

Climate Sensitivity of Franz Josef Glacier, New Zealand, as Revealed by Numerical Modeling

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Abstract

The sensitivity of Franz Josef Glacier is studied with a numerical ice-flow model. The model calculates ice mass flux along a central flow line and deals with the three-dimensional geometry in a parameterized way. Forcing is provided through a mass balance model that generates specific balance from climatological input data. Because of the very large mass turnover, *e*-folding response times are short: about 15 yr for ice volume and about 25 yr for glacier length. The sensitivity of glacier length to uniform warming is about 1.5 km K⁻¹. The sensitivity to uniform changes in precipitation is about 0.05 km %⁻¹, implying that a 30% increase in precipitation would be needed to compensate for a 1 K warming.

Introduction

Because of its location, climatologists are very interested in understanding the historic fluctuations of Franz Josef Glacier, New Zealand. Information on climate change on the century time scale from this part of the world is scanty, so any proxy climate indicator seems welcome. A number of studies have been carried out in which possible connections between the behavior of this glacier and patterns of climatic change have been discussed (e.g. Brazier et al., 1992; Fitzharris et al., 1992; Woo and Fitzharris, 1992). These authors have suggested that Franz Josef Glacier is notably sensitive to changes in precipitation and changes in atmospheric circulation. One of the goals of this study is to see if these ideas can be supported by model calculations.

The response of the position of a glacier front to climate change is the outcome of a process that has several steps. Modeling this response requires an approach in which two main modules are coupled. A *mass-balance model* should translate changes in meteorological conditions into changes in specific balance. This should serve as forcing for an *ice-flow model*, which delivers changes in glacier geometry in the course of time.

Here results are presented from a combined mass balance-ice flow model. Mass balance profiles generated with a meteorological model will be discussed and imposed to the flow model. This will provide insight in the basic sensitivity of the glacier, i.e. changes in geometry in response to changes in temperature and precipitation. Also response times will be considered. Finally an order-of-magnitude estimate will be made of the magnitude of climate change needed to produce the observed glacier retreat since the little ice age.

More detailed time-dependent simulations, a comparison of glacier behavior with instrumental records of meteorological quantities, and projection into the 21st century for various climate-change scenarios will be presented in a later paper.

Franz Josef Glacier

It is likely that Abel Tasman (a Dutch explorer who discovered Tasmania and New Zealand) was the first person from the western world to see the glacier (in 1642; Chinn, 1989). The glacier is located at 43°28'S, 170°11'E, in a maritime climate

with very high precipitation amounts. It has a wide upper basin and a narrow tongue flowing in north-westerly direction (Fig. 1). Denton and Hendy (1994) referred to an estimated area of 37 km²; Brazier et al. (1992) mentioned 34 km² (excluding the Salisbury snowfield).

Investigations by explorers/scientists did not start until the second half of the 19th century. On the basis of geomorphological field evidence, maximum stands have been identified around A.D. 1750 and 1820. More data points on the length of the glacier are available since 1850 (see Fig. 2). The 1750 stand most likely represents the Neoglacial maximum. There appears to have been steady retreat since that time, with a tremendous acceleration starting around 1935. The Neoglacial minimum was reached in the early 1980s and since that time a significant advance took place.

Although a number of interesting scientific projects have been carried out on Franz Josef Glacier (e.g. Ishikawa et al., 1992), long-term systematic investigations on mass balance or ice dynamics have not been done. It is generally thought that Franz Josef Glacier has a small response time, because it is fairly steep and has a tremendous mass turnover (precipitation in the 4 to 10 m a⁻¹ range).

The Ice-Flow Model

Information on the geometry and dynamics of Franz Josef Glacier is really scanty, and one may wonder if a modeling effort is worthwhile. In this study the challenge is to see how far one can get if the only information is a topographic map and data from a nearby climate station.

The ice-flow model used in this study has already been used in earlier studies (e.g. for Nigardsbreen: Oerlemans, 1986, 1992, 1997; Rhonegletscher: Stroeven et al., 1989; Hintereisferner: Greuell, 1992). Here only a brief discussion is given. The model is basically one-dimensional (along a flow-line, the *x*-axis, see Fig. 1), but takes into account the effect of geometry on mass conservation. More specifically, the width of the flow line may depend on *x* and ice thickness (Fig. 3). The choice of the flow line in the upper part of the glacier is rather ambiguous but not very critical to what happens at the glacier snout. It is important, however, to retain the area-elevation distribution.

Simple shearing flow and sliding is considered. The lateral

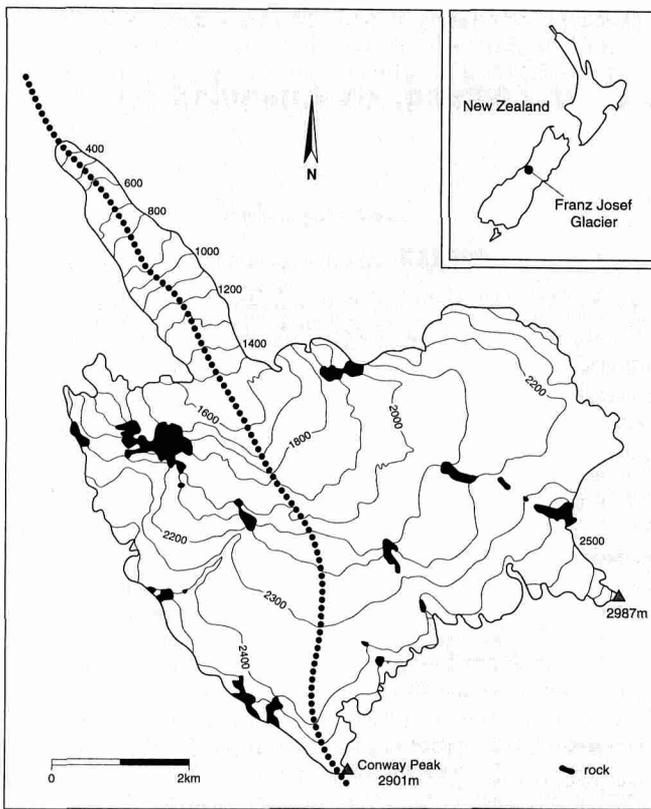


FIGURE 1. A map of Franz Josef glacier, based on topographic maps H34, G35, H35, and H36 (Dept. of Survey and Land Information, Upper Hutt, New Zealand). The flow line used in the numerical model, with grid points, is shown in (the) map.

geometry is parameterized by a trapezoidal cross section, having two degrees of freedom. Shape factors (e.g. Paterson, 1994) are not used. At the head of the glacier $x = 0$ by definition.

Denoting the cross sectional area by S , conservation of mass (for constant density) can be formulated as

$$\frac{\partial S}{\partial t} = -\frac{\partial(US)}{\partial x} + wB \quad (1)$$

where t is time and U is the vertical mean ice velocity. Here B is the specific balance and w the glacier width at the surface, written as:

$$w = w_0 + \lambda H \quad (2)$$

where H is the ice thickness.

For a trapezoidal geometry the area of the cross section is

$$S = H(w_0 + \frac{1}{2}\lambda H) \quad (3)$$

Substituting equation (3) in (1) yields the rate equation for ice thickness:

$$\frac{\partial H}{\partial t} = \frac{-1}{w_0 + \lambda H} \frac{\partial}{\partial x} \left[\left(w_0 + \frac{1}{2}\lambda H \right) UH \right] + B. \quad (4)$$

The vertical mean ice velocity U is entirely determined by the local "driving stress" τ , being proportional to the ice thickness H and surface slope (h is surface elevation):

$$\tau = -\rho g H \frac{\partial h}{\partial x}. \quad (5)$$

Following a Weertman-type law for sliding and assuming that the basal water pressure is a constant fraction of the ice over-

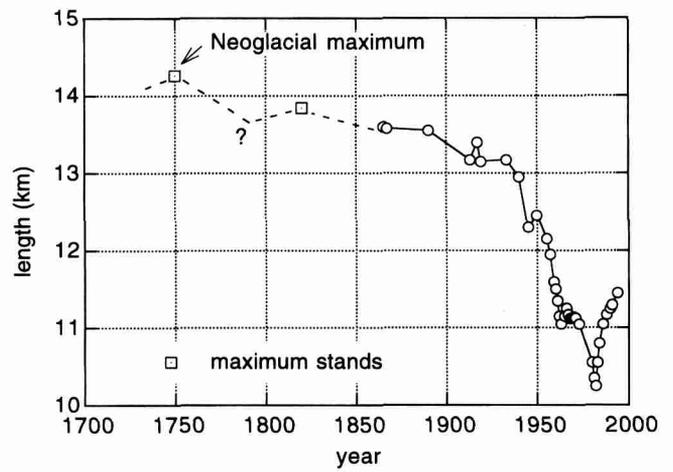


FIGURE 2. Record of glacier length based on Chinn (1989), Woo and Fitzharris (1992) and Fitzharris (pers. comm., 1996).

burden than leads to the following formulation for the vertical mean ice velocity (Budd et al., 1979):

$$U = U_d + U_s = f_d H \tau^3 + \frac{f_s \tau^3}{H}. \quad (6)$$

Here U_d and U_s are the contributions from deformation and sliding, and f_d and f_s are the corresponding flow parameters. The values suggested by Budd et al. (1979) are $f_d = 1.9 \cdot 10^{-24} \text{ Pa}^{-3} \text{ s}^{-1}$ and $f_s = 5.7 \cdot 10^{-20} \text{ Pa}^{-3} \text{ m}^2 \text{ s}^{-1}$. These are used in the present study.

With the aid of equation (5) the ice velocity can be expressed in thickness and surface slope:

$$U = \left\{ f_d \gamma H^4 \left(\frac{\partial h}{\partial x} \right)^2 + f_s \gamma H^2 \left(\frac{\partial h}{\partial x} \right)^2 \right\} \frac{\partial h}{\partial x}, \quad \text{where } \gamma = (\rho g)^3. \quad (7)$$

Substitution into equation (4) now shows that the change of ice thickness is governed by a nonlinear diffusion equation:

$$\frac{\partial H}{\partial t} = \frac{-1}{w_0 + \lambda H} \frac{\partial}{\partial x} \left[D \frac{\partial(b + H)}{\partial x} \right] + B. \quad (8)$$

where the diffusivity is

$$D = \left(w_0 + \frac{1}{2}\lambda H \right) \left\{ f_d \gamma H^5 \left(\frac{\partial h}{\partial x} \right)^2 + f_s \gamma H^3 \left(\frac{\partial h}{\partial x} \right)^2 \right\}. \quad (9)$$

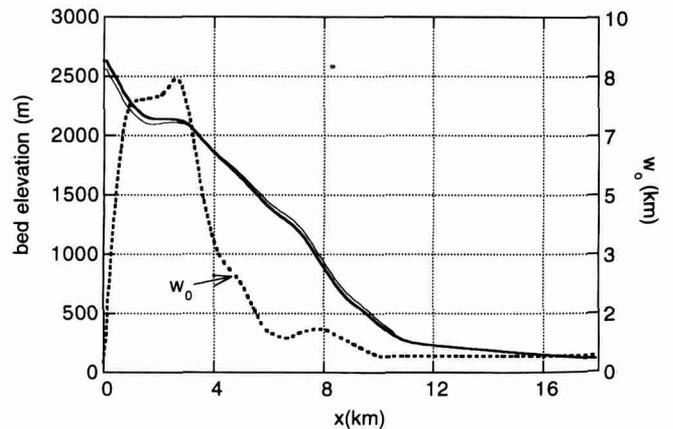


FIGURE 3. Geometric input data (glacier width at the bed and orography of the bed) for the flow-line model. The thin line shows the initial bed profile (after smoothing), the solid line the profile after correction for the variation of the basal stress as explained in the text.

Equation (8) is solved on a 100-m grid. Time integration is done with a forward explicit scheme, which is known to be stable for parabolic equations if the CFL-condition is met (e.g. Smith, 1978). The flow line has 180 grid points.

The topography of the bed of Franz Josef Glacier is unknown, except for the part of the valley now deglaciated. Consequently, an estimate of bed elevation was obtained by assuming that surface slope times ice thickness is constant (this would be true for perfectly plastic behavior; e.g. Oerlemans and Van der Veen, 1984). The bed elevation at grid point i is therefore calculated as

$$b_i = h_i - \frac{\tau_y}{\rho g s_i} \quad (10)$$

Here s_i is the surface slope estimated from the map and τ_y a constant yield stress. Practice has shown that for continental glaciers with moderate mass turnover a value of 10^5 Pa for τ_y is a good choice. For maritime glaciers with large mass flow a value of 1.5 to $2 \cdot 10^5$ Pa is preferable. Here a value of $2 \cdot 10^5$ Pa has been used. Once an initial bed profile $b(x)$ has been obtained smoothing in space should be applied, because estimated slopes may have large errors. Also, the assumptions underlying equation (6), in which the effect of longitudinal stresses is totally ignored, are only justified when interest is in length scales that are several times the characteristic ice thickness.

After having calculated a model glacier in steady state (in fact the state diagnosed in Fig. 8), the calculated pattern of basal stress was used to make a correction to the original bed profile. This was done by fitting a third order polynomial through the calculated stress field and use this through equation (10) to make a "correction" to the initial bed topography. Although this does not necessarily lead to a bed profile that approaches the real unknown profile better, it will provide an idea on how critical the assumption of constant base stress is. If the "corrected" profile would deviate substantially from the first estimate, the procedure should be rejected. Fortunately, the "correction" turned out to be small, as shown in Figure 3. In this figure the input parameters $b(x)$, $w_b(x)$, and $\lambda(x)$ are plotted in one frame.

In spite of the fact that the "corrected" bed profile deviates only slightly from the initial one, sensitivity experiments were done with both bed profiles. It appeared that results are not sensitive to the precise choice of orography. Only the broad scale undulations are of significance.

Calculation of the Specific Balance

The mass balance model used has been documented in a study of Norwegian glaciers (Oerlemans, 1992). Here a summary is given. The treatment of the various energy transfers between atmosphere and glacier surface is described qualitatively.

The basic equation is

$$B = \int_{\text{year}} \{(1 - f) \min(0; -F\downarrow/L + P^*)\} dt \quad (11)$$

Here B is the specific balance, $F\downarrow$ the energy flux at the surface, L the latent heat of melting, and P^* the rate of precipitation/evaporation in solid form. The fraction f accounts for meltwater that does not runoff but refreezes in the firn. Equation (11) is based on the assumption that melting occurs as soon as the surface energy flux becomes positive. For mid-latitude glaciers this assumption is justified as the subsurface storage of heat is small (Greuell and Oerlemans, 1987).

$F\downarrow$ then has four significant components:

$$F\downarrow = (1 - \alpha)G + L\uparrow + L\downarrow + H_{se} + H_{la} \quad (12)$$

The first term on the right-hand side represents absorbed solar radiation (G is global radiation, α albedo), the second and third term upwelling and downwelling infrared radiation, the third and fourth term turbulent exchange of sensible heat and latent heat (water vapor).

The calculation of the global radiation on an inclined surface of a valley glacier is very complicated. In fact, a detailed calculation can only be done if a digital terrain model is available (which, to the knowledge of the author, does not exist for Franz Josef Glacier). In the current model a two-stream approximation is used, that is, solar radiation is split into a direct part and an isotropic diffusive part. The partition depends on cloudiness (more diffusive radiation when cloudiness is larger). This partition makes it possible to take orientation and slope of the glacier surface into account (only the amount of direct radiation is affected by this). For Franz Josef Glacier the exposition was taken into account in a crude way by using a surface slope of 0.2 and an exposure of 320° in all radiation calculations. In energy-balance calculations the treatment of albedo is crucial. Albedo varies strongly in space and time, depending on the melt and accumulation history. Because significant feedbacks are involved, albedo should be generated internally if one wants to climate change experiments. This is difficult, however, as the albedo depends in a complicated way on crystal structure, ice and snow morphology, dust concentrations, morainic material, liquid water in veins, water running across the surface, solar elevation, cloudiness, etc. The best one can do is to construct a simple scheme, in which the gross features show up and broadly match available data from valley glaciers.

The basis of the scheme is formed by a background albedo profile, giving the albedo in dependence of altitude relative to the equilibrium-line altitude. This can be considered as the characteristic albedo at the end of the ablation season. Values range from 0.2 on the glacier tongue to about 0.6 in the accumulation basin. Then the albedo increases when snow is present. A simple formulation is used to give a smooth transition between ice and snow albedo, depending on snow depth.

The calculation of incoming longwave radiation follows the approach suggested by Kimball et al. (1982). Two contributions are distinguished: one from the clear-sky atmosphere, and one originating at the base of clouds and transmitted in the 8- to 14- μm band (atmospheric window).

Turbulent fluxes over melting ice/snow surfaces can be quite large in spite of the stable stratification normally encountered. Compared to other atmospheric boundary layers, much of the turbulent kinetic energy is concentrated in relatively small scales, and it appears that existing schemes for stable boundary layers underestimate the turbulent fluxes. As for most cases surface roughness and climatological wind conditions are unknown, a constant exchange coefficient is used in the hope that the bulk effect of the fluxes is captured.

In order to include the daily cycle, a 15-min time step is used to integrate equation (11). Precipitation is assumed to fall in solid form when air temperature is below 2°C . Rain is assumed to runoff immediately, so it does not contribute to the mass balance.

Model parameters like turbulent exchange coefficients, cloud radiative properties, and albedo values of snow and ice were initially given the same values as in the study of Norwegian glaciers (Oerlemans, 1992). Climatological input quantities taken from the nearby climate station (National Park Headquarters,

TABLE 1

Meteorological input quantities used for the calculation of the specific balance^a

Annual temperature (°C)	11.1
Seasonal temperature range (K)	7.4
Daily temperature range (K)	6
Temperature lapse rate (K m ⁻¹)	0.0065
Mean cloudiness	0.78
Mean relative humidity (%)	90
Annual precipitation (m)	$5 + 0.008z - 3.2 \times 10^{-6}z^2$
Annual precipitation* (m)	$5 + 0.0048z - 1.92 \times 10^{-6}z^2$

^a In the expression for the precipitation, z is altitude (in m a.s.l.). The expression finally used is marked with an asterisk (see text).

Franz Josef village, about 6 km from the current position of the glacier snout) are given in Table 1. In addition, estimates of temperature lapse rate and precipitation are needed. The lapse rate was simply taken constant and set to 0.0065 K m⁻¹. Precipitation is difficult, of course. The climate station mentioned above has an annual precipitation of slightly over 5 m a⁻¹. It is known that on the western slopes precipitation increases rapidly with height and reaches a maximum value. Brazier et al. (1992) mention a maximum value of 10 m a⁻¹ at an elevation of 1250 m. As a first estimate, precipitation was formulated as a parabola (in terms of elevation) going through the z,P points {0 m, 5 m a⁻¹}, {1250 m, 5 m a⁻¹}, {2500 m, 5 m a⁻¹}, i.e.

$$P = 5 + 0.008z - 0.000032z^2 \text{ m a}^{-1}. \quad (13)$$

This results in lower accumulation in the higher parts of the catchment, in line with the general belief that large amounts of drifting snow are lost across the divide because of the prevailing strong westerly winds (Woo and Fitzharris, 1992).

The calculated balance profile obtained with the input described above was imposed on the flow model. This resulted in a glacier that grew unrealistically large. In principle, this can be caused by unrealistic values of the flow parameters, or by a too inaccurate estimate of the bed topography. A number of tests were carried out to see where the large sensitivities are. This undoubtedly appeared to be in the specification of the climatological input and in the albedo values used in the calculation of the energy budget. It was decided to:

- accept first of all that the quality of the input data does not allow any absolute results (e.g. the current mean specific balance) to be obtained;
- lower the ice albedo by 0.06 and to set the precipitation maximum (at 1250 m elevation) at 8 m a⁻¹ instead of 10 m a⁻¹.

Most likely, values for ice albedo and precipitation are still well within their range of uncertainty. In any case, no further attempts were made to adjust bed geometry or flow parameters. The reference case thus defined still has a large glacier as equilibrium state: the steady state length is 12.55 km (compare Fig. 2). However, note that the choice of reference case is not of direct relevance, as in the following only differences associated with different climatic states will be discussed.

To illustrate the type of output produced by the mass balance model, Figure 4 shows the cumulative balance at five different altitudes. Apparently, for mean climatological conditions, there is always melting at 450 and 950 m. At 2450 m altitude there is very little melt, even in summer. Note that for the first part of the balance year the cumulative balance is larger at 1950

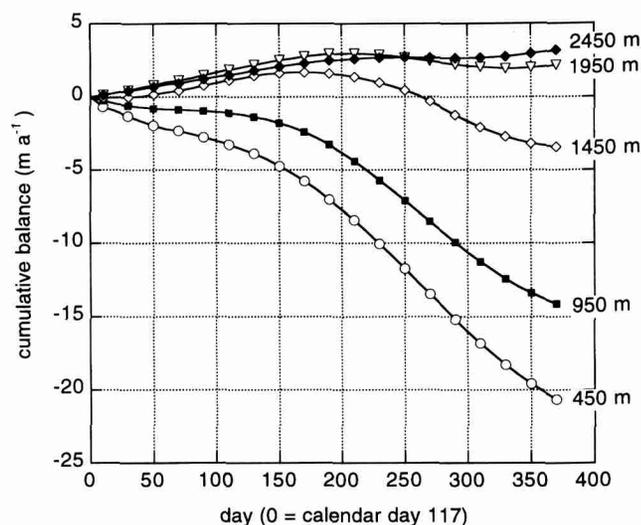


FIGURE 4. Cumulative balance at some selected grid points. At the end of the balance year, the specific balance B is equal to the cumulative balance by definition. Note that, for climatological input data, the cumulative balance is always negative on the glacier tongue. This is typical for (very) maritime glaciers because of relatively high winter temperatures.

m than at 2450 m. This is due to the shape of the precipitation curve, implying decreasing snowfall above 1500 m (see Fig. 5).

Equilibrium States in Dependence of Climatic Conditions

The sensitivity of the mass-balance profiles for uniform changes in annual temperature is shown in Figure 5. Labelled temperature changes are relative to the reference case defined above (this also applies to Fig. 6). Changes are largest in the mid-elevation range. This is a result of the highest amounts of precipitation found here (implying the strongest albedo feedback and a large shift in the distribution of the precipitation over snow and rain). It is in contrast to the results for other glaciers, where changes tend to increase further down the glacier tongue. The changes in equilibrium-line altitude are very large: about 210 m for a 1 K temperature change. In absolute terms, changes in the specific balance are impressive and it is not surprising to see a large sensitivity of the equilibrium length of the glacier to annual mean temperature (Fig. 6).

The 0 K case is typical for the state of the glacier in the last century. The corresponding equilibrium-line altitude is about 1680 m, in broad agreement with the findings of Woo and Fitzharris (1992). The range in glacier length seen in the historic record since 1850 can be explained by changes of air temperature within 1.5 K (assuming steady states).

Calculated changes in the mass balance profile caused by changes in precipitation are also shown in Figure 5. Although the imposed changes are quite large ($\pm 10\%$), the effect is negligible in the lower reaches of the glacier (where rain dominates) and moderate in the upper reaches. Consequently, resulting changes in equilibrium glacier length are less pronounced. In fact, a 10% increase in precipitation has the same effect as a 0.5 K cooling; a 10% decrease in precipitation has the same effect as a 0.4 K warming. On the basis of these results, we cannot support the idea that Franz Josef Glacier is notably sensitive to changes in precipitation. Rather, our findings seem to be in line with the theory that sensitivity to air temperature increases when the climatic regime is wetter (Oerlemans and Fortuin, 1992).

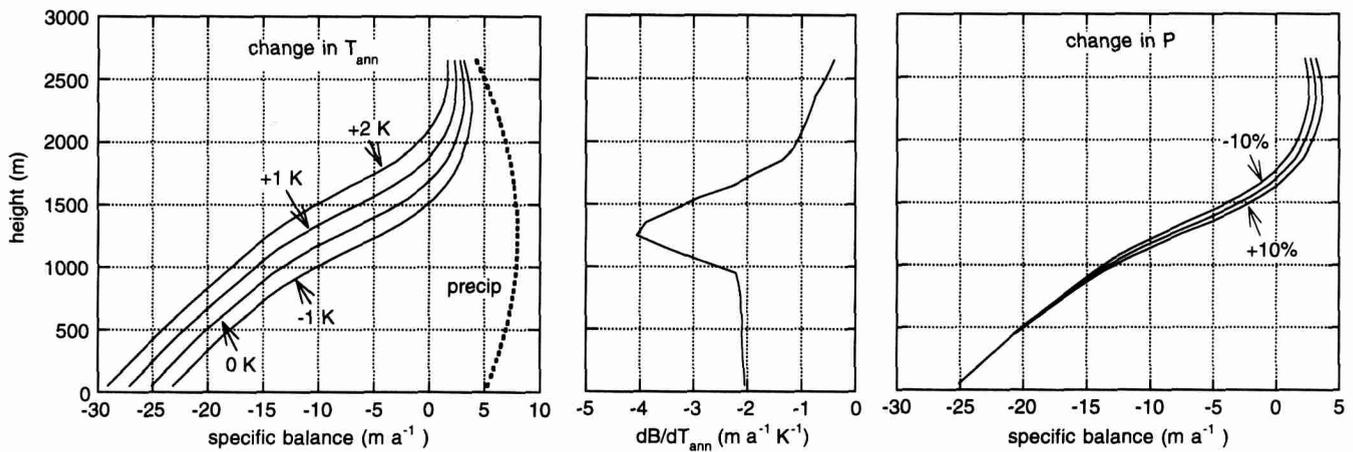


FIGURE 5. Left: calculated mass balance profiles for the reference case (0 K) and for some uniform changes in air temperature. The imposed precipitation is shown by the dotted line. Middle: the sensitivity parameter dB/dT_{ann} based on these calculations. Right: effect of a change in precipitation on the mass balance profile.

In Figure 7 results of the steady-state experiments are summarized. For a 3 K range in temperature, glacier length shows a variation of 9 km and glacier volume of about 4 km^3 . Figure 8 provides more detailed model output for the reference case. In the middle and lower part of the glacier the driving stress is high: between 2 and 3 bar. Ice velocities are notably high in the steep part of the glacier between $x = 6$ and 8.5 km; a characteristic value is 800 m a^{-1} . Sliding dominates when ice thickness is small. This should be considered a direct consequence of equation (6) and not an inherent modeling result. The high ice velocities are due to the large balance gradient, of course. By definition, in a steady state the downward mass flux is largest at the equilibrium line. The maximum in ice velocity is found a bit farther downstream, because glacier width decreases rapidly below the equilibrium line (see Fig. 3).

Response Time

In glaciology, the concept of response time is not always used in a consistent manner. Here reference is made to an e -

folding response time. For ice volume V , the definition is based on the existence of two steady states with volume V_1 (the reference state) and V_2 , corresponding to climatic states C_1 and C_2 . When, in some sense, $|C_1 - C_2|$ is small, the response would be that of a perturbed linear system, and would thus follow an exponential curve. In this case it is obvious that the volume response time should be defined as:

$$\tau_V = t \left(V = V_2 - \frac{V_2 - V_1}{e} \right). \quad (14)$$

Similarly, the response time for glacier length L is written as

$$\tau_L = t \left(L = L_2 - \frac{L_2 - L_1}{e} \right). \quad (15)$$

Here L_1 and L_2 are the equilibrium glacier lengths.

When $|C_1 - C_2|$ is large enough to make nonlinear effects important, and this is generally the case, there is no mathematical or physical reason to define response times in the same way. Nevertheless, for the sake of clarity it seems best to use the same formulation.

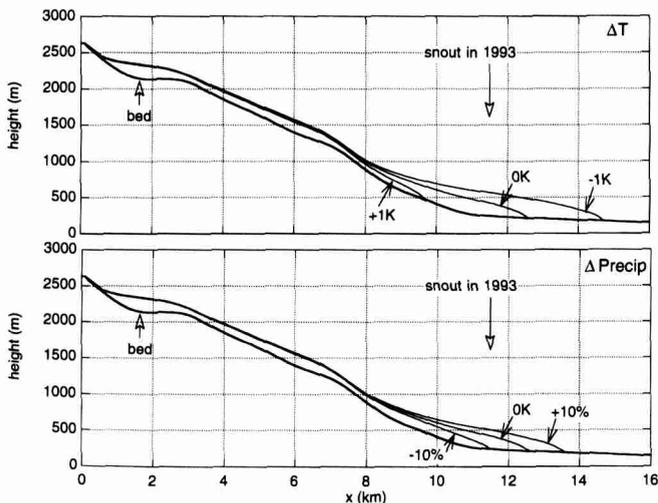


FIGURE 6. Equilibrium states of the model glacier for changes in temperature (upper panel) and precipitation (lower panel). The corresponding mass balance profiles are shown in Figure 5.

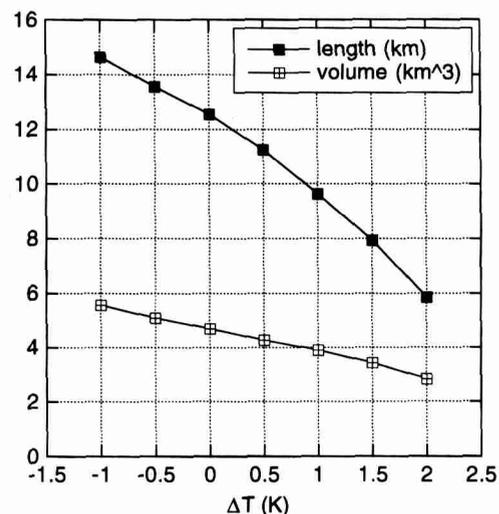


FIGURE 7. Equilibrium glacier length and volume in dependence of air temperature (relative to the reference case defined in the text).

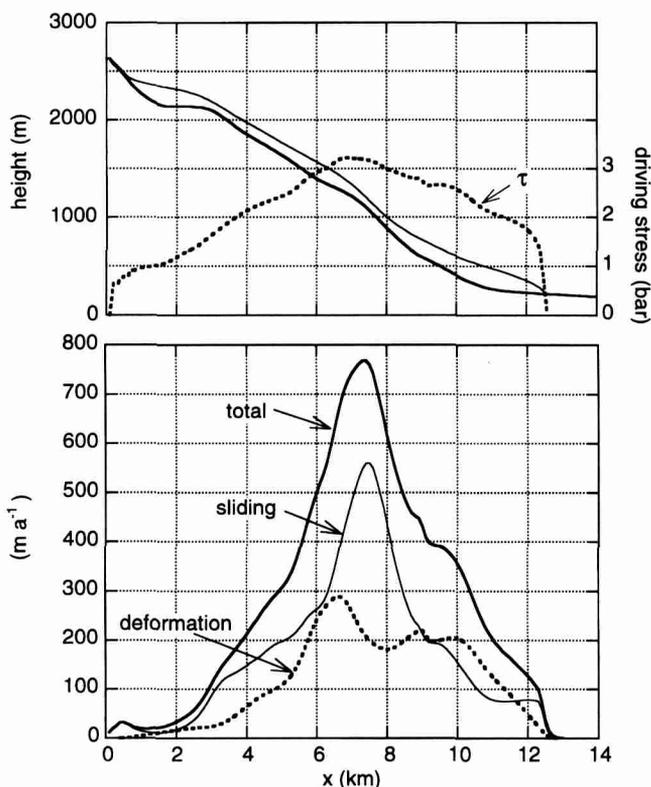


FIGURE 8. Diagnosis of the reference state ($\delta T = 0$). The upper panel shows driving stress, reaching maximum values just below the equilibrium line. The lower panel shows ice velocities.

It should be stressed that in defining response time it is essential to refer to equilibrium states, in spite of the fact that glaciers are normally in a transient state. One could of course also consider the response time of a glacier in transient state, but then response time is not a physical property of a glacier, but depends on the climatic history.

To obtain an order-of-magnitude estimate of τ_V and τ_L for Franz Josef Glacier, a steady state was first calculated and then perturbed with an instantaneous change in climatic conditions (a temperature change of 0.5 and -0.5 K, independent of altitude and season). Results are shown in Figure 9. It appears that τ_V is smaller than τ_L , a result also found in other studies and readily understandable as ice volume is more directly affected by changes in the specific balance.

It is interesting to compare the response times found here with theoretical estimates. Jóhannesson et al. (1989) have suggested that a volume time scale can be estimated from the expression $\tau_V = H^*/-B_{term}$, where H^* is a characteristic ice thickness and B_{term} the specific balance on the glacier snout. With $B_{term} = -25 \text{ m a}^{-1}$ and $H^* = 121 \text{ m}$ (calculated mean ice thickness over the entire glacier for the reference case), this yields a time scale of only 5 a. This is significantly smaller than the characteristic value of 15 a as found from the numerical model. The difference is probably related to the fact that the height-mass balance feedback, not taken into account in the estimate of Jóhannesson et al. (1989), is significant for Franz Josef Glacier. This feedback makes the response time longer.

Epilogue

Preferably, modeling natural systems like a glacier should be based on physical laws. Here an attempt has been made to

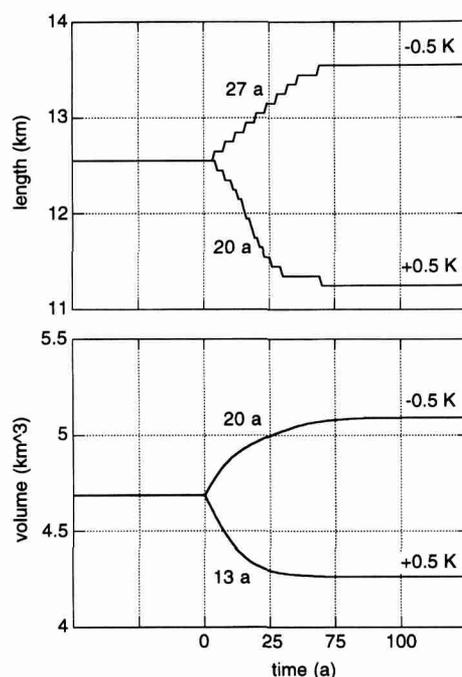


FIGURE 9. Response of glacier length and volume to a sudden change in air temperature imposed at $t = 0$. The initial equilibrium state corresponds to $\delta T = 0$.

do this for a glacier for which little is known. It is thus necessary to accept that the degree of detail is limited, and that it is not possible to make an absolute quantitative assessment of the current state of balance. Still, meaningful results about the sensitivity to external forcing can be obtained. As the basic hypsometry and the mean slope are the most important factors determining the response of a glacier to climate change, the broad results of the present study should be robust.

Some degree of calibration was necessary. Fortunately, this could be limited to the precipitation profile (Table 1) and the characteristic albedo of a snow-free glacier surface.

I plan to expand this work by using climate records from the New Zealand region. After careful selection and tests on data homogeneity, these will be used to feed the mass balance model which, in turn, will force the ice flow model. To set the stage, Figure 10 shows a few results from runs with imposed constant warming rates. If the global retreat of Franz-Josef Glacier since the Neoglacial maximum around A.D. 1750 were to be explained

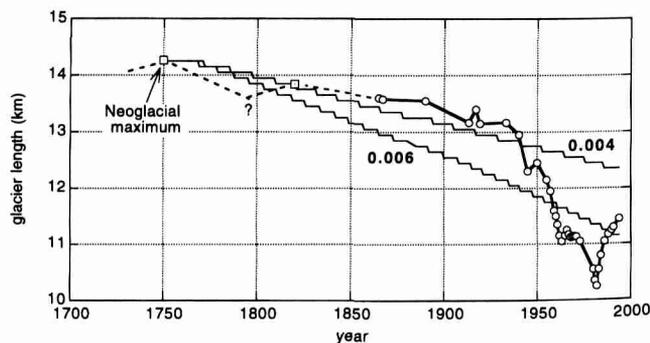


FIGURE 10. Calculate glacier length for two climate change scenarios with constant warming rates (see labels). The initial state at A.D. 1750 is an equilibrium state (corresponding to $\delta T = -0.8$ K, see Fig. 7). The "observed" length record is shown by the dashed and solid lines.

by a constant warming rate, 0.006 K a^{-1} (i.e. $0.6 \text{ K per century}$) would be the number! However, it will be interesting to study this in more detail and single out the relative importance of secular changes in seasonal anomalies of temperature and precipitation.

Acknowledgments

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