4. A MANUAL TO THE TABLES

As a prerequisite to an understanding of the remaining chapters, the reader is here made familiar with the terminology of the *multistepped* and *single-stepped* formats of various lunar tables. These concepts address the way in which the tables are put to use. They have nothing to do with the way by which their coefficients have been fixed, and also one should not associate them with some kind of an iterative scheme. The term 'multistep' refers to the fact that the computation of a lunar position, by way of these tables, goes through several stages (as will be made fully clear further down). 'Single-step', on the other hand, refers to a computational procedure of only one stage.

After an introduction to the tables and an explanation of the multistep format, an example computation is given. Section 4.3 on time and calendar conventions is not important for an understanding of the later chapters or of the format, but it is perhaps helpful to anyone who intends to work with the tables.

We will use the tables of the `rede` edition here,[1] because these tables were more widely distributed and because they fit better than the `kil` version to the end result of Mayer's lunar theory, which we encounter in chapters 5 and 7. The publications of both the `kil` and `rede` editions were accompanied by an example calculation.[2] The last section of this chapter points out in what way the application of those two sets of tables differs.

4.1 ON TABLES

Several tables are needed for a single position calculation of the moon. Some of these tables represent a mean position or mean motion: these tables are entered with a date/time argument. Other tables represent an equAtion: these tables are entered with an argument in angular measure. Since the lunar equAtions depend on the position of the sun, the sun's position is computed from tables *en passant*. Appendix A lists all the mean motions and equAtions of several table versions including `rede`.

We remind that from a modern mathematical point of view, an equAtion is a function; all equAtions of longitude are of the form $\sum_{k=1}^{n} c_k \sin(k\alpha)$ with $n \leq 4$. The c_k are coefficients and α is called the argument of the equAtion. Mayer's tables are effectively lists of function argument and function value pairs: see figure 4.1 for examples. A table is used to look up the value of the function for a specific value

1 [Mayer, 1770].

2 Mayer had warned that the example computation included with the 1753 `kil` tables was based on a slightly older version, which I could identify as `zwin`.

of its argument, this value is then added to some other argument which it serves to correct, or to equAte in the old terminology.

The functions that the mean motion and epoch tables represent, are linear functions, while the table for secular acceleration provides a quadratic term. An example of the epoch tables showing the mean positions at the beginning of the years 1780 to 1807 is contained in figure 4.1.

The following is a complete list of the 22 tables in `rede` necessary to compute the longitude of the moon. Because we will not consider the calculations for the moon's parallax and latitude, which are somewhat similar to the longitude computation, there is no need to list the tables for their calculation here. The solar tables are likewise omitted, although an example comutation of the sun's longitude is given.

- Mean motion of the moon's longitude, longitude of the apogee, and longitude of the ascending node for integer numbers of elapsed Julian years (i.e, years of 365 days, and a leap day for every fourth year; note carefully that these are time intervals, not years in the Julian calendar).
- Mean epochs of longitude, apogee, and node for the beginning of the year (more precisely, for mean noon on the preceding December 31) on the Greenwich meridian. These were listed for selected years in both the Julian and Gregorian calenders.[3]
- Mean motions of longitude, apogee, and node for months and days.
- Mean motions of longitude, apogee, and node for hours, minutes, and seconds.
- Secular acceleration, from 900 BC to AD 4300.
- Annual equAtion of the anomaly as a function of mean solar anomaly.
- Annual equAtion of the mean ascending node as a function of mean solar anomaly.
- Thirteen tables (numbered I to XIII) for just as many equAtions, all with different arguments. These include the annual equAtion of longitude, equAtion of centre, evection, and variation. Further details are provided in the following example.
- Two more tables, to adjust the frame of reference rather than the position of the moon. The first is for the so-called reduction, which converts the moon's longitude in its orbital plane to the plane of the ecliptic. The second is the equAtion of the equinoxes. It shifts the origin of the longitude scale from the mean equinox of date to the true equinox of date, thus correcting for nutation. There is no need for the reduction from the mean equinox of epoch to the mean equinox of date, because the mean motion tables already include precession. Earlier tables of Mayer's such as `kil` had no nutation correction.

3 The Julian calender table listed epochs of every century from 600 BC to AD 1400, every 20 years from 1460 to 1700, and every year from 1701 to 1752 (when Great Britain switched to the Gregorian calender); the Gregorian calender table listed epochs of every 20 years from 1600 to 1700, and every year from 1701 to 1807.

Epochæ mediorum motuum LUNÆ,

Tempore medio currente sub meridiano Observatorii GRENOVICENSIS.

Stylo Gregoriano.		Long. med. ☽	Long. Apog. ☽	Longit. ☊
		s ° ′ ″	s ° ′ ″	s ° ′ ″
Anni post C. N.	1780	7. 5. 19. 46	10. 11. 44. 37	2. 0. 2. 10
	1781	11. 14. 42. 51	11. 22. 24. 27	1. 10. 42. 27
	1782	3. 24. 5. 57	1. 3. 4. 18	0. 21. 22. 44
	1783	8. 3. 29. 2	2. 13. 44. 8	0. 2. 3. 1
	1784	0. 26. 2. 43	3. 24. 30. 40	11. 12. 40. 7
	1785	5. 5. 25. 48	5. 5. 10. 30	10. 23. 20. 24
	1786	9. 14. 48. 53	6. 15. 50. 21	10. 4. 0. 41
	1787	1. 24. 11. 59	7. 26. 30. 11	9. 14. 40. 58
	1788	6. 16. 45. 39	9. 7. 16. 43	8. 25. 18. 4
	1789	10. 26. 8. 45	10. 17. 56. 33	8. 5. 58. 21
	1790	3. 5. 31. 50	11. 28. 36. 24	7. 16. 38. 38
	1791	7. 14. 54. 55	1. 9. 16. 14	6. 27. 18. 55
	1792	0. 7. 28. 36	2. 20. 1. 46	6. 7. 56. 1
	1793	4. 16. 51. 41	4. 0. 42. 36	5. 18. 36. 18
	1794	8. 26. 14. 47	5. 11. 22. 27	4. 29. 16. 35
	1795	1. 5. 37. 52	6. 22. 2. 17	4. 9. 56. 52
	1796	5. 28. 11. 32	8. 2. 48. 49	3. 20. 33. 58
	1797	10. 7. 34. 38	9. 13. 28. 39	3. 1. 14. 15
	1798	2. 16. 57. 43	10. 24. 8. 30	2. 11. 54. 32
	1799	6. 26. 20. 48	0. 4. 48. 20	1. 22. 34. 49
	1800	11. 5. 43. 54	1. 15. 28. 10	1. 3. 14. 6
	1801	3. 15. 6. 59	2. 26. 8. 0	0. 13. 54. 23
	1802	7. 24. 30. 5	4. 6. 47. 51	11. 24. 34. 40
	1803	0. 3. 53. 10	5. 17. 27. 41	11. 5. 14. 57
	1804	4. 26. 26. 51	6. 28. 14. 13	10. 15. 52. 3
	1805	9. 5. 49. 56	8. 8. 54. 3	9. 26. 32. 20
	1806	1. 15. 13. 1	9. 19. 33. 54	9. 7. 12. 37
	1807	5. 24. 36. 7	11. 0. 13. 44	8. 17. 52. 54

I. Pro longitudine Lunæ.

Argum. I.
Anomalia media Solis.

S.	0	1	2	3	4	5	S.
	+	+	+	+	+	+	
°	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	°
0	0. 0	5. 35	9. 42	11. 16	9. 49	5. 42	30
1	0. 12	5. 45	9. 48	11. 16	9. 43	5. 31	29
2	0. 23	5. 54	9. 54	11. 16	9. 37	5. 21	28
3	0. 35	6. 4	9. 59	11. 15	9. 31	5. 10	27
4	0. 47	6. 14	10. 5	11. 14	9. 24	4. 59	26
5	0. 58	6. 24	10. 10	11. 14	9. 18	4. 49	25
6	1. 10	6. 33	10. 14	11. 13	9. 11	4. 38	24
7	1. 21	6. 43	10. 19	11. 12	9. 4	4. 27	23
8	1. 33	6. 52	10. 24	11. 11	8. 57	4. 16	22
9	1. 45	7. 1	10. 28	11. 9	8. 49	4. 5	21
10	1. 56	7. 10	10. 33	11. 7	8. 42	3. 54	20
11	2. 7	7. 19	10. 37	11. 5	8. 34	3. 43	19
12	2. 19	7. 28	10. 41	11. 3	8. 26	3. 31	18
13	2. 30	7. 37	10. 44	11. 0	8. 18	3. 20	17
14	2. 42	7. 46	10. 48	10. 58	8. 10	3. 8	16
15	2. 53	7. 54	10. 51	10. 55	8. 2	2. 57	15
16	3. 4	8. 2	10. 54	10. 52	7. 54	2. 45	14
17	3. 15	8. 10	10. 57	10. 49	7. 45	2. 33	13
18	3. 26	8. 18	10. 59	10. 45	7. 36	2. 21	12
19	3. 38	8. 26	11. 2	10. 42	7. 27	2. 10	11
20	3. 49	8. 34	11. 4	10. 38	7. 18	1. 59	10
21	3. 59	8. 41	11. 6	10. 34	7. 9	1. 47	9
22	4. 10	8. 49	11. 8	10. 30	7. 0	1. 35	8
23	4. 21	8. 56	11. 10	10. 25	6. 51	1. 23	7
24	4. 32	9. 3	11. 11	10. 21	6. 41	1. 11	6
25	4. 43	9. 10	11. 13	10. 16	6. 32	1. 0	5
26	4. 53	9. 16	11. 14	10. 11	6. 22	0. 48	4
27	5. 4	9. 23	11. 15	10. 6	6. 12	0. 36	3
28	5. 14	9. 30	11. 15	10. 0	6. 2	0. 24	2
29	5. 24	9. 36	11. 16	9. 55	5. 52	0. 12	1
30	5. 35	9. 42	11. 16	9. 49	5. 42	0. 0	0
	—	—	—	—	—	—	
S.	11	10	9	8	7	6	S.

Figure 4.1: Examples from [Mayer, 1770]. Top: p. XLII, epochs. The errata mentioned that the node positions of 1800 and later were 1′ too small, but an identical error in the 1792 apogee went unnoticed. Bottom: p. LIII, the annual equAtion, with mean solar anomaly as argument.

4.2 SINGLE-STEP AND MULTISTEP PROCEDURES

Modern lunar theories[4] are usually formulated in such a way that all the equAtions (i.e., all their periodic terms) are computed directly from the mean motion arguments. This is what we here call the single-stepped procedure. Alternatively, one may imagine that the arguments are equAted, i.e., adjusted, after each equAtion is computed and before the next one, or, to put it differently, that the position of the moon is improved step by step from the mean towards the true place. Each next equAtion is then computed with updated arguments. Indeed, Newton's lunar theory of which we shall speak in chapter 6 is such a theory.

The lunar tables of Mayer may be distinguished according to the procedure that should be followed when using them. His older tables up to early in 1752 are of the 'modern' single-stepped form: all their equAtions should be computed with the same mean motion arguments. The lunar tables of his colleagues Euler, Clairaut, and d'Alembert were of this kind, too. Even Mayer's treatise *Theoria Lunae*, written in 1755, initially developed a single-stepped theory.

But from the spring of 1752 onwards Mayer's tables occupy a position in between the single-stepped form and the step-by-step form of Newton's theory. Leaving it to chapter 6 to demonstrate the link between Newton's and Mayer's theories, we may here presuppose that Mayer reordered Newton's equAtions and lumped several of them together in a smaller number of 'steps'. Such is the idea of what I have termed the multistep form of his tables.

Among those multistepped tables, different varieties may be distinguished once more. The most important variants are the `rede` tables, which we will employ in our example calculation, and the `kil` version. They differ by the way in which the equAtions are distributed over the steps. Incidentally, this also has consequences for the magnitudes of the equAtions's coefficients (to be explored further in chapters 6 and 7).

The format to be used with the `rede` tables is as explained below. It will be compared to the format of `kil` in the last section of the chapter.

First step Compute the mean motion arguments, ten so-called 'minor equations', and two annual equAtions of apogee and node.

Second step Update the arguments with the ten minor and the annual equAtions, then use the updated anomaly as argument for the equAtion of centre.

Third step Update the lunar longitude with the equAtion of centre. Use the thus updated longitude to find the variation.

Fourth step Update the longitude again, and find out the value of the XIIIth equAtion, which depends on longitude, node, and anomaly.

Fifth step Update again, and find the reduction and the equAtion of the equinoxes before finally arriving at the true longitude of the moon.

4 Examples include almost all dynamical theories such as ELP2000 or Laplace, described in [Chapront-Touzé and Chapront, 1983] and [Laplace, 1802], respectively.

The essence of the multistep procedure is that the arguments in each step differ from the arguments in the previous one. It is, however, not essential whether we decide that a new step begins just before or just after the arguments are updated. The example computation to be discussed below follows Maskelyne's rules, where the updating of individual arguments is delayed until they are needed. This streamlines the work flow without affecting the end result.[5]

4.3 CALENDAR CONVENTIONS

The time to be used with the epoch and mean motion tables is mean solar time on the Greenwich meridian. The epochs in Mayer's tables were made up for mean time on the meridian of the Royal Observatory in Paris, but Nevil Maskelyne adjusted the values to fit his own workplace at the Royal Greenwich Observatory. The astronomical day customarily started at 12 noon, half a day later than the civil day, so that hours from 0 to 12 refer to an event *post meridiem*, while hours from 12 to 24 likewise refer to an event *ante meridiem*.

Maskelyne's time scale is roughly equivalent to the definition of Greenwich Mean Time (GMT) prior to 1925. From 1925 on, day start was redefined at midnight, and soon afterwards Universal Time (UT) was introduced, satisfying the relation UT − GMT = 12 hours. The GMT and UT time scales are both coupled to the rotation of the earth. In the middle of the nineteenth century, Adams and Delauney confirmed earlier suspicions that the earth's rotation is not uniform. Accuracy demands have since then led to an increasing call for a uniform time scale on which to base orbit computations. This has resulted in the 1950's in the design of Terrestrial Time (TT), to be used for computations of apparent geocentric ephemerides.

The non-uniformity of earth's rotation is responsible for the difference $\Delta T =$ TT − UT. The value of ΔT has been slowly increasing over the past centuries, as a consequence of the general slowing down of the earth's rotation. The ΔT-value for 1755 was +15s, for 2002 +64s. This difference matters when comparing historical lunar observations and computations to modern computations, because in 1 minute time the moon moves about $30''$ arc with respect to the stars. The discrepancies between a uniform timescale (TT) and a non-uniform one (UT) show most prominently in the position of the fastest moving celestial body, the moon.[6]

Leap days were intercalated in Mayer's tables as the first day in the year (Jan 0, so to say), instead of between February 28 and March 1. A user of the tables had to subtract one day if the date for which he made a computation happened to be in the months January and February of a leap year.

5 Maskelyne included both Mayer's rules and his own modifications in the explanatory chapter of [Mayer, 1770]. The earlier tables in [Mayer, 1753b] had no explanation of procedure, only an example computation. A sketch of the procedure is also in [Forbes and Wilson, 1995, pp. 64–65]. Maskelyne also published instructions and an example in [Maskelyne, 1763, pp. 28–37], but these applied to the `kil` tables contained in that work.

6 The concepts of time are covered in [Seidelmann, 1992, Ch. 2].

4.4 AN EXAMPLE COMPUTATION

The following example shows in detail how to compute the ecliptic longitude of the moon for 2002 November 15, 08:00:00 UT from the `rede` tables [Mayer, 1770]. The sun's position is also computed, because several arguments of the lunar tables depend on its position. The description of the calculation is a paraphrase of Maskelyne's translation of Mayer's instructions. For ease of reference to the cited source, the numbering in square brackets of the following paragraphs is Maskelyne's.[7] His numbering has nothing to do with the division in steps of the multistepped procedure.

4.4.1 To find the sun's longitude

The solar tables in [Mayer, 1770] are Mayer's solar tables adjusted to the time of the Greenwich meridian; Mayer's tables in turn are a slight modification from Lacaille's solar tables.[8]

The tables use sexagesimal notation of degrees, minutes and seconds. Along the ecliptic, however, arcs were further subdivided into *signs*, obeying an age-old practice, with 1 sign $= 1^s = 30°$. Full circles of 12^s are silently discarded from calculations, or, phrased differently, angles along the ecliptic and equator are taken modulo 12^s.

The following explanation of the computation of the sun's true longitude and mean anomaly can be quickly scanned over by a reader interested in the workings of the lunar tables only. But since computations for the sun are so much easier, this part could be of help when difficulties are experienced with the lunar part of the calculations.

Display 4.1 shows the completed calculation for the sun's true longitude. The numbers on the left side of the display correspond to the numbered instructions of the explanation.

[1] *(Epoch)* Make one column for the sun's longitude, and one for its apogee and anomaly. Look up in the tables the mean longitude of the sun and of its apogee for the given year. (If the year is outside the range of the table, then take the values for a convenient earlier year, and look up in the table for single (Julian) years the mean motions for the remaining years.)

[2] *(Mean motion)* Take from the tables the mean motions of the longitude and apogee for the month, day, and time of day, and write them down under the mean longitude and apogee just found.

[3] *(Sun's mean position)* Add the numbers in the columns to find the mean longitude of the sun and of its apogee for the given date and time (remember to work modulo 12^s).

7 Cf. [Mayer, 1770], pp. 101–102 for the sun, pp. 121–127 for the moon.

8 [Wilson, 1980, pp. 184-188].

		longitude	apog./anom.
[1]	epoch 1802	$9^s\ 9°25'56.6''$t	$3^s\ 9°34'46''$t
[1]	200 years	$0^s\ 1°32'46.0''$t	$0^s\ 3°40'\ 0''$t
[2]	Nov 13	$10^s\ 12°27'\ 0.9''$t	$57''$t
[2]	20^h	$49'16.9''$t	
[3]	mean longitude	$7^s\ 24°15'\ 0.4''$	$3^s\ 13°15'43''$
[4]	mean anomaly ς		$4^s\ 10°59'17''$
[5]	eqn. of centre	$-1°28'24.6''$t	
[6]	small eqns	$-5.1''$t	
[7]	true longitude ⊙	$7^s\ 22°46'30.7''$	

Display 4.1: The completed calculation of the solar longitude.

[4] *(Mean anomaly)* Subtract the longitude of apogee from the longitude of sun, and write the result under the apogee column. This is the mean anomaly of the sun.

[5] *(EquAtion of centre)* With this mean anomaly as argument, enter the table of the equAtion of the centre, look up the value of this equAtion and place it under the sun's mean longitude.

[6] *(Planetary perturbations)* From appropriate columns in the solar mean motion tables, find three small equAtions due to the perturbing effects of the moon, Jupiter, and Venus on the earth, and one for the equAtion of the equinoxes. For brevity and clarity, I omit the details of these small effects.

Note that at this stage we encounter two equAtions depending on the moon, whose exact position is not known without recourse to the *lunar* tables. But since the absolute values of these equAtions are at most $8''$ and $18''$, respectively, it suffices to take the position of the mean moon here.

[7] *(Sun's true longitude)* Add the equAtions of [6] to the mean longitude of the sun to arrive at its true longitude.

Our specific example of 2002 November 15, 08:00 UT runs well outside the range that Nevil Maskelyne had catered for in his epoch table, which went no further than 1807. Therefore we take in [1] the epoch year 1802 and add 200 Julian years from the table for single years. Then we subtract one day from the given date, because 1900 is a leap year in the Julian calendar, but not in the Gregorian calendar. In [2] we also subtract 12 hours from the given time, because the astronomical day started conventionally twelve hours after midnight. Thus, we enter the tables with November 13, 20:00 GMT.

4.4.2 To find the moon's longitude

We recall that we were to compute the lunar true longitude on 2002 November 15, 08:00 UT, and that we enter the tables with November 13, 20:00 GMT because of the calendar conventions. In the following computation, one readily recognizes the

		longitude	apogee	node
[2]	sec. accel	$1'21''$t		
[2]	epoch 1802	$7^s 24^\circ 30'\ 5''$t	$4^s\ 6^\circ 47' 51''$t	$11^s 24^\circ 35' 40''$t
[2]	200 years	$8^s 15^\circ 47' 10''$t	$7^s\ 8^\circ 22' 30''$t	$8^s 28^\circ 22' 30''$t
[2]	Nov 13	$7^s\ 6^\circ 55'\ 4''$t	$1^s\ 5^\circ 18' 59''$t	$16^\circ 47' 12''$t
[2]	20^h	$10^\circ 58' 49''$t	$5' 34''$t	$2' 39''$t
[3]				$9^s 15^\circ 12' 21''$
[3]	mean long ☽	$11^s 28^\circ 12' 29''$	$0^s 20^\circ 34' 54''$	$2^s\ 9^\circ 23' 19''$ ☊
[4]	mean anom p		$11^s\ 7^\circ 37' 35''$	
[4]	elongation ω	$4^s\ 5^\circ 25' 58''$		

Display 4.2: Calculation of mean arguments of the moon. The letter 't' indicates values that have been looked up in a table.

equAtion of centre, evection, variation, and the annual equAtion. Displays 4.2–4.4 show several stadia of the computation. As before, the numbers in square brackets correspond to Maskelyne's explanations in [Mayer, 1770]; they do not match the sequence of steps outlined above. The completed calculation is shown in display 4.4.

[1] *(Sun's true longitude and mean anomaly)* Find the true longitude and mean anomaly of the sun as already explained.

[2] *(Epoch and mean motion)* The first part of the computation is analogous to finding the sun's mean position. Look up the desired year in the moon's epoch tables and take out the mean longitudes of the moon, her apogee, and ascending node; write these numbers next to each other in three columns. Place under them the mean motions in longitude, apogee, and node taken from the mean motion tables for the month, day, and time of day. Prefix the secular acceleration (given in a separate table) to the column of the moon's longitude.[9]

[3] *(Mean position)* Add the numbers in the first column to find the moon's mean longitude, and add the numbers in the second column to find the mean longitude of apogee. Then find the mean longitude of the ascending node ☊ by *subtracting* the mean motions in the third column from the epoch of the node. These must be subtracted because the node moves retrograde or backwards.[10]

[4] *(Anomaly and elongation, and ten minor equAtions)* Subtract the longitude of apogee from the longitude of the moon. The difference is the mean anomaly of the moon; write this number under the apogee column. Also subtract the true longitude of the sun from the mean longitude of the moon, the result is the elongation

9 Strictly, what is here called the 'longitude' of the moon is an angle in the lunar orbit, not along the ecliptic. Thus it is not one of the standard ecliptic coordinates until the reduction in [8] below has been applied.

10 To make the calculations easier and more uniform, some later tables (e.g., [Lalande, 1764]; [Hell and Pilgram, 1772]) tabulated the complement of the longitude of the ascending node, which increases in time just like longitude and mean apogee.

	table	argument		value
	I	ς	$4^s\,10°\,59'$	$+\,8'\,34''$
	II	$2\omega+\varsigma$	$0^s\,21°\,51'$	$-\,0'\,20''$
	III	$2\omega-\varsigma$	$3^s\,29°\,53'$	$-\,1'\;\;0''$
	IV	$2\omega+p$	$7^s\,18°\,30'$	$-\,0'\,40''$
	V	$2\omega-p$	$9^s\;\;3°\,14'$	$+1°20'\,21''$
	VI	$2\omega-p+\varsigma$	$1^s\,14°\,13'$	$+\,1'\,30''$
	VII	$2\omega-p-\varsigma$	$4^s\,22°\,15'$	$+\,0'\,30''$
	VIII	$p-\varsigma$	$6^s\,26°\,39'$	$-\,0'\,15''$
	IX	$\Omega-\odot$	$6^s\,16°\,37'$	$+\,0'\,32''$
	X	$\omega-p$	$4^s\,27°\,48'$	$+\,1'\;\;2''$
[4]				$+1°30'\,14''$

Display 4.3: The minor equAtions in the computation of lunar longitude. All values in the right hand column have been looked up in tables.

of the moon.[11] Display 4.2 shows the calculation for the three basic mean arguments: mean lunar longitude, mean anomaly, and mean longitude of the ascending node. With the mean motion values thus found, form ten different arguments for ten different tables of what Mayer called the *minor equAtions*. Nine of these ten arguments are integer linear combinations of the three arguments that we have computed already (see display 4.1): the mean solar anomaly ς, elongation ω, and mean lunar anomaly p; an additional argument, $\Omega-\odot$ (see displays 4.1 and 4.2), expresses the orientation of the lunar line of nodes with respect to the direction of the sun. Look up the equAtion values in the tables for each of these ten equAtions, and write them down as in display 4.3. Add the equAtion values and write their sum at the bottom.

The Roman numerals correspond to the Roman numbering of the tables. The Ist equAtion is the annual equAtion of longitude; the Vth is the evection.

[5] *(Annual equAtions of anomaly and node, and equAtion of centre)* With argument ς, look up the annual equAtions of the moon's mean anomaly and ascending node. Next, we enter the second step of the multistep procedure, because we will now update some of the arguments with the equAtions found thus far. Add the annual equAtion of anomaly and the sum of the ten minor equAtions just found to the moon's mean anomaly. This will yield the corrected anomaly of the moon denoted by $\tilde{p}$. Use this as the argument to look up the equAtion of centre in table XI.

[6] *(Variation)* Add the sum of the ten minor equAtions and the equAtion of centre to the elongation ω, to get the corrected elongation $\tilde{\omega}$. Here begins the third step of the multistepped procedure. With $\tilde{\omega}$ look up the equAtion of the variation in table XII. Add this variation, the equAtion of centre, and the ten minor equAtions to the mean longitude to obtain the corrected longitude.

11 Actually, the term *elongation* is ambiguous, since it could mean either the difference in longitude of a body and the sun, or the angular separation of a body from the sun (see also [Meeus, 1998, p. 253]). In this text, elongation will be the difference of the ecliptic longitude of the true sun and the longitude of the mean moon in its orbit.

		longitude	apogee	node
[2]	sec. accel	$1'21''$ t		
[2]	epoch 1802	$7^s 24° 30' 5''$ t	$4^s 6° 47' 51''$ t	$11^s 24° 35' 40''$ t
[2]	200 years	$8^s 15° 47' 10''$ t	$7^s 8° 22' 30''$ t	$8^s 28° 22' 30''$ t
[2]	Nov 13	$7^s 6° 55' 4''$ t	$1^s 5° 18' 59''$ t	$16° 47' 12''$ t
[2]	20^h	$10° 58' 49''$ t	$5' 34''$ t	$2' 39''$ t
[3]				$9^s 15° 12' 21''$
[3]	mean long	$11^s 28° 12' 29''$	$20° 34' 54''$	$2^s 9° 23' 19''$
[4]	mean anom p		$11^s 7° 37' 35''$	
[4]	elongation ω	$4^s 5° 25' 58''$		
[5]	annual eqns.		$17' 36''$ t	$6' 42''$ t
[5]	minor eqns.	$1° 30' 14''$	$1° 30' 14''$	
[5]	cor. anom $\tilde{p}$		$11^s 9° 25' 25''$	
[5]	eqn. centre	$2° 5' 0''$ t		
[6,7]	cor. elong $\tilde{\omega}$	$4^s 9° 1' 12''$		$2^s 9° 30' 01''$ cor. Ω
[6]	variation	$-36' 21''$ t		
[6]	cor. long	$0^s 1° 11' 22''$		$0^s 1° 11' 22''$
[7]	δ			$9^s 21° 41' 21''$
[7]	$2\delta - \tilde{p}$			$8^s 3° 57' 17''$
[7]	eqn. XIII	$-1' 15''$ t		$-1' 15''$ t
[8]	reduction	$4' 36''$ t		$9^s 21° 40' 6''$ $\tilde{\delta}$
[9]	eqn. equinox	$-17''$ t		
[10]	eclipt. long	$0^s 1° 14' 26''$		

Display 4.4: The completed calculation of lunar longitude. The letter ‘t’ indicates values that have been looked up in a table.

[7] *(EquAtions to the node)* Now we come to the fourth step of the multistepped procedure. Correct the mean longitude of the ascending node by its annual equAtion found in [5]. Subtract the result from the moon’s corrected longitude, to get the equAted distance of the moon from the node δ. Compute $2\delta - \tilde{p}$, this is the argument of the equAtion listed in the XIIIth table. Write the value of this equAtion under the moon’s corrected longitude, and also under the node column.

[8] *(Reduction to the ecliptic)* Entering the fifth and final step of the multistepped procedure, add the XIIIth equAtion to the equAted distance from the node to get $\tilde{\delta}$, with which look up the equAtion of the reduction to the ecliptic in the XIVth table. Write the result under the longitude column.

[9] *(EquAtion of the equinoxes)* With the mean longitude of the node Ω as argument, look up the equAtion of the equinoxes in the XVth table. Write the result under the value obtained in the previous step.

[10] *(True longitude of the moon)* Add the last three equAtions to the corrected longitude of the moon to get its true longitude in the ecliptic.

4.5 A NOTE ON ACCURACY

We found that the tables predict the longitude of the moon in the ecliptic on 2002 November 15, 08:00 UT to be $1^\circ 14' 27''$, and the sun's longitude $232^\circ 46' 50''$. A computation using modern software[12] yields $1^\circ 10' 23''$ for the longitude of the moon, and $232^\circ 46' 38''$ for the sun's longitude. The difference is $4' 4''$ for the moon, and $12''$ for the sun.

It is well known from dynamical systems theory that every lunar theory will eventually diverge from the real position of the moon. In this example computation for a date in 2002, we see that the prediction according to the 18th-century lunar theory differs by $4' 4''$ from the modern prediction. That is almost $\frac{1}{7}$ of the moon's diameter, and comparable to the thickness of one or two coins seen from a distance of ten meter. Such a perhaps seemingly small divergence would make Mayer's lunar theory useless as the basis for the lunar distance method of finding geographical longitudes today—even if we would be content with the same margins that were acceptable in 1760. The result is also significantly worse than the accuracy of about $30''$ to $1'$ which Mayer aimed at.

Would the result of this single example be typical for the error values? To answer this and similar questions, I made a computer program to simulate some of the different lunar tables and theories of Mayer's. The program was first used to generate daily predictions (according to these same `rede` tables) a few months before and after 2002 November 15, and then to compare these to the modern predictions. The differences showed an almost monthly recurring pattern ranging from about $-1\frac{1}{2}'$ to $+4\frac{1}{2}'$. The result for the example above happens to be near an extreme.

Most of the error is brought about by the mean motions, which are slightly off the correct values, leading to a slowly increasing error in mean longitude, apogee, and node. Particularly the accumulated discrepancy in mean apogee (about $20'$) leads to a periodically varying error in lunar longitude, as is fully explained in appendix B, page 200. In modern times Mayer's `rede` tables predict the position of the moon generally better than our single example calculation suggested, but still not as accurate as these tables were in their own times.

The same procedure repeated for a time span in the 1750's shows only incidentally errors in excess of $1'$; table 6.3 on page 106 lists a standard deviation of $30''$ for these tables. We may draw the conclusion that Mayer's tables fulfilled his proclaimed expectations during his own era, but that their accuracy gets diluted over time through the drift in the mean motions, in particular the mean anomaly.

The prediction error that we found for the sun's position was a little less than $12''$ (about 0.6% of its diameter), which is much more accurate than that for the moon. This need not surprise us, because the motion of the sun is much more regular and much easier to predict than the moon's.

12 JPL HORIZONS 3.12 [Giorgini et al., 1996]

	`kil`	`rede`
I	ς	ς
II	$2\omega+\varsigma$	$2\omega+\varsigma$
III	$2\omega-\varsigma$	$2\omega-\varsigma$
IV	$2\omega-p+\varsigma$	$2\omega+p$
V	$2\omega-p-\varsigma$	$2\omega-p$
VI	$2\omega+p$	$2\omega-p+\varsigma$
VII	$2\delta-p$	$2\omega-p-\varsigma$
VIII	$p-\varsigma$	$p-\varsigma$
IX	$2\delta-2\omega$	$2\delta-2\omega$
X	$\omega-p$	$\omega-p$
XI	p	p
XII	$2\omega-p$	ω
XIII	ω	$2\delta-p$
XIV	2δ	2δ

Display 4.5: The sequence of equAtions in `kil` tables of 1753 (left) and `rede` tables of 1770 (right); only the arguments of the equAtions listed.

4.6 THE WANDERINGS OF EVECTION

The `kil` tables of 1753 and the `rede` tables of 1770 use the same multistep procedure of computation, with only a seemingly small change in the sequence of the equAtions, apart from differences in the coefficients.

The difference is illustrated in display 4.5, which lists the arguments of the equAtions in their `kil` and `rede` sequences, with steps separated by horizontal lines (mean motions are not shown). Since the reordering of equAtions within a step is inconsequential by our definition of a step, the noteworthy changes from `kil` to `rede` concern the positions of the arguments $2\omega-p$ (the argument of evection) and $2\delta-p$, which both move into another step. Table versions intermediate between `kil` and `rede` show that the move of the $2\delta-p$ equation took effect before March 1754, but that evection moved at about November 1754.

Why would Mayer have done this? In chapter 6 it will be made apparent that `kil`'s combined equAtion of centre and evection in its second step was a direct inheritance from Newton's lunar theory. For reasons that will also be made clear there, Mayer might possibly have had uneasy feelings about this when Bradley asked him to submit the theoretical basis of his tables. Therefore, the breaking up of the combination may have been appealing to Mayer.

But an other reason can be given, one that is more in accord with Mayer's own words: the change in the order of equAtions leads to more elegant operations for the table user.[13] In the new situation, argument $2\omega-p$ takes its place next to $2\omega+$

13 Mayer's words on this subject are 'But because the form of these [i.e., the tables in the `kil` format] is less appropriate, and [because] the calculation on behalf of the many equAtions that

p, nicely preceded by the two arguments $2\omega \pm \varsigma$ and nicely followed by the pair $2\omega - p \pm \varsigma$. This arrangement is easier for the human mind to work with, especially during bulk computations for many different dates. But a similar argument for the move of $2\delta - p$ is less convincing.

These changes are almost trivial for the table user, but they are far from trivial for the table maker. They have consequences for the magnitudes of the equAtion coefficients: not only of those equAtions that are moved around, but others as well. The technique to quantify the consequences is the subject matter of chapter 7; let it suffice here to provide some intuition of what is at hand.

Evection is an equAtion with an extreme value of $\approx 1°21'$, only second in magnitude to the equAtion of centre with maximum $\approx 6°32'$, and followed by number three, variation, which may reach $\approx 43'$. The change that we are considering is whether evection is applied before or after the equAtion of centre; in particular, whether or not evection affects the argument p. If the equAtions would be maintained with the same coefficients, then the resulting value of the equAtion of centre might differ by as much as $8'$, which is clearly devastating if the required accuracy of the end result is $\frac{1}{2}'$. To find the correct values of the coefficients, one needs to consider also combined terms with arguments such as $(2\omega - p) - p = 2(\omega - p)$, which affects a term of equation X, and $(2\omega - p) + p = 2\omega$, which affects the main term of the variation. Both these terms change by about $4'$, which is so much that in general equAtion X reverses sign. The equation with argument $2\delta - p$ is much easier to move around because its coefficient is much smaller.

It is now time first to turn to Mayer's account of his lunar theory; thereafter we have the full and right perspective to tackle the ins and outs of the multistepped procedure.

have thence to be taken out causes much trouble, I do not want to present them, and therefore I have sent only those that are easier to use. (*'Sed cum forma his sit minus commoda, et calculus propter plures aequationes inde excerpendas plus negotii facescat, nolui eas praeferre, atque adeo transmisi istas, quae usu faciliores sunt.'*) Mayer to Michaelis, 14 April 1755, reproduced in [Mayer, 1770, p. 43].