A. LUNAR EQUATIONS: VERSIONS AND ALIASES

This appendix contains information on the various versions of Mayer's lunar equAtions, in what locations they appear, and in what form they appear. Display A.1 lists this information, and is followed by a list of coefficients of some of the most prominent versions.

In most cases there is an indication of the form in which the equAtions appear: 'tables', as ready to use tables; 'list', as a list of coefficients and arguments; 'comparison', as a column in a tableau where different versions are put next to each other. Locations are abbreviated as EM for Euler-Mayer Correspondence [Forbes, 1971a], NA for Nautical Almanac, TL for *Theoria Lunae* [Mayer, 1767], and the usual manuscript abbreviations of appendix C. Dates are mostly indicative. The following remarks apply to the individual instances of lunar tables and equAtions.

`Duin` is Mayer's manuscript copy of Euler's lunar tables [Euler, 1746]. `Terp`, like `duin`, uses eccentric anomaly, which Mayer uses nowhere else in lunar tables. Therefore (and because `duin` and `terp` occur in the same manuscript) they can be considered as near-contemporary. The coefficients of `kreek` are not known, but the characteristics that Mayer reported to De l'Isle of these lost tables are met by certain position calculations (as mentioned on p. 27); the same characteristics are not met by any other versions described here. In principle, then, the `kreek` coefficients could be reconstructed from those position calculations.

`Zand`, `diep`, and `gors` are very nearly identical; Mayer reported `zand` in a letter to Euler written on 6 January 1752, therefore we have an upper bound for its dating. According to the table headings in Cod. $\mu^{\sharp}_{15}$ (see fig. 6.4), `gors` was a fit of `geul` to observations. Therefore `geul` must date from (late) 1751.

In reply to Mayer's letter containing `zand`, Euler sent Clairaut's coefficients, which Mayer included in Cod. $\mu^{\sharp}_{15}$. This helps to date `griend` and `grond`, and additionally it puts a lower bound on the date of `geer`, the version that clearly started the multistepped development stage as described in section 6.7.

The development from `geer` via `gat`, `put`, and `plaat` into `zwin`, `pas`, and finally `kil` can be traced out by analysis of the position calculations in Cod. $\mu^{\sharp}_{41}$ in combination with a double folio of spreadsheets in Cod. $\mu^{\sharp}_{28}$. `Zwin` was reported in a letter to Euler on 7 January 1753. Only slight changes separate `zwin` from the `kil` tables that were published in spring in [Mayer, 1753b]. The example computation that went with the `kil` edition still employed the `zwin` tables, and Mayer warned his readers that it was based on an older version.

The post-`kil` tables and lists can be divided into two groups: the ones that have, like `kil`, the equAtion of centre and evection together in the middle step, and the newer ones with evection relocated in the first step. Even for incomplete lists or tables it is often possible to determine to which group they belong by looking at the coefficients for the equAtions with arguments 2ω or $2(\omega - p)$, if present.

The tables in the first group include `val`, `vlije`, `vlakte`, and `mui`. These all have the equAtion of argument $2\delta - p$ moved to the end of the procedure. `Vlakte` and `vlije` form a pair that complement each other, not only by their equAtions but also by their physical location as two pamphlets stuck together. `Vlakte` was in use in March 1754, when Mayer answered to Euler the question about an averred Chinese eclipse reported by Gaubil. Mayer's computations regarding the question are extant in one of the last folios of Cod. $\mu^{\sharp}_{41}$.[1]

Apparently `mui` was used with the computations in Cod. μ_5. Early in that manuscript, there are dates 1754 March and April; later Mayer added a calculation for 2 Oct 1754. This bounds `mui`'s date.

Computations necessary to move evection away from the equAtion of centre are on foll. 52–53 of Cod. μ_7. These start out with a variant of the `vlakte` and `mui` coefficients; they are immediately followed by a list of the new coefficients of `ley`, now with evection among the minor equAtions (albeit in position VII instead of V).

The version that Nevil Maskelyne repeatedly referred to as the 'first manuscript tables' were the `rak` tables that Mayer sent to England around the beginning of December, 1754. These are almost identical to `ley` and have evection as equAtion number V. The relocation computations of Cod. μ_7 just mentioned thus took place in October or November 1754.

Quickly afterwards, in April of the following year, Mayer wrote a letter stating not only his reasons for the relocation of evection, but also several further amendments of the coefficients, which perfectly match the transition from `rak` to `wijd`.[2]

`Wantij`, which is the multistepped form computed from the single-stepped theoretical solution `waard` of *Theoria Lunae*, was originally computed in Cod. $\mu^{\sharp}_{28}$. `Wad` is `wantij` with certain adjustments obtained from observations: these adjustments, as Mayer dutifully reported in *Theoria Lunae*,[3] consist of a revised eccentricity and solar parallax on the basis of the maximum observed elliptic and parallactic equAtions respectively, and a changed evection coefficient. It is not the result of a complete fit of all the coefficients independently to observations. *Theoria Lunae* included a comparison between `wad` and `wijd` (his latest fitted version at that moment) in order to show to what extent theory and tables agreed. (As we have seen in chapter 5, the computation of the coefficients in the theory made such an abundant use of fitted table coefficients, that we may doubt whether the test was unbiased).

There remains one set of tables in Cod. $\mu^{\sharp}_{49}$, `veen`. It is of the kind that has evection with the minor equations, unlike the other versions contained in that man-

1 This episode is recounted in [Forbes, 1980, p. 147], the exchange between Euler and Mayer is recorded in [Forbes, 1971a, pp. 79, 80, 84], and Mayer's fuller account can be read in [Forbes, 1972, I, pp. 96–98].

2 The letter was published in [Mayer, 1770, pp. 43–45]; the reasons are quoted on p. 4.6 above. Coefficients of `wijd` are also included in *Theoria Lunae* [Mayer, 1767, p. 52].

3 Quoted on p. 133 above.

uscript. It is practically identical to `rede`, the ‘last manuscript tables’ as Maskelyne called them, which were sent by Mayer’s widow to England in 1763. The `rede` tables were printed in [Mayer, 1770]; the original manuscript tables without table headings are in RGO4/125 (second quire), the crudely written headings in mixed Latin and German are in RGO4/130.[4] Maskelyne listed the coefficients of `kil` and `rede` in the Nautical Almanac for the year 1774, together with two variant coefficient lists devised by Bradley and Morris after Mayer’s tables. One page was torn out of the `veen` quire and sent to England together with the pack of last manuscript tables, because it contained a corrected secular acceleration table on one side. On the reverse side was half of `veen`’s evection table.

`Oever` seems to be a pre-version of `wantij`. The first few pages of Cod. μ_8 show the transformation of multistepped `rede` into the equivalent but single-stepped `schor`. The latter was then listed under the heading *juxta tabb. noviss.* next to `sloe`, which, headed by *juxta calc. theor. corr.*, looks like a revised version of the very similar `waard`. The list also included the coefficients of the theories of d’Alembert and Clairaut and was presumably meant to compare the various results. Finally, the coefficients of `balg` (in particular, the coefficients of equAtions $2\omega - 2p$ and 2ω) suggest that it belongs to the same family as `rede`, but otherwise `balg` is quite a-typical. It appears in Cod. μ_4 after pages of lunar theory calculated at a later date than *Theoria Lunae* was written.

Display A.2 The following remarks apply to display A.2 of the coefficients inof the most important versions of tables and equAtions.

In the list of `mui` coefficients, the symbol † indicates those coefficients that have been reconstructed from position calculations; the double entries occur because some of the `mui` tables had been corrected by a pasted-on new version.

The `kil` epochs and mean motions were extracted from the published version of 1753, theand subsequently they were adjusted from the Paris meridian to the Greenwich meridian (by $+9^{\mathrm{m}}16^{\mathrm{s}}$), and then $2''$ were added to the lunar longitude on account of the secular acceleration.

The `rak` epochs were copied from the preface of the Nautical Almanac for the year 1774.

The `rede` epochs and mean motions were taken from [Mayer, 1770]; their epochs agree to what is published in the Nautical Almanac just quoted. The stated 1750.0 epochs include secular acceleration.

4 Actually two sets of the `rede` tables are extant in RGO4/125; one was sent by the widow two years later [Forbes, 1966, p. 109]. There are also revised mean motion tables referred to the Greenwich meridian, presumably made by Maskelyne or an assistant.

alias	date	source	comments
`duin`	1746	Cod. $\mu_{14}^{\sharp}$	tables, copy of Euler's 1746
`terp`		Cod. $\mu_{14}^{\sharp}$, f.14+	tables
`kreek`	Jan. 1751	lost	reported in correspondence
`geul`	1751	Cod. $\mu_{15}^{\sharp}$, f.8v	list *calculus m. ex theor.* (fig. 6.4)
`gors`		Cod. $\mu_{15}^{\sharp}$, f.8v	list *corr ex observ* (fig. 6.4)
`diep`		Cod. $\mu_{14}^{\sharp}$	list
`zand`	Jan. 1752	EM, p.48	list, from correspondence
`griend`	Mar. 1752	Cod. $\mu_{15}^{\sharp}$, f.8v	list *tabb* ☽ *calc* (fig. 6.4)
`grond`	Mar. 1752	Cod. $\mu_{15}^{\sharp}$, ff.9-16v	tables
`geer`	later 1752	Cod. $\mu_{15}^{\sharp}$, f.17r	*Entwurf* (display 6.2)
`gat`		Cod. $\mu_{15}^{\sharp}$, f.17v+	tables, very simple
`put`		Cod. $\mu_{41}^{\sharp}$, f.11-30	reconstructed from computations
`plaat`		Cod. $\mu_{41}^{\sharp}$, f.108	tables, draft
`zwin`	Jan. 1753	EM, p.62	list, from correspondence
`pas`		Cod. $\mu_{41}^{\sharp}$, f.150v	list
`kil`	spring '53	[Mayer, 1753b]	tables published $<$ May 7, 1753
`val`	1754	Cod. $\mu_{49}^{\sharp}$	tables, 1st booklet, neat, almost complete
`vlakte`		Cod. $\mu_{49}^{\sharp}$	tables, 3d booklet
`vlije`	Mar. 1754	Cod. $\mu_{49}^{\sharp}$	tables, booklet stuck inside `vlakte`
`mui`		Cod. $\mu_{30}^{\sharp}$	tables, only one sheet
`ley`	Oct/Nov	Cod. μ_7, f.54v	list
`rak`	Nov. 1754	NA 1774	list, 'last manuscript tables'
`wijd`	Apr. 1755	TL p.52	list, *tabulas… ultimam correctionem*
`waard`	mid '55	TL p.46-47	list, single-stepped, see display 7.1
`wantij`	1755	Cod. $\mu_{28}^{\sharp}$,II	list, also in *Theoria Lunae*, see display 7.1
`wad`	1755	TL p.52	list *ex calc*, improved by observations
`veen`		Cod. $\mu_{49}^{\sharp}$	tables, one p. dislocated to RGO4/125(3)
`rede`	after '55	NA 1774	'last manuscript tables'
`oever`	1755?	Cod. $\mu_{28}^{\sharp}$,III,50	list
`schor`	post-rede	Cod. μ_8, f.7r	list, *juxta tabb. noviss*
`sloe`		Cod. μ_8, f.7r	list, *juxta calc. theor. corr.*
`balg`		Cod. μ_4, f.58-59	list

Display A.1: Versions of lunar equations found among Mayer's papers.

	`kil`		`vlakte`	`vlije`	`mui`	
epoch	$6^s\,8°22'11''$					long
1750.0	$5^s20°56'50''$					apog
Greenw	$9^s10°19'\,8''$					node
mean	$1^s10°43'24''$					long
motion	$9^s11°30'45''$					apog
/60JY	$2^s20°30'45''$					node
An.A	$+20'36''$	An.A	$+25'15''$	$+22'28''$		$\sin(\varsigma)$
	$-18''$		$-16''$	$-14''$		$\sin(2\varsigma)$
			$1'12''$			$\sin(\Omega)$
An.N	$+10'18''$	An.N	?	$+9'12''$		$\sin(\varsigma)$
	$-9''$		?	$-6''$		$\sin(2\varsigma)$
I	$+11'20''$	I	$+11'14''$	$+11'14''$		$\sin(\varsigma)$
	$-10''$		$-\,7''$	$-7''$		$\sin(2\varsigma)$
II	$-54''$	II	$-56''$	$-58''$	$\dagger{-56''}$	$\sin(2\omega+\varsigma)$
III	$-1'\,2''$	III		$-1'6''$	$-1'\,6''$	$\sin(2\omega-\varsigma)$
IV	$+1'48''$	IV		$+1'50''$	$+1'50''$	$\sin(2\omega-p+\varsigma)$
V	$+1'12''$	V		$+57''$	$+57,+49''$	$\sin(2\omega-p-\varsigma)$
VI	$+1'30''$	VI		$+1'\,7''$	$+1'\,4''$	$\sin(2\omega+p)$
VII	$+58''$	XIV	$+1'25''$	$+1'25''$		$\sin(2\delta-p)$
VIII	$+40''$	VII		$+30''$	$+14,+28''$	$\sin(p-\varsigma)$
IX	$-47''$	VIII		$-56''$	$-56''$	$\sin(2\delta-2\omega)$
X	$+29''$	IX	$+16''$	?	$\dagger{+23''}$	$\sin(\omega-p)$
	$+3'24''$		$+3'31''$		$\dagger{+3'26''}$	$\sin(2\omega-2p)$
		X	$+7''$			$\sin(2\omega-2s)$
		Xa	$-6''$			$\sin(2\omega-3p)$
			$4''$			$\sin(\Omega)$
XI	$-6°18'26''$	XI	$-6°18'18''$		$\dagger{-6°18'18''}$	$\sin(p)$
	$+13'\,0''$		$+12'53''$		$\dagger{+12'53''}$	$\sin(2p)$
	$-36''$		$-35''$		$\dagger{-34''}$	$\sin(3p)$
XII	$-1°20'42''$	XII	$-1°20'50''$		$\dagger{-1°20'52''}$	$\sin(2\omega-p)$
	$+35''$		$+27''$		$\dagger{+32''}$	$\sin(4\omega-2p)$
		XIIa	$+5''$			$\sin(4\omega-p)$
XIII	$-1'55''$	XIII	$-1'55''$			$\sin(\omega)$
	$+40'21''$		$+40'15''$			$\sin(2\omega)$
	$+2''$		$+2''$			$\sin(3\omega)$
	$+17''$		$+14''$			$\sin(4\omega)$
XIV	$-6'57''$	XIV	$-6'51''$	$-6'51''$		$\sin(2\delta)$

Display A.2: Coefficients of the most prominent versions (continued on next page).

	`wad`	`rak`	`wijd`	`veen`	`rede`	
epoch	$6^s\,8^\circ22'39''$				$6^s\,8^\circ22'26''$	long
1750.0	$5^s20^\circ55'59''$				$5^s20^\circ55'54''$	apog
Greenw	$9^s10^\circ18'38''$				$9^s10^\circ19'\,8''$	node
mean					$1^s10^\circ44'\,9''$	long
motion					$9^s11^\circ30'45''$	apog
/60JY					$2^s20^\circ30'45''$	node
An.A	$+22'\,6''$	$+25'15''$		$+23'12''$	$+23'12''$	$\sin(\varsigma)$
	$-15''$	$-16''$		$-6''$	$-6''$	$\sin(2\varsigma)$
An.N		$+9'13''$		??	$+8'50''$	$\sin(\varsigma)$
		$-6''$		??	$-2''$	$\sin(2\varsigma)$
I	$+11'39''$	$+11'14''$	$+11'14''$	$+11'18''$	$+11'16''$	$\sin(\varsigma)$
	$-10''$	$-4''$	$-4''$	$-4''$	$-4''$	$\sin(2\varsigma)$
II	$-58''$	$-56''$	$-56''$	$-53''$	$-54''$	$\sin(2\omega+\varsigma)$
III	$-1'13''$	$-1'\,6''$	$-1'\,6''$	$-1'\,8''$	$-1'\,9''$	$\sin(2\omega-\varsigma)$
IV	$+55''$	$+53''$	$+49''$	$+53''$	$+54''$	$\sin(2\omega+p)$
V	$-1^\circ20'\,8''$	$-1^\circ20'36''$	$-1^\circ20'36''$	$-1^\circ20'33''$	$-1^\circ20'33''$	$\sin(2\omega-p)$
	$+38''$	$+38''$	$+26''$	$+35''$	$+35''$	$\sin(4\omega-2p)$
VI	$+2'11''$	$+2'\,0''$	$+2'\,0''$	$+2'\,8''$	$+2'\,9''$	$\sin(2\omega-p+\varsigma)$
VII	$+44''$	$+39''$	$+47''$	$+47''$	$+49''$	$\sin(2\omega-p-\varsigma)$
VIII	$+40''$	$+28''$	$+28''$	$+34''$	$+34''$	$\sin(p-\varsigma)$
IX	$-1'26''$	$-56''$	$-51''$	$-56''$	$-56''$	$\sin(2\delta-2\omega)$
X	$+8''$	$+16''$	$+16''$	$+16''$	$+16''$	$\sin(\omega-p)$
	$-1'\,2''$	$-57''$	$-1'\,0''$	$-1'\,0''$	$-1'\,0''$	$\sin(2\omega-2p)$
Xa			$+4''$			$\sin(\Omega)$
XI	$-6^\circ18'11''$	$-6^\circ18'11''$	$-6^\circ18'11''$	$-6^\circ18'17''$	$-6^\circ18'15''$	$\sin(p)$
	$+13'\,2''$	$+12'52''$	$+12'52''$	$+13'\,0''$	$+13'\,0''$	$\sin(2p)$
	$-36''$	$-37''$	$-37''$	$-37''$	$-37''$	$\sin(3p)$
				$+2''$	$+2''$	$\sin(4p)$
XII	$-1'55''$	$-1'55''$	$-1'55''$	$-1'56''$	$-1'55''$	$\sin(\omega)$
	$+35'42''$	$+35'47''$	$+35'47''$	$+35'44''$	$+35'43''$	$\sin(2\omega)$
	$+1''$	$+2''$	$+2''$	$+1.3''$	$+2''$	$\sin(3\omega)$
	$+5''$	$+14''$	$+14''$	$+11''$	$+12''$	$\sin(4\omega)$
XIII	$+1'15''$	$+1'25''$	$+1'26''$	$+1'25''$	$+1'23''$	$\sin(2\delta-p)$
XIV	$-6'51''$	$-6'51''$	$-6'51''$	$-6'44''$	$-6'43''$	$\sin(2\delta)$