605b, pp. 144–150].

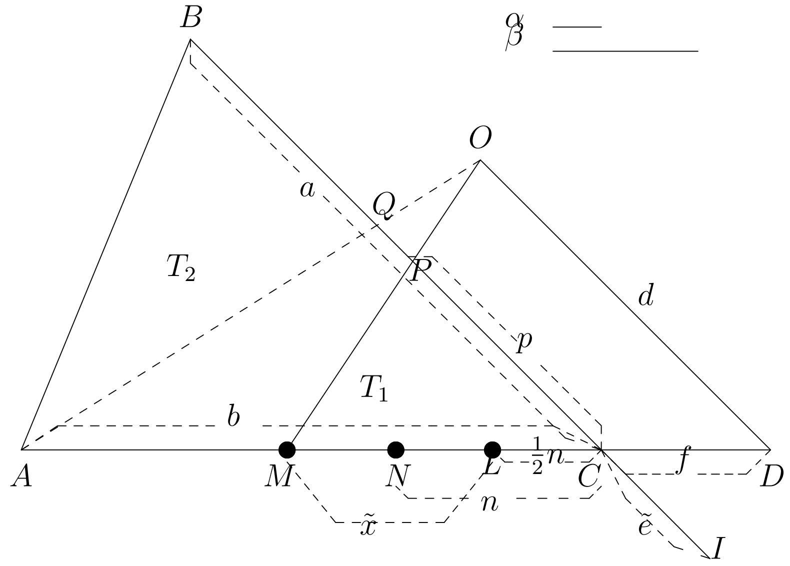

Figure 6.4: Triangle division problem, Stevin's solution (*Meetdaet*)

the figure are referred to).

Problem 6.3 (Stevin, *Meetdaet*) *Given a triangle ABC, a point O outside the triangle and a ratio $\rho = \alpha : \beta$. Cut the triangle in two parts T_1 and T_2 by means of a line through O such that $T_1 : T_2 = \rho$ (see figure 6.4).*

Construction:

1. Prolong either AB or AC.[33] To determine which of these, draw a line from O to A and call its intersection with BC Q. If $\triangle(AQC) : \triangle(ABQ) = \alpha : \beta$, the problem is solved; if $\cdots > \cdots$, prolong AC, otherwise AB. Assume we have to prolong AC. Draw a line parallel to BC through O and call the intersection point of this line and AC prolonged D. [Call OD d and CD f.]

2. Determine I on BC prolonged such that $(\alpha + \beta) : \alpha = BC : CI$. [If we call BC a and CI $\tilde{e}$, then $\tilde{e} = \frac{\alpha a}{\alpha+\beta}$.]

3. Determine N on AC such that $OD : CI = AC : NC$. [If we call AC b and NC n, then $n = \frac{\tilde{e}b}{d}$.]

[33] The construction cannot be applied if O falls outsides the angles $\angle BAC$, $\angle ABC$ and $\angle ACB$, which case is not discussed by Stevin.

4. Construct L at equal distance from N and C. [$LC = \frac{1}{2}n$.]

5. Determine the mean proportional ML of LC and $LC + 2CD$, that is, $LC : ML = ML : (LC + 2CD)$ (Stevin refers to the construction in *Meetdaet* 4.3; a reference to *Elements* VI.13 is given there). [Call ML $\tilde{x}$. Then $\tilde{x} = \sqrt{\frac{1}{2}n(\frac{1}{2}n + 2f)}$.] Draw M on AC (M closer to A than L).

6. Construct OM; this line solves the problem. Call the intersection point of BC and OM P. Now $T_1 = \triangle MPC, T_2 =$ quadrangle $ABPM$.

Proof:

1. rect (AC, BC) : rect $(MC, PC) = \triangle(ABC) : \triangle(MPC)$, because the two triangles have one angle in common (*Elements* VI.4).

2. $(\alpha+\beta) : \alpha = BC : CI =$ rect (AC, BC) : rect (AC, CI) [$= ba : b\tilde{e}$] because of construction step 2.

3. rect $(OD, NC) =$ rect (AC, CI) [$dn = b\tilde{e}$] because of construction step 3.

4. rect (MC, MN) = rect (CD, NC) because of construction steps 4 and 5. [See below for a discussion of this step; $\tilde{x}^2 = \frac{1}{2}n(\frac{1}{2}n + 2f)$, hence $(\tilde{x} + \frac{1}{2}n)(\tilde{x} - \frac{1}{2}n) = fn$.]

5. $CD : MC = MN : NC$ (proof step 4), therefore (*componendo*) $MD : MC = MC : NC$. Also $MD : MC = OD : PC$ because $\triangle(MOD) \sim \triangle(MPC)$.

   Now $MC : NC = OD : PC$, hence rect $(MC, PC) =$ rect $(NC, OD) =$ rect (CI, AC) (proof step 3). Because of proof step 2 we have:

$$\begin{aligned}(\alpha + \beta) : \alpha &= \text{rect } (AC, BC) : \text{rect } (AC, CI)\\ = \text{rect } (AC, BC) : \text{rect } (MC, PC) &= \triangle(ABC) : \triangle(MPC)\end{aligned}$$

   (proof step 1).

   Therefore, $\alpha : \beta = \triangle(MPC) :$ quadrangle $(ABPM)$.

The general outline of Stevin's construction is similar to Snellius's. Stevin gave a longwinded explanation of the proof steps, covering over three folio pages. The part that is most at variance with Snellius's solution and most remarkable altogether is proof step 4, where again the problem equivalent to the solution of a second degree equation had to be tackled. Stevin's explanation of step 4 consists of several parts. His purpose was not only to show that his inferences were right in this case, but also to explain how to find a geometrical solution if a similar problem appeared.

In order to achieve this, Stevin made a detour through algebra, to obtain what he called 'a geometrical operation drawn from an algebraical operation'.[34] He first gave an example: he assigned numerical values to most of the line segments and showed that in this case he had to solve the problem 'to find two numbers, one of which is $\frac{3}{2}$ smaller than the other, and the outcome of their multiplication is $\frac{8}{3}$'.[35] In this way, he could get rid of the geometrical context and interpretation for a moment and he solved the problem by calling the unknown MC 1① (in the notation of my explanation of the construction, $MC = ML + LC = \tilde{x} + \frac{1}{2}n$) and applying the standard algorithm for finding the root (he only considers the positive one) of this second degree equation. He then repeated the steps of the procedure, explicitly referring to the names of the line segments, still using the same numerical values, but thus enabling the reader to apply the algorithm with other numbers. Finally, he returned to geometrical terms, explained that this procedure was not the shortest in this case and construed why construction steps 4 and 5 yielded the desired result.

Stevin's concern about didactics made him give very long and detailed explanations. Although his construction is of the same degree of complexity as Snellius's 1599 solution, Stevin spent many more pages to his solution. His clarification helps the reader, yet sometimes it can lead to confusion because the status of the information is not completely clear, as is the case in step 4 of the proof, where Stevin does not make explicit whether an algebraic procedure counts as part of the proof of the construction of a geometrical problem or only helps to find a way towards a proper geometrical solution. Moreover, his constructions are well explained in general, but they are sometimes difficult to understand because of a number of mistakes.

Stevin paid some attention to the non-solvability of division problems: he explained that for some internal points and ratios no solution exists. As an example, he mentioned that if the line has to pass through the 'centre of gravity' of the triangle, the area that is cut off is always between $\frac{4}{9}$ and $\frac{5}{9}$ of the total area of the triangle, without proving this assertion.[36]

Snellius knew the *Meetdaet* well, because he had translated it into Latin.[37] When we compare the part of the *Meetdaet* discussed above to Snellius's rendering of it, we see that he made an almost literal translation. He made some small corrections, but left at least one mistake (in proof step 4, a wrong line

[34] 'meetconstighe wercking ghetrocken uyt een stelreghelsche wercking/ [in margin:] Geometrica operatio sumpta ex Algebraica operatione.' [Stevin, 1605b, p. 147].

[35] 'Te vinden twee ghetalen, teen 3/2 cleender als t'ander, die t'samen ghemenichvuldicht uytbrenghen 8/3.' [Stevin, 1605b, p. 147].

[36] 'een driehoucx swaerheyts middelpunt', [Stevin, 1605b, p. 150].

[37] Snellius translated Stevin's *Wisconstige Gedachtenissen* into Latin, under the title *Hypomnemata Mathematica*, [Stevin, 1608a], part of which is the *Meetdaet*, translated as *De Geometriae Praxi*, [Stevin, 1605a].

segment was designated).[38] He also added a reference to a proposition in a work by Vitello to Stevin's explanation of step 4 of the proof, by which 'it could be explained very easily'. Apparently, Snellius considered Stevin's explanation as not completely satisfying.[39]

To summarize, in the decades around 1600, the problem of dividing triangles and other polygons received ever more attention. Several relevant texts became gradually more known, stimulating mathematicians to add their own solutions. Snellius is a representative of this group of eager students of the problem. He certainly knew Van Ceulen's and Stevin's work. Whether he had also been studying Clavius is unclear, but it is not unlikely, considering that Clavius was one of the most renowned mathematicians of his era.

6.4 Better and better: Snellius's continuous involvement

6.4.1 Snellius in *Cutting off* (1607): 'order of nature'

Snellius contributed again to the solution of the triangle division problem in his own treatise Περὶ λόγου ἀποτομῆς, καὶ περὶ χωρίου ἀποτομῆς *resuscitata geometria* of 1607 (see p. 53; the title is abbreviated here as *Cutting off*). This was his reconstruction of two lost treatises on problems from pure geometry by Apollonius of Perga, *Cutting off of a Ratio* and *Cutting off of an Area*.[40]

Snellius addressed the second dedication letter of *Cutting off* to Stevin, and immediately mentioned triangle division in it:

> There is no doubt that the division of rectilinear figures has been described properly and rightly by the ancients and that therefore the section from a given point inside or outside the triangle was by no means unattempted by or unknown to them (which I remember you too to have remarked elsewhere)[41]. Indeed, there existed a small appendix to this work *On the Cutting off of an Area*, as can be inferred from Pappus. And for that reason I have also restored that little corollary here: because I believed that it is better to draw these things from the sources than to pursue them in small brooks.[42]

[38][Stevin, 1605a, pp. 133–139].

[39]'potuit autem commodissime expediri per 127 propos. lib. 1. Vitellonis.' [Stevin, 1605a, p. 135].

[40]See section 2.8 for an overview of Snellius's reconstructions of Apollonian treatises.

[41]Stevin had written that the Greeks had not known this type of problems (see previous section), so we should probably take Snellius's 'ancients' to be both the Pristine Sages and the Greeks from Antiquity—Snellius seems to indicate the latter.

[42]This expression was used by Cicero several times.

> And the more so because not so long ago, I have seen a miserable discussion of this problem about the division of a triangle from a given point outside it.[43]

We see that Snellius considered the triangle division problem as belonging to the subject matter of *Cutting off of an Area*, for which reason it should also be discussed in his reconstruction. I have not been able to find any reference to the appendix which Snellius mentioned, either in modern editions of Pappus, or in the Latin translation by Commandino read in Snellius's time. Therefore I presume that it was Snellius himself who saw the connection between Apollonius's treatises and triangle division, and only after having done so saw some statement in Pappus's summaries which he could interpret as an indication of the former existence of such an appendix. It is even conceivable that Snellius only referred to Pappus to convince the reader that this addition was in truly Apollonian spirit.

Snellius then adopted a polemical attitude, attacking someone whom he called Bambalio:

> Indeed, a certain Bambalio does not cease to stammer, after he had copied off that demonstration from somewhere else, nor does he dare—or is he able—to clearly say the general [rule]. Because it is of no consequence where the given point is situated (as long as it is outside the triangle): for in one position it requires no other interpretation of the work than in another.[44]

It is not certain to whom the nickname 'Bambalio' refers. It is very well possible that Snellius taunted Tartaglia. *Bambalio* was a Latin 'cognomen' (surname), meaning something like the stammerer[45], and the Italian *Tartaglia* means the same. Tartaglia had indeed discussed several problems in which O was situated outside the triangle, but this feature is not typical for him. In fact, the critique applied to Van Ceulen's presentation of the division problem as well, but it is difficult to imagine that Snellius would have criticized his teacher so severely.

In the sequel to this statement, Snellius made his thoughts on clear mathematics more explicit:

[43]'Tributionem figurarum rectilinearum a veteribus ordine et via descriptam: atque adeo sectionem, a dato intra vel extra triangulum puncto, neutiquam illis intentatam, aut ignoratam (quod aliubi tibi quoque notatum memini) non est dubium. Fuit enim appendicula huius περὶ χωρίου ἀποτομῆς, quemadmodum ex Pappo coniicere est. ideoque etiam consectariolum illud huc revocavi: satius enim credidi ista ex fontibus haurire, quam in rivulis consectari. idque eo magis, quod problema hoc, de sectione trianguli abs dato externo puncto, misere non ita pridem tractatum viderem.' [Snellius, 1607b, p. 15].

[44]'enimvero Bambalio quidam, cum demonstrationem illam alicunde descripsisset, balbutire tamen non desinit, nec aperte audet, aut novit dicere τὸ καθολικὸν. nam nihil interest datum punctum (si modo extra triangulum est) ubi ubi sit: neque enim aliam operis exegesin hoc situ, quam illo postulat.' [Snellius, 1607b, p. 15].

[45][Lewis and Short, 1879, p. 221].

> Or would it not be strange to split into many parts what nature wanted to be one, and to tear it into small pieces against the precepts of science? I have therefore bestowed care upon the solution of this division, not only that from a given point outside the triangle, but also from one inside it, in one single problem: I have sought to attain the simplest [solution] indeed, the analytical one, according to the order of nature.
>
> The small distinctions of this kind must certainly be reduced to a unity, because they are maintained in science against scientific truth: we have learned that the ancient writers have always done this by dealing with one and the same proposition, with several cases. They were named Great Geometers because of their more general propositions, and because of the divinity of their intellect they are even placed above the stars.[46]

Thus, according to Snellius, there is a superhuman norm for good mathematics, of which nature itself has determined the structure. This was understood well by the Greek mathematicians, whose works were still relevant as praiseworthy examples for Snellius's contemporaries for that reason. He claimed that in good mathematics, propositions had to be general. If a problem could not be solved generally, different cases could be admitted.[47]

This methodological point of departure can indeed be recognized in Snellius's solution to the triangle division problem in *Cutting off*. Snellius discussed the following variant of the problem (type IV and V):[48]

Corollary 6.4 *Given a triangle ABC, a point O inside or outside the triangle and a ratio ρ. Cut the triangle in two parts by means of a line through O so that the resulting parts have ratio ρ to each other.*

Snellius did not include the case where O is located on the triangle explicitly,

[46]'Annon igitur, quod natura unum esse voluit, illud πολυσχιδεῖν, inque minutias discerpere contra artis praecepta ἄτοπον fuerit? Quamobrem dedi operam ut sectionem, non solum a dato extra, sed etiam intra triangulum puncto unico problemate absolverem: simplicissimam enim ἀναλυτικὴν secundum ordinem naturae, secutus sum. istiusmodi vero distinctiunculae, quia in arte contra artem retinentur, ad unitatem revocandae sunt: quod veteres in una eademque propositione, casibus differente, factitasse didicimus. qui ob enuntiationes καθολικωτέρας Magni Geometrae audiebant, et ob ingenii divinitatem supra ipsa astra collocati sunt.' [Snellius, 1607b, p. 15].

[47]Cp. the highly similar view by the modern Dutch mathematician Hendrik Lenstra in his inaugural lecture: 'Zo is het met een goed bewijs ook, je koerst zonder overbodige omwegen op het doel af, en je gebruikt alleen middelen die je echt nodig hebt. [...] Het geldt ook als lelijk om te veel verschillende gevallen te onderscheiden, wiskunde is niet hetzelfde als puzzelen. Het is veel beter, en meestal ook begrijpelijker, als je alle gevallen allemaal onder één hoedje kunt vangen.' [Lenstra, 2001, p. 40].

[48][Snellius, 1607b, pp. 19–20].

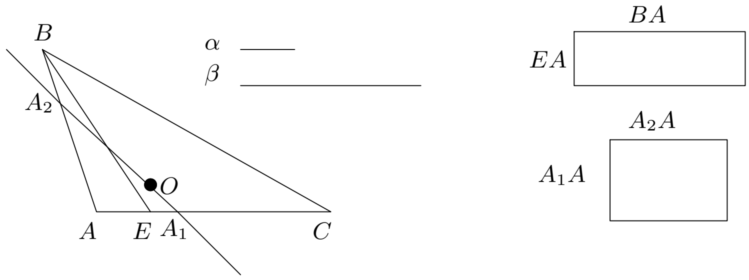

Figure 6.5: The triangle division problem in *Cutting off of an Area*

but his solution is valid for that case as well, so that we can perceive his problem as a rare example of type I.

Before giving his construction, Snellius repeated Cicero's sentence that it was better to draw these things from the sources than to pursue them in small brooks. He seems to mean that the triangle division problem had its natural position here in the framework of Apollonius's work, both because that was ancient, and because the problem could be solved without much extra effort.

Snellius's construction is as follows (see figure 6.5):

Construction:

1. Divide the base AC of the triangle in E, such that $AE : EC = \rho$.

2. Use the second problem of *Cutting off of an Area* (see problem 6.5 below) to determine A_1 on AC, A_2 on AB such that rect $(AE, AB) =$ rect (AA_1, AA_2) and O, A_1, A_2 collinear. The line through O, A_1 and A_2 solves the problem.

Proof:

rect $(AE, AB) =$ rect (AA_1, AA_2), hence $AA_1 : AB = AE : AA_2$, and hence $\triangle AEB$ and $\triangle AA_1A_2$, which have one angle in common, have equal sizes. Because of construction step 1 $\triangle AEB : \triangle CEB = \rho$.

We see that the triangle division problem is solved here in a corollary to the second problem of *Cutting off of an Area*, which is:

Problem 6.5 *Given a point O, two lines in position l_1 and l_2, U the intersection of l_1 and l_2 and an area* $\mathbf{G}$. *It is required to determine points A_1 on l_1 and A_2 on l_2 collinear with O, such that rect $(A_1U, A_2U) =$* $\mathbf{G}$ *(see figure 6.6).*

This is a special case of the general problem of *Cutting off of an Area*, the formulation of which is the same, apart from the fact that instead of U, U_1

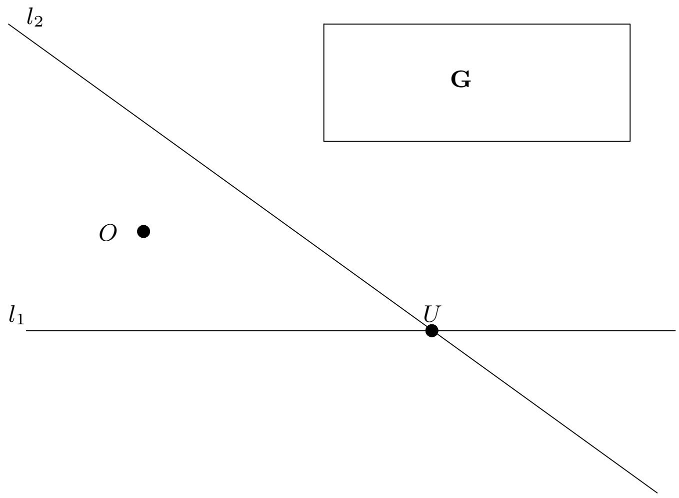

Figure 6.6: The givens of problem 6.5

and U_2 are given, located somewhere on l_1 and l_2 respectively (see figure 6.7). If we substitute l_1 by AC prolonged, l_2 by AB prolonged, $U = A$ and $\mathbf{G} =$ rect (AE, AB) (see figure 6.5), we see that the solution to this problem is indeed applicable to the triangle division problem.

Snellius solved problem 6.5 as follows (see figure 6.8):[49]

Construction:

1. Draw a line through O parallel to l_2 and call its intersection point with l_1 I [notation: $OI = d$].
2. Determine T on l_1 such that rect $(UT, OI) = \mathbf{G}$. [$UT = \frac{\mathbf{G}}{d}$, call this a.]
3. Determine A_1 on l_1 such that sq $(A_1U) =$ rect (A_1I, UT) by means of one of the problems of *Apollonius Batavus*. [If we call $A_1U = x$ and $IU = e$, this is equivalent to solving $x^2 = a(e - x)$.]
4. Draw a line through O and A_1; its intersection point with l_2 is A_2.

Proof: Since sq $(A_1U) =$ rect (A_1I, UT), we have $UT : A_1U = A_1U : A_1I$; moreover (similar triangles) $A_1U : A_1I = A_2U : OI$, hence $UT : A_1U = A_2U : OI$. Therefore, rect $(A_1U, A_2U) =$ rect $(UT, OI) = \mathbf{G}$.

In step 3 of the above construction, Snellius used another reduction, namely to the first problem of *Apollonius Batavus*. The most important aspect of this

[49] [Snellius, 1607b, p. 19].

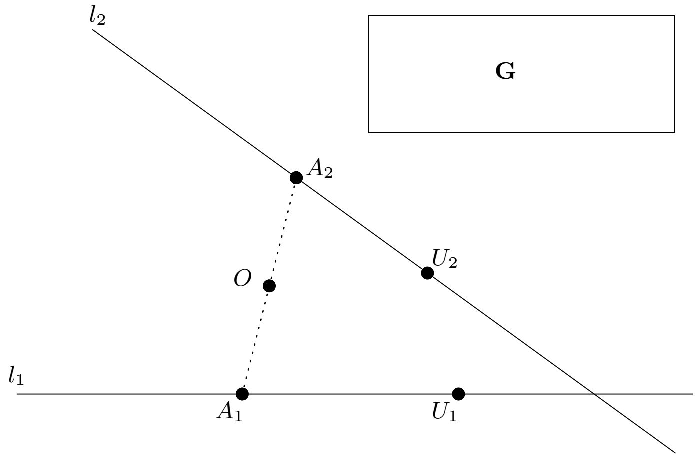

Figure 6.7: The general problem of *Cutting off of an Area*

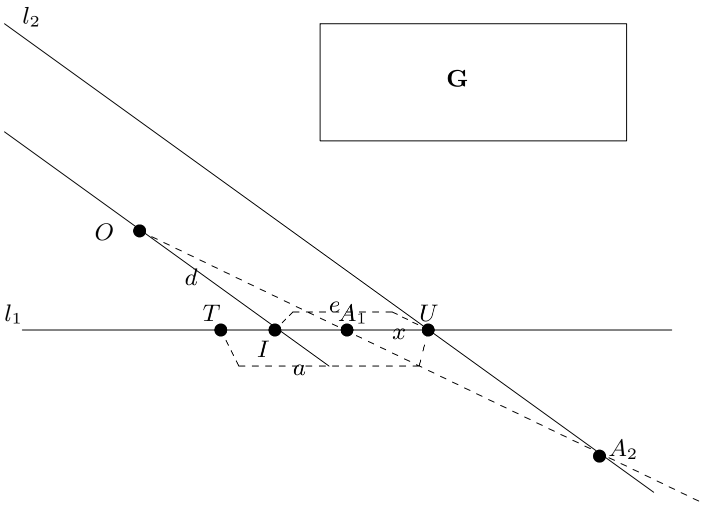

Figure 6.8: Problem 6.5 with one of its solutions

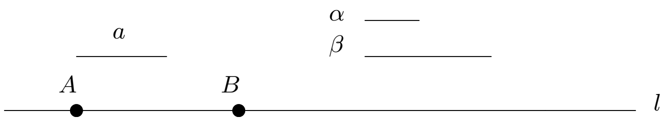

Figure 6.9: The givens of the first problem of *Apollonius Batavus*

problem is that now all givens and the required point are on one line, instead of on two:[50]

Problem 6.6 *Given the points A and B on a line l, a ratio $\rho = \alpha : \beta$ and a line segment a. Determine the point O on l such that $sq\,(AO) : rect\,(BO, a) = \rho$ (see figure 6.9).*

This can be applied to step 3 of the construction of problem 6.5 if we substitute l by l_1, A by U, B by I, O by A_1, a by UT and if we take $\rho = 1 : 1$ (see figure 6.8).

Problem 6.6 can be interpreted as the representation of several classes of second degree equations in one unknown: if $AO = x$ and $AB = b\,(> 0)$, and if we take A to the left of B (the other case is analogous), we must find x'es that obey one of the following three equations (interpret ρ now as a number):

$$x^2 = \rho a(b + x), \tag{6.1}$$
$$x^2 = \rho a(b - x), \tag{6.2}$$
$$x^2 = \rho a(x - b). \tag{6.3}$$

Equation (6.1) yields one solution O to the left of A, eq. (6.2) one solution O between A and B and eq. (6.3) zero, one or two solutions O to the right of B. This number of solutions follows from the fact that $a, b, \rho, x > 0$. Snellius found all these solutions by means of a geometrical construction involving two circles, one of which always cuts the line l in two points, thus yielding two solutions, the other circle either situated above the line, or cutting it in one (the limit case) or two points.[51] He also gave the condition for the limit case. The different cases of problem 6.6 were relevant to the solution of problem 6.5 and therefore also to the solution of the triangle division problem 6.4.

The construction-and-proof tree discussed above can be represented in a diagram, which also contains some branches undiscussed here:

[50][Snellius, 1608a, p. 12].

[51]The construction is almost completely the same as that discussed in section 6.4.3, to which I refer for more details.

Triangle division problem		
cor. 6.4 ↓		
Special case of problem of	General problem of	General problem of
Cutting off of an Area	*Cutting off of an Area*	*Cutting off of a Ratio*
prob. 6.5	fig. 6.7	prob. 2.1
↓	↙ (*condition*) ↓	↓
Problem *Apollonius Batavus*, var. 1		Problem *Apollonius Batavus*, var. 2[52]
prob. 6.6		

↓: is reduced to

This figure shows that Snellius tackled the triangle division problem by placing it in a hierarchy of problems, in which the solutions of problems of the higher layers depended on those of the lower layers. In this way, he could solve the division problem at once, without having to discuss separate cases for the point O inside, on, or outside the triangle on this layer. This arrangement suggests that Snellius was looking for the most general means of solving problems.

Pappus did not connect the problems of *Cutting off* and the *Determinate Section* in Snellius's way, giving the summary of the *Cutting off*-treatises before that of the *Determinate Section*, which formed the basis of the *Apollonius Batavus*-treatise. He must have used other criteria for his ordering, maybe the complexity of the subject matter. Snellius's *Cutting off* was published one year before *Apollonius Batavus*, hence it seems that Snellius discovered this structure while working and decided to publish first what had seemed to be first when he started his work.

Snellius's own way of solving the triangle division problem shows that he allows for a multi-layered solution, where the different cases of the lowest layer influence the upper layers. By embedding the triangle division problem in this structure, Snellius saved the reader some effort because the work done in one layer also applied to the other layers; yet the reader only interested in triangles could lose track more easily and had to understand more than if he only had had to apply one of the traditional solutions discussed before.

The antithesis between the presentation of the solution of *Cutting off* and that of the older ones, both of Snellius and others, can be described in modern terms as 'global' versus 'local'. Only knowledge of the propositions of the *Elements* was needed for the understanding of a local presentation. On the other hand, a global presentation was part of a larger structure, of which a part had to be worked through to understand the correctness of the solution. The foundations of the edifice were again the *Elements*. Snellius connected the three Apollonius-treatises under consideration in his reconstruction of them, deducing

[52] Given the points A, B and C on a line l, a ratio ρ and a line segment a. Determine the point O on l such that rect (AO, a) : rect $(BO, CO) = \rho$ (the names of the points can be permutated).

the solution of the triangle division problem as an ornament of his building.

Other Dutchmen also continued to work on division problems. The practitioner Sybrandt Hansz Cardinael devoted several of his *Hondert Geometrische questien met hare solutiën* ('Hundred Geometrical Questions with their Solutions', 1612) to the division of triangles and other polygons. Cardinael solved the problem of dividing a triangle into two equal parts by means of a line through a point inside the triangle (type IVa) by giving a construction based on *Elements* VI.31, a generalization of Pythagoras's Theorem.[53] His construction is different from Snellius's. Cardinael formulated the assignment in terms of a practical surveying problem: the triangle was a field that had to be divided into equal parts by a straight ditch through a given point.[54]

6.4.2 *The professor and his teacher: the* Fundamenta

In section 6.3.1, Van Ceulen's work on triangle division and Snellius's juvenile contribution to it have been discussed. Snellius translated the *Fondamenten* into Latin as *Fundamenta Arithmetica et Geometrica* (1615).[55] As said before, Snellius did not translate Van Ceulen's text literally: he added (sometimes rather large) comments, which were printed in italics, he corrected mistakes and changed Van Ceulen's presentation in many places, in this way creating a new work that bears his mark distinctly. Yet he was restricted by the fact that there was no occasion to have new figures produced; about this Snellius complained openly in several places in the book.

This reworking can be discerned very clearly when the first section of the *Fondamenten* on division and its translation are compared. Almost always, Van Ceulen gave specific examples, with special values for ρ expressed in numbers. Snellius formulated the problems in a more general way. A comparison between Van Ceulen's and Snellius's formulations in a specific case shows this difference clearly. Van Ceulen wrote: 'One wants to divide this triangle that is put here, drawn by ABC, into two equal parts, with a straight line drawn from vertex C', whereas Snellius asked 'to cut a given triangle in a given ratio with a straight line drawn from a vertex'. Snellius used the classical concept of 'given' twice.[56]

Although Snellius's formulation was more general, he followed Van Ceulen in his treatment: in fact, he gave Van Ceulen's examples after his general enunciations, which enabled him to use the old diagrams. However, more often than

[53] [Sitters, 2003, p. 315]. For Cardinael's biography see [Sitters, 2004].

[54] [Sems and Dou, s a, App., p. 101].

[55] See section 2.9.3 for the publication history of the *Fundamenta* and section 5.3 for a general comparison of the Latin and Dutch edition.

[56] 'Desen hier tegen-gestelden Tryangel, geteeckent met ABC, wilmen deelen in twee gelijcke deelen, met een rechte linie getrocken wt den hoeck C.' [van Ceulen, 1615a, p. 119]. 'Datum triangulum recta ex angulo ducta secare ratione data.' [van Ceulen, 1615b, p. 92].

Van Ceulen, he ended the discussion of the problems by remarking that the given construction could be used for any ratio. Snellius stressed the general features of some of the problems in another way as well: by combining several of Van Ceulen's separate problems into one enunciation. And where Van Ceulen discussed various polygons in separate examples, Snellius resumed these in one proposition about 'triangulata'. This Ramist term designated a rectilinear area consisting of triangles.[57]

Snellius used this same approach in his translation of Van Ceulen's second section on division problems. He bundled Van Ceulen's nine examples into two problems, one with the point O outside the triangle and one with O inside it. Some obscurities were clarified, yet a few minor mistakes were maintained. Because Van Ceulen had not proved the correctness of his construction to solve the problem of type IVa, Snellius supplemented his translation with a proof, remarking that if a figure to his own liking had been available, he could have provided a much easier construction and a more elegant proof.[58] It seems that Snellius wanted to 'purify' Van Ceulen's work, which means to make it more Euclidean by imposing a clearer structure on it, by diminishing the role of numerical exemplary values in the geometrical problems and by having the correctness of all constructions proved. This purification would make the work fit better into the classical mathematical tradition.

Snellius added an interesting comment to his translation of Van Ceulen's rendering of Snellius's own solution to the problem. He drew the attention to his own work in *Cutting off*, and announced some more work on the same topic:

> I have reduced [the problem of] dividing a given triangle from a given point outside or inside it in a given ratio, as a corollary, to the second problem in the *Geometry about the section of the area* of Apollonius as restored by me, where much more general [problems] than these are presented. However, because the division that is displayed here is somewhat tedious and less elegant, I would now have shown the way that I usually follow in these matters, had it been allowed by a figure, which will be provided in the second edition.[59]

[57][van Ceulen, 1615a, pp. 127–128], [van Ceulen, 1615b, pp. 100–102], [Verdonk, 1966, p. 315].

[58]'Si diagramma ex arbitrio nostro deformatum fuisset non tantum fabrica longe foret expeditior, sed et ipsa demonstratio non paulo concinnior: nunc aliena lineamenta nobis fuerunt consectanda.' [van Ceulen, 1615b, p. 205].

[59]'Datum triangulum e puncto extra aut intra ipsum dato data ratione secare, tanquam consectarium rettuli ad problema secundum in suscitata a nobis Apollonii de spatii sectione Geometria, ubi longe istis generaliora praestantur. Veruntamen quia factio, quae hic exhibetur nonnihil taedii et minus concinnitatis habet, illam quam in istis soleo insistere viam nunc ostenderem, si per diagramma licitum esset, quod iterata editione curabitur.' [van Ceulen, 1615b, p. 200].

He criticized his own first attempt here, preferring now the more general approach of *Cutting off*.

Snellius ended this addition to Van Ceulen's text with a reference to Stevin's *Hypomnemata Mathematica* for the reader who wanted to see some other solutions to the same problems. The interested reader could also find there the problem of subtracting a given area S_2 from a (larger) area S_1 by means of a line through a point inside or outside S_1. Stevin had 'shown [these solutions] fully and most clearly'.[60] Thus while Snellius drew the attention to his own work, he also pointed out the merits of other solutions of division problems.

6.4.3 Snellius in De Circulo (1619): 'easier and more elegant'

Although Snellius wishfully referred to a second edition of the *Fundamenta*, in which he could further develop some of his improvements of and variations on Van Ceulen's work, such a book did not appear. Yet one of the projects which he had announced did reach the public in a sort of minimalist version of such a second edition. He added a small appendix to another Latin translation of a book by Van Ceulen, *De Circulo* (1619), with another solution to the division problem, illustrated by a new figure.

In fact, this book contained, to a large extent, different material from the original Dutch book called *Vanden Circkel*, in which Van Ceulen had discussed the quadrature of the circle and other topics. Part of the material of the *Fundamenta* was recycled in *De Circulo*, among which the part on triangle division, and therefore Snellius thought it expedient to add an appendix about the same problem, which contained two new figures. He added two references to the appendix in the main text, one of them at the end of the reprinted quotation cited at the end of the previous subsection.[61]

He solved the problems of dividing the triangle by means of a line through a point outside (type V) and a point inside it (type IV) separately in this appendix. The constructions are to a large extent similar. I give the problem of type V in full to make a comparison possible with problem 6.2, which is almost equivalent.

Problem 6.7 (Snellius, 1619) *Given a triangle ABC, a point O outside the triangle and a ratio $\rho = \alpha : \beta$. Cut the triangle in two parts T_1 and T_2 by means of a line through O such that $T_1 : T_2 = \rho$ (see figure 6.10).*

Construction:[62]

[60] '[...] luculentissime commonstravit [...]', [van Ceulen, 1615b, p. 200].

[61] Appendix in [van Ceulen, 1619, 1, pp. 219–220], references in [van Ceulen, 1619, 1, pp. 155, 159].

[62] I have adjusted the notation to that of problem 6.2.

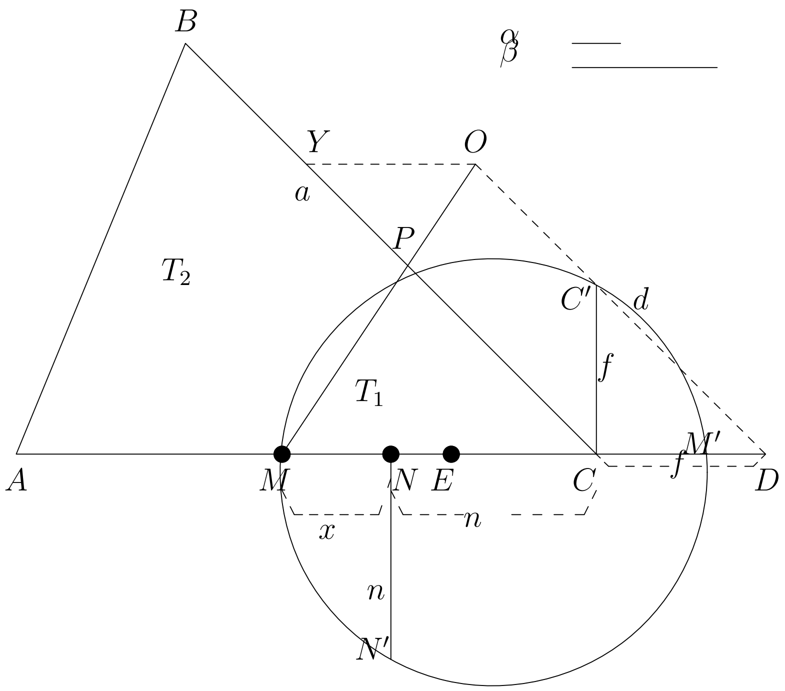

Figure 6.10: Triangle division problem, Snellius's solution (1619)

1. Draw a line through O, parallel to BC, and call its intersection point with AC prolonged D. [If necessary, permutate the names A, B and C to arrange that O is closest to side BC. Call DO d.]

2. Mark Y on BC such that $YC = OD[= d]$.

3. Determine N on AC such that $\triangle NYC : \triangle ABC = \alpha : (\alpha + \beta)$. $[NC = n.]$

   [Snellius does not explain how to do this. Cp. construction steps 1 and 3 of problem 6.2: take E on AC such that $AE : EC = \beta : \alpha$ and construct triangle EBC. Then $\triangle ABE : \triangle EBC = \beta : \alpha$. Call BC a. Determine N such that $BC : YC = NC : EC$, then $\triangle EBC = \triangle NYC$, because they have one angle in common. This construction solves the division problem when O is located on one of the sides of the triangle.]

4. Construct N', such that $NN' \perp AC$, N' under AC, $NN' = NC[= n]$ and construct C', such that $CC' \perp AC$, C' above AC, $CC' = CD$. [Call CD f.] $(*)$

5. Construct a circle of which $C'N'$ is a diameter; call its intersection points with AC (prolonged) M and M', M to the left of M'.

   [In a Cartesian coordinate system with N as the origin, the circle can be described by

   $$(x - \frac{1}{2}n)^2 + (y - \frac{1}{2}(f - n))^2 = \frac{1}{4}(n + f)^2 + \frac{1}{4}n^2.$$

   Thus the coordinates of M are $(\frac{n}{2} - \sqrt{n(\frac{n}{4} + f)}, 0)$, of $M' : (\frac{n}{2} + \sqrt{n(\frac{n}{4} + f)}, 0)$. M' is always located beyond C and can further be disregarded. M is always located to the left of N. If M is located beyond A, the construction does not work.]

6. Connect M and O; call the intersection point of this line segment with BC P. The line OPM solves the problem.

Proof:[63]

1. If we can show that $\triangle MPC = \triangle NYC$, we are ready because of construction step 3. In order to do so, we use a figure from *Apollonius Batavus* (see figure 6.11):[64]

   - $\angle N'M'C' = \frac{\pi}{2}$ because $N'C'$ is a diameter of the circle. This angle is equal to $\angle N'M'N + \angle C'M'N$.

[63] Extended version of Snellius's own proof.

[64] [Snellius, 1608a, pp. 12–13], part of the solution of problem 6.6.

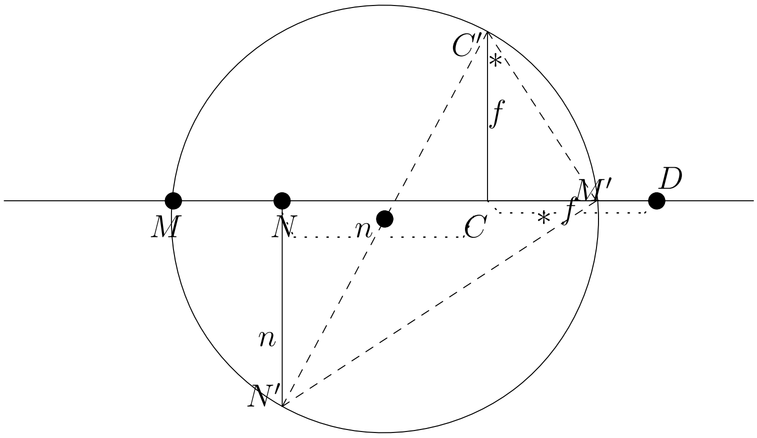

Figure 6.11: Auxiliary steps from *Apollonius Batavus*

- Because also $\angle CC'M' + \angle C'M'C = \frac{\pi}{2}$, $\angle N'M'N = \angle CC'M'$ (both marked with $*$ in the figure).
- Thus, $\triangle N'M'N \sim \triangle M'C'C$, hence $NN' : CM' = NM' : CC'$.
- Then rect (NN', CC') = rect (CM', NM'). The left-hand side is equal to rect $(NC, CD)[= nf]$. The right-hand side is equal to rect (MN, MC) because of the symmetry of the figure.

2. Because rect (MN, MC) = rect (NC, CD), we have: $MN : NC = CD : MC$. By *componendo*: $MC : NC = MD : MC$.

3. Because $\triangle MPC \sim \triangle MOD$ and $OD = YC$, $MD : MC = YC : PC$. Combination with the previous step yields $MC : NC = YC : PC$.

4. Because $\triangle MPC$ and $\triangle NYC$ have one angle in common, and $MC : NC = YC : PC$, their areas are equal, as had to be proved.

   [A shorter proof can be given by means of algebra. To prove that $\triangle MPC$ and $\triangle NYC$ have equal areas, note that because of their shared angle, $\triangle MPC : \triangle NYC =$ rect (MC, PC) : rect (NC, YC). Now $MC = \frac{n+\sqrt{n^2+4fn}}{2}$; use this and the similarity of $\triangle MPC$ and $\triangle MOD$ to calculate that $PC = \frac{YC \cdot MC}{MD} = \frac{d(n+\sqrt{n^2+4fn})}{n+\sqrt{n^2+4fn}+2f}$. It follows that rect (MC, PC) : rect $(NC, YC) = dn : dn = 1 : 1$.]

If the point O is situated inside the triangle, the division problem does not always have a solution. The construction that Snellius proposed in this case is almost the same as the one above, only C' has to be constructed under instead of above AC in step $(*)$. The proof is almost the same as before; the reasoning in the part borrowed from *Apollonius Batavus* has to be slightly changed to yield again the two similar triangles $\triangle N'M'N$ and $\triangle M'C'C$.

Snellius did not explain that this last construction would not have yielded a solution if O is outside the triangle, nor would the first construction (with both perpendiculars to different sides) if O is inside the triangle. He did mention, however, that the circle does not always cut AC if O is inside the triangle: 'when the circle cuts or touches the base, the [solution of the] problem is possible'.[65] Snellius in fact claimed here that if his construction did not yield a solution, the problem was not solvable and it was useless to look for another construction. This claim could be proven, but Snellius did not do so. This approach fits in the normal style of early modern geometrical problem solving in which giving solutions and demonstrating their validity was the focus of activities, not the exploration of limitations, which is different from Snellius's Apollonius treatises, where a meticulous survey of all different cases is given for all problems.

The construction of problem 6.7 cannot be applied if O is in the section between BA prolonged (beyond A) and CA prolonged (beyond A), AB prolonged and CB prolonged, or AC prolonged and BC prolonged. Snellius did not discuss this case. If P happens to fall outside the line segment BC, the argumentation of the proof does not work, but Snellius does not address the question how the construction should be changed in that case.

At some moment, Snellius must have noticed the similarity between one of the steps of his triangle division solution of 1599 and part of the construction of one of the Apollonian problems, realizing that he could easily find an ingenious new solution by this connection. In fact, his 1619 solution is very similar to his 1607 solution (both involving a similar construction by means of circles), yet this new one is presented 'locally'. The reader could only appreciate this clever connection with *Apollonius Batavus* if he knew where to find this construction in this treatise; Snellius did not give him a page number. It is not easy to trace it because the problem to be solved in step $(*)$ does not correspond to any of those solved in *Apollonius Batavus*; the relevant construction is part of a larger solution. Snellius's assertion that it is easy to derive the equality of rect (MN, MC) and rect (NC, CD) (proof step 1) can only hold for a rather advanced reader. If this were evident, his explanation in *Apollonius Batavus* would also have been superfluous.

Why, then, did Snellius consider this solution easier and more elegant than that by Van Ceulen and why did he not stick to his own solution of 1599? The main reason for this seems to have been that the new solution of the problem

[65]'[...] cum circulus [...] basin [...] secabit aut continget problema est possibile [...]', [van Ceulen, 1619, 1, p. 220].

can be drawn in one figure in its entirety, which Snellius may have preferred to using auxiliary figures.[66] The similarity of the constructions for O inside and outside the triangle may also have appealed to him.

Another reason might be that Snellius followed a criterion of Pappus for elegant mathematics. Pappus had written in his summary of Apollonius's *Determinate Section* that Apollonius had solved the problems of that book in two ways: by straight lines, 'trying the more beaten path', and 'ingeniously by means of semicircles'. Snellius had quoted this paragraph in *Apollonius Batavus*.[67] He may have been sensitive to the judgment of Pappus and have tried to follow the more sophisticated way involving circles in his own solution to the triangle division problem. The connection to *Apollonius Batavus* and thus to the highly-rated mathematics of Apollonius may also have been an appealing feature.

No definitive explanation of Snellius's statement can be given, because 'easier and more elegant' are not objective mathematical criteria. It can be considered as an advertorial claim, drawing the attention to yet another, not necessarily better solution of the well-known triangle division problem. To grasp Snellius's 1619 construction, some insight into the equality of angles is needed and the proof is somewhat longer than that of 1599, but this makes no essential difference to earlier solutions.

6.5 Conclusion

The triangle division problem is a typical specimen from the tradition of early modern geometrical problem solving. A number of mathematicians studied it around 1600, discovering sources with older solutions and devising their own constructions in the same period. Snellius was one of those. He invented several new solutions of the triangle division problem and introduced others to relevant works by Benedictus and Apollonius. The problem clearly occupied Snellius and can therefore serve as a telling example of Snellius's activity in pure geometry and his relationship to other mathematicians.

Snellius presented his own and Van Ceulen's solutions in a strictly geometrical framework in Euclidean spirit, in which he kept numbers separated from the geometrical exposition as much as possible. Snellius seemed to want to show the reader that he had surpassed his former teacher Van Ceulen in exactness and elegance.

[66]Cp.: 'Yet there is an attempt at elegance to be discerned in Ghetaldi's constructions; he tried as much as possible to combine the different steps of a construction in one figure, using the minimum number of auxiliary lines and circles. If he had made the separate steps in separate drawings, the matter would have been structurally more clear, but the construction would have been less direct and simple.' [Bos, 2001a, p. 106].

[67]Translation quoted from [Pappus of Alexandria, 1986, pp. 90–91]; [Snellius, 1608a, p. 9].

The presentation of the solution in the Apollonius reconstructions is of a different nature from that of the other solutions by Snellius: the former could be characterized as global (the solution was embedded in a general structure), as opposed to the other, local presentations. With his global approach, Snellius distinguished himself from the other triangle dividers. His local presentations date both from before and after the Apollonius treatise, thus we cannot summarize the development in Snellius's work as one towards greater generality. The context of the different books and their audiences is probably an explanatory factor for the choice of a local or a global approach. *Cutting off* and *Apollonius Batavus* were mainly intended for specialist mathematicians with a firm knowledge of classical mathematics. Snellius's predilection for a unified approach is supported by his interpretation of the classical sources. Practical applications played no role.

Van Ceulen's books were much closer to the tradition of practical problem solving, and their audience may have consisted for a large part of practitioners, surveyors and *Rechenmeister*. Even when Snellius had translated them into Latin and had made them accessible for an international learned audience in that way, their roots in down-to-earth problem solving, with transparent constructions without much mathematical subtleties, did not disappear. Therefore, a multi-layered solution would have been out of place in *De Circulo*. Besides, the market for that kind of more advanced mathematics was only small. If Snellius aimed to reach his fellow country men, the contact with whom was more relevant to everyday life than that with mathematicians abroad, he certainly had to take care to keep his mathematics accessible.

Snellius confined himself to the division of triangles, although other mathematicians also studied cognate problems, such as the division of other rectilinear figures, or the division of figures by a line parallel to one of its sides. He always discussed the problem in frameworks which did not give him the room for an elaborate exposition. Moreover, he may have preferred to restrict himself to the essence instead of discussing a whole series of almost equal problems, which he would have considered as tedious.

The mathematicians interested in the division of figures formed a more diverse group than, for instance, that of the Apollonius-reconstructors. It contained experts of the classical tradition such as Clavius and Snellius and practitioners such as Van Ceulen and Cardinael. This topic was accessible to a somewhat broader group of mathematicians than the more advanced works of the Greek period, because it only involved elementary techniques. The triangle dividers approached problems in almost the same way. The most striking difference is that the practitioners formulated and solved the problems more often with respect to special (numerical) values of the givens than the classicists. Stevin, who could also be seen as a practitioner, even included the discussion of an equation in which the unknown was a number in the explanation of one of the geometrical steps—without making the correspondence between the algebra and the geometry completely clear. Only rarely was the problem presented in a surveying context.

To appreciate all the different contributions, it is necessary to reconstruct what knowledge about earlier division constructions was available to whom, and what was contrived by the participants. Exactly in the period when Snellius was active, much new knowledge became available. This is for instance shown in the different contributions of Stevin, who only solved the easiest cases at first, and after having studied more authors, was also able to solve the more complicated cases. Some authors refer to other mathematicians and thus make it possible to reconstruct the genealogy of the solutions of the division problem.

A study of the different solutions of the problems shows that there were no technical innovations. All solutions discussed here are comparable in the sense that they only use 'plane' geometrical means. Yet the vigour of the activities shows that new methods were not necessary as a motor of action. The question remains what stimulated early modern mathematicians to add new solutions to the old problem of polygon dividing. Sometimes, real improvements were attained: more complicated variants could be solved or constructions were made simpler. In many cases, the value of the new solutions is not so evident. The practitioners seem to have been excited by constructions considered to be more elegant than already existing ones, or by noticing unexpected connections to other geometrical problems. They were not deterred from problems already solved. On the contrary, giving a new solution to an old problem counted as a token of ingenuity and virtuosity and could serve as an advertisement. We cannot conclude that Snellius's contemporaries saw a progression in quality in the various solutions that were presented.

As a last point, it has to be remarked that these mathematicians were almost exclusively interested in finding geometrical solutions that could be applied in most cases. No general questions were formulated about conditions for the existence of solutions, their number, or whether different constructions of the same problem always yielded the same solutions. Some authors made a single remark about these issues, or discussed several configurations, but the questions were not addressed generally—although while doing their research the mathematicians must have realized that solutions to certain problems could be found which were not produced by one of their constructions, or that their method sometimes did not yield a solution. Snellius is no exception in this respect.

Chapter 7

Dimensional scruples: the areas of triangles and cyclic quadrilaterals

7.1 Introduction

What is good mathematics? Does every algorithm have to be proved elaborately before it can be applied in practice? And can a proof be valid if the interpretation of some of the individual steps is problematic? Snellius's answers to these general questions, which are relevant to the understanding of the assumptions hidden behind many mathematical texts, will be given here for two particular geometrical topics in which he was interested: Heron's Theorem and its proof, and cyclic quadrilaterals. Ramus's comments on Heron's Theorem will also be discussed, because Snellius was influenced by it. This theorem about the area of a triangle was already very old, yet it had not been proved to the satisfaction of Ramus and Snellius. Their discontent is remarkable, because—as far as I know—no older sources exist in which the theorem and its proof were criticized, although it had been treated many times since Antiquity.

In the first part of the chapter, the history of Heron's Theorem and its proofs will be sketched, because some background is necessary to properly assess Ramus's and Snellius's contribution to this old subject. This survey is followed by a detailed explanation of Ramus's and Snellius's objections to, and modifications of the traditional proof, which mainly concerned the handling of four-dimensional magnitudes. This case study is relevant for a better understanding of Snellius's geometrical work, because it provides us with insight into some of his main criteria for good mathematics. Besides, it enlightens us about the connections

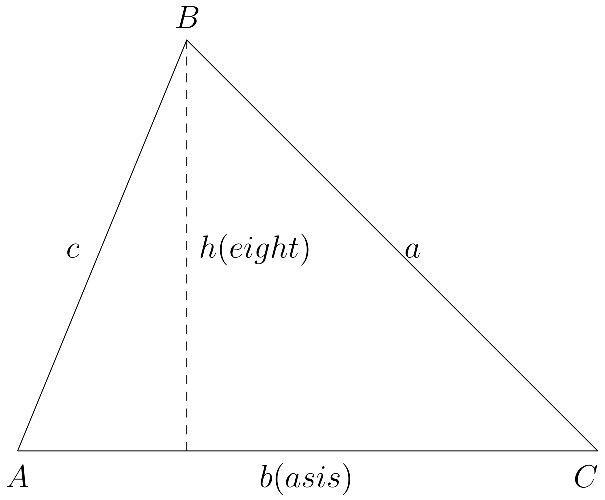

Figure 7.1: A general triangle

between Ramus and Snellius.

Snellius also found an expression for the area of cyclic quadrilaterals that was very similar to Heron's Theorem, and he studied their construction. This part of his work adds more insight into his use of higher-dimensional magnitudes in geometry.

7.2 Heron's Theorem before Ramus: tradition

7.2.1 The area of a triangle in Antiquity

The area of a triangle can be determined by the following theorem:[1]

Theorem 7.1 *(See figure 7.1) Given a triangle ABC. If $b = AC$ is its base, h its height and $\mathbf{A}$ its area, then:* $\mathbf{A} = \frac{1}{2}b \cdot h$.

Already in Antiquity, an alternative method for finding the area of a triangle was known, for which the height of the triangle did not have to be determined:

[1]Note that most of the writers discussed below do not explicitly distinguish between geometrical magnitudes and their sizes expressed in numbers or otherwise. This renders it difficult to formulate the theorem both exactly and in a way that does justice to the sources. In this chapter, the reaction of Ramus and Snellius to an ambiguity resulting from the absence of this distinction will be central. The aim of the chapter is not to fully explore the intricacies of all concepts and operations involved.

Theorem 7.2 (Heron's Theorem) *(See figure 7.1) Given a triangle ABC. If a, b and c are the lengths of its sides, $s = \frac{a+b+c}{2}$ and $\mathbf{A}$ its area, then:*

$$\mathbf{A} = \sqrt{s(s-a)(s-b)(s-c)}.$$

This last theorem was very convenient for application in practice, because it was often easier to measure the sides of a triangular area (for instance because this was a field containing an obstacle such as a building or a pond) than to determine the length of the perpendicular from the vertex to the base.

The theorem was of a mixed nature, involving geometry and arithmetic: its object was geometrical (the triangle), but the input consisted of numbers (the lengths of the sides), on which arithmetical operations were performed. However, the proof of this 'mixed' theorem had to be purely geometrical, because before the development of algebra and its adjustment to geometry, there was no set of tools that could construct proofs as certain as those of geometry.

Neither theorem 7.1 nor 7.2 is discussed in Euclid's *Elements*. This is not surprising because they belong to the field of what I will call 'numerical geometry', in which numbers that express the measure of geometrical magnitudes with respect to some unit are used. This handling of geometrical objects is fundamentally different from the purely geometrical, Euclidean one, in which magnitudes can be 'known', which means that their size (and sometimes their position in a figure) can be constructed on the basis of given magnitudes. No number expressing their length is associated with them.

It is of course possible to reformulate the assignment of finding the area of a triangle in geometrical terms, which could for instance mean: construct a square with the same area as a given triangle (a rectangle with the same area would not be unique). From a purely geometrical point of view, the construction of such a square by the first theorem is easier than by the second theorem. Nonetheless, neither construction is proposed in the *Elements*, although all necessary building blocks are present there. The Greek word for 'area' (ἐμβαδόν) does not even occur in this work. When Euclid wanted to discuss statements on areas, he used the notion of equal figures.[2]

A difficulty connected to the application of either of the two mixed theorems 7.1 and 7.2 is that it is not always possible to assign measures to the sides of a triangle, because two line segments can be incommensurable, that is, without a common measure. If this is the case, one of the line segments can be expressed by an arbitrary integer number, but whatever number is chosen, no rational number exists that can express exactly the length of the other line segment. In practical geometry, this restriction did not play a role because the lengths of

[2]For this last fact, see Ian Mueller in [Proclus, 1970, p. 183].

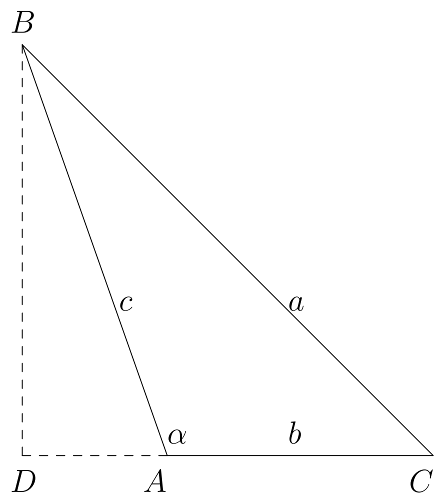

Figure 7.2: *El.* II.12: if α obtuse, then $\mathrm{sq}\ (a) - (\mathrm{sq}\ (c) + \mathrm{sq}\ (b)) = 2\ \mathrm{rect}\ (b, AD)$

the line segments were measured and then approximated by means of rational numbers. Indeed, this difficulty is generally passed over by the authors treated here. In pure geometry, this problem was not relevant either, because if two line segments were given, their ratio was also given and all Euclidean constructions could be performed. However, these different approaches contributed to keeping practical and pure geometry apart.

Although Euclid devoted the huge tenth book of the *Elements* to (in)commensurability, this did not help to unite the two approaches.[3] Euclid may not have included theorems 7.1 and 7.2 in the *Elements* because of the non-unicity of an area in geometrical terms and the incommensurability-problem. This seems to be a more likely explanation for their absence than his not knowing them.

Theorem 7.1 was known to at least some mathematicians in Antiquity. Although the height of a figure was only defined explicitly in book VI of the *Elements* ('the perpendicular drawn from the vertex to the base'[4]), propositions II.12 and II.13 were used by Heron and others to determine the area of a triangle given the lengths of its sides (see figures 7.2 and 7.3 for these propositions).[5]

[3]Cp. section 5.4 for Snellius and *Elements* X.

[4][Euclid, s a, 2, p. 188].

[5]See [Heath, 1921, II, pp. 320–321]. I have adjusted the lettering in such a way that the same letter refers to the same point in all figures in the following theorems, so as to make visible where the constructions differ. Explanations are added between square brackets.

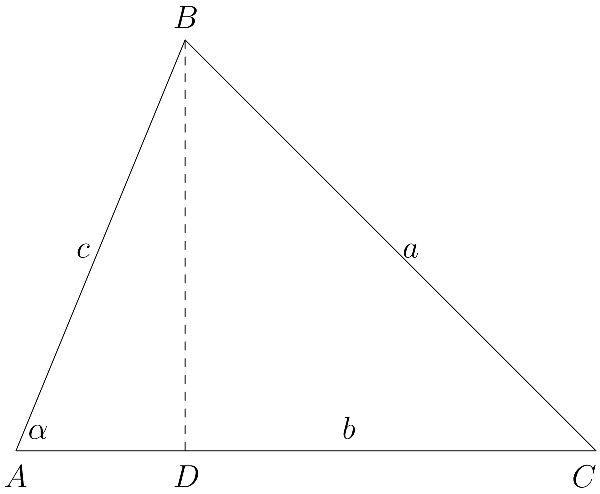

Figure 7.3: *El.* II.13: if α acute, then $(\text{sq }(c) + \text{sq }(b)) - \text{sq }(a) = 2 \text{ rect }(b, AD)$

In order to explain this, I introduce the following abbreviations, which are also used in the rest of this chapter:

$$\begin{aligned}
\mathbf{T} &= \triangle ABC, \\
r &= \text{ the radius of the circle inscribed in } \mathbf{T}, \\
\alpha &= \angle BAC, \quad \beta = \angle ABC, \quad \gamma = \angle ACB, \\
a &= BC, \quad b = AC, \quad c = AB, \\
s &= \frac{a+b+c}{2}, \\
\mathbf{A} &= \text{area of } \mathbf{T}.
\end{aligned}$$

If a, b and c are given, AD can be calculated, by II.12 if α is obtuse and by II.13 if α is sharp (if $\alpha = \frac{\pi}{2}$, then $AD = 0$). Now the height of $\mathbf{T}$, BD, can be determined by applying the Theorem of Pythagoras in the triangle ABD. The area of $\mathbf{T}$ is equal to $\frac{1}{2}b \cdot BD$ (see theorem 7.1).

Heron's Theorem (theorem 7.2) appeared for the first time in the work of Heron of Alexandria, who probably lived in the first century AD.[6] He wrote textbooks about topics in mathematics, mechanics, physics and pneumatics,

[6]This period has been argued convincingly by O. Neugebauer, [Drachmann and Mahoney, 1972, p. 310]. Heath had earlier concluded that Heron lived in the third century AD. See for an elaborate discussion of the arguments for this [Heath, 1921, II, pp. 298–306]. For a summary of Heron's works, see [Drachmann and Mahoney, 1972] or [Heath, 1921, II, pp. 307–354].

maybe as a teacher in the Alexandrian Museum. Apart from his works, nothing is known about him. His interest in applied mathematics resulted in what Michael Mahoney described as 'ambiguity towards the rigor and theoretical fine points of classical Greek geometry'[7], as can for instance be seen in the theorem under discussion here (see below). According to an Arab source, the theorem had been found earlier, by Archimedes (third c. BC), but no contemporary sources confirm this.[8]

It is important to realize that the text of the version of the theorem and proof attributed to Heron were virtually unknown after Antiquity until they were rediscovered in the nineteenth century in the *Dioptra*, a treatise on surveying, which was part of applied geometry. Later in that century, they were included by F. Hultsch in his critical edition of Heron's work. Hultsch tried to restore the original text, which had become corrupt and unclear in the process of transmission. Thus, Heron's text as we now know it was only constituted in the nineteenth century, but it is probably true to the original.[9]

The next editor of the text, Schöne, argued against Hultsch that the theorem and proof were not a later interpolation in the *Dioptra*.[10] Schöne also edited the text of Heron's *Metrica*, of which a complete manuscript was found only in 1896. In this work, Heron's geometrical achievements can be discerned most clearly. It contained Heron's Theorem and its proof as well, in words almost identical to those in the *Dioptra*.[11] The complete text had survived the ages in one copy only, which was found in Constantinople.

In the *Metrica*, Heron introduced the problem of finding the area of a triangle of which the sides are known by giving a (numerical) example, which was treated in the form of an algorithm. This was the $7-8-9$ triangle, the area of which cannot be expressed by a rational number. Therefore, Heron had to explain a technique to approximate the value of a square root. He then announced that it is possible to find the area of a triangle by calculating its height, but this was not done in his solution. The word *calculating* puts the problem immediately within the realm of numerical geometry, where a measure, that is, a number, is assigned to the height.

[7]Mahoney in [Drachmann and Mahoney, 1972, p. 314].

[8][Lorch and Folkerts, 1989, p. 73].

[9]Hultsch devoted an article to the theorem and the rediscovery of its connection to Heron, in which the proof is also discussed elaborately: [Hultsch, 1864]. The oldest critical edition of Heron's work is Hultsch's; a summary of the problems connected to Heron's Theorem is given in its preface, [Hultsch, 1864, pp. XVII–XVIII]; the theorem is in [Hultsch, 1864, pp. 235–237].

[10][Schöne, 1903, pp. XIX–XX].

[11]They are found in *Dioptra* 30 and *Metrica* I.8 in Schöne's edition: [Schöne, 1903, pp. 18–25, 280–285], with a German translation.

Heron's Theorem can also be found in [Thomas (with an English translation), 1941, pp. 470–477] (Greek text with an English translation; it follows Schöne's edition of the *Metrica*); non-literal renderings are given in [Euclid, s a, 2, pp. 87–88], [Heilbron, 1998, pp. 213, 215, 269–271], [Heath, 1921, pp. 321–323].

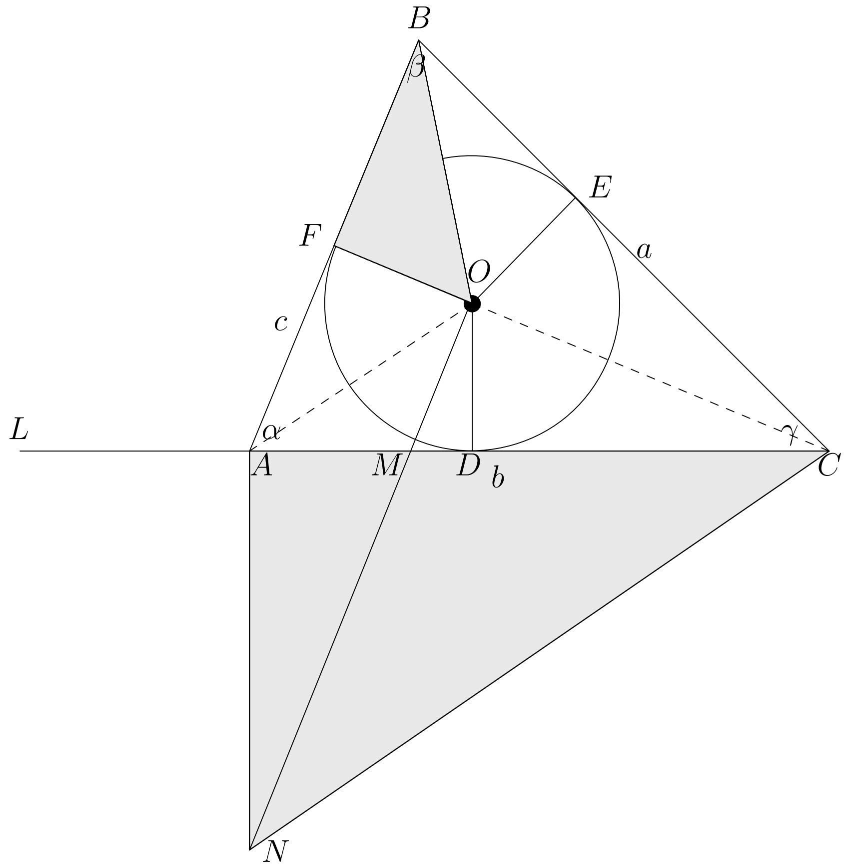

Figure 7.4: Heron's construction

He did not formulate his 'theorem' explicitly, but he did give a general proof of it, which is the following (see theorem 7.2 and figure 7.4):

Proof (Heron):

1. Construct the inscribed circle in **T**: bisect α, β, γ, call the point of intersection of the three bisectors O and draw perpendiculars from O to the three sides. Construct D, the point of intersection of the perpendicular to AC through O and AC, and likewise E on BC and F on AB. Describe the circle with centre O and radius $OD = OE = OF$ [it can be proved that these are equal, *Elements* IV.4].

2. Now

$$\begin{array}{rcl} \text{rect } (AB, FO) & = & 2\triangle AOB \\ \text{rect } (AC, DO) & = & 2\triangle AOC \\ \text{rect } (BC, EO) & = & 2\triangle BOC \\ \hline \text{Taking their sums yields: rect } (2s, r) & = & 2\mathbf{A}. \end{array}$$

3. Produce CA and mark L on it such that $AL = BF$. Then, because of the similarity of $\triangle AFO$ and $\triangle ADO$, and of $\triangle CDO$ and $\triangle CEO$, $AF = AD$ and $DC = EC$, and we have $CA + AL = CL = s$. Now:[12]

$$\begin{array}{rcl} \text{rect } (CL, DO) & = & \text{rect } (s, r) = \mathbf{A}. \\ \text{But also rect } (CL, DO) & = & \sqrt{\text{sq } (CL) \cdot \text{sq } (DO)}. \end{array} \tag{7.1}$$

Therefore,

$$\mathbf{A}^2 = \text{sq } (CL) \cdot \text{sq } (DO). \tag{7.2}$$

4. Draw a line perpendicular to CO through O and call its intersection point with AC M. Draw a line perpendicular to AC through A and call the intersection point of the two new lines N. Because $\angle CON = \angle CAN = \frac{\pi}{2}$, a circle can be described about the quadrilateral $COAN$ [*El.* III.31]. Therefore [*El.* III.22],

$$\angle COA + \angle CNA = \pi. \tag{7.3}$$

5. $\angle COA + \angle BOF = \angle COB + \angle AOF$, $\angle COA + \angle BOF + \angle COB + \angle AOF = 2\pi$ and therefore $\angle COA + \angle BOF = \pi$. Combining with (7.3) yields $\angle BOF = \angle CNA$. Also, $\angle BFO = \angle CAN = \frac{\pi}{2}$. Therefore, $\triangle BOF \sim \triangle CNA$ (the two shaded triangles in fig. 7.4).

6. Therefore,

$$\begin{array}{rcl} AC : AN & = & FB : FO \\ (FB = AL, FO = DO) & & \\ & = & AL : DO. \end{array}$$

Permutando:[13]

$$\begin{array}{rcl} AC : AL & = & AN : DO \\ (\triangle ANM \sim \triangle DOM) & & \\ & = & AM : DM. \end{array}$$

[12] The following statements have to be understood in numerical geometrical mode. They will be discussed after the proof.

[13] See *Elements* V.16: $A : B = C : D \Rightarrow A : C = B : D$.

Componendo:[14]

$$CL : AL = AD : DM.$$

Multiplying both terms of the left side by CL, of the right side by CD yields $(*)$

$$\begin{aligned}
\text{sq }(CL) : \text{rect }(CL, AL) &= \text{rect }(AD, CD) : \text{rect }(DM, CD) \\
(\angle MOC = \frac{\pi}{2}) & \\
&= \text{rect }(AD, CD) : \text{sq }(DO),
\end{aligned}$$

from which follows:

$$\begin{aligned}
\text{sq }(CL) \cdot \text{sq }(DO) &= \text{rect }(CL, AL) \cdot \text{rect }(AD, CD) \qquad (7.4) \\
&= [s \cdot (s-b) \cdot (s-a) \cdot (s-c)].
\end{aligned}$$

Substituting (7.2) yields:

$$\mathbf{A}^2 = s(s-a)(s-b)(s-c).$$

Although Heron started out with a numerical calculation, his proof does not involve numbers, but a construction that is Euclidean in spirit. He first constructed an inscribed circle in the triangle with radius r and showed that we have: $\mathbf{A} = \text{rect }(r, s)$. Now a puzzling part of the proof follows: Heron first stated that a rectangle was equal to the root of the product of two squares in equation (7.1), which could only be understood geometrically if the right side was interpreted as rect $(\sqrt{\text{sq }(CL)}, \sqrt{\text{sq }(DO)})$. Then he concluded that (see (7.2))

$$\mathbf{A}^2 = \text{sq }(s) \cdot \text{sq }(r). \qquad (7.5)$$

He expressed this statement in words, not having symbols at his disposal, but they leave no room for another interpretation than this one.[15] The statement would be true if considered in the context of numbers, where squaring and square root extraction are inverse operations (for positive magnitudes), that is: if A, s and r were numbers (≥ 0), then

$$\mathbf{A} = s \cdot r \Leftrightarrow \mathbf{A}^2 = (s \cdot r)^2 = s^2 \cdot r^2.$$

Equation (7.5), however, cannot be interpreted geometrically, because both at the left-hand side and at the right-hand side of the equation, we obtain the product of two (two-dimensional) areas, which should be a four-dimensional

[14] See *Elements* V.18: $A : B = C : D \Rightarrow (A + B) : B = (C + D) : D$.

[15] 'ἔσται ἄρα τοῦ ΑΒΓ τριγώνου τὸ ἐμβαδὸν ἐφ' ἑαυτὸ γενόμενον ἴσον τῷ ἀπὸ τῆς ΘΓ ἐπὶ τὸ ἀπὸ τῆς ΕΗ.' [Thomas (with an English translation), 1941, p. 474].

entity. This is inconceivable in classical, Euclidean geometry, where a magnitude could have at most three dimensions.

Euclid was always very careful in working with magnitudes of different dimensions, not multiplying magnitudes with each other at all.[16] The step taken in the proof at $(*)$ is therefore alien to Euclidean geometry. Other classical mathematicians were somewhat less strict. Precisely Heron's proof was used to argue that 'the ancient geometers were prepared to admit fourth-dimensional terms within informal derivations', as Wilbur Knorr writes.[17]

Euclid's careful style also had adherents, such as the late classical mathematician Pappus. In his explanation of the so-called 'Pappus problem', he pointed out that if more than six lines were involved,

> one can no longer say 'the ratio is given of the something contained by four to that by the rest', since there is nothing contained by more than three dimensions.[18]

Pappus then rejected the habit of his 'immediate predecessors' (probably writers on algebra such as Diophantus) to 'admit meaning to such things', that is, to higher dimensional magnitudes. He preferred the use of compound ratios.[19]

The confusion in the interpretation of Heron's proof is caused by the difference between geometry and arithmetic. In the latter domain, dimensions do not exist and therefore one can multiply as many terms as one likes without problems of interpretation. If the lengths of the sides of the triangle are expressed in numbers, and its area as well, then statement (7.5) is true and no difficulties of interpretation arise. Because Heron did not formulate the proposition explicitly, the four-dimensionality is hidden in the proof, but implicitly it is already present in the algorithm used to determine the area: the root of the product of four factors has to be found. If (7.5) is accepted, the outcome of the proof is acceptable as well, because the other steps of the proof are completely straightforward (apart from (7.4), which is similar to (7.5)).

Snellius and his contemporaries could not consult any published texts of Heron's work. Although some manuscript copies were circulating in Europe, there is no indication that they reached Ramus and Snellius. Some incomplete manuscripts of the *Metrica* were in France in Ramus's time, but he did not refer

[16]This point is stressed by Grattan-Guinness, who adds: 'In other words, *the square on the side is not the side squared:* it is a *region*, and Euclid was not concerned with its area.' [Grattan-Guinness, 1997, pp. 58–59].

[17][Knorr, 1986, p. 271], where he concludes: 'Since Hero's rule is arguably of Archimedean provenance, we may infer that geometers long knew of this ploy and could exploit it for the analysis of problems'. There does not seem to be enough support for this conclusion.

[18][Pappus of Alexandria, 1986, pp. 122–123].

[19]For the Pappus problem, see [Pappus of Alexandria, 1986, 1, pp. 120–123; 2, p. 404] (*Collection* 7, 35–40). The compound ratio of $a : b$ and $P : Q$ is $a : c$ with c determined such that $P : Q = b : c$ (it is equivalent to the arithmetical product $\frac{a}{b} \cdot \frac{P}{Q}$). [Bos, 2001a, pp. 125–126].

to them, nor is there any sign that he knew the *Dioptra*.[20] They must have known the theorem and proof by a more indirect transmission, to which we now turn.

7.2.2 The tradition until the sixteenth century

In the centuries after Heron's death, his works suffered the same fate as so many other texts from Antiquity: they became extremely rare and only survived thanks to some lucky circumstances. Heron's Theorem appeared in different texts in the Roman, Jewish and Indian world, and was studied more closely by Arab scholars. The theorem itself was explained either by a numerical example or in the form of an algorithm, in which it seemed to be assumed that a (numerical) measure of the sides was known.[21]

The first known proof of the theorem in Latin dates from the twelfth century. It was included in the *Verba Filiorum*, the Latin translation by Gerard of Cremona of a treatise by the Banū Mūsā, three ninth-century Arab mathematicians (and brothers). The proof differed on a number of points from Heron's. It may already have been altered in Antiquity for some reason: maybe part of the text had become corrupt and had to be emended, maybe part of it was regarded as unclear or maybe some mathematician unsuccessfully tried to get rid of the four-dimensionality in the proof. Whatever the reason, the proof that was most commonly given from then on was essentially the same as Heron's, but was slightly more complicated: it contained the inscribed circle and a four-dimensional statement like Heron's, but used an auxiliary pentagon instead of a triangle, which made the proof longer. I call this proof 'the traditional proof', because it dominated the tradition to which Ramus and Snellius reacted.

An important link in the transmission of the theorem was Leonardo Pisano (Fibonacci), an Italian mathematician (active around 1200) who acted as an intermediary between Arab and Western science in many cases. He based himself on the Banū Mūsā text when he gave the proof in the *Practica Geometrie* (1220).[22] Thence, the theorem was spread through Italy and Europe. Not all authors who discussed it considered it necessary to give its proof. For example Leonardo Mainardi from Cremona (fifteenth century) only gave an algorithm

[20] [Verdonk, 1966, p. 408].

[21] See [Hultsch, 1864, pp. 237–249], [Tropfke, 1923, pp. 86–89] and [Lorch and Folkerts, 1989, pp. 73–78] for a history of the transmission of the theorem and proof. [Lorch and Folkerts, 1989, pp. 73–74] mention some Arab proofs, which were probably unknown in the West European tradition.

I use the name 'Heron's Theorem' for simplicity's sake, although the authors discussed in this section did not know it as such.

[22] See [Clagett, 1964, pp. 278–289] for the Latin translation of the Banū Mūsā proof and an English translation of it, and see [Clagett, 1964, pp. 223–224] for a description of its contents and its relationship to Leonardo Pisano.

and a numerical example to demonstrate its use in his *Practica Geometriae*.[23]

A proof essentially the same as Pisano's was given in the work of two later Italian mathematicians, Luca Pacioli and Tartaglia, both of whom were later mentioned by Snellius when he discussed Heron's Theorem. Luca Pacioli gave the oldest printed proof in his *Summa* of 1494, using the $13 - 14 - 15$ triangle as an example. In his *General trattato de' numeri et misure*, Tartaglia used the same example and added that of the $6 - 6 - 4$ triangle; his proof is almost literally the same as Pacioli's (using the same letters for the same points in the figure).[24] In the last step of the argument, they manipulated with proportions as though they were fractional numbers, so that the four-dimensionality easily slipped in. Tartaglia wrote:

> Therefore, the proportion of ae to al is equal to the proportion of the square of et to the product of eb and bl. Multiplying the square of et and al yields the same as multiplying ae and the product of be and bl. [...] And the multiplication of the square of et and the square of al yields the same result as the multiplication of ae and the product of eb and bl, and its result [multiplied] by al. But the multiplication of the square of et and the square of al is equal to the square of the area of triangle abg.[25]

The fact that Tartaglia spoke of the product of two line segments instead of of the rectangle contained by the line segments shows that the framework is more arithmetical than geometrical—he even extends this use of the word to the product of four line segments. He also multiplied squares by each other. Apparently, Tartaglia did not find fault with this aspect of Heron's Theorem.

In the Middle Ages, another variant of the proof of Heron's Theorem circulated as well. It was more similar to Heron's proof, with an auxiliary triangle.

[23] See [Curtze, 1902, pp. 386–387] for the Italian text of Leonardo Mainardi and a German translation.

[24] For Tartaglia's theorem and proof, see [Tartaglia, 1560a, fol. 7^r–8^r]. For Luca Pacioli's version, see the translation and commentary in [Hultsch, 1864, pp. 242–246] (taken from *Summa de arithmetica geometria*, 1523, fol. 9^v ff). Snellius's proof, for which see section 7.5, is very similar to all these proofs.

[25] 'La proportione adunque del .ae. al .al. è come la proportione del quadrato del .et. al dutto del. eb. in .bl. multiplicato adunque il quadrato del .et. in .al. è come multiplicato .ae. nel produtto del .be. in .bl. [...] Et la multiplicatione del quadrato del .et. nel quadrato del .al. è come la multiplicatione del .ae. nel produtto del .eb. in .bl. & quello che fa in .al. Ma la multiplicatione del quadrato del .et. nel quadrato del .al. è come il quadrato della superficie del triangolo .abg. [...]', [Tartaglia, 1560a, fol. 7^v].

The word *come* ('as') must mean that the two entities left and right of 'è come' are not identical, but are expressions for different magnitudes that have the same size and are therefore in this context equivalent. In the notation of section 7.5: $a = B, e = F, l = G, t = O, b = A$. The steps in the quote given here are equivalent to those in the last two steps of Snellius's proof (see p. 275; see p. 265 for a further analysis of this part of Tartaglia's proof).

The variant was associated with Jordanus de Nemore, a thirteenth-century mathematician, but may not have been his. In one of the extant manuscript copies, some worry about the dimension problem is voiced:

> Since in the rule the third multiplication is evidently that of a line by a solid, which cannot exist among continuous magnitudes, one ought to assume in it a relation of numbers.[26]

This concern, however, did not prevent the author from constructing a four-dimensional magnitude, just like his predecessors had done.

Although Heron's own text remained hidden in the shadows for a long period, the conspicuous four-dimensional statement was repeated by all the later students of the theorem. They tried to use a feature of arithmetic, namely the absence of dimensions, without being able to give a full arithmetical proof. This did not cause any serious doubts about the validity of the proof.

7.3 Ramus against obscurity

Finally, the general atmosphere of satisfaction with the proof of Heron's Theorem was disturbed by Petrus Ramus. This French philosopher and educator[27] had no scruples about attacking some till then generally acknowledged ideas in his attempt to reform all sciences. He reacted to the proof tradition of Heron's Theorem very critically, giving the theorem in his geometry textbook (*Geometria*), and writing down what was according to him wrong with it in the *Scholae Mathematicae*. This book was devoted to a large extent devoted to a critical assessment of Euclid's geometry. The first editions of both these works were published in 1569.[28]

In the *Geometria*, Ramus gave the theorem in the form of an algorithm, explained by an example (XII.9). Remarkably, he knew that the theorem originated from Heron, and he seems to have been the first West European to connect the theorem to him.[29] He thought that Heron had not given a proof, which is easily explained by his lack of knowledge of the Heron text as we now have it. He did study some Heron manuscripts,[30] one of which contained some fragments of the *Geodaesia*. In this work Heron also stated the rule to find the area of a

[26] 'Regula [...], in qua quoniam tertia multiplicatio, videlicet linee in solidum, que in continuis non habetur, rationem in ea sumere oportet numerorum.' [Clagett, 1964, pp. 642–643]. See [Clagett, 1964, pp. 635–657] for the 'Jordanus' proof (in Latin, with an English translation) and an overview of the history of the problem.

[27] For Ramus and mathematics, cp. section 2.5.2. [Verdonk, 1966, p. 316] mentions Ramus's discussion of this theorem briefly.

[28] [Verdonk, 1966, p. 421].

[29] I have not found an earlier reference. Tropfke also stated that Ramus was the first to connect the theorem to Heron, [Tropfke, 1923, pp. 86–87].

[30] [Ramus, 1569, p. 35].

triangle generally, accompanied by the example of the $3-4-5$ triangle, but without a proof. It is most likely that Ramus found the method there. Indeed, Ramus gave almost the same example: the $6-8-10$ triangle.[31]

Ramus did not prove the method either, not because he did not know a proof, but because 'the cause [of this measurement] is more appropriate to a riddle than to a theorem.'[32] The *Geometria* was meant for students and therefore the explanations should confer insight, not confuse them. Yet he did not want to withhold his opinion on this proof from the interested reader in a more appropriate place: the *Scholae Mathematicae*, which was full of his own views on mathematics. He started his discussion on the last page of this book by giving the example of the $13-14-15$ triangle, which Tartaglia had also used. Ramus referred to the (in his view) unacceptable proofs of Jordanus and Tartaglia. The proof that he discussed belonged to the tradition of the Banū Mūsā and Tartaglia. It was probably borrowed from Tartaglia's proof, which it greatly resembles.

It is unclear to which Jordanus text Ramus referred. He may have found a reference to 'Jordanus's' statement of the theorem, but there is no indication that he knew the alternative proof tradition represented by him. In any case, he did not distinguish between Tartaglia's and Jordanus's proofs.[33]

Ramus did not explicitly mark the points where the proof was unacceptable in his view, which makes it difficult to evaluate his comments. Instead, he made some general sneering remarks:

> Jordanus and Tartaglia tried to find the geometrical cause of this calculation, with strange circumlocutions however, and by far exceeding the logic of Apollonius. [...] Therefore, this demonstration of Jordanus and Tartaglia shows the splendid intelligence of its authors in mathematical issues, but I wish it demonstrated a splendid logic together with that as well.[34]

Apollonius of Perga, one of the greatest Greek mathematicians, is used here

[31] *Geodaesia* 19, [Hultsch, 1864, pp. 151–152].

[32] '[...] quamvis si caussam requiras aenigmati potius, quam theoremati germana sit.' [Ramus, 1580, G, p. 101].

[33] Cp. [Clagett, 1964, p. 636]: 'My guess is that [Ramus] found the proof in a work of Tartaglia, who perhaps got it from Pacioli. I suspect that Tartaglia also saw the different proof that circulated with the *De ratione ponderis* of Jordanus and that perhaps he mentioned such a proof without reproducing it, reproducing rather the proof of the Banū Mūsā as given by Pacioli.'

This seems likely, as Tartaglia owned a manuscript copy of *De ratione ponderis*, which was published by Curtius Trojanus, and from which Tartaglia included a number of propositions in *Questii ed invenzioni diverse*, [Grant, 1973, p. 174]. Nevertheless, I have not been able to find such a reference in any of these books.

[34] 'huius numerationis geometricam caussam Iordanus et Tartalea quaesivere, sed miris ambagibus, et Apollonii logicam longissime superantibus. [...] Haec igitur demonstratio jordani et Tartaleae egregiam suorum authorum in mathematicis intelligentiam demonstrat, at vellem etiam egregiam logicam una demonstraret.' [Ramus, 1569, pp. 319–320].

as an example of someone who had a thorough command of the use of logic in mathematics, and his approach is opposed ('exceeding' is used ironically) to the strange logic of Tartaglia and Jordanus. He is praised in the *Scholae* as an excellent mathematician more often, and in Ramus's eyes, he seems to have been worthy of even more praise than Euclid himself.[35]

Ramus's summary of the proof is itself somewhat obscure because of its brevity. His objections seem to boil down to two major points, the first of which is the four-dimensionality in the proof. He felt uneasiness about the use of magnitudes beyond solids, that is, with more than three dimensions, in geometry. He wrote in the *Geometria*:

> Jordanus and his successors have tried to prove [this theorem], but they did so obscurely and by means of stereometry, which seems also to be indicated by the continuous multiplication of the half [sc. s] and the others [sc. three magnitudes, $(s-a)(s-b)(s-c)$] in the theorem itself.[36]

The term *stereometry* ('measurement of volumes') refers here not to three-dimensional, but to four-dimensional magnitudes.

The second objection is the lack of logic in the proof, or, to use Ramus's expression, the *obscure hysterology*.[37] *Hysterology*, one of Ramus's favourite terms, meant the precedence of what is logically later to what is logically earlier. According to Ramus, who was very preoccupied with method and order, this was usually a serious mistake, because it violated his third law, the *lex de universali*, which stated that the more general precepts should precede the more specific ones and that 'the general has to be discussed generally and the specific specifically.'[38]

In mathematics, Ramus used 'hysterology' to describe a whole range of (to his mind) errors, for instance:

- the assignment of properties to a special class of entities instead of to all entities to which they belonged (e.g. Ramus wanted to define parallelism in such a way that it did not only apply to straight lines, but to all lines);
- the enunciation of special propositions before their general versions (e.g. the special proposition *Elements* I.35—parallelograms on the same base and

[35]See [Ramus, 1569, p. 33].

[36]'Iordanus et Iordano posteriores demonstrare conati sunt, sed obscure et per stereometriam, quam etiam continuatio multiplicationis e dimidio et reliquis in ipso theoremate videtur indicare.' [Ramus, 1580, G, p. 101].

[37]'Demonstratio geodaesiae triangularis, huc reiecta propter hysterologiam obscuram.' [Ramus, 1569, p. 319]. *Hysterologia* is the Latinized version of ὑστερολογία, which meant in Antiquity the figure of speech *hysteron proteron* in which the later precedes the earlier, [Lewis and Short, 1879, p. 874].

[38]'generalissima [...] generaliter, quae vero specialia, specialiter.' [Snellius, 1596i, p. 19], quoted in [Sellberg, 1979, p. 65]. See further [Verdonk, 1966, p. 323]; cp. section 2.5.

in the same parallels are equal—was given before the more general one I.36—parallelograms on equal bases and in the same parallels are equal; in this case, as in many others, Ramus did not consider the deduction of I.36 from I.35);

- definitions and axioms being given in the beginning instead of where they were first needed.[39]

The concept 'hysterology' summarizes much of Ramus's criticism of the *Elements*, part of which stemmed from his misunderstanding of the axiomatic-deductive structure of Euclid's work.

Ramus did not tell the reader explicitly where the hysterology appears in the proof of Heron's Theorem. There are two possible interpretations. The first is a wrong order of the elements of the proof. Ramus may have found it unacceptable that Tartaglia had put off the proof of one of the steps of his argument until the end,[40] because he did not consider the postponement of an explanation as mathematically equivalent to direct settlement, instead wanting every mathematician to follow the order which he accepted as the only right one from a didactical point of view. If this interpretation is correct, Ramus's view on correct mathematics is markedly different from the traditional axiomatic-deductive one, where the logic of the elements of the proof makes it compelling and not their order of presentation. Although this interpretation fits well in Ramus's concepts, it seems to be less likely than the second one, which is closer to his usual use of the term 'hysterology' in geometry.

This second interpretation is that he meant a confusion of terms. In order to understand this, some terminology which Ramus used in the *Geometria* has to be explained first. He dichotomized the concept of a parallelogram, as he generally did with concepts:[41]

$$\textit{parallelogrammum}\left\{\begin{array}{l} \textit{rectangulum} \quad \left\{\begin{array}{l}\textit{quadratum}\\ \textit{oblongum}\end{array}\right. \\ \textit{obliquangulum}\end{array}\right.$$

Thus, he divided the general category of 'parallelograms' into a special class, the 'rectangles' (parallelograms with a right angle), and a general class, the 'oblique-angles' (all parallelograms with a non-right angle). The rectangles were further divided into the special class of squares (having all sides of equal length) and oblongs (the rest). He also defined the meanings of these terms. The terms and structure had been borrowed from the post-Euclidean tradition.[42]

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[39] See e.g. [Ramus, 1569, pp. 101–104, 193, 196, 291, 301], [Verdonk, 1966, p. 331], [Loget, 2004, p. 14].

[40] 'come in fine dimostraremo'. [Tartaglia, 1560a, fol. 7^v].

[41] [Ramus, 1580, G, pp. 90, 93].

[42] Ramus's classification is equal to a part of the classification of quadrilaterals attributed by Proclus to Posidonius and also given in Heron's *Definitions*, [Euclid, s a, 1, p. 189].

Ramus was sensitive about the correct use of these terms, as he showed for instance in his discussion of *Elements* II.3[43] in the *Scholae*, where he wanted to replace the word *rectangulum* (rectangle) by *oblongum* (non-square rectangle). Euclid had caused a hysterology, because 'the oblong shape was placed on a par with its genus [i.e. the rectangle]'.[44]

The 'hysterology' is probably hidden in the last part of Ramus's version of Tartaglia's proof, because the rest of the steps are straightforward, and Ramus followed Tartaglia's chain of reasoning closely there. In the last part of the proof, some proportions had to be manipulated. Proportions were used to deal with all sorts of magnitudes: general magnitudes, numbers or geometrical magnitudes. Most of the operations on them were introduced in *Elements* V. All of these could be applied to both numbers and geometrical magnitudes without any precautions. Yet some early modern mathematicians viewed proportions as fractional numbers and extended the range of permissible operations. The geometrical dimension difficulties were caused in our case, as was often the case, by this interpretation, that is, when from $a : b = c : d$ it was inferred that $ad = bc$ (not a Euclidean rule). This, however, is only true if ad and bc exist, that is, are well-defined. If a and b are three-dimensional solids and c and d are line segments, this is not the case.

Tartaglia had expressed the last part of his proof mainly in ambiguous geometrical-arithmetical terms, thus 'hiding' the fact that four-dimensional magnitudes are involved. In the first part, he had introduced several similar triangles (his construction is represented by figure 7.5; $\triangle AFO \sim \triangle KGA$ and $\triangle BFO \sim \triangle BGK$). The last part can be summarized as follows:[45]

1. $BF : BG =$ quad $FO :$ product (AF, GA);
2. quad $FO \cdot BG = BF \cdot$ product (AF, GA);
3. quad $FO \cdot$ quad $BG = BF \cdot$ product $(AF, GA) \cdot BG$;
4. But also quad $FO \cdot$ quad $BG =$ quad $\mathbf{A}$. [Tartaglia shows this at the end of his proof.]
5. Therefore $BF \cdot AF \cdot AG \cdot BG =$ quad $\mathbf{A}$.

[43] 'If a straight line be cut at random, the rectangle contained by the whole and one of the segments is equal to the rectangle contained by the segments and the square on the aforesaid segment.' [Euclid, s a, 1, p. 378]. In Ramus's formulation: 'Oblongum e tota recta et altero ipsius segmento aequatur rectangulo utriusque segmenti, et praedicti quadrato.' [Ramus, 1569, p. 196].

[44] 'Ac tum (ut antea monui) hysterologia percipietur, hic enim oblongum cum genere comparatur.' [Ramus, 1569, p. 196].

[45] I give a schematic representation here, staying close to Tartaglia's terminology. 'quadrato' is abbreviated as 'quad', '(pro)dutto' is translated as 'product'. For the main part of Tartaglia's own words and their translation see p. 260. I use the notation of theorem 7.4 and its proof. What follows here, is equivalent to step 7–8 of Snellius's proof on p. 275.

In this sequence, the dimension of the magnitudes under consideration is 1 and 2 in the first step, 3 in the second step, and then it is raised to 4 in the third step. Tartaglia did not refer to the three- and four-dimensional solids with special terms.

Ramus presented this part of the proof in a different way. He used more distinct terms to denote the various magnitudes involved. His reasoning is the following:[46]

1. oblongum $(FO, GK) =$ oblongum (FA, GA);[47]

2. sq (FO) : rect $(FO, GK) =$ sq (FO) : rect $(FA, GA) = FO : GK = BF : BG$;

3. oblongum (sq $(FO), BG)) =$ oblongum (rect $(FA, GA), BF)$
$=$ planus $(s - a, s - b, s - c)$.

4. Multiplying both *plani* by s yields:
planus (sq $(FO), BG, s) =$ planus $(s, s - a, s - b, s - c)$.

5. The left side is equal to planus (sq (OF), sq (s)); this is equal to sq $(\mathbf{A})$ because a triangle with base $2s$ and height OF has the same area as the original triangle.[48] This can also be expressed as a parallelogram with

[46]The emendations of the citation are borrowed from the edition of the *Scholae* by Lazarus Schonerus, see [Ramus (L. Schonerus ed.), 1627, pp. 313–314].

'Quare triangula *ose* et *elm* sunt aequiangula, ideoque similia per 4 p 6, oblongumque extremorum *os* et *ml* aequatur oblongo mediorum per 16 p 6. Atque ut quadratum ex *os* ad rectangulum extremorum, id est mediorum, sic perpendicularis *os* ad rectam *ml*. Recta enim est ad rectam, ut quadratum alterius ad rectangulum utriusque per 1 p 6, sic item *as* ad *al*. Oblongum igitur extremorum aequatur oblongo mediorum, id est plano trium differentiarum. Atque uterque planus multiplicatus per semiperimetrum facit duos iterum planos aequales. Hic rursus planus differentiarum multiplicatur per dimidium [Schonerus has 'semiperimetrum'], et planus e quadrato perpendicularis [Schonerus adds 'et semiperimetro'] per semiperimetrum est planus e quadrato perpendicularis et e quadrato semiperimetri. At planus e duobus quadratis aequat quadratum ex area trianguli. Nam si triangulum sumatur aequalis tum basis dati trianguli perimetro, tum altitudinis perpendiculari aequabit per 1 p 6 datum triangulum: itemque parallelogrammum aequealtum et basis dimidiae: quod ipsum erit medium proportionale per 1 p 6 ad quadratum e lateribus ipsius [Schonerus has instead of 'ad ... ipsius': 'inter quadrata suorum laterum']. Itaque multiplicare haec duo laterum quadrata est quadrare datum triangulum, eiusque quadrati latus erit area dati trianguli.' [Ramus, 1569, p. 320].

In the notation of section 7.5: $o = O, s = F, e = A, m = K, l = G, a = B$.

Ramus's definition of *planus*: 'Figuratus rectanguli rationalis appellatur planus.' [Ramus, 1580, G, p. 92]. He refers to *Elements* VII, def. 16: 'And, when two numbers having multiplied one another make some number, the number so produced is called plane, and its sides are the numbers which have multiplied one another.' [Euclid, s a, 2, p. 278].

[47]For this step, not in my summary of Tartaglia's proof, cp. step 7 of Snellius's proof on p. 275.

[48]Ramus does not actually prove that this is true; it is the step that Tartaglia had postponed until the end of his proof and had proved there.

base s and height OF, which has the same area as the mean proportional between sq (s) and sq (OF).

6. $\mathbf{A} = \sqrt{\text{planus } (\text{sq } (OF), \text{sq } (s))}$.

Ramus seems to have tried to translate Tartaglia's proof in his own terminology, attempting to use the proper terms and explicitly mentioning all operations, to show what was wrong with this proof. He used some terms (*oblongum*, *planus*) that had no equivalent in Tartaglia's version. In the *Geometria*, Ramus had defined *planus* as the product of two numbers; this meaning is extended here to the product of three and four numbers. It is not always completely clear how he determined the equivalents of Tartaglia's terms in his own language. He denoted for instance the same magnitudes by different terms (see the first two steps, where the two *oblonga* are mathematically the same as the two rectangles), which does not seem to be motivated by the original terms. Ramus used the term *planus* for both a three-dimensional and a four-dimensional magnitude. *Oblongum* denoted both a two- and a three-dimensional solid.

All things considered, the most likely interpretation of Ramus's use of *hysterologia* in this context is that he did not manage to make a consistent translation of the elements of Tartaglia's proof in his own conceptual language because Tartaglia was sloppy in distinguishing between arithmetical and geometrical terms in the proof. This was a sign of a more serious error in Ramus's eyes, that is, Tartaglia's not dealing properly with the underlying concepts, in particular not distinguishing between magnitudes of different dimensions. Therefore, Ramus got himself trapped into a knot of concepts that could hardly be disentangled, a situation which violated his own demand that the proper term should be used in the proper place.

Remarkably, Ramus did not give a better proof himself, although the traditional proof did not have to be altered much to obey the rules of classical geometry,[49] nor did he formulate the theorem in geometrical terms. An explanation for this may be that his concept of proof was different from that of other mathematicians. A proof that was logically correct did not satisfy him; as a didactician, he wanted a proof that conferred insight in some sense. His use of the word *causa* ('cause') instead of the *demonstratio* ('demonstration, proof') is very telling: cause is used in the Aristotelian sense of the word, as a true explanation of the characteristics of an entity.[50] He probably found the proof not only obscure because of its *hysterologia*, but also because it was too complicated to 'see through' it at once, and he was not able to create a simpler, more satisfying proof.

Ramus's confusion about the need for Euclidean-style proofs in geometry in general and in this case in particular can be more easily understood when com-

[49] This will be shown in section 7.5.
[50] Cp. section 4.4.1.

paring it to some related lack of clarity in one of his sources, namely Scheubelius's edition of Euclid's *Elements*, a book which Ramus often consulted.[51] He may well have borrowed the $6-8-10$ example from it. In an appendix to proposition I.34,[52] Scheubelius had stated Heron's Theorem, omitting the proof and giving a number of examples instead. Apparently, he had not realized the deductive structure of Euclidean geometry. His reason for including the theorem was:

> Because however this proposition 34, and many following it as well, have been found to be true in numbers—that is, in the discrete quantity no less than in the continuous quantity—, it was necessary, in order to show this more easily, to propose a general rule, by which the areas of all kinds of triangles can be found as long as their sides are known [...].[53]

According to Verdonk, the specialist on Ramus's mathematics, Ramus was not so much led astray by the absence of a proof, but by the fact that Scheubelius had not shown how he had calculated his examples.

Ramus was also aware of the existence of the method to calculate the area of a triangle based on *Elements* II.12 and II.13.[54] He presented both methods to his students and readers. Even though he considered the proof of Heron's Theorem too obscure to be of any help in understanding why this theorem was true, its use was undebated. After all, geometry was 'the art of measuring well' for Ramus, and the theorem was most suitable for this purpose.[55]

7.4 Continuation of the tradition

The work of some mathematicians of the next generation shows that Ramus's irritation at the traditional proof of Heron's Theorem was not shared by all. The approaches of two Jesuit mathematicians, Clavius and Grienberger, and—closer to Snellius—the relevant work of Rudolph Snellius, Ludolph van Ceulen, and Sems and Dou are briefly discussed.

In the *Geometria Practica*, Clavius gave the two traditional methods for determining the area of a triangle, Heron's Theorem and the formula based on

[51] J. Scheybl in German. For Scheubelius's influence on Ramus in general and in this particular case, see [Verdonk, 1966, pp. 232, 380].

[52] 'In parallelogrammic areas the opposite sides and angles are equal to one another, and the diameter bisects the areas.' [Euclid, s a, 1, p. 323].

[53] 'Quoniam autem haec propositio 34, et multae etiam sequentes, in numeris, quantitate nimirum discreta, non minus atque in quantitate continua, verae esse reperiuntur, quo id ostendamus commodius, canonem quendam generalem, per quem omnis generis triangulorum (modo latera eorum nota fuerint) areae inveniri possent, subiicere necesse fuit, his verbis.' [Euclid and Scheubelius, 1550, p. 110].

[54] [Ramus, 1580, G, pp. 101, 103–104].

[55] 'Geometria est ars bene metiendi.' [Ramus, 1580, G, p. 53].

height and base. He called this first one 'very accurate',[56] without explaining why this would be so, and while recognizing that sometimes the number found as the solution would be an approximation of the true solution.

In the last part of the proof of Heron's Theorem, Clavius consciously switched to numbers: 'if these lines are turned into numbers ...', and then spoke about the products of numbers, instead of of rectangles and solids. Apparently, he aimed to avoid the construction of a geometrically inconceivable solid, yet he did not explain to his readers why this detour was necessary.[57]

In his edition of the *Elements*, Clavius distinguished himself from most other mathematicians by drawing attention to a theoretical difficulty. He was conscious that in an arbitrary triangle, not all sides would be commensurable in general. This would make the determination of the numerical value of the area impossible. Therefore, he gave some conditions for triangles to have all sides commensurable, apparently not caring that the theorem could not be applied to the large majority of triangles.[58]

Christoph Grienberger wrote a manuscript about the problem of finding the area of a triangle when its sides are given, and had it defended by one of his students (1594).[59] Grienberger attributed 'Heron's Theorem' to Leonardo Pisano, not connecting it to Heron, as Ramus had done. He further referred to Luca Pacioli and Clavius.[60]

He remarked that

> Almost all interpreters of Euclid derive the solution of the problem [of finding the area of a triangle with given sides] from propositions 12 and 13 from book II.[61]

This method for the calculation of the area of a triangle is indeed closer to Euclid's work than Heron's Theorem.

Grienberger was not troubled by the use of numbers in the theorem and proof. He introduced the method by a numerical example and did not formulate it as a theorem. The first part of the proof is purely geometrical, but in the last part, the geometrical magnitudes are replaced by numbers and concrete numerical values are alternated with general magnitudes.[62]

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[56] 'accuratissima', [Clavius, 1604, p. 175].

[57] 'si lineae hae ad numeros contrahantur [...] erit numerus, qui sit ex DE, in HK, aequalis numero, qui sit ex EB, in BH.' [Clavius, 1604, p. 177].

[58] [Clavius, 1589a, pp. 292–293].

[59] [Gorman, 2003, pp. 15–16].

[60] [Grienberger (attr.), 1594, fol. 17^v]. Part of the first page is edited by Michael John Gorman with a short introduction in [Gorman, 2003, pp. 48–49]. The whole document is reproduced in facsimile by Gorman on the Internet: http://shl.stanford.edu/Eyes/modesty/III/index.djvu.

[61] 'Solutionem problematis [sc. Datis lateribus Aream trianguli inquirere] omnes fere Euclidis interpretes conficiunt ex propositionibus 12, et 13, secundi libri [...]', [Gorman, 2003, p. 48].

[62] [Grienberger (attr.), 1594, fol. 20^v–21^v].

Clearly, Clavius and Grienberger were aware of the geometrical deficiency of the proof, and tried to get rid of it by changing the character of the magnitudes involved from geometrical to arithmetical, halfway through the proof. Grienberger made a remark about the status of the numbers involved that he crossed out: he wrote that it would be enough to consider the geometrical magnitudes as numbers, they need not be given in numbers. This ambiguous solution undoubtedly would not have pleased Ramus, and Snellius would choose a different way out.

Snellius's father Rudolph wrote a commentary on Ramus's *Geometry*. He showed no awareness of any difficulties connected to Heron's Theorem and did not refer to its proof, but only explained the algorithm in mixed geometrical-arithmetical terminology.[63]

Ludolph van Ceulen did not present a rigorous solution either. He gave the theorem and proved it in the *Arithmetische en Geometrische Fondamenten*, with essentially the same construction as in the older proofs. Apparently, Ramus's critique was unknown or irrelevant to him. He did not pay much attention to the geometrical character of the theorem and proof either: he intertwined a numerical example and the geometrical demonstration, changing his language from geometrical to arithmetical during the proof, thus obscuring the four-dimensional statement.[64]

The theorem is also presented in the surveyors' handbook of Johan Sems and Jan Dou, but Heron's name is not attached to it. They also circumvented the troublesome part by switching to a numerical example instead of consistently arguing about general magnitudes.[65]

7.5 Snellius: 'stereometry of planoplanes'

The previous sections show that Heron's Theorem and its proof were a well-known geometrical topic by Snellius's time. Snellius considered it worthy of a considerable amount of attention, discussing it twice in his publications: in the *Fundamenta* (1615) and in the *Meetkonst* (1622). The *Fundamenta Arithmetica et Geometrica* was Snellius's translation of Van Ceulen's *Fondamenten*, to which Snellius added his own elaborate comments. Heron's Theorem was also the subject of Snellius's appendix to the *Meetkonst*, the translation of Ramus's *Geometria* that Snellius had initiated and supervised. Indeed Snellius was as dissatisfied as Ramus had been.[66]

[63] [Snellius, 1596e, pp. 267–268].

[64] See [van Ceulen, 1615a, pp. 146–147], [van Ceulen, 1615b, pp. 122–123].

[65] [Sems and Dou, s a, pp. 92–97].

[66] For the *Fundamenta* see section 5.3. Snellius gave the method without a proof or commentary in his summary of Ramus's *Geometria*, [Ramus (W. Snellius), 1612, pp. 32–33].

topic	Ramus	Snellius
sources	Heron, Jordanus, Tartaglia	Heron, Jordanus, Tartaglia, Luca Pacioli, Ramus
classical standard	e.g. Apollonius	e.g. Apollonius, Archimedes
four-dimensionality	'stereometry'	'stereometry of planoplanes'
arithmetic in geometry	implicit objection	explicit objection
logic	error of 'hysterology'	reference to Ramus
correction	—	own version theorem, proof
additions	—	three corollaries and one lemma

Table 7.1: Ramus's and Snellius's reactions to Heron's Theorem

The connection between Ramus's and Snellius's negative judgements is clear, from their content, from Snellius's actual mentioning of Ramus and from the place where Snellius discussed the proof. Ramus and Snellius seem to have been the only two early modern mathematicians who really bothered about the points which they considered as awkward. Even so, Snellius's reaction is not identical to Ramus's and Snellius's own input is easily discernible. The differences and similarities, which will be explained in more detail in the rest of this section, are summarized in table 7.1. The 'sources' and 'classical standard' are authors mentioned by them, 'four-dimensionality' gives their term for that phenomenon, 'arithmetic in geometry' gives their reaction to the use of arithmetical concepts in a geometrical theorem, 'logic' their viewpoint on the logical correctness of the proof, 'corrections' indicates whether they improved the formulation of the theorem and proof and 'additions' whether they added new results.

The *Fundamenta* was a natural place for resistance against the traditional proof, because this was embodied in Van Ceulen's rendering of it. Snellius translated it before he gave his own opinion in a commentary and then gave an alternative. He made some minor changes to Van Ceulen's proof in his translation, correcting an inconsistency between the text and the figure, changing the order of the steps, and giving sometimes more, sometimes less explanation than Van Ceulen.[67]

Snellius's main point of censure is against the error of dimensions made in the traditional proof:

> Whether they [sc. mathematicians from present and past] have copied this demonstration from Jordanus or Tartaglia [or have not copied it], they have all insisted on the same method, and they cheat the reader by I-don't-know-what stereometry of planoplanes.[68]

[67] [van Ceulen, 1615b, pp. 122–125] and [Ramus, 1622, fol. $Ee4^v$–$Ff2^v$].

[68] 'Quotquot hanc demonstrationem e Iordano aut Tartalea descripserunt omnes eandem insistunt viam, et pernescio quam planoplanorum stereometriam lectorem circumducunt [...]',

Snellius mentioned the same sources as Ramus in this quote. He added a reference to Luca Pacioli in the *Meetkonst*, which shows that he had used more sources than just Ramus's books.[69]

The term which Snellius used to indicate four-dimensionality was 'stereometry of planoplanes'; 'planoplanes' was the product of two (two-dimensional) planes. He had probably borrowed this term from François Viète, the most innovative mathematician of the era, who created a framework which enabled the handling of more-dimensional magnitudes. Snellius, however, only borrowed the name, not the tools connected to it, preferring instead the familiar world of classical geometry.[70] Snellius meant the 'stereometry' that appeared in the end of the traditional proof (see e.g. steps 3–5 of Tartaglia's proof on p. 265). Viète's terminology gave him the opportunity to express the error of the traditional proof in exact terms.

Evidently, Ramus had not used the term 'planoplanes' that was coined after his death and was necessitated to use a more ambiguous term. Moreover, Snellius explained the dimension objection more clearly than Ramus. He elaborated on it in the *Meetkonst*:

> Here we make the remarkable observation that they seek a quantity of four dimensions, because four terms are multiplied there, which is however nowhere to be found in the entire realm of geometry, where the composition of two lines makes a surface and the composition of three lines a volume. The domain of geometry ends there.[71]

In the *Fundamenta*, Snellius continued his argumentation as a true humanist by referring to the convictions of classical mathematicians, who would never have made the error of constructing an impossible solid. Although the essence of this remark was also contained in Ramus's discussion, Snellius explained his meaning in more detail:

> This is very different from what those ancient mathematicians did, who emphatically dismissed that 'transition to a further kind' and either removed it by application to a common altitude, or solved it by [the use of] proportions, or interpreted it by the compounding of ratios, just as can often be seen in the work of Archimedes, Apollonius

[van Ceulen, 1615b, p. 123].

[69]'frater Lucas', [Ramus, 1622, fol. Ee 4^v]. Pacioli's proof was in fact equal to Tartaglia's.

[70][Vieta, 1646b, p. 3], cp. [Bos, 2001a, p. 149]. Snellius was well aware of Viète's oeuvre and merits; cp. p. 8.3.2.

[71]'Alwaer men merckelijcken siet dat men een quantiteyt soeckt van vier * [in margin: * dimensionum.] metinghen/ want daer vier palen door malkanderen menichvuldicht werden/ de welcke nochtans in de gantsche meetkonst niet en is te vinden/ alwaer het begrijp van twee linien een plat/ en van drie een lichaem maken/ en daer heeft de meetkonst zijn uyterste eynde.' [Ramus, 1622, fol. Ee 4^v].

and others.[72]

'Transition to a further kind' is a reference to Aristotle's *De Caelo* I.1. In this section, Aristotle explained

> Of magnitude, that divisible in one way is line, that in two surface, that in three body; and besides these there is no other magnitude, because three is equivalent to all and 'in three ways' to 'in all'. [...] This, however, is clear: there cannot be a transition to another kind of magnitude, as from length to surface, and from surface to body; for magnitude of such a kind would no longer be complete.[73]

Aristotle stated that nature itself has made three the absolute maximum number of dimensions of a magnitude. Snellius's use of this quotation leads us to believe that like Aristotle, he pointed to the nature of the universe to explain correctness within mathematics.

However, Snellius's confident reference to the good example of the ancients shows that he was not aware of the fact that Heron's proof had already contained a four-dimensional trick. Like Tartaglia, Ramus and Van Ceulen, Snellius did not know this proof. Therefore, he was unaware that his argument disqualified Heron himself and in his ignorance he could oppose the shortcomings of the traditional proof as he knew it, to a classical Greek ideal, to which he tried to adjust. Just as Ramus had pointed to Apollonius as a model mathematician, so did Snellius in this quote. He added the hero of classical pure and applied mathematics, Archimedes.

The three methods which Snellius proposed could indeed be used to handle 'products' of geometrical magnitudes. If a line segment and an area are given, applying the area means to construct an area (in particular a parallelogram) which is equal in size to the given area and has the line segment as one of its sides (it is equivalent to the operation of division).[74] If two areas are applied to the same line segment, the difference in areas can be expressed as a difference in lengths and thus, two-dimensional magnitudes are reduced to one-dimensional magnitudes. By repeating this operation, the increase of dimensions can be prevented. The use of proportions for all sorts of magnitudes was a very common practice (cp. p. 265). The compounding of ratios in cases where more than three line segments were involved had been approved of by Pappus (see p. 258). Snellius chose the second option, working with proportions, in his own proof. If

[72]'longe aliter quam veteres illi mathematici, qui studiose declinabant illam μετάβασιν εἰς ἄλλο γένος, eamque vel applicatione ad communem altitudinem tollunt, vel proportione dissolvunt, vel rationum compositione interpretantur, quemadmodum apud Archimedem Apollonium aliosque saepe videre est.' [van Ceulen, 1615b, p. 123].

[73][Aristotle, 1995, pp. 48–49].

[74][Heath, 1921, I, p. 150].

a^3 and b^3 are three-dimensional solids and c and d are line segments, Snellius would not accept the inference $a^3 : b^3 = c : d \Rightarrow a^3d = b^3c$.

In the same passage, Snellius also indicated his indebtedness to Ramus explicitly:

> Anyhow, Petrus Ramus [...] was the first to note this breach of logic. Thanks to him I have long ago dissipated this obscurity and fog, and dealt with plane magnitudes in a plane way, that is, I have brought them back in their own class and order.[75]

Snellius referred here to some praise that François Viète had bestowed on Ramus, probably because he wanted to avert the disapproval to which Ramus had so often been subjected. Snellius seems to have interpreted Ramus's comment on the issue of the logic in the proof in such a way that it referred only to the four-dimensionality, not to any confusion of terms or order. Thus, Snellius's term ἀλογία ('breach of logic') was not completely identical to Ramus's hysterology. Snellius had used this last term in the Ramist sense in his *Theses*, where he had claimed that Euclid had introduced some concepts in the wrong order.[76]

Snellius went further than Ramus by showing how exactly the theorem should be enunciated and proved. He attached less value to the insight that a proof yielded than Ramus, and therefore, not sharing Ramus's impediments to give his own proof, he did give his own version. Snellius's wish to treat all magnitudes in their own class and order, that is, in their proper category, also reminds one of Ramus's methodology, but the predominance of the right method is much less than in Ramus's work.

The contents of Snellius's two discussions in the *Fundamenta* and the *Meetkonst* are almost identical and therefore they will be dealt with together.[77] Snellius's proof is to a large extent similar to that of his predecessors: he used essentially the same construction as Luca Pacioli, Tartaglia and Van Ceulen. The theorem and proof are prepared by one lemma.

Lemma 7.3 *Given line segments a, b, c and d. If $a : b = c : d$, then: rect (a, b) : rect (b, c) = rect (b, c) : rect (c, d).*[78]

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[75]'Sed istam ἀλογίαν Petrus Ramus (quem hominum λογίκωτατον vocat vir subtilissimus Franciscus Vieta quondam libellorum supplicum regius magister) primus notavit. cuius gratia olim hanc obscuritatem et caliginem discussimus et plana tractavimus plane, hoc est in suam classem et ordinem reduximus [...].' [van Ceulen, 1615b, p. 123].

[76][Snellius, 1608b, fol. $a3^{r-v}$].

[77]The *Meetkonst* version contains references to Ramus's propositions in the *Geometria*, the *Fundamenta* version contains hardly any references, but the underlying material is in both cases the *Elements*.

[78][van Ceulen, 1615b, p. 123] (explained by means of a numerical example, not proved) and [Ramus, 1622, fol. Ff 2^r] (only given after the theorem and proof, proved by means of a proposition from the main text and followed by a numerical example).

Theorem 7.4 *Given a triangle ABC. Its sides are a, b and c. Call* $\mathbf{A}$ *its area,* $s = \frac{a+b+c}{2}$. *Then: rect* $(s, s-b) : \mathbf{A} = \mathbf{A} :$ *rect* $(s-a, s-c)$.[79]

Proof: (see figure 7.5)

1. See Heron's proof nr. 1 (p. 255). $BF = BE, AF = AD, CD = CE$.

2. Prolong BA and BC and bisect the angles between each of these prolonged segments and AC. Call K the intersection point of the bisectors. Connect OK. Construct G on AB prolonged such that $AG \perp GK$, construct H on BC prolonged such that $AH \perp HK$ and construct I on AC such that $AC \perp IK$. In triangles AGK and AIK AK is common, $\angle GAK = \angle IAK$ and $\angle AGK = \angle AIK = \frac{\pi}{2}$, therefore $\triangle AGK$ and $\triangle AIK$ are congruent, from which follows that $KG = KI$ and $AG = AI$. Similarly, it can be proved that $KI = KH$ and $CI = CH$.

3. It follows that $BG+BH = a+b+c = 2s$. Now, $KG = KI = KH$. Because $\angle GAC = \beta + \gamma$, $\angle KAC = \frac{1}{2}\angle GAC = \frac{\beta+\gamma}{2}$. $\angle KAO = \angle KAC + \angle OAC = \frac{\beta+\gamma}{2} + \frac{\alpha}{2} = \frac{\pi}{2}$; $\angle KCO$ is also right (similar reasoning). Because in the quadrangle $AOCK$ the opposite angles $\angle KAO$ and $\angle KCO$ are right, the four points A, O, C, K are on a circle and $\angle AKC + \angle AOC = \pi$.

4. Call Q the intersection point of AC and OK. Then $\triangle AQK \sim \triangle OQC$[80] and it follows that $\angle KOC = \angle KAC = \frac{\beta+\gamma}{2}$. Combining this last result with $\angle BOE = \frac{\alpha+\gamma}{2}$, $\angle EOC = \frac{\alpha+\beta}{2}$ yields that $\angle KOB = \pi$ and therefore K, O and B are on one straight line.

5. KB is common to $\triangle GBK$ and $\triangle HBK$, $KG = KH$, therefore $\triangle GBK \cong \triangle HBK$. Because $BG + BH = 2s$, $BG = BH = s$. $\angle AGK = \angle CHK = \frac{\pi}{2}$ by construction.

6. $\angle OAK = \frac{\pi}{2}, \angle GAK + \angle FAO = \pi - \angle OAK = \frac{\pi}{2}$, and $\angle FAO + \angle FOA = \frac{\pi}{2}$, which yields that $\angle GAK = \angle FOA$. Therefore, $\triangle AFO \sim \triangle KGA$.

7. It follows that $KG : GA = AF : FO$, which yields rect $(KG, FO) =$ rect (GA, AF). Because FO and GK are parallel perpendiculars, $\triangle BFO \sim \triangle BGK$ and $BF : BG = FO : GK$. Applying the lemma we get rect (BF, BG) : rect $(BG, FO) =$ rect (BG, FO) : rect $(FO, GK) =$ rect (BG, FO) : rect (GA, AF).

[79] 'Si de dimidio collectorum laterum dati trianguli latera sigillatim subducantur erit ut rectangulum sub dimidio et differentia quacunque ad aream trianguli, sic eadem area ad rectangulum sub reliquis differentiis comprehensum.' [van Ceulen, 1615b, p. 123] or (equivalently, in Dutch) [Ramus, 1622, fol. Ee 4^v]. a, b and c can be permutated. I follow the proof of [van Ceulen, 1615b, p. 124].

[80] Cp. p. 282 for this property of triangles in a cyclic quadrilateral.

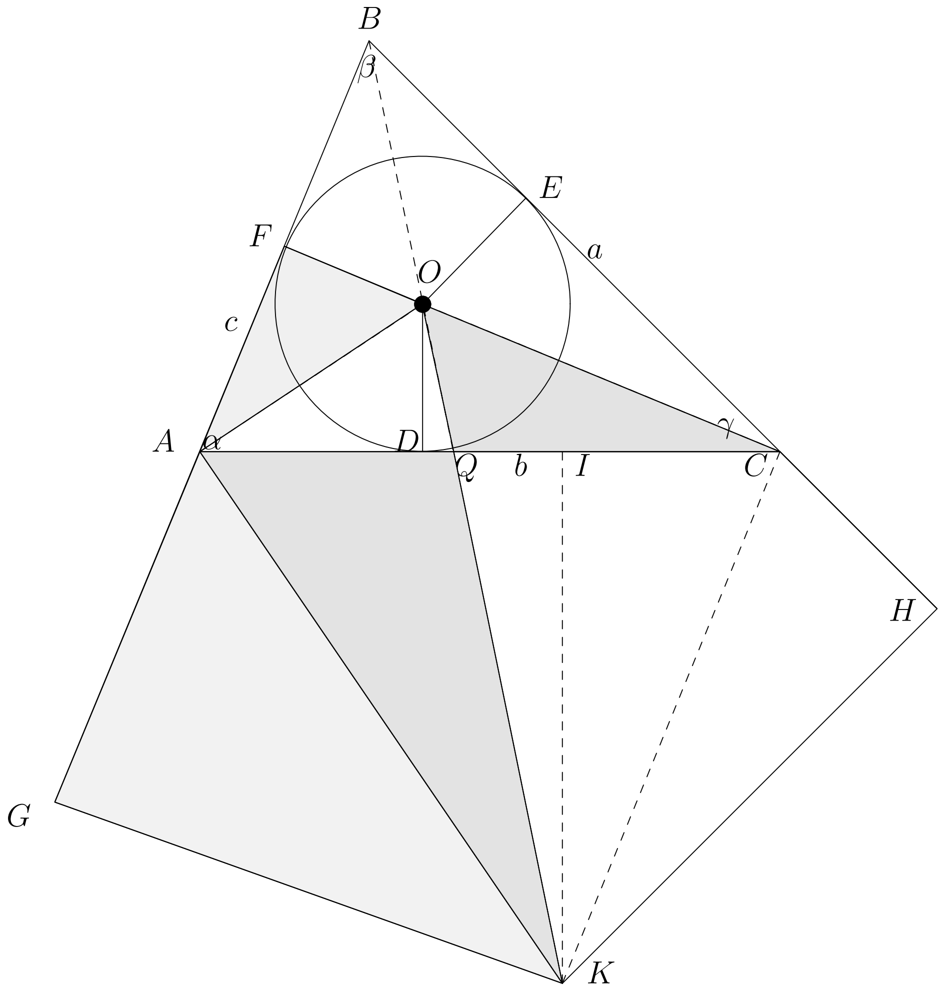

Figure 7.5: Snellius's proof in the *Fundamenta*

8. Now[81] rect $(BG, FO) = \mathbf{A}, BG = s, AF = s - a, BF = s - b, GA = s - c$. The substitution of these yields the result.

Snellius's own statement of the theorem avoided the pitfall of higher dimensionality. He claimed that he had reformed the theorem in good geometrical fashion,[82] and indeed, his version did not contain magnitudes or operations alien to Euclidean geometry. Yet in fact, this reformulation made the theorem somewhat different from the traditional theorem. Notably, Snellius lost the symmetry of the expression in a, b and c. If the expression in his theorem is interpreted arithmetically, it can be rewritten as:

$$\mathbf{A}^2 = s(s - a)(s - b)(s - c),$$

which is equivalent to the version of the theorem that we started with (theorem 7.2). The last step of Snellius's proof shows that he consistently only considered proportions and did not multiply terms by each other, in this way avoiding the use of four-dimensional or three-dimensional magnitudes and expressing all in lines and areas instead.

Snellius elaborated on the cause of the common error of the four-dimensionality. He brought forward explicitly what Ramus had only alluded to: the use of arithmetical concepts in a geometrical proof was to blame:

> They [sc. Jordanus, Luca Pacioli, Tartaglia and others] have in their proofs erred from the true nature of the geometrical proof, and while they started out geometrically, they ended arithmetically. In this way, they have used a strange and for the ancients unusual manner, from which has originated the great obscurity of this proof. Therefore, it has been heeded by few people and understood by even fewer.[83]

He continued by pointing out that this misunderstanding was already shown in the formulation of the theorem.

Snellius rightly realized that the four-dimensionality to which he objected might more easily be implied in the theorem and proof if numbers were allowed

[81] In the preceding problem, 'How to find the radius of a circle inscribed in a given triangle', it had been proved that $\mathbf{A} = rs$. [van Ceulen, 1615b, pp. 120–121]. The equality of the segments AF, BF and GA to $s - a, s - b, s - c$ is not pointed out explicitly either by Van Ceulen or by Snellius.

[82] '[...] nunc illud [sc. theorema] reformatum bene Geometrice ita concipio et enuntio.' [van Ceulen, 1615b, p. 123].

[83] 'soo zijn zy nochtans daer in van de waren aert des meetkonstich bewijs ter zyden afghedwaelt/ ende meetkonstich beginnende telkonstich gheendicht/ en hebben alsoo een vreemden/ en by den ouden onghebruyckte maniere van doen ter handt ghenomen. waer uyt dan ghesproten is een groote duysternisse van dit bewijs/ alsoo dat het by weynighe ter herten ghenomen/ en van noch weynigher verstaen zy/ [...]', [Ramus, 1622, fol. Ee 4^v].

to represent geometrical magnitudes, because numbers have no dimensions. If geometrical magnitudes were manipulated as though they were numbers, a non-defined entity could be created erroneously. Again classical mathematicians played a role as advocates of purity.

In the *Fundamenta*, when pointing out the step in Van Ceulen's proof where two areas were multiplied by each other, which created a product of four factors, Snellius wrote: 'although this is justly allowed by the arithmetician, it is rightly rejected by the geometer.'[84] His conclusion in the *Meetkonst* was somewhat different. After he had given the 'natural proof' there, he was willing to present the theorem in arithmetical terms. In the end, he did not turn down the convenience of the arithmetical formulation of Heron's Theorem, but he was convinced that it first should be proved in a strictly geometrical way before it could be accepted as arithmetically valid.[85]

Snellius exhausted the opportunities that the subject offered, by giving some relationships between the lengths of the sides, the height of the triangle, its area, and the radius of its inscribed circle in the shape of corollaries. He announced them in the *Fundamenta*, promising to give all but one on a proper occasion.[86] He found room for one of them in the *Fundamenta*:

Corollary 7.5 *(a, b, c and s are defined as in theorem 7.4.) If x and y are determined in such a way that $s : x = x : (s-a)$ and $(s-b) : y = y : (s-c)$, then: rect $(x, y) = \mathbf{A}$.*[87]

Two others were added to the *Meetkonst*:

Corollary 7.6 $s : (s-a) = \textit{rect}\ (s-b, s-c) : \textit{sq}\ (r)$.[88]

He also showed how the theorem could be used to find the height of a triangle, given the lengths of its sides, thus relating the two traditional ways for determining the area of a triangle to each other:

Corollary 7.7 $\textit{sq}\ (\frac{b}{2}) : \textit{rect}\ (s-a, s) = \textit{rect}\ (s-b, s-c) : \textit{sq}\ (h)$.[89]

[84] '[...] numerus quatuor numerorum continua multiplicatione factus, quod ut ab Arithmetico bene conceditur ita ab Geometra iure repudiatur'. [van Ceulen, 1615b, p. 124].

[85] 'Dit was dan het natuerlijck bewijs om de waerheydt van dit selve voorstel duydelijck en meetkonstich te bewysen. En hier uyt is ghenomen dat telkonstich uytspreken van dit selve voorstel/ om bequamelijck/ sonder de ghedachten met eenich naebedencken te quellen/ den inhoudt des ghegeven driehoecks te vinden aldus:

Indien yder zyde des ghegeven driehoecks afghetrocken werde van de helft aller zyden, soo sal de vierkantwortel van't ghetal uyt de menichvuldinghe van de helft en de drie overblyfsels doorgaens ghemaeckt, den inhoudt zijn van den ghegeven driehoeck.' [Ramus, 1622, fol. Ff 1^v].

[86] [van Ceulen, 1615b, pp. 124–125], [Ramus, 1622, fol. Ff 2^r]. I will not give the proof of any of these, because they are all elementary.

[87] a, b and c can be permutated.

[88] [Ramus, 1622, fol. Ff 2^r]. a, b and c can be permutated.

[89] [Ramus, 1622, fol. Ff 2^r]. a, b and c can be permutated. If b is replaced by a or c, the height has to be adjusted.

Any remark about commensurability is absent in this whole discussion of Heron's Theorem, although Snellius must have been aware that the assignment of rational numbers to all relevant line segments was only possible in special cases.

7.6 Quadrilaterals: the expediency of numbers

One of Snellius's most renowned contributions to mathematics is his formulation of a theorem expressing the area of a cyclic quadrilateral (a quadrilateral that can be inscribed in a circle) in terms of its sides. This theorem will be discussed here in the context in which it figures, which is Van Ceulen's construction of quadrilaterals and Snellius's comments. Although this section of the *Fundamenta* does not help us much to understand how Snellius discovered the theorem, it gives some more food for thought about the relationship between numbers, algebra and geometry in Snellius's work.

7.6.1 The construction of a cyclic quadrilateral

The quadrilateral became a topic in geometrical problem solving in Europe in the fifteenth century. Some of the contributors were Joseph Scaliger, François Viète and Joannes Praetorius ('second to none by his knowledge of this field' according to Snellius).[90] The work of these last three scholars was known to Snellius, who devoted one of his commentaries in Van Ceulen's *Fundamenta* to the quadrilateral.[91] The subject received Snellius's special attention because he was encouraged by Van Ceulen, Praetorius (whom he had met during his travels) and some other friends to study the construction of a cyclic quadrilateral on the basis of four given line segments. Scaliger had been one of Snellius's teachers, but his contribution was slighted by Snellius, who wrote: 'the problem [of the construction of a cyclic quadrilateral] was made notorious by a remarkable falsely-drawn figure by Joseph Scaliger, who in his *Cyclometrica* tried to convince others by a miraculous false reasoning that he had delivered a good proof of this.'[92]

The problem under consideration is this:

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[90] See [Tropfke, 1940, pp. 156–165] for the history of the study of the cyclic quadrilateral until the first part of the seventeenth century.

'Clarissimus Ioachimus Praetorius harum artium scientia nulli secundus [...]', [van Ceulen, 1615b, p. 188]. Oddly, Snellius gives 'Joachim' as the first name of Praetorius, although he certainly meant Joannes, to whose book on cyclic quadrilaterals he refers in this place. Two pages later, he does refer to Joannes Praetorius.

[91] [van Ceulen, 1615b, pp. 188–190].

[92] 'Problema insigni pseudographemate Iosephi Scaligeri nobilitatum, qui in Cyclometricis suis mirabili paralogismo bene a se id praestitum aliis persuadere conatus est [...]', [van Ceulen, 1615b, p. 188].

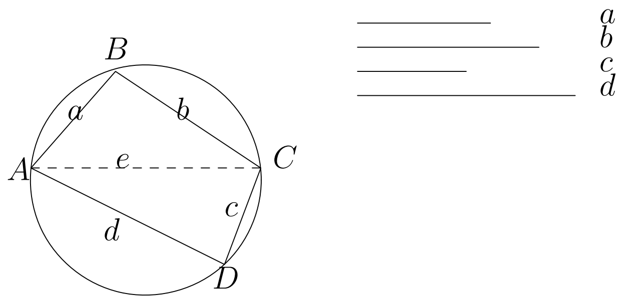

Figure 7.6: A cyclic quadrilateral

Problem 7.8 *Given four line segments a, b, c and d. Construct a quadrilateral with sides a, b, c, d, inscribed in a circle, and such that a is opposite c, b is opposite d (see figure 7.6).*

Snellius's commentary followed Ludolph van Ceulen's two constructions of the problem.

Both Van Ceulen's and Snellius's constructions were based on the idea that the problem was solved if the length of one diagonal was determined. For instance if $a = AB, b = BC$, and the diagonal $e = AC$ is determined, a triangle can be constructed with sides a, b and e. Precisely one circle can be drawn through the points A, B, C. The segments $c = CD$ and $d = DA$ can now be placed in the circle (or a triangle with sides e, c and d with side AC in common with the first triangle can be constructed), and the quadrilateral is constructed (that c and d actually meet on the circle, in point D, follows from the analysis below).[93]

Van Ceulen's first construction is formulated completely in arithmetical terms.[94] He took exemplary values for the line segments and explained which calculations have to be made with them. If we generalize this, we obtain the following:

Construction:

1. Calculate $\frac{b}{d}$ and $\frac{a}{c}$ and add the results. The outcome is I.

2. Calculate $\frac{a\frac{b}{d}}{c} + 1$ and multiply by I. The outcome is II.

3. Calculate $ac + bd$, divide by II, take the square root and multiply by I.

[93]The triangle and therefore the quadrilateral are not unique, because a, b and c, d can be ordered clockwise or anticlockwise. Following Van Ceulen and Snellius, I assume that the segments are ordered clockwise. The solution is then unique. If the order of the line segments is changed, the radius of the circle in which the quadrilateral is inscribed remains the same.

[94][van Ceulen, 1615a, pp. 203–204], [van Ceulen, 1615b, pp. 185–186].

The result is the length of the diagonal AC.[95]

$$[AC = \sqrt{\frac{ac+bd}{(\frac{ab}{dc}+1)(\frac{b}{d}+\frac{a}{c})}}\left(\frac{b}{d}+\frac{a}{c}\right).] \qquad (7.6)$$

In the introduction to this problem, Van Ceulen wrote that it had been sent to him by Johan Pouwelsz twenty years earlier, and that he had first solved it by means of the rule of cos (algebra) and then found another method for those not knowing cos. He gave no proof, but only a reference to two propositions of the *Elements* underlying the construction (I.15 and VI.21).

Van Ceulen then gave a second solution of the same problem, this time borrowed from Cornelis Pietersz.[96] The solution is formulated in geometrical terms and no proof is given. I add some algebraic notation between square brackets to facilitate comparison to the other constructions.[97]

Construction:

1. Construct the mean proportional EF between a and b ($a : EF = EF : b$). [$EF = \sqrt{ab}$.]
2. Construct the mean proportional FG between c and d. [$FG = \sqrt{cd}$.]
3. Construct a triangle with sides EF and FG and a right angle between them; its hypotenuse is EG. [$EG = \sqrt{ab+cd}$.]
4. Construct the mean proportional HI between a and d. [$HI = \sqrt{ad}$.]
5. Construct the mean proportional IJ between b and c. [$IJ = \sqrt{bc}$.]
6. Construct a triangle with sides HI and IJ and a right angle between them; its hypotenuse is HJ. [$HJ = \sqrt{ad+bc}$.]
7. Construct the mean proportional KL between a and c. [$KL = \sqrt{ac}$.]
8. Construct the mean proportional LM between b and d. [$LM = \sqrt{bd}$.]
9. Construct a triangle with sides KL and LM and a right angle between them; its hypotenuse is KM. [$KM = \sqrt{ac+bd}$.]

[95] According to Van Ceulen, the outcome is the diagonal BD, but this is wrong, as is shown by the proof.

[96] Pouwelsz and Pieterz also communicated with Van Ceulen about the triangle division problem, see p. 221.

[97] [van Ceulen, 1615a, pp. 204–205], [van Ceulen, 1615b, pp. 186–187]. The original lettering and figures are slightly confusing, therefore I have partly changed them.

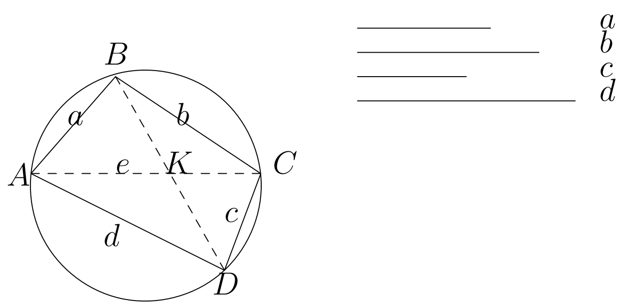

Figure 7.7: A cyclic quadrilateral (Snellius's analysis)

10. Determine the fourth proportional e of EG, HJ, KM ($EG : HJ = KM : e$). e is the length of the required diagonal AC.

$$[e = \sqrt{\frac{(ad+bc)(ac+bd)}{ab+cd}}.] \tag{7.7}$$

In algebraical terms, Van Ceulen's two constructions yield the same solution. It can easily be verified that

$$\sqrt{\frac{ac+bd}{(\frac{ab}{dc}+1)(\frac{b}{d}+\frac{a}{c})}}\left(\frac{b}{d}+\frac{a}{c}\right) = \sqrt{\frac{(ad+bc)(ac+bd)}{ab+cd}}.$$

However, the steps of the two constructions are not equivalent.

Snellius supplemented Van Ceulen's construction with an *analysis* (see p. 184 for this concept), by means of which he wanted to elucidate Van Ceulen's undemonstrated construction, as he explained to the reader, thus implicitly criticizing his former teacher.[98]

Analysis: (see figure 7.7)

1. Four line segments have again been given: $a = AB, b = BC, c = CD, d = DA$. Suppose that the cyclic quadrilateral with a, b, c, d as its sides has been constructed. Call the point of intersection of its diagonals AC and BD K. Now $\triangle AKB \sim \triangle DKC, \triangle BKC \sim \triangle AKD$. [These similarities are not explained by Snellius. They can be proved by means of *Elements* I.15 and III.35.]

2. Therefore,

[98][van Ceulen, 1615b, pp. 188–189].

$$\begin{aligned}
DK : AK = DC : AB &= \text{rect } (DC, AD) : \text{rect } (AB, AD),\\
AK : BK = AD : BC &= \text{rect } (AD, AB) : \text{rect } (BC, AB),\\
BK : CK = AB : DC &= \text{rect } (AB, BC) : \text{rect } (DC, BC).
\end{aligned}$$

3. Combining these three proportions yields
   $DK : AK : BK : CK =$
   rect (DC, AD) : rect (AB, AD) : rect (AB, BC) : rect (DC, BC).$(*)$
   Moreover, $(BK + KD) : (AK + KC) = BD : AC$,
   hence (add up the components of $(*)$ in pairs)

$$\begin{aligned}
BD &: AC =\\
(\text{rect } (DC, AD) + \text{rect } (AB, BC)) &: (\text{rect } (AB, AD) + \text{rect } (DC, BC)).
\end{aligned}$$

4. This last equation is an expression of the ratio of the two diagonals. Now transform the last two pairs of rectangles into one square each (this can be done by *Elements* I.45 and II.14); call these sq (MM_1) and sq (NN_1) respectively. [$MM_1 = \sqrt{ab + cd}$, $NN_1 = \sqrt{ad + bc}$.]

5. It follows that

$$\begin{aligned}
AC : BD &= \text{rect } (BD, AC) : \text{sq } (BD) =\\
(\text{sq } (MM_1) \cdot \text{sq } (NN_1)) : {MM_1}^4 &= \text{sq } (NN_1) : \text{sq } (MM_1). \qquad (7.8)
\end{aligned}$$

6. Ptolemy's Theorem states that rect $(BD, AC) =$ rect (AB, CD)+rect (AD, BC). Construct a square with sides OO_1 equal in area to this last sum. [$OO_1 = \sqrt{ac + bd}$.]

7. Combining the last two results yields sq (NN_1) : sq $(MM_1) =$ sq (OO_1) : sq (BD), and therefore $NN_1 : MM_1 = OO_1 : BD$. Now BD, the required diagonal, is the fourth proportional of three known magnitudes (based on the given line segments) and can therefore be determined.

$$[BD = \sqrt{\frac{(ab + cd)(ac + bd)}{ad + bc}}.] \qquad (7.9)$$

Snellius concluded this analysis by remarking that the actual construction of the problem, the *synthesis*, was easy now. It is indeed not difficult to reverse the steps of his argument: they only entailed elementary geometrical constructions. Snellius observed however

> If you consider lines, the construction of this problem is rather laborious, but it is very easy when dealt with by means of numbers.[99]

This analysis and the passage following it contain several intriguing features. In the first place, there is no direct correspondence between Van Ceulen's two constructions and Snellius's analysis, and therefore the question is whether Snellius's addition indeed enlightened his readers. Even after his explanation, the first two constructions can to a great extent be considered as magical: they work, but most readers would not understand why and certainly would not be able to explain how Van Ceulen and Pietersz found them.

Even if the correspondence is somewhat hidden, Snellius's analysis does indeed prove that the first two constructions yield a correct result. This can be seen most easily by comparing the three algebraical expressions (7.6), (7.7) and (7.9) of the diagonals,[100] which are equivalent. This correspondence is more difficult to see for someone with a strictly geometrical frame of mind. The operations in Van Ceulen's first construction are arithmetical, those in his second problem and in Snellius's analysis are geometrical. Is it too bold to surmise that Snellius assumed a mixed algebraical-geometrical mindset of his audience—or an arithmetical-geometrical mindset with numerical values instead of indeterminates?

There are several indications for the validity of this interpretation. Snellius introduced four-dimensional magnitudes in his analysis (see (7.8)), again referring to them with Vietean terminology: 'plano-planum', 'quadrato-quadratum'.[101] They were only used in an intermediate step, but still it is a remarkable feature after his statements about Heron's Theorem. The reason for his greater liberality here must be that the analysis did not have the status of a proof and that therefore the arguments used could be somewhat looser.

This use of higher dimensional magnitudes in the analysis shows that Snellius did not want to keep the realms of arithmetic and geometry separated in all circumstances. This was also true for the main result of the analysis, which was expressed in geometrical terms, but illustrated by a numerical example. Still, he preferred a construction without higher dimensional magnitudes:

> However, another much more elegant geometrical construction could also be derived from the same analysis, in which we avoid this stereometry of planoplanes in the proof: and although we lack a figure that is suitable for this matter, I will still state it as clearly as possible,

[99]'Constructio problematis si lineas spectes satis operosa, in numerorum autem pragmatia valde expedita [...]', [van Ceulen, 1615b, p. 189].

[100]If Snellius had considered AC, he would have found $AC = \sqrt{\frac{(ad+bc)(ac+bd)}{ab+cd}}$.

[101]'plano-planum sub quadrato ML in quadratum EH ad quadrato-quadratum ab ML [...]', [van Ceulen, 1615b, p. 188].

and in the second edition I will take care of figures more zealously.[102]

Snellius asserts that he did not actually give this proof completely, because the figure needed to explain it had not yet been made. Would this be the real reason or an excuse to avoid a tedious explanation very similar to the analysis? As the solution involved only standard constructions, it would have been easy to write it down without using three- or four-dimensional magnitudes.

Snellius then gave another way to determine the diagonal AC, now phrased in geometrical terms,[103] telling the reader to deduce the demonstration from his analysis, and again explaining that he could not give it for want of a proper figure. He warned the reader that everyone took for granted that the converse of Ptolemy's Theorem was also true (if $ac + bd$ = product of the diagonals in a quadrilateral, then it must be cyclic), although it had not been proved by anybody.[104]

7.6.2 The area of a quadrilateral: 'a new little theorem'

Before further discussing Snellius's section on quadrilaterals, a short detour to the study of the area of cyclic quadrilaterals is convenient. A theorem analogous to Heron's Theorem exists for cyclic quadrilaterals, expressing their area in terms of the lengths of their sides.

Theorem 7.9 *If a, b, c and d are the lengths of the sides of a cyclic quadrilateral, $s = \frac{a+b+c+d}{2}$ and $\mathbf{A}$ its area, then: $\mathbf{A} = \sqrt{(s-a)(s-b)(s-c)(s-d)}$.*

Unlike Heron's Theorem, this theorem did not stem from Greek mathematics. Its first appearance was in India, in the work of Brahmagupta (seventh century AD). He may have created the formula inductively from Heron's Theorem. There is no evidence that a proof was known at the time. This was only found in the Arab world in the eleventh century. The knowledge of the theorem and its proof

[102] 'Potuit autem ex eadem analysi alia etiam longe concinnior geometrica fabrica derivari in qua illam planoplanorum stereometriam in demonstratione declinemus: et quamvis diagrammate ad hanc rem idoneo distituamur, dicam tamen quam potero clarissime, iterata editione diagrammata maiore sedulitate procuraturi.' [van Ceulen, 1615b, p. 189].

Tropfke translates this sentence thus: 'Aus derselben Analysis kann man aber eine weit klarere geometrische Methode ableiten, in der wir die Stereometrie vierdimensionaler Gebilde in der Darlegung benutzen und, obgleich wir hierfür kaum geeignete Anschauungsformen besitzen, verspreche ich bei einer neuen Auflage so klar wie möglich mit größerem Fleiß die Anschauungsform zu beschaffen.' [Tropfke, 1940, p. 164].

Tropfke seems to have been mistaken both in the meaning of 'declinemus' and of 'diagrammata', which he takes proverbial instead of literal. However his summary of Snellius's analysis is useful.

[103] He wrote BD, but that seems to be a mistake.

[104] 'id a nemine hactenus unquam fuerit geometricis demonstrationum mominibus firmatum [...]', [van Ceulen, 1615b, p. 189].

disappeared again in the following centuries, and none of Snellius's European predecessors referred to it.[105]

This same theorem 7.9 was presented by Snellius:

> If from half of the sum of the sides of a given quadrangle inscribed in a circle the sides are subtracted one by one, the root of the product of the four differences is the area.[106]

Snellius was the first European who published this theorem,[107] and he was certainly aware of the weight of his discovery. He claimed it as his own invention, not knowing of the older tradition, telling the reader 'we have a very elegant theorem of this kind' and then referring to it as 'this new little theorem of mine'.[108]

Snellius did not give a proof, however. The first correct proof in Western Europe was only found later.[109] Where or how, then, did Snellius find the theorem? Probably, his reasoning was similar to Brahmagupta's. Snellius may have guessed the expression as an analogue to Heron's Theorem, checking its truth in a number of cases by calculating the area of the two triangles into which any quadrilateral can be divided, and noting the restriction to cyclic quadrilaterals in this way—a restriction without an analogue for triangles. In the case of cyclic quadrilaterals, he could calculate the length of a diagonal in the way discussed immediately before the presentation of his new result. Then it was easy to calculate the areas of the two composing triangles by means of Heron's Theorem. However, it was not easy to find a general expression for the area of the cyclic quadrilateral in this way.

Any reference to a proof is conspicuously absent, which makes it likely that Snellius did not have one. Another possibility is that he discovered one through algebraical manipulations, but did not manage to translate that into proper geometrical constructions. The context of Van Ceulen's constructions suggests this option, but without definite indications it cannot become more than a suggestion. The geometrical interpretation of this theorem was problematic anyway, because it involved—again—the product of four line-segments. Snellius gave a numerical example, maybe to distract the reader from this problem.

Apparently, the strict criteria which Snellius had advocated in the Heron case—the theorem had to be proved by correct, geometrical means—did not hold

[105] [Tropfke, 1940, pp. 152–163].

[106] 'Si de dimidio collectorum laterum dati quadranguli in circulum inscripti latera sigillatim subducantur, latus continue a quatuor differentiis facti erit area.' [van Ceulen, 1615b, p. 189].

[107] That Snellius was the first, is assessed by [Tropfke, 1940, p. 165], [de Waard, 1927b, c. 1157] and [Struik, 1975, p. 500].

[108] 'theorema huiusmodi habemus valde scitum.' [van Ceulen, 1615b, p. 189]. 'novum hoc nostrum theoremation', [van Ceulen, 1615b, p. 190].

[109] In 1727 according to Tropfke; however, dr. Eisso Atzema found an earlier proof in Abraham de Graaff's *Inleiding tot de Wiskonst*, sec. edition, 1706. See [Heilbron, 1998, pp. 219–220] for a proof using trigonometrical functions.

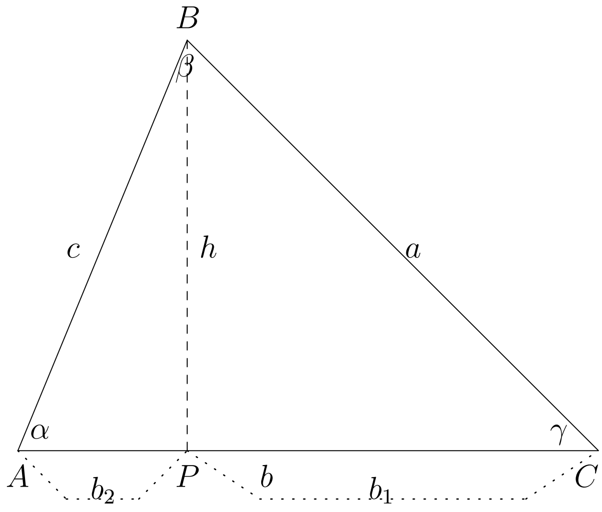

Figure 7.8: A modern approach

here, or rather, now that Snellius had to choose between claiming an invention as his own or withholding it because he was not able to formulate and prove it correctly, he chose the first alternative.

After this interlude, Snellius gave one of three of his own solutions to the problem of constructing a cyclic quadrilateral if its sides are given. This was different from those discussed previously. A few pages later, Snellius again advertised all his own inventions regarding the quadrilateral.[110]

Although Snellius's new theorem received some attention in secondary literature as one of his contributions to the development of mathematics, no study of its context, notably of its role in the development of an arithmetical-algebraical line of thought in geometry, existed until now.

7.7 Epilogue: trigonometrical functions

After Snellius's time, trigonometrical functions and algebra were used to prove theorem 7.2, which simplified the proof considerably.[111] If trigonometrical functions are used, the auxiliary triangle or quadrilateral can be dispensed with:

[110] [van Ceulen, 1615b, pp. 190, 194].
[111] [Tropfke, 1923, p. 88].

Proof:[112] (see theorem 7.2 and figure 7.8)

$$\mathbf{A} = \frac{1}{2}bh. \tag{7.10}$$

$$\frac{h}{a} = \sin\gamma. \tag{7.11}$$

Apply the law of cosines (proved by applying the theorem of Pythagoras to $\triangle ABP$ and $\triangle BCP$ and by substituting $b_1 = a\cos\gamma$ and $b_2 = b - b_1$):

$$c^2 = a^2 + b^2 - 2ab\cos\gamma. \tag{7.12}$$

The combination of (7.10)–(7.12) and the property $\sin^2\gamma + \cos^2\gamma = 1$ yields

$$\mathbf{A}^2 = \frac{b^2h^2}{4} = \frac{(a+b+c)}{2}\frac{(a+b-c)}{2}\frac{(a-b+c)}{2}\frac{(-a+b+c)}{2} = s(s-a)(s-b)(s-c).$$

This approach makes the proof much shorter and less ingenious. The amateurs of traditional geometrical problem solving could consider this as less rewarding. A speculative explication of the lack of a proof of theorem 7.9 can be based on this observation.

For this, it has to be noted that Snellius's posthumous publication *Doctrina Triangulorum* teaches us that he was not ignorant of the manipulation of trigonometrical functions. He derived for example (in modern notation, cp. theorem 7.1): $1 : \sin\alpha = bc : 2\mathbf{A}$.[113] Snellius may have been able to prove the theorem by means of trigonometrical functions, or at least to ground his conjecture about the area, yet he might not have seen a short way to translate this into a traditional geometrical proof. Probably he considered trigonometrical functions as expedient calculating aids, but not as proper means for proving geometrical assertions. He may have reckoned on another occasion to continue his work on his invention (the printers were now hurrying him) and publish a more elaborate version, which has never appeared.

One of the problems studied in *Doctrina Triangulorum* is that of finding the area of a (non-cyclic) quadrilateral if its four sides and one angle are given. Snellius discussed a numerical example. The quadrilateral is divided into two triangles, the areas of which are calculated by means of Ramus's XII.9, which is nothing else than Heron's Theorem in the version of Ramus's *Geometria*. The answer is given in numbers.[114] This shows again that Snellius was willing to allow the expression of geometrical magnitudes in numbers, if this was suitable in the context.

[112] [Heilbron, 1998, p. 271].
[113] [Snellius, 1627, p. 90].
[114] [Snellius, 1627, pp. 90–92].

7.8 Conclusion

This chapter shows the struggle of early modern mathematicians with the use of arithmetical operations in geometry, focusing on Ramus's and Snellius's difficulties with Heron's Theorem. Snellius was the first to modify this theorem into a proposition meaningful in a Euclidean geometrical framework and to prove it in the same framework. Ramus's and Snellius's criticism of the proof of this theorem discloses some of their criteria for good mathematics: first, it had to be in accordance with ancient examples and second, theorems had to be proved according to strict rules, whether they were meant for theory or for practice. Moreover, geometrical and arithmetical concepts must not be confused.

The main reason for this confusion, and also for some difficulties in interpreting Ramus's objections and determining Snellius's point of view exactly, seems to be a certain vagueness of the terms used by early modern and earlier mathematicians. The meaning and domain of terms are in general not explicitly defined. It is for instance not always clear when these mathematicians mean a rectangle and when a product of two numbers. The two operations of rectangle formation and multiplying are related, yet not equal. The precise nature of this relationship cannot be caught in the terminology of Snellius and his predecessors.

Snellius changed the traditional proof just enough to make it satisfy his standards of good geometry, that is, a geometry with a restricted place for arithmetical concepts. His endeavours could be described as an 'exactification' of canonical knowledge. His concern shows that he wanted mathematics to be correct for its own sake and according to its own norms, because his adjustment of the theorem and proof did not make any difference to the practical use of the algorithm.

Snellius's work on the quadrilateral modifies the view that he would be opposed to the use of algebraic concepts or numbers in geometry altogether. Rather, he seems to have given them the status of auxiliary means in the search for solutions, his final goal being a traditional geometrical construction and proof. His careful distinction between those stages makes his work more exact than Van Ceulen's, and he consciously adjusted Van Ceulen's work to make it meet higher standards of exactness. Snellius's formulation of the theorem in which the area of a cyclic quadrilateral is expressed is an exception to this general exactness, however. His presentation shows that he was well aware of his achievement in discovering a new theorem in an old field. In this case the extension of knowledge took priority over exactification.

Questions concerning Snellius's attitude towards exactness have never been raised in the discussions of Snellius's new theorem, yet the comparison between Snellius's discussion of the area of a triangle and of a quadrilateral give some good indications of rival approaches within geometry and Snellius's struggle with them. Precisely because these theorems are of a mixed arithmetical-geometrical nature, they furnish an excellent example of Snellius's position in the changing world of geometry in his time, in which the increasing role of algebraic-arithmetical tools did not only help to solve problems, but also caused questions and confusion.

Chapter 8

Characteristics of Snellius as a mathematician

> Ismaël Boulliau wrote about Snellius [...]: 'Whose thinking was more profound, whose research was more acute, who was more sharp-sighted in perceiving the mathematical truth, and who was more precise in solving problems?'[1]

8.1 Introduction

The introduction of this thesis started with some seemingly unconnected images from Snellius's life. Now that many stories have been told and many aspects of Snellius's work and deeds have been analysed in the previous chapters, it is time to connect all these in one single picture. This painting has different layers: when seen from a distance, only the general characteristics are discerned (discussed in section 8.2), but when the observer approaches, he will be able to see repetitions of patterns in various parts of the picture. Much attention is drawn to the lively colours of pure mathematics (section 8.3), from which the eye strays to the rest of Snellius's work (section 8.4). Some themes relevant to all of Snellius's work will be discussed in the last sections (patronage, network and audiences in section 8.5,

[1] 'De quo [sc. Snellio] sic Ismaël Bullialdus, Prolegomenis in Philolaicam Astronomiam: "Quis aut cogitatione profundior, aut sagacior in investigando, aut ad veritatis Mathematicae perspicientiam acutior, et in problematis solvendis subtilior? Astronomiae quoque operam dedit: et, nisi morte praematura praeventus fuisset, tanta erat ingenii facilitate praeditus, ut plurima deperdita in lucem revocare potuerit, et repertis quamplurima nova addere." Magnus omnino vir fuit: sed non suo pretio aestimatus a suis. Nempe iis fere moribus nunc vivitur, ut vix aliud laudent, nisi quo ipsi valeant; caetera vero studia, ne minores habeantur ob inscitiam, per eos pedibus trahantur licet.' [Vossius, 1650, p. 202].

humanism in 8.6, and Ramist aspects as part of humanism in 8.6.4). Although all shapes are borrowed from the previous chapters, some details, figurines and colours are only added in this chapter. When starting in one corner of the painting, a different interpretation can arise than if one starts in another corner. Although it is therefore not possible to obtain one single, coherent interpretation of the whole work, each viewer should be able to go home with a satisfied feeling of understanding the meaning of the picture.

8.2 General characteristics

The first characteristic of Snellius's mathematics is its broadness. He covered almost the whole field of the mathematical sciences of his day, publishing on geometry, arithmetic, astronomy, navigation, surveying; preparing a book on optics; and teaching, corresponding and advising on many mathematical topics. He had a profound knowledge of all these domains. The cross-fertilization of his endeavours in different parts of the mathematical sciences, combined with his talents, his high output rate, and 'dedication to the progress of mathematics'[2], made him contribute valuable results to mathematics.

Snellius was educated by a very learned and eager father, and was surrounded by able scholars and practitioners from his youth onward. His father also gave him the opportunity to meet men of learning abroad. By the time Rudolph Snellius was dead, Willebrord had acquired his own position in Leiden and was thus able to continue the contacts with scholars there and elsewhere. Snellius took full advantage of this network: his intellectual background enabled him to study classical and contemporaneous sources successfully and he exchanged knowledge with others.

Almost all his works discuss problems with a classical origin: geometrical problem solving, the nature of irrational magnitudes, the quadrature of the circle, the size of the earth, the origin of comets and the behaviour of light were all topics with ancient Greek roots. Since the problems concerning these topics had not been solved to satisfaction in Antiquity, they continued to invite Snellius and his contemporaries to consider them. This does not mean that nothing worthwhile had happened in the meantime in mathematics: e.g. many trigonometrical results had only been found in the sixteenth century. Snellius also took into account some contributions of more recent predecessors. He was less interested in other traditions, but did not neglect them altogether: e.g. he used a small amount of algebra, included a chapter on Arabic surveying in the *Eratosthenes Batavus*, studied the optical work of Alhazen and was aware of the fact that the four books of the *Conics* that were not available in Greek existed

[2]'[...] sic te bonis hisce Artibus promovendis deditum coniicio.' Letter of Gassendi to Snellius, [Gassendi, 1964, p. 4].

in an Arabic translation, yet he did not know their exact contents.[3]

The problems studied by Snellius were old and had a long scholarly tradition. Therefore, it is not surprising that Snellius's contributions, although good, were not revolutionary. He had soaked up the scholarship of men of great intellect, was able to use it imaginatively, react to it critically, yet he did not break away from it completely. In this way, he remained within the discourse of his time. His aim was not the revolution, but the restoration of older knowledge. His triangulation method, for example, leaned on earlier examples, but by developing the theory further, applying it in practice and solving a number of practical problems while working, he produced the first accurate, extended triangulation network. Snellius's preference for popular problems must have been a conscious choice in general: he could show the value of his work most easily by contributing solutions to problems also studied by others. Sometimes people around him, such as Van Ceulen, stimulated him to look at certain problems.

Snellius's selection of the topics that he studied was partly steered from within, and partly from without: from within, because he was genuinely interested in the solution of mathematical problems and eager to to prove his capability, and from without, because he responded to the stimuli of people around him, and took into account the interests of different audiences. Although he presented usefulness as a leading stimulus, his works never offer complete solutions to practical problems; at most they offer elements which could contribute to these solutions, maybe for fear of being perceived as a practitioner.

8.2.1 Snellius's method

When trying to grasp the correspondence in Snellius's approach of different problems, I found that I could describe his method as a *combination strategy*: he achieved new results by using different instruments, combining them cleverly while also using his own intellect to process his data, in this way reaching further than contemporary mathematicians in many respects. These instruments were books, from his own library or borrowed; his network, of which scholars, practitioners, regents and two princes were members; and scientific instruments, such as telescopes, quadrants, a clock and his own naked eyes. Some examples may suffice to show the diversity of his instruments: Scaliger lent a manuscript to him, Van Ceulen taught him how to solve geometrical problems, Beeckman and Gassendi sent him observations, Snellius's university stipend allowed him to buy better instruments which he probably used in the second round of measurements for the determination of the circumference of the earth, he looked out for rainbows when he travelled and he built and used his own instrument for the weighing of water. His network also played a role in his work by helping him to

[3]'reliquos [sc. libros Conicorum] in Arabicam linguam conversos Romae in Vaticano adhuc adservari rumor est.' [Snellius, 1607b, p. 7].

improve his position in the university: Cunaeus wrote preliminary poems for his books, Kepler publicly asked for a patron for him and Rosendalius interceded for Snellius. The variety of these instruments and the extent to which he mastered all of them at the same time was very exceptional (although elements of this strategy can also be traced in the work of other specialized mathematicians of the period).

Two examples, one from pure and one from mixed mathematics, will illustrate his combination strategy further. In pure mathematics, the fact that he contributed three solutions to the triangle division problem, one of which was of a different type than the regular ones, can be explained by his mastering of this strategy. He received stimulation and input from people around him from different backgrounds (Van Ceulen, Stevin, Scaliger), had good literature at his disposal (Benedictus, Pappus), had sufficient mathematical skills and insight and a clear opinion on good mathematics. All these together stimulated him to find new solutions. In mixed mathematics, we can consider his rendering of his standard unit of measure in no less than four different ways, at least one of which was very original: through the determination of the relative density of water by means of an instrument consisting of two cylinders. For these four, he needed knowledge of old sources, information gathered by himself and his acquaintances, inventiveness and technical skills. Although the four ways are in principle independent, they reinforce each other: none of the methods is exact, but a combination of them diminishes the margins of uncertainty. Moreover, they challenge the writer and reader to compare them and thus to think more deeply about the problem and its possible solutions.

Some compliments paid to Snellius show that this strategy had also caught the attention of his contemporaries. Gassendi asked him for advice when he needed two observational instruments, praised him for 'adding excellent observations to the unremitting perusal of books' (see p. 93) and Boulliau esteemed his intelligence, propensity to investigate, insight into mathematics and ability to solve problems (see the quotation at the beginning of this chapter).

Snellius did not only combine different external input elements, he also put the fruits of his efforts to work in different contexts, e.g. by often including cross-references to his own works, both to books that had already appeared and still unfinished works, in this way advertising them. These were self-conscious efforts to bring his whole oeuvre into the limelight. In his edition of Ramus's *Arithmetica* for example, he mentioned several of his results from *Apollonius Batavus*, clearly proud of them. He proposed a change in his discussion of the determination of geographical positions for the new edition of *Eratosthenes Batavus*: in the original edition, he had referred to the contributions of Petrus Nonius, the 'very learned Stevin' and others; in the new edition, he planned to replace Stevin's work by his own *Tiphys Batavus*, adding his new word 'loxodromia'. In the same *Eratosthenes Batavus*, he also announced the *Doctrina Triangulorum*, and he made use of both his own *De Re Nummaria* and Scaliger's book with the same title, which he had edited.[4] In the Latin edition of Van Ceulen's work on the quadrature of the circle, Snellius advertised his own forthcoming volume on the same subject matter, boasting that he had improved Archimedes's method. In the *Fundamenta*, he announced that he would present his own work on geometry on a later occasion and in the same book, he declared proudly that his work on the cyclic quadrilateral was more convenient and easier than anything else.[5] Moreover, some of Snellius's publications were related to his other activities, e.g. to his teaching (the edition of Ramus's *Arithmetica*) or his advisory role (*Tiphys Batavus*).

8.3 Characteristics of Snellius's pure mathematics

Snellius published in both traditional branches of pure mathematics, geometry and arithmetic. The lack of a 'research' tradition in arithmetic is mirrored in Snellius's work: he only made an edition of Ramus's *Arithmetica*, meant for educational purposes. His geometrical work is more original, reflecting the lively activities in that field in his time. He reconstructed several of Apollonius's lost works, included much material in his translation of Van Ceulen's *Fundamenta*, added interesting appendices to *De Circulo* and *Meetkonst*, wrote on the quadrature of the circle in *Cyclometricus* and discussed trigonometry in *Doctrina Triangulorum*. His covering of pure mathematics was not exhaustive: he did not work with conic sections and hardly wrote on algebra. Moreover, the only topic from stereometry which he treated was spherical trigonometry, which was the most relevant part for astronomy and navigation. Pure mathematics had various functions in Snellius's time: it was studied as an intellectual pursuit for its own sake, served to train the mind of students and was developed for the needs of mixed mathematics. All these roles are present in Snellius's works, and their sometimes conflicting demands are reflected in it.

Snellius's thoughts about mathematics can be derived from two kinds of texts: the mathematical cores of his treatises, and their dedicatory letters and prefaces. His philosophy of mathematics must mainly be derived from his mathematical practice, because unlike other authors, he did not devote treatises of a more

[4][Ramus (W. Snellius ed.), 1613, pp. 51, 63]; [Snellius, 1617c, p. 239]; [Snellius, 1617c, ad p. 222-2]; [Snellius, 1617b, pp. 144, 145, 149].

[5][van Ceulen, 1619, p. 32]; [van Ceulen, 1615b, p. 121]; 'Ad diagoniorum in numeris inventionem non esse expeditiorem aliam ullam rationem ea, quam supra primo loco in nostris commentariis ostendimus [...] Secundo ad investigationem areae quadranguli in circulo nihil parabilius fieri potest isto quem exhibuimus modo [...] Tertio ad inveniendas laterum concurrentium continuationes facilem suppeditat viam secunda nostra huius problematis solutio.' [van Ceulen, 1615b, p. 194].

philosophical character to the nature of mathematics. Rudolph Snellius showed more interest in explicit qualitative considerations regarding mathematics than his son.

8.3.1 Exactness

In the introduction, I have claimed that 'exactness'[6] is a key word to understand Snellius's mathematics. A reasonable case will now be made for this assertion by means of the results of the previous chapters. Snellius made known his criteria of exactness both implicitly—by showing what he thought of as a good approach—as explicitly—by giving arguments for this approach. His remarks are often short and sometimes ambiguous and the picture is not completely consistent, notably because these remarks are to some extent context-related and are therefore not meant to be taken out of their context and combined into one coherent system. Even so, some features stand out. Part of the motivation behind his search for exactness must be found in the classroom: like Ramus, he favoured the teaching of clear, useful mathematics.

It has to be remembered that arguments for exactness are extra-mathematical and therefore cannot be judged by 'objective' mathematical standards. The subjectivity of the arguments is for instance seen in Snellius's frequent use of the terms 'elegant' and 'easy' to recommend his own solutions to problems, which can be interpreted as suitable, short, inventive, not boring and attractive.[7] Even though the arguments may not always be convincing, they play a role in mathematical discourse and research. E.g. Snellius drew much attention to his triangle division solution in the Apollonius treatises, because it fitted into his conception of a good solution.

Snellius's concept of exactness was based on five key elements, by means of which he demarcated correct mathematics from the rest:

1. classification,
2. structure,
3. purity,
4. ancient examples,
5. use.

Ad 1. As explained in the geometry chapter, geometrical problems were divided into three classes, plane, solid and line-like. Snellius dealt almost exclusively with the first of these, where ruler and compass sufficed for solving

[6] Note that exactness is here used in a restricted meaning, as defined by Bos; see p. 11.

[7] See e.g. [van Ceulen, 1615b, p. 205], [Snellius, 1627, p. 15].

problems. He mentioned this pair explicitly a few times as tools for finding constructions in a geometrical way, which made him an exception among his contemporaries in giving a positive criterion to distinguish true geometrical procedures from unacceptable ones.[8]

Only a few references to line-like problems are found in Snellius's work. He paid some attention to non-plane solutions of the quadrature of the circle in his preface to the reader of the *Cyclometricus*, where he gave the ancient classification of geometrical problems borrowed from Pappus. He explained that line-like curves could be generated by two moving curves or straight lines, of which the points of intersection traced the new curve, mentioning some classical curves of the kind such as the spiral and the quadratrix. According to Snellius, this concept of moving could not be translated into a geometrical operation by ruler and compass and could therefore not count as geometrically acceptable. For this reason, pointwise construction should only be used if a 'legitimate solution' by plane or solid means was impossible.[9] This was a strict vision: Snellius preferred not to challenge the ancient demarcation rather than to extend the field of geometrically acceptable constructions, and he insisted on the principal place granted to Euclidean straight-line-and-circle methods.

Ad 2. Snellius's strict application of the classification is an example of his general predilection for structure. A number of examples and statements show that he wanted to organize geometrical material in such a way that it could be discussed on as general a level as possible. This may very well be a translation of Ramus's rule to discuss 'the most general things generally, but specific things specifically'. Snellius claimed to be very fond of general statements, when he presented a more general version of a problem by Van Ceulen in the *Fundamenta*.[10] This same book yields many examples of Snellius's connection of Van Ceulen's separate problems. He favoured brevity when possible.

The Apollonius treatises clearly illustrate what structure means to Snellius. He wanted to organize the material into a limited number of problems:

> All the material that Apollonius had discussed in an extraordinary labyrinth of many propositions, I have tried to present and prove more generally in just a few problems.[11]

[8][Bos, 2001a, p. 220]. Some relevant quotes from Snellius are given there as well. Cp. 'Age igitur regulam ac circinum expediamus', [Snellius, 1607b, p. 9]. Cp. [Brigagli and Nastasi, 1986, p. 84] on Viète's emphasis on the importance of ruler and compass, as opposed to the ingenious inventions of his predecessors for solving geometrical problems, which suggests an influence of Viète on Snellius.

[9][Snellius, 1621, fol. $**1^v$–$**6^v$], summarized in [Bos, 2001a, p. 218].

[10]'valde enim me generalia delectant', [van Ceulen, 1615b, p. 163].

[11]'Quaeque ab eo [sc. Apollonio] multarum propositionum mirabili labyrintho tractata fuerant, pauculis problematis καθολικότερον efferre et demonstrare conati sumus.' [Snellius, 1607b, p. 7].

In the dedicatory letter to *Cutting off*, he described how he had struggled with 'the excessive hair-splitting in the individual problems', but had managed to reduce the 'the countless repetitions of indetical things' by 'the instruments of purer logic' (see p. 59). The nature of the problems entailed the use of a series of cases. Snellius also wanted to avoid unnecessary case distinctions: 'Or would it not have been strange to split into many parts what nature has wanted to be one, and to tear it into small pieces against the precepts of the science?' (see p. 232). Moreover, the use of short and perspicuous proofs as advocated in the dedicatory letter to *Apollonius Batavus* also served to keep track of the general structure of the treatise.

Snellius reduced the general problems of *Cutting off of a Ratio* and *Cutting off of an Area* to the problems of *Apollonius Batavus*, thus avoiding any redundancy in his treatment. In this way, he made *Cutting off* dependent on *Apollonius Batavus*, which is inconsistent with the order in which Pappus had described them and the order in which Snellius had published his reconstructions. This shows that Snellius took Pappus's summary as his point of departure, allowing himself to digress if his mathematical conscience prodded him to do so.

The solution to the triangle division problem as proposed in *Cutting off* illustrates this same preference for larger structures in which individual problems could be solved with as little effort as possible. Snellius claimed to follow the 'order of nature' when he solved the problem by considering it as a corollary to another problem. In this case, he even seemed to prefer a multi-layered solution (the solutions to some other problems had to be applied) to a more straightforward 'local' solution.

Ad 3. Snellius's strife for purity is most clearly seen in his demarcation of the realms of geometry and arithmetic, for which see the next section. It also entails precision in the use of concepts. All steps of proofs or theorems, whether they are meant for theory or for practice, must have an exact interpretation. Snellius's discussion of Heron's Theorem illustrates this best. Moreover, the *Fundamenta* as a whole is a 'purified' version of the Dutch original: Snellius modified much of the material in such a way that it fitted better into the Euclidean model.

Some exceptions to this exact mindset can be found in Snellius's commentary on the cyclic quadrilateral. His analysis of the construction of a cyclic quadrilateral shows that more freedom of means was allowed in an analysis than in a proof. Snellius's theorem that expresses the area of a cyclic quadrilateral in terms of its sides is a most notable exception: it was not proved, and was not pure either, because the product of four line segments cannot be interpreted in strictly geometrical terms. This suggests that in some cases Snellius felt his own choice for exactness was a hindrance to experimenting and exploring new tools.

Ad 4. The authority of ancient authors induced Snellius to choose certain approaches (see further section 8.6.3). He accepted some modifications of the classical tradition, as is e.g. shown by his support of Van Ceulen's approach in the dedicatory letter to the *Fundamenta*. Elsewhere, he defended his views by references to 'the classical writers', and something 'different from what those classical mathematicians did' was wrong. Because in many cases he did not give references, it is not always clear whom he had in mind. In some cases, he may not even have meant a specific authoritative text, referring to the ancients in general because he was convinced that they could or should have defended his point.

Ad 5. Snellius stressed time and again that mathematics should be useful, not obscure (see further section 8.6.1). The study of topics was only relevant if they were connected to other parts of the mathematical sciences, to other sciences or to practical matters. Although these connections were not always made explicit, and although the professed attention to usefulness was partly rhetorical, Snellius does indeed seem to have selected his material with this criterium in the back of his mind. It explains for instance why he did not delve to deeply into matters of which he did not see much relevance, such as conic sections or Vietean algebra. In the *Doctrina Triangulorum* for instance, he remarked that some mathematicians, among whom was Viète, had determined the sine of one minute 'more solidly and elegantly by means of analytical equations' than by continuous bisection and proportions. Snellius, however, presented his own trigonometrical approach as 'simpler and suitable for use'.[12]

The criteria for exactness are most visible and feasible in geometrical problem solving. Trigonometry without numbers would have been useless, and thus the separation of arithmetical and geometrical concepts is absent from *Doctrina Triangulorum*. Even Heron's Theorem is applied there to determine the area of two triangles without further comment.[13]

Snellius's pursuit of exactness is a sign of the emergence of a specialist class of professional mathematicians, who were not satisfied with results that seemed to be practical, yet had been formulated and proved sloppily, but wanted mathematics to be correct according to its own norms. However, in now way was pure mathematics an isolated field for him, but a part of the mathematical sciences that had both roles of queen and servant.

8.3.2 Arithmetical concepts in geometry

Snellius's attention to exact geometry is most clearly visible in his careful use of arithmetical concepts in geometry. Their use threatened the purity and preciseness of geometry, not only because they were alien, but also because the key

[12] 'Alii solidius elegantiusque per aequationes analyticas; [...] Nos nostra secuti sumus tanquam simpliciora et usui oportuna.' [Snellius, 1627, p. 17].

[13] [Snellius, 1627, pp. 92–93]. As this book was not published by Snellius himself, it cannot be said with certainty that the material is presented in exactly the way in which Snellius would have wanted it.

concept 'number' was not well defined. Moreover, a practical impediment to the use of numbers was that there were no widely accepted units of measure. These objections explain partly why the merging of geometry, arithmetic and algebra was a troublesome process in the seventeenth century.[14] Although the use of numbers in geometry was in practice unavoidable, this fusion was still far from acceptable in pure geometry, where loss of the clarity and irrefutability of classical geometry threatened.

Snellius wanted Heron's Theorem to be stated and proved without arithmetical concepts, because due to their lack of dimensions, their use led to uninterpretable (four-dimensional) geometrical volumes. The 'obscurity of the proof' was taken away by his own improved, strictly geometrical proof. Only after this had been given, the theorem could actually be used to express the area of a triangle in terms of the lengths of its sides, that is, as a number.

Snellius left numbers out of the Apollonius reconstructions, because they were not necessary and their use would not have been proper and historical (arithmetic had been absent from Greek pure geometry). Contrary to some other mathematicians, he also solved the triangle division problem by means of constructions without assigning numbers to the line segments, and he was critical about Van Ceulen's procedures and terminology when introducing the four elementary operations for segment-number pairs.

Snellius's defence of the use of numbers in geometry in his dedicatory letter to the *Fundamenta* seems to be strongly deviant from this number-averse opinion. However, this must be seen in the first place as the advocacy of Van Ceulen's practice, more than of Snellius's own. One of the reasons for his suspicion about their application is implied in his abhorrence of Euclid's Book X professed in the same letter: the calculation with numbers, notably with irrational numbers, could become exceedingly difficult, which distracted from the principal geometrical problem at stake. The background of this is that Snellius and his contemporaries did not calculate with indeterminate magnitudes, but with exemplary values.

When numbers facilitated the solution of problems, Snellius did not hesitate to use them; after all, usefulness was also a leading concern. However, his approximative solution of the quadrature of the circle involved numbers and trigonometry. This was an example of the popular large-scale reckoning. Another example of this is found in *Eratosthenes Batavus*, where he attached numbers which represented distances to line segments and made calculations with these without justifying this procedure. This was the common thing to do in mixed mathematics, because in practice it was always possible to find a rational approximation of the length of a line segment by measuring it, and the difficult

[14] Cp. [Bos, 2001a, pp. 131–132] for 'obstacles to the merging of arithmetic, geometry, algebra, and analysis'. Cp. section 5.2.3.

theory of incommensurability was irrelevant. In the *Cyclometricus*, he solved a problem 'mechanically', which involved the use of a proportional compass, and he used 314 : 100 as a practical approximation of the ratio of the circumference and the diameter of a circle. He was loose in the use of the terms 'equal' and 'approximately equal' there.[15] Snellius also approved of Vanden Brouck's invention of a procedure to find approximations of the values of cubic roots by a combination of geometrical and arithmetical means.

Moreover, Van Ceulen had stimulated Snellius's use of numbers in geometrical problem solving, which is visible in the *Fundamenta*. Snellius wrote for instance that when he had proposed a geometrical problem to Van Ceulen, the solutions of both of them (the lengths of the sides of a certain triangle, in numbers) had seemed to be different. Snellius had discovered that they were in fact equivalent and had warned Van Ceulen that he had been wrong in not expressing the numbers as simply as possible—again a token of Snellius's strife for simplicity. Van Ceulen, however, had not corrected his solution, according to Snellius either because he had found the calculation too boring, or because this task had slipped from his mind. A third possibility is that Van Ceulen did not agree with his idea of the simplest expression of the number. Snellius then showed the equivalency directly and by means of trigonometry.[16] He proposed and solved other geometrical problems in which numbers figured as well.[17]

Snellius also claimed to have found an instrument to determine 'very easily and accurately' roots of degree 2^n. He did not have occasion to describe it and therefore it is not clear whether the results were exact or approximative. These roots were in principle constructible by ruler and compass on the basis of a given line segment.[18]

In his commentary to another geometrical problem posed by him to Van Ceulen, Snellius criticized him for stating that the numbers that were to solve the problem were too complicated: Snellius denied this and assumed that Van Ceulen either had made a calculation mistake, or had chosen a less suitable construction. He continued:

> For however much the principle of every construction by means of geometrical lines is true and necessary, not everyone finds the method of numbers equally quick and easy.[19]

The opinion of 'everyone' may well be Snellius's own: numbers can sometimes behave very messily, especially when they are applied in a construction inade-

[15] [Snellius, 1621, pp. 62–64, 90].
[16] [van Ceulen, 1615b, p. 216].
[17] [van Ceulen, 1615b, pp. 222–223, 232–233].
[18] [van Ceulen, 1615b, p. 109].
[19] 'quamvis enim omnis geometricorum lineamentorum constructionis formula vera sit et necessaria, non tamen secundum omnes aeque est expedita ac facilis numerationis via.' [van Ceulen, 1615b, p. 235].

quate for that purpose, whereas a true geometrical construction is always valid and thus of a higher level. Because Van Ceulen, his former teacher, was such a virtuoso with numbers and had taught Snellius to solve similar problems, he could not voice his doubts too explicitly, but they must have been present.

Only a short section has to be devoted to algebra, because Snellius hardly used it. He added some algebra (expressed in cossist symbols) in Van Ceulen's *De Circulo*,[20] but none is used in his own *Cyclometricus*. It may not have appealed to him very much because it did not originate from the classical tradition. Moreover, algebra was only rarely applied in geometrical problem solving in his time. The triangle division problem illustrates nicely why algebra was not a popular tool: it was not evident at all how to translate the different givens into equations. Numerous different cases, depending on the form of the triangle and on the other givens, would have to be distinguished. Algebra could have helped to solve general questions about for instance existence and unicity of solutions, but these did not interest early modern geometers much. Moreover, the relation between manipulating an equation and finding a geometrical construction was not clear. Some mathematicians may even have seen the use of algebra as spoiling the fun of finding creative solutions.

Tellingly, although Snellius's multitudinous references to the work of 'the great chorus-leader Viète'[21] in different books prove that he knew and appreciated it, nowhere did he use Viète's new algebra.

8.4 Mixed mathematics and more

Snellius covered a large part of mixed mathematics. Especially astronomy occupied him frequently. As a young man, he had taught Ptolemy's *Almagest* and had met some of the famous astronomers of the period during his *peregrinatio academica*: Maestlin, Brahe and Kepler. Snellius discussed sunspots in an anonymous work. His main astronomical works were *Observationes Hassiacae* and *Descriptio Cometae*. They were both dedicated to Maurice of Hessen and the second one was even explicitly ordered by him. Thus, they were elements of a patronage relationship, in which the position of court astronomer may have been at stake.

Astronomical issues also play a role, albeit less central, in several other projects of Snellius. The purpose of *Eratosthenes Batavus* was to establish the circumference of the earth, the value of which he used for a calculation in *De-*

[20] [van Ceulen, 1619, pp. 49–50].

[21] 'Choragus Magnus ille Vieta', [Snellius, 1627, p. 17]. See other examples in the case studies and e.g. [Snellius, 1617b, pp. 116, 212], [van Ceulen, 1615b, pp. 191–194], [Lansbergius, 1628, fol. $*4^v$], [Snellius, 1627, pp. 35, 73].

scriptio Cometae. His interest in astronomy is reflected in his correspondence, notably in that with Rosendalius, Bainbridge and Gassendi. With Rosendalius he discussed the newly-invented telescopes, Bainbridge complimented him on his astronomical work and Gassendi provided Snellius with astronomical data in exchange for his involvement in the construction of two instruments. Other mixed disciplines discussed by Snellius were surveying (the development of a triangulation network in *Eratosthenes Batavus*), optics (not published) and navigation (in *Tiphys Batavus*).

All in all, Snellius's mixed mathematics has the character of a hotchpotch: it contains qualitative and quantitative aspects; some observations, some calculations, some more technical parts and other more 'philosophical', which included a discussion of astrology. The topics are much interwoven. The outcomes of the *Eratosthenes Batavus*-project, for instance, could be used in surveying, astronomy and navigation. Snellius's studies of the behaviour of light were relevant for optics, but also helped him to interpret astronomical observations, and his astronomical studies were indirectly relevant for the determination of position at sea. Trigonometry was an auxiliary science for most of these topics.

Snellius's mixed mathematics offers the best illustrations of his 'combination strategy'. He borrowed much material from books. His framework was classical, mainly Aristotelian in astronomy (although Snellius had also much against Aristotle). He also knew some results from Arabic scholarship and some newer developments. His broad knowledge and critical lecture are for instance testified by the historical survey in *Eratosthenes Batavus*, his study of the nature of comets in *Descriptio Cometae* and his notes to the *Optica* of Risnerus. His own and other newer insights made him sometimes dissent from the ancients, e.g. about the character of comets, or the real existence of planetary orbs.[22] However, he did not take a stand on metaphysical statements when it was not necessary for his argumentation (e.g. in the cases of heliocentricity and sunspots), not being inclined to speculation. More than other comet researchers, he based his findings on observations and mathematical reasoning, because qualitative argumentations 'cannot force such an unwilling and reluctant person to approve them, as something near the truth, or take away every doubt. Therefore, we will have to derive our arguments with respect to this matter from Geometry.' (see p. 164).

The input material of mixed mathematics was much more diverse than in pure mathematics, where some books, a scrap of paper, a ruler and a compass sufficed. This less bookish side of Snellius is more hidden because of the nature of the source material—mainly books. He used the different ways to find or generate this input extensively and creatively and was thus able to make a number of contributions by input of his 'reason and observation' (see p. 165). On

[22]See section 4.4; [Snellius, 1608b, fol. $A5^r$].

earth, in the air and at home, he took his own measurements and observations, for which he had collected a number of good instruments. Gassendi's letters show that he was recognized as a specialist in this field. Yet although he owned a telescope, no observations done with such an instrument are known.

He undertook a complicated expedition to collect data for the *Eratosthenes Batavus*: he had to travel to a number of places, some of which were situated in hostile territory. He always kept his eyes open to observe phenomena in the open air, such as rainbows, and he did some experiments, e.g. to determine the weight of a cubic Rhenish foot. In the winter of 1622, he took advantage of the coldness in two ways: he experimented with the behaviour of light in relation to ice, and he measured a new base line for his triangulation network outside Leiden. He also involved his students in his triangulation measurements. This combination of theory and practical aspects reflects Simon Stevin's adage of the importance of both 'spiegheling en daet' (theoretical investigation and practice).

Snellius also collected facts from other sources like observation reports and letters. Thanks to his network, he had access to unpublished sources such as Tycho Brahe's observations. He had data sent to him as well, e.g. some angular distances by Isaac Beeckman, and Houtman told Snellius about a strange phenomenon that he had observed. Snellius devoted much energy to all necessary calculations. He may have made some mistakes in the huge amount of calculations in *Eratosthenes Batavus*, but the mathematics underlying them was correct.

Snellius went on some excursions outside the mathematical sciences. His development of an (again) exact measuring device for the mass of water shows his engineering side. Moreover, he studied several quantitative topics from Antiquity, which resulted in two books on ancient money, one edited and one written by him, and discussions with Cunaeus about the correct interpretation of the Jubilee year and the unit of measure 'arura'. The auction catalogue of his library suggests that he owned books that covered many more fields than mathematics. His *Theses Philosophicae* did not only discuss many mathematical sciences, but also show his knowledge of the trivial arts (grammar, rhetoric, dialectic) and ethics.

8.4.1 Comparison of pure and mixed mathematics

Both in pure and mixed mathematics, Snellius selected popular topics. In both branches, his concept of exactness is visible, but less clearly in mixed mathematics, where the demands of the goal and material left less space for considerations about demarcation, purity and structure. Yet his pursuit of exactness is visible in his critical scrutiny of older work, the precise determination of his unit of length in *Eratosthenes Batavus*, his lack of speculation about metaphysics but instead a restriction to the domain about which he was knowledgeable. The mixed discussions lack a certain mathematical subtlety. In his pure works, Snellius wanted mathematics to be correct for its own sake, not for a better application, whereas geometry is only used as a tool in mixed mathematics. His mixed pursuits may have kept him off 'obscure' pure geometry that was too far removed from applications. This practical attitude is most clearly seen in the mixture of arithmetic and geometry in mixed mathematics, e.g. in the *Eratosthenes Batavus*. He used plane instead of spherical trigonometry for the surface of the earth, which indeed yielded a reasonable approximation for his purpose.

Pappus's classification of geometrical problems also played a role in mixed mathematics. In the manuscript which contains the outline of Snellius's optical treatise, he wrote that the so-called curve of refraction, which describes the series of apparent positions of an object on a fixed distance from the interface between two media, had a conchoid shape. He remarked that the curve belonging to the denser medium had a shape different from every conic section and wondered whether the same was true for the curve of the rarer medium. Another problem related to refraction was also 'line-like' according to him.[23]

8.5 The inner and outer circle

Some of the people with whom Snellius was in contact were crucial in shaping his career and his work. The role of others was more at a distance, yet as (potential) readers they also gave him direction. Snellius's work was known among his peer group of well-educated scholars. He also taught a number of students, yet he did not have the chance to gather a large following, because almost all his students only met him on their way through the arts curriculum on to the higher faculties. The relation of all these persons to his work will be reviewed in this section.

8.5.1 Patronage and network

As almost all early modern scientists, Snellius needed patrons. Their function for him was threefold: they gave subsidies for his publications (e.g. the States General), they helped to improve his career prospects (e.g. Rosendalius) and they enhanced his status by the glory that reflected on him (e.g. Maurice of Hessen). The clearest evidence of patronage is found in the dedicatory letters of Snellius's publications. In most cases the influence that the patrons had on the contents of the books seems to have been small. Sometimes they may have suggested the topic. It is likely that in other cases Snellius only selected the dedicatees after having decided on the content of the books, either choosing a

[23][de Waard, 1935, pp. 67, 69], [Hentschel, 2001, pp. 306–308]. For the construction of the conchoid by means of an instrument consisting of three rulers, one of which moving, see [Bos, 2001a, p. 30].

dedicatee suitable to the contents or someone of whom he expected benefits. The clearest example of this last category is the lawyer Rosendalius, to whom the *Fundamenta* was dedicated. In exchange, he was asked to assist Snellius in obtaining a promotion in Leiden.

Snellius's patronage network was typically Dutch. In the Dutch Republic, the power was divided between a number of political bodies, involving a rather large group of persons. Snellius covered a substantial part of them by the dedications of his books: the Leiden curators, the States General, the States of Holland and Count Maurice of Nassau. He also showed his value to them in more practical ways, for instance by assisting the States General in judging methods for the determination of longitude at sea. In this period, it seems that the political support of scholarship, also of the kind of which the country did not profit directly, was considerable.

Snellius also dedicated books to private persons with whom he was well acquainted and with whom he shared scholarly interests: Adrianus Romanus, Rudolph Snellius, Simon Stevin, Hugo Grotius and Aemilius Rosendalius. With the exception of Rosendalius, these persons probably could not offer any financial advantages, but they could help Snellius to become an inhabitant of a universe of learning and consequence. Rosendalius offered the happy combination of genuine interest in Snellius's activities, a family connection and a high rank in society, so that Snellius's requests to him to help him rise in the academic hierarchy were not in vain. Moreover, Rosendalius offered him material help by lending his own coin collection to Snellius.

Almost all Snellius's professional activities took place in the Dutch Republic, but thanks to the German period in his father's career, he had one patron abroad. This was Landgrave Maurice of Hessen, a patron of true distinction. He favoured Rudolph Snellius because of their shared Ramist ideology, and invited Willebrord Snellius to continue the Kassel astronomical tradition. In this case we have clear evidence that a patron induced Snellius to write a book (the treatise on the comet of 1618) and in this way influenced the contents of his work.

Maurice of Hessen was not the only princely patron of the Snellii, who were also connected to Maurice of Nassau. A comparison between these patronage relationships can hardly be made, because the character of the second is virtually unknown. Maurice of Nassau was an admirer of mathematics, but because he had Simon Stevin as his personal adviser in this domain, there was less room left for Willebrord Snellius and it was therefore attractive for Snellius to have a foreign prince as a patron as well. A connection to any of the Maurices, both adherents of the orthodox current in the Reformed church, could have proved useful in the period around 1618 when the Remonstrants were ruled out in the Dutch Republic.

Snellius had a network of mathematicians with whom he corresponded and shared scientific results. Evidence shows that it contained Lansbergen, Anderson, Vanden Brouck, Bainbridge, Gassendi and Beeckman. Because only a small number of letters by Snellius is still in existence, it is very likely that his network was in fact much larger. The members of the network sometimes exchanged material: e.g. Gassendi supplied data, and Snellius had an instrument made for Gassendi. Moreover, Gassendi served as his patron (his own term) by interceding with Galileo on Snellius's behalf. Snellius's connections to humanists in and around Leiden were so good that they wrote contributions to the preliminary matter of his books and for his wedding. Hence we have poems by Cunaeus, Scriverius, Scaliger, Vossius, Letting and Vulcanius. His only Dutch publication was adorned by no less than Joost van den Vondel.

Among Snellius's patrons and the members of his network were many men of high rank, which reflects his high status as a scholar. Snellius enlarged the profit that he could draw from such friends by publicly claiming their good relationships in several of his books. In the *Eratosthenes Batavus* for example, he devoted $1\frac{1}{2}$ pages to the famous men who were born, had lived or died on or near the parallel of Mainz and Prague; these included his 'very dear friend' Adrianus Romanus, and Joannes Praetorius, Maestlin and Kepler, with whom 'I enjoyed an intimate acquaintance when they were still alive, although I was only a very young man'.[24] This excursion bears no clear relation to the main text.

To flatter his patrons and to enhance his own status, Snellius used the tool available to men of learning: the highly developed rhetoric of humanist scholarship. This is mainly visible in the dedicatory letters. In this way, Snellius 'fashioned himself' as a worthy member of both the *Respublica Litterarum* and the Dutch Republic. Snellius was a successful player of the patronage game: he first became an extraordinary professor, then an *ordinarius*, received regular increases in salary and had a good reputation. Or had he hoped for more, like becoming a court astronomer?

The role of non-mathematical factors, notably patronism, in the genesis of Snellius's works is exemplified by the publication history of the *Fundamenta*. We have seen Snellius struggling for recognition, dealing with a reluctant widow, impatient publishers and a thrifty university, and feeling obliged to keep the memory of an old friend alive, although he did not have enough time for the working load. These difficult circumstances hampered his efforts both to make a Latin edition to his own satisfaction and to use the book as a career tool. This is the human side of Snellius, and of mathematics, of which we often do not know

[24]'Adrianum Romanum [...] dum viveret nobis amicissimum.' [Snellius, 1617b, p. 228]. '[...] dum viverent familiaritate, etsi adolescentulus, usus essem.' [Snellius, 1617b, pp. 229–230]. Snellius in fact wrote 'Ioachimus Praetorius', but this must be a mistake, because he evidently referred to his acquaintance Joannes Praetorius, who was born in Joachimstal, which Snellius mentions here as Praetorius's place of birth. No mathematician called Joachim Praetorius is known. Moreover, see for the same error p. 279.

much because it is hidden under the surface of mathematical books.

Although an understanding of the mechanisms of patronage and networking is necessary to explain a part of the contents and style of Snellius's work and his motivation, they certainly cannot explain all. Many of the mathematical intricacies cannot have been appreciated by the patrons and they must be explained by other factors, especially by the demands of exact mathematics.

8.5.2 Audiences

The persons mentioned in the previous section constituted an important part of the audience of Snellius's works. Because he restricted himself to Latin in his publications (with the exception of the *Meetkonst*), his audience was limited to men of learning. This is remarkable, because many of his results were also of potential interest to several groups of practitioners. For example his triangulation method and the distances between places that he had calculated could have helped surveyors, but they were unaware of this for a long time. Yet sometimes the results of Snellius's works reached practitioners through other ways than his own books. Navigators learned for instance about the Snellius mile through its discussion in a book by Jarichs, who had personally heard from Snellius about his endeavours. Other practically inclined mathematicians, such as Stevin, Van Ceulen and Metius, did make their results accessible to people who only knew the vernacular. Snellius must have made this choice for Latin, and the scholarly presentation of his work that belonged to that choice, consciously, to show that his level was above that of the practitioners.

Snellius did not intend the core parts of his books for the same audiences as the preliminary matter. Whereas the core was mainly meant for specialized mathematicians, the aim of the titles, dedicatory letters, prefaces and laudatory poems was to impress patrons and to explain Snellius's ideas about the nature and use of mathematics and his own contribution to the field. His series of *Batavus*-books must have appealed to the patriotic feelings of many of his Dutch readers; they proudly expressed how the Dutchman Snellius had emulated the works of the classical heroes Apollonius, Eratosthenes, Tiphys and Menelaos.

The audience of the core parts was probably very small, because Snellius's 'peer-group' of specialized mathematicians from all over Europe was small. The audience of the less technical parts could also have consisted of scholars without a specific mathematical inclination. Some friction between this restricted accessibility and Snellius's claim to usefulness may have been felt by his contemporaries. Moreover, to make his works as understandable and attractive as possible for the elite, Snellius had to be careful not to get entangled in too subtle issues, which would be incomprehensible for these people, making them less convinced of the usefulness of mathematics. After all, they determined his academic position and not his colleagues abroad. Kepler's angry reaction to Snellius's Book X-statements shows that it could be difficult to please both groups. Both the small size of the expert audience and the need to take the divergent wishes of potential readers into account may partly explain why Snellius's success was considerable, but not overwhelming. This is also expressed by Vossius, who complained that Snellius 'was not appraised at his true value by the people around him' (see the quotation at the beginning of this chapter).

8.6 Humanist aspects of Snellius's mathematics

Snellius's own description of himself as

> One of those who are led by admiration for the inventions of the ancients or find delight in the investigation of new things,[25]

that is, a scholar both open to the rich treasuries which the classical authors had to offer and to the new discoveries of Snellius's own age, makes it evident that he can be considered as a humanist scholar.[26] In his humanism, the scholarly side is dominant over the educational side, which is common for humanism of the period.

Several characteristics of Snellius's work that further corroborate the view on Snellius as a humanist and the goal of this way of doing mathematics will be discussed in this section. The treatment here is not meant to be exhaustive, but to point the way to a sensible approach of Snellius's mathematics, which could be applied in further research.

8.6.1 The rhetoric of mathematics

A good humanist scholar was able to persuade the audience of his message by his mastery of rhetorical means. In Snellius's case, these skills are mainly visible in his dedicatory letters, because the mathematical cores of his works had a more technical style. He was indeed able to convey a subtle message, as the dedication letter of the *Fundamenta* shows.

In this section, the focus will be on Snellius's defence of the usefulness of mathematics as found in most of his dedicatory letters. The eulogy of mathematics was a common theme in orations, prefaces and dedicatory letters of mathematicians and other scholars, which had counterparts for other sciences. Snellius's arguments are not new, which was equally the case for other authors in this genre: their purpose was not to be original, but to present the well-known arguments inventively and phrase them attractively. The main stock arguments

[25]'unus aliquis ex eorum numero, qui ducuntur admiratione earum rerum, quae sunt ab antiquis repertae, vel delectantur investigatione novarum [...]', [Snellius, 1607b, p. 4].

[26]Cp. for this topic [de Wreede, 2006].

for the worthiness of the mathematical sciences were that they prepared the mind for the study of philosophy and that they yielded many useful applications.[27] These functions may seem to be opposite, yet they complemented each other in the view of Snellius and others.

The first main argument for the usefulness of mathematics is that it helps to elevate the soul, by training it to contemplate higher entities. This traditional argument originated from Plato's *Republic*, and Snellius indeed quoted him in the dedicatory letter to the *Fundamenta*: 'the human soul "is purified and rekindled" by the mathematical sciences' (see p. 189). In the *Meetkonst*, the translator Houtman, a clergyman, gave a related, but this time explicitly Christian, argument for the use of mathematics: God had 'ordered all things in measure, number, and weight'.[28] Besides, he argued that mathematics teaches children to think reasonably and to understand that God had organized all affairs according to his laws, which should help them avoid all kinds of temptations.[29]

In the same book, Snellius claimed that mathematics was the 'foster mother' of all sciences.[30] In *Cutting off*, Snellius also motivated the study of mathematics by its role in other sciences, after he had stressed that usefulness should be the leading concern for scholars:

> I say use: for I have not taken this burden on my shoulders because I longed to know all things and of what kind they are, or because they all showed a kind of riddle—for that is something for curious people—but because the contemplation of greater things led me to the perfection of science, for usefulness and results that could thence spread to other sciences were my only aim in this.[31]

[27]Cp. [Imhausen and Remmert, 2006, p. 75]; see the same article for an edition and translation of a typical representative of the genre by Martinus Hortensius.

[28]'om dat GODT alles in ghetal, ghewicht, en mate geschapen heeft, also de Wyseman zeydt.' [Ramus, 1622, p. *2^r]. The reference is to the Book of Wisdom, 11:21.

[29]'op dat door haer onwedersprekelijcke sekerheydt het oordeel der Leerlinghen alsoo van joncx op ghestelt en gheformeert mochte werden, dat zy niet licht yets, sonder, ofte tegens reden souden leeren ghelooven. En diens volgende haer tot ydelheydt, afgoderie der schepselen, en andere ongherijmtheydt niet lichtelijck souden begeven; ofte door bedroch (hoe geestich en aerdich die oock mocht opghepronckt zijn) haer daer toe laten verleyden. Om dat zy hier uyt leeren souden dat alle dinck met natuerlijcke ordre, nae de wetten, die zy van GODT ontfanghen hebben, hier toe moeten gaen, en werden volbracht.' [Ramus, 1622, pp. *2^r–*2^v].

[30]'Nademael dat nu, door een bysondere ghenade des Heeren, de kloecke verstanden in dese onse Nederlanden, sich begeven om in alle fraye konsten en wetenschappen haer selven te oeffenen, alsoo dat zy de Natien en Volckeren die voor de kloeckste vermaert zijn weynich of niet en wijcken; en de * Wiskonsten [* Mathemata [sic] in margin] daer de Voester-moeder toe zijn [...]', [Ramus, 1622, p. *4^r].

[31]'usu inquam: non enim in hanc curam incubui, quod omnia scire cuiusque modi essent, aut quia problematis speciem aliquam prae se ferrent, cuperem, id enim curiosorum est: sed quod ducerer maiorum rerum contemplatione, ad artis perfectionem, usus enim et fructus, qui inde in alias artes dimanare posset, mihi hic solummodo propositus fuit. [...] nam ne ipsa quidem sapientia, quae ars vivendi putanda est, expeteretur, si nihil efficeret, ait orator.'

In *Apollonius Batavus*, he complained how

> these shady scholars in their shady scholastic cells separate the use, which is the juice and blood, from science as though it were something very alien.[32]

This anti-scholastic rhetoric was common among humanists. Snellius may not have had specific scholars in mind, but merely have made up some adversaries to advertise his own approach.

The second main argument for the usefulness of mathematics was that it had many applications, both in other sciences and in daily life. Examples are taken from, among other things, astronomy, optics, the production of weapons and ships and perspective. Snellius adjusted his examples to the dedicatee: most attention was paid to applications of pure and mixed mathematics in warfare in the letters for Maurice of Nassau in *Apollonius Batavus* and *Hypomnemata Mathematica*, whereas the lawyer Rosendalius was regaled with legal examples. Snellius's praise of Maurice's active involvement in mathematics is similar to Ramus's compliments for several German princes who supported mathematics.[33] Moreover, the stress of *use*, the anti-scholasticism and the use of several other typical terms make Ramus's influence on these dedicatory letters very plausible.

The dedication letter for Stevin in *Cutting off* is also adjusted to its addressee: Snellius told Stevin how harmful the abhorrence of applications of some scholars was. This led to the unhappy circumstance that theory was taught by other people than practice, although the same geometrical knowledge formed the basis of both.[34] To reinforce his point, Snellius used some literary means here, like an *s*-alliteration. After his rejection of the wrong way of doing mathematics, he praised Archimedes elaborately, who embodied the ideal of the practical use of mathematics, with his war machines trying to repulse the Romans who attacked Sicily. Stevin was then praised similarly because he had 'proved that practice is improved by science, and science by practice' both in words and in deeds.[35]

[Snellius, 1607b, p. 5].

[32]'[...] umbratici isti Philosophi in scholasticis umbraculis [...] usum, hoc est succum et sanguinem, ab arte, tanquam alienissimum, seiungunt.' [Snellius, 1608a, p. 4].

[33][Ramus, 1569, pp. 67 ff.]; cp. 146.

[34]'[...] ii, qui harum artium studiis liberalissimis sunt doctrinisque versati, minimam partem artis affectui, aut usui dederint [...] Nam opinio ea est, non modo vulgi, sed etiam hominum non leviter eruditorum, id hic maxime excellere, quod longissime sit ab imperitorum intelligentia sensuque disiunctum. [...] Hinc dissidium illud extitit, quasi animi et corporis, absurdum sane, et inutile, et reprehendendum, ut alii nos theoriam, alii usum docerent. cum tamen Geometria quocunque incedat eodem sit instructu, ornatuque comitata, sive coeli cardines describat, sideribusque fixis ac vagis domicilia ac sedes designet; seu terrae regiones emetiatur in solido solo, singulisque suos assignet limites, seu in fluctuante salo maria permetiatur, portusque ac sedes optatas ostendat, sive castra muniat, seu oppugnet [...]', [Snellius, 1607b, pp. 15–16].

[35]'Tu, tu inquam usum arte, artem usu firmari non magis scriptis, quam factis probasti.' [Snellius, 1607b, p. 16].

This glowing speech for a science in which theory and practice collaborate fruitfully reflects an ideal, which is not necessarily visible in all mathematical works. Indeed, it is not evident how the Apollonius reconstructions themselves are to contribute to mathematical practice.

In the dedicatory letter to the *Meetkonst*, Houtman gave a long series of applications of mathematics, some of which were rather farfetched, such as 'the transposition of houses and towers' and 'the counting of the uncountable'.[36]

In conclusion, it can be remarked that Snellius devoted ample space to the issue of the use of mathematics, yet in many cases he gave generic arguments, which did not specifically apply to the work which they preceded. This indicates that he tried to 'sell' his works by means of elaborate claims of their usefulness, which were not necessarily relevant for the core. This aspiration to use may indeed explain only part of his motivation. For instance Snellius's claim that the *Eratosthenes Batavus*-enterprise was useful for the longitude at sea problem did not return in the main matter of the book and therefore mainly seems to have served as an advertisement for a project that Snellius considered as scientifically challenging and rewarding in the first place.

Although most visible in the preliminary matter, the rhetorical presentation was not altogether absent from the main matter either. An example of this is Snellius's attempt to evoke the sympathy of the reader by telling the tragic story of the destruction of Oudewater. His description is phrased to resemble a spontaneous emotional outburst, but must in fact have been well designed, and Snellius carefully envisaged an extension of this section in the second edition.[37]

8.6.2 Languages: Latin, Greek, Dutch

In his written communication, Snellius served himself almost exclusively of Latin, the international language of learning. Even when corresponding with fellow Dutchmen such as Rosendalius and Scriverius, he used Latin. Thus, his public *persona* was inseparable of his scholarly identity. Tellingly, even in the local

[36]'Dese wetenschappen dan, zijn oock ten anderen by Koningen, Princen en Republijcken in eeren altijdts gehouden, om 't groot gebruyck der selven, 't welck in 's menschen leven soo noodwendich is, datter niet en is 't welck sonder dese wetenschappen can bestaen. want selfs de tyden van 't jaer, hare daghen, maenden, uren, en alle veranderinghe van dien; de ghestaltenisse en beweginghe des hemels, werdt hier door tot over veel hondert jaren bekent en voorzeydt: de verscheydenheydt des aertrijcx in ghelegentheyt tot koude en hitte; het reysen over de wilde Zee; het bewegen van onbewegelijcke swaerte; het versetten van huysen en torens; het vervoeren van vrachten, het bewegen der schepen; haer werckinghe in tyden van oorlogh; haer gebruyck in tyden van vrede te water en te lande, by Schipper en Bouwman; het ghesicht der onsichtbare; het tellen der ontelbare, en andere ontallicke profyten meer hier uyt spruytende, hebben dese konsten niet alleen, gelijck gezeydt is, in waerde ghehouden, maer oock mede begeerte en lust in veel singuliere verstanden verweckt.' [Ramus, 1622, p. *2^v].

[37][Snellius, 1617c, pp. 178–179].

Leiden archives his name is often given as 'Snellius' and not as 'Snel van Royen'.[38]

His mastery of Latin on a high level, as shown in his letters, dedicatory letters and the discursive parts of his books, is an important indication of the humanist character of his work. This mastery was not restricted to knowledge of the language: he must also have known the works of many classical authors, among whom were not only scholars (see e.g. in the first part of *Eratosthenes Batavus*), but also literary authors. As an example of Snellius's familiarity with the classics, consider a section from the dedicatory letter to *Apollonius Batavus* which is very dense with classical references. First Snellius compared himself to Aesculapius, the Greek god of medicine, because he revived the 'patient', that is, the almost lost Apollonian treatise, and next he compared Apollonius's mind to the Nile, which fertilized the fields when overflowing. Snellius then quoted a modified half line of the Roman poet Lucanus and borrowed the wordings of Horace, Ovid, Petronius and Cicero to express his opinion about the value of Apollonius's work and their loss.[39] This network of quotations contains an element of showing off, of proving to master the game of humanist scholars to phrase one own's thoughts elegantly with the help of appropriate quotations. He may either have collected these sentences during his own lecture of the classical works, or he may have used a printed quotation book (a 'commonplace book') to find some relevant phrases.

On other occasions, classical citations were given for their content instead of their formulation. Snellius's reference to Aristotle in the dedicatory letter to the *Fundamenta* shows that this use was sometimes very creative: the citation was interpreted in a way that could not be justified when considering the original context.

The linguistic side of Snellius's mathematics also shows that his work can not be considered as mainstream mathematics. In the technical mathematical parts, the role of a well-phrased literary formulation, an essential ingredient of normal humanist texts, was minimal: not language had to convince the reader of the truth of what he read, but mathematical constructions or observations. Snellius

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[38] Therefore, the habit of much English-language literature to call Snellius 'Snel(l)' does not do justice to Snellius's own preferred identity.

[39] 'De cuius tanta tamque multiplici eruditione apud Pappum utilissimorum et subtilissimorum librorum argumenta duntaxat extant,

— *ut magni nominis umbrae.*

Qui ut ad posteritatis memoriam traderentur non merebantur solum; sed etiam universae mathesios intererat: atqui haec omnia quia optima erant damnosa dies et edax tempus absumpsere. Quamobrem ite nunc mortales et magnis cogitationibus pectora vestra implete, Non cum vitae tempore esse demittendam commemorationem nominis nostri, sed cum omni posteritate ad adaequandam.' [Snellius, 1608a, p. 11].

Lucanus, *Bellum Civile* 1.135 has 'Stat magni nominis umbra'; 'damnosa dies' is from Horace, *Carmina* 3.6.45; 'edax tempus' from Ovid, *Met.* 15.234 and *Epist. ex Ponto* 4.10.7. 'Ite ...implete' ('vestra' supplied by Snellius) is from Petronius, *Satyrica* 115.14, the rest of the sentence from Cicero, *Pro Archia* 29 (without 'ad').

was aware of this special position of mathematics, as is for instance shown by his statement that 'though the mathematician is, when proving, well provided with a variety of facts and cases, he is more sparing with words' (see p. 62).

Not only did Snellius have a thorough command of Latin, he also mastered Greek, which he showed frequently by spicing up his Latin text with Greek interspersions. He even reported about his new baby to Rosendalius in Greek. His emendations of some sentences of Pappus in the Apollonius-reconstructions are on a more scholarly level. Snellius included quotes from the Greek Pappus text elsewhere in these treatises.

Even though Snellius preferred Latin for his scholarly publications, the *Meetkonst* shows that he was aware of the potential role of the vernacular in education. The main topic which he addressed in his letter to the reader is the defence of the importance of the use of Dutch words for mathematical terms. This shows his interest in the *Questione della Lingua*, the question of the suitability of the vernacular for various literary genres, a matter which had also occupied Ramus, who had advocated the use of French in rhetoric.[40] He extended the practicability of Dutch by the creation and use of technical terms. His argument that Dutch is eminently suitable for the coining of new words because the language is rich and compositions of words can be made easily betrays the influence of Simon Stevin.[41]

There is no indication that Snellius mastered other living or dead languages, for which he had no real need. Nevertheless, he apologized to Gassendi for not writing into French to him.

8.6.3 Authority and originality

As already argued in section 8.2, Snellius's work was largely inspired by and based on problems and methods originating from classical scholarship, a truly humanist characteristic. His wish to transmit, enlarge and correct ancient knowledge, without abolishing the foundations, was a key element of his humanist conception of mathematics. His concern for usefulness for his own time, either in a more scholarly sense or more practical, is also humanist. Yet thanks to his combination strategy, Snellius could incorporate more than the benefits of the

[40] [Meerhoff, 1986, pp. 34–40], [Meerhoff, 2001b, pp. 370–371].

[41] 'Het en moet uw niet vreempt geven dat de Latijnsche kunstwoorden, hier in't duyts ghebruyckt en overgheset, wat vreemt luyden; want selfs de Latynen (in welcker tale dese konsten beschreven worden) soodanighe woorden, by den ouden onghehoort, versieren en maken, jae door haer onbequaemheydt en ghebreck van de Griecken ontleenen moeten. Maer alsoo onse tael een van de rijckste is, en tot maken en t'samen setten van nieuwe woorden seer bequaem; hebbe ick (daer in volgende de voetstappen van andere onnut en onnodigh gheacht van dese nabueren te gaen leenen, 't gene wy aen ons selven hadden. want het seer ongherijmt is den Nederlantschen yveraer beyde met woorden en daet te bekommeren; hebbe niet te min de Latijnsche en Grieckse woorden aen de kant gestelt, om te beter dese woorden in't ghebruyck te brenghen.' [Ramus, 1622, pp. 7–8].

humanist approach in his scholarship, gaining also from other new developments in science and from his own observations. This could also be considered as the *aemulatio*-part of his work: he surpassed the classical examples there. His observations of nature may be an expression of a belief in the then popular Book of Nature-philosophy: the idea that God's plan could be derived from a careful study of all small and great natural phenomena.

Some compliments of contemporaries show that they thought that the ingredients of Snellius's work complemented each other: Gassendi explained how much he liked those few (among whom was Snellius) 'who, sometimes setting foot outside their tiny studies, contemplate the majesty itself of nature with open face', disdaining bookish scholars who neglected the real matter,[42] whereas Boulliau claimed that if Snellius had not died untimely, he would both have managed to revive lost astronomical knowledge and add new findings to that.[43]

Snellius usually devoted much attention to the history of old problems before embarking on the development of his own solution. In the *Cyclometricus* for instance, he wrote a long preface on the history of the quadrature of the circle, focusing on Greek contributions, but also mentioning the work of Adrianus Romanus, Franciscus Viète and Ludolph van Ceulen.[44] In the book proper, he compared his own achievements in this field with those of Archimedes, and he proudly let the reader know that his method yielded more digits of π when using the same regular polygon as Archimedes.[45] The *Tiphys Batavus* contains a long preface about navigation in Antiquity, at the end of which the recent history of the topic is also briefly discussed, and in *Eratosthenes Batavus* even more space is devoted to the history of the topic of the book.

The input of these different elements makes his style eclectic: sometimes, he let the authority of the ancients prevail while on other occasions he used more recent material. It is not possible to discern a general, rigid system underlying his choices for either older or newer authorities, or his own input. Snellius must have granted himself this freedom consciously: thus, he could follow interesting tracks, picking out the elements which he liked without being unnecessarily confined. He for instance did not make known publicly whether he believed in heliocentricity. For a mathematician, such a system-free way of operating may have been more easily accepted than for a natural philosopher, let alone a theologian.

A telling example of his varying attitude is found in his use of Aristotle. Although Snellius criticized him vehemently in *Descriptio Cometae*, his own

[42]'[...] umbraticos istos eruditos [...] magnifacere non possim, quod veritatem, et scientiam rerum venentur dumtaxat ex libris, res ipsas prorsus negligant: [...] qui pedem interdum ducentes ex Musaeolis, ipsam Naturae maiestatem aperta facie contemplantur.' [Gassendi, 1964, p. 3].

[43]See quotation on p. 291.

[44][Snellius, 1621, fol. $*4^v$–$**6^v$].

[45][Snellius, 1621, p. 52].

viewpoints on cometology were in fact not far removed from Aristotle's. Aristotle was quoted with consent in the dedicatory letter to the *Fundamenta*, yet Snellius also supported Gassendi's anti-Aristotelianism. These inconsistencies can be understood by considering the function of the statements in each of their contexts. Snellius was not pro or contra Aristotle in an absolute sense, but some of his viewpoints were based on Aristotle, whereas others, originating elsewhere, could attractively be presented by contrasting them against the great master's. This hypercritical approach of ancient authorities, and especially of Aristotle, was much advertised by Ramus. Other scholars of the period showed a similar ambiguity towards Aristotle.[46] Snellius's attack on Euclid's *Elements* X shows the same attitude towards an author otherwise appreciated by him. The different reactions of early modern mathematicians to this book also make clear that a study of the same material could lead to completely different viewpoints.

Snellius did not prefer old or new scholarship principally, but rather he reached his viewpoint through a careful consideration of all relevant ingredients and once he had reached it, he defended it by seeking support in the works of other authors or evidence that was closest to his viewpoint. In this way, he mentioned the good practice of the ancients in the Heron case, when he wanted to argue that all modern authors were wrong, yet novelty was a positive criterium when he recommended his own inventions.[47] And although he praised the 'exactness, clearness and brevity' in the proofs of the ancients (see p. 59), he allowed himself to improve the structure and proofs of the Apollonius treatises, because he considered the order in which Pappus had presented them as wrong. And while clearly not satisfied with what had been achieved by Pappus, his point of departure remained the work of the ancients: 'but only that material seems to deserve recommendation to me, in which the industry of the ancients was present and at work',[48] and he seems to have followed Pappus in his preference for the solution of geometrical problems by means of circles to that by straight lines in his last solution of the triangle division problem. On another occasion, he used data from mariners to correct Ptolemy's determination of the length (in degrees) of the Mediterrean Sea.[49]

The proportion between the elements classical authors – modern authors – observations – calculations – own reasonings did not only depend on Snellius's taste, but also on the genre. He for instance filled the margins of his copy of Risnerus's *Optica* with classical quotations about the nature of light, because this fitted in a work with a strongly qualitative character. The quotations stemmed

[46]'The great scientific revolutionaries rejected Aristotle; but it was their academic grounding in Aristotle that gave them the ability to do it.' [Porter, 1996, p. 551].

[47]See e.g. 'epichirema nostrum, non minus utilitate sua, quam inventi novitate commendatum [...]', [Snellius, 1621, fol. $**6^v$].

[48]'sed ea demum ad commendationem digna est mihi visa materia, in qua veterum industria esset versata et exercita.' [Snellius, 1608a, p. 10].

[49][Gassendi, 1964, p. 393].

from literary and scholarly authors. He added some of his own observations and ideas. This volume gives us some insight into a Snellian work in progress. The proportion between the input elements may have been different in his own completed work. In books in which geometrical problems had to be solved, much less space was devoted to explicit references to predecessors.

Mathematics seems to distinguish itself from other branches of scholarship by the comparatively small role that authority plays in it. In principle, mathematical theorems are proved to be true by logical deduction independent of whomever gives it, and moral issues play no role (and analogously, mathematical problems are solved by constructions that can be proved to be valid). These qualities make mathematics somewhat different from the more mainstream humanist pursuits. Nevertheless, this difference should not be overestimated, as Snellius's work distinctly shows. As in other fields, the topics studied, the methods used and the acceptability of solutions were largely determined by tradition and authority.

8.6.4 Ramist aspects of Snellius's mathematics

Although Ramism is a fluid concept, which makes it impossible to pin down the exact influence of Ramus's thought on Snellius, the evidence that such an influence existed is so large that it makes sense to consider him as a partial Ramist. Indeed, Kepler called him 'the faithful disciple of Ramus'. Rudolph Snellius, by far the most outspoken Dutch Ramist, was crucial in familiarizing his son with Ramus's views, which continued to occupy Willebrord after his father's death. Yet Willebrord Snellius was certainly no thoughtless epigone of Ramus. This was impossible, because Ramus had not been a specialist in mathematics, producing just introductory books to pure mathematics and discussing only a small part of mixed mathematics. Snellius's knowledge of and interest in mathematics was much more profound and therefore Ramus could only direct a small part of the content of Snellius's books. Moreover, Ramus and Rudolph Snellius were primarily driven by the needs of education, and for this reason concentrated on rearranging and presenting anew existent knowledge, whereas Willebrord aimed to create new knowledge besides spreading what was already known.

Three kinds of reception of Ramus's work can be distinguished in Willebrord Snellius's oeuvre. In the first place, he made available several works by Ramus: he published a summarized version of the *Geometria*, an edition with commentary of the *Arithmetica* and a Dutch translation of the *Geometria*. These publications made Ramus's mathematics available to a wider audience. Snellius presented Ramus's ideas virtually unchanged in these books. Moreover, he stimulated Maurice of Hessen to publish more works of Ramus and Risnerus because 'this right method places the outlines of the whole structure clearly before the eyes' (see p. 153).

In the second place, Snellius shared some specific insights with Ramus. Two cases are especially significant. Firstly, Snellius denounced the traditional proof of Heron's Theorem, like Ramus and unlike their mathematical contemporaries. Where Ramus was unwilling to explain the difficulties calmly, Snellius did give a clear argument and also corrected the proof. Secondly, Snellius adopted Ramus's hostility towards Book X of the *Elements*. In this case, Snellius's heated rhetoric was not followed by an explanation of the alternative. This was not necessary, because the polemic had primarily a rhetorical function, expressing Snellius's position in the mathematical field. In these two examples, Snellius did not only inherit Ramus's points of view, but also his willingness to involve himself in debates.

In other places, Snellius also included Ramist elements. In his dedicatory letter to his father of *Cutting off*, he recommended Ramus's method for organizing a seemingly shapeless pile of material. The publication of observations and the abhorrence of Aristotle's domination of science as found in *Descriptio Cometae* both fitted in Ramus's programme. When objecting to the order in which Pappus had presented the Apollonian treatises, he called this wrong order a 'hysterologia', a favourite term of Ramus.[50] He considered Ramus 'most sensible' in *Doctrina Triangulorum*[51] and also referred to him in other works, e.g. in *Observationes Hassiacae*, and (without mentioning him explicitly) in *Cutting off* and his *Theses*. He studied Risnerus's Ramist *Optica* profoundly. Yet Snellius did not adopt Ramus's main mathematical innovation, his rearrangement of Euclidean mathematics, in his own works, because the abandonment of the deductive method did not appeal to him, as was the case with other mathematicians as well.[52]

In the third place, Ramus influenced Snellius's mathematical ideology. Ramus had argued that mathematics was eminently useful and would be acknowledged as such if only its obscure exposition would be abolished, which topics Snellius endorsed as well. Snellius seems to have inherited Ramus's conviction that mathematics had to play a central part in the liberal arts and that it was especially important in the humanist education of students. Their common wish to make mathematics accessible to students caused their impatience for oversubtle issues. Ramus had promoted the knowledge of Greek and Greek mathematics, whereas Snellius did indeed study little-known Greek works. Moreover, Snellius favoured mixed mathematics. Although Ramus had not paid much attention to this part of the mathematical sciences, his predilection for mathematics that could be applied in practice could have stimulated Snellius to go into this direction.

[50] [Snellius, 1608a, p. 10].
[51] 'λογικώτατου illius P. Rami', [Snellius, 1627, p. 63].
[52] [Verdonk, 1966, p. 370].

8.6.5 The goal of humanist mathematics

The characteristics of Snellius's style as described above sketch a picture of a mathematician thoroughly familiar with classical mathematics, its problems and solutions. They also show how his mathematics was a part of a unified world of learning, in which the humanist approach could, with different accents, be applied to all fields of knowledge. This unity also explains why Snellius could extend his studies to ancient money and other ancient topics without being hindered by methodological boundaries.[53]

All in all, Snellius's mastery of the humanist way of approaching mathematics was such that he should be considered as one of the most, perhaps even the most, outspoken representatives of the current of humanist mathematics. Moreover, the survey of his work given so far shows that he had a programme, which consisted of dealing with all mathematical sciences in a humanist way so as to show that mathematics was a worthy topic, the importance of which stretched far beyond the realm of propaedeutical educational needs. The reason for the pursuit of the programme must have been twofold. In the first place, the framework of humanism offered an abundant amount of material and methods for the study of interesting mathematical problems, connecting it to other lively parts of scholarship.

In the second place, Snellius must have been driven by his wish to acquire a good position in Leiden University, which had had a very strong tradition in humanist scholarship since its foundation in 1575. Where other universities were primarily teaching institutions, research played an important role in Leiden, for which reason it was a fertile soil for the programme. Snellius must have used the humanist style as a strategy for having mathematics recognized as a proper academic discipline and thus for convincing the other professors and the board of the university of the value of his work and simultaneously of himself. It was not easy to obtain a fixed position, as is shown by the fact that Snellius had already been teaching in the university for thirteen years before he became an *extraordinarius*. After becoming a professor, he could still improve his position: a mathematics professor only belonged to the *artes* faculty and earned less than the professors in the higher faculties.

8.7 General conclusion: between three fires

It was no trivial matter to pursue Snellius's style of mathematics and still remain good friends to everyone. We can see him as trapped between three fires, that of mathematical practice as personified by Ludolph van Ceulen, that of humanist scholarship as embodied by J. J. Scaliger, and that of Rudolph Snellius's Ramist

[53] Cp. [de Wreede, 2001].

pedagogy. They were indeed fires for Snellius: these men attracted him by their knowledge, of which he benefited through their private lessons, and their support, which stimulated him to create a wealth of works, yet his attraction to them also resembled that of a moth to a flame: when he would come too close, he could be burnt, that is, his reputation could be tarnished.

Van Ceulen had a large knowledge of and original ideas about the front-line of geometrical research: applying algebra to geometrical problems and calculating with huge numbers. Yet the fact that he was rooted in the world of the mathematical practitioners, in which they quibbled for ever about the correct solution of trivial problems, made him less suitable as a companion for Snellius, who longed for a higher status in society. Moreover, Van Ceulen did not master classical languages and could therefore not access the treasuries of Greek mathematics himself.

On the other hand, the status of Scaliger was more than high enough and he initiated Snellius in the riches of classical scholarship as no one else would have been able to. Yet although he considered himself as knowledgeable in mathematical issues, unfortunately he was not acknowledged as such by real experts among whom was Van Ceulen, whom he tried to disparage by calling him a mere boxer. The great humanist did appreciate his pupil Snellius and would certainly have supported his further development, had he not died in 1609.

Rudolph Snellius was the first to initiate Willebrord into the world of learning, by his education and his connections. His adherence of Ramus must have stimulated his son's interest in mathematics, yet he could not help him develop into a mathematical specialist. Van Ceulen died in 1610 and Rudolph Snellius in 1613, leaving Willebrord intellectually orphaned, but in this way also giving him the freedom to develop his own style.

When Snellius developed into a mature mathematician, he selected the best of what these three teachers had to offer as the foundations for his own work. Because none of them could act as a role model to show him how a mathematical scholar should do his work, he had to invent that for himself. Snellius may also have been driven by an ambition to surpass his father as a scholar, which he did not utter explicitly, because he was aware of all the fruits that he had reaped from Rudolph.

For the rest of his life, Willebrord Snellius was challenged to combine the styles of his teachers into a new unity. The wish to discover how both specialized and less advanced mathematics should look in a humanist guise must have been a leading stimulus for his work. His whole adult life is characterized by a certain restlessness. First, when young, he felt 'bound' (his own word) in Leiden. Later, he kept publishing his books, alongside many other activities, with high speed, frequently redirecting his attention to various disciplines within or close to the mathematical sciences. It seems that on the one hand he did not grant himself the leisure to study a few parts of the mathematical sciences more profoundly, because he wanted to hurry to other parts to see how he should incorporate them in his system of humanist mathematics. Yet on the other hand he strived after exactness to create more certainty in his difficult project and he returned sometimes to problems that he had studied previously because he still had something to say about their solution, such as in the case of the new edition of *Eratosthenes Batavus* or of the different triangle division solutions.

Snellius must have been lonely sometimes: without siblings or adult children, he had to find his own path after his father's death. Although many of his contemporaries were in some way involved in mathematics, they had little knowledge and interests in common due to the wideness of the field. Moreover, most members of this group only applied existing knowledge and did not add new results to the corpus of mathematics. Within the Dutch Republic, only a few other scholars had a knowledge of mathematics that was almost on a par with Snellius's. The technical, non-linguistic aspect of mathematics prevented it from becoming part of mainstream humanist scholarship. No wonder that Snellius's friend Vossius remarked after his death that Snellius had not always been appreciated at his proper value, although he had been a great man.[54]

The *Fundamenta* offers an excellent illustration of how Snellius adjusted Van Ceulen's mathematics to a humanist framework, elevating and 'exactifying' it. Snellius had some reason to distance himself from Van Ceulen in this book, because he wanted to use it to improve his position in the university and therefore had to show that instead of being a practitioner like Van Ceulen, he was as much a scholar as Scaliger and the other university professors. We see here a good example of the influence of external factors on the content of pure mathematics.

Snellius and Van Ceulen can both be considered as specialized mathematicians, a very rare species then, which entailed that there were no generally accepted rules for playing the game. There were indeed crucial differences in their approaches: Van Ceulen tried to explore and enlarge the fields that he studied and break through the boundaries between them, showing taste for experimenting, whereas Snellius raised standards of exactness within the field of geometry, trying to grasp difficult concepts and striving for profound knowledge. Snellius's close ties with classical mathematics caused his results to be of 'bounded originality': within the framework set by old and some new authorities, he developed his own thoughts and added some new elements, and he was only open to moderate innovations. Because he formulated his statements about good mathematics explicitly, his work is a rich source for understanding the mathematical challenges and action of his time.

Snellius's personal choices can also be considered in the broader perspective of the 'aristocratization' of science. There are strong indications that the social

[54]'Magnus omnino vir fuit: sed non suo pretio aestimatus a suis.' See p. 291.

distance between the university professors and engineers was growing in Snellius's time and afterward, which made it more important for scientists to try to align with the higher social classes.[55]

What, then, is the significance of Snellius? Some of his contributions to mathematics have given him the reputation of an outstanding Dutch mathematician from his own time to the present. This is illustrated by the fact that the building of the Mathematical Institute of Leiden University was baptized 'Snellius' some years ago. These achievements were the invention of the Law of Refraction, the development of the method of triangulation, the development of a faster method for calculating π and the determination of the area of a quadrilateral.

It is true that these are admirable results. Yet if one considers them in isolation, one could easily overlook his main contributions to mathematics: by extending and spreading mathematical knowledge, developing his own style and showing the value of mathematics as a discipline in its own right, he contributed to its emancipation and growing importance. Seen in this light, Snellius's oeuvre has to be considered as a whole of which the members complement each other.

Moreover, his 'exactification', the stress that he put on the need for mathematics to be correct for its own sake and according to its own norms, also contributed to the development of mathematics into an independent discipline. Snellius could be described as a transitional figure: he contributed to the development of one general kind of (humanist) scholarship into many different disciplines which all had their own methods.

The direct results of Snellius's programme to show that mathematics had to be esteemed as a humanist science were small. After his death, his chair even remained vacant for three years. The modesty of this success was caused mainly by factors outside Snellius's control. The structure of early modern universities, with their rigid restriction of the doctoral degree to three disciplines only, was an obstacle to a large increase in the status of mathematics. Snellius's peer group of classically trained scholars with a profound knowledge of mathematics was very small. The other groups to which he directed his attention were so diverse that it was impossible to satisfy them all, and he could not take away the friction that he must have felt between his heartfelt interest in mathematics and the need to secure a good position to be able to take care of himself and his steadily growing family.

Nevertheless, this lack of visible consequences of his experiment does not diminish the significance of Snellius's work. He explored the connections between

[55][van Berkel, 1998, pp. 46–47], [van Berkel, 1999, p. 37].

Cp.: '[Rudolph] Snellius and his successors needed some other legitimation [than repeating the courses of the engineering school in Latin] for their chairs. Instead of exposing the practical use of their field, they tried to stress its theoretical and scholarly aspects. This strategy also offered perspectives to align themselves with scholars and give their discipline a new respectability.' [Vermij, 1996, pp. 86–87]. However, in the case of Willebrord, there is no opposition between (practical) use and scholarly aspects, but a fusion.

humanism and mathematics more than almost anybody else. In his publications on pure and mixed mathematics he intended to show that mathematics was a valuable and serious intellectual pursuit, to be treated on equal footing with accepted humanist topics such as ethics, rhetoric and history, and he stubbornly continued his quest, even if he did not have much immediate success. His efforts were not exclusively directed at a small public of scholars: he also paid attention to mathematical education, and he tried to cross the boundaries between mathematical scholarship and its practical application. By doing so at Leiden University—which was becoming one of the most important scientific institutions in Europe—and in the Dutch Republic—which gained in importance in its Golden Age—, he contributed substantially to mathematics.

8.8 Epilogue

In this thesis, I have tried to develop a fruitful method for describing and explain the characteristics of Snellius's mathematics and his acts as a mathematician. In particular, I have focused on his style, humanist mathematics, his development of that style and the way in which he applied it to obtain new results. In most chapters, case studies have been discussed, because only a close study of detailed (technical) points can yield an accurate description of the characteristics of the style and Snellius's concept of exactness. This leaves enough topics for further research. More parts of Snellius's work could be studied in detail and placed in the traditions to which they belong, and more unknown manuscripts could be hunted for in archives and libraries in the Netherlands and abroad.

The reception of his work in his own and later times could be researched more systematically than I have done. It would be especially worthwhile to know more about the connections between Snellius and the Dutch practitioners, who in general could not read his books because they only mastered the vernacular, but might have exchanged ideas with him orally or through intermediaries. For this purpose, not only books should be studied, but also archival evidence, maps and measuring instruments. Such a study could also shed light on the question to what extent Snellius wanted his mathematics to be applicable outside academia.

Further, this study has completely been concentrated on Snellius. When more insight into the lives, style, motivation, criteria of exactness, network and audiences of other mathematicians (or other scholars) has been collected, either from existing literature or afresh from the sources, Snellius could be compared to these others, so that we would know more precisely which part of his characteristics is really typical for him, and which he shared with others. New questions could then also be asked, e.g. about different schools of mathematicians determined by their different styles, the influence of different local circumstances on mathematics and viceversa, or about developments in the debates about exactness.

Samenvatting

Hoewel Willebrord Snellius één van de bekendste Nederlandse wetenschappers uit de Gouden Eeuw is, is er nooit eerder een volledig boek aan zijn leven en werken gewijd. Mijn proefschrift voorziet in die lacune, door een overzicht van leven en werk van Snellius te geven en een aantal casussen meer in de diepte te analyseren. Daarbij wordt zowel nieuw licht geworpen op bekende onderwerpen (zoals de door Snellius gevonden optische Brekingswet en zijn landmeetkundige methode), als voorheen onderbelichte onderwerpen geanalyseerd (zoals Snellius' opvattingen over het gebruik van arithmetische concepten in de meetkunde en zijn correspondentie), wat een aantal nieuwe inzichten over zijn prestaties, motivatie en positie in zijn eigen tijd opgeleverd heeft. In dit boek ligt de nadruk op Snellius' ontwikkeling van de stijl van humanistische wiskunde en de wortels van die stijl, met name de invloed van zijn Leidse leermeesters. Ook zijn voorliefde voor 'exactheid', dat wil zeggen de duidelijke afbakening van toegestane constructies in de meetkunde, en de invloed hiervan op Snellius' meetkunde krijgen speciale aandacht.

Het proefschrift bestaat uit een biografisch deel, vijf casushoofdstukken en een afsluitend hoofdstuk. In het biografische deel van het proefschrift bespreek ik Snellius' leven en werk. Ook komen daar de achtergronden aan bod die nodig zijn om zijn wiskundig oeuvre te kunnen begrijpen: de Leidse universiteit, de humanistische wetenschap zoals die daar beoefend werd, wiskundige activiteiten in de rest van de Republiek en het gedachtegoed van Petrus Ramus. Ook wordt uitgebreid aandacht besteed aan leven en werken van Willebrords vader Rudolf Snellius, de meest uitgesproken Ramist van de Nederlanden, eerste leermeester van zijn zoon en diens voorganger als hoogleraar wiskunde in Leiden.

Willebrord Snellius, geboren in 1580, studeerde aan de Leidse universiteit, onder andere bij de grote humanist J. J. Scaliger. Daarbuiten kreeg hij wiskundeonderwijs van Ludolf van Ceulen. Hij gaf zijn eerste colleges aan de universiteit al in 1600, over de *Almagest* van Ptolemaeus. Daarna trok hij de wereld in omdat, naar eigen zeggen, Leiden hem benauwde. Hij reisde naar Duitsland, waar hij onder andere de wiskundige Adrianus Romanus leerde kennen. Daarna liep hij stage bij de topastronoom Tycho Brahe in Bohemen, bij wie hij ook Johannes

Kepler tegen kwam. Daarna keerde hij terug naar Leiden, en trok vervolgens naar Parijs om rechten te gaan studeren. Deze studie onderbrak hij toen zijn vader hem vroeg naar het hof van zijn patroon Maurits Landgraaf van Hessen te komen. Na een bezoek aan een aantal andere plaatsen keerde Willebrord terug naar Leiden, dat hij nadien slechts sporadisch zou verlaten.

Hij werd eerst onderwijsassistent van zijn vader, in 1613 (toen zijn vader gepensioneerd was) buitengewoon hoogleraar en in 1615 gewoon hoogleraar. Het verkrijgen van een behoorlijke aanstelling met bijbehorend salaris was geen eenvoudige zaak, en Snellius moest veel moeite doen om zijn positie te verbeteren. Zo droeg hij de Latijnse vertaling die hij van Ludolf van Ceulens *Arithmetische en Geometrische Fondamenten* gemaakt had (*Fundamenta Arithmetica en Geometrica*) op aan zijn maatschappelijk vooraanstaande familielid Aemilius Rosendalius, waarbij hij hem expliciet vroeg een goed woordje voor hem te doen bij de bestuurders van de universiteit. Snellius' wens serieus genomen te worden in de kringen van de Leidse geleerden is ook één van de belangrijkste verklaringen voor zijn humanistische stijl van wiskundebeoefening.

Snellius trouwde in 1608 met Maria de Langhe. Volgens de overlevering kregen ze achttien kinderen, maar dit aantal lijkt te hoog te zijn. Archiefbronnen wijzen op het bestaan van zeven kinderen, waarnaast er nog enkele jonggestorvenen zouden kunnen zijn geweest. Snellius heeft geen van zijn kinderen als volwassene meegemaakt.

Snellius was een bijzonder actief geleerde. Hij gaf onderwijs, waarbij hij zijn studenten wel eens meenam naar buiten voor praktisch onderricht in de landmeetkunde, was wetenschappelijk adviseur voor collega's en 's lands regering, en publiceerde veel: zowel eigen werk als uitgaven en vertalingen van het werk van anderen, alle in het Latijn op één na (de door hem geïnitieerde vertaling van Ramus' *Geometria* in het Nederlands). Met zijn werk bestreek hij bijna alle toenmalige wiskundige wetenschappen: meetkunde, arithmetica, sterrenkunde, navigatie, landmeetkunde en optica. Hij stierf in 1626, betreurd door collega's en vrienden in binnen- en buitenland.

In het eerste casushoofdstuk wordt *Eratosthenes Batavus* geanalyseerd, waarin Snellius de omtrek van de aarde nauwkeuriger berekende dan zijn voorgangers. De methode waarmee hij zijn resultaten bereikte wordt uiteengezet. Vooral opvallend is het aantal verschillende bronnen en vaardigheden die Snellius gebruikte: hij bestudeerde oude en nieuwe literatuur zorgvuldig, verzamelde gegevens in zijn omgeving en via andere geleerden, deed een groot aantal metingen van relatieve posities van steden in Holland en in de Zuidelijke Nederlanden, en maakte met dit alles ingewikkelde berekeningen. De excursus in zijn boek die Snellius wijdde aan een instrument om het soortelijk gewicht van water vast te stellen werpt nieuw licht op zijn ingenieursinteresses.

In het hoofdstuk over astronomie en patronage wordt de patronageverhouding tussen Maurits van Hessen, en Rudolf en Willebrord Snellius onder de loupe genomen, grotendeels aan de hand van tot nog toe onbekende brieven. Deze verhouding leidde tot twee publicaties van Willebrord Snellius, één uitgave van een aantal astronomische observaties van anderen, en één gewijd aan de komeet van 1618. Dit laatste werk had Snellius op Maurits' expliciete verzoek geschreven. Het is anti-Aristotelisch van strekking, en benadrukt sterk het belang dat Snellius hechtte aan goede observaties en redelijke gevolgtrekkingen.

De volgende drie hoofdstukken behandelen onderwerpen uit de zuivere meetkunde. In het eerste daarvan worden, na een korte inleiding op de stand van zaken in de meetkunde in Snellius' tijd, twee onderwerpen uit Van Ceulens *Fundamenta* behandeld. Dit boek is een belangrijke bron voor Snellius' opvattingen over wat goede wiskunde is, omdat hij in zijn vertaling Van Ceulens werk soms aanpaste of verbeterde en vaak zijn eigen commentaar toevoegde, wat het de vorm geeft van een dialoog tussen een vertegenwordiger van de rekenmeesterstraditie en één van de academische wiskundige traditie. Bovendien geeft de opdrachtbrief, die geanalyseerd wordt naar inhoud en retorische strategie, Snellius' opvattingen over wiskunde in een notedop, in het bijzonder over het gebruik van getallen in de meetkunde. Daarna bespreek ik hoe Van Ceulen de vier elementaire operaties (optellen, aftrekken, vermenigvuldigen en delen) invoert voor lijnsegmenten waarvan de lengte uitgedrukt wordt door een getal.

De volgende casus betreft driehoeksdeling, een populair probleem in Snellius' tijd. De opgave is een gegeven driehoek te verdelen in twee delen waarvan de oppervlaktes een gegeven verhouding hebben, door middel van een lijn door een gegeven punt. Verschillende varianten van dit probleem, bijvoorbeeld met het gegeven punt binnen of buiten de driehoek, werden bestudeerd. Dit probleem stamde uit de Oudheid en intrigeerde ook Snellius en zijn tijdgenoten, die langzamerhand alle relevante bronnen (her)ontdekten. Het probleem behoort tot de vlakke meetkunde, wat betekent dat het met alleen passer en liniaal opgelost kan worden. Snellius gaf drie verschillende oplossingen van het probleem in de loop van zijn leven. De aard van één van zijn oplossingen verschilt sterk van de traditionele oplossingen: Snellius bedt het probleem in in een hiërarchie van problemen in zijn reconstructies van het verloren werk van Apollonius van Perga en geeft het daarmee een plaats in een grotere structuur.

In het laatste meetkundehoofdstuk staat de Stelling van Heron centraal, die de oppervlakte van een driehoek relateert aan de lengtes van de zijden. Deze stelling werd sinds de Oudheid bewezen en gebruikt zonder kritiek van wiskundigen, wat veranderde toen Ramus zich in de zestiende eeuw beklaagde over de duisternis van het bewijs. Snellius borduurde voort op Ramus' kritiek en maakte expliciet waaraan het traditionele bewijs mank ging: in een meetkundige context werd een vier-dimensionale grootheid gebruikt, wat toen onvoorstelbaar was. Dit werd volgens Snellius veroorzaakt doordat arithmetische en meetkundige begrippen door elkaar werden gebruikt. Snellius herformuleerde stelling en bewijs in zuiver meetkundige termen met vermijding van de vier-dimensionaliteit.

In hetzelfde hoofdstuk wordt Snellius' commentaar op Van Ceulens constructie van een koordenvierhoek besproken, waarin Snellius ook, als eerste in West-Europa, de formule voor de oppervlakte van een koordenvierhoek gaf, een analogon van de Stelling van Heron. In dit deel van zijn werk lijkt Snellius meer open te staan voor een arithmetisch-algebraïsche benadering van meetkundige problemen dan in de hiervoor genoemde gevallen.

Tenslotte worden in het afsluitende karakteristiekenhoofdstuk alle resultaten van de voorafgaande hoofdstukken samengevoegd tot een coherent beeld van Snellius' wiskunde en Snellius als wiskundige. Bijzondere aandacht is er voor 'exactheid' (de precieze afbakening van acceptable wiskunde), zijn 'combinatiemethode' van onderzoek doen (waarmee hij goede, maar geen revolutionaire resultaten boekte door op intelligente wijze gebruik te maken van alle kennis die hij in zijn omgeving op kon doen), zijn netwerk en patronagerelaties, en de humanistische en Ramistische aspecten van zijn werk.

Zijn humanistische aanpak blijkt vooral uit zijn beheersing van het Latijn op hoog niveau, goed zichtbaar in de inleidende delen van zijn werken, waarin hij ook blijk gaf van beheersing van retorische middelen. Bovendien was zijn kennis van de klassieke bronnen groot: zowel Latijnse als Griekse, wiskundige als niet-wiskundige. Ze dienden voor hem als ijkpunt van goede wiskunde. Zijn werk is meestal een bewuste *aemulatio*: hij liet zien dat hij verder kwam met het oplossen van problemen waar de Ouden al mee worstelden dan zijn voorgangers. De verhouding tussen de inbreng van klassieke schrijvers-moderne schrijvers-observaties-berekeningen-eigen redeneringen hing af van Snellius' smaak en van het genre.

Concluderend kan Willebrord Snellius gezien worden als gevangen tussen drie vuren, de drie leermeesters van zijn jeugd Rudolf Snellius, Scaliger en Van Ceulen. Deze mannen trokken hem aan door hun kennis en ondersteuning, maar geen van hen kon dienen als voorbeeld van hoe een gespecialiseerd wiskundige zijn werk moest doen. Daarom moest Snellius zijn eigen stijl ontwikkelen, waarbij hij zich baseerde op wat hij geleerd had, maar waarmee hij zelf een nieuwe aanpak van de wiskunde ontwikkelde. Hierin ligt ook zijn belangrijkste bijdrage aan de wetenschap.

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[Snellius, 1596d] Snellius, R. (1596d). *In P. Rami Arithmeticam Explicationes lectissimae: Lazari Schoneri, Bernh. Salignaci, et Christiani Urstisii, commentationibus passim locupletatae.* Ex Officina Typographica Ioannis Saurii, impensis haeredum Petri Fischeri, Francofurti. UBL 197 E 27.

[Snellius, 1596e] Snellius, R. (1596e). *In P. Rami Geometriam praelectiones; cum lectissimis Lazari Schoeneri, et Iohannis Thomae Freigii explicationibus, ad praecipua quaeque elementa statim adiectis, illustratae.* Ex Officina Typographica Ioannis Saurii, impensis haeredum Petri Fischeri, Francofurti. SUB Göttingen 8 MATH III, 585 (1).

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[Snellius, 1596g] Snellius, R. (1596g). *In Sphaeram Cornelii Valerii praelectiones; cum lectissimis aliorum commentationibus et animadversionibus sedulo collatae. Ad calcem adiectae sunt collationes et animadversiones quaedam necessariae.* Ex Officina Typographica Ioannis Saurii, impensis haeredum Petri Fischeri, Francofurti. UBU Rariora Duod 84 con: 1.

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[Snellius, 1596i] Snellius, R. (1596i). *Snellio-Ramaeum Philosophiae Syntagma, tomis aliquot separatis distinctum; quibus continentur: 1. Generales sincerioris philosophiae Ramaeae informationes: 2. Dialectica; 3. Rhetorica; 4. Arithmetica; 5. Geometria; 6. Sphaera, seu Astronomia; 7. Physica; 8. Psychologia; 9. Ethica; Rhodolphi Snellii, excellentissimi Philosophi; commentationibus succinctis et accuratis; sed cum aliorum huius aevi praecellentissimorum virorum lectissimis observationibus ac notis studiose passim collatis et auctis; explicatae. Quibus praefatio D. Rhodolphi Goclenii, celeberrimi in Marpurgensi Academia Philosophi, praefixa est. Tomo autem primo hic post Praefationem subiungitur Vera Vereque Ramea Recte Philosophandi Ratio: Sequentibus vero ac separatis Tomis reliquae sequentes disciplinae.* Ex Officina Typographica Ioannis Saurii, impensis haeredum Petri Fischeri, Francofurti. HAB Wolfenbüttel 6.1 Log. (2).

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[Snellius, 1614] Snellius, R. (1614). *Rudolphi Snellii Veter-aquinatis Partitiones Physicae, methodi Rameae legibus informatae: Exceptae olim ex dictantis eius ore in Schola Marpurgensi. Editio prioribus correctior et emendatior.* Apud Haeredes Guilielmi Antonii, Hanoviae. UBL 189 F 28.1.

[Snellius, 1617a] Snellius, R. (1617a). *Rodolphi Snellii Veteraquinatis Commentarius in Rhetoricam Audomari Talaei.* Apud Ioannem Maire, Lugduni Batavorum. UBL 722 G 3.

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[Snellius, 1617c] Snellius, W. (1617c). *Eratosthenes Batavus De Terrae ambitus vera quantitate, a Willebrordo Snellio, διὰ τῶν ἐξ ἀποστημάτων μετρουσῶν διοπτρῶν, suscitatus.* Apud Iodocum a Colster, Lugduni Batavorum. TUDelft CB TRK 9804211: photocopy of Royal Library Brussels 8365 G 04, which contains handwritten notes by Snellius.

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Curriculum Vitae

Liesbeth de Wreede is geboren op 22 april 1974 in Rotterdam. In 1992 behaalde ze haar diploma Gymnasium-α en -β aan Scholengemeenschap 'De Krimpenerwaard' te Krimpen aan den IJssel, waarna ze wiskunde ging studeren aan de Universiteit Leiden. Een jaar later begon ze daarnaast aan de studie Griekse en Latijnse Talen en Culturen, ook in Leiden. In 1998 studeerde ze af in de wiskunde en een jaar later in de klassieke talen (*cum laude*), beide keren op een onderwerp uit de geschiedenis van de wiskunde. Van november 1999 tot februari 2005 was ze aio aan het Mathematisch Instituut van de Universiteit Utrecht. Naast haar onderzoeks- en onderwijswerkzaamheden was ze onder meer lid van het onderwijsbestuur van het Mathematisch Instituut en van het BAU-bestuur (de Utrechtse promovendi-belangenvereniging), co-organisator van de *Novembertagung* van 2004 (een congres voor jonge historici van de wiskunde) en (mede-)oprichter en lid van verschillende promovendiwerkgezelschappen. In 2000 kreeg ze het Dr Ted Meijer Stipendium van het Koninklijk Nederlands Instituut te Rome, dat haar in staat stelde drie maanden oriënterend onderzoek naar vroeg-moderne wetenschapscultuur te doen in Rome.

Vanaf 2005 was ze docent van een aantal cursussen op het gebied van de geschiedenis van de wiskunde aan de Leidse Universiteit. Vanaf 2006 werkt ze als wetenschappelijk onderzoeker bij de afdeling Medische Statistiek en Bioinformatica van het Leids Universitair Medisch Centrum.