

Some Results of a Numerical Experiment Concerning the Wind-Driven Flow through the Straits of Dover

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Summary

In this paper the wind-driven flow of water through the Straits of Dover is studied with a numerical model. The main goal is to get some insight into the effect of the specific configuration of the Straits of Dover region on the wind-driven flow from the Channel into the North Sea. For 18 wind directions the steady-state solutions and response times are presented. The results of a 21-day model run are compared with those of computing the transport with the steady-state solutions. Except for periods with a very rapidly changing wind field, the steady-state approximation appears to be good.

Einige Ergebnisse eines numerischen Experiments bezüglich der winderzeugten Strömung in der Straße von Dover (Zusammenfassung)

In dieser Arbeit wird die winderzeugte Strömung in der Straße von Dover anhand eines numerischen Modells studiert. Das Hauptziel dabei ist die Gewinnung einiger Kenntnisse über die Wirkung der besonderen Struktur der Straße von Dover auf die winderzeugte Strömung vom Kanal in die Nordsee. Für 18 Windrichtungen werden die Lösungen für den stationären Fall und auch die Reaktionszeiten dargestellt. Die Ergebnisse eines 21tägigen Modellversuchs werden mit jenen verglichen, die aus der Berechnung des Transports mit den Lösungen für den stationären Fall gewonnen wurden. Außer für Perioden mit einem sehr schnell wechselnden Windfeld scheint die Näherungslösung für den stationären Fall geeignet zu sein.

Quelques résultats d'une expérience numérique concernant le courant induit par le vent dans le Pas-de-Calais (Résumé)

Dans cet article, le flux du courant induit par le vent à travers le Pas-de-Calais est étudié sur un modèle numérique. Le but principal est d'obtenir un aperçu de l'effet de la configuration spécifique de la région du Pas-de-Calais sur le flux dû au vent de la Manche vers la mer du Nord. Pour dix-huit directions de vent, les solutions en régime permanent et les temps de réponse sont présentés. Les résultats de vingt-et-un jours de fonctionnement du modèle sont comparés à l'estimation du transport déduite des solutions en régime permanent. Sauf pour les périodes présentant un champ de vent à variations très rapides, l'approximation du régime permanent paraît bonne.

Introduction

Most numerical models of shallow european seas were developed to study and predict storm surges. For this kind of models it is desirable to incorporate not only the effect of the wind-stress field, but also of atmospheric pressure gradients (e.g. Timmerman [1975]) and interaction of those effects with the tides (e.g. Davies and Flather [1977]). Apart from the problem of surge prediction a growing interest exists in residual transport patterns, in particular in the southern North Sea. Residual transports are important with regard to budget studies of passive biological species and pollution injected into the sea by rivers like the Rhine,

Scheldt and Thames. Although the JONSDAP-experiments provided a lot of measurements, residual circulations for various meteorological conditions are only known roughly. The problem is now tackled with numerical models that eventually can be verified with the JONSDAP-measurements.

The residual circulation in the southern North Sea is mainly determined by (in probable order of importance) the mean wind-stress field, nonlinear effects associated with the diurnal and semi-diurnal tides, very long tidal components (i.e. components with periods that are large compared to the time span over which the residual transport is defined) and density gradients caused by differences in salinity and temperature. Ronday [1975] studied the tidal contribution in detail and proposed to describe this contribution by the introduction of a so-called tidal stress. Incorporated in a model, this tidal stress gives an extra external forcing. If the model is linear, the effect of the tidal stress can be computed separately and added to the circulation induced by the wind stress field. Therefore, conventional storm-surge models without tides can be used to study residual transports. The model of the Koninklijk Nederlands Meteorologisch Instituut (KNMI-model, see Timmerman [1977] for a documentation) is used to study residual flow in the southern part of the North Sea (Riepma [1976]).

With regard to the southern North Sea, a serious problem arises. The open southern boundary of this region, the Straits of Dover, plays an important role in the water budget but has such a small width that it is badly resolved by grids used in models for operational surge prediction. Therefore, it is desirable to have some insight into the effects of the specific configuration of the Straits and the adjacent parts of the North Sea and the Channel. In this paper we will discuss some results of a numerical model covering the eastern part of the Channel and the southern part of the North Sea with a mesh size of 15 km. We will deal with the wind-driven part of the transport through the Straits and consider the dependence on wind direction, the response time of the transport for various wind directions and the possibility to compute the transport by using the asymptotic solutions only.

Brief description of the model

The numerical model used in this study is based on the equations of motion and the continuity equation for a barotropic fluid of constant density ϱ . The equations are averaged from the bottom to the surface of the sea. The derivation of the set of equations that determine the model will not be reproduced here. For a detailed discussion of the various approximations involved, see, among others, Fischer [1959].

In a x, y, t -frame, with the x -axis pointing in west-east direction and the y -axis in south-north direction, the set of equations reads

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -g \frac{\partial \zeta}{\partial x} + fv + (\tau_x - Ru)/(\zeta + H), \quad (1)$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -g \frac{\partial \zeta}{\partial y} - fu + (\tau_y - Rv)/(\zeta + H), \quad (2)$$

$$\frac{\partial \zeta}{\partial t} + \frac{\partial}{\partial x} (H + \zeta) u + \frac{\partial}{\partial y} (H + \zeta) v = 0, \quad (3)$$

where u, v x and y component of the vertical mean velocity,

ζ elevation of the sea surface with respect to the equilibrium state (no forcing),

H depth of the sea in the equilibrium state,

R frictional constant,

f Coriolis parameter,

g acceleration of gravity,

τ_x, τ_y components of the wind stress at the sea surface divided by ϱ .

Dissipation is represented by means of a linear bottom friction term and the only forcing function is the wind-stress field. Since relative large velocity gradients may be expected in

the vicinity of the Straits of Dover, the nonlinear advection terms were not omitted immediately, as is the usual procedure. Moreover, at first $\zeta + H$ was not approximated by H .

Fig. 1 shows the domain of the model. The grid has a mesh size of 15 km. Since it permits a more accurate description of the coastline, a conventional grid is used instead of a staggered

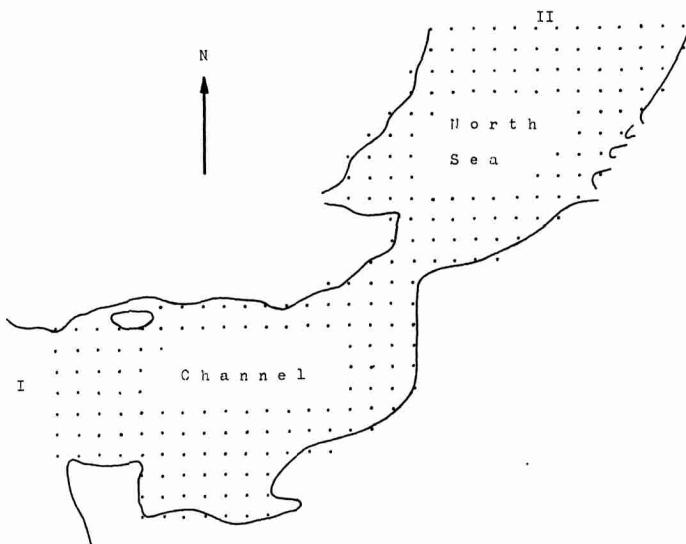


Fig. 1. The grid of the model; the mesh size is 15 km

grid. The grid points on or near the coastline represent areas that are less than 225 km^2 . The appropriate areas were carefully taken into account in the construction of the difference scheme. The model has two open boundaries denoted by I and II. At these boundaries ζ is prescribed as a constant or a function of time. The boundary condition at the coast is formulated in such a way that there is no mass flux perpendicular to the coastline. Central differences are applied for the discretization in space; a semi-implicit method is used to integrate the equations in time, the time step being 150 seconds.

Stationary solutions

The model has got two constants that should be fixed: the frictional constant R and the drag coefficient C_d which is needed for the computation of the wind-stress field. After some experimentation R was set at $0,0024 \text{ m/s}$ and C_d at $3 \cdot 10^{-3}$. These values give the best fit to the empirical relation between the wind field and the transport through the Straits of Dover as derived by Bowden [1956]. In this section we will discuss experiments performed with a constant and homogeneous wind-stress field of magnitude $5 \cdot 10^{-4} \text{ m}^2/\text{s}^2$, so the results do not depend on the particular choice of C_d . With the value of C_d mentioned above the corresponding wind field has a speed of about 10 m/s.

Starting with the equilibrium state the evolution of the flow pattern was computed for 18 wind directions until a steady-state was reached. In experiment A the condition $\zeta = 0$ was applied at boundaries I and II. Experiment B was carried out with $\zeta = 0$ at boundary I whereas at boundary II ζ was obtained from the KNMI-model (mesh size 42 km). More specific, the grid points at boundary II got the appropriate values from the solution of the KNMI-model for the same wind-stress field. Fig. 2 shows the steady-state transports through the Straits as a function of wind direction θ . We see that in both cases the θ -dependency of the transport can be described very well by a sine function. In experiment B the maximum transport is larger than in experiment A. There also is a slight difference in the wind direction of maximum

transport between A and B. If we truncate a Fourier-analysis on the 18 points of curve B at the first harmonic we get the following expression for the transport T :

$$T(\theta) = [41,0 \cdot \cos(\theta - 204^\circ) - 0,3] \cdot 10^4 \text{ m}^3/\text{s} \quad (4)$$

A positive value of T denotes a transport from the Channel to the North Sea, a negative value a transport in opposite direction. The rms-difference between (4) and the computed transport with respect to θ is $0,84 \cdot 10^4 \text{ m}^3/\text{s}$ indicating that (4) is a very good approximation. The maximum transport occurs at a wind direction of $\theta_{\max} = 204^\circ$; we will call this direction the principal axis of the Straits of Dover region. It is in excellent agreement with the empirical

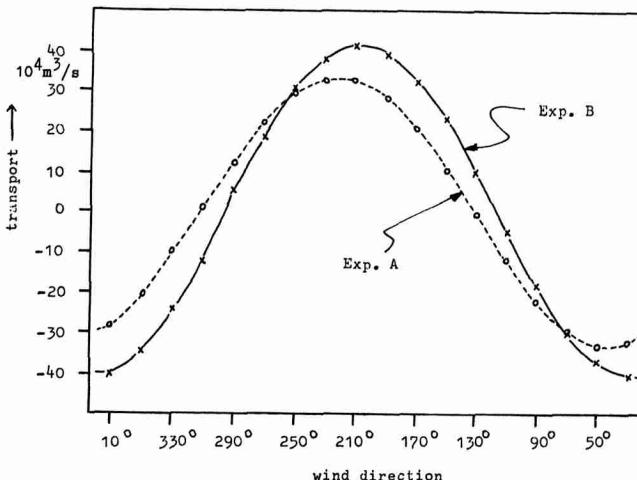


Fig. 2. The transport through the Straits as a function of wind direction for experiments A and B (see text)

result of Bowden [1956] who found $\theta_{\max} = 202^\circ$. It should be emphasized that this agreement is not due to a particular choice of R . Experiments of other values of R brought to light that θ_{\max} is independent of R . This already points towards a linear behaviour of the model.

Variation of the magnitude of the wind stress for various wind directions revealed a fully linear relation between steady-state transport and wind stress. In order to estimate the importance of the nonlinear terms in (1) to (3), some runs were carried out without those terms. In all cases (5 runs of 24 hours of simulated time) the evolution of the flow pattern did hardly show any deviations from that of the corresponding nonlinear run. At any moment, the difference in transport through the Straits was less than 2 %. Consequently, the nonlinear terms were dropped and further experiments were carried out with the remaining linear model. The steady-state transport through the Straits may now be written

$$T(\theta, \tau) = 8,2 \cdot 10^8 \cdot \tau \cdot \cos(\theta - 204^\circ) \text{ m}^3/\text{s}. \quad (5)$$

Response of the Straits of Dover region

One of the basic characteristics we like to know is the response time of the flow pattern with regard to changes in the forcing, i.e. in the wind stress field. Here we will consider the transport through the Straits only.

The configuration of the Straits of Dover region suggests a rather strong dependence of the response time on wind direction. To investigate this we define a 90 % response time t_r by

$$T_{t=t_r} = 0,9 \cdot T_{t=\infty}. \quad (6)$$

Starting with an equilibrium state and the full wind field, t_r simply is the time at which the transport is 90 % of its final asymptotic value. Since we now deal with a linear model, t_r will

not vary with the magnitude of the forcing. Fig. 3 is based on experiment A discussed in the foregoing section. It displays t_r for the 18 wind directions. Two peaks in t_r are apparent; they occur at $\theta = 310^\circ$ and $\theta = 130^\circ$. In fact, we may expect two points where t_r becomes infinite, namely, the directions perpendicular to the principal axis. However, it did not seem worthwhile to spent a lot of computertime to refine the picture. We conclude that the response time is between 5 and 9 hours, except for wind directions that are almost perpendicular to the principal axis. We note that the significance of large values of t_r is not great: the associated transports are small.

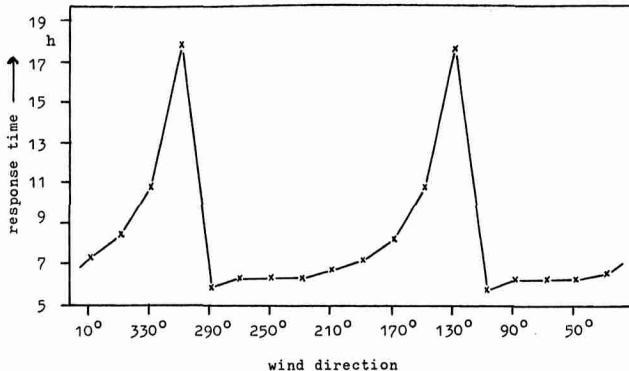


Fig. 3. The response time t_r as a function of wind direction

The values of t_r for various wind directions constitute a basic characteristic of the model and, as we hope, of the region considered in this study. We should not forget, however, that interaction with the adjacent seas is always present. In general, these seas have larger response times, so the adjustment of T to a changed wind field will be somewhat slower.

The case of a changing wind field

It is important to know the consequences of the properties discussed so far for the computation of the transport in the case of a time-dependent wind field. Since the region considered has rather small dimensions, the problem of the homogeneity of the wind field is less serious. However, if interaction with the adjacent seas becomes dominating the spatial variability of the wind field may be important. Anyway, we will deal with homogeneous wind fields and focus on the following question: to what extent can eq. (5) be used for the computation of the wind-driven transport for real (changing) wind fields?

Let t_{ad} be a characteristic response time of the adjacent seas. According to Weenink [1956] the North Sea has a typical response time of about 30 hours. We accept this value to be the order of magnitude of t_{ad} . A third time-scale is fixed by the speed at which the wind field changes, we denote it by t_{wi} . Based on these time-scales we can define a few typical situations. First we note that $t_r < t_{ad}$. With this in mind we consider four cases:

- $t_{wi} < t_r$; in such a situation we may expect that the interaction with the adjacent seas is small because they hardly react. Eq. (5) will approximate the transport through the Straits very poorly, but it can be computed with the model with boundary conditions $\zeta = 0$.
- $t_{wi} \approx t_r$; interaction with the adjacent seas will occur. Prescribing stationary boundary conditions will represent a poor approximation. Eq. (5) will perform slightly better than in case a).
- $t_{wi} \approx t_{ad}$; it probably is a good approximation to assume stationary boundary conditions and since eq. (5) was computed with such conditions it will provide a reasonable estimate of the transport.
- $t_{wi} > t_{ad}$; in this case we may expect a nearly steady-state to exist and there is no advantage in using the numerical model instead of eq. (5).

In the real atmosphere all the cases mentioned may occur. To give a few examples: wind fields of mesoscale systems (including sea breezes) come under a), of rapidly moving depressions under b) or c), of anticyclones or depressions in their final stage under d). The atmospheric kinetic energy available at the surface in time-scales smaller than t_r is rather small. Kinetic energy spectra of 200 m winds measured at Cabauw (Netherlands) show a f^{-2} shape (f is frequency) in the 2 hours to 1 day range (Van den Dool [1975]). Although these 200 m winds are not fully representative for sea-surface winds, they give a better idea than 500 mbar wind spectra that are subject to most studies. Anyway, we may state that the importance of wind fluctuations decreases rapidly with their time-scale. This means that cases c) and d) dominate and that eq. (5) should give a reasonable estimate of the transport through the Straits.

In order to test this idea the following experiment was carried out. A model run of 21 days was performed with a homogeneous wind field changing every 2 hours. The wind field chosen was an observed one and typical for western Europe: rather variable with a dominating southwesterly component. At boundary I, ζ was kept at 0; at boundary II, ζ was prescribed as a function of time according to

$$\zeta(t) = \frac{1}{2}(B' + B) + \frac{1}{2}(B' - B) \cos(\pi t/t_{ad}). \quad (7)$$

Here, B' and B are the boundary conditions for the asymptotic states corresponding to the mean wind stress fields of the preceding and actual day respectively. Eq. (7) merely provides a smooth adjustment of ζ . The solid curve in Fig. 4 shows the computed daily transports. The graphs at the bottom of the picture represent the daily mean wind stress vector.

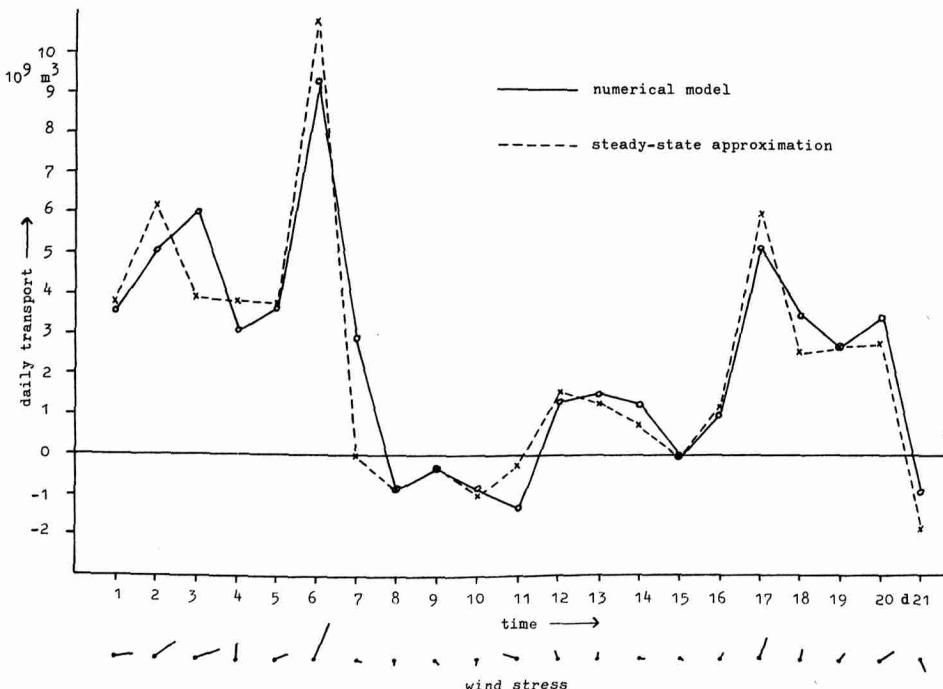


Fig. 4. A comparison between daily transports computed with the numerical model (solid line) and estimated with the steady-state approximation (dashed line). The daily mean wind-stress vectors are shown at the bottom

The transport can also be computed with eq. (5) by assuming that at each day a nearly steady-state exists that corresponds to the mean wind stress of that day. The transport computed in this way is shown by the dashed line in Fig. 4. We see that eq. (5) gives a useful estimate of the transport: the curves essentially show the same behaviour. There are a few days, however, with marked differences: day 3, 7 and 11. The total transport computed with the numerical model was $49,30 \cdot 10^9 \text{ m}^3$, whereas eq. (5) gave $46,07 \cdot 10^9 \text{ m}^3$. With respect to daily transports, the rms-difference between the results of the numerical method and the steady-state approximation appeared to be $1,03 \cdot 10^9 \text{ m}^3$.

Discussion

The results presented in the foregoing sections revealed some properties of the wind-driven flow through the Straits of Dover. Since the behaviour of this flow is simple enough to describe it to a considerable extent by its asymptotic solutions and response times for a set of wind directions, only part of the output of the numerical experiment was discussed.

An important result is the $\cos \theta$ -dependency of the transport in the steady state. It confirms the empirical result of Bowden [1956], although this is not strictly valid for (fictive) steady states. The 21-day run brought to light that eq. (5) provides a useful estimate of the transport. Large errors occurred only for days on which the wind vector differed markedly from that of the preceding day (see Fig. 4). Apparently, the inertia of the water masses plays an important role in such cases, or, in other words, on such days the flow is far from a steady state.

If one is interested in more accurate estimates of the transport in periods with a rapidly changing wind field, it seems promising to compute the transport with the following model:

$$C(\theta) \frac{\partial T}{\partial t} = \alpha(\tau \cdot k) - R^* T. \quad (8)$$

In (8), k is a unit vector along the principal axis, R^* is a frictional constant, $C(\theta)$ a function that describes the inertia of the system corresponding to the results shown in Fig. 3, and α a coefficient that fixes the steady-state transport in accordance with eq. (5). However, no matter what kind of model is used, computation of T for practical purposes requires a calibration that introduces errors. In addition, an accurate determination of the wind-stress field from real measurements is very difficult. In view of these facts it is not obvious that a refinement of the steady-state method by means of eq. (8) makes sense. This problem constitutes a subject for further study, anyway.

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