# THE HEAT BUDGET OF THE ROSS DRAINAGE BASIN

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## ABSTRACT

Integration of the thermodynamic equation over an entire drainage basin yields a fairly simple expression for the steady-state heat balance. This stems from the fact that dissipative heating can be calculated directly from the release of gravitational energy. When mass balance, surface temperature and geothermal input are known, the mean ice temperature at the grounding line can be obtained as a residual.

The procedure is applied to the drainage basin feeding the Ross Ice Shelf. The resulting mean outlet temperature is -16.2 $^{o}C$. The heating rates making the balance turn out to be (in 0.0001 K/yr): dissipation 8.2, advective flux divergence -13.5 and geothermal heating 5.3. The method also reveals how the mean outlet temperature depends on mass balance, surface elevation, etc.

## INTRODUCTION

Calculation of the temperature field in ice masses is a major problem in ice-sheet modelling. In the first place there exists a significant coupling between mechanics and thermodynamics (for a given stress, strain rates may vary over an order of magnitude for a 5 or 10 K temperature difference), making it difficult to handle one aspect in isolation. Secondly, in most cases boundary conditions are not known accurately.

In a general approach, the nonlinear advection terms are most difficult to handle. They require numerical methods with large processor times (Jenssen,1977; Young,1981; Oerlemans,1982). Although detailed integrations are certainly useful, much insight can be gained already from simpler models to which an analytic approach is possible. Early studies concerning temperature distributions in ice sheets considered local vertical profiles, thereby neglecting horizontal advection (or variations in horizontal advection) and using simple prescribed distributions of dissipative heating. Examples of such studies are Robin (1955), Weertman (1968) and Lliboutry (1979).



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Another way of arriving at simple models is by integration of the heat equation over an entire drainage basin. This yields a tractable equation because the advection terms and the dissipation take a simple form. Since, for polar ice sheets, the energy loss due to conductive transport in the upper layer is very small, the mean temperature of the ice leaving the drainage basin (at the grounding line) can be obtained as a residual. It is likely that this quantity ($\Theta$) controls to some extent the discharge , so it is worthwhile to study the factors that contribute most to changes in $\Theta$.

In this contribution an attempt is made to apply this procedure to the Ross drainage basin. Input data from various sources, mainly Atlas Antarktiki (1967) and Drewry (1983), are used to estimate the various components of the heat budget.

## THE HEAT BUDGET

We denote the area of the drainage basin by A, the average geothermal heat input by G, ice density (constant) by $\rho$, mass balance by M and surface temperature by T.

When the drainage basin is in equilibrium, the change in internal energy due to accumulation and calving is

$$\rho c \int_A MT \, dA \; - \; \rho c \Theta \int_A M \, dA \qquad . \qquad (1)$$

In the following we will refer to this contribution as 'advection'. Surface temperature is obtained from a sea-level temperature $T_0$ and a lapse rate $\gamma$, both appropriate for the basin under study. So

$$T = T_0 - \gamma h \qquad , \qquad (2)$$

where h is surface elevation.

The dissipative heating can be calculated from the release of gravitational energy. Denoting the mean height (relative to sea level) at which ice leaves the drainage system by $h_e$, the release of energy by downward motion equals

$$\rho g \left[ \int_A Mh \, dA \; - \; h_e \int_A M \, dA \right] \qquad . \qquad (3)$$

Setting the geothermal heat input to AG and neglecting the conductive flux through the top layer of the ice sheet, the heat budget reads:

$$\rho c \, (\bar{M}T_0 - \gamma\overline{Mh} - \bar{M}\Theta) + \rho g \, (\overline{Mh} - \bar{M}h_e) + G = 0 \qquad . \qquad (4)$$

The overbar denotes averaging over the drainage basin. We now solve for $\Theta$, yielding

$$\Theta = T_o - gh_e/c + (g/c-\gamma)\overline{Mh}/\bar{M} + G/(\rho c\bar{M}) \quad . \qquad (5)$$

This equation illustrates in a transparent way how the various components of the heat budget contribute to the mean outlet temperature.

First of all, it is interesting to note that $g/c = 0.0046$ K/m, i.e. always smaller than $\Theta$ (which is in the 0.006 to 0.012 K/m range). So the third term on the right-hand side is negative. Writing $\overline{Mh} = r\ \bar{M}\ \bar{h}$, where r measures the correlation between mass balance and surface elevation and is assumed to be invariant under climatic change, a higher surface elevation thus implies a lower outlet temperature. Apparently, the cooling of the ice mass due to lower surface temperature is larger than the increase in dissipation. This seems to be an important conclusion.

With the assumption of constant r, the third term becomes independent of $\bar{M}$. Changes in the mass balance are thus determined by the last term in equation (5) (ignoring at this point associated changes in ice thickness). We conclude that an increasing mass balance, all other things being equal, leads to lower outlet temperatures.

## THERMAL REGIME OF THE ROSS DRAINAGE BASIN

The theory described above is now applied to the Ross basin. From Atlas Antarktiki (1967) and Drewry (1983), the following values were obtained (by summing gridded values) for the various quantities needed to calculate the heat budget:

$\bar{M}$ = 0.183 m ice depth per year ,
$\bar{h}$ = 1220 m ,
$r$ = 1.01 ,
$h_e$ = −320 m ,
$T_o$ = −20 °C ,
$\gamma$ = 0.007 K/km .

It is striking that r is about 1, reflecting the fact that the mass balance hardly depends on surface elevation. For almost all other drainage basins of the Antarctic Ice Sheet this is not the case; a typical value for r is 0.75 (Oerlemans,1985).

The lapse rate of 7 K/km is typical for West Antarctica, a mean value for East Antarctica is about 10 K/km. The estimate of $h_e$ is made with the assumption that the ice velocity at the grounding line is independent of height, i.e. most of the motion is by basal slip. Of course, a correction could be made for any prescribed velocity profile, but this would hardly affect the results. Changes in ice velocity along the grounding line form a more serious problem: deeper outlet regions probably have higher discharge rates: this might reduce the effective value of $h_e$ (estimated error: 50 m). However, in view of the uncertainties in position of the grounding line and associated depth, corrections were not made.

With $G = 0.0544\ Wm^{-2}$ (Sclater et al.,1980) the calculated mean outlet temperature is:

$$\Theta = -16.2\ ^oC \quad .$$

It is interesting to compare the contributions to the budget in terms of heating rates (in 0.0001 K/yr):

dissipation: 8.2
geothermal: 5.3
advection: -13.5

So dissipation and geothermal input are of roughly equal importance.

At first sight the mean outlet temperature found here seems to be rather low, but in fact it is in good agreement with measurements and with more detailed calculations made by Young (1981). He computed the temperature and velocity fields along Ice Stream B and was able to match his results very well with the few measured temperature profiles on the Ross Ice Shelf. At the grounding line Young's model gives a vertical mean ice temperature of about -16.0 $^oC$, and the difference with the mean value obtained in this study is certainly acceptable.

Next we consider the sensitivity of $\Theta$ to environmental changes. Inserting r = 1 in equation (5) gives:

$$\Theta - T_o = -0.0046\ h_e - 0.0024\ \bar{h} + 0.91/\bar{M} \quad . \qquad (6)$$

Here $h_e$ and $\bar{h}$ are in m, $\bar{M}$ in m ice depth per year, $\Theta - T_o$ in K. On the basis of this expression some inferences can be made concerning the effect of changing surface elevation and mass balance. For drainage basins of more or less fixed size, $\bar{M}$ and $\bar{h}$ can be related by including global mechanics (Oerlemans and Van der Veen,1984). However, for drainage basins with subtle grounding-line dynamics such an approach makes no sense, so we simply consider a curve calculated directly from equation (6).

Figure 1 shows how the mean outlet temperature depends on the mass balance. For low values of the mass balance the outlet temperature increases dramatically. In reality, such high values will of course be prevented by extensive basal melting, but the curve illustrates the point. Just to show the sensitivity of $\Theta$ to changes in mean surface elevation h, another calculation was done with an imposed relation between mass balance perturbation (i.e. relative to the present value) and change in $\bar{h}$ :

$$\bar{h}' = \bar{h}_o + 5000\ \bar{M}' \quad . \qquad (7)$$

So it is simply assumed that a 10 cm change in the mass balance corresponds to a 500 m increase in elevation. The effect turns out to be small, as can be seen in Table 1.
The second column shows values that are also displayed in figure 1, the third column those obtained with equation (7).

| $\bar{M}$ | $\Theta - T_o$ | $(\Theta - T_o)'$ |
|---|---|---|
| 0.05 | 16.7 | 18.3 |
| 0.1 | 7.6 | 8.6 |
| 0.15 | 4.6 | 5.0 |
| 0.2 | 3.1 | 2.9 |
| 0.25 | 2.2 | 1.4 |
| 0.3 | 1.6 | 0.2 |
| 0.4 | 0.8 | -1.8 |

Table 1. Mean outlet temperature (relative to sea-level temperature $T_o$, in K) for various values of the mass balance $\bar{M}$ in m ice depth per year).

It is noteworthy that the area A of the drainage basin does not appear in the equation for $\Theta$. The outlet temperature for situations in which the West Antarctic Ice Sheet extends much further in the Ross Sea is therefore thought to be controlled mainly by $T_o$ and $\bar{M}$, unless associated changes in h and $h_e$ are very large [according to Denton and Hughes (1981), this is unlikely]. The effect of lower environmental temperatures during glacial conditions will partly be cancelled by the effect of lower accumulation rates. This suggest that the long-term thermal regime of the West Antarctic Ice Sheet is probably not so sensitive to climatic change as is sometimes assumed.

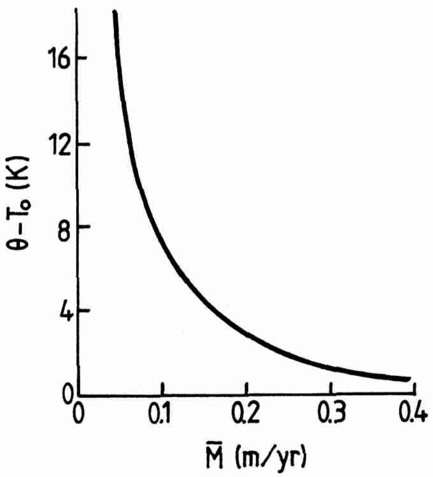


Figure 1. The relation between mean outlet temperature and mass balance for the Ross drainage basin. It has been assumed that the mean ice thickness is constant.

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