THE CONTEXTUAL AND MODAL CHARACTER OF QUANTUM MECHANICS

A FORMAL AND PHILOSOPHICAL ANALYSIS IN THE FOUNDATIONS OF PHYSICS

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Colofon

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The Contextual and Modal Character of Quantum Mechanics

A Formal and Philosophical Analysis in the Foundations of Physics

Het contextuele en modale karakter van de kwantummechanica

Een formele en filosofische analyse op het gebied van de fundamenten van de fysica

(met een samenvatting in het Nederlands)

Proefschrift

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door

Christian de Ronde

geboren op 1 september 1976
te Buenos Aires, Argentinië

Promotors: Prof.dr. D.G.B.J. Dieks
Prof.dr. D. Aerts

Para mi mamá y mi papá.
Para Vincent, Gustavo y Acha.

Out yonder there was this huge world, which exists independently
of us human beings and which stands before us like a great,
eternal riddle, at least partially accessible to our inspection.

If you wish to learn from the theoretical physicist anything
about the methods which he uses, I would give you the following
piece of advice: Don't listen to his words, examine his achievements.
For to the discoverer in that field, the constructions of his
imagination appear so necessary and so natural that he is apt
to treat them not as the creations of his thoughts but as given realities.

You know, it would be sufficient to really understand the electron.

Albert Einstein.

Preface

Since I was a child, the question which has always caught my attention, that which gives a weird shivering going up my back, is the question of existence. Why is it the case that we exist, rather than we don't? Does the world in which we live really exist? And in that case, what is the meaning of this world? Does it have any meaning at all? These questions have always remained at the back of my head, as my deepest interests. It is because of these questions that I decided to study physics and philosophy. To find my own way through the foggy fields which restrict these regions of thought has been for me a an incredible and beautiful trip.

I'm half Argentine and half Dutch, for that reason some of my friends in the quantum world, used to tell me I'm an *improper mixture*; this, despite how it sounds, I always took as a compliment. I consider myself very fortunate to have lived in between two continents and worlds, different in many aspects, similar in some. In this respect, this last years in Europe have given me the opportunity also to mix different schools related to quantum theory.

Around 2001, I decided I would end my physics degree with a (master) thesis in the interpretation of quantum mechanics, the field that had kept my attention through my studies. Talking advantage of the help of my family both in Argentina and in Holland, I went then into an adventure of some weeks to Europe, trying to find someone who could direct such a research. I had no idea of the field I was getting into. I went to many Universities before being able to converge to the *Institute for the History and Foundations of Science* at Utrecht University and Professor Dennis Dieks. Professor Dieks was not only kind to take me as a student, but also encouraged me to continue my investigations through many long discussions. He has taught me most of what I know about the quantum world, its problems and possibilities, and has remained for me a referent to try to copy in many aspects. In Utrecht I also met Michiel Seevinck of whom I became a friend, to discuss not only about contextuality and multipartite quantum correlations, but also about life and death. My master thesis was called: "The Perspectival Version of the Modal Interpretation: A Story About Correlations and Holism" and was defended in 2003. But the story did not end then.

After that great adventure, I found a position in Brussels, with Diederik Aerts as my promotor in the *Center Leo Apostel.* Back in Buenos Aires, I had studied the 1976 green book of Piron together with a then small group of people[1] —Graciela Domenech and Andrea Costa— interested in foundational issues of physics. Now, I had the opportunity to meet "live" the direct continuation of the Geneva School, which went back to Wolfgang Pauli —someone I regard as maybe one of the most interesting of the amazing characters which took part in the quantum revolution. I've always been passionate in discussions relating to quantum mechanics, maybe because I feel so attached— or "entangled" —to the theory. In Brussels I had the great opportunity to discuss with so many interesting people. In the first place of course, Diederik Aerts has been a mentor and a guide for my work and also for my life. I have benefitted so much from endless talks at the CLEA house or by phone, not only about

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[1] A group which has grown today into the Quantum Structures Group of Buenos Aires.

quantum mechanics, but also about philosophy, politics, economics, music and all... I also have to mention Karin Verelst, her passion, her power and her soul (which go all together but separate), Sven Aerts with his calm but always deep remarks, Wim Christiaens who is a true thinker, and Sonja Smets, one of the most knowledgeable people in the logic of the quantum. Apart from my stay in Europe and the possibility to travel to every conference I wanted to go to, I was able, once in a while to go back to Buenos Aires, and continue my endless discussions with Graciela Domenech, without whom nothing of this could have been possible for me in the first place. Graciela has been not only co-author in many of my papers, but has always been in the right —and so rare— balance, criticism and incisive but open to new ideas. Last but not least, Federico Holik, a passionate student of Graciela with whom I shared many discussions. I consider myself incredibly lucky to have been able to interact with so many interesting people. My dissertation, is also a mixture of these different lines of thought which travel around the globe. An improper mixture as well!

I consider myself a believer. I believe in finding a solution to the problem of the quantum, in understanding Nature, in changing the world. I also believe that it is important to take positions, not only in life, but also in science and philosophy, which are but a reflection of life itself. I consider these two elements, 'believing' and 'taking a position', as deeply important to create a path. When you are lost, you need not only to choose a direction, but also to believe that you will succeed in getting somewhere. A right balance between believing and remaining critical is of course difficult to achieve. You need to be strong to go against the wind and calm to see the signs and detours which the path offers to you. Regarding my own path, I remember an episode of which I thought quite a lot afterwards and has related very closely to my work. In September 2004, close to my birthday, I had the opportunity to attend to the *13th UK meeting on the foundations of physics* at the beautiful city of York. There was one of the talks in the conference which caught the attention of the public, suddenly some kind of battle started among those present, arguments for and against the presentation were growing in strength and power. I had not seen this before so much in the conferences I had attended, but it seemed this talk had awaken a lot of passion. People started getting very excited and suddenly there were two quite definite groups or bands trying to get control of the situation. At a certain moment, when the discussion seemed to have gone out of control and there was shouting and screaming all around the room, Tony Sudbery, who was the host of the conference, tried to calm down the situation and find a way out. He then asked those who were present the following question: "Do you believe that in the future, say ten or twenty years from now, quantum mechanics will change into a different theory? That we will find a formalism capable of taking into account both classical and quantum and answer the questions which today seem so obscure? Or, do you believe that the formal structure of quantum mechanics will remain basically the same?", and he added, "Let's vote!" It was very funny. To the first question, which seemed quite more revolutionary, professors and the older generation raised their hands. To the second question, which, at first glance, seemed more conservative, —because implied that no new formalism would enter the scene— a smaller group, only the very young people who were making their PhD's by then, raised their hand. Tony saw this immediately and commented on the fact, we had a good laugh and continued the discussions in the bar. The discussion and especially these two positions which were explicitly exposed really caught my attention. I remember discussing that night with Michiel, I had a clear idea that the fact that the formalism would change would mean that quantum mechanics would then be thought without all its weirdness. But as I believed then, and still today, it it is exactly the weirdness of the quantum which is most interesting. If the formalism would change it would mean that we would have found a "way out" and that we could recover our classical conception of the world, while, if the formalism was not to change, something very strange would need to happen, we would need to think in a radically different manner, we would need to create a different way to understand the world. From this perspective, of course the most revolutionary position would be to think that the quantum formalism will not change in its basic aspects, rather, a conceptual break through will need to take place such that we understand the world in different terms.

In relation to this most interesting problem, I believe that the modal interpretation provides very interesting elements, in order to think, both about the possibilities and impossibilities of quantum theory. I remember very vividly some of my first discussions with Professor Dieks, telling me "One of the most important features of modal interpretations is that they stay close to the standard formalism, that nothing is added by hand!" At the time I did not really understood why this was so important, today I see this point as central in order to advance towards a consistent interpretation of quantum mechanics. My dissertation deals with these problems, the formal structure of quantum mechanics, its interpretation, its possibilities and impossibilities. It is also about the different positions taken and how I understand these very different choices.

It is not so strange that the questions which I address in this work, are to great extent, still the same basic questions which I posed to myself when I was a child. One could say I've traveled a long way to go back to the same place. Anyhow, I've always thought that it is not so much important where you get to, it is much more important how you travel. My motto has always been: when the trip is good, you cannot get to a wrong place. And I must admit in any case, that this trip has been great.

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I want to thank my mom for her love. I want to thank my father for his inspiration and coherency. I want to thank Acha for showing me the world in so many different ways. I want to thank very especially my cousin Vinnie without whom I would have not been able to accomplish many of the things I did —in particular, this dissertation.

In schools about the foundations of quantum theory, but especially after conferences, in bars and restaurants, I have deeply profitted from something you do not find in books: the reflections, comments and answers by many of the main figures in the foundations of quantum mechanics domain. I am especially grateful for their time, kindness and attention. Michel Bitbol, Guido Bacciagaluppi, Jeffrey Bub, Vassilios Karakostas, Bas van Fraassen, David Mermin, Pieter Vermaas, Itmar Pitowsky, Victor Rodriguez, Carlo Rovelli, Mauricio Suárez, Holger Lyre, Miklos Redei, Jeremy Butterfield, Pekka Lahti, Bob Coecke, Sonja Smets, Thomas Durt, Esther Diaz, Charles Alunni, Alejandro Cassini, Decio Krause and Newton da Costa.

I want to thank my fellow companions in the road to get a PhD. The many endless discussions in the Y.Q.M., conferences and, of course, bars, have developed my work substantially. I am truly indebt to Karin Verelst, Michel Seevinck, Sven Aerts, Giovanni Valente, Soazig Le Bihan, Alexei Grinbaum, Patricia Kauark, Adrien Barton, Hernan Pringe, Gabriel Catren, Romina Zuppone and Albert Solé.

Quiero agradecer a mis amigos que son también mi familia: Pablo, Joaquin, Gamba, Tomas, Jota, Edgardo, Ezequiel y Maxi, al Guaton y a Marto, a Gustavo, a Santi, Nahuel, Walterino y Andrea.

A Luciana que me acompaño estos últimos años de correcciones infructuosas, todo lo que soy.

My co-authors in several papers: Wim Christiaens, Federico Holik, Bart D'Hooghe, Vincent Bontems, Garciela Domenech and Hector Freytes. Thanks for the many working hours!

Last but not least, two of the people I admire the most in the filed known as foundations of quantum mechanics. I am indebt to them not only academically but also personally: Dennis Dieks and Diederik Aerts.

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I

INTRODUCTION

Introduction to the Dissertation

This dissertation derives from my work in the last years, namely from a set of papers some of which have been jointly authored with Diederik Aerts, Bart D'Hooghe, Graciela Domenech, Hector Freytes, Federico Holik, Wim Christiaens and Vincent Bontems. The common underlying theme of these articles is the search for understanding of the formal structure of quantum mechanics, its possibilities and impossibilities.

I learned about modal interpretations when I came to Utrecht in 2002. Quite immediately, I felt close to the attempts and spirit which I believed continued the questions addressed by the founding fathers of the theory. One of the most interesting features regarding modal interpretations is for me, the idea that one should stay close to the standard quantum formalism. At that time, Professor Dieks together with Gyula Bene had presented a paper called *A Perspectival Version of the Modal Interpretation*, in which they attempted to present a relational theory which would be able to avoid difficulties regarding the definiteness of properties in modal interpretation. The study of this interpretation and of the modal interpretation in general allowed me to think about and study in depth the *mode of being* of properties in the formal structure of quantum mechanics.

I came to Brussels in 2004, where I had the opportunity to work in the Geneva school approach put forward by Josef-Maria Jauch and Constantin Piron. There, I learned about the subtleties involved in the relation between the logical aspects of both classical and quantum theory. Also, the realistic operational approach of Piron and Aerts gave me a completely different angle to think about the problems involved in considering *elements of physical reality*. In this scheme, the notions of *possibility* and *potentiality*, which are closely related to the modal interpretation, were understood in very definite way.

Also in Buenos Aires, together with Graciela Domenech and Hector Freytes, we started asking a question which related both to the contextual aspect of the theory and to the notion of *possibility*. Although quantum mechanics has always been related to possibility —specially through the notion of probability— the KS theorem constraints only actual properties to have definite values. The Kochen-Specker theorem does not talk about possibility but rather about the actuality of definite properties. The question which we posed to ourselves was quite simple, does the Kochen-Specker theorem have something to say about possible properties? This question is extensively discussed in this dissertation.

More generally, the question which underpins this dissertation is that of interpretation itself. What are the constraints to interpret quantum mechanics in terms of our classical conception of physics? How can we characterize such a classical conception of physics? Which are the elements that can allow us to construct a coherent interpretation of quantum mechanics?

1.1 The Problems Addressed

This doctoral dissertation contains four main elements:

1. We put forward an interpretational **map** of quantum mechanics in general and of the modal interpretation in particular based on metaphysical and anti-metaphysical stances.
2. In contraposition to what we characterize as the anti-metaphysical and classical metaphysical stances we collect arguments in favor of a **constructive metaphysical stance**.
3. We analyze the meaning of **contextuality** and **possibility** in quantum theory in general, and in the modal interpretation in particular.
4. Finally, we end up with a proposal for the development of a possible constructive metaphysical scheme based on the **notion of potentiality**.

Although some of these problems are well known in the literature we attempt to cast new light on the discussion through the analysis of the concepts involved and their relation to the formalism. We attempt to make explicit the tension in between the theoretical conditions and the conceptual structure of the theory, in order to discuss and disclose the metaphysical ideas involved within the different interpretational problems of quantum mechanics. We believe that this analysis can help us to understand much of the implicit background of such theoretical conditions.

The dissertation centers itself on the modal and contextual character of quantum mechanics. On the one hand, we discuss the contextual character of *quantum possibility* through the development of a Modal Kochen-Specker theorem, and on the other hand, we analyze the contextual character of quantum correlations through the distinction of properties. In the last part we discuss some more speculative ideas related to the possible metaphysical developments of quantum mechanics based on the notion of potentiality.

The list of publications which relate to the present work is the following:

1. Aerts, D., de Ronde, C. and D'Hooghe B., 2011, "Compatibility and Separability for Classical and Quantum Entanglement", In *Worldviews, Science and Us: Bridging knowledge and its implications for our perspectives of the world*, D. Aerts, B. D'Hooghe and N. Note (Eds.), World Scientific, Singapore, forthcoming.
2. Christiaens, W. and de Ronde, C, 2009, *Fysica & wiskunde*, In E. Romein, M. Schuilenburg, S. van Tuinen (Eds.), 328-344, Boom, Amsterdam.
3. Christiaens, W., de Ronde, C., D'Hooghe, B. and Holik, F., 2010, "Some Remarks on the Notion of Separability within the Creation Discovery View", *International Journal of Theoretical Physics*, **49**, 3061-3068.

4. Domenech, G., Freytes, H. and de Ronde, C., 2006, “Scopes and limits of modality in quantum mechanics”, *Annalen der Physik*, **15**, 853-860.
5. Domenech, G., Freytes, H. and de Ronde, C., 2008, “A topological study of contextuality and modality in quantum mechanics”, *International Journal of Theoretical Physics*, **47**, 168-174.
6. Domenech, G., Freytes, H. and de Ronde, C., 2009, “Many worlds and modality in the interpretation of quantum mechanics: an algebraic approach”, *Journal of Mathematical Physics*, **50**, 072108.
7. Domenech, G. and de Ronde, C., 2010, “Non-Individuality in the Formal Structure of Quantum Mechanics”, *Manuscrito*, **33**, 207-222.
8. de Ronde, C., 2005, “Potencialidad ontológica y teoría cuántica”, In *Volumen 11 de Epistemología e Historia de la Ciencia*, V. Rodríguez and L. Salvatico (Eds.), Universidad Nacional de Córdoba, Córdoba.
9. de Ronde, C., 2009, “El enfoque de descripciones complementarias: en búsqueda de un desarrollo expresivo de la realidad física”, *Perspectivas Metodológicas*, **9**, 126-138.
10. de Ronde, C., 2010, “No Entity, No Identity”, In *Filosofía e história da ciência no Cone Sul. Seleçâo de trabalhos do 6 Encontro*, 176-183, R. Martins, L. Lewowicz, J. Ferreira, C. Silva, L. Martins (Eds.), Associaçâo de Filosofia e História da Ciência do Cone Sul, Campinas.
11. de Ronde, C., 2010, “Metaphysical Issues in the Philosophical Foundation of Quantum Mechanics”, *Philosophica*, **83**, 5-14.
12. de Ronde, C., 2010, “For and Against Metaphysics in the Modal Interpretation of Quantum Mechanics”, *Philosophica*, **83**, 85-117.
13. de Ronde, C., 2011, “Understanding Quantum Mechanics Through the Complementary Descriptions”, In *Probing the Meaning of Quantum Mechanics: Physical, Philosophical, Mathematical and Logical Perspectives*, C. de Ronde, S. Aerts, D. Aerts (Eds.), forthcoming.
14. de Ronde, C., 2011, “La noción de potencialidad ontológica en la interpretación modal de la mecánica cuántica”, *Scientiae Studia*, forthcoming.
15. de Ronde, C. and Bontems, V., 2011, “La notion d’entité en tant qu’obstacle épistémologique: Bachelard, la mécanique quantique et la logique”, *Bulletin des Amis de Gaston Bachelard*, forthcoming.
16. de Ronde, C., Domenech, G., Holik, F. and Freytes, H., 2011, “Entities, Identity and the Formal Structure of Quantum Mechanics”, In *Contactforum Sructure and Identity at the Koninklijke Vlaamse Academie van België*, W. Christiaens and K. Verelst (Eds.), forthcoming.

1.2 Overview

The thesis is divided in five parts. **Part I**, the INTRODUCTION, presents the main ideas of the dissertation. In **chapter 1**, we put forward the general scheme of the dissertation and we discuss the main problems which will be addressed through this work. In **chapter 2**, following van Fraassen, we present philosophy in terms of a stance, as an existential enterprise. Within this context, we discuss the need of interpretation in physical theories. We also present briefly some aspects of the stance which will be developed through the dissertation, according to which, apart from the empirical elements and the theoretical structure, special emphasis must be given to the representational conceptual scheme of a physical theory.

Part II, QUANTUM MECHANICS AND THE INTERPRETATION OF PHYSICAL REALITY, starts to configure the three main stances which characterize the map we put forward regarding the interpretation of quantum theory, namely, the anti-metaphysical stance, the classical metaphysical stance and the constructive metaphysical stance. **Chapter 3** analyzes the role of contemporary analytic philosophy of physics and philosophy of quantum mechanics in particular. In this context we discuss the need of discussing the problem of representation of reality within physical theories. We also present a representational realist account of physical theories. In **chapter 4**, we argue that Mach's positivistic philosophy and the criticism of the *a priori* Kantian scheme was a key element for the development of the quantum revolution. We discuss the positions taken by Einstein, Heisenberg and Pauli in relation to both physics and metaphysics. We analyze Heisenberg's indeterminacy principle and Bhor's complementarity approach. More specifically, we interpret Bohr's philosophy of physics as introducing physics into the philosophical development of the 20th century known as the "linguistic turn". **Chapter 5** goes back to quantum mechanics and discusses the metaphysical conditions, related to the formalism, implicitly imposed on the determination and construction of two of the most paradigmatic interpretational problems, namely, Bell inequalities and the Kochen-Specker theorem. We also distinguish between *actual observable*, *actual preexistent*, *actual observable statistical* and *actual preexistent statistical* properties. Within our stance we argue, in **chapter 6**, for a constructive metaphysical position.

Part III, MODALITY, discusses the notions of *possibility* and *actuality* within the formal structure of quantum mechanics, and more specifically, in relation to the so called modal interpretation. We also put forward our methodological distinction and place our arguments for a constructive metaphysical stance. In **chapter 7**, we present the modal interpretation of quantum mechanics and discuss some of its different versions. We propose the distinction between *Modal Interpretations which start from Metaphysical Conditions* (MIMP) and *Modal Interpretations which start from the Mathematical Formalism* (MIMF). **Chapter 8**, discusses the *limits* of considering *actual properties* within the modal interpretation through several KS type theorems, such as that of Bacciagaluppi and Clifton, and the preferred factorization problem. In **chapter 9** we analyze the "metaphysically tenable conditions" imposed by the different versions and put forward our strategy to interpret quantum mechanics based on a constructive metaphysical stance.

Part IV, CONTEXTUALITY, analyzes the meaning of the contextual character of the formal structure of quantum mechanics. We also discuss the contextual character of the notions of *quantum possibility* and *quantum correlations.* In **chapter 10**, we discuss the meaning of contextuality in terms of the problem of representation of physical reality itself. We also relate this discussion to the classical and constructive metaphysical stances. **Chapter 11**, discusses, based on the original introduction of a Modal Kochen Specker type theorem, the contextual character of *quantum possibility.* Based on this theorem, we argue that the notion of possibility which appears in the orthodox structure of quantum mechanics is not classical and that, from a representational realist perspective, modal interpretations must take this aspect into account when discussing the interpretational development of the theory, also in relation to MIMP and MIMF. **Chapter 12** analyzes the contextual character of *quantum correlations* taking into account Mermin's proposal to consider "correlations as actual elements of physical reality". In order to clarify several seemingly contradictory aspects of the discussion, such as the co-existence of the subsystem correlation (SSC) theorem on the one hand and Cabello's and Seevinck's theorems on the other, we go back to our earlier distinction of properties.

Part V, MODALITY REVISITED, analyzes in formal and metaphysical terms the notion of possibility and actuality. It also discusses, within modal interpretations, a proposal to interpret quantum possibility in terms of the notion of rational potentiality. In **Chapter 13** we recall the modes of being of actuality and potentiality in Aristotle's metaphysics. We analyze the realms of actuality and potentiality in classical physics and recall Heisenberg's interpretation which takes into account the notion of *potentia* in quantum mechanics. In **Chapter 14** we present a review and analysis of the different meanings of quantum possibility in quantum mechanics. We relate these interpretations to the multiple interpretations of quantum superpositions. To finalize, in **Chapter 15**, we provide some concluding remarks regarding our representational realist proposal and the constructive metaphysical strategy of developing a coherent interpretation of quantum mechanics.

Philosophy as a Stance

In one of his last books, *The Empirical Stance*, Bas van Fraassen makes a remarkable claim:

> "The problem of appearance and reality affects first of all philosophy itself. I argue for a view of philosophy as a stance, as existential." B. van Fraassen (Quoted from [256], p. xviii)

Personally, I believe that this remark is of the utmost importance. To consider philosophy as a stance, as existential, means that our analysis is not void of values, intentions and presuppositions. These must be made explicit in order to be honest about the limits of our arguments. Van Fraassen continues:

> "Philosophy itself is a value- and attitude-driven enterprise; philosophy is in false consciousness when it sees itself otherwise. To me philosophy is of overriding importance, to our culture, to our civilization, to us individually. For it is the enterprise in which we, in every century, interpret ourselves anew. But unless it so understands itself, it degenerates into an arid play of mere forms." B. van Fraassen (Quoted from [256], p. 17)

In this work we are concerned with the metaphysical presuppositions involved in the analysis of quantum mechanics and its possible interpretation. Following van Fraassen, we also consider philosophy as a stance, so let's be honest about ours, or at least, lets make some points clear. We understand physics, like the Greeks, as a question which relates to existence, to *Being*. This is the question which has driven me to study physics and philosophy, the possibility to try to understand the world. This is to me an amazing and fascinating question. If physics would be only about developing technique, about finding out an algorithm which provides the correct outcomes, I would have preferred to study music. I dismiss the idea that quantum mechanics does not refer or express physical reality. If there is no connection to the world, to the real, despite all the problems which arise from this connection, it simply makes no sense to me that quantum mechanics works at all. The interesting question is how to understand the relation between the theory and reality, and not just how to build up technical developments through algorithmic structures which we do not understand.

In this sense, the debate which took place between Einstein and Bohr regarding the interpretation of quantum mechanics reveals the tension present, not only in 20th century physics but also philosophy, between two antagonic stances. Although both analyzed the conditions of possibility to consider physical experience they developed two antagonic positions. While Einstein engaged in a metaphysical quest and considered the problem of reality as a guiding line for physics, Bohr, in consonance with his time, was ready to give up on metaphysics and the ontological questions which had guided Western thought —at least until Immanuel Kant had rightly criticized the limits of such attempts. Instead of Einstein's ontological quest, Bhor advanced into a pragmatic scheme based on language and communicability. These two orthogonal lines of thought determine the plane on which quantum mechanics has been discussed ever since. We understand this debate, not only in relation to the well known discussions which took place —regarding mainly the double-slit experiment— in the Solvay conferences, but also as a much broader confrontation between two distinct positions which relate physics to epistemology and ontology. It is the tension exposed through these two figures which has dominated the agenda of both the physical and philosophical analysis of quantum mechanics in the 20th century.

Einstein and Bohr took different stances in the analysis of the meaning of quantum mechanics and physics itself. Their positions are reflected in their writings, that is what we have left. We could of course provide many different interpretations regarding their intentions and hidden agendas; more interesting than that is to coherently place their thoughts and ideas within a stance which comprises their own philosophical schemes. In this sense, we are not interested so much in what Einstein and Bohr *believed* but rather, we are interested in a particular interpretation which we will put forward and which will help us to understand the consequences, the relations and the processes of development of ideas which took place in the 20th century within the many attempts to understand quantum mechanics.

2.1 The Problem of Interpretation

Regarding its formal structure we could say that quantum mechanics seems to be a "finished theory". In terms of empirical adequacy, it provides outstanding results, its mathematical structure —developed in the first three decades of the 20th century by people like Werner Heisenberg, Pascual Jordan, Max Born, Erwin Schrödinger and Paul Dirac— seems able to provide until now the adequate modeling to any experiment we can think of. However, apart from its fantastic accuracy, even today its physical interpretation remains an open problem. In the standard formulation, quantum mechanics assigns a quantum mechanical state to a system, but 'the state' has a meaning only in terms of the outcomes of the measurements performed and not in terms of 'something' which one can coherently relate to physical reality. It is not at all clear, apart from measurement outcomes, what is the *referent* of this quantum state, in particular, and of the formal structure, in general. If we are to ask too many questions, problems start to pop up and simple answers seem doomed to inconsistency.

At the beginning of the 20th century, physics tackled the metaphysical conception of the world. As noted by Bohr ([48], p. 53): "[the essence of quantum theory] may be expressed in the so-called quantum postulate, which attributes to any atomic process an essential discontinuity, or rather individuality, completely foreign to the classical theories and symbolized by Plank's quantum of action." The nature of the problems raised by the new quantum theory revealed a critical confrontation with the classical worldview, it would soon become clear that the return to the departed land was if not impossible, a very difficult task to accomplish.

Let it be a particle, a wave or a field, a *physical entity* is a machinery which allows to unify the multiple phenomena under study. In physical theories the physical object remains a primitive concept acting as a *presupposition* for describing phenomena. The "simple" idea of analyzing the trajectory of a body presupposes "right from the start" the existence of 'the body', an object as the referent to which the properties can be attached. However, the contextual character of quantum mechanics poses problems for this classical interpretation of physical reality. As noticed a long time ago by Schrödinger:

> "[...] if I wish to ascribe to the model at each moment a definite (merely not exactly known to me) state, or (which is the same) to *all* determining parts definite (merely not exactly known to me) numerical values, then there is no supposition as to these numerical values *to be imagined* that would not conflict with some portion of quantum theoretical assertions." E. Schrödinger (Quoted from [234], p. 156)

We will argue in the chapters to come that the discussion regarding physical reality and existence within quantum mechanics was a central issue for the founding fathers and no clear 'common position' was ever approached. Even between those who were later on considered to have a common view of the problem —such as Pauli, Heisenberg and Bohr—, there was no consensus but rather debate and discussion. The subtleties involved in the many debates of the founding fathers of the theory have been obscured by the simplistic translation into the 'realist-antirelist' debate still present in analytic philosophy of science. For example, as noticed by Kalervo Laurikainen:

> "It is not generally known that there was a profound difference in the philosophical attitudes of Niels Bohr and Wolfgang Pauli (Laurikainen 1985b, section 3). In his address at the Second Centenary of Columbia University in 1954, "The Unity of Knowledge", Bohr claimed that the observer even in quantum mechanics can be considered 'detached' provided we understand the observation in the right way (Bohr 1955, p. 83). An observation includes a detailed description of all the experimental arrangements which can have an influence upon the phenomenon under investigation, and it is finished only when a registered result is obtained which everybody can verify afterwards. In this sense, Bohr said, an observation is quite *objective* (which for Bohr means 'intersubjective'), and the observer does not have any influence on the result in any other way than by choosing the method of observation. The result is explicitly associated with a given method of observation. If physics is understood as a system which makes it possible to govern such objective observational results —which, however, is only possible in probabilistic sense— then physics, according to Bohr, can even in atomic physics be considered quite objective and the observer is 'detached' in exactly the same way as in classical physics." K. Laurikainen (Quoted from [171], p. 42)

Bohr had sent the manuscript of the paper to Pauli in order to receive his critics and comments. In his reply, dated February 15, 1955, Pauli pointed out explicitly —in line with Einstein— that the role of the observer in classical mechanics is essentially different from that in quantum theory.

> "[...] it seems to me quite appropriate to call the conceptual description of nature in classical physics, which Einstein so emphatically wishes to retain, 'the ideal of the detached

> observer'. To put it drastically the observer has according to this ideal to disappear entirely in a discrete manner as hidden spectator, never as actor, nature being left alone in a predetermined course of events, independent of the way in which phenomena are observed. 'Like the moon has a definite position' Einstein said to me last winter, 'whether or not we look at the moon, the same must also hold for the atomic objects, as there is no sharp distinction possible between these and macroscopic objects. Observation cannot *create* an element of reality like position, there must be something contained in the complete description of physical reality which corresponds to the *possibility* of observing a position, already before the observation has been actually made.' I hope, that I quoted Einstein correctly; it is always difficult to quote somebody out of memory with whom one does not agree. It is precisely this kind of postulate which I call the ideal of the detached observer.
>
> In quantum mechanics, on the contrary, an observation *hic et nunc* changes in general the 'state' of the observed system, in a way not contained in the mathematical formulated *laws*, which only apply to the automatical time dependence of the state of a *closed* system. I think here of the passage to a new phenomenon of observation which is taken into account by the so-called 'reduction of the wave packets'. As it is allowed to consider the instruments of observation as a kind of prolongation of the sense organs of the observer, I consider the impredictable change of the state by a single observation —in spite of the objective character of the results of every observation and notwithstanding the statistical laws of frequencies of repeated observation under equal conditions— to be *an abandonment of the idea of the isolation (detachment) of the observer from the course of physical events outside himself.*
>
> To put it in nontechnical common language one can compare the role of the observer in quantum theory with that of a person, who by his freely chosen experimental arrangements and recordings brings forth a considerable 'trouble' in nature, without being able to influence its unpredictable outcome and results which afterwards can be objectively checked by everyone." W. Pauli (Quoted from [170], p. 60, emphasis added)

Since the debate of the founding fathers many interpretations have attempted to provide a coherent account of quantum mechanics, an account which refers to objective physical reality. Still today, the many discussions in the literature and the ongoing battle between different interpretations seem to point to the intrinsic difficulty of the problem raised.

> "*Einstein*'s opposition to [quantum mechanics] is again reflected in his papers which he published, at first in collaboration with *Rosen* and *Podolsky*, and later alone, as a critique of the concept of reality in quantum mechanics. We often discussed these questions together, and I invariably profited very greatly even when I could not agree with *Einstein*'s view. 'Physics is after all the description of reality' he said to me, continuing, with a sarcastic glance in my direction 'or should I perhaps say physics is the description of what one merely imagines?' This question clearly shows *Einstein*'s concern that the objective character of physics might be lost through a theory of the type of quantum mechanics, in that as a consequence of a wider conception of the objectivity of an explanation of nature the difference between physical reality and dream or hallucination might become blurred."
> W. Pauli (Quoted from [193], p. 122)

Related to the question of physical reality in quantum mechanics, from the very beginning of the voyage, the problem was to find a picture (an *anschauliche* content), a physical representation which would explain *what* quantum mechanics was talking about. Due to the impossibility to provide such an account, very soon more pragmatic stances were developed. For example, Paul Dirac writes in his book, *The Principles of Quantum Mechanics*:

> "[...] *the main object of physical science is not the provision of physical pictures, but is the formulation of laws governing phenomena and the application of these laws to the discovery of new phenomena.* If a picture exists, so much the better; but whether a picture exists or not is a matter of only secondary importance." P. Dirac (Quoted from [102], p. 10, emphasis added)

As we shall argue in chapter 4, Niels Bohr might have been the most influential figure who tried —and succeeded to great extent— to expel metaphysical questions from the interpretation of quantum mechanics. This line of thought, which considers a physical theory in pragmatic terms, ends up in a path which seems difficult to maintain —at least in the case we are still willing to state that there is something like "physical reality" about which our theories talk. In a spirit very similar to the ideas expressed by Dirac, and after endless discussions regarding the meaning of the quantum, in the year 2000, exactly one century after the beginning of the voyage, Christopher Fuchs and Asher Peres finally took this line to its unavoidable conclusion in a paper entitled: *Quantum Theory Needs no 'Interpretation'.* There, they wrote:

> "[...] quantum theory does not describe physical reality. What it does is provide an algorithm for computing probabilities for the macroscopic events ("detector clicks") that are the consequences of experimental interventions. This strict definition of the scope of quantum theory is the only interpretation ever needed, whether by experimenters or theorists." C. Fuchs and A. Peres (Quoted from [129], p. 1)

Of course this emphasis on prediction to the detriment of description can be severely questioned. The main objection against this instrumentalistic point of view is that the success of a theory can not be explained, that is to say, we do not know how and why quantum physics is in general able to carry out predictions (and in particular with such a fantastic accuracy). Undoubtedly a "hard" instrumentalist may simply refuse to look for such an explanation, since it is in fact the mere effectiveness of a theory which justifies it, so that he may not be interested in advancing towards a justification of that effectiveness. If one takes such a position there is nothing left to say. Just like the Oracle of Delphi provided always the right answer to the ancient Greeks, quantum mechanics provides us with the correct probability distribution for every experiment we can think of. So there is nothing else we need; that is all we can ask from a physical theory and there is no need to supplement it with an *interpretation.*

In physics we deal, not only with mathematical equations, we also need to take into account their *interpretation.* A mathematical scheme needs an interpretation in order to refer to physical experience for there is no physics without a conceptual structure that supports phenomena. We agree with Sunny Auyang when she writes:

> "Interpretation usually applies to texts. Unlike discourses, which are rooted in concrete situations and surroundings to which demonstratives and other indexical terms refer, texts are to some degree decontextualized. A text addresses unknown audiences and frees itself from the conditions of its production, thus opening the possibility for various readings. Reading is receptive but not passive, and genuine reading is interpretative. The reader contributes by adopting, fixing the references, finding meaning for textual descriptions in

> his own comprehension, entering the world jointly opened by the text and his anticipation, thereby gaining an understanding of the text. Understanding, interpretation, and application are inseparable elements of a hermeneutical act.
>
> No application of a theory is possible without some interpretation. Quantum mechanics is a physical theory, not pure mathematics draped in meaning assigned by correspondence rules. Therefore there is no pure quantum formalism, no 'neutral' presentation of the structure of quantum mechanics." S. Auyang (Quoted from [20], p. 61)

The history of Western metaphysics seems to indicate that reality is not something "self evident", but can be only determined and justified from a set of presuppositions and constraints given by a conceptual scheme.[2] In this sense, already classical physics is a (metaphysical) interpretation of the world. Interpretation supports a creative field, it does not mean to force a formalism into presupposed metaphysical or anti-metaphysical schemes, rather, it means to create the conceptual links which can allow us to express reality. A problem only arises when metaphysics is turned into dogma. When definite presuppositions are turned into closed, unmovable and static sculptures of a world maybe too human. When metaphysics starts to turn against the creative field of interpretation and presents limits to that which can be thought. We claim that a proper understanding of quantum mechanics as such, will be only provided in the case we can answer coherently the question: what is quantum mechanics talking about?

2.2 Our Stance

To expose one's own position is of great importance for an intellectually honest discussion and to open the possibility of the creation of new ideas. This work argues for a definite stance, a stance which we attempt to develop through our analysis of both the formalism and the metaphysical principles involved in quantum mechanics. This stance will become clear as we advance through the chapters. However, there are some ideas which we can already put forward. The point of departure for the analysis we attempt to provide in this dissertation regards the idea of putting forward an interpretation of quantum mechanics which takes into account what we call *representational realism.* According to this stance physical theories are necessarily related to a *formal* and *conceptual* scheme, a physical representation which allows, and is at the same time a precondition, to consider *physical experience.* There is no experment nor meaning of a 'physical situation' —or even a 'physical property'— without the presupposition of a conceptual scheme which provides a representation of physical reality. Instead of the fixed *a priories* present in the Kantian architectonic, our stance proposes to reflect about the possibility of considering constructive *a priories*; i.e. metaphysical conditions which are developed in physical theories in order to access reality.[3] Rather than concentrating in the question of truth —which is mainly addressed in the realism-antirealism debate in philosophy of science— we are interested in discussing the role played by metaphysical presuppositions within ontological interpretations of quantum theory.

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[2] As an interesting example of the shift in between metaphysical presuppositions see [61], where a deep analysis of the different presuppositions involved in science in the Middle Ages and Modernity is presented.

[3] This stress of the relation between observation and the conceptual structure of the theory can be certainly linked to the neo-Kantian tradition expressed today by Mittelstaedt, Bitbol, Pettitot and many others. See [164] for a complete map of the neo-Kantian tradition within the interpretation of quantum mechanics.

A main notion which we discuss —in order to get closer to the meaning of possibility— is that of *actuality* and its different meanings within philosophical stances. While empiricism —attempting to escape representation— refers to actuality in relation to observation, to the *hic et nunc*, realism relates actuality to preexistence (i.e. existence independent of observation) through representation avoiding the need of referring to measurement results (see section 3.3). We understand preexistence as a representational concept which considers existence with a temporal identity; it is this which allows to go beyond the *hic et nunc* and represent existence in metaphysical terms, as a formal and conceptual structure which represents the world. Physical theories —until quantum mechanics— have always considered such ideal as a basic presupposition in order to discuss about the dynamic and evolution of systems. Empiricism can do without dynamics, for it does not believe that a formal structure describes the world. For the empiricist it is only the actual observation which matters.

In order to advance with a representational realistic approach, we also need to clearly characterize the limits of the classical metaphysical conception of the world. Indeed, as Mauro Dorato [110] argues: "[...] in favor of the view that if physics is to become a coherent metaphysics of nature, it needs an interpretation, namely (i) a clear formulation of its ontological/metaphysical claims and (ii) a precise understanding of how such claims are related to the world of our physical experience, which is the most important reservoir of traditional, merely aprioristic metaphysical speculations." Our stance is that physics relates to reality and phenomena. The stance which we attempt to put forward is based on the possibility to develop a constructive metaphysical scheme which allows to interpret quantum mechanics without the presuppositions involved in the classical conception of the world.

We attempt to link our attempt of a representational realistic stance to the proposal of Heisenberg of *closed theories*. As remarked by Bokulich: "The German phrase that Heisenberg uses is *abgeschlossene Theorie*, where *abgeschlossene* can be translated as 'closed', 'locked', 'isolated', or 'self-contained'." Heisenberg (see [141], chapter 6 and [53] for discussion) understands 'closed theories' as a relation of tight interconnected concepts, definitions and laws whereby a large field of phenomena can be described.[4] As noticed by Carl Friedrich von Weizsäcker:

> "Heisenberg's probably most important contribution to the philosophy of science [is] the concept of the progress of physics as a sequence of 'closed theories' (abgeschlossene Theorien; Heisenberg 1948). This concept places the systematic achievements of physics (closed theories) into an historic concept (sequence). The concept originated in a reflection on what Heisenberg himself had achieved twenty years earlier when he was lucky enough to lay the foundation of the latest closed theory which so far has emerged in physics. A 'closed' theory is not an ultimate, all embracing theory, but a theory which cannot be improved any more by small changes. Classical mechanics, thermodynamics, electrodynamics, special and general relativity, quantum mechanics are examples. A new closed theory differs from its predecessors not only by new assertions but by new concepts. The closed theories of the 20th century have confined the earlier ones —now called 'classical'— to limited fields

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[4] As noticed by Alisa Bokulich further insight into Heisenberg's philosophy of science can be gained through a comparison of his views to those of Thomas Kuhn on scientific revolutions [168]. See also ([32], p. 288) and especially the very interesting paper by Alisa Bokulich: 'Heisenberg Meets Kuhn: Closed Theories and Paradigms' [54].

> of application. Heisenberg did not consider quantum mechanics as ultimate; he aimed at finding a closed theory beyond it." C. F. von Weizsäcker (Quoted from [271], p. 278)

From this perspective there is no need of a reductionistic conception of concepts,[5] for every physical theory needs to develop its own conceptual scheme —conceptual schemes which are independent of another different (closed) theory. As remarked by Heisenberg in an interview by Thomas Kuhn (quoted from [54], p. 98): "The decisive step is always a rather discontinuous step. You can never hope to go by small steps nearer and nearer to the real theory; at one point you are bound to jump, you must really leave the old concepts and try something new... in any case you can't keep the old concepts." The only important aspect to consider a physical theory as 'closed' is the internal *coherency* between the formal mathematical elements, the conceptual structure and the physical experience involved.[6] The coherency reflects a wholeness present in every 'closed theory' which Heisenberg expresses in the following manner:

> "One finds [in closed theories] structures so linked and entangled with each other that it is really impossible to make further changes at any point without calling all the connections into question [...] We are reminded here of the artistic ribbon decorations of an Arab mosque, in which so many symmetries are realized all at once that it would be impossible to alter a single leaf without crucially disturbing the connection of the whole." W. Heisenberg (Quoted from [54], p. 95)

Heisenberg took quantum mechanics to be a 'closed theory'. Already in 1927 in a paper with Max Born (quoted from [54], p. 92) they write: "We wish to emphasize that [...] we consider quantum mechanics to be a closed theory, whose fundamental physical and mathematical assumptions are no longer susceptible to any modification." This is one of the main points in which our approach takes distance from Heisenberg's ideas for we do not consider we have still been able to build up a proper interpretation, we have not found the new concepts which make a coherent bridge to the formalism and physical experience.

Just like Everett [117], we believe that quantum mechanics should be able to find it own interpretation. The formalism of quantum mechanics has a lot to say, its symmetries, its formal features are crying for new concepts which can explain what quantum mechanics is telling us about the world. Coherency must respect the formal features and make sense of them without *ad hoc* moves or dogmatic (classical) presuppositions. In this sense, a main problem which has been present in the interpretation of quantum mechanics since its very origin regards the meaning of probability and possibility. It is well known that the non-Kolmogorovian structure of the formalism puts limits to an ignorance or epistemic interpretation of quantum

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[5] An important example to understand this non-reductionistic characterization is how Newtonian mechanics has to be understood in relation, for example, to relativity theory. According to Heisenberg ([141], pp. 97-98): "New phenomena that had been observed could only be understood by new concepts which were adapted to the new phenomena. [...] These new concepts again could be connected in a closed system. [...] This problem arose at once when the theory of special relativity had been discovered. The concepts of space and time belonged to both Newtonian mechanics and to the theory of relativity. But space and time in Newtonian mechanics were independent; in the theory of relativity they were connected."

[6] It is important to remark that the coherency to which we relate in this case is not the one discussed in "coherence theory of truths" in philosophy of science (see for example [277]). While coherence theory of truths discuss about the relation between *propositions* within a physical theory, we discuss about the relation between *concepts*, *mathematical expressions* and *physical experience*.

systems. Still today there are many attempts in the literature which provide an interpretation of probability [6; 202; 209]. Modal interpretations of quantum mechanics have discussed the meaning of possibility in formal terms, allowing to analyze the question from a very broad perspective, it is exactly this analysis which we attempt to continue in this dissertation.

II

Quantum Mechanics and the Interpretation of Physical Reality

Philosophy of Science: For or Against Metaphysics?

In the literature the term “metaphysics” comprises a series of many different definitions. For some, it can be considered as a supreme form of knowledge, while for others, it remains an occupation constituted by unfruitful discussions. As noticed by Gilles Gaston Granger the development of metaphysics is intrinsically related to the different questions which have determined the path of Western thought:

> “le problème moderne: qu’est-ce que le réel ? n’est pas posé en ces tremes par les deux grands philosophes classiques Platon et Aristote. La question, héritée sans doute de Parménide, est alors très généralement exprimée par eux sous la forme : qu’est-ce que l’Être ?” G G. Granger (Quoted from [132], p. 13)

Metaphysics designates a controversial field which has accompanied the development of Western thought since Plato and Aristotle. However, there is already a huge distance between these two main figures and creators of metaphysical thought. Within the analysis proposed by Granger in his book *Sciences et réalité*, Plato proposes an interpretation of the real as participation of the absolute Being ([132], p. 22). “Cet Être absolu est ‘Idée’, c’est-à-dire non point le produit d’une pensée mais une entité accessible seulement par un acte de ‘vision’ et non par un acte discursif de l’entendement, à travers un langage. L’Être absolu est normatif, en ce sens qu’il est le *modèle* et la *source* de toutes ses *réalisations* imparfaites; c’est pourquoi l’Idée par excellence est l’Idée du bien et du beau en soi.” The proposal of Aristotle already takes distance from Plato and proposes in a very different style an interpretation of the real which can be already considered as much closer to ours.

> “En ce qui concerne la question qui nous ocupe, la grande innovation aristotélicienne est double. D’une part, Aristote découvre et systématise *à travers les formes du langage* une structure et une organisation de l’Être ; d’autre part, il réintroduit de plein droit dans l’Être, et decrit comme type d’objet connaissable, un *individuel* qui n’est pas celui très abstrait des idées platoniciennes.” G G. Granger (Quoted from [132], pp. 23-24)

Aristotle defines metaphysics as a theory of “being *qua* being” (Aristotle, Metaphysics 1003a20) a theory about what it means or implies to “be” in its different senses. Since then, it has become clear that the importance of metaphysical thought within physical theories can be hardly underestimated. As noticed by Edwin Arthur Burtt ([61], p. 224): "[...] there is no escape from metaphysics, that is, from the final implications of any proposition or set of propositions. The only way to avoid becoming a metaphysician is to say nothing." It is Grager who remarks that, apart from both Plato and Aristotle, it is Leibniz who, through a new idea of the possible, is capable of introducing the specifical sense of the question about reality and existence ([132], p. 14). According to Wolff —who gave, following Leibniz, a classic definition of what is metaphysics from the perspective of 17th century rationalism— *metaphysica specialis* is divided in four main regions (see for example [275] and references therein). Rational theology, which discusses the existence and attributes of God, rational psychology, which studies the soul as a simple non-extended substance, rational cosmology, which discusses the world as a whole, and metaphysical generalis or ontology, which discusses traits of the existent in general, the Being qua Being. This characterization of metaphysics was severely criticized by Immanuel Kant who reconfigured the discussions regarding metaphysical thought and the limits of knowledge. According to Kant —in the "Preface" to the Critique of Pure Reason— Wolff is "the greatest of all dogmatic philosophers". Kant wanted to escape metaphysics, which meant for him the possibility to go beyond dogma and belief, to understand the *finite* access with which every human being is confronted.[7] By understanding the limits of human knowledge metaphysics would finally follow the secure path of science and show how (scientific) knowledge is possible.

Within philosophy of science, as a part of the analytic tradition, the tension between metaphysical and anti-metaphysical positions did not disappear but remained at the center of gravity of many discussions. As remarked by van Fraassen, although analytic philosophy had begun as a revolt against metaphysics, this movement was very soon subverted. As if it were a repetition of the Einstein-Bohr debate, but now in the context of analytic philosophy of science, positions for and against were also developed in the realm of quantum mechanics and its interpretation. In the following chapter we address these different positions and advance towards the problem of the justification of physical experience as related to a conceptual scheme. We shall also present a position, namely, representational realism, from which we attempt to use to discuss several notions and interpretational questions within quantum mechanics, in the rest of the dissertation.

3.1 Empiricism and Analytic Philosophy: Against Metaphysics?

In order to understand oneself one needs to know one's own history and traditions. Where do we come from? Who was our father? What religion did he profess? Who was our grandfather? In which war did he fight? The same applies to philosophical stances which are always connected to traditions of thought, to lineages, to fights and battles which go back in time. In particular, one could interpret the history of occidental philosophy as a confrontation between two main forces. On the one side the metaphysical or ontological force, which seeks to answer the question of *Being qua Being*; and on the other side, an anti-metaphysical or epistemological force, which focus its attention on the limits and constrains of such a question. Analytic

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[7] See in this respect the very interesting paper by Michel Bitbol [40] where he discusses the importance of the notion of metaphysics within Kant and its relation to quantum mechanics.

philosophy has been clearly, not only from an historical perspective but also methodologically, part of this second force.

Empiricism and logicism are two of the main sources of the origin of analytic philosophy. An idea often found in empiricism is that science should use theories as an instrument and should renounce the quest for explanation. It should be clear that the search for such explanations is a metaphysical enterprise. As noticed by van Fraassen ([256], p. xviii) "Empiricist philosophers have always concentrated on epistemology, the study of knowledge, belief, and opinion, with a distinct tendency to advocate the importance of opinion." Against the ontological concerns of the metaphysicians, analytic philosophers engaged in epistemological issues. Escaping from the true statements of the metaphysicians, of *episteme*, analytic philosophy remained closer to opinion and *doxa* [66]. True knowledge was regarded with suspicion, as a dogma of the past, as a metaphysical idol with no proper fundament. This is what Burtt calls the "central position of positivism itself", the idea that it is possible to "acquire truths about things without presupposing any theory of their ultimate nature; or more simply, it is possible to have a correct knowledge of the part without knowing the nature of the whole." According to van Fraassen, the history of analytic philosophy is also directly connected to a criticism of and reaction against the dominant metaphysical attitude in Continental Europe in the 17th century.

> "The story of empiricism is a story of recurrent rebellion against a certain systematizing and theorizing tendency in philosophy: a recurrent rebelion against the metaphysicians." B. van Fraassen (Quoted from [256], p. 36)

However, even though analytic philosophy started from a revolution against metaphysics, the introduction of metaphysical questions reappeared very soon within analytic philosophy itself.

> "As I see it, analytic philosophy —which is the strand to which I belong— began with a revolution that was subverted by reactionary forces. I am speaking here of reversion to a seventeenth-century style of metaphysics. I do not reject all metaphysics, but this reversion I see as disastrous. Paradoxically, this disaster seems to be worst in two areas that scarcely relate to each other at all. I mean, on one hand, the area loosely characterized as "science and religion" studies and, on the other, academic analytic philosophy. Both suffer from unacknowledged as well as explicit metaphysics." B. van Fraassen (Quoted from [256], p. xviii)

As noticed by van Fraassen, one of the most interesting and subversive starting points of analytic philosophy was very soon turned upsidedown.

> "[...] with the rise of analytic philosophy something paradoxical happened. This movement began in a series of revolts, across Europe and America, against all forms of metaphysics. And lo, even before mid-century, some of its ablest adherents began to make the world safe for metaphysics again. Since then we have seen the growth of analytic ontology, analytic metaphysics, and it thrives today.
>
> Or so it seems. I say that metaphysics is dead. What I see is false consciousness, a philosophy that has genuinely advanced beyond the past, but a philosophy that misunderstands itself." B. van Fraassen (Quoted from [256], pp. 3-4)

3.2 Philosophy of Quantum Mechanics Today

Since the second world war the philosophical analysis of science, and of quantum theory in particular, has been an almost exclusive field owned by analytic philosophy —in distinction to the so called "continental" philosophical tradition of thought which "has discussed the large spiritual problems that are concern of every thinking person: the meaning of life, the nature of humanity, the character of a good society" ([127], p. 9). Although the analytical tradition is the inheritor —via logical positivism and logical empiricism— of a deep criticism of metaphysics, strangely enough, the return to metaphysics within this philosophy seems to be a recursive element in the analysis of physics in general and of the interpretation of quantum mechanics in particular. The criticism by van Fraassen to analytic philosophy can be clearly explained within the philosophy of quantum mechanics today. It is interesting to point out that, within this context, something very similar to the history of analytic philosophy itself happened in relation to the metaphysical presuppositions very soon imposed on the formal structure of the theory. The position of Bohr, which can be very well regarded in close continuation to analytic concerns, was soon replaced by much more metaphysical approaches, such as, for example, Bohmian mechanics and DeWitt's many worlds interpretation. While Bohr attempted to analyze the logical structure of the theory and concentrated on the analysis of phenomena, such attempts as those of many worlds and Bohmian mechanics intended to recover the metaphysical conditions under which one could talk, for example, about classical properties and trajectories. It seems in this case a bit ironic that the aversion professed by many philosophers of physics to Bohr's ideas does not recognize the profound connection of his thought to analytic philosophy itself. These same philosophers choose —knowingly or not— for metaphysical schemes going against their own tradition. In the case of many worlds interpretation the metaphysical step goes as far as to propose non-observable entities in order to explain the formal aspects of quantum mechanics. Also, from a metaphysical point of view, the many worlds attempt seems to end up in an extreme violation of Ockham's principle: "Entities are not to be multiplied beyond necessity".[8] In the case of Bohmian mechanics the metaphysical dogma relates to particles with trajectories. Bitbol notices in this respect ([40], p. 8) that: "Bohm's original theory of 1952 is likely to be the most metaphysical (in the strongest, speculative, sense) of all readings of quantum mechanics. It posits free particle trajectories in space-time, that are unobservable in virtue of the theory itself." Furthermore, that which should play the role of space-time in the mathematical formalism varies its dimension with the addition or substraction of particles breaking the initial attempt to recover trajectories in space-time. It is not at all clear that these kinds of attempts bring more solutions than problems. From this perspective, it might seem obvious why van Fraassen has chosen for Bohr rather than these new lines of thought, which to great extent, go against many of the analytical *a priori* concerns and methodology. The Danish physicist remained agnostic regarding the

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[8] Although Lev Vaidman [249] claims that: "in judging physical theories one could reasonably argue that one should not multiply physical laws beyond necessity either (such a verion of Ockham's Razor has been applied in the past), and in this respect the many worlds interpretations is the most economical theory. Indeed, it has all the laws of the standard quantum theory, but without the collapse postulate, the most problematic of physical laws." One could argue however, that due to the existence of modal interpretations, which are also no collapse interpretations and share the same formal structure as many worlds, there is no clear argument why one should be forced into this expensive metaphysical extension.

metaphysical concerns raised especially by Einstein, but also those of Heisenberg and Pauli. He tried by all means to restrict his analysis to the empirical data as exposed by classical physical theories and language, and not go beyond the interpretation of the formalism in terms of a new conceptual scheme —an aspect which is shared by the semantical approach to theories. Contrary to this analysis, Bohmian mechanics and many worlds interpretations, two of the most important interpretational lines of investigation today —especially in the United States and in United Kingdom, there where analytic philosophy is strongest— compose their analysis with heavy metaphysical commitments. Rather than starting from the analysis of the logical formal structure of the theory, the metaphysical presuppositions constitute the very foundation and center of gravity of such interpretations. Even attempting in some cases to change the formalism in order to recover —at least some of— our classical (metaphysical) conception of the world.

3.3 Observation and Representation *within* Physical Reality

In the first half of the twentieth century, the logical positivists and their successors, the logical empiricists, approached the issue of scientific realism reflecting on the role of observation. Within their scheme, as remarked by Curd and Cover ([76], p. 1227), "it was natural for the logical empiricists to emphasize a distinction between the observational components of a theory, which refer to objects and properties that are directly observable, and the theoretical components which apparently refer to objects and properties that are not directly observable." Philosophers of science in the second half of the twentieth century based themselves, in great measure, on these same grounds.

> "Logical positivism is dead and logical empiricism is no longer an avowed school of philosophical thought. But despite our historical and philosophical distance from logical positivism and empiricism, their influence can be felt. An important part of their legacy is observational-theoretical distinction itself, which continues to play a central role in debates about scientific realism." M. Curd and J. Cover (Quoted from [76], p. 1228)

The realism-antirealism debate stands in close relation to the observational-theory distinction. As noticed by Alan Musgrave:

> "As usually understood, the realism-antirealism issue centers precisely on the question of truth. Positivists deny the existence of 'theoretical entities' of science, and think that any theory which asserts the existence of such entities is *false.* Instrumentalists think that scientific theories are tools or rules which are *neither true nor false.* Empistemological antirealists like van Fraassen or Laudan concede that theories have truth-values, even that some of them might be true, but insist that no theory should be *accepted as true.*" A. Musgrave (Quoted from [76], pp. 1209-1210)

Regardless of the different positions it is clear that the center of gravity of these discussions is the notion of truth. The relevant conception of truth is a version of the common-sense correspondence theory of truth. As remarked by Musgrave ([76], p. 1221): "In traditional discussions of scientific realism, common sense realism regarding tables and chairs (or the moon) is accepted as unproblematic by both sides. Attention is focused on the difficulties of scientific realism regarding 'unobservables' like electrons." In this context one of the most important antirealist positions has been developed by van Fraassen with his constructive empiricism. According to him: "science aims to give us theories which are empirically adequate: an acceptance of a theory involves as belief only that it is empirically adequate." A theory is empirically adequate when it 'saves the phenomena' —when what it says about *observables* objects, events, and properties is true. "The respect in which van Fraassen's antirealism departs both from logical empiricism and from scientific realism is thus apparent. To accept (hold) a theory is to claim that it accurately describes observable phenomena; this does not entail that talk of theoretical entities is meaningless, nor does it entail that such entities are fictional or real. By distinguishing in this way between accepting a theory and believing it to be true, the constructive empiricist recommends a position of agnosticism about the theoretical." Van Fraassen agrees at the same time that all language is theory-infected, but he denies that this shows anything about scientific realism. At the same time he claims that:

> "To be an empiricist is to withhold belief in anything that goes beyond the actual, observable phenomena, and to recognize no objective modality in nature. To develop an empiricist account of science is to depict it as involving a search for truth only about the empirical world, about what is actual and observable." B. Van Fraassen (Quoted from [253], pp. 202-203)

The role of observation in this account has been criticized by Musgrave and others (see for example [76]).

After the revolution brought by positivism, one can hardly deny the importance of empirical observation in physics. Einstein ([89], p. 175) was very clear regarding this point: "[...] the distinction between 'direct observable' and 'not directly observable' has no ontological significance [...] the only decisive factor for the question whether or not to accept a particular physical theory is its empirical success." However, Einstein knew very well that the uncritical consideration of observation was out of the question.[9] The interrelation between metaphysics and the description of physical reality seemed to remain a central problem for Einstein, whom in a letter to Schrödinger in the summer of 1935 wrote that:

> "The problem is that physics is a kind of metaphysics; physics describes 'reality'. But we do not know what 'reality' is. We know it only through physical description..." A. Einstein (Quoted from ([76], p. 1196)

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[9] This can be seen from the very interesting discussion between Heisenberg and Einstein were the latter explains: "I have no wish to appear as an advocate of a naive form of realism; I know that these are very difficult questions, but then I consider Mach's concept of observation also much too naive. He pretends that we know perfectly well what the word 'observe' means, and thinks this exempts him from having to discriminate between 'objective' and 'subjective' phenomena. No wonder his principle has so suspiciously commercial a name: 'thought economy.' His idea of simplicity is much too subjective for me. In reality, the simplicity of natural laws is an objective fact as well, and the correct conceptual scheme must balance the subjective side of this simplicity with the objective. But that is a very difficult task." (A. Einstein quoted by W. Heisenberg in [142], p. 66). Einstein was part, willingly or not, of the neo-Kantian tradition (see [150]) and, as noticed by Howard: "he was not the friend of any simple realism" ([149], p. 206).

It is clear that, even from an empiricist account, we must recognize the importance of metaphysical schemes.[10] But independently of the importance of metaphysics as related to empirical theories and stepping outside the realist-antirealist debate, we claim that we must also consider the dominant role played by metaphysical schemes *within* physical experience itself, for a world of pure sensation remains outside the limits of language and expression. This has been expressed by Jorge Luis Borges in a beautiful story called 'Funes el memorioso' [57].

From our metaphysical constructive stance we stress the need of considering the conceptual scheme which relates to the mathematical structure and physical phenomena. According to this position there are no 'naked facts'. Physics relates and is necessarily involved with metaphysical schemes which constitute and configure physical experience. Remaining in the limits of empirical evidence one cannot access physical representation for, in order to describe any 'observed actuality' we are necessarily committed to a conceptual scheme. The conceptual representation of the actually given, the *hic et nunc*, is always —implicitly or explicitly— needed in order to describe a state of affairs and remains from our perspective maybe the most problematic issue within physics itself. The metaphysical choices that one introduces for such a description configure and constitute the possibility of a particular physical experience. From this standpoint, we consider concepts as creations, creations through which the physicist relates to reality and physical experience.

> "Concepts that have proven useful in ordering things easily achieve such an authority over us that we forget their earthly origins and accept them as unalterable givens. Thus they come to be stamped as 'necessities of thought,' 'a priori givens,' etc. The path of scientific advance is often made impossible for a long time through such errors. For that reason, it is by no means an idle game if we become practiced in analyzing the long commonplace concepts and exhibiting those circumstances upon which their justification and usefulness depend, how they have grown up, individually, out of the givens of experience. By this means, their all-too-great authority will be broken. They will be removed if they cannot be properly legitimated, corrected if their correlation with given things be far too superfluous, replaced by others if a new system can be established that we prefer for whatever reason." A. Einstein (Quoted from [153])

Following this line of thought, the concept of physical entity or object must be also considered as a creation, a conceptual representation which has played a major role in the history of Western thought. However, and independently of the unquestionable development which this concept has undergone through more than twenty centuries, it is not self evident weather this notion is also well suited to account for what quantum mechanics is telling us about the world.

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[10] As it has been stressed already by Feyerabend ([81], pp. 943-944): "A good empiricist will not rest content with the theory that is in the center of attention and with those tests of the theory which can be carried out in a direct manner. Knowing that the most fundamental and the most general criticism is the criticism produced with the help of alternatives, he will try to invent such alternatives. [...] His first step will therefore be the formulation of fairly general assumptions which are not yet directly connected with observations; this means that his first step will be invention of a new *metaphysics.* This metaphysics must then be elaborated in sufficient detail in order to be able to compete with the theory to be investigated as regards generality, details of prediction, precision of formulation. We may sum up both activities by saying that a good empiricist must be a critical metaphysician. Elimination of all metaphysics, far from increasing the empirical content of the remaining theories, is liable to turn these theories into dogmas."

> "In one of his lectures on the development of physics Max Planck said: 'In the history of science a new concept never springs up in complete and final form as in the ancient Greek myth, Pallas Athene sprang up from the head of Zeus.' The history of physics is not only a sequence of experimental discoveries and observations, followed by their mathematical description; it is also a history of concepts. *For an understanding of the phenomena the first condition is the introduction of adequate concepts. Only with the help of correct concepts can we really know what has been observed.*" W. Heisenberg (Quoted from [143], p. 264, emphasis added)

The problem, from this perspective, becomes the justification of the relation between conceptual schemes and reality. Intimately connected with this problematic is the problem of representation. What is the theory representing? This question has been addressed in the context of philosophy of science in the last decades. Unfortunately, as stressed by Mauricio Suárez, the community has not been able to achieve agreement with respect to what is meant by 'representation':

> "Many philosophers of science would agree that a primary aim of science is to represent the world (Cartwright (2000), Giere (1988, 2000), Friedman (1982, chapter VI), Kitcher (1983), Morrison (2001, chapter II), Morrison and Morgan (1999), Van Fraassen (1981, 1987); a well known dissenter is Ian Hacking (1983)). What those philosophers understand by 'represent' is however a lot less clear. No account of representation in science is well-established." M. Suárez (Quoted from ([245], p. 1)

We believe an understanding of this important question within the philosophy of science could shed new light on the problem of interpretation of quantum mechanics, for if representation through classical concepts is at stake and not just uncritically accepted there would be no need to "restore a classical way of thinking about what there is" and new conceptual schemes could be developed without the resistance of present physicists and philosophers of science —who either turn their back to the question of interpretation or attempt to return to the most classical metaphysical scheme allowed by the formalism. Turning our attention back to quantum theory, the abolition of the possibility to develop new conceptual schemes might be in great measure, as remarked by Feyerabend,[11] the responsibility of Niels Bohr. The position of Bohr and its relation to the present interpretation of quantum mechanics is summarized very clearly by Arthur Fine:

> "These instrumentalist moves, away from a realist construal of the emerging quantum theory, were given particular force by Bohr's so-called 'philosophy of complementarity'; and this nonrealist position was consolidated at the time of the famous Solvay conference, in October of 1927, and is firmly in place today. Such quantum nonrealism is part of what

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[11] According to Feyerabend Bohr turned this position into unquestionable dogma (see [81], p. 938). We agree with Feyerabend that, in political terms, the influence of Bohr can be hardly underestimated. However, we must also stress that we strongly disagree with Feyerabend who used to criticize within the term "the Copenhagen mafia", not only Bohr, but also Heisenberg, Pauli, Born and many others. As we have stressed above, we believe that Bohr's position was not uniform but rather part of a much broader debate with strong disagreements. We believe there has been a misinterpretation and oversimplification of Bohr's philosophical stances.

> every graduate physicist learns and practices. It is the conceptual backdrop to all the brilliant success in atomic, nuclear, and particle physics over the past fifty years. Physicists have learned to think about their theory in a highly nonrealist way, and doing just that has brought about the most marvelous predictive success in the history of science." A. Fine (Quoted from [76], p. 1195)

As we shall argue, although Bohr recognized the importance of representation within phenomena he restricted representation to classical physics and language, for according to Bohr ([274], p. 7) "[...] the unambiguous interpretation of any measurement must be essentially framed in terms of classical physical theories, and we may say that in this sense the language of Newton and Maxwell will remain the language of physicists for all time." More importantly, "it would be a misconception to believe that the difficulties of the atomic theory may be evaded by eventually replacing the concepts of classical physics by new conceptual forms." Bohr's philosophical scheme is based, on the one side, on the recognition of classical discourse to account for phenomena, and on the other side, on the necessity of an intersubjective account of physical experience. The interpretational gap in between the quantum formalism and the empirical substructures explained in classical terms is resolved by evading the question about the manner in which the world —and not the measurement outcomes— is represented according to the quantum formalism. While holding fast to classical representations,[12] Bohr is forced to abandon representation in the quantum realm. In order not to step outside his original (classical) conceptual scheme he is forced to grant that the quantum wave function Ψ is only an *algorithm*. The notion of *complementarity*, taken as a *regulative principle*, is able to dissolve the contradiction between the same quantum wave function Ψ and its possible representations in terms of particles or waves.[13] The price to pay is that the quantum wave function must be left without a conceptual scheme that supports it. Detached from the classical world it must stand as an algorithm outside physical (quantum) reality. Feyerabend was again very critical of Bohr's attempt to close this discussion using as an argument that quantum mechanics is able to "account for an immense body of experience":

> "the semblance of absolute truth *is nothing but the result of an absolute conformism.* For how can we possibly test, or improve upon, the truth of a theory if it is built in such a manner that any conceivable event can be described, and explained, in terms of its principles? The *only* way of investigating such all embracing principles is to compare them with a different set of *equally all-embracing* principles —but this way has been excluded from the very beginning. The myth is therefore of no objective relevance, it continues to exist solely as the result of the effort of the community of believers and of their leaders, be these now priests or Nobel prize winners. *Its 'success' is entirely manmade.* This I think, is the most decisive argument against any method that encourages uniformity, be it now empirical or not. Any such method is in the last resort a method of deception. It enforces an unenlightened conformism, and speaks of truth; it leads to a deterioration of intellectual capabilities, of the power of imagination, and speaks of deep insight; it destroys the most precious gift of the young, their tremendous power of imagination, and speaks of education." P. Feyerabend (Quoted from [81], pp. 938-939)

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[12] See also the very interesting discussion related to the importance of classical mechanics in Bohr's philosophy of physics in [55].

[13] I am grateful to Hernan Pringe for the many discussions regarding this subject. See also [210].

An algorithm is a set of finite instructions or steps which allow to execute or resolve a problem, a calculus machine through which one obtains results. Bohr's ideas mistaken in this radical form end up in the statement phrased explicitly by Fuchs and Peres that: "quantum theory does not describe physical reality." According to them, quantum theory: "provides an algorithm for computing probabilities for the macroscopic events." The danger of this position lies in the complete obturation of any possible creative solution to the problem of interpreting quantum mechanics in relation to physical reality.

3.4 Realism: Physical Theories as Descriptions of the World

Realism in philosophy of science is a stance which considers physical theories as being able to describe the world independently of consciousness and observers. As it is prior to observation, from now on we shall call this type of existence related to a general realist stance: *preexistence*. Contrary to empiricism, which founds itself in actual observation, realism takes observation *hic et nunc* as discovering a preexistent reality —'pre' with respect to measurement— described by theory. This determines that realism is necessarily linked to the presupposition of the existence of a world independent of our consciousness and the possibility to represent it.

The problem with realism is the lack of justification which can provide an internal link between the world and its representation. The solution proposed by Kant in his *Critic of Pure Reason* considered representation as internal to the knowing subject. In this case representation is given through a fixed set of *a priori* categories and forms of intuition. The problem of this solution which has haunted philosophy ever since, is that 'objective knowledge', namely, the knowledge provided by the transcendental subject, does not refer anymore to the world *as it is*. Metaphysical questions of the type: 'what is the world *in itself*?' are, form this perspective, completely meaningless. On the other hand, scientific realism —still today a main stance in present philosophy of science— considers there is a *true* story of how the world *is*, and that physics is able to find out what exactly is this story about. At the end of the road there is a true representation which describes reality; there is a *correspondence* between the concepts involved in representation and the world as it is. This ideal faces scientific realism with deep questions for it is not clear how such representation can be linked to reality *as it is*.

3.5 Representational Realism: Physical Theories as Expressions of the World

According to our approach it is not possibility to discuss in physical terms about the features of the world independently of representation. This means that a statement such as 'the world is deterministic' is meaningless, for it is only through representation that one can discuss about 'the world'. The notion of 'determinism' is a concept —and as such, part of a representation— not something we find in the world. One can say instead: 'according to classical physics the world is deterministic'. In this case, the notion refers to the world only indirectly, through a specific formal and conceptual representation. What are the conceptual presuppositions involved in classical physics? That the world is constituted by objects, that these objects are logically founded on the principles of existence, non-contradiction and identity, that they exist in space-time, etc. These particular presuppositions are a way to configure phenomena and not something which can be inferred from phenomena. For example, the notion of 'identity' is a presupposition to talk about objects, but we never find 'identity' in the world. We use identity as a presupposition to deal with phenomena and to constitute the notion of object, and so when we see a chair, we presuppose it remains the same through time; and even though we might change our perspective and not see the same face of the chair we keep in mind we are seeing the same chair. Classical physics talks about a particular metaphysical world built up from classical objects. Does quantum mechanics talks about the same metaphysical world? Only if that is the case, we would arrive at a contradiction by comparing the statements: 'according to quantum physics the world is indeterministic' and 'according to classical physics the world is deterministic'. The contradiction would only arise if both theories were talking about the same metaphysical representation of the world. In that case, they would be assuming the same presuppositions to expose phenomena. What we know up to today about quantum mechanics does not seem to point in this direction.

The belief in the convergence of representations relates to the idea that there is a fundamental representation which lies somewhere at the end of the road. That there is a set of true concepts which describe the world *as it is*. This idea is also based on the presupposition that the findings of a theory can be translated to a different theory. The idea is to avoid representation and arrive to the true represented world —which means implicitly that representation can be avoided. But this idea goes against our starting point, which is that we cannot access the world without representation. The idea of 'closed theories' also goes completely against these reductionistic presuppositions. It assumes that new theories determine intrinsically new phenomena, phenomena which cannot be seen from a different theory —and thus, cannot be translated. A fundamentally new theory would be one that arrives at completely new phenomena through intrinsically different formal and conceptual presuppositions. According to our stance we have always relied on certain classically based presuppositions, but quantum mechanics is a completely new fundamental theory which breaks down the basic conception of a classical world. Thus, from this perspective, the problem is not to find a bridge between quantum mechanics and classical mechanics —what is known today to be the 'quantum to classical limit' and was first developed by Bohr in terms of his 'correspondence principle'. We need not explain why the world is classical. What we need is to find a conceptual structure which allows to expose quantum experience in all its strength, find the metaphysical presuppositions which coherently —namely, without any *ad hoc* moves or unobservable metaphysical objects— relate to the mathematical formalism and are able to explain phenomena. Each concept must find a meaning from *within* the theory itself. The meaning of 'time' is not the same in Newton as in Aristotle, the meaning of 'space' is not the same in classical mechanics as in relativity theory —something which has been clearly shown by both Thomas Kuhn and Paul Feyerabend. One cannot *export* metaphysical presuppositions of any kind, for every theory must be independent and closed under its own concepts and sets of possible physical experiences. When Galileo thought of an object in the void with no resistance on an infinite inclined plane, he imagined, through new concepts (void, an infinite plane, etc.) and a mathematical formalism, a completely new physical experience which could not be imagined within Aristotelian physics. In this sense is that we claim that physical experiences are not suddenly discovered, we do not get hit by new physical experience if we are not prepared. There is a creative aspect involved within representation which allows us to set up the conditions of possibility for new physical experience to be found. A subtle interplay between creation and discovery which allows physical representation to expose an aspect of the world.[14]

There are of course physical theories which are not empirically adequate. It is obvious that not every physical representation can be empirically sound. But what about those physical theories which are empirically adequate? If representation as a creation lies in between phenomena and the world how can we understand representation? Is it possible to talk about the world —independent of consciousness, of human beings— from within representation itself? We believe there is a strategy which might allow us to answer this question positively. The realism that we propose is based on the metaphysics of Spinoza who conceived Being as a singular substance with infinite modes. Modes are the way in which the attributes of the one, infinite substance manifests its essence; in other words, everything we know is a mode of the eternal substance manifesting in itself. Each mode is capable of expressing an attribute. While the attribute 'extension' is expressed by 'physical bodies'; the attribute 'thought' is expressed by 'ideas'. There are other modes or expressions of Being but we, as human beings, know only these two. Because extension and thought have nothing in common, the two realms of matter and mind are causally closed systems. However, both are expressions of the same substance. Although being different modes, the attributes are independent and equal, it is the same modification of the substance which is expressed in one mode or the other. In other words, the attributes are parallel expressions of Being. In analogous fashion to the way in which Spinoza claims that the attribute extension and the attribute thought express univocally one and the same substance, we could think —using Spinoza's metaphysical scheme— that each closed physical representation provides an expression of one and the same world. From this standpoint we could investigate the idea that classical physics and quantum physics are not talking about the same 'things' but nevertheless express the same world. Following this line of thought, each closed physical theory expresses the world through representation and adequate physical experiences. An adequate physical experience is one which can be coherently configured from a particular theory and exposes the world through empirical adequacy. Every phenomenon is *local* in the sense that the presuppositions involved can be only applied to the specific designed physical experience, but never to the world in itself. To believe that such presuppositions talk about the world in a "correspondence" manner implies once again a non-representational scheme of thought —which is exactly the jump we want to avoid. Although every phenomena is perspectival in the sense that it depends directly on the theory from which it is observed, there is no relativism involved in our scheme simply because every statement arising from an empirically adequate, closed and coherent theory expresses the world. Furthermore, from our approach, statements which pertain to intrinsically different theories cannot be compared. Such comparison would imply a translation, the presupposition that one can find, for any two representations of the world, an encompassing representation or theory which takes them both into account. Neither, it can be claimed that "anything works!" for the final judge of physical expression is always physical experience. Classical physics is just a particular metaphysical scheme which expresses the world and through which we have found an amazing range of physical experiences. We know what classical physics talks about. Quantum physics

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[14] In line with this remark about the importance of creation, as noticed by Granger ([132], p. 10): "Un autre aspect de la connaissance scientifique est qu'elle consiste pour une bonne part à *suivre des règles*. Elle est en ce sens encore pensée. Mias on en manquerait un caractère essentiel si l'on ne remarquait qu'elle consiste aussi à *échapper* à des règles préétablies, à en créer de nouvelles, à figurer des exceptions."

stands, till today, incoherently related to physical experience awaiting a metaphysical scheme which allows us to explain what it is talking about. Only when we answer this question in a closed manner, we will be able to say that we have "understood" quantum mechanics.

It is only the theory which can tell you what can be observed. From our perspective, a phenomenon is not independent of the particular theory which lies down the conditions of possibility to account for physical experience. Each particular physical phenomenon lies within the limits of the particular physical theory which contains it as a possibility. Thus, to point out there is a 'click' in a detector is not enough to provide an adequate account of phenomena. Contrary to radical empiricism, we do not agree that a 'click' in a detector can be regarded as an observation voided of theoretical content. A 'click' can be understood from within different, mutually incompatible, physical theories. One cannot presuppose that physical experience appears itself naked. Thus, a 'click' in a detector must not necessarily be considered as limited by classical physical concepts. We leave open the possibility that new concepts can allow us to configure a conceptual scheme which closes the circle connecting the orthodox mathematical formulation of quantum mechanics to physical experience. Unlike Bohr, we do not agree that, for example in the double slit experiment, the observable quantum phenomena must be necessarily considered as "classical phenomena" simply because there is a 'click' in a detector and thus a space-time event. In this case, it is the representation of a *singularity*[15] which configures physical experience in classical terms by considering the detector as a classical object in space-time, etc. However, it is the 'click' itself which cannot be configured under such classical presuppositions in a closed manner, in other words, the 'click' does not seem to come from a 'classical object'. Even when the particular experimental set up is chosen, there are many problems when interpreting the quantum wave function in terms of a 'particle' or a 'wave' which exist in space-time. According to our scheme, a phenomenon can be considered as "classical" only in case the presuppositions, involved to coherently arrive to this particular phenomena, would be those of classical physics.

The meaning of explanation and understanding in physics, within our approach, relates directly to the capability of a physical theory to internally account —in the sense of a 'closed theory'— for a given phenomena. Another example of the relation between a particular theory and phenomena comes from the Bell inequality (which we shall discuss more in detail on chapter 5). The Bell inequality provides statistical limits to the outcomes of classical physical experience and proves that quantum mechanics cannot be subsumed under such classical presuppositions. From this perspective, the analysis of a 'click' in a photographic plate presupposes the conditions under which such 'click' arises. Only then can we talk of phenomena. Only in the case the classical objects would be able to account for the result and explain the 'click' we can say we are talking about classical phenomena. To say it differently, not every set of 'clicks' can be seen as arising from a classical theory. The question for us is: what are the conditions under which we can explain, in both conceptual and formal terms, the 'click' of which quantum theory is talking about? We need to be able to close the gap in between the formalism —which provides a mathematical representation of the theory— and the concepts —which provides a conceptual representation of the theory —in order to properly account for phenomena. Thus, in the same way we use a point in phase space to

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[15] We take a singularity to be the meeting point between the *hic et nunc* and *physical representation*. It is in this point were Being is expressed through physical representation.

describe the trajectory of an object in classical space-time —both of which, namely, ‘the point in phase space’ and ‘the object in space-time’, are part of the mathematical and conceptual representation of classical physics—, we need to find out in a coherent manner what is a quantum superposition describing in conceptual terms (we will come back to this important question in chapter 14).

According to our representational realist stance, there is no ‘physical world’ nor ‘physical context’, what there is instead is a ‘physical representation’ given by a particular theory which allows to configure and consider a particular physical experience. The problem of a representational realist remains to build up a representational conceptual scheme which would allow us to relate coherently the quantum formalism to the empirical structure predicted by the theory in a closed manner. The concepts must be internally defined by the theory itself and not presupposed or self-evident extensions of a different theory. This means, for example, that it is not obvious to us that the notion of possibility used in classical physics is, or should be considered as the same notion of possibility used in quantum theory. The concepts which allow us to provide a coherent account of quantum phenomena must be able to provide a story about how the world *is* according to quantum mechanics. In turn, these same concepts might be even capable of developing new physical experiences.

Reconsidering the Early Debate on Quantum Mechanics and the Meaning of Physical Reality

With the rise of philosophy, the Greeks placed the fundament of thought in the external Cosmos, knowledge of which could be achieved through contemplation. Physics was understood as a field which allowed to *discover* Nature. With the advent of Newtonian Mechanics and its empirical achievements, philosophy posed the problem of justification of a physical theory as true knowledge, *episteme* (see for discussion [186]). Both the rationalists and the empiricists attempted to provide an answer to this question. Later on, the analysis of David Hume would expose the problem in all its depth. But it was Immanuel Kant who was able, in his *Critic of Pure Reason*, to present an architectonic which would lay the new foundations of philosophical thought and objectivity itself. In order to overcome the problem of justification of the relation between 'object' and *phenomena*, Kant developed a philosophy in which the notion of object was not an *external* aspect of discovery but rather part of an *internal* scheme through which the *transcendental subject* was able to bring into unity the chaotic sensations of experience. According to Kant, objectivity came from the interplay of phenomena through the forms of intuition and the *a priori* categories. It is the transcendental subject, and the way through which he experiences that the object is brought into stage. Kant wanted to overcome the metaphysics of his time which, according to him, had turned thought into dogma. His critical analysis attempted to place philosophy in the secure path of science, exposing the limits of human knowledge. However, his own scheme of thought, based on the *a priori* categories and forms of intuition would be very soon turned itself into new (metaphysical) dogma. The *a priori* categories and forms of intuition through which Kantian philosophy had constrained physics were questioned and criticized at the end of the 19th century. From the ashes left by this crisis, at the beginning of the 20th century, two physical theories —relativity theory and quantum mechanics— emerged in this new region of thought which confronted the very basic principles of Kantian metaphysics. It is clear that in order to understand the creation of quantum mechanics, one would need to provide an analysis which is not limited to physics. The crisis in the foundations which developed already at the end of the 19th century manifested itself in many fields of human knowledge. Science, philosophy and art expressed in different manners one of the most important crises in metaphysical thought, a crisis which had in its kernel the problem of representation and language. The crisis in the foundation of critical philosophy determined what has been considered by many the end of metaphysical thought itself [261]. The 20th century witnessed a crisis of metaphysics and the parting of the ways between what has been known as the analytic school of thought and continental philosophy [127]. Regardless of their differences, both philosophies have shown a deep interest in the relation of language to reality. Some call this new period, in which the fundament of thought has been related to language itself the linguistic turn [233].

Philosophy of physics has faced a tension between the classical conception of an objective physical reality and what was taught by quantum theory. Although there have been many proposals to solve this problem,[16] the tension, at least for many, has not yet been resolved. In this chapter we are interested in showing that the discussion between Einstein, Bohr, Pauli and Heisenberg was centered in the problems posed by metaphysical and anti-metaphysical positions regarding the meaning and understanding of physical reality, the role played by concepts and language within physical theories and the consideration of what is meant by a 'physical situation'. While Einstein embraced a metaphysical stance, Bohr developed a philosophy of physics directly linked to anti-metaphysical commitments. Heisenberg, just in between the two giants, remained with contradictory positions, and even though recognized the importance of metaphysics, was pushed by the complementarity scheme to embrace epistemology rather than ontology.[17] According to our interpretation Bohr's complementarity scheme can be seen today, in the fight which took place in the 20th century, as the triumphant (anti-metaphysical) position.

4.1 Mach's Positivism and the Quantum Revolution

Already in the mid 19th century criticism of metaphysics had appeared explicitly in the positivistic philosophy of the French Auguste Compte and the British John Stuart Mill. Positivism derived from Enlightenment thinkers like Pierre-Simon Laplace and many others, but was firstly systematically theorized by Compte, who saw the scientific method as replacing metaphysics in the history of thought. Positivism is a philosophy which states that the only authentic knowledge is knowledge that is based on actual sense experience. Such knowledge can come only from affirmation of theories through strict scientific method. Metaphysical speculation is avoided.[18]

Mach's critical positivism played a significant role within the scientific revolutions that took place at the beginning of the 20th century. After some centuries, the categories and forms of intuition had become exactly what Kant had striven to attack in the metaphysics of his time, dogmatic and unquestionable elements. Kant had fought 17th century metaphysics, but his own philosophy had turned itself into new dogma. As noted by van Fraassen ([256] p. 2): "Kant exposed the illusions of Reason, the way in which reason overreaches itself in traditional metaphysics, and the limits of what can be achieved within the limits of reason alone. But on one hand Kant's arguments were not faultless, and on the other there was a

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[16] In quantum mechanics the problem with objectivity has become a motor for the development of different solutions and interpretations of the formalism. Many, like Bohr himself, have attempted to change the notion of objectivity, others have provided more classical type interpretations attempting to recover a classical idea of objectivity.

[17] Also Mara Beller [32] has called the attention about the contradictory aspects of Heisenberg's discourse.

[18] The development of positivistic ideas goes from "social positivism" of Compte to Mach's "critical positivism" and "logical positivists" later on (see [126] for further discussion).

positive part to Kant's project that, in his successors, engaged a new metaphysics. About a century later the widespread rebellions against the Idealist tradition expressed the complaint that Reason had returned to its cherished Illusions, if perhaps in different ways."

Ernst Mach is maybe one of the most influential positivistic thinkers of the 19th century. His ideas and criticisms are deeply engaged with the development of physics that took place at the beginning of the 20th century.[19] Mach, a physicist himself, was primarily interested in the nature of physical knowledge. His investigations led him to the conclusion that science is nothing but the systematic and synoptical recording of data of experience.

> "Nature consists of the elements given by the senses. Primitive man first takes out of them certain complexes of these elements that present themselves with a certain stability and are most important to him. The first and oldest words are names for 'things'. [...] The sensations are no 'symbols of things'. On the contrary the 'thing' is a mental symbol for a sensation-complex of relative stability. Not the things, the bodies, but colors, sounds, pressures, times (what we usually call sensations) are the true elements of the world." E. Mach (Quoted from [181])

In close analogy to Darwinistic ideas Mach conceived the evolution of knowledge in physical theories as a process of "struggle for life" and "survival of the fittest". In his *Analysis of Sensations* [181], Mach concluded that primary sensations constitute the ultimate building blocks of science, inferring at the same time that scientific concepts are only admissible if they can be defined in terms of sensations. Although Mach had been himself a neo-Kantian, within his neo-positivist conception of science, he stated that we should reject every *a priori* element in the constitution of our knowledge about things. Science would be then nothing but a conceptual reflection of the facts which are provided by sensations. Scientific propositions should be empirically verifiable and as a consequence, within this doctrine there is no place left for *a priori* concepts.

The incisive criticism of Mach on the lack of foundation of the physical concepts in the theories of his time allowed a complete reformulation of the meaning and applicability of physical concepts. It is well known that the philosophical ideas of Mach had a great influence on the development of special and general relativity. Albert Einstein indicated on several occasions the relations of his own ideas to those of Mach, in whom he recognized a guide. Moreover, the importance of Mach's thought should not be underestimated in relation to the development of quantum theory. As noticed by Charles Enz ([115], p. 250): "Clearly, positivism played an important part in the creation of quantum mechanics as is evident, e.g., from the letter [93] by Heisenberg to Pauli quoted in Ref. 16. And, as mentioned earlier, Pauli never refuted his positivistic argument about the relation between field strength and test charge contained in his third paper of 1919." Heisenberg also discussed together with Einstein and Pauli the relation between Machian philosophy and the ontological and epistemological constitution of atomic theory. The richness of this debate, which comprises several questions such as the meaning of observation, the question of pragmatics, realism and the problem of interpretation can hardly be underestimated (see [142], pp. 63-66).

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[19] An interesting work which discusses the relation of Mach's philosophy to the previous lines of thought in the 19th century is that of Massimo Cacciari [64]. See also [88].

For us, the importance of Mach's positivistic ideas regards mainly the deconstruction of Kant's *a priori* structure of thought. His analysis opened small cracks in the basic physical presuppositions connected with the metaphysics of his time, and so, prepared a period where those who followed were able to go beyond the presuppositions and impositions of classical physics. Only after Mach and his criticism of the *a priori*; the concepts of 'space', 'time', 'substance', 'causality', etc. could be discussed and deconstructed one by one opening at the same time the doors of new concepts. As noticed by Bohr ([274], p. 106) himself, Heisenberg had succeeded "in emancipating himself completely from the classical concept of motion by replacing from the very start the ordinary kinematical and mechanical quantities by symbols which refer directly to the individual processes demanded by the quantum postulate." The Machian epistemological principle had broken the chains of the fixed Kantian *a priories*. A new physical experience was disclosed, a new region of thought had been created.

4.2 Einstein, Heisenberg and Pauli: Positivism and the Problem of Reality

Although Einstein, Heisenberg and Pauli[20] were close followers of Mach regarding the criticism of the *a priori* conditions of understanding —which is the very subversive element involved in positivism— and the influence of positivistic ideas can be witnessed in Einstein's interpretation of the photoelectric effect and Heisenberg's interpretation of the cloud chamber or the determination of the indeterminacy relations,[21] it might go too far to consider them as positivists. Still, the problem which all these thinkers confronted was that of *physical reality* and in this sense it is not strange to find out that the development of positivistic ideas regarding the meaning of science in the context of the Vienna circle were criticized by all three of them on many occasions.[22]

As explicitly remarked in its manifesto [66], the Vienna Circle is characterized "essentially by two features. First, it is empiricist and positivist: there is knowledge only from experi-

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[20] Wolfgang Ernst Pauli, the godson of Mach and one of the most influential physicists in the quantum revolution, took —like Einstein had done in the past— many tools from positivism which he certainly used methodologically in order to advance in his investigations. One of the most important points in this respect regards of course the abandonment of the Kantian (metaphysical) *a priori* elements of his philosophy: "In many respects the present appears as a time of insecurity of the fundamentals, of shaky foundations. Even the development of the exact sciences has not entirely escaped this mood of insecurity, as appears, for instance, in the phrases 'crisis in the foundations' in mathematics, or 'revolution in our picture of the universe' in physics. Indeed many concepts apparently derived directly from intuitive forms borrowed from sense-perceptions, formerly taken as matters of course or trivial or directly obvious, appear to the modern physicist to be of limited applicability. The modern physicist regards with scepticism philosophical systems which, while imagining that they have definitively recognized the *a priori* conditions of human understanding itself, have in fact succeeded only in setting up the *a priori* conditions of the systems of mathematics and the exact sciences of a particular epoch." ([193], p. 95).

[21] As noticed by Arthur Fine ([76], p. 1195): "Heisenberg's seminal paper of 1925 is prefaced by the following abstract, announcing, in effect, his philosophical stance: 'In this paper an attempt will be made to obtain bases for a quantum-theoretical mechanics based exclusively on relations between quantities observable in principle'."

[22] The importance of the problem of reality in Heisenberg's interpretation, which is maybe the less explicit —very possibly due to Bohr's influence— of the three characters, can be witnessed for example in chapter 10 of his book *Physics and Philosophy* [141]. There he explicitly discusses the problem of the relation between language and reality making clear that the question of reality remains for him fundamental.

ence [...] Second, the scientific world-conception is marked by the application of a certain method, namely logical analysis." Following Frege, Russell and Carnap, logical analysis is the method of clarification of philosophical problems and the task of philosophy lies in the clarification of problems and assertions. Logical analysis shows that there are two different kinds of statements; one kind includes statements reducible to simpler statements about the empirically given; the other kind includes statements which cannot be reduced to statements about experience and thus they are devoid of meaning. Metaphysical statements belong to this second kind and therefore they are meaningless. Within the Vienna Circle many philosophical problems are rejected as pseudo-problems which arise from categorical mistakes or the use of pseudo-concepts, while others are re-interpreted as empirical statements and thus become the subject of scientific inquiries. The source of the logical mistakes —which are at the origins of metaphysics— resides in the ambiguity of natural language. The Vienna Circle appeared also as an answer to the Kantian scheme and criticized "the notion that thinking can either lead to knowledge out of its own resources without using any empirical material, or at least arrive at new contents by an inference from given states of affair." The only two kinds of statements accepted by the Vienna Circle are *synthetic statements a posteriori* (i.e. scientific statements) and *analytic statements a priori* (i.e. logical and mathematical statements). The Kantian idea according to which there are *synthetic statements a priori* that expand knowledge without using experience was rejected. Mathematics, which according to Kant is an example of necessarily valid synthetic knowledge derived from pure reason alone, has instead a tautological character, that is, its statements are analytical statements. The Vienna Circle embraced the return to the Sophistic philosophy in which the notion of truth was now imprisoned in logic and language.

Regarding these anti-metaphysical elements, Einstein remained at a distance from logico-positivism. As noted by Howard:

> "Einstein was dismayed by the Vienna Circle's ever more stridently anti-metaphysical doctrine. The group dismissed as metaphysical any element of theory whose connection to experience could not be demonstrated clearly enough. But Einstein's disagreement with the Vienna Circle went deeper. It involved fundamental questions about the empirical interpretation and testing of theories." D. Howard (Quoted from [152], p. 73)

For Einstein ([274], p. vii), the starting point for physics was also a metaphysical stance: "Out yonder there was this huge world, which exists independently of us human beings and which stands before us like a great, eternal riddle, at least partially accessible to our inspection." According to him, the guiding line of physics was to be described in the following terms:

> "[...] it is the purpose of theoretical physics to achieve understanding of physical reality which exists independently of the observer, and for which the distinction between 'direct observable' and 'not directly observable' has no ontological significance; this aim furnishes the physicist at least part of the motivation for his work; but the only decisive factor for the question whether or not to accept a particular physical theory is its empirical success." A. Einstein (Quoted from [89], p. 175)

As noticed by Vassilios Karakostas ([160], p. 15): “[...] the concept of mind-independent reality is not strictly scientific; it is metaphysical by nature. It concerns the existence of things in themselves, absolutely independent of any act of perception or observation. Hence, it does not apply to empirical science proper because, by definition, it excludes the empirical testing of its existence. It may be viewed, however, as a regulative principle in physics research, as a conviction which gives direction and motive to the scientific quest.”

Heisenberg also took positivism as developed by the Vienna Circle to be a definite aim of attack. In his autobiography he writes:

> “Positivist insistence on conceptual clarity is, of course, something I fully endorse, but their prohibition of any discussion of the wider issues, simply because we lack clear-cut enough concepts in this realm, does not seem very useful to me —this same ban would prevent our understanding of quantum theory.” W. Heisenberg (Quoted from [142], p. 208)

And continues later on:

> “The positivists have a simple solution: the world must be divided into that which we can say clearly and the rest, which we had better pass over in silence. But can anyone conceive of a more pointless philosophy, seeing that what we can say clearly amounts to next to nothing? If we omitted all that is unclear, we would probably be left with completely uninteresting and trivial tautologies.” W. Heisenberg (Quoted from [142], p. 213)

It seems that for both Heisenberg and Pauli, the position they had against positivism was directly related to the denial of logical positivism to the metaphysical questions involved within the problem of interpretation of quantum mechanics. As recalled by Heisenberg, Bohr once declared when coming back from a conference:

> “Some time ago there was a meeting of philosophers, most of them positivists, here in Copenhagen, during which members of the Vienna Circle played a prominent part. I was asked to address them on the interpretation of quantum theory. After my lecture, no one raised any objections or asked any embarrassing questions, but I must say this very fact proved a terrible disappointment to me. For those who are not shocked when they first come across quantum theory cannot possibly have understood it. Probably I spoke so badly that no one knew what I was talking about.
>
> [Pauli then replied] The fault need not necessarily have been yours. It is part and parcel of the positivist creed that facts must be taken for granted, sight unseen, so to speak. As far as I remember, Wittgenstein says: ‘The world is everything that is the case.’ ‘The world is the totality of facts, not of things.’ Now if you start from that premise, you are bound to welcome any theory representative of the ‘case.’ The positivists have gathered that quantum mechanics describes atomic phenomena correctly, and so they have no cause for complaint. What else we have had to add —complementarity, interference of probabilities, uncertainty relations, separation of subject and object, etc.— strikes them as just so many embellishments, mere relapses into prescientific thought, bits of idle chatter that do not have to be taken seriously. Perhaps this attitude is logically defensible, but, if

> it is, I for one can no longer tell what we mean when we say we have understood nature."
> N. Bohr and W. Pauli (Quoted by W. Heisenberg in [142], pp. 205-206)

Einstein, Heisenberg and Pauli expose the tension between the critical analysis of the meaning of the concepts involved in any physical representation, and a deep concern for not losing the *reference* to which physics is connected —Nature and Being.[23] Recalling the discussion between Einstein and Heisenberg:

> "I believe, just like you, that the simplicity of natural laws has an objective character, that it is not just the result of thought economy. If nature leads us to mathematical forms of great simplicity and beauty —by forms I am referring to coherent systems of hypotheses, axioms, etc.— to forms that no one has previously encountered, we cannot help thinking that they are 'true,' that they reveal a genuine feature of nature. It may be that these forms also cover our subjective relationship to nature, that they reflect elements of our own thought economy. But the mere fact that we could never have arrived at these forms by ourselves, that they were revealed to us by nature, suggests strongly that they must be part of reality itself, not just of our thoughts about reality." W. Heisenberg (Quoted from [142], pp. 67-68, emphasis added)

It is also interesting to remark that in Heisenberg's philosophy this concern for the problem of reality on the one hand, and holding on to Bohr's complementarity approach on the other, would led him to severe inconsistencies.[24] Inconsistencies which Bohr was very careful to avoid.

Of all three of them, it was Pauli who remained the most radical thinker. Pauli was ready to leave aside the Kantian *a priori* preconditions of understanding and replace them by new —still to be developed— concepts. As explicitly expressed by him, the crisis with which 20th century physics and philosophy confronts us, against the Kantian claim and its very different proponents, relates to a proper development of the meaning of reality itself.

> "When the layman says 'reality' he usually thinks that he is speaking about something which is self-evidently known; while to me it appears to be specifically the most important

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[23] It is interesting to notice in this respect the connection between Einstein, Haisenberg and Pauli to Spinoza, Plato and Schopenahuer, respectively; three philosophers of the *absolute*, of the *infinite*, three metaphysicians. As remarked by C. F. von Weizsäcker [271]: "The classical philosophical answers are best represented by great philosophers, rather than by abstract terms (like "realism", "positivism", "idealism") which stay themselves in need of further semantical clarification. Einstein, I think, described himself rightly as a follower of Spinoza who, in the great tradition of ontology, asked that which *is*. Bohr, I feel, is rather a follower of Kant who asked what we can know; Bohr's questions are epistemological, Socratic. Heisenberg, especially in his later years, recognized Plato as the leading thinker, the poet of the "central order". Pauli referred to Schopenhauer, and he learnt most from the Psychology of C.G. Jung."

[24] In this respect we could take as an example Heisenberg's relation to concepts in physics. On the one hand, Heisenberg ([141], pp. 80-81) seemed to accept Bohr's desiderata, that we should hold fast to classical concepts: "The Copenhagen interpretation of quantum theory starts from a paradox. Any experiment in physics, whether it refers to the phenomena of daily life or to atomic events, is to be described in the terms of classical physics. The concepts of classical physics form the language by which we describe the arrangements of our experiments and state the results. *We cannot and should not try to replace these concepts by any others.* Still the application of these concepts is limited by the relations of uncertainty. We must keep in mind this limited range of applicability of the classical concepts while using them, but we cannot and should not try to improve them." On the other hand, as we have seen above, his own ideas regarding the development of physics in terms of closed theories went completely against this position.

> and extremely difficult task of our time to work on the elaboration of a new idea of reality."
> W. Pauli (Quoted from [171], p. 193)

It is exactly this path which we attempt to recover through our constructive metaphysical stance.

4.3 Heisenberg's Indeterminacy Principle and Bohr's Complementarity Approach

Werner Heisenberg [140] presented in 1927 one of the most important papers of the 20th century. In this paper called: *"Uber den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik"*, Heisenberg derived from Planck's quantum postulate, the *indeterminacy relations.* These relations —or principle, depending on the interpretation given to this mathematical formal expression of the theory— exposed the impossibility of assigning exact simultaneous values to the position and momentum of a particle. In mathematical terms this is expressed in the following manner:

$$\triangle\, x \triangle\, y \geq \hbar \tag{4.1}$$

There are radically different interpretations of this mathematical expression. The different readings choose between 'principle' or 'relation', 'indetermination' or 'uncertainty', according to their own agendas. Heisenberg, following the positivistic principle that no element of the theory should be thought to exist independently of observation, regarded the notion of trajectory as superfluous. Thus, the properties are not determined until the measurement has taken place. In his own words: "I believe that one can formulate the emergence of the classical 'path' of a particle pregnantly as follows: *the 'path' comes into being only because we observe it.*" As recalled by Heisenberg himself it was Einstein's recommendation which guided his quest:

> "[in the transformation theory by Dirac and Jordan] one could transform from $\psi(q)$ to $\psi(p)$, and it was natural to assume that the square $|\psi(p)|^2$ would be the probability to find the electron with momentum p. So gradually one acquired the notion that the square of the wave function, which by the way was not the wave function in three-dimensional space but in configuration space, meant the probability for something. With this knowledge we returned to the electron in the cloud chamber. Could it be that we had asked the wrong question? I remember Einstein telling me, 'it is always the theory which decides what can be observed.' And that meant, if it was taken seriously, that we should not ask: 'How can we represent the path of the electron in the cloud chamber?' We should ask instead: 'Is it not perhaps true that in nature only such situations occur which can be represented in quantum mechanics or wave mechanics?" W. Heisenberg (Quoted from [143], p. 269)

If taken to its last consequences, Einstein's recommendation means that 'the theory' expresses the *conditions of possibility* to determine what is to be considered 'experience'. Our conception of reality is modeled in this way by the theory itself which determines the ontological and epistemological conditions over which it provides 'meaning'. It is the theory which determines the limits of what is to be considered experience and physical reality. Following this line of thought, in terms of an *ontological interpretation*, the **principle of indetermination** has nothing to do with ignorance, but rather provides the conditions under which that which is analyzed can be discussed. However, as we shall see in the following section, Bohr's pressure to subsume Heisenberg's principle under his own complementarity scheme forced the subsequent *epistemological discussions* in terms of the limitations of classical concepts within experimental arrangements. Heisenberg had started by analyzing experiments, but after having found a consistent way of recovering "the observed" through his mathematical scheme of matrix mechanics, he was stopped from going further and taking this same principle as a guiding line to determine future experience. Heisenberg had always emphasized the discreteness of quantum theory. This conception brought him to the indeterminacy relations in February 1927 when Bohr left Copenhagen for a holiday. At his return, Bohr did not accept very enthusiastically the priority of the particle picture over the wave picture in the scheme presented by Heisenberg, and discussions followed in which Pauli had to defend his young fellow who ended up in tears. Heisenberg, very cleverly, had already sent the manuscript, he had to apologize for this.[25] Later, Heisenberg returned on his footsteps and remained within the limits imposed by classicality. Instead of taking his principle along the ontological road of Einstein, Heisenberg followed Bohr's epistemological path. This trip had no other goal than to justify quantum theory from the heights of classical thought. Indeed, as noted by Hilgevoord and Uffink [146]: "[...] it is remarkable that in his later years Heisenberg put a somewhat different gloss on his relations. In his autobiography, he described how he had found his relations inspired by a remark by Einstein that 'it is the theory which decides what one can observe' —thus giving precedence to theory above experience, rather than the other way around." Most interestingly is the fact remarked by Hilgevoord and Uffink, that "some years later he even admitted that his famous discussions of thought experiments were actually trivial since '[...] if the process of observation itself is subject to the laws of quantum theory, it must be possible to represent its result in the mathematical scheme of this theory'."

For Bohr the starting point was the classical description of experimental arrangements univocally described in classical language (with the aid of physics). This is why he always started from the wave-particle duality in which both wave behavior and particle behavior pertain to well defined classical representation of experimental contexts. Bohr considered the

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[25] The pressure of Bohr can be read in the "Addition in Proof" to Heisenberg's foundational paper ([274], p. 83): "After the conclusion of the forgoing paper, more recent investigations of Bohr have led to a point of view which permits an essential deepening and sharpening of the analysis of quantum-mechanical correlations attempted in this work. In this connection Bohr has brought to my attention that I have overlooked essential points in the course of several discussions in this paper. Above all, the uncertainty in our observation does not arise exclusively from the occurrence of discontinuities, but is tied directly to the demand that we ascribe equal validity to the quite different experiments which show up in corpuscular theory in the one hand, and in the wave theory in the other hand. [...] I owe great thanks to Professor Bohr for sharing with me at an early stage the results of these more recent investigations of his-to appear soon in a paper on the conceptual structure of quantum theory- and for discussing them with me." Later on, in an interview with Kuhn, Hiesenberg ([54], p. 96) would express a very different position: "Bohr was very much inclined . . . to go forth and back between wave and particle picture. That was a thing which I didnt like too much because I felt that at least quantum theory seems to be a consistent scheme [...] For me it was clear that ultimately there was no dualism and after all, we had a closed mathematical scheme [...] Therefore I was always a bit upset by this tendency of Bohr of putting it into a dualistic scheme".

wave-particle duality present in the double-slit experiment as expressing the most important character of quantum theory. What Bohr had in mind was to resolve this duality through the **principle of complementarity**. Bohr's agenda was focused in fulfilling the consistency requirements of the quantum formalism to apply the well known classical scheme. The discussions which followed took Heisenberg's principle only as providing the limits of certainty and applicability of classical concepts as such. The classical scheme would then remain that which secured the knowledge provided by quantum theory, and analogously, Heisenberg's *uncertainty relations* that which secured the knowledge provided by the more general principle of complementarity.[26]

> "On the one hand, Bohr was quite enthusiastic about Heisenberg's ideas which seemed to fit wonderfully with his own thinking. Indeed, in his subsequent work, Bohr always presented the uncertainty relations as the symbolic expression of his complementarity viewpoint. On the other hand, he criticized Heisenberg severely for his suggestion that these relations were due to discontinuous changes occurring during a measurement process. Rather, Bohr argued, their proper derivation should start from the indispensability of both particle and wave concepts. He pointed out that the uncertainties in the experiment did not exclusively arise from the discontinuities but also from the fact that in the experiment we need to take into account both the particle theory and the wave theory." J. Hilgevoord and J. Uffink (Quoted from [146], section 3)

According to Leon Rosenfeld:

> "Bohr wanted to pursue the epistemological analysis one step further [than Heisenberg], and in particular to understand the logical nature of the mutual exclusion of the aspects opposed in the particle-wave dualism. From this point of view the indeterminacy relations appear in a new light. [...] The indeterminacy relations are therefore essential to ensure the consistency of the theory, by assigning the limits within which the use of classical concepts belonging to the two extreme pictures may be applied without contradiction. For this novel logical relationship, which called in Bohr's mind echoes of his philosophical meditations over the duality of our mental activity, he proposed the name 'complementarity', conscious that he was here breaking new ground in epistemology." L. Rosenfeld (Quoted from [274], p. 59)

If taken through the lines of thought of Einstein himself, acknowledging that in physics "there is no difference between observable and non-observable", Heisenberg's principle appears in a completely new light, referring to a different *mode of existence* to that of actuality (preexistence). But instead of developing a conceptual scheme that would allow to further understand the meaning of the principle.[27] Bohr used the uncertainty relations to place the limits of knowledge, and classical language to close the gates of any future conceptual development.

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[26] It is important to notice that Heisenberg's relations can be directly derived from the mathematical scheme of the theory, as a direct consequence of the quantum postulate. Today, we have more elements to make precise the relation between both principles, see for example, the analysis of Pekka Lahti in his thesis [175].

[27] Heisenberg intended such a development in terms of Aristotelian concept of *potentia* but he did not advance into a closed interpretation (see section 13.3).

> "[...] it must above all be recognized that, however far quantum effects transcend the scope of classical physical analysis, the account of the experimental arrangement and the record of the observations must always be expressed in common language supplemented with the terminology of classical physics." N. Bohr (Quoted from [51], p. 313)

Bohr's interpretation of quantum mechanics is then restricted to the consistent account of phenomena:

> "*The entire formalism is to be considered as a tool for deriving predictions*, of definite or statistical character, as regards information obtainable under experimental conditions described in classical terms and specified by means of parameters entering into the algebraic or differential equations of which the matrices or the wave-functions, respectively, are solutions. These symbols themselves, as is indicated already by the use of imaginary numbers, are not susceptible to pictorial interpretation; and even derived real functions like densities and currents are only to be regarded as expressing the probabilities for the occurrence of individual events observable under well-defined experimental conditions." N. Bohr (Quoted from [51], p. 314, emphasis added)

4.4 Bohr's Philosophy of Physics and the Importance of Language

We believe that the debate which took place between Einstein and Bohr can be only understood as part of the neo-Kantian tradition and discussion which was taking place in German speaking countries at the end of the 19th and beginning of the 20th century. This discussion is very well exposed by Michael Friedman in his beautiful book, *A Parting of the Ways.* From this perspective, both Einstein and Bohr were discussing from within representation, considering specifically the conditions of possibility to access phenomena. According to our reading of Bohr,[28] this preeminence of language within his own philosophical scheme can be only understood in relation to the bigger philosophical movement which was taking place in Europe and has been called the linguistic turn.[29] So even though quantum mechanics arose in relation to the criticism of Kant's epistemology —escaping from the domains of the *a priori* categories and forms of intuition—, it was soon *re-covered* by the neo-Kantian scheme of thought. It was the Danish physicist who was capable of introducing physics into the philosophical movement of the linguistic turn, shifting quantum theory from ontological concerns into epistemological ones. Niels Bohr was the first to initiate the re-turn of physics back into the domain of philosophy after the revolution produced by relativity theory and quantum mechanics. Just like Immanuel Kant did with Newtonian mechanics, turning upside down the relation of power between physics and philosophy (see [186]), Bohr was able to constrain the strength of physics

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[28] We acknowledge there are almost as many interpretations of Bohr as physicists and philosophers of science. Even though the orthodoxy has been to interpret Bohr from a neo-Kantian perspective, there are also ontological interpretations of Bohr such as those proposed by Folse [123] and Dieks [90]. In particular, Dieks interprets complementarity as an ontological notion which relates 'experimental situations'.

[29] We take the lingüistic turn to be a moment with a very different lines of philosophical investigation which can be comprised by the importance of language as a fundament. See the interesting analysis of Dardo Scavino in [233].

within the limitations imposed by that which would now play the role of the *a priori*: classical language.[30]

Niels Bohr's ideas have played a central role in the development of physics in the 20th century, placing the discipline within the main philosophical line of discussion of the period, this is, the problem of language and its relation to ontology and epistemology. The linguistic turn is a technical term in the history of philosophy (according to which all problems in philosophy are problems of language). We do not claim that Bohr knew that movement or was explicitly part of it. Rather, we point to the fact that, quite independently of this movement, Bohr took for himself many of the discussions and problems involved within such philosophical stance. A clear statement regarding this point is the famous quotation by Aage Petersen. According to the long time assistant of Bohr, when asked whether the quantum theory could be considered as somehow mirroring an underlying quantum reality Bohr declared the following:

> "There is no quantum world. There is only an abstract quantum physical description. It is wrong to think that the task of physics is to find out how nature is. Physics concerns what we can say about nature" N. Bohr (Quoted by A. Petersen in [274], p. 8)

Just like with Kant, in Bohr's philosophy there is no reference outside the scheme which determines phenomena.

> "On the lines of objective description [I advocate using] the word *phenomenon* to refer only to observations under circumstances whose description includes an account of the whole experimental arrangement. In such terminology, the observational problem in quantum physics is deprived of any special intricacy and we are, moreover, directly reminded that every atomic phenomenon is closed in the sense that its observation is based on registrations obtained by means of suitable amplification devices with irreversible functioning such as, for example, permanent marks on a photographic plate, caused by the penetration of electrons into the emulsion. In this connection, it is important to realize that the quantum-mechanical formalism permits well defined applications referring only to such closed phenomena." N. Bohr (Quoted from [274], p. 3)

But while for Kant, the scheme was structured through the transcendental subject, for Bohr, the problem was to consistently link phenomena through the use of language. 'Consistency' can be then translated into 'objectivity', or even better, as 'inter-subjectivity'.

> "The description of atomic phenomena has in these respects a perfectly objective character, in the sense that no explicit reference is made to any individual observer and that therefore... no ambiguity is involved in the communication of observation." N. Bohr (Quoted from [80], p. 98)

The problem was now how to secure *communication.* Instead of going directly back into Kant's *a priori*, Bohr made a detour into the realm of language. Bohr found then his cornerstone, his fundament, his "clear and distinct idea" in the the language used by classical physics.

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[30] It is then not a surprise to notice that the philosophy of Niels Bohr has been directly engaged with philosophers like Ludwig Wittgentstein (see [190]), Jaques Derrida (see [205]), and of course, Immanuel Kant (see [164] for an extensive review of the relation between Bohr and Kant's philosophy).

> "Even when the phenomena transcend the scope of classical physical theories, the account of the experimental arrangement and the recording of observations must be given in plain language, suitably supplemented by technical physical terminology. This is a clear logical demand, since the very word *experiment* refers to a situation where we can tell others what we have done and what we have learned." N. Bohr (Quoted from [274], p. 7)

As noted by Aage Petersen:

> "Traditional philosophy has accustomed us to regard language as something secondary and reality as something primary. Bohr considered this attitude toward the relation between language and reality inappropriate. When one said to him that it cannot be language which is fundamental, but that it must be reality which, so to speak, lies beneath language, and of which language is a picture, he would reply, "We are suspended in language in such a way that we cannot say what is up and what is down. The word 'reality' is also a word, a word which we must learn to use correctly" Bohr was not puzzled by ontological problems or by questions as to how concepts are related to reality. Such questions seemed sterile to him. He saw the problem of knowledge in a different light." A. Petersen (Quoted from [196], p. 11)

The relation to phenomena would then be secured by this 'static linguistic framework', according to Bohr ([274], p. 7) "[...] the unambiguous interpretation of any measurement must be essentially framed in terms of classical physical theories, and we may say that in this sense the language of Newton and Maxwell will remain the language of physicists *for all time.*" Bohr had found a new *a priori* —classical language— which would serve to secure objectivity. But, in order to close the circle, no "new language" was allowed from now on to enter the scene. Above all, it would be wrong to think that a new conceptual scheme would allow us to understand quantum mechanics: "it would be a misconception to believe that the difficulties of the atomic theory may be evaded by eventually replacing the concepts of classical physics by new conceptual forms."[31] Bohr had closed the doors of any future development based on new concepts. In this respect, John Hendry presents an interesting picture regarding the tense position maintained by Bohr:

> "Of all those actively involved in the search for a new quantum mechanics in the 1920s, Bohr was at once the most radical and the most conservative. He had been initially responsible for the idea that classical mechanics and kinematical concepts were incapable of describing quantum phenomena, and he had continued to believe this throughout. But he had also held fast to the belief that these concepts, and especially those of the classical wave theory of light could not be replaced." J. Hendry (Quoted from [145], p. 119)

Bohr's characterization of physics goes then together with his linguistically based pragmatic account:

> "Physics is to be regarded not so much as the study of something a priori given, but rather as *the development of methods of ordering and surveying human experience.* In this respect our task must be to account for such experience in a manner independent of

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[31] See also N. Bohr as quoted by Heisenberg in ([142], p. 162).

> individual subjective judgement and therefor *objective in the sense that it can be unambiguously communicated in ordinary human language.*" N. Bohr (Quoted from [52], emphasis added)

Thus, in Bohr's account of physics, the truly metaphysical questions and the problem of reality are regarded as unfruitful questions. One could in this case make a clear analogy with that which Kant called "ideas of reason", the World, the Soul and God (i.e., metaphysical ideas which go beyond the limits of possible experience). Bohr was not interested in discussing ontology. In his interpretation, the connection with whatever is called 'reality' or 'thing in itself', remains a meaningless question, a *faux question* which must be avoided. This is why Bohr persistently evades any direct engagement with the question of 'reality', such rejection is a requirement to consistently discuss classical phenomena in quantum theory.[32] In Bohr, the focus shifts from ontology to language, epistemology and inter-subjective communication. Language and pragmatism guide his quest, "reality" becomes then just another word in a linguistic scheme.

> "I am quite prepared to talk of the spiritual life of an electronic computer; to say that it is considering or that it is in a bad mood. What really matters is the unambiguous description of its behavior, which is what we observe. The question as to whether the machine *really* feels, or whether it merely looks as though it did, is absolutely as meaningless as to whether light is "in reality" waves or particles. *We must never forget that "reality" too is a human word just like "wave" or "consciousness." Our task is to learn to use these words correctly —that is, unambiguously and consistently.*" N. Bohr (Quoted by Kalckar in [274], p. 5, emphasis added)

If taken through these lines, Bohr's position is quite subtle and difficult to tackle. Bohr developed quantum mechanics in strict relation to language. He taught his followers not only to stay away from ill posed questions but also to remain within the limits imposed by language itself. Because there is no reference outside Bohr's epistemological scheme the problem of reality is dissolved. A 'physical situation' is part of a linguistic scheme and its analysis must be framed in terms of the communicability and intersubjectivity of statements. Within the realm of quantum mechanics, Bohr reconfigured the problem of objectivity not in terms of objects, but rather in terms of *communicability*, presenting the entity only as a discursive element with no further ontological commitment. The quantum state need not refer to objects but rather to the knowledge of how phenomena behaved and related through classical discourse. As recalled by Petersen:

> "The chief characteristic of the sort of description we seek in science as well as in practical life is objectivity. In Bohr's usage, an objective message was an unambiguous message,

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[32] It is interesting to notice that this very important point in Bohr's philosophical scheme seems to have not been understood by Heisenberg [274] who made the mistake of relating for example complementarity to reality: "The other way of approach was Bohr's concept of complementarity. Schrödinger had described the atom as a system not of a nucleus and electrons but of a nucleus and matter waves. This picture of the matter waves certainly also contained an element of truth. Bohr considered the two pictures —particle picture and wave picture– as two complementary descriptions of the same reality. Any of these descriptions can be only partially true, there must be limitations to the use of the particle concept as well as of the wave concept, else one could not avoid contradictions. If one takes into account those limitations which can be expressed by the uncertainty relations, the contradictions disappear."

> one that could not be misunderstood. If our communications are to be understood, their content must be clearly delineated. There must be, so to speak, a partition between the subject which communicates and the object which is the content of the communication. This partition is indispensable in every objective description, and Bohr saw in it the core of the problem of knowledge." A. Petersen (Quoted from [196], p. 11)

Bohr was able to recover an objective account of quantum theory in terms of language —which meant that the propositions of the theory should be robust only under *intersubjective agreement.* This is clearly related to what D'Espagnat [80] has called *weakly objective* statements or stated more explicitly what we shall call "linguistic" objectivity: *the way in which classical language provides an intersubjective account of phenomena.* Bohr had regained objectivity by watching quantum theory from a distance, standing on the well known heights of classical language. However, the problem remains highly controversial. On the one hand, there is no referent, no object about which one is being objective about, on the other, the idea that agreement, communicability, is the fundament of science appears as highly suspicious. If we believe that physics is about Nature, the objective character of a theory should be secured by the theory itself, or, in case it is an appendix of a different theory, one should clearly understand their relationship. As it stands, the position of Bohr forces us into a very unclear relation between the classical world and the quantum formalism, which does not seem to have a place in the classical conception of the world, but nevertheless, talks about it in terms of measurement outcomes.

As in the Kantian scheme, the Being is left aside, remaining as the unspeakable void of which one should remain silent. But now, contrary to the Kantian scheme in which even metaphysics plays a significant role, the object is buried as a ghost of the past. Classical language relates to phenomena giving place to a *consistent discourse*, detached of reality and Being. The search of Einstein is in this sense much closer to our understanding of the problem to which quantum mechanics confronts us. Physics is not, to put it in Wittgenstein's terms, just a "game of language". We do not agree with Rorty when he says that physicists and philosophers are but "vigourous poets".

Classical Experience and the Quantum Formalism

As we discussed above, according to our representational realistic stance, a necessary condition to account for any physical phenomenon is the explicit reference to a physical theory, i.e. a conceptual and formal scheme supplied by a field of physical experience. In this chapter we want to discuss some important presuppositions involved in the analysis of interpretational questions in quantum mechanics. We will concentrate on two main paradigmatic problems which have been extensively discussed in the literature: the Bell inequality and the KS theorem. We attempt to make explicit which are the formal and the metaphysical constraints that are at the basis of these problems, their scopes and limits.

5.1 Bell Inequalities: Metaphysical Conditions for Possible (Classical) Experience

At the very end of the EPR paper —due to the conclusion, that quantum mechanics is an incomplete theory— one finds the following proposed line of investigation:

> "While we have thus shown that the wave function does not provide a complete description of the physical reality, we left open the question of whether or not such a description exists. We believe, however, that such a theory is possible." A. Einstein, B. Podolsky and N. Rosen (Quoted from [113])

The reading of John Bell ([30], p. 195) was in this respect quite direct: "The paradox of Einstein, Podolsky and Rosen was advanced as an argument that quantum mechanics could not be a complete theory but *should be supplemented by additional variables.* These additional variables were to restore to the theory causality and locality." Thus, allowing to provide a representational scheme about what there is in terms of systems with preexistent properties; i.e. systems possessing properties with definite values at every —future and past— instant of time —independently of observation. However, in spite of his own desires, he himself proved that such a hidden variable theory could not match the statistical predictions of quantum mechanics. Bell derived an inequality which must be satisfied by any *local realistic* and *deterministic* hidden-variable theory, and he also showed that quantum mechanics violated this inequality.

This, he concluded, showed that no local realistic and deterministic hidden-variable theory can reproduce all the quantum mechanical experimental predictions.

The scope of the Bell inequality and its relation to quantum mechanics is often not recognized in the literature.[33] It is often claimed that Bell inequality talks about quantum mechanics, however, the specificity of the statements involved and its relation to quantum theory is not always stated explicitly. The Bell inequality is a statistical statement which is based on a set of (metaphysical) presuppositions implied by our classical conception of the world. This fact appears as obvious if we acknowledge that an equivalent version was already derived by Boole in 1864, long before quantum mechanics was even created.

> "In the mid-nineteenth century George Boole formulated his 'conditions of possible experience'. These are equations and inequalities that the relative frequencies of (logically connected) events must satisfy. Some of Boole's conditions have been rediscovered in more recent years by physicists, including Bell inequalities, Clauser Horne inequalities, and many others." I. Pitowsky (Quoted from [203], p. 95)

As remarked by Pitowsky, Boole's research problem can be phrased in modern terminology as follows: we are given a set of rational numbers $p_1, p_2, ...p_n$ which represent the relative frequencies of n logically connected events. The problem is to specify necessary and sufficient conditions that these numbers can be realized as probabilities in *some* probability space. These conditions were called by Boole for obvious reasons, *conditions of possible experience.*

> "Boole's problem is simple: we are given rational numbers which indicate the relative frequencies of certain events. If no logical relations obtain among the events, then the only constraints imposed on these numbers are that they each can be non-negative and less than one. If however, the events are logically interconnected, there are further equalities or inequalities that obtain among the numbers. The problem thus is to determine the numerical relations among frequencies, in terms of equalities and inequalities which are induced by a set of logical relations among other events. The equalities and inequalities are called 'conditions of possible experience'. [...] From a a mathematical point of view Boole's achievement lies in the realization that all the 'conditions of possible experience' are *linear* in the probabilities. In other words, the inequalities and equalities never involve expressions such as p^2 or 2^q when p, q are probabilities. Moreover, given a finite set of events, with (obviously) finitely many logical relations obtaining among them, there is only a finite set of conditions which hold. To be more precise, there is a finite set of equalities and inequalities, from which all other valid conditions logically follow. These... may be termed conditions of possible experience. When satisfied they indicate that the data may have, when not satisfied they indicate that the data cannot have resulted from an actual observation." I. Pitowsky (Quoted from [203], p. 100)

As noticed by Pitowsky the conditions of possible experience were rediscovered by physics in 1964 by John Bell. Bell intended to formally prove the possibility of the existence of a hidden variable theory —such as the already known to him Bohm's theory— which would allow to account for the seemingly weird results predicted by quantum mechanics, opening

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[33] I am grateful to Sven Aerts for pointing this out to me.

the doors for the restitution of our understanding of phenomena in classical terms. Contrary to his expectations he proved that, in order to reproduce quantum predictions, such hidden variable models must violate either *locality* or *realism*, two necessary conditions for recovering the classical picture —in the case of Bohm it was locality which had to be given up. Boole's conditions of possible experience can be derived from very elementary assumptions, either those of probability theory or alternatively those of propositional logic. This points in the direction that the puzzling aspects of quantum mechanics reside in the phenomena themselves and not just in the theory which predicts them. This seems to point in the direction that we are dealing here with a puzzle which does not depend on the details of a complex physical theory but rather, on a completely new type of phenomena which cannot be reduced to our classical conception of the world. The important point which we wanted to make explicit here is the fact that the Bell inequality talks about classical experience, it limits the relation between average sets of measurement outcomes predicted by a physical theory which attempts to provide a classical representation of the world. What Boole found, and later on Bell, is that any theory which goes beyond the results of the Bell inequality will have problems with such classical representation of physical experience. The Bell (or Boole) inequality makes explicit what we explained regarding the relation between representation and physical experience, for in this case the limits of experience of a physical theory are directly derived from the formalism itself.

5.2 The Kochen-Specker Theorem: Formal Constraints of the Quantum Formalism

A complete set of properties of a system that may be simultaneously predicated allows to construct a Boolean propositional system. In quantum mechanics this is usually referred to as a *context*.[34] In terms of operators, it is in correspondence with a complete set of commuting observables, C.S.C.O. for short. When making reference to a single context, assigning values (a set of their corresponding eigenvalues) to these magnitudes poses no difficulties. But when, considering all the possible contexts of inquiry simultaneously, and we try to interpret eigenvalues as the actual values of the physical properties of a system, we are facing all kind of no-go theorems that preclude this possibility. Most remarkably is the Kochen-Specker (KS) theorem that rules out the non-contextual assignment of values to physical magnitudes [166]. An explicit statement of the KS theorem reads [144]:

Theorem 5.2.1. *Let $\mathcal{H}$ be a Hilbert space of dimension greater than 2 of the states of the system and M be a set of observables, represented by operators on $\mathcal{H}$. Then, the following two assumptions are contradictory:*

1. *All members of M simultaneously have values, i.e. are unambiguously mapped onto real unique numbers (designated, for observables* $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, *... by* $v(\mathbf{A})$, $v(\mathbf{B})$, $v(\mathbf{C})$, *...).*

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[34] We already call here the attention that the term 'context' is, as we shall see in chapter 10, extremely problematic for it relates in an incomplete manner the formal level —as related to mathematical operators— and the conceptual level —as related to a metaphysical conception of what there is in terms of an experimental arrangement.

2. Values of observables conform to the following constrains:

If $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ *are all compatible and* $\mathbf{C} = \mathbf{A} + \mathbf{B}$, *then* $v(\mathbf{C}) = v(\mathbf{A}) + v(\mathbf{B})$;

if $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ *are all compatible and* $\mathbf{C} = \mathbf{AB}$, *then* $v(\mathbf{C}) = v(\mathbf{A})v(\mathbf{B})$. □

Let $\mathcal{H}$ be the Hilbert space associated to the physical system and $L(\mathcal{H})$ be the set of closed subspaces on $\mathcal{H}$. If we consider the set of these subspaces ordered by inclusion, then $L(\mathcal{H})$ is a complete orthomodular lattice [183]. It is well known that each self-adjoint operator $\mathbf{A}$ has an associated Boolean sublattice W_A of $L(\mathcal{H})$. More precisely, W_A is the Boolean algebra of projectors $\mathbf{P}_i$ of the spectral decomposition $\mathbf{A} = \sum_i a_i \mathbf{P}_i$. We will refer to W_A as the spectral algebra of the operator $\mathbf{A}$. Any proposition about the system is represented by an element of $L(\mathcal{H})$ which is the algebra of quantum logic introduced by George David Birkhoff and John von Neumann [42]. In the frame of quantum logic, as Michael Dickson remarks in [86], the KS theorem and the absence of a valuation from the orthomodular lattice to $\mathbf{2}$ can be understood as a consequence of the failure of the distributive law in $L(\mathcal{H})$ (see [78; 212]).

Assigning values to a physical quantity A is equivalent to establishing a Boolean homomorphism $v : W_A \to \mathbf{2}$ [154], being $\mathbf{2}$ the two elements Boolean algebra. So it is natural to consider the following definition:

Definition 5.2.2. *Let* $(W_i)_{i \in I}$ *be the family of Boolean sublattices of* $L(\mathcal{H})$. *A global valuation over* $L(\mathcal{H})$ *is a family of Boolean homomorphisms* $(v_i : W_i \to \mathbf{2})_{i \in I}$ *such that* $v_i \mid W_i \cap W_j = v_j \mid W_i \cap W_j$ *for each* $i, j \in I$

This global valuation would give the values of all magnitudes at the same time maintaining a **compatibility condition** in the sense that whenever two magnitudes share one or more projectors, the values assigned to those projectors are the same from every context. But the KS theorem assures that we cannot assign real numbers pertaining to their spectra to operators $\mathbf{A}$ in such a way to satisfy the functional composition principle (FUNC) which is the expression of the "natural" requirement mentioned by Dirac that, for any operator $\mathbf{A}$ representing a dynamical variable and any real-valued function $f(\mathbf{A})$, the value of $f(\mathbf{A})$ is the corresponding function of the value of $\mathbf{A}$.

> "The expression that an observable 'has a particular value' for a particular state is permissible in quantum mechanics in the special case when a measurement of the observable is certain to lead to the particular value, so that the state is an eigenstate of the observable. It may easily be verified from the algebra that, with this restricted meaning for an observable 'having a value', if two observables have values for a particular state, then for this state the sum of the two observables (if the sum is an observable) has a value equal to the sum of the values of the two observables separately and the product of the two observables (if this product is an observable) has a value equal to the product of the values of the two observables separately." P. Dirac (Quoted from [102], p. 46)

This is a very restrictive constraint because it does not allow to assign values to all possible physical quantities or to assign true-false as truth values to all propositions about the system. The KS theorem means that, if we demand a valuation to satisfy FUNC, then it is forbidden to define it in a non-contextual fashion for subsets of quantities represented by commuting operators. In the algebraic terms of the previous definition [103], the KS theorem reads:

Theorem 5.2.3. *If $\mathcal{H}$ is a Hilbert space such that $dim(\mathcal{H}) > 2$, then a global valuation over $L(\mathcal{H})$ is not possible.* ☐

Contextual valuations allow us to refer to sets of actual properties. Algebraically, a *contextual valuation* is a Boolean valuation over one chosen spectral algebra. In classical particle mechanics it is possible to define a Boolean valuation of all propositions, that is to say, it is possible to give a value to all the properties in such a way of satisfying FUNC. This possibility is lost in the quantum case. And it is not a matter of interpretation, it is the underlying mathematical structure that enables this possibility for classical mechanics and forbids it in the quantum case. The impossibility to assign values to the properties at the same time satisfying FUNC is a weighty obstacle for almost any interpretation of the formalism as something more than a mere instrument. KS talks about the formalism of quantum mechanics, its inner symmetries and relations, it puts limits to the possible interpretation of the projection operators —which are the mathematical elements of the theory interpreted as the properties which pertain to quantum systems.

5.3 *Internal* and *External* Statements Regarding Quantum Mechanics

We shall refer to statements in relation to the physical theory they discuss about. While **internal statements** refer to the theory itself and provide direct knowledge about *what the theory is*, enlightening characters and features of the formalism or the concepts involved within the theory; **external statements** are those which refer to *what the theory is not*, statements which relate to different theories and make reference to the original theory only in an indirect manner. As we have seen, the Bell inequality is directly derived from a set of classical (metaphysical) presuppositions and does not relate explicitly to the quantum formalism, rather, it is a *statistical statement* which relates to a metaphysical conception of physical reality in terms of a set of definite valued properties. The Bell inequality talks about classical properties, they do not talk about how quantum properties relate, but rather about the limits which quantum properties must obey if they are to be considered as classical. On the contrary, the argument of the KS theorem begins "right from the start" from the formalism of quantum mechanics itself. One could argue that there are also classical metaphysical presuppositions involved in the argument for the idea of 'property' and 'system' are presupposed, however, the presuppositions appear as showing the limits of the concepts which might be applied to interpret the formalism and not *vice versa*. The theorem does not start by presupposing the *mode of existence* of the properties but rather, provides constraints to the possible interpretation of projection operators as related to the mode of existence of quantum properties. While KS theorem tells us something about how to interpret the quantum formalism, the Bell inequality talks about the limits of quantum mechanics with respect to classical physical experience. From this perspective, while the Bell inequality is an internal statement with respect to classical physics and an external statement for quantum mechanics, the KS theorem —since it is derived from the formalism of the theory itself— is an internal statement for quantum mechanics.

From our representational realist stance the mode of existence of properties and the level of discourse involved within a given theory are not self-evident characters but need to be explicitly stated. A statistical property such as 'temperature' need not supervene or make reference to a more 'profound level' of non-statistical properties such as velocity of particles. At least one is not necessary committed to such metaphysical reductionism. From our stance, which also takes into account the idea of 'closed theories' such reductionism is part of the illusion of the naive realist that physical theories can evade representation and provide a description of the world *as it is*. Each theory needs to provide the constraints under which its properties are understood. Bell inequality talks about statistical properties which supervene on a classical definite state of affairs, i.e. there is in the derivation the presupposition of the system as constituted by a set of definite valued properties. However, average values are not necessarily related to an actual set of definite valued properties. One could in principle have statistical properties which are not supervinient on a definite state of affairs (described by a set of definite valued properties). The KS theorem, on the other hand, talks about definite valued properties, i.e. about the impossibility to interpret the projection operators as preexistent *actual* properties.

In order to be clear about the limit of statements regarding a given theory we need to be explicit, for example, of the mode of existence of the properties involved within such statement. The following definitions go in order:

Definition 5.3.1. Actual Observed Properties*: The observation hic et nunc of a given property. Actual observed properties do not imply a representational level nor the mode of being of the properties considered as preexistent.*

Definition 5.3.2. Actual Preexistent Properties*: Properties which are part of a representational scheme and exist through time as being definite independently of observation. The mode of being of these properties is preexistence.*

Statistical or average properties do not necessarily need to restrict themselves to the constraints imposed by states of affairs described in terms of a set of definite actual preexistent properties. Bell inequality, does commit itself to the discussion about an actual preexistent state of affairs for, as we have seen, it implies the classical metaphysical presupposition —as very clearly shown by Pitowsky— of a classical system that which exists quite independently of quantum mechanics. We can distinguish thus between *actual observed statistical properties*, which have no ontological commitment whatsoever, and *actual preexistent statistical properties* which directly relate to the objective existence of an actual preexistent state of affairs on which these properties supervene:

Definition 5.3.3. Actual Observed Statistical Properties*: The mean values, at a time, of a given set of observations of properties.*

Definition 5.3.4. Actual Preexistent Statistical Properties*: The (non-contextual*[35]*) mean values, at a time, which supervene on a set of actual preexistent properties.*

We will come back to these definitions later on in chapter 8 and 12 where they will play an important role and it will become clear why such specific characterization needs to be made when discussing interpretational questions.

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[35] The relation between contextuality, preexistence and representation will be further discussed on chapter 10.

A Methodological Map: Metaphysical Conditions vs the Mathematical Formalism

We believe that an interesting distinction that can help us to understand the huge interpretational map of quantum mechanics relates to the position one takes with respect to metaphysics. This controversial relation between physics and metaphysics displaces the problem of truth to a secondary stage and concentrates its analysis in the conditions of possibility to access and distinguish physical phenomena. Metaphysical schemes provide the coordinates through which the representational map of realistic stances can be developed. Among those who attempt to provide a metaphysical account of quantum mechanics there is a first group that tries, in different ways, to "restore a classical way of thinking about *what there is.*". Staying close to at least some of the classical notions of physics (space-time, causality, objects, etc.) these approaches have no problem to give up the orthodox formulation of quantum mechanics. A second group also interested in the metaphysical question regarding quantum mechanics attempts to begin "right from the start" with the successful mathematical formalism in its orthodox form, trying to learn about its structure and internal features in order to find a metaphysical scheme which is able to fit the formalism. We might consider the first group as going from metaphysics into the formal structure while the second group goes from the formal structure into the metaphysical scheme.[36]

6.1 From Classical Metaphysical Principles into the Mathematical Formalism

There are several examples in the literature which begin their analysis "right from the start" imposing classical metaphysical features which they consider self-evident characters of a physical theory. Maybe the most known example which follows this line of thought is David Bohm's interpretation [44; 45]. In order to restore the classical way of thinking about what there is and discuss about 'positions' and 'fields', instead of 'undefinite properties' and weird 'quantum states', Bohm's interpretation is forced to change the formalism with seemingly *ad hoc* moves;

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[36] Strange as it might seem the most "metaphysical" positions regarding this map come from approaches which have an important reputation in anglosaxon regions, many worlds interpretation in England and Bohmian mechanics in the United States. While the less metaphysical positions which relate to quantum logic and modal interpretations have their center of action on the European Continent.

moves which can be only justified in relation to the *prior* metaphysical commitments.[37] But quite independently of the discussion regarding the possibility to coherently develop such a classically based realistic scheme it is important above all to recognize the agenda of such an attempt. As Bacciagaluppi ([22], p. 74)) explicitly notes: “to restore a classical way of thinking about *what there is*.”

6.2 From the Mathematical Formalism into Classical Metaphysical Schemes

There is also a second position which begins its analysis “right from the start” from the orthodox formalism of quantum mechanics. This line of thought can be traced to the original work of Birkhoff and von Neumann [42] who in 1935 wrote a fundamental paper where they discussed the logical structure of quantum mechanics and gave birth to what has been called ever since quantum logic. This line of investigation continued with the development of Carl Friedrich Von Weizsäcker, and later on Pieter Mittelstaedt, Joseff Maria Jauch, Constantin Prion, among many others. From a different perspective, Hugh Everett also attempted to find an interpretation looking into the formal structure of the theory. According to Everett “quantum mechanics needs to find its own interpretation.” Later on, the reading of DeWitt of Everett’s interpretation, recovered the classical world at the price of creating an infinity of multiple non-observable worlds... [191] Although many of these attempts recognize the importance of the orthodox quantum formalism, and its departure of the classical world view, many of them have ended their voyage with a return to the basic classical metaphysical conception of the world.

6.3 From the Mathematical Formalism into Constructive Metaphysical Schemes

In the case of those interpretations which attempt to begin their analysis from the formalism the problem arises where to go to? As we have seen, there are many interpretations which fall pray of the classical metaphysical scheme. But, is there any other possibility which one could

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[37] Heisenberg ([141], pp. 116-117) was very critical in this respect: “One consequence of this interpretation is, as Pauli has emphasized, that the electrons in the ground states of many atoms should be at rest, not performing any orbital motion around the atomic nucleus. This looks like a contradiction of the experiments, since measurements of the velocity of the electrons in the ground state (for instance, by means of the Compton effect) reveal always a velocity distribution in the ground state, which is —in conformity with the rules of quantum mechanics— given by the square of the wave function in momentum or velocity space. But here Bohm can argue that the measurement can no longer be evaluated by the ordinary laws. He agrees that the normal evaluation of the measurement would indeed lead to a velocity distribution; but when the quantum theory for the measuring equipment is taken into account —especially some strange quantum potentials introduced ad hoc by Bohm— then the statement is admissible that the electrons ‘really’ always are at rest. In measurements of the position of the particle, Bohm takes the ordinary interpretation of the experiments as correct; in measurements of the velocity he rejects it. At this price Bohm considers himself able to assert: ‘We do not need to abandon the precise, rational and objective description of individual systems in the realm of quantum theory.’ This objective description, however, reveals itself as a kind of ‘ideological superstructure,’ which has little to do with immediate physical reality; for the hidden parameters of Bohm’s interpretation are of such a kind that they can never occur in the description of real processes, if quantum theory remains unchanged.”

think of? For us, modal interpretations introduce a key question which can open the door to a new understanding of quantum mechanics. The question is: How can we think of modality from within quantum theory?

As we shall see, modal interpretations have a controversial agenda and have been understood in the literature from two extreme and opposite perspectives. Van Fraassen's original proposal of an empiricist anti-metaphysical interpretation very soon turned itself into the realist classical metaphysical agenda of the hidden variable program. His criticism against analytic metaphysics could be understood also in relation to what philosophers of science had done with his own attempt to understand quantum mechanics. As we have argued, we believe that modal interpretations can also open the door to a more radical stance which is able to account for the world according to quantum mechanics but does not necessarily rely on classical metaphysical presuppositions. Our stance attempts to remain within the metaphysical question which relates physics with reality, taking very seriously the critics of 20th century analytic philosophy (see also [163]), but avoiding the dogmatic presuppositions imposed through the ideal of a classical conception of the world. According to our constructive metaphysical stance, reality should not be a pre-established concept nor a prejudice in observing and relating empirical data, but rather an open concept which should be transformed and developed according to the necessities of each physical theory. We should not expect reality to be as we would like it to be. We must be able to constantly develop new conceptual and formal frameworks which are able to express new phenomena. Following the main idea, which led Einstein to the special theory of relativity, we should not conclude experiments from reality, but we should neither change a successful formalism just because it does not expresses reality as we (metaphysically) believe it should be. The physicist and the philosopher should remain humble; not presupposing that they already know what reality is about.

As noticed by van Fraassen "a philosophical position can consist in something other than a belief in what the world is like. [...] A philosophical position can consist in a stance (attitude, commitment, approach, a cluster of such —possibly including some propositional attitudes such as beliefs as well). Such a stance can of course be expressed, and may involve or presuppose some beliefs as well, but cannot be simply equated with having beliefs or making assertions about what there is." We are now ready to advance in a series of desiderata which characterize our own stance regarding the possible interpretation of quantum theory:

1. **Metaphysical Stance:** *Quantum mechanics relates to reality.* We are open to the fact quantum mechanics might be able to express new possibly revolutionary features of reality. This means we are not relying on *classical concepts* (e.g. the notion of object) in order to determine the conceptual structure of quantum theory. As noted by Dieks ([91], p. 1417): "This would deny the possibility of really new fundamental theories, conceptually independent of classical physics."

2. **Closed Representational Stance:** Each physical representation is closed under its own formal and conceptual structure and provides access to a specific set of phenomena. The physical theory also provides the constraints to consider, explain and understand physical phenomenon. The understanding of a phenomena is always *local* for it refers to the closed structure given by the physical theory from which it is observed.

3. **Formalism and Empirical Adequacy:** The formalism of quantum mechanics is able

to provide (outstanding) empirically adequate results. Empirical adequacy determines the success of a theory and not its commitment to a certain presupposed conception of the world. Thus, it seems to us that the problem is not to find a new formalism. On the contrary, as also remarked by Dieks in relation to modal interpretations, the 'road signs' point in the direction that we must *stay close to the orthodox quantum formalism.*

4. **Coherency of the Interpretation:** Until today, there is no agreement regarding the interpretation of quantum mechanics. All interpretations until today seem to have different unsolved problems and questions. These problems can be seen as the impossibility to relate the mathematical structure of the theory to a conceptual scheme which allows to account in a *coherent* manner to quantum experience.

5. **Constructive Stance:** In order to learn about the limits of the classical concepts — such as: properties, apparatuses, systems— from quantum mechanics, we need to work out the limits of the formalism, the symmetries, its invariances. To learn about what the formalism is telling us about reality and how to express this in more adequate terms we might be in need of *creating new concepts.*

Within our constructive metaphysical stance, modal interpretations could be understood as providing a twofold development. A critical analysis which exposes the limits of classical language with respect to the quantum formalism and a positive analysis which attempts to understand the meaning of modality within quantum mechanics. From this account, the problem regarding the interpretation of quantum mechanics relates to the creation of an adequate metaphysical scheme, or in other words, to answer the question: what is quantum mechanics talking about?

III

MODALITY

The Modal Interpretation of Quantum Mechanics

According to our constructive metaphysical approach, in order to provide a coherent account of phenomena the mathematical structure of a physical theory must be linked to the conceptual structure in a way that allows to explain and understand phenomena. *Probability* and *possibility* are two of the key notions involved within the description of quantum phenomena. Thus, a realistic representational interpretation of quantum mechanics must provide an internal understanding of such concepts. It has many times been claimed that "quantum mechanics is a theory about probability and possibility". This is explicit already from Born's interpretation of the quantum wave function in terms of a density of probability to find the particle in a determinate position.

> "Almost simultaneously with the appearance of Schrödinger's fourth communication, a new interpretation of the ψ-function was published that had far-reaching consequences for modern physics not only from the purely technical point of view but also with respect to the philosophical significance of its content. Only four days after Schrödinger's concluding contribution had been sent to the editor of the *Annalen der Physik* the publishers of the *Zeitschrift für Physik* received a paper, less than five pages long, titled 'On the Quantum Mechanics of Collision Processes' in which Max Born proposed, for the first time, a probabilistic interpretation of the wave function implying thereby that microphysics must be considered a probabilistic theory." M. Jammer (Quoted from [155], p. 38)

However, together with this statement, we also find the warning that we do not know, in the case of quantum mechanics, what 'probability' exactly means. In his original article, Born begins by explicitly characterizing quantum mechanics in modal terms, emphasizing on the one hand its probabilistic character, but stressing at the same time the indeterministic element present in the theory:

> "Schrödinger's quantum mechanics [therefore] gives quite a definite answer to the question of the effect of the collision; but there is no question of any causal description. One gets no answer to the question, 'what is the state after the collision' but only to the question, 'how probable is a specified outcome of the collision'.
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> Here the whole problem of determinism comes up. *From the standpoint of our quantum mechanics there is no quantity which in any individual case causally fixes the consequence*

> *of the collision; but also experimentally we have so far no reason to believe that there are some inner properties of the atom which condition a definite outcome for the collision.* [...] I myself am inclined to give up determinism in the world of the atoms. But that is a philosophical question for which physical arguments alone are not decisive." M. Born (Quoted from [274], p. 57, emphasis added)

In this paper Max Born formulated the now-standard interpretation of the $\psi(x)$ in the Schrödinger equation of quantum mechanics, as encoding a probability density function for a certain particle to be found at a given region. The wave function is a complex-valued function of a continuous variable. According to Born's interpretation the physical meaning is provided via $|\psi(x)|^2 = \psi(x)^*\psi(x)$. For a state ψ, the associated probability density function is $\psi^*\psi$, which is equal to $|\psi(x)|^2$. If $|\psi(x)|^2$ has a finite integral over the whole of three-dimensional space, then it is possible to choose a normalizing constant. The probability that a particle is within a particular region V is the integral over V of $|\psi(x)|^2$. However, even though this interpretation worked fairly well, it soon became evident that the concept of probability in the new theory departed from the notion considered in classical statistical mechanics —as *lack of knowledge* about a preexistent state of affairs described in terms of definite valued properties.

In the history of physics the development of probability took place through a concrete physical problem and has a long history which goes back to the 18th century. The physical problem with which probability dealt was the problem of characterizing a state of affairs of which there is an incomplete knowledge, or in other words, gambling. This physical problem was connected later on to a mathematical theory developed by Laplace and others. But it was only after Kolmogorov that this mathematical theory found a closed set of axioms [167]. Although there are still today many interpretational problems regarding the physical understanding of classical probability, when a realist physicist talks about probability in statistical mechanics he is discussing about the (average values of) properties of an uncertain —but existent— state of affairs.[38] This is why the problem to determine a definite state of affairs in quantum mechanics —the sets of definite valued properties which characterize the quantum system— poses also problems to the interpretation of probability and possibility within the theory itself. As noticed by Schrödinger in a letter to Einstein:

> "It seems to me that the concept of probability is terribly mishandled these days. Probability surely has as its substance a statement as to whether something *is* or *is not* the case —an uncertain statement, to be sure. But nevertheless it has meaning only if one is indeed convinced that the something in question quite definitely *is* or *is not* the case. A probabilistic assertion presupposes the full reality of its subject." E. Schrödinger (Quoted from [59], p. 115)

Mathematics, contrary to physics, is a non-representational art which respects no metaphysical limits whatsoever. The mathematician does not need to constrain himself to any sort of metaphysical principle but only to the internal structure of the mathematical theory itself. 'Probability' is regarded by the mathematician as a "theory of mathematics" and in this

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[38] In this respect it is important to remark that the orthodox interpretation of probability in terms of relative frequencies, although provides a conceptual framework to relate to measurement outcomes, refers to 'events' and to 'properties of a system'; in this sense it is not necessarily linked to a realistic physical representation but rather supports an empiricist account of the observed measurement results.

sense departs from any meaning provided by the (representational realist) physicist who has a physical concept attached to the formal structure. A mathematician thinks of a probability model as the set of axioms which fit a mathematical structure and wonders about the internal consistency rather than about how this structure relates and can be interpreted in relation to experience and physical reality. As noticed by Hans Primas:

> "Mathematical probability theory is just a branch of pure mathematics, based on some axioms devoid of any interpretation. In this framework, the concepts 'probability', 'independence', etc. are conceptually unexplained notions, they have a purely mathematical meaning. While there is a widespread agreement concerning the essential features of the calculus of probability, there are widely diverging opinions what the referent of mathematical probability theory is." H. Primas (Quoted from [209], p. 582)

The important point is that when a mathematician and a physicist talk about 'probability' they need not refer to the *same* concept. While for the mathematician the question of the relation between the mathematical structure of probability and experience plays no significant role, for the physicist, according to our stance, the question of probability is *necessarily* related to experience and physical reality.

Luigi Accardi proved in 1981 that there is a direct relation between Bell inequalities and probability models [1]. The theorem of Accardi states that any theory which violates Bell inequality has a non-Kolmogorovian probability model. Since only Kolmogorovian models can be interpreted in terms of referring to a degree of ignorance of a presupposed state of affairs given by a set of definite valued preexistent properties, this means that quantum mechanics possesses a probability model which cannot be interpreted in terms of ignorance of such preexistent reality. The fact that quantum mechanics possesses a non-Kolmogorovian probability model is not such a big issue from a mathematical perspective: many mathematicians work with these probability structures and do not get astonished in any way by them. But from a realistic representational physical perspective, the question which arises is very deep, namely, what is the meaning of a concept of probability which does not talk about the degree of knowledge of a definite state of affairs? From our perspective, if such a question is not properly acknowledged, the statement "quantum mechanics is a theory about probabilities" looses all physical content. According to our stance, if we are to understand quantum mechanics as a physical theory, and not merely as a mathematical structure or an algorithm, then it is clear that we still need to provide a clear link between the mathematical structure and the concepts used to provide an account of quantum phenomena. An analogous problem appears when one discusses the notion of possibility. A logically possible proposition is one that can be asserted without implying a logical contradiction. This is to say that a proposition is logically possible if there is some coherent way for the world to be, under which the proposition would be true. Logical possibility is a logical concept which seems, at first hand, quite intuitive. Anyone seems to know what is the meaning of possibility, however when we discuss this notion within the realm of quantum mechanics and its formalism, problems (once again) arise. The notion of possibility is closely related to that of probability. Both notions have been analyzed in the literature since the early discussions which took place between the founding fathers of the theory. However, the formal aspect of possibility was not addressed more rigorously until, in the seventies, Bas van Fraassen used modal logic in order to analyze formally what was known until then about possibility in quantum mechanics [250; 251]. From this investigation a new interpretation arose on the map of the quantum called, for obvious reasons, the modal interpretation of quantum mechanics. In the following chapter we attempt to present this interpretation and investigate its possibilities and impossibilities to deliver a consistent view of what quantum mechanics is talking about.

7.1 Modality within the Modal Interpretation

We believe that the most important and controversial notion of quantum mechanics is the notion of *possibility*. Quantum mechanics seems to refer to possible or probable states of affairs in a way which has no analogue in classical physical theories. The meaning of the notion of quantum possibility relates directly to the interpretation given to another main formal expression which appears in the quantum formalism: the quantum superposition.

In classical mechanics the representation of the state of the physical system is given by a point in phase space Γ and the physical magnitudes are represented by real functions over Γ. These functions commute with each other and can be interpreted as possessing definite values independently of measurement, i.e. each function can be interpreted as being actual. The term actual refers here to *preexistence* (within the transcendent representation) and not to the observation *hic et nunc*. In the orthodox formulation of quantum mechanics, the representation of the state of a system is given by a ray in Hilbert space $\mathcal{H}$. But, contrary to the classical scheme, physical magnitudes are represented by operators on $\mathcal{H}$ that, in general, do not commute. This mathematical fact has interpretational consequences for it is then extremely problematic to affirm that these quantum magnitudes are simultaneously preexistent —in the sense of being actual independently of observation. In order to restrict the discourse to different sets of commuting magnitudes, various Complete Sets of Commuting Operators (CSCO) may be chosen. The choice of a particular representation (given by a CSCO) determines the basis in which the observables diagonalize and in which the ray can be expressed. Thus, the ray can be written as different linear combinations of states:

$$\alpha_i|\varphi_i^{B1} > +\alpha_j|\varphi_j^{B1} >= |\varphi_q^{B2} >= \beta_m|\varphi_m^{B3} > +\beta_n|\varphi_n^{B3} > +\beta_o|\varphi_o^{B3} > \tag{7.1}$$

For example if we consider a spin $\frac{1}{2}$ system:

$$\frac{1}{\sqrt{2}}[| \uparrow_x> +| \downarrow_x>] = | \uparrow_z>= \frac{1}{\sqrt{2}}[| \uparrow_y> +| \downarrow_y>] \tag{7.2}$$

The linear combinations of states are also called quantum superpositions. From the point of view of the theory of vector spaces, the fact that a vector can have different mutually equivalent mathematical representations poses no difficulty, but the fact that there is a consistent mathematical framework does not mean that we also have a physical interpretation of such a formal scheme. Recalling Dirac:

> “The nature of the relationships which the superposition principle requires to exist between the states of any system is of a kind that cannot be explained in terms of familiar physical concepts. One cannot in the classical sense picture a system being partly in each of two states and see the equivalence of this to the system being completely in some other

> state. There is an entirely new idea involved, to which one must get accustomed and in terms of which one must proceed to build up an exact mathematical theory, without having any detailed classical picture." P. Dirac (Quoted from [102], p. 12)

The question which arises, due to the non-commutative structure of the formalism, is the meaning of the representation of the state in different bases and the equivalence implied in equation 7.1 between the different representations of the state of affairs. In particular, it is important to notice that formally, the representation of the state by one single term has no priority with respect to representations in which there exist more than one term. Furthermore, if the representations are thought to describe or express something physically real then it is not obvious why one of the representations might be considered as more real than any other. This goes back to the question posed by Dirac of the meaning of the combination of states referring to different sets of incompatible properties. If the system is in the state $\mid \uparrow_z >$, what does it mean that, due to the mathematical equivalence of equation 7.2, it is also in the (same) state formed by $\mid \uparrow_x >$ and $\mid \downarrow_x >$? It is not clear how the same state can comprise the representation of mutually incompatible sets of properties if we believe at the same time that this state describes a unique world. The mathematical equivalence of equation 7.2 is commonly interpreted in the literature as saying that each representation expresses the 'same' state. The problem already arises with the use of language, for the word 'same' implies a particular interpretation of a mathematical expression. The word 'same' implies that we can 'put together' the multiple representations as making reference to 'something'. At the mathematical level this seems to be explicitly noticed by the equality sign '='. However, it should be clear that a mathematical sign involves no metaphysical commitment whatsoever. Rather, it is part of an abstract non-representational structure with interdefined inner relations. At the conceptual level of interpretation, the word 'same' accompanied by the word 'state'[39] gives line to an interpretation in terms of entities or physical objects —understood in terms of Aristotle, as a notion which arises from the logical and ontological principles of existence, non-contradiction and identity. It is exactly this interpretation in terms of objects, which rises severe problems when related to the formalism of quantum mechanics.

As we shall argue, there is a deep relation in between the interpretation of possibility on the one side and the interpretation of quantum superpositions on the other. In the quantum superposition, as noted by Dieks, "all possible outcomes occur on an equal footing in the superposition of the final state, so that there is no sign that any one of them is more real than any other." The physical consequences for a representation in terms of a superposition ($\Psi = \psi_1 + \psi_2$) and the case in which the situation is given by a state which is not a superposition but a case of ignorance (*either* ψ_1 or ψ_2, but we do not know which) are completely different (see for discussion [100]). In the quantum phenomena every term in the quantum superposition plays an indispensable role in bringing about the measurement result.

> "Both components are needed for the interference pattern; removing one of them, by

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[39] As remarked by Michel Bitbol ([40], p. 72): "The tendency to reify state vectors manifests itself in the use of the very word 'state'. The 'grammar' (in Wittgenstein's sense) of the word 'state' requires that this is the state of something; that it belongs to something; that it characterizes this something independently of anything else. Such grammar, and the conception associated to it, is sufficient to generate one of the major aspects of the measurement problem."

> changing the experimental set-up (e.g., by blocking one of the slits [in the double-slit experiment]) will result in a change in the observable pattern on the photographic plate. One could perhaps say that each term represents a part of the total cause of the final result. [T]his result cannot be regarded as the sum of what would be brought about by the partial causes separately: there is interference, so the 'causes' apparently interact with each other. Anyway, it can be concluded that both ψ_1 and ψ_2 refer to aspects of what actually exists —indeed, something that does not exist cannot be causally relevant.[...]
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> In classical physics the most fundamental description of a physical system (a point in phase space) reflects only the actual, and nothing that is merely possible. It is true that sometimes states involving probabilities occur in classical physics: think of the probability distributions ρ in statistical mechanics. But the occurrence of possibilities in such cases merely reflects *our ignorance* about what is actual. The statistical states do not correspond to features of the actual system (unlike the case of the quantum mechanical superpositions), but quantify our lack of knowledge of those actual features. This relates to the essential point of difference between quantum mechanics and classical mechanics that we have already noted: in quantum mechanics the possibilities contained in the superposition state may interfere with each other. There is nothing comparable in classical physics. In statistical mechanics the possibilities contained in ρ evolve separately from each other and do not have any mutual influence. Only one of these possibilities corresponds to the actual situation. The above (putative) argument for the reality of modalities can therefore not be repeated for the case of classical physics." D. Dieks (Quoted from [100], pp. 124-125)

Non-collapse interpretations such as the modal approach state that superpositions remain in the possible realm always intact, independent of the actual observation. The "collapse" of the quantum wave function is not considered as a physical interaction:

> "Now, in the older literature it is often said that at some stage during the measurement interaction the state 'collapses': that all terms suddenly disappear except the one corresponding to the actually realized outcome. Such a collapse would single out the actual from the merely possible. Collapses constitute, however, a process of evolution that conflicts with the evolution governed by the Schrödinger equation. And this raises the question of exactly when during the measurement process such a collapse could take place or, in other words, of when the Schrödinger equation is suspended. This question has become very urgent in the last couple of decades, during which sophisticated experiments have clearly demonstrated that in interaction processes on the sub-microscopic, microscopic and mesoscopic scales collapses are never encountered." D. Dieks (Quoted from [100], p. 120)

According to Dieks ([89], p. 182) "[...] there is no need for the projection postulate. On the theoretical level the full superposition of states is always maintained, and the time evolution is unitary. One could say that the 'projection' has been shifted from the level of the theoretical formalism to the semantics: it is only the empirical interpretation of the superposition that the component terms sometimes, and to some extent, receive an independent status." Thus, in modal interpretations, the passage from the potential to the actual is given through different interpretational rules according to the different versions. However, as noted by Vermaas, there is a non-standard character of how possibility is considered within the modal interpretation:

> "The name 'modal' is in my opinion suited because one may understand it as pointing to the fact that modal interpretations interpret quantum mechanics by slightly changing the standard understanding of the modalities 'actuality' and 'possibility'. [...] [In classical statistical theories] non-actualized possibilities are standardly removed from states. In modal interpretations the state is now not updated if a certain state of affairs becomes actual. The non-actualized possibilities are not removed from the description of a system and this state therefor codifies not only what is presently actual but also what was presently possible. These non-actualized possibilities can, as a consequence, in principle still affect the course of later events." P. Vermaas (Quoted from [266] p. 295)

In the following we attempt to present the modal interpretation, we will come back to the meaning of quantum possibility in the chapters that follow (especially in chapter 14 were we discuss its relation to the interpretation of the quantum superposition).

7.2 The (Very) Different Modal Interpretations

The study of the modal character of quantum mechanics was explicitly formalized in the seventies and eighties by a group of physicists and philosophers of science. Bas van Fraassen was the first one to formally include the reasoning of modal logic in quantum mechanics. He presented a modal interpretation of quantum logic in terms of its semantical analysis [250; 251; 252; 254] which had the purpose to clarify which properties among those of the complete set structured in the lattice of subspaces of Hilbert space pertain to the system. In general terms van Fraassen's position can be closely related, as he does himself, to Bohr's interpretation. Later on, Simon Kochen presented his own modal version in 1985 [165], in one of the famous conferences organized by Kalervo Laurikainen in Finland.[40] This interpretation of quantum mechanics was also a continuation of the discussions between the founding fathers of the theory. Carl Friedrich von Weizsäcker and Theodor Görnitz ([131], p. 357) referred specifically to it in a paper entitled "Remarks on S. Kochen's Interpretation of Quantum Mechanics". In this paper they state: "We consider it is an illuminating clarification of the mathematical structure of the theory, especially apt to describe the measuring process. We would, however feel that it means not an alternative but a continuation to the Copenhagen interpretation (Bohr and, to some extent, Heisenberg)." Also Dieks' interpretation can be considered as a continuation and a formal account of Bohr's ideas on complementarity and measurement.[41] I make this remark because later reviews of modal interpretations such as those of Clifton, Bub, Dickson and Bacciagaluppi neglect almost completely this tradition, the starting point and agenda of these authors. After van Fraassen's first papers Dieks went further in relation to the metaphysical presuppositions involved, making explicit that modal interpretations [87; 89; 91; 98] could be considered from a realistic perspective as describing systems with properties. If considered from this perspective, modal interpretations face the problem of finding an objective reading of the accepted mathematical formalism of the theory, a reading "in terms of properties possessed by physical systems, independently of consciousness and measurements (in the sense of human interventions)" [98]. The main problem modal

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[40] Kalervo Laurikainen organized a series of conferences which provided a fertile ground of discussions from the mid-eighties till the mid-nineties [172; 173; 174; 176; 177].

[41] Private discussion, Utrecht, December 2009.

interpretations then have to face is the determination of the set of definite valued properties possessed by a physical system. The way in which they attack the problem rests on the distinction between the levels of possibility and actuality.

In this chapter we intend to discuss different versions of the modal interpretation. We concentrate on van Fraassen's Copenhagen variant, Kochen-Dieks modal interpretation, Bub's Bohmian variant and the atomic interpretation of Bacciagaluppi and Dickson (see also [138; 14; 15; 266; 35; 87; 101; 220] for general reviews). In particular, we want to analyze the interpretational rules which account for the path from the *possible* to the *actual*, and the meaning of modality.

7.2.1 Van Fraassen's Copenhagen Variant

As noted by Dirac in the first chapter of his famous book, the existence of superpositions is responsible for the striking difference of quantum behavior from the classical one. In fact, the photon being in a superposition of states must be accepted if we want to explain interference effects [102]. Superpositions are also central when dealing with the measurement process, where the various terms associated with the possible outcomes of a measurement must be assumed to be present together in the description. This fact leads van Fraassen to the distinction between *value-attributing propositions* and *state-attributing propositions*, between *value-states* and *dynamic-states*:

> "[...] a *state*, which is in the scope of quantum mechanics, gives us only probabilities for actual occurrence of *events* which are outside that scope. They can't be entirely outside the scope, since the events are surely described if they are assigned probabilities; but at least they are not the same things as the states which assign the probability.
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> In other words, the state delimits what can and cannot occur, and how likely it is —it delimits possibility, impossibility, and probability of occurrence— but does not say what actually occurs." B. van Fraassen (Quoted from [255], p. 279)

So van Fraassen distinguishes propositions about events and propositions about states. Propositions about events are *value-attributing propositions* $< \mathbf{A}, \sigma >$, they say that 'Observable $\mathbf{A}$ has a certain value belonging to a set σ'. Propositions about states are of the form 'The system is in a state of this or that type' (in a pure state, in some mixture of pure states, in a state such that...). A *state-attribution proposition* $[\mathbf{A}, \sigma]$ gives a probability of the value-attribution proposition, it states that $\mathbf{A}$ will have a value in σ, with a certain probability. *Value-states* are specified by stating which observables have values and what these values are. *Dynamic-states* state how the system will develop. This is endowed with the following interpretation:

> "The interpretation says that, if a system X has dynamic state ρ at t, then the state-attributions $[\mathbf{A}, \sigma]$ which are true are those that $Tr(\rho \mathbf{P}^{\mathbf{A}}_{\sigma}) = 1$. [$\mathbf{P}^{\mathbf{A}}_{\sigma}$ is the projector over the corresponding subspace.] About the value-attributions, it says that they cannot be deduced from the dynamic state, but are constrained in three ways:
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> 1. If $[\mathbf{A}, \sigma]$ is true then so is the value-attribution $< \mathbf{A}, \sigma >$: observable $\mathbf{A}$ has value in σ.
> 2. All the true value-attributions should have Born probability 1 together.

> 3. The set of true value-attributions is maximal with respect to the feature (2.)" B. van Fraassen (Quoted from [255], p. 281).

This distinction between value-attribution propositions and state-attribution propositions allows van Fraassen to face the measurement problem from a new position. The way out proposed by von Neumann, of the contradiction between the presence of various results associated to the different terms in a superposition and the appearance of only one result, is the so called "projection postulate" which determines the non-causal state transition from the quantum state into a single term. In his spirit, an observable pertaining to a system has a value if and only if the system is in a corresponding eigenstate of the observable (the eigenstate-eigenvalue link). So, the observable, say $\mathbf{A}$, has a value if and only if a measurement of $\mathbf{A}$ is certain to have a certain outcome. If the outcome of the measurement is uncertain, which is the case when the state is in a superposition of eigenstates of the observable, then the observable has no value. Van Fraassen, on the contrary, proposes to emphasize this modal character of the theory via the rôle of the state:

> "[...] the transition from the possible to the actual is not a transition *of* state, but a transition *described by* the state." B. van Fraassen (Quoted from [255], p. 279)

and to interpret the emergence of a result in a new light:

> "[...] [the emergence of a result is] *as if the Projection Postulate were correct.* For at the end of a measurement of $\mathbf{A}$ on system X, it is indeed true that $\mathbf{A}$ has the actual value which is the measurement outcome. But, of course, the Projection Postulate is not really correct: there has been a transition from possible to actual value, so what it entailed about values of observables is correct, but that is all. There has been no acausal state transition." B. van Fraassen (Quoted from [255], p. 288)

The main aspects of van Fraassen's modal interpretation in terms of quantum logic ([255], chapter 9) are as follows. In the modal interpretation, the probabilities are of events, each describable as 'an observable having a certain value', corresponding to value states. If w is a physical situation in which system X exists, then X has both a *dynamic state* φ and a *value state* λ, i.e. $w =< \varphi, \lambda >$. A *value state* λ is a map of observable $\mathbf{A}$ into non-empty Borel sets σ such that it assigns $\{1\}$ to $1_\sigma \mathbf{A}$. 1_σ is the characteristic function of the set σ of values. So, if the observable $1_\sigma \mathbf{A}$ has value 1, then it is impossible that $\mathbf{A}$ has a value outside σ. The proposition $< \mathbf{A}, \sigma >= \{w : \lambda(w)(\mathbf{A}) \subseteq \sigma\}$ assigns values to physical magnitudes, it is a *value-attribution proposition* and is read as '$\mathbf{A}$ (actually) has value in σ'. $\mathcal{V}$ is called the set of value attributions $\mathcal{V} = \{< \mathbf{A}, \sigma >$: $\mathbf{A}$ an observable and σ a Borel set$\}$. The logic operations among value-attribution propositions are defined as: $< \mathbf{A}, \sigma >^\perp =< \mathbf{A}, \Re - \sigma >$, $< \mathbf{A}, \sigma > \wedge < \mathbf{A}, \theta > =< \mathbf{A}, \sigma \cap \theta >$, $< \mathbf{A}, \sigma > \vee < \mathbf{A}, \theta > =< \mathbf{A}, \sigma \cup \theta >$ and $\wedge\{< \mathbf{A}, \sigma_i >: i \in \mathcal{N}\} =< \mathbf{A}, \cap\{\sigma_i : i \in \mathcal{N}\} >$. With all this, $\mathcal{V}$ is the union of a family of Boolean sigma algebras $< \mathbf{A} >$ with common unit and zero equal to $< \mathbf{A}, S(\mathbf{A}) >$ and $< \mathbf{A}, \wedge >$ respectively. The Law of Excluded Middle is satisfied: every situation w belongs to $q \vee q^\perp$, but not the Law of Bivalence: situation w may belong neither to q nor to $q^\perp$.

A *dynamic state* φ is a function from $\mathcal{V}$ into [0, 1], whose restriction to each Boolean sigma algebra $< \mathbf{A} >$ is a probability measure. The relation between dynamic and value state is the following: φ and λ are a dynamic state and a value state respectively, only if there exist possible situations w and w' such that $\varphi = \varphi(w)$, $\lambda = \lambda(w')$. Here, φ is an eigenstate of $\mathbf{A}$, with corresponding eigenvalue a, exactly if $\varphi(< \mathbf{A}, \{a\} >) = 1$. The *state-attribution proposition* $[\mathbf{A}, \sigma]$ is defined as: $[\mathbf{A}, \sigma] = \{w : \varphi(w)(< \mathbf{A}, \sigma >) = 1\}$ and means '$\mathbf{A}$ must have value in σ'. $\mathcal{P}$ denotes the set of state-attribution propositions: $\mathcal{P} = \{[\mathbf{A}, \sigma] : \mathbf{A} \text{ an observable, } \sigma \text{ a Borel set}\}$. Partial order between them is given by $[\mathbf{A}, \sigma] \subseteq [\mathbf{A}^{'}, \sigma^{'}]$ only if, for all dynamical states φ, $\varphi(< \mathbf{A}, \sigma >) \leq \varphi(< \mathbf{A}^{'}, \sigma^{'} >)$ and the logic operations are (well) defined as: $[\mathbf{A}, \sigma]^{\perp} = [\mathbf{A}, \Re - \sigma]$, $[\mathbf{A}, \sigma] \uplus [\mathbf{A}, \theta] = [\mathbf{A}, \sigma \cup \theta]$ and $[\mathbf{A}, \sigma] \cap [\mathbf{A}, \theta] = [\mathbf{A}, \sigma \cap \theta]$. With all this, $< \mathcal{P}, \subseteq, ^{\perp} >$ is an orthoposet, the orthoposet formed by 'pasting together' a family of Boolean algebras in which whole operations coincide in areas of overlap. It may be enriched to approach the lattice of subspaces of Hilbert space.

One may recognize a modal relation between both kind of propositions. One starts denying the collapse in the measurement process and recognizing that the observable has one of the possible eigenvalues. Then it may be asked what may be inferred with respect to those values when one knows the dynamic state. The answer van Fraassen gives is that, in the case that $\varphi(w)$ is an eigenstate of the observable $\mathbf{A}$ with eigenvalue a, then $\mathbf{A}$ actually does have value a. This means that in this case, the measurement 'reveals' the value the observable already had. He generalizes this idea and postulates that $[\mathbf{A}, \sigma]$ implies $< \mathbf{A}, \sigma >$. With this assumption and the rejection of an ignorance interpretation of the uncertainty principle, he is able to prove that $[\mathbf{A}, \sigma] = \Box < \mathbf{A}, \sigma >$. $\Box$ is defined by $\Box Q = \{w : \text{ for all } w', \text{ if } wRw' \text{ then } w' \in Q\}$, where Q is any proposition and R the relative possibility relation: w' is possible relative to w exactly if, for all Q in $\mathcal{V}$, if w is in Q then w' is in Q. So, $[\mathbf{A}, \sigma]$ may be read as 'necessarily, $< \mathbf{A}, \sigma >$'. This says that the dynamic state assigns 1 to $< \mathbf{A}, \sigma >$ if and only if the value state that accompanies any relatively possible dynamic state makes $< \mathbf{A}, \sigma >$ true. Instead of the transitive possibility relation R, one may use an equivalence relation to define the necessity operator $\tilde{\Box}$. In this case, van Fraassen maintains that the map $[\mathbf{A}, \sigma] \rightarrow < \mathbf{A}, \sigma >$ is an isomorphism of posets $< \mathcal{P}, \subseteq >$ and $< \mathcal{V}, \subseteq >$ and, when orthocomplementation if defined, it becomes an isomorphism between the orthoposets. Thus, the logic of $\mathcal{V}$ is that of $\mathcal{P}$, i.e., quantum logic. Endowed with these tools, van Fraassen gives an interpretation of the probabilities of the measurement outcomes which is in agreement with the Born rule.

7.2.2 Kochen-Dieks Modal Interpretation

The next modal interpretation we would like to review is due to Simon Kochen and Dennis Dieks (K-D, for short), who proposed to use the so called biorthogonal decomposition theorem (also called Schmidt theorem) in order to describe the correlations between the quantum system and the apparatus considering the measurement and the actual observation as a special case of this representation. This possibility to refer to the properties of a quantum system independently of measurement is the realistic move which attempts to consider the quantum formalism —through the Schmidt decomposition— as a physical representation of reality which goes beyond the prediction of measurement outcomes:

Theorem 7.2.1. *Given a state* $|\Psi_{\alpha\beta}\rangle$ *in* $\mathcal{H} = \mathcal{H}_{\alpha} \otimes \mathcal{H}_{\beta}$. *The Schmidt theorem assures there always exist orthonormal bases for* $\mathcal{H}_{\alpha}$ *and* $\mathcal{H}_{\beta}$, $\{|a_i\rangle\}$ *and* $\{|b_j\rangle\}$ *such that* $|\Psi_{\alpha\beta}\rangle$ *can be written as*

$$|\Psi_{\alpha\beta}\rangle = \sum c_j |a_j\rangle \otimes |b_j\rangle.$$

The different values in $\{|c_j|^2\}$ *represent the spectrum of the state. Every* λ_j *represents a projection in* $\mathcal{H}_\alpha$ *and a projection in* $\mathcal{H}_\beta$ *defined as* $P_\alpha(\lambda_j) = \sum |a_j\rangle\langle a_j|$ *and* $P_\beta(\lambda_j) = \sum |b_j\rangle\langle b_j|$*, respectively. Furthermore, if the* $\{|c_j|^2\}$ *are non degenerate, there is a one-to-one correlation between the projections* $P_\alpha = \sum |a_j\rangle\langle a_j|$ *and* $P_\beta = \sum |b_j\rangle\langle b_j|$ *pertaining to subsystems* $\mathcal{H}_\alpha$ *and* $\mathcal{H}_\beta$ *given by each value of the spectrum.* ☐

If we assume non-degeneracy the modal interpretation based on the Schmidt decomposition establishes a one-to-one correlation between the reduced states of system and apparatus.[42] Through this theorem one is able to distinguish, by tracing over the degrees of freedom of the subspace $\mathcal{H}_\alpha$ or the subspace $\mathcal{H}_\beta$, between system and apparatus (for a proof of the theorem see [22], section 2.3). As noted by Kochen ([165], p. 152): “Every interaction gives rise to a unique correlation between certain canonically defined properties of the two interacting systems. These properties form a Boolean algebra and so obey the laws of classical logic.” The biorthogonal decomposition gives in this way a one to one relation between the apparatus and the quantum system and the following interpretation: *The system* α *possibly possesses one of the properties* $\{|a_j\rangle\langle a_j|\}$*, and the actual possessed property* $|a_k\rangle\langle a_k|$ *is determined by the device possessing the reading* $|b_k\rangle\langle b_k|$*.*[43] However, it is important to remark that by tracing over the degrees of freedom of the system, one obtains an *improper mixture*. It is well known that improper mixtures cannot be interpreted in terms of ignorance [79], and thus, one comes back to the problem of interpreting modalities (see also [85]). Following van Fraassen’s distinction between *value states* and *dynamical states*, Dieks solves the problem of putting together the seemingly incompatible character of improper mixtures and ignorance via the distinction between different levels of discourse. For example, with respect to Schrödinger’s cat, Dieks ([89], p. 189) states that: “It is the state vector which is in a superposition, not the cat itself. ‘State vector’ and ‘cat’ are two concepts at different levels of discourse.” In order to explicitly take into account this remark, Vermaas and Dieks distinguish between *physical states* and *mathematical states*. Still, the main problem, as we shall see in the following, remains that of interpreting possibility and its relation to actuality (see 11). According to Dieks:

> “[...] an irreducible statistical theory only speaks about possible outcomes, not about the actual one; this predicts only probability distributions of all outcomes, and says nothing about the result which really will be realized in a single case. In brief, *such a theory is not about what is real and actual but only about what could be the case*.” D. Dieks (Quoted from [89], p. 177, emphasis added)

Kochen (1985, p. 152) and Dieks do not consider the collapse of the state vector as a physical process: “[...] the collapse of the wave function is not a real physical effect in this interpretation but is simply the result of a change in perspective from our witnessing system to another.”

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[42] In the case of degeneracy it is also possible to define new multi-dimensional projections and recompose this one-to-one correlation between subsystems [93].

[43] As Bacciagaluppi [22] points out with respect to Kochen’s interpretation: “It seems that he conceives [the states in the Schmidt decomposition] rather as states that are relative to each other. It seems that he espouses an Everettian view that systems have states only relative to each other but that he considers the ascription of relative states (in Everett sense) only in the symmetrical situation in which not only the $|a_j\rangle$ are relative to the $|a_i\rangle$, but at the same time the $|a_j\rangle$ are relative to the $|a_i\rangle$.”

However, they must give an account of the emergence of a single result, they thus appeal to an interpretational rule in an analogous fashion to van Fraassen's proposal:

> "I now propose the following interpretational rule: as soon as there is a unique decomposition of the form (2), the partial system represented by the $|\phi_k\rangle$, taken by itself, can be described as possessing one of the values of the physical quantity corresponding to the set $|\phi_k\rangle$, with probability $|c_k|^2$. [Eq (2) is $|\Psi_{\alpha\beta}\rangle = \sum c_j|\phi_j\rangle \otimes |R_j\rangle$]
>
> This rule is intended to have the following important consequence. Experimental data that pertain only to the object system, and that say it possesses the property associated with, e.g., $|\phi_1\rangle$, not only count as support for the theoretical description $|\phi_1\rangle|R_1\rangle$ but also as empirical support for the theoretical description (2)." D. Dieks (Quoted from [90], p. 39)

It is important to notice that the seemingly *ad hoc* move of using a preferred basis (such as the Schmidt basis) can be given a physical motivation. It has been proved by Dieks [95] that, given the following two conditions:

1. **One-to-one correlation**: we require a one to one correlation between the definite properties of the system and the definite properties of its environment,
2. **No hidden variables**: the Hilbert space formalism, with the usual representation of physical magnitudes by observables, should be completely respected,

the only basis that accomplishes these two conditions is the Schmidt basis. The first demand appears as obvious when reflecting on the preconditions which allow us to talk about measurement. The second demand can be considered as a commitment to the early interpretation of Bohr, Born, Heisenberg and Pauli; to consider the quantum description as providing all there is to know with respect to atomic events.

It is important to stress at this point that, given the complete system and its corresponding Hilbert space, the choice of what is the system under study and what is the apparatus determines the factorization of the complete space that leads to the set of definite properties given by the Schmidt decomposition. In spite of the fact that this cut is (mathematically) not fixed *a priori* in the formalism, the (physical) choice of the apparatus determines explicitly the context. As we shall see in 8, it is exactly this possibility —of having different incompatible contexts given by the choice of mutually incompatible apparatuses— which in turn determines KS type contradictions within the modal interpretation (For a related discussion see [160], section 6.1).

The modal interpretation is able to ascribe properties to systems in a consistent manner via the interpretational rule. Thus, the problems raised by the projection postulate are solved. Measurements can be characterized as normal physical interactions, there is again the unitary evolution of states and time symmetry is restored; furthermore, superpositions are never reduced to one single term and there is a new connection between the mathematical formalism and results of observation, in such a way that a superposition corresponds to one unique result of observation on the object system. No reference is made to what is found in a measurement. Instead, physical properties are attributed to systems. Although what kind of properties are attributable according to $\sum c_j|a_j\rangle \otimes |b_j\rangle$ depends on a system's interaction, the attribution is wholly objective in the sense that whether or not some conscious being becomes aware of the properties in question is completely irrelevant. It is central to the proposed interpretation that a system always possesses exactly one of the possible values of the physical magnitude singled out by the form $\sum c_j|a_j\rangle\otimes|b_j\rangle$. In particular, in the context of a successfully completed ideal measurement, the interpretation assigns a definite value of the measured quantity to the system. Thus, a system is always in a definite physical state as long as a unique decomposition of the form $\sum c_j|a_j\rangle \otimes |b_j\rangle$ applies. As Dieks point out, it is important to notice that:

> "The argument presented here [the Interpretational Rule] is not that the reduced density matrix for the object system is of the form of a mixture, and that we can therefore for all practical purposes treat the system by itself with a classical mixture description. Such an argument can rightly be criticized for not addressing the conceptual problem of the occurrence, in any given experiment of a definite result. The purpose of our approach is precisely to solve this conceptual problem by denying that there is contradiction between the mathematical description (2) and the occurrence of definite measurement outcomes."
> D. Dieks (Quoted from [89], p. 1407)

According to Dieks, with the proposed interpretational rule, there is an objective physical criterion whether or not a system by itself can be described by a property related to a single term of a superposition. The decisive factor is whether or not the coherence between the various $|a_j\rangle$ is broken through the coupling with other system –whether the relative states $|b_j\rangle$ are mutually orthogonal (this depends on the form of $\mathcal{H}$ and the initial state of the system).

7.2.3 Dieks-Vermaas Generalization to Density Operators

The Dieks-Vermaas (D-V) interpretation generalizes the K-D interpretation for density operators. This means that the property ascription is now generalized to any system α which might be a mixture. This new formulation is based on the well known spectral decomposition theorem which is given in the finite-dimensional case by:

> **Spectral Theorem:** Every normal operator has a set of complete orthonormal eigenvectors, the so called eigenbasis with the corresponding set of eigenvalues, and conversely every basis with eigenvalues determines uniquely a normal operator.

Consider the spectral resolution of the state ρ^α. States are always trace class operators, so the spectral resolution of ρ^α is:

$$\rho^\alpha = \sum w_j P_j^\alpha \tag{7.3}$$

with $w_j \neq w_k$ for $j \neq k$, $P_j^\alpha P_k^\alpha = 0$ for $j \neq k$ and P_j^α denote the possibly multi-dim eigenprojection of ρ^α that corresponds to the eigenvalue w_j. The core projections of α are given by the P_j^α that correspond to the $w_j \neq 0$ of ρ^α. The probability that one of this P_a^α a has the value 1 is:

$$p(P_a^\alpha) = Tr_\alpha(\rho^\alpha P_a^\alpha) \tag{7.4}$$

The most important point of the formulation given by Dieks and Vermaas [268] is that it ascribes properties to ρ^α even if it is not part of a composite that is in a pure state; moreover, it gives correlations between the properties of arbitrary sets of disjoint systems. Take N disjoint systems $\alpha, \beta, ..., \gamma$, the joint probability that the core properties ascriptions to these systems are simultaneously $[P_a^\alpha] = [P_B^\beta] = ... = [P_c^\gamma] = 1...$ is:

$$p(P_a^\alpha, P_B^\beta, P_c^\gamma) = Tr_\omega(\rho^\omega[P_a^\alpha \otimes P_B^\beta \otimes P_c^\gamma]) \tag{7.5}$$

core properties are formulated by means of the spectral resolution. The D-V Property Ascription then ascribes to a system, properties represented by the eigenprojectors of the reduced state:

> **Property Ascription (PA)**: Take a system α with associated Hilbert space H^α. Let the state of α be given at time t by a density operator $\rho^\alpha = \sum c_j P_j^\alpha$ defined in H^α. Then the following holds: System α possesses at time t one of the properties represented by the eigenprojections P_j^α of ρ^α and with probability $p(P_a^\alpha) = Tr_\alpha(\rho^\alpha P_a^\alpha)$ possesses actually the property P_a^α. When $Tr_\alpha(\rho^\alpha P_a^\alpha)$ is defined in the Hilbert space H^α, $p(P_a^\alpha)$ is the usual Born probability.

The D-V formulation is a generalisation of the K-D interpretation. That is, if one accepts the two constraints: (i) to only ascribe properties to systems α which are part of a composite system ω in a pure state, and (ii) to only correlate the properties of two systems α and β if their composite $\alpha\beta$ is in a pure state $|\Psi^{\alpha\beta}\rangle$, then the D-V interpretation becomes equivalent to the K-D interpretation. In the D-V modal interpretation, the density operators always result from tracing out degrees of freedom of larger systems, thus they are improper mixtures. Furthermore, as we have seen above, modal interpretations describe all physical systems quantum mechanically so there is no room for an outside experimenter to mix states who is not included in the description. On the other hand, as pointed out by Dieks and Vermaas:

> "...the interpretation given to density operators is in terms of ignorance: [...] the interpretation gives the probabilities of various possible states of affairs of which it is assumed that only one is actually realized. From the point of view of the standard interpretation this combination of improper mixture and an ignorance interpretation may seem paradoxical.
>
> To resolve the paradox, it is crucial to notice the distinction which is made in the modal interpretation between *physical properties* and *mathematical states*. The latter are defined in Hilbert space and encode probabilistic information about the properties (values of physical magnitudes) which are present in the system itself. *There is no one-to-one relation between physical properties and mathematical states.* Now, the ignorance inherent in the modal interpretation pertains to physical properties; not to mathematical states. It is not assumed that the description by ρ is incomplete and that some other state (namely a pure state) would give a more complete description. The modal interpretation is therefore only in a quite specific sense similar to the standard ignorance interpretation and the density operators are partial traces, and in this sense improper mixtures. They cannot be considered as incomplete mathematical state specifications (in contradiction to proper mixtures in the standard approach). But there still is room for ignorance about the values of physical magnitudes!" P. Vermaas and D. Dieks (Quoted from [268], p. 155, emphasis added)

The D-V interpretation aims at assigning joint probabilities to mutually non-disjoint systems. From this perspective, the modal project would be concluded if one could also determine correlations for non-disjoint systems. This would allow to determine what are the actual pre-existent properties according to the formalism, independently of observation. Anyhow, it was found through several no-go theorems that this possibility is precluded for modal interpretations [21; 264].

As noted by Dieks in a recent paper [98]: "According to the modal interpretation the state in Hilbert space thus is about possibilities, about what may be the case; about modalities. But there is also a second aspect to ψ: it is the theoretical quantity that occurs in the evolution equation, and its evolution governs deterministically how the set of definite valued quantities changes. This double role of ψ, on the one hand probabilistic and on the other dynamical and deterministic, is a well-known feature of the Bohm interpretation." We will now turn our attention to Bub's Bohmian variant in order to discuss further the relation between Bohmian mechanics and the modal interpretation.

7.2.4 Bub's Bohmian Variant

The modal version of J. Bub reminds of D. Bohm's interpretation and proposes to take some observable, $\mathbf{R}$, as *always* possessing a definite value. In this way one can avoid KS contradictions and maintain a consistent discourse about statements which pertain to the sub-lattice determined by the *preferred observable* $\mathbf{R}$. As van Fraassen's and Vermaas and Dieks' interpretations, Bub's proposal distinguishes between *dynamical states* and *property* or *value states*, in his case with the purpose of interpreting the wave function as defining a Kolmogorovian probability measure over a restricted subalgebra of the lattice $\mathcal{L}(\mathcal{H})$ of projection operations (corresponding to yes-no experiments) over the state space. It is this distinction between property states and dynamical states that gives a modal character to the interpretation:

> "The idea behind a 'modal' interpretation of quantum mechanics is that quantum states, unlike classical states, constrain possibilities rather than actualities —which leaves open the question of whether one can introduce property states [...] that attribute values to (some) observables of the theory, or equivalently, truth values to the corresponding propositions." J. Bub (Quoted from [59], p. 173)

In precise terms, as $\mathcal{L}(\mathcal{H})$ does not admit a global family of compatible valuations, and thus not all propositions about the system are determinately true or false, probabilities defined by the (pure) state cannot be interpreted epistemically [59] (p. 119). But, if one chooses, for a given state $|e\rangle$, a "preferred observable" $\mathbf{R}$, these properties can be taken as determinate since the propositions associated with $\mathbf{R}$, i.e., with the projectors in which $\mathbf{R}$ decomposes, generate a Boolean algebra. Bub constructs the maximal sublattices $\mathcal{D}(|e\rangle,\ \mathbf{R}) \subseteq \mathcal{L}(\mathcal{H})$ to which truth values can be assigned via a 2-valued homomorphism and demonstrates a uniqueness theorem that allows the construction of the preferred observable.

In Bub's proposal, a *property state* is a maximal specification of the properties of the system at a particular time, defined by a Boolean homomorphism from the determined sublattice to the Boolean algebra of two elements. On the other hand, a *dynamical state* is an atom of $\mathcal{L}(\mathcal{H})$ that evolves unitarily in time following the Schrödinger equation. So, dynamical states do not coincide with property states. Given a dynamical state represented by the atom $|e\rangle \in \mathcal{L}(\mathcal{H})$, one constructs the sublattice $\mathcal{D}(|e\rangle,\ \mathbf{R})$ with Kolmogorovian probabilities defined over alternative subsets of properties in the sublattice. They are the properties of the system, and the probabilities defined by $|e\rangle$ evolve (via the evolution of $|e\rangle$) in time. If the preferred observable is the identity operator $\mathbf{I}$, the atoms in $\mathcal{D}(|e\rangle,\ \mathbf{I})$ may be pictured as a "fan" of its projectors generated by the "handle" $|e\rangle$ (Quoted from [58], p. 751) or an "umbrella" with state $|e\rangle$ again as the handle and the rays in $(|e\rangle)^{\perp}$ as the spines. When observable $\mathbf{R} \neq \mathbf{I}$, there is a set of handles $\{|e_{r_i}\rangle, i = 1...k\}$ given by the nonzero projections of $|e\rangle$ onto the eigenspaces of $\mathbf{R}$ and the spines represented by all the rays in the orthogonal complement of the subspace generated by the handles. When $dim(\mathcal{H}) > 2$, there are k 2-valued homomorphisms which map each of the handles onto 1 and the remaining atoms onto 0. The determinate sublattice (that changes with the dynamics of the system) is a partial Boolean algebra, i.e., the union of a family of Boolean algebras pasted together in such a way that the maximum and minimum elements of each one, and eventually other elements, are identified and, for every n-uple of pair-wise compatible elements, there exists a Boolean algebra in the family containing the n elements. The possibility of constructing a probability space with respect to which the Born probabilities generated by $|e\rangle$ can be thought as measures over subsets of property states depends on the existence of sufficiently many property states defined as 2-valued homomorphisms over $\mathcal{D}(|e\rangle,\ \mathbf{R})$. This is guaranteed by a uniqueness theorem that characterizes $\mathcal{D}(|e\rangle,\ \mathbf{R})$ (Quoted from [59], p. 126). Thus constructed, the structure avoids KS-type theorems. Then, given a system S and a measuring apparatus M,

> "[...] if some quantity $\mathbf{R}$ of M is designated as always determinate, and M interacts with S via an interaction that sets up a correlation between the values of $\mathbf{R}$ and the values of some quantity $\mathbf{A}$ of S, then $\mathbf{A}$ becomes determinate in the interaction. Moreover, the quantum state can be interpreted as assigning probabilities to the different possible ways in which the set of determinate quantities can have values, where *one particular set of values represents the actual but unknown values of these quantities.*" J. Bub (Quoted from [58], p. 750, emphasis added)

The problem with this interpretation is that, in the case of an isolated system, there is no single element in the formalism of quantum mechanics which allows us to choose an observable, $\mathbf{R}$, rather than other. This is why the move seems flagrantly *ad hoc*. Were we dealing with an apparatus, there would be a preferred observable, namely the pointer position. But the quantum wave function contains in itself mutually *incompatible representations* (choices of apparatuses) each of which provides non-trivial information of the state of affairs.

Our research is focused exactly on the meaning of modality. It is thus interesting to go back to Bohm himself and recover the ideas which allowed him to develop his "causal interpretation". As noted by Bub with respect to Bohmian mechanics:

> "[...] the change in the quantum state $|\psi\rangle$ manifests itself directly at a *modal* level –the level of possibility rather than actuality– through the determinate sublattice defined by $|\psi\rangle$ and position in configuration space as the preferred determinate observable." J. Bub (Quoted from [59], p. 170)

In [46] Bohm assumes a probability density in his interpretation that equals the quantum probability density $|\psi|^2$. In this paper he clearly describes the main distinction between the usual interpretation, provided by Bohr, Born, Heisenberg and Pauli, and his own causal interpretation:

> "The arbitrariness of the usual interpretation in the description of the behavior of an individual system is closely related to the assumption, already stated, that the wave function determines all physically significant properties of the system. Now, in the case of the uranium nucleus, the wave function takes the form of a packet initially entirely within the nucleus, which gradually 'leaks' through the barrier and thereafter rapidly spreads without limit in all directions. Clearly, although this wave function is supposed to describe *all* physically significant properties of the system, it cannot explain the fact that each α-particle is actually detected in a comparatively small region of space and at a fairly well-defined instant of time. *The usual interpretation states that this phenomenon must be simply accepted as an event that somehow manages to occur but in a way that is as a matter of principle forever beyond the possibility of a simultaneous and detailed 'space-time and causal description.'* Indeed, even to ask for such a description is said to be a meaningless question within the framework of the usual interpretation of the quantum theory. *In the causal interpretation, however, the postulated particles with precisely defined positions explain in a natural way why an α-particle can be detected as a fairly definite place and time, on the basis of the simple assumption that the particle existed all the time and just moved from its original location to the place where it was finally found.* Thus, even though we cannot yet observe the precise locations of our postulated particles, they already perform a real function in the theory, namely, to explain certain properties of *individual* systems which are said in the usual interpretation to be just empirically given and forever unexplainable." D. Bohm (Quoted from [46], p. 464, emphasis added)

The usual interpretation follows the second demand given also by Dieks [95], that no deterministic picture provides a more complete description of the state of affairs. Bohm's causal interpretation, on the other hand, wishes to retain this classical feature.

> "[...] *in the usual interpretation two completely different kinds of statistics are needed.* First, there is the ordinary statistical mechanics, which treats of the distortion of systems among the quantum states, resulting from various chaotic factors such as collisions. The need of this type of statistics could in principle be avoided by means of more accurate measurements which would supply more detailed information about the quantum state, but in systems of appreciable complexity, such measurements would be impracticably difficult. Secondly, however, there is the fundamental and irreducible probability distribution, $P(x) = |\psi(x)|^2$ [...]. The need of this type of statistics cannot even in principle be avoided by means of better measurements, nor can it be explained in terms of the effects of random collision processes. [...] On the other hand, *the causal interpretation requires only one kind of probability.* For as we have seen, we can deduce the probability distribution $P(x) = |\psi(x)|^2$ as a consequence of the same random collision processes that give rise to the statistical distributions among the quantum states." D. Bohm (Quoted from [46], p. 465, emphasis added)

These two statistics to which Bohm refers are at the two levels of discourse present in modal interpretations (dynamical and value state). Bohm wishes to recover the classical concept of probability as lack of knowledge; however, this is not achievable in general within the orthodox formulation of quantum mechanics. Possibility in quantum mechanics is a contextual concept: a set of shared possibilities corresponding to disjoint sets of actual properties cannot be non-contextually actualized without contradictions (we will come back to this point in 11). Classical probability can only be recovered as lack of knowledge once a definite context has been chosen or imposed.

7.2.5 Atomic Modal Interpretation

The atomic modal interpretation is due to Guido Bacciagaluppi and Michael Dickson [25]. It intends, via a factorization, to separate the state space of the system $\mathcal{H}$ in disjoint spaces $\mathcal{H}_k$. A factorization Φ of a Hilbert space $\mathcal{H}$ into a tensor product of two Hilbert spaces $\mathcal{H}_1 \otimes \mathcal{H}_2$ is given by an equivalence class of isomorphisms differing only by a basis transformation of the factor spaces onto themselves. It may be proved that there are many different factorizations. The question becomes now whether, by letting Φ vary, the definite properties pertaining to the different factorizations will admit a truth valuation. Bacciagaluppi has proved that this question must be answered negatively because these properties include the set of properties for which Kochen and Specker have shown that it is not allowed an homomorphism to the Boolean algebra **2** [21]. In order to escape this no-go theorem, Bacciagaluppi and Dickson assume that there exists in Nature a special set of disjoint sub-spaces $\mathcal{H}_k$ which are the building blocks of all physical systems; i.e. a *preferred factorization* of the Hilbert space of the whole Universe:

> "[...] we note that the idea of a preferred factorization is not, perhaps, as *ad hoc* as it might first appear. After all assuming that the universe is really made of, say, electrons, quarks, and so on, it makes good sense to take these objects to be 'real' constituents of the universe, i.e. the bearers of properties that do not supervene on the properties of subsystems." G. Bacciagaluppi and M. Dickson (Quoted from [25], p. 3)

The core property ascription to an atom α_k in the atomic modal interpretation is the same as in the Dieks-Vermaas modal interpretation. Thus, the core projections are the eigenprojections $P_j^{\alpha_k}$ of the state of α_k and P^{α_k} has the value 1 with probability

$$p(P_a^{\alpha_k}) = Tr_{\alpha_k}(\rho^{\alpha_k} P_a^{\alpha_k}) \tag{7.6}$$

Moreover, the joint probability that the core property ascriptions to a set of different atoms $\alpha_q, \alpha_r, \alpha_s$... are simultaneously $[P_a^{\alpha_k}] = 1, [P_b^{\alpha_r}] = 1$, etc. is equal to:

$$p(P_a^{\alpha_q}, P_b^{\alpha_r}, P_c^{\alpha_s}) = Tr_{\alpha_k}(\rho_a^{\alpha_q}, \rho_b^{\alpha_r}, \rho_c^{\alpha_s}[P^{\alpha_{qa}} \otimes P^{\alpha_{rb}} \otimes P^{\alpha_{sc}} \otimes ...]) \tag{7.7}$$

Now, the core property ascription for the properties of all possible subsystems of a composite of atoms is quite different from the one in the D-V modal interpretation.[44] The recipe is to assign properties to all atomic systems and postulate that the properties of the different molecules (composed of non-atomic systems) follows from property composition (see [71]).[45]

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[44] Dieks assumes that a Hilbert space can be, in principle, factorized in infinitely many ways

[45] The main idea behind this property ascription is to avoid the so called Kochen-Specker contradiction and obtain an expression for the joint probability for the projectors corresponding to non-disjoint subsystems. We will come back to this problem in the next chapter.

Let us consider a system $\delta = \alpha\beta\gamma$ then P^{α}, P^{β} and P^{γ} will be a set of commutative operators and we can define a joint probability for the set P^{α}, P^{β}, P^{γ}. If we take also into account the projectors corresponding to the non-disjoint subsystems $\alpha\beta$, $\alpha\gamma$ and $\beta\gamma$, they will not define a set of commutative operators, i.e. there will be no expression for the joint probability of the set P^{α}, P^{β}, P^{γ}, $P^{\alpha\beta}$, $P^{\alpha\gamma}$, $P^{\beta\gamma}$, $P^{\alpha\beta\gamma}$. The atomic interpretation is now able to design a joint probability distribution by assigning to each molecule (consisting of different atoms) the properties of the different atoms in the following way:

$$P^{\alpha\beta} = P^{\alpha} \otimes P^{\beta} \tag{7.8}$$

$$P^{\alpha\gamma} = P^{\alpha} \otimes P^{\gamma} \tag{7.9}$$

$$P^{\beta\gamma} = P^{\beta} \otimes P^{\gamma} \tag{7.10}$$

$$P^{\alpha\beta\gamma} = P^{\alpha} \otimes P^{\beta} \otimes P^{\gamma} \tag{7.11}$$

This is also called Property Composition:

> **Property composition**: If we have the system $\delta = \alpha_1,\ \alpha_2,\ \alpha_3, ...,\ \alpha_n$ then the eigenprojector assigned to the molecule $\alpha_i,\ \alpha_j, ...,\ \alpha_k$ will be $P^{ij...k} = P^i \otimes P^j \otimes ... \otimes P^k$ the products of the eigenprojections of the states of the atoms in δ. The core property ascription to δ assigns the value 1 to the projection $P^i_a \otimes P^j_b \otimes ... \otimes P^k_c$ if and only if the core property ascription to the atoms in δ assigns simultaneously the value 1 to $P^i_a,\ P^j_b$, etc.
>
> $$p(P^i_a \otimes P^j_b \otimes ... \otimes P^k_c, P^{\delta}_{a'b'...}) = \delta_{aa'}\delta_{bb'}...Tr_{\delta}(\rho^{\delta} P^{\delta}_{abc...})$$
>
> it follows that the core property ascription to only δ is $[P^{\delta}_{abc...}] = 1$ with probability $p(P^{\delta}_{ab...}) = Tr_{\delta}(\rho^{\delta} P^{\delta}_{abc...})$

In this way the atomic modal interpretation is able to give the correlations between the properties of non-disjoint subsystems, i.e. it correlates all the possible molecules derived from a collection of atoms. However, the main drawback of this interpretation is that it does not take into account the holistic nature of quantum mechanics and the emergence of properties from different possible factorizations, i.e. that it is essential to quantum mechanics that systems can possess properties that do not supervene on the properties of their parts (see [96] for discussion of the possibility of the emergence of properties in a preferred factorization). In this interpretation the structure of the probability assignment becomes classical, i.e. one can define a classical joint probability distribution for any set of chosen properties. As a consequence, probability can be interpreted in terms of ignorance. In the atomic interpretation there is a *single context* given by the preferred factorization and thus, as in classical physics, the KS theorem does not apply. In this interpretation there are no complementary physical experiences, due to the fact that every property is determined, and this is why according to Bacciagaluppi modal interpretations are in the end a theory about actualities.

Bacciagaluppi's account of modal interpretations [22], appears in an analogous fashion to Bub's proposal, closely related to Bohm's causal interpretation.

> "The properties possessed by a system in the modal interpretation are possessed *in addition* to the properties possessed by the system according to quantum mechanics. It is thus natural to call these properties 'hidden variables'. Hidden variables theories do not represent a return to classical, pre-quantum physics. Indeed, the no-go theorems for hidden variables theories show not that hidden variables are impossible, but that they must be in important ways different from classical physics (e.g. they are non-local). On the other hand, hidden variables theories always restore a classical way of thinking about *what there is.* In particular, the logical and probabilistic structure of a hidden variables theory is always classical: there is no 'complementarity' of hidden variables, and probabilities are rigorously Kolmogorovian. [...] The status of probabilities in the modal interpretation, however, has been the subject of some debate, I would presume partially because the original modal interpretation of van Fraassen was not intended as a theory about what there is, but, indeed, as a theory about possibilities. Thus, Dickson (1995) has wondered whether the modal interpretation really is a no-collapse interpretation, and Healey (1995) has expressed reservations about the desirability of introducing a dynamics for the proposed properties. However, for our proposes, I would claim that, despite the name, the modal interpretation in the version of Vermaas and Dieks is a theory about actualities —albeit a stochastic one." G. Bacciagaluppi (Quoted from [22], p. 74)

Bacciagaluppi points out that van Fraassen modal interpretation "is not intended as a theory about what there is", by this he means a theory which represents reality beyond measurement outcomes —although van Fraassen could argue that this is not true for what there is, is actual observations. Bacciagaluppi is right to point out that Vermaas and Dieks modal interpretation is a theory about actuality, but here the meaning of actuality has to be understood in realistic terms (as preexistent and independent of measurement). In the modal interpretation "the complete state is a pair (ρ, P_i), consisting of a quantum state ρ and one eigenprojection P_i of ρ representing the property of the system that generates the set of properties that are actually possessed." Actuality does not refer here to the actual observation but to the representation of existence through the mathematical structure and the idea that this structure can be related to systems which possess properties.

The distinction between dynamical state and value state in van Fraassen's interpretation provides a non-Kolmogorovian feature regarding properties, the 'two statistics' needed in the usual interpretation, and criticized by Bohm. The mode of existence regarding the properties in the dynamical state (or in the mathematical state) is what provides a formal picture which remains non-classical, and which cannot be interpreted in terms of preexistence. While van Fraassen is not interested in advancing towards a realistic account Dieks has provided an interpretation in terms of a system with a definite valued property (a property which is always actual independent of measurement), this is why Bacciagaluppi claims that Dieks interpretation is in the end part of the hidden variable program. Alike Bohm's interpretation Dieks version provides an account of reality in terms of definite valued properties. However, unlike Bohm's interpretation Dieks attempt tries to base its interpretation in the symmetries and features of the orthodox quantum formalism and is against a transformation of the formal structure. The meaning of what is a formalism or, to what extent one departs such formalism through the introduction of interpretational rules, are in themselves a subtle matter of debate. We will come back to these questions through the dissertation.

7.3 A Methodological Map for Modal Interpretations: MIMF vs MIMP

The interpretational map we proposed in chapter 6 can be also used to analyze the very different versions of the modal interpretation. Contrary to many interpretations which possess a clear agenda of what they attempt to provide in metaphysical or anti-metaphysical terms, within modal interpretations the agenda is in itself a matter of debate. Strangely enough, two of the main philosophical stances which have fought through centuries in the philosophical arena seem to find a place within what is considered to be a single interpretation. Although the original idea of van Fraassen was to evade metaphysics and provide an empiricist account of quantum mechanics, later moves, first by Dieks and later on by Bub, Clifton, Bacciagaluppi and Dickson —introducing explicitly the hidden variables program within the modal interpretation—, have turned the agenda of modal interpretations into the realist stance. In this sense, the problems introduced since then have centered their attention in the possibility to consider a physical representation in which it is clear what are the sets of definite valued properties which constitute quantum systems; i.e. which are the preexistent elements of the formalism through which one can refer to a representation beyond observation. Also the question of dynamics has been introduced in these new versions, for a realistic interpretation of quantum mechanics which attempts to describe *what there is* needs to provide a representation through time. It is through this metaphysical move that the realist can exceed the description of the measurement process and actual observations *hic et nunc* (see discussion in chapter 3). Empiricism, on the contrary, is not forced to provide a representational scheme, and though a metaphysical interpretation might be of use, an empiricist —as explicitly stated by van Fraassen— is only worried about actual observations.

Following our interpretational map we can consider a first group of modal interpretations which attempts to begin “right from the start” from the orthodox mathematical formalism and find a consistent interpretation through the development of a metaphysical scheme, we shall call these versions: *Modal Interpretations (which start from) the Mathematical Formalism* (MIMF). Van Fraassen’s Copenhagen variant can be seen as part of this original attempt followed later on by Dieks. It becomes clear that MIMF might, or might not be, interpreted following a representational realistic scheme. Following van Fraassen’s empiricist consideration, that outside of measurement contexts “anything is possible” ([255], p. 294), there is no need of considering a dynamical evolution —or explaining the path from the *dynamic state* to the *value state*. Van Fraassen also remains within the orthodox position that the state of the system is all there is to know and there is no need of hidden variables to be added. On the other hand, the version of Dieks attempts to provide a realistic interpretation of quantum mechanics starting from the symmetries encountered within the formalism. Using the Schmidt basis he is able to account for the definite valued properties within a measurement context. Dieks proposal can be thus considered as part of MIMF. Dieks wishes to interpret quantum mechanics respecting the orthodox formalism, the symmetries and features of the formal structure, without adding *ad hoc* rules. In this sense, although there is a clear classical metaphysical scheme involved of ‘systems which possess sets of definite valued properties’, Dieks remains —like van Fraassen— very open to any metaphysical development which can do the job of providing a story about what quantum mechanics is telling us about the world.[46]

Another set of versions begin their analysis with much more solid metaphysical conditions than Dieks and Van Fraassen's proposals. We shall call these versions, *Modal Interpretations (which start from) Metaphysical Principles* (MIMP). These approaches, based on the hidden variable program, attempt to provide a classical metaphysically tenable characterization of what is going on according to quantum mechanics. Although these conditions might vary from one version to the other, the will to retain a classical metaphysical picture is always present. Bub's Bohmian version and the atomic proposal of Bacciagaluppi and Dickson can be accounted for what we call MIMP. In the case of MIMP there can be —as in the case of Bohm— explicit formal deviations from the orthodox formulation of quantum mechanics. Bacciagaluppi and Dickson regard modal interpretations as referring to actual objective properties and this is why they look for a dynamical picture that governs the evolution of these properties. Developing a stochastic scheme they attempt to provide for both, the empirical adequacy of the orthodox formulation, and an explanation of the path from the possible to the actual [25]. These attempts might be considered to constitute not only an interpretation of the formal structure of quantum mechanics, but a new theory in itself. This important difference in the account provided by the versions of modal interpretations draws a clear distinction which was characterized by Laura Ruetsche [231] in terms of *Modal Interpretations with Semantic Probabilities* (MISP) and *Modal Interpretations with Hidden Variables* (MIHV). Ruetsche takes a stance and argues for MIHV by stating that: "I urge that we adopt a principle of leeway according to which the interpretation of quantum mechanics needn't be a purely semantic project. This principle frees interpretations from the obligation to adjust their semantics to the state space of quantum mechanics innocently construed; they may fiddle with that state space, or unitary dynamics, or both." Contrary to this position, there are many who claim that changing the formalism is not part of "interpreting" a theory. As noticed by Healey ([138], p. 24): "[...] I cannot accept a hidden-variable theory as an interpretation of quantum mechanics. A hidden-variable theory is, fundamentally, a separate and distinct theory from quantum mechanics. To offer such a theory is not to present an interpretation of quantum mechanics, but to change the subject."

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[46] Even though Dieks has started with a quite classically based metaphysical interpretation of systems with properties, he is not unwilling to change his classical metaphysical presumptions into less orthodox ones, as in the case of his relational version of the modal interpretation [35; 99]. Dieks ([91], p. 1416) argues that: "The proposal is to conceive the mathematical structure of quantum theory as a representation of the physical structure of the world [...] there is an additional source of meaning from the relation with experience. A number of substructures of the total structure are identified with isomorphous structures constructed from empirical data. As a result the structure makes contact with the world [...]"

Actual (Preexistent) Properties within the Modal Interpretations

As we have argued above we are interested in discussing modal interpretations from a realistic stance which considers formal and conceptual representation as a precondition to discuss about physical phenomena. From this perspective, realism considers existence beyond observation and through representation. A basic idea of a physical theory is that it is able to predict phenomena, this can be only done through a representation which allows to consider some existent element through time. This means there are certain elements, within the formalism, which are thought to describe the world as identities which exist within the representation through time. This existence related to representation —presupposed independently of observation— is what we have called *preexistence* (see for discussion sections 3.4 and 3.5). For the realist, the observation simply exposes something already existent —which we call preexistent— before and after measurement. And so, when we talk about actual properties in classical physics we refer to the properties which the representation describes as existent —with a definite value independent of observation. Furthermore, in classical theories the possible or probable are intrinsically defined in terms of preexistent actuality. The presupposition is that there is an actual state of affairs which will evolve into another actual state of affairs. Independently of knowing or not what the actual situation *is*, in classical physics we assume —as an ontological presupposition— *there is* a well defined preexistent state of affairs which can be described according to the theory (see quotation on page 60). In quantum mechanics if we are to understand modality, we also need to understand the limits of what can be considered preexistent actuality and physical representation.

Within the different versions of the modal interpretation there is a double meaning of the word ‘actual’. In the first place, from an empiricist perspective, as in the Copenhagen variant put forward by van Fraassen, the term ‘actual’ refers to an observation *hic et nunc*. In this case the stress is placed on the measurement results and the empirical structure. In the second place, from a classical realistic perspective such as that put forward by the hidden variable program of Bub, Bacciagaluppi and Dickson, the term ‘actual’ is meant to refer to the *preexistence* —independent of observation— of the properties which make reference to a physical description of the world. As we shall see in the following chapter, realist modal interpretations which attempt to discuss about preexistent properties face many difficulties when related to the formal structure and the contextual character of quantum mechanics. Empiricist interpretations evade this problem for the focus is not placed in a metaphysical interpretation of the formalism but rather on the particular observation of measurement outcomes.

8.1 Bacciagaluppi's Kochen-Specker Type Theorem for Modal Interpretations

A complete set of properties of a system that may be simultaneously predicated allows to construct a Boolean propositional system. Assigning values (a set of their corresponding eigenvalues) to these magnitudes in a contextual manner (i.e. choosing a particular context of inquiry) poses no difficulties. But if we try to interpret eigenvalues as the actual (preexistent) values of the physical properties of a system, we are faced to all kind of no-go theorems that preclude this possibility. Regarding the specific scheme of the proposed K-D modal interpretation, Bacciagaluppi and Clifton were able to derive a KS type contradiction which showed that one cannot extend the set of definite valued properties to non-disjoint sub-systems. In the K-D interpretation one considers arbitrary factorizations as defining systems to which one can ascribe definite valued properties. As we have showed in 7.2.2, K-D are able to ascribe properties to every quantum mechanical system, that is, to any subsystem appearing in any possible factorization of the Hilbert space of the Universe. As noted by Vermaas [266], the KS theorem by Bacciagaluppi [21] is a constraint on any explicit rule correlating properties of subsystems belonging to different factorizations. In order to derive a contradiction Bacciagaluppi takes a composite ω defined on a 9-dimensional Hilbert space and considers a number of factorizations $\omega = \alpha_i\beta_i, \ i = 1, 2, ...$ of ω in subsystems $\{\alpha_i\}$ and $\{\beta_i\}$ defined on three dimensional Hilbert spaces. Then he considers the core properties $\{P_j^{\alpha_i}\}$ and $\{P_j^{\beta_i}\}$ ascribed to these subsystems by the K-D modal interpretation. Next, he ascribes these properties via *property composition* to ω. Bacciagaluppi is able to prove that all the properties ascribed to ω include the set of properties for which the KS theorem shows that it does not allow an homomorphism to the Boolean algebra of $\{0, 1\}$.

It is important to notice that in the K-D interpretation the *choice of the factorization* plays exactly the same role as the *choice of the basis* in the orthodox interpretation of quantum mechanics (we will come back to this point later in section 8.3). In this case, in order to avoid the determination of that which exists through *choice*, in order to recover a *detached observer*, there would have to be an objective way to arrive to a *preferred factorization*; this is exactly the idea of the atomic interpretation which begins "right from the start" with such preferred "cut" of the complete Hilbert space of the Universe (see section 7.2.5). Another possibility, proposed by Dieks, Bacciagaluppi, Hemmo and Bub, is to take into account the *principle of decoherence* in order to provide such choice [93; 26; 23; 59; 60]. Both proposals face serious difficulties.

8.2 Clifton's Lesson for Modal Interpretations

In the case of the atomic modal interpretation the theorem does not apply because one denies from the start the possibility of choosing an alternative factorization for the system, i.e. to assume that one can freely factorize any given system into pairs of subsystems. However, Clifton [73] has also proven that, even in the case one has one definite factorization, one does not have a proper property ascription. Clifton considers a composite system ω which can be factorized into only two subsystems α and β. By ascribing properties with the K-D modal interpretation to both subsystems α and β and the complete system ω and by employing *property composition*, he derived that ω possesses a set of properties for which a Boolean valuation is not allowed.[47] The work of Rob Clifton focuses on a very interesting lesson which should be learned about modal interpretations. That is, modal interpretation cannot reason classically about actual (preexistent) properties. Regarding his own theorem, Clifton states that:

> "When the quantum state of a system is an eigenstate of **P**, the certainty that a measurement of **P** will yield a positive (or negative) outcome should not generally be grounded in the fact that the proposition corresponding to **P** picks out a categorical property possessed (or lacked) by the system.
>
> Bohm's and Bub's modal interpretations abide by this moral, since they take all a system's measurement dispositions to be reducible to the premeasurement value of one particular preferred observable regardless of which observable is 'measured'. But the above moral clashes with all the other modal interpretations I have discussed that are saddled with metaphysically indefensible violations of property intersection and composition. So if my moral is taken on board, they will have to be reworked." R. Clifton (Quoted from [73], p. 397)

We agree that if modal interpretations are to be taken as the constraints to the consistent discourse regarding the quantum formalism and actual properties (as preexistent), this lesson must be taken into account. However, it is not necessarily the case that modal interpretations should be restricted to the classical metaphysical scheme presupposed by Clifton. As we shall discuss in the following chapter, modal interpretations do not seem to be necessarily committed to these metaphysical desiderata.

8.3 Perspectives and the Preferred Factorization Problem

The so called 'preferred basis problem' exposes the fact that within quantum theory there is no clear formal nor conceptual justification of the path from the multiple set of possible basis to the one particular basis which gets actualized. The mutual incompatibility between the different basis, together with the impossibility to escape the choice (of a particular basis) determines the loss of the *detachness of the description*, and thus, the impossibility to account for the multiple representations as describing a singular physical reality. In order to "solve" this problem there are different approaches. The most obvious might be that of Bohm who selects a preferred observable and thus a *preferred basis* "right from the start". Bohm attempts to recover a classical description of quantum mechanics by presupposing that the only physical basis is that given by the observable 'position'. This move can be of course criticized for being *ad hoc* and for destroying the symmetries in the formal structure of quantum theory, apart from not being even able to recover the space-time basic geometrical structure needed to discuss about 'position' in the first place. A more subtle position is taken by Dieks who attempts to ground this choice digging in the structure of quantum theory itself, analyzing

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[47] For a more extensive discussion, see [27].

the symmetries and invariance of the formalism. In this case Dieks proposes the Schmidt basis through quite convincing physical arguments regarding the meaning of 'measurement' itself (see section 7.2.2). However, in order to have such 'preferred basis' one first needs to "cut" the whole system into two subsystems. This 'cut' represents the distinction between the measurement apparatus (plus environment) and the system. (For a related discussion on what is to be considered the Heisenberg's cut see [208].) In such case, *the choice of the basis* is translated into the *choice of the cut*; i.e. *the choice of the factorization* of the initial Hilbert space which represents the whole system into the subsystems $\mathcal{H} = \mathcal{H}_\alpha \otimes \mathcal{H}_\beta$.

Although we might agree that K-D modal interpretation is able to provide a physical motivation for the particular choice of the Schmidt basis evading the so called "preferred basis problem" we argue that any modal realistic version which attempts to describe systems with definite properties relying on the biorthogonal decomposition faces an analogous interpretational problem which we will call *preferred factorization problem.*

This problem is explicitly noticed in the introduction of Bacciagaluppi's 1995 paper:

> "[...] the versions of Healey and Dieks diverge at the point of deciding *which* factorizations are allowed as defining subsystems to which properties are ascribed. Healey argues that certain factorizations are *physically preferred*, and ascribes properties *only* to the systems defined by those factorizations. Dieks considers instead *arbitrary* factorizations as defining systems that are ascribed properties. Thus Dieks ascribes properties to *every* quantum mechanical system, that is, to any subsystem appearing in any possible factorization of the Hilbert space of the universe. However, he has not yet formulated an explicit rule for correlations between properties of subsystems belonging to different factorizations." G. Bacciagaluppi (Quoted from [21], p. 1209).

Bacciagaluppi's KS type theorem is in fact a constraint on any such rule for correlations between properties associated with different factorizations. In ([21], p. 1215) he concludes that: "Dieks has to introduce a new kind of metaphysics, one in which the notion of a quantum mechanics system becomes primary, and one in which properties of systems belonging to different factorizations seem not to be correlated." The question then becomes: are modal interpretations attempting to describe a state of affairs in terms of sets of preexistent properties? This might seem to be the case for Dieks early realist proposal of the modal interpretation [89; 90] in which the programme was to find a joint probability distribution of properties for non-disjoint subsystems. However, after the no-go theorems of Bacciagaluppi, Clifton and Vermaas [21; 71; 264] it was clear that this attempt had to be reformulated. The relational proposal of Bene and Dieks attempts to overcome the difficulties and at the same time still describe an objective account of physical reality. The motivation and aim of the new perspectival version goes in line with the earlier proposals of Dieks.

> "The motivation for introducing states that correspond to physical properties is the wish to give descriptions of systems, and thus to transcend the traditional interpretational framework in which systems are only discussed in terms of possible measurement results. [...] We want to treat measurements as ordinary physical interactions, and measurement outcomes as properties of measuring devices or displays, and thus to remove any mysterious aspects of the concept of quantum measurement." G. Bene and D. Dieks (Quoted from [35], p. 1).

As we shall discuss in more detail in chapter 12, if *relations* or *correlations* are to be conceived in terms preexistent of elements of physical reality, they will also fall prey to contextuality; i.e. the impossibility to account for a preexistent set of properties independently of a choice between mutually incompatible quantum representations. In order to overcome this problem Bene and Dieks preclude the possibility to refer to non-disjoint subsystems.

> "We do not define joint probabilities if the systems are not pair-wise disjoint. This is in accordance with the no-go theorem by Vermaas. More generally, joint probabilities cannot always be defined within the present approach because states that are defined with respect to different quantum reference systems need not be commensurable." G. Bene and D. Dieks (Quoted from [35], p. 2).

The problem is the justification of this interpretational maneuver. Avoiding to refer to non-compatible observables succeeds in escaping the no-go theorems, but this can be also done in a non-relational scheme where we simply deny the possibility to refer to properties in a non-contextual manner. Gyula Bene has studied different contextual interpretations and claims: "...one expects that existing things, even if they are defined with respect to different reference systems, must somehow be comparable. This expectation is actually based on classical experience, and its failure does not violate any well founded physical principle..." ([34], p. 1) And continues: "One cannot think of reality as a big book where all the states of any systems with respect to any quantum reference systems are carefully registered. Such a registration would readily imply that the simultaneous existence of any state can always be checked, i.e. any states are comparable. Precisely this is impossible." ([34], p. 1) States of different systems that are defined with respect to different quantum reference systems cannot be compared, not even in principle.[48] The problem is, as very clearly noticed by Stefano Osnaghi:

> "Van Fraassen's main concern with the Copenhagen interpretation is that "the appearance of the term 'measurement' in the Born Rule bears its anthropocentric connotations essentially", and seems to imply that "we cannot think of quantum theory as a putative autonomous description of the world in neutral physical terms." (Van Fraassen 1991a, p. 284) Indeed, even on the minimal hypothesis that the theory only deals with measurement outcomes, one must assume that there are definite outcomes. This seems to require that certain macroscopic observables have values (exactly those values that correspond to the "pointer states" of a measuring device). The problem is that, in the Copenhagen interpretation, the possibility of attributing a value to a quantum observable is in general contextual, i.e. it depends on the set of experiments that one considers. Hence, in order to fulfil the above requirement, one must conjecture that certain macroscopic observables always appear to us as if they had a value, which, in turn, amounts to assuming that the observers who look at the readings of the measuring instruments are bound to choose certain "observation contexts" and not others. Unless one manages to describe the observers as part of a "closed system inside of which the contextual selections are determined by purely physical factors" (van Fraassen 1991b, p. 499), the consistency of the Copenhagen interpretation seems therefore to rest on a formal partition of the physical world into a portion obeying quantum mechanics and a portion, including the measuring instruments,

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[48] This idea goes in line to the proposal of Rovelli [230] which we shall analyze in chapter 12.

> in which the phenomena fit de jure the conceptual framework of human beings. Moreover, the "reduction" of the state vector seems to be necessary in order to provide a coherent link between these two kinds of description." S. Osnaghi (Quoted from [191], p. 165)

One can develop no-go theorems which explicitly show that different factorizations of the original system give rise to mutually incompatible values of possible sets of properties. This inconsistency or 'factorization-dependent determination of the values of properties' precludes the possibility of recovering a detached account of the representation of physical reality. One could argue following Bacciagaluppi and Dickson that "the idea of a preferred factorization is not, perhaps, as *ad hoc* as it might first appear", however, such move also runs into problems, the most important one being the fact that a fixed factorization destroys the structure itself of quantum mechanics and it is not at all clear how such proposal would recover the quantum behavior nor the emergent properties (see for discussion [96]). We could argue instead that the perspectival version has a way to get to such preferred factorization through some general principle such as decoherence [220]. But it seems doubtful that this principle, which has in itself already many problems, can give a consistent answer (see for discussion [26; 114]). Furthermore, the level on which the multiple factorizations co-exist remains unexplained and without a metaphysical fundament. This question might seem uninteresting from an empiricist perspective but must be answered from a realist one. A question arising from a realist stance is thus, what would be the metaphysical scheme which supports a relational account of physical reality? If we understand that metaphysics and physics are related, the problem of factorization exposes, due to the contextual character of quantum mechanics and in an analogous way to the preferred basis problem, a deep difficulty to maintain our classical notion of physical reality.

Metaphysically Tenable Interpretations of Quantum Mechanics?

The number of interpretations of quantum mechanics has kept growing in the last decades with many different attempts to "make sense" of the theory. Within many interpretations the questions related to the possibility of providing a "metaphysically tenable interpretation" have been discussed. As we have argued (see chapter 6), in the realm of the quantum we can summarize this discussion in two main groups of interpretations, those which begin "right from the start" from metaphysical presuppositions and those which tend to begin with the analysis of the formalism. Also, within the modal interpretation, this question has been addressed on many occasions, and while MIMP take metaphysical presuppositions as the very starting point of departure of their analysis, MIMF present a much more agnostic position regarding metaphysical principles. In the following we shall discuss and analyze more deeply some of these positions.

9.1 Van Fraassen's Empiricism

As we mentioned earlier, Van Fraassen position remains close to the tradition inaugurated by Niels Bohr and his interpretation of quantum mechanics. Van Fraassen wishes to remain close to what he characterizes as the Copenhagen agnosticism. The relation of van Fraassen's interpretation to the orthodox view can be seen as a consequence of maintaining a "conservative" position regarding the values of definite properties:

> "The interpretational question facing us is exactly: in general, which value attributions are true? The response to this question can be very conservative or very liberal. Both court later puzzles. I take it that the Copenhagen interpretation —really, a roughly correlated set of attitudes expressed by members of the Copenhagen school, and not a precise interpretation— introduced great conservatism in this respect. Copenhagen scientists appeared to doubt or deny that observables even have values, unless their state forces to say so. I shall accordingly refer to the following very cautious answer as the *Copenhagen variant* of the modal interpretation. It is the variant I prefer." B. van Fraassen (Quoted from [255], p. 280)

Van Frassen's position —as he himself explicitly notices— is close in many aspects to Bohr's interpretation and anti-metaphysical commitment. In particular, the agnosticism of properties proposed by him should be considered as an attempt to remain outside any type of metaphysical constraint. However, contrary to Bohr, van Fraassen respects the possibility to interpret the formalism of quantum mechanics in metaphysical terms. The justification of van Fraassen's agnosticism is based on his pragmatic and empirical account of physics. But while for Bohr the definition of phenomena relies explicitly on the language and concepts used in classical physics, and this, implicitly —as we have pointed out in section 4.4— places him within the linguistic turn, van Fraassen expresses a radical aversion to the linguistic movement.[49] Also, contrary to Bohr, who wanted to close the doors of any possible future conceptual development of the interpretation of quantum mechanics, van Fraassen is open to any metaphysical interpretation which would provide an answer to the (metaphysical) question, what would be the world like if quantum mechanics were to be true.[50] It is remarkable in this sense that, even though he was the main creator of the modal interpretation —and even called the first version "Copenhagen variant" in clear allusion to Bohr's trend of thought— the following versions —especially those of Bub and Bacciagaluppi and Dickson— reverted his own scheme and shifted the attention to the hidden variable program.

9.2 Dieks: From Empiricism to Realism

Dieks' interpretation of quantum mechanics was the first attempt to go beyond the empiricist account of van Fraassen and turn the modal proposal towards the realistic agenda. Dieks introduced the idea of discussing, within the quantum realm, of "systems which possess properties", making explicit at the same time the need to remain within the formal scheme of orthodox quantum mechanics. "The ascribed properties are thus not fixed by something which is not part of the quantum formalism —they are not put in 'by hand', for instance." ([266], p. 43) Contrary to van Fraassen, and in accordance to the ideal of the realist of describing the world through physics, Dieks does attempt to construct a dynamical picture of the definite valued properties (see [263]). Dieks remains empiricist regarding the interpretation of possibility but realist regarding the existence of systems and properties, as that of which quantum mechanics is talking about [89; 90; 91]. It is important to notice that Dieks's attempt, contrary to Bub's position and the atomic interpretation, relies on the formalism itself, its structure and symmetries. Also, there is an attempt to describe reality in terms of the orthodox formalism. This methodological attitude is an essential character of Dieks interpretation which has even investigated a different metaphysical scheme in his relational version of the modal interpretation (see [35] and [220; 99] for discussion). The approach of Dieks seems suitable for going a new way, which allows to interpret the formalism of quantum mechanics in terms of an objective account of physical reality and make sense of empirical data at the same time. It is also important to remark the fact that Dieks takes the modal interpretation to be only

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[49] Van Fraassen ([255], p. 6) states with respect to the linguistic turn: "Despite certain undoubted successes, the linguistic turn in analytic philosophy was eventually a burden to philosophy of science." Against the linguistic turn van Fraassen ([255], p. 7) takes the path of the semantic approach: "This new tradition is called the 'semantic approach' or 'semantic view'. It has been developed, since the mid-1960s, by a number of writers, some scientific realist (like Giere) and some not. In this approach the role of language (and especially syntax or questions of axiomatization) is resolutely de-emphasized."

[50] In this respect one can see the very interesting discussion of van Fraassen with respect to Rovelli's relational attempt [259]. See also [258] for an analysis of the relation between van Fraassen philosophical stance and transcendental approaches.

one of the many possible valid interpretations of quantum mechanics. Within this kind of pluralistic pragmatical stance, according to Dieks, each of these valid interpretations can be used for definite purposes depending on the specific situation analyzed.[51] Depending on the particular situation, either modal interpretations or Bohmian mechanics, consistent histories, many worlds or GRW might be suitted to solve problems. It is not claimed by Dieks that the modal interpretation is the true interpretation of quantum mechanics.

9.3 Clifton's and Vermaas' (Classical) Metaphysical Desiderata

Contrary to van Fraassen and Dieks, Rob Clifton is one of the clearest proponents of taking into account "right from the start" metaphysical considerations when discussing the possible interpretation of the quantum formalism. According to Clifton, modal interpretations "aim to tell a systematic story about what the categorical properties of quantum systems are that is not built upon the eigenstate eigenvalue link." Clifton ([73], p. 382) states that modal interpretations are based on a series of desiderata, the first of which is that: "The set of categorical property ascriptions to systems in any given quantum mechanical situation at any given time should be metaphysically tenable." Clifton makes explicit this characterization:

1. The set of categorical property ascriptions to systems in any given quantum mechanical situation at any given time should be metaphysically tenable.
2. It should be possible for the probabilities dictated by the quantum formalism for measurement results to be recovered as measures over the different possible property ascriptions applicable in the special case of measurement interactions.
3. It should be possible to give a sensible deterministic or stochastic dynamics for the evolution of properties and their probabilities over time that is consistent with the Schrödinger evolution of quantum states.
4. Property ascriptions to macroscopic objects should be sufficient to recover our everyday perceptions of those objects.
5. It should be possible to achieve all of the above without necessarily breaking Lorentz invariance.

The conditions imposed by Clifton advance then by characterizing the relation between quantum properties: "as part of satisfying desideratum (1), property intersection should be imposed on modal interpretations [...]" ([73], p. 382). If we recall the idea that modal interpretations must remain close to the standard formalism, this seems an impossible mission to accomplish. It is far from obvious how to recover classical properties and at the same time maintain the orthodox quantum formalism. It is remarkable, if we recall van Fraassen's original ideas, to see the shift proposed by Clifton regarding the characterization itself of what was to be named a "modal" interpretation. These desiderata go completely against the ideas

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[51] Private discussion, Utrecht 2010.

and spirit of van Fraassen's original proposal.

A much more moderate position is endorsed by Pieter Vermaas. Although Vermaas presents a set of metaphysically tenable conditions for developing modal interpretations, he remains at the same time very cautious of imposing conditions on unobservable states of affairs. Vermaas characterizes the realist attitude in the following manner:

> "[If] one adopts a [...] realist attitude towards quantum mechanics and assumes that it is a theory about electrons, protons, etc., which exist independently of us and independently of the performance of measurements, then the standard formulation can only be a beginning. In the realist conception, a true physical theory about elementary particles, aims at (literally) describing the properties of those particles as they exist out there." P. Vermaas (Quoted from [266], p. 16)

And continues his analysis by stating that, if one accepts the need of an interpretation for quantum mechanics, the question arises of what such an interpretation should provide. In particular, according to Vermaas, the following demands should be accomplished:

1. The interpretation should give a description of reality in which things like positions, spin and energy are normal physical magnitudes which pertain to systems and which exist independently of the notion of observation or measurement. An interpretation should ascribe properties to systems, meaning, that it should yield a fully flagged theory of these properties.

2. The description of reality given by an interpretation should be empirically adequate, meaning, that the interpretation should reproduce the predictions of the standard formulation of quantum mechanics with regard to the outcomes of measurements.

3. The interpretation should give a metaphysically tenable description of the magnitudes and properties of systems.

Vermaas characterizes his analysis regarding metaphysically tenable descriptions in the following manner:

> "The third demand that a modal interpretation should yield a metaphysically tenable description of reality surpasses the first two demands because a fully developed and empirically adequate description of reality *can still give a totally weird and unacceptable description of the properties of non-observed quantum systems.* [...] [B]ecause modal interpretations describe states of affairs which are in principle unobservable, one should be careful about discarding modal descriptions of reality as metaphysically untenable. And it seems to me that it is incorrect to impose intuitions about descriptions of what is observable on descriptions of what is, in principle, unobservable. The criteria I propose for metaphysical tenability are thus very sparse:
>
> *Consistency:* The description of reality should be free of contradiction.
>
> *Internal completeness:* The description of reality by an interpretation should be complete with regard to the standards set by that interpretation: that is, an interpretation should deliver the description that it promises to deliver." P. Vermaas (Quoted from [266], p. 34, emphasis added)

Regarding the first point proposed by Vermaas, it is interesting to note that *demand 1* might be regarded already as a metaphysical demand. Vermaas states that modal interpretations should "give a description of reality in which things like positions, spin and energy are normal physical magnitudes which pertain to systems and which exist independently of the notion of observation or measurement." The question we pose is why should we have necessarily such a classical conceptual structure surrounding quantum mechanics? It might be the case that quantum mechanics could be interpreted in terms of properties and systems, and that the properties behave as in the case of classical physics, but, isn't classical physics based already in a metaphysical construction? So what is the justification for asking the theory to accomplish *demands*? Isn't this already part of a metaphysical stance?

9.4 Metaphysical Constructivism: Beyond the Representational Realm of Actuality?

According to Dieks:

> "[...] there is no visualizable model encompassing the whole structure [of quantum theory], the demand that there should be a visualizable model would be tantamount to demand that classical physics should determine the conceptual tools of new theories. This would deny the possibility of really new fundamental theories, conceptually independent of classical physics" D. Dieks (Quoted from [91], p. 1417)

In line with this remark, from our constructive metaphysical approach, our aim is to seek for a conceptual representational scheme of the formal structure of quantum mechanics which allows to provide a metaphysical account of the world according to the theory (metaphysical stance). However, we do not regard classical physics to be the only possible metaphysical scheme (constructive stance). Due to the fact the formalism of quantum mechanics is able to provide outstanding empirically adequate results (empirical adequacy) and that this determines the success of a theory —and not its commitment to a certain presupposed conception of the world—, it seems to us that the problem is not to find a new formalism. What we need to provide is a clear understanding of all concepts with the quantum realm (coherency of the interpretation). As we mentioned above, within our constructive metaphysical stance, modal interpretations could be understood as providing a twofold development. A critical analysis which exposes the limits of classical language with respect to the quantum formalism and a constructive analysis which attempts to understand the meaning of the conceptual structure which can provide an understanding of quantum mechanics. From this account, the problem regarding the interpretation of quantum mechanics relates directly to the creation of an adequate metaphysical scheme.

As we have argued above, regarding the representation of quantum theory, one of the main concepts at stake is that of possibility. What is the meaning of quantum possibility? And what is its relation to both, the actual observation, on the one hand, and the actuality present within representation (preexistence) on the other? In the following we shall continue our investigation in formal and metaphysical terms. Another question which arises is if it would be possible to consider, within a representational realist physical theory, a realm of existence independent of preexistent actuality? We will come back to this question later in the final part of the dissertation.

IV

CONTEXTUALITY

Incompatible Mathematical Representations in the Quantum Formalism

The notion of quantum contextuality is a concept completely foreign to classical theories: it refers on the one hand to the role played by the measurement context in quantum experiments and on the other hand to the non-commutative formal structure of the theory. What is exactly the meaning of 'contextuality' is a subject of deep and subtle investigation in itself. Its analysis goes back to the debate between Einstein and Bohr about the double-slit *gedankenexperiment* in the famous Solvay conference in Brussels. These discussions continued with renovated strength in the so called "EPR paper" [113] and the immediate answer of Bohr [49] and Schrödinger [234]. In the following, we attempt to discuss the notion of contextuality in relation to our representational realist stance. We attempt to show that the problem of contextuality is intrinsically linked to that of physical representation. We will also address the relation between contextuality and the classical and constructive metaphysical stances.

10.1 Contextuality and Representation

As we argued before, we take the realist conception of physics as providing a description or expression of reality. Within representation, the idea that a *preexistent* —independent of observation— set of definite properties constitute reality is one of the basic ideas which remains the fundament of all classical physical theories and determines the possibility to discuss about an independent objective reality. A reality which does not depend on our choices or consciousness, a reality which can be conceived and analyzed in terms of a theory independently of actual observation. This description of reality has problems in quantum mechanics. The structure of the theory posses limits on the interpretation of properties as preexistent. In formal terms, consider three observables A, B and C. Quantum contextuality exposes the fact that the value of A depends on the *choice* of the context of inquiry; i.e. if A is measured together with B or together with C (see [195], chapter 7). This formal fact of the theory has been related on many occasions to a seemingly subjective aspect.[52]

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[52] For example Karl Popper ([206], p. 7) wrote when presenting his well known objective propensity interpretation: "This is an attempt to exorcize the ghost called 'consciousness' or 'the observer' from quantum mechanics, and to show that quantum mechanics is as 'objective' a theory as, say, classical statistical mechanics... The opposite view, usually called the Copenhagen interpretation of quantum mechanics, is almost universally accepted. In brief it says that 'objective reality has evaporated', and that quantum mechanics does not represent particles, but rather our knowledge, our observations, or our consciousness of particles."

Certainly, contextuality determines a necessary reconsideration of the meaning of objectivity and physical reality in quantum mechanics. However, as we argued in section 3.5, according to our stance, it must be clear that this problem does not relate to the level of actual observation, but rather, to the fundamental possibility given within physical representation of providing meaningful counterfactual statements.

From a classical realist stance we could argue, following Bohr, that 'contexts', considered as classical measurement situations, exist. This position seems to take us down the path of Bohr, who interpreted the formalism as an algorithmic structure, or down the path of Bohm, who was forced to change the formalism in order to recover a description in terms of well defined properties and systems. It is this statement which we do not want to take for granted. From our representational realist stance an important point is not to confuse the formal representation given by the mathematical structure on the one hand, and the conceptual representation of a theory given by the conceptual structure on which such theory stands on the other. Following the remark of Einstein that *it is only the theory which can tell us what can be observed*, we take the actual observation to be meaningful only when being part of a conceptual scheme which allows to represent this actual physical experience. According to our stance, there are no *naked facts* and thus, a phenomenon is only such when conceived from within a coherent theory. We cannot presuppose the existence of a 'situation' without knowing what the theory is talking about for it is the situation itself which must be accounted for by the theory.

The consistency requirements satisfied by preexistent properties stand on the realist ideal that physics does not only rely on the *hic et nunc* but rather, as any metaphysical attempt, relates to the *representation* of (physical) reality through a conceptual scheme. According to our stance, physical experience is a particular aspect of the physical discourse in which the *hic et nunc* is expressed and described. The physical description exceeds through thought and representation the particular *singularity* exposed in an actual observation. In the classical case, the presupposition of having sets of definite valued properties which constitute systems is the condition of possibility for an objective physical discourse. The object is in this case the concept which unifies sense data. This is what allows us to think in classical mechanics —in a counterfactual manner— of the possible results of experiments and draw conclusions about an object without the need of actually performing the experiment. According to the realist stance all the possible experience of an object is implied and can be derived by knowledge of its properties. Thus, we know a glass is 'breakable' through its *a priori* conceptualization and representation, we need not break it in order to prove it has this property. Already in the definition of 'glass' we presuppose the properties related to the object. If we would need to break it in order to known it is breakable physical discourse would loose its inherent capability of discussing about states of affairs.

To 'represent' means to *bring into presence.* This notion has of course a deep metaphysical presupposition at play, that is, that by representing one is disclosing a feature already present, already existent, already actual. To 'bring into presence' means to expose something already *preexistent* —to observation. That there is a preexistent reality, a transcendent description to the *hic et nunc*, is one of the basic metaphysical presuppositions of classical physics. This is a feature shared by all physical theories that we have known until quantum mechanics. It is the principle of identity which allow us to extend through time the set of definite properties which constitute systems and think in terms of a dynamical picture. However, the problem of contextuality in quantum theory must be thought independently of evolution, at a single instant of time, for it is at each instant that such classical ideal of representation is precluded due to the incompatible or non-commutative structure of the formalism.[53] Contextuality can be then directly related to the impossibility to represent a piece of the world as constituted by a set of definite valued properties independently of the choice of the context considered in terms of its formal representation. In quantum mechanics although we have a well defined *mathematical object*, namely the quantum wave function Ψ, which provides us fantastically accurate empirical results, this mathematical object has no clear *conceptual reference*. Our attempt to think of this mathematical object in terms of a "classical object" seems doomed to fail. This failure is due to the constraints provided by the formalism of the theory itself and has been extensively discussed in the literature. In the following we attempt to offer some new analysis of the importance and understanding of contextuality from different metaphysical stances. Later on we shall discuss the implications of contextuality with respect to the meaning of 'possibility' and 'correlations' in quantum theory.

10.2 Contextuality and Classical Metaphysical Stances

Contextuality exposes a contradictory aspect of the formalism when attempting to discuss about a single object, for it associates multiple mutually inconsistent representations with the same quantum wave function Ψ, a quantum wave function which is thought to provide a description of a given state of affairs. We have characterized classical metaphysical stances as those interpretations of quantum mechanics which attempt to restore a classical way of thinking about *what there is*. From a metaphysical point of view, it is a complete revolution to have found a theory which, due to its formal structure, seems to resist a straightforward interpretation (i.e. without violating its symmetries and invariance) in terms of this familiar classical account. Classical metaphysical stances have a right to follow this path, for the importance of these metaphysical presuppositions in the history of physics can be hardly overestimated and the difficulties which arise when attempting to construct a different ontological scheme are immense.

The discussion about the necessity of adding hidden variables to standard physical magnitudes in quantum mechanics in order to provide a complete account of physical reality began with the famous EPR paper [113]. A possible reading of the EPR conclusion was that it is necessary to complete the quantum description with *hidden* magnitudes which would allow to restore the classical way of thinking about what there is in terms of a set of preexistent properties which describe the world. Against such attempts, von Neumann developed a theorem which seemed to preclude hidden variables due to the inexistence of dispersion free states (DFS, i.e. states for which $< \mathbf{A} >^2 = < \mathbf{A}^2 >$) compatible with the mathematical structure of the theory [269].[54] Given the authority of von Neumann in the academic community, his

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[53] Notice that in the KS theorem there is no time involved, the derivation deals with the compatibility of properties within an instant of time, no evolution is taken into account.

[54] Von Neumann considered the measurement of a physical magnitude over an ensemble of systems in the same state. Quantum mechanics predicts that, in the general case, each measurement will give as a result any of the eigenvalues of theoperator representing the magnitude. Thus, we obtain different results for the measurement of the same quantity although all the systems are in the same state. According to von Neumann,

proof was considered as conclusive, and it was believed that the hidden variable attempts were doomed to failure. However, Bohmian mechanics [44; 45] flagrantly contradicted von Neumann's theorem, thus opening the analysis of the strength of the presuppositions involved in the theorem. Observing this situation, Bell reconsidered the hidden variable program. But contrary to his own expectations he himself proved that no local, realistic hidden variable theory would be able to reproduce the statistical predictions of quantum mechanics (see section 5.1). Bohmian mechanics could do so only at the price of giving up locality. Bell's theorem had proven that the set of outcomes which arise in EPR type experiment precludes a local realistic hidden variable model to exist, in order to keep alive the hidden variable program, either some physical presupposition had to be given up or at least some part of the formalism had to be changed.

Although the properties might behave in a non-local manner, it is interesting to notice also that Bohm's interpretation is able to recover the ideal of a world described as a set of actual preexistent definite valued properties.[55] Independently of the criticism which might apply, Bohm's causal interpretation is able to get rid of contextuality. There is one single context given by the quantum wave function —which now describes a quantum field— in which all properties are well defined (we will come back to this point on chapter 14). A very different attempt to recover a classical metaphysical scheme is the so called many worlds interpretation. The many worlds interpretation of quantum mechanics [84] has become one of the most important lines of investigation and is considered to be a direct continuation of Everett's first proposal in terms of 'relative states' [117; 118]. Everett's idea was to let quantum mechanics find its own interpretation, doing justice to the symmetries inherent in the Hilbert space formalism in a simple and convincing way (for discussion see for example [29], [82], [98], [137] and [272]). The main idea behind the many worlds interpretation is that superpositions refer to

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this is so either because there are some hidden variables which the quantum description does not take into account or because, though the systems are really in the same state, the dispersion of measured values is due to Nature itself. If quantum mechanics were to be described by hidden variables, the ensemble would have to contain as many sub-ensembles as there are different eingevalues, with every system in a sub-ensemble in a DFS characterized by a particular value of each hidden variable. Starting from a set of assumptions he considered plausible, von Neumann proved that there does not exist a state which makes all variables dispersion free. An extension of von Neumann's theorem was developed by Jauch and Piron [157; 198] relaxing the conditions in the definition of the states.

[55] For some the price to pay, namely, non-locality, might be to high. Heisenberg ([141], pp. 115-116) was in this respect very critical of Bohm's interpretation: "Bohm has made a counter-proposal to the Copenhagen interpretation, which has recently been taken up to some extent also by de Broglie. Bohm's interpretation has been worked out in detail. It may therefore serve here as a basis for the discussions. Bohm considers the particles as 'objectively real' structures, like the point masses in Newtonian mechanics. The waves in configuration space are in his interpretation 'objectively real' too, like electric fields. Configuration space is a space of many dimensions referring to the different co-ordinates of all the particles belonging to the system. Here we meet a first difficulty: what does it mean to call waves in configuration space 'real'? This space is a very abstract space. The word 'real' goes back to the Latin word 'res,' which means 'thing'; but things are in the ordinary three-dimensional space, not in an abstract configuration space. One may call the waves in configuration space 'objective' when one wants to say that these waves do not depend on any observer; but one can scarcely call them 'real' unless one is willing to change the meaning of the word. Bohm goes on defining the lines perpendicular to the surfaces of constant wave-phase as the possible orbits of the particles. Which of these lines is the 'real' orbit depends, according to him, on the history of the system and the measuring apparatus and cannot be decided without knowing more about the system and the measuring equipment than actually can be known. This history contains in fact the hidden parameters, the 'actual orbit' before the experiment started."

collections of worlds, in each of which exactly one value of an observable, which corresponds to one of the terms in the superposition, is realized. Apart from being simple, the claim is that it possesses a natural fit to the formalism, respecting its symmetries. This provides a solution to the measurement problem by assuming that each one of the terms in the superposition is *actual* in its own correspondent world. Thus, it is not only the single value which we see in 'our world' which gets actualized but rather, that a branching of worlds takes place in every measurement, giving rise to a multiplicity of worlds with their corresponding actual values. The possible splits of the worlds are determined by the laws of quantum mechanics. In relation to the methodological map we proposed in chapter 6 many worlds start from the formalism but go back again into the classical metaphysical scheme and rescue the classical world in each branch. However, although many worlds interpretation provide an interpretation of quantum superpositions arising from each representation, contextuality remains a problem for it is not clear which of the possible representations should be taken into account (something which is called in the literature: the basis problem). We will come back to this point on chapter 14.

From the point of view of classical metaphysical stances, quantum contextuality is regarded as a problem. As a feature of the formalism which needs to be avoided. As we shall see in the following section, there is a completely different way through which conetxtuality can be taken into account, not as a feature which needs to be bypassed or evaded, but rather as an internal aspect of the theory which needs to be interpreted coherently through the development of a conceptual scheme.

10.3 Contextuality and Constructive Metaphysical Stances

To introduce the relation between contextuality in quantum mechanics and the position taken by constructive metaphysical stances we might provide an interesting analogy with relativity theory and its relation to Non-Euclidean geometry. For a long time it was thought that Euclidean geometry was the only meaningful set of axioms which could be conceived of in relation to the physical world. Dropping the fifth axiom was quite a counter-intuitive exercise which did not seem to relate in any way to physical reality. But independently of such intuitions, it was through non-Euclidean geometry that it was possible to develop one of the most revolutionary theories of physics. The development of relativity theory was only possible through the interpretation of this mathematical non-classical structure which led at the same time to new concepts of space and time. The appearance of quantum mechanics determines a confrontation with our classical conception and understanding of the world. Contrary to what happened with relativity, there has been no Einstein able to interpret the quantum formalism coherently. As it was the case for relativity theory the mathematical scheme is consistent, the formalism is capable of being empirically sound, but still, no one has been able to "make sense" of what the theory is talking about. It is the constructive metaphysical stance which attempts to develop a new (non-classical) metaphysical scheme which would allow us to understand quantum theory.

We believe, as Pauli did, that the impossibility to understand quantum mechanics points in the direction of a needed deep reconfiguration of our representation of physical reality. We also believe, as Heisenberg did, that such reconfiguration deals exclusively with quantum theory and, as a closed theory, need not imply a change of our classical conception of the world which will rest valid within its own limits.[56]

Following Constantin Piron's remark, quantum mechanics seems to be still part of a revolution which did not take place [201]. The experimental evidence [16; 17; 128; 148] urges the development of metaphysical presumptions which go against our classical understanding of the world. Constructive metaphysical schemes have strived to think anew all these phenomena. Among many proposals we may mention Mermin's attempt to interpret quantum mechanics in terms of "correlations without correlata" —putting forward a relational account of correlations (we shall come back to Mermin's interpretation in chapter 12). As we mentioned earlier, Dennis Dieks together with Gyula Bene, also proposed a relational account of quantum mechanics based on the modal interpretation and relational properties (see [35] and [220]).[57] We must also call special attention to Constantin Piron's proposal to recover the Aristotelian scheme, an attempt which remains to us one of the most interesting proposals to understand the quantum formalism [198; 199; 200]. A development of Piron's ideas has been also put forward by Diederik Aerts stressing and developing different elements of the Geneva School [2; 5; 7; 9; 10] (see also for discussion [12; 242; 70]). Even within pragmatic anti-metaphysical schemes one can find proposals which can help in the development of new ideas which do not see metaphysics as a barrier or useless questioning. This can be justified as noticed by Mauro Dorato ([110], p. 8) from the fact that: "[the] interpretation of the 'interpretation of physics' (which has variously been defended by van Fraassen 1980, Giere 1988 and Lange 2002) does not require truth from our physical theories, but can be embarked upon also by instrumentalists, since the whole interpretative task rests on a conditional statement ('if the theories are at least approximately true')." In this line of thought we may note the work of Michel Bitbol who has developed a very interesting position which he calls "Reflective Metaphysics" [40].[58] Also van Fraassen is interested in new possible metaphysical schemes. His interest comes from realizing the importance of metaphysical schemes even within pragmatic accounts.[59]

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[56] As noticed by Heisenberg ([54], p. 94): "Newtonian mechanics is a limited description of nature and in that limited field it is perfectly accurate. It can never be improved. All attempts to improve Newtonian mechanics are just fruitless [...] Since it is a closed axiomatic system I think it should be left as it is [...] Of course it doesn't cover the whole of physics. There are other schemes. Already Maxwell theory is entirely different from it and again that is a closed scheme and cannot be improved."

[57] Recently, Dieks has developed these ideas to try to put forward a new idea of objectivity. "The key idea is to adapt the notion of objectivity itself, by introducing *relational* or *perspectival* properties. It seems that such a 'relational perspective' offers prospects of overcoming some of the long-standing problems in the interpretation of quantum mechanics" ([99], p. 1).

[58] According to Bitbol ([40], p. 79): "As Kant wrote to his friend Markus Herz in May 11, 1781: '[The Critique of Pure Reason] contains the metaphysics of metaphysics'. Critical philosophy goes beyond metaphysics by inquiring beneath its preconditions. By doing so, it annihilates standard metaphysical claims of knowledge, and at the same time it elucidates the origins of many paradoxes of rational inquiry that remained unfathomable as long as they were hidden in the smoke of conceptual reifications. This work of clarification is useful in every sector of scientific research, where it has recently taken the form of a 'philosophy of experiment' inspired by pragmatism. But it is truly indispensible in quantum mechanics, where the preconditions of knowledge are also preconditions for the emergence of the empirical material of this knowledge."

[59] An interesting example is a recent paper entitled "Rovelli's World" where van Fraassen discusses the metaphysical consequences of Rovelli's interpretation [259]. According to van Fraassen: "[Rovelli's relational interpretation] provides a new vision of what the world of quantum mechanics is like, and it offers a program to derive the theory's formalism from a set of simple postulates pertaining to information processing. I propose here to concentrate entirely on the former, to explore the world of quantum mechanics as Rovelli depicts it."

Contrary to classical realistic stances we cannot state from our representational realistic stance that: “there is a context”, in the sense there is an actual state of affairs described by classical concepts. Such statement would presuppose from our perspective that we already know what quantum mechanics is talking about and that there is a clear explanation of the relation in between such ‘context’, understood in terms of classical concepts, and the formalism of quantum mechanics. We still need to find a conceptual structure which allows us to understand quantum mechanics. Contrary to classical metaphysical approaches, constructive metaphysical approaches take contextuality, not as a problem, but rater as an intrinsic feature of quantum mechanics itself. A feature which should be taken into account for a proper development of an interpretation which allows us to discuss what is quantum mechanics talking about.

The Contextual Character of Quantum Possibility

In [104], respecting the orthodox quantum formalism, we developed a generalized modal scheme in order to take into account both actual and possible properties within the same formal structure. Using algebraic tools, we showed an embedding of the orthomodular system in a modal one. Differently from van Fraassen's treatment of possible and actual propositions as belonging to two different structures, our unified system gives an adequate frame to picture the path from possibility to actuality. Furthermore, it also allows to represent the Born rule for the probability of actualization of possible properties. Within this enlarged structure we studied the constraints imposed by the quantum formalism on the discourse about modal properties. The conclusion we draw from this analysis is that one cannot consider the possible realm, described in terms of the quantum formalism, as being able to escape contextuality.

11.1 A Modal Kochen-Specker Theorem

In order to stay away from inconsistencies when speaking about properties which pertain to the system, one must acknowledge the limitations imposed by the KS theorem. To do so, modal interpretations assign to the system only a limited set of definite properties. However, as we have seen, this is not achievable when talking about properties while combining different contexts (see for discussion [27]). At first sight it might seem paradoxical that, even though modal interpretations of quantum mechanics talk about modalities, the KS theorem refers only to actual values of physical properties. Elsewhere, and following the line of thought of quantum logic, we have investigated the question whether the KS theorem has something to say about *possibility* and its relation to *actuality* ([104; 105]). The answer was provided via a characterization of the relations between actual and possible properties pertaining to different contexts. By applying algebraic and topological tools we studied the structure of the orthomodular lattice of actual propositions enriched with modal propositions. Let us briefly recall the results. As usual, given a proposition about the system, it is possible to define a context from which one can predicate with certainty about it (and about a set of propositions that are compatible with it) and predicate probabilities about the other ones. This is to say that one may predicate truth or falsity of all possibilities at the same time, i.e. possibilities allow an interpretation in a classical system of propositions. In order to describe rigorously the formalism which allows to capture all propositions in a single structure, let $\mathcal{L}$ be an orthomodular lattice. Given a, b, c in L, we write: $(a, b, c)D$ iff $(a \vee b) \wedge c = (a \wedge c) \vee (b \wedge c)$; $(a, b, c)D^*$ iff $(a \wedge b) \vee c = (a \vee c) \wedge (b \vee c)$ and $(a, b, c)T$ iff $(a, b, c)D$, (a,b,c)D^* hold for all permutations of a, b, c. An element $z \in \mathcal{L}$ is called *central* iff for all elements $a, b \in L$ we have $(a, b, z)T$. We denote by $Z(\mathcal{L})$ the set of all central elements of $\mathcal{L}$ and it is called the *center* of $\mathcal{L}$. $Z(\mathcal{L})$ is a Boolean sublattice of L ([183], Theorem 4.15).

Let P be a proposition about a system and consider it as an element of an orthomodular lattice $\mathcal{L}$. If we refer with $\Diamond P$ to the possibility of P, then $\Diamond P$ will be a central element of $\mathcal{L}$. This interpretation of the possibility in terms of the Boolean algebra of central elements of $\mathcal{L}$ reflects the fact that one can simultaneously predicate about all possibilities. This is so because it is always possible to establish Boolean homomorphisms of the form $v : Z(\mathcal{L}) \rightarrow \mathbf{2}$. Therefore, the key idea is to expand the orthomodular structure in such a way to include propositions about possibility. This expansion is performed in the following way: If P is a proposition about the system and P occurs, then it is trivially possible that P occurs. This is expressed as $P \leq \Diamond P$. If we identify P with the value-attribution proposition $< \mathbf{A}, \sigma >$ as defined by van Fraassen, we may say that the classical consequences of P coincide with those of its correspondent state-attribution proposition $[\mathbf{A}, \sigma]$. In fact, to assume an actual property and a complete set of properties that are compatible with it determines a context in which the classical discourse holds. Classical consequences that are compatible with it, for example probability assignments to the actuality of other propositions, shear the classical frame. These consequences are the same ones as those which would be obtained by considering the original actual property as a possible property. This is interpreted as, if P is a property of the system, $\Diamond P$ is the smallest central element greater than P. With these tools, we are able to give an extension of the orthomodular structure by adding a possibility operator that fulfills the mentioned requirements. More precisely, the extension is a class of algebras, called Boolean saturated orthomodular lattices, that admits the orthomodular structure as a reduct and we demonstrate that they are a variety, i.e., definable by equations. Complete orthomodular lattices are examples of them [104]. This algebraic construction is what allows us to consistently expand the structure of actual properties to include modal properties. This is so because we have proved that every orthomodular lattice may be embedded in a Boolean saturated orthomodular one.

If $\mathcal{L}$ is an orthomodular lattice and $\mathcal{L}^\Diamond$ a Boolean saturated orthomodular one such that $\mathcal{L}$ can be embedded in $\mathcal{L}^\Diamond$, we say that $\mathcal{L}^\Diamond$ is a modal extension of $\mathcal{L}$. Given $\mathcal{L}$ and a modal extension $\mathcal{L}^\Diamond$, we define the *possibility space* as the subalgebra of $\mathcal{L}^\Diamond$ generated by $\{\Diamond P : P \in \mathcal{L}\}$. We denote by $\Diamond \mathcal{L}$ this space and it may be proved that it is a Boolean subalgebra of the modal extension. The possibility space represents the modal content added to the discourse about properties of the system. Within this frame, the actualization of a possible property acquires a rigorous meaning. Let $\mathcal{L}$ be an orthomodular lattice, $(W_i)_{i \in I}$ the family of Boolean sublattices of $\mathcal{L}$ and $\mathcal{L}^\Diamond$ a modal extension of $\mathcal{L}$. If $f : \Diamond \mathcal{L} \rightarrow \mathbf{2}$ is a Boolean homomorphism, an actualization compatible with f is a global valuation $(v_i : W_i \rightarrow \mathbf{2})_{i \in I}$ such that $v_i \mid W_i \cap \Diamond \mathcal{L} = f \mid W_i \cap \Diamond \mathcal{L}$ for each $i \in I$. A kind of converse of this possibility of actualizing properties may be read as an algebraic representation of the Born rule, something that has no place in the orthomodular lattice alone [104]:

Theorem 11.1.1. *Let $\mathcal{L}$ be an orthomodular lattice, W a Boolean sublattice of $\mathcal{L}$ and $f : W \rightarrow \mathbf{2}$ a Boolean homomorphism. If we consider a modal extension $\mathcal{L}^\Diamond$ of $\mathcal{L}$ then there exists* *a Boolean homomorphism* $f^* : \langle W \cup \Diamond\mathcal{L}\rangle_{\mathcal{L}^\Diamond} \rightarrow \mathbf{2}$ *such that* $f^* \mid W = f$. □

Compatible actualizations represent the passage from possibility to actuality, they may be regarded as formal constrains when applying the interpretational rule proposed by Dieks. When taking into account compatible actualizations from different contexts, the following KS theorem for modalities can be proved [104]:

Theorem 11.1.2. *Let* $\mathcal{L}$ *be an orthomodular lattice. Then* $\mathcal{L}$ *admits a global valuation iff for each possibility space there exists a Boolean homomorphism* $f : \Diamond\mathcal{L} \rightarrow \mathbf{2}$ *that admits a compatible actualization.* □

This theorem shows that no enrichment of the orthomodular lattice with modal propositions allows to circumvent the contextual character of the quantum language. This is why we have called theorem 11.1.2 the Modal Kochen-Specker (MKS) theorem. As in the case regarding actual propositions, the MKS theorem may be demonstrated with topological tools [105]. It is important to remark that our formalism also provides a formal meaning in an algebraic frame to the Born rule, something that has been discussed recently by Dieks in relation to the possible derivation of a preferred probability measure [98].

11.2 Classical Possibility vs Quantum Possibility

Our interest in KS-type theorems in relation to modalities stems from the fact that quantum mechanics has referred, from its very beginning, not only to actuality but also to possibility. In this sense, our problem has been to clarify through the formalism itself the relation between contextuality and modality in quantum mechanics. We have chosen a logical approach because, though the main contribution of a logical calculus is rather technical, it makes visible the structure of which propositions are part and provides bounds to a consistent discourse. It is also a powerful tool when dealing with non-classical propositions. In this frame we have shown that the addition of modalities to the discourse about *the properties of a quantum system genuinely enlarges its expressive power*. More precisely, the usual orthomodular propositional structure $\mathcal{L}$ that does not contain modal elements is embedded in a Boolean saturated orthomodular lattice $\mathcal{L}^\Diamond$, its modal extension, to obtain a common frame. But in view of MKS theorem, a global actualization that would correspond to a family of compatible valuations is prohibited. Thus, *contextuality remains a central feature of quantum systems even when possibilities are taken into account by enriching the structure with modal propositions.*

At least in classical Aristotelian logic, there is no genuine frame to account for possibility as independent of its actualization. Our investigation has been focused on the modal aspects of quantum mechanics and makes clear that the distinction between the classical and quantum structures of actual properties is inherited by the set of possible properties through their actualization. Precisely, let $\mathcal{OML}$ be the variety of orthomodular lattices and $\mathcal{B}$ be the variety of Boolean lattices. It is well known that $\mathcal{B} = \mathcal{OML} + \{x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z))\}$ and then $\mathcal{B} \subset \mathcal{OML}$. Thus considered, the operator $\Diamond$ is the identity in each Boolean algebra, i.e. Boolean saturated Boolean algebras are the Boolean algebras such that their signature is trivially expanded by the identity homomorphism. This framework captures the idea that in the classical domain there exists a compatible family of Boolean valuations to $\mathbf{2}$

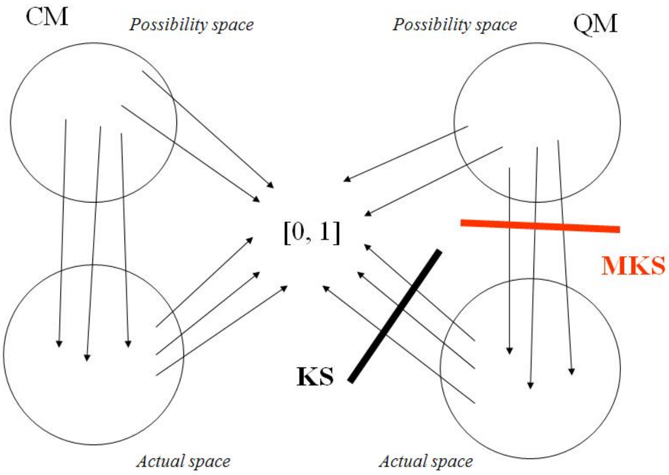

Figure 11.1: In this graphic we schematize the relation between possible and actual spaces in both classical and quantum mechanics. While the KS theorem precludes the non-contextual valuation of actual properties to the [0,1], the MKS theorem precludes the non-contextual actualization of multiple families of possible properties.

and that, with this definition of possibility, the passage from actuality to possibility is non-contextual. In the quantum case, the possibility space $\Diamond\mathcal{L}$ is the center of the whole $\mathcal{L}^{\Diamond}$, thus all quantum possibilities also admit, as in the classical case, Boolean valuations. However, when we consider logical possibility in relation to actuality, *the MKS theorem precludes the existence of a compatible family of actualizations.* In this case, there is no state of affairs described in terms of sets of actual properties that may be the domain of a valuation that arises from a family of actualizations, exception made for the trivial case in which the domain is a single context —in such case, quantum possibility has collapsed into classical possibility.

The following diagram (figure 11.1) shows that while the KS theorem precludes the existence of mutually compatible contexts, the MKS theorem precludes the actualization of possible contexts into an actual state of affairs. We may conclude that there exist deep differences in the logical notions of possibility as formally described in classical systems and quantum systems. More intuitively, we could explain the difference between classical possibility and quantum possibility in relation to the *actualization* of a possible state of affairs. Classical possibility has no constraints whatsoever with respect to the different states of affairs which might turn to become actual, any state of affairs might become actual and in this sense when we talk about a “possible state of affairs” we need not constrain our discourse with respect to other distinct “possible states of affairs”. But in quantum theory the notion of possibility is constrained by the mutually incompatible states of affairs and their actualizations. Thus, when considering possible states of affairs in the quantum realm we must also take into account the contextual character of such possibilities.

Within the representational realist stance we have discussed explicitly the relation between the formalism and the concepts involved within physical theories. In this respect we take the position that each particular “closed theory” should be able to coherently relate the conceptual structure to the mathematical formalism and physical experience internally. The concepts should find their ground of definition and justification within the theory itself. Since we do not accept a reductionistic relation in between theories, it makes no sense for us to "export" notions from external —in principle, incompatible— theories. Such interrelation in between notions which pertain to different theories would need, according to our stance, a clear justification, for it is such notions which provide the metaphysical ground of theories and allow us to consider what these theories are telling us about the world.

The question is whether quantum mechanics implies a different notion of possibility to that of classical physics. If we consider the formalism —as we do— to be directly linked to the formal structure of physical concepts, our MKS theorem answers this question positively. Regarding its specific relation to modal interpretations, the scope of the MKS theorem and its topological version supersedes the usual KS-type theorems of Bacciagaluppi and Clifton (see chapter 8), which only refer to the actual values of physical properties. In spite of the fact that at first sight it may be thought that referring to possibility could help to circumvent contextuality, allowing to refer to possible physical properties belonging to the system in an objective way that resembles the classical picture, our theorem shows that this cannot be consistently maintained. According to the constructive metaphysical strategy to interpret quantum mechanics we should start from the orthodox formalism and try to derive what is meant by the theory to have a state of affairs. In quantum mechanics, the formalism seems to show that one cannot presuppose the existence of contexts in a compatible manner, even when considering the possible realm. The concept of possibility which we use in classical (Aristotelian) logic is not the same concept which appears in the orthodox structure of quantum mechanics. This allows us to conclude a new lesson for modal interpretations, namely, that modal interpretations must abandon, not only classical reasoning about actual properties —as proposed by Clifton—, but also *must abandon classical reasoning about possible properties.* The range of applicability of this conclusion will be discussed in the following section.

11.3 A New Lesson for Modal Interpretations

Regarding modal interpretations it is important to specify the range of applicability of the MKS theorem. The general conclusion is that possibility is a non-classical contextual concept: a set of shared possibilities corresponding to disjoint sets of actual properties cannot be non-contextually actualized without contradictions and thus, there is an incompatibility with the idea that quantum possibility might be able to provide an account of a state of affairs univocally described in terms of a set of actual properties. This means that if we take into account the orthodox formalism, quantum possibility cannot bypass contextuality. We would like now to go more in detail regarding the conclusion already provided in relation to the theorem and modal interpretations, so let's go more deeply into the analysis taking advantage of the distinction between MIMP and MIMF.

The MKS theorem only applies to the orthodox formulation of quantum mechanics in relation to possibility, so MIMF —which are entitled to respect such formalism— are completely subject to the theorem. Thus, the first conclusion we can derive from our reasoning in relation to MIMF takes the following form: *MIMF must abandon classical reasoning about possible quantum properties.* However, following van Fraassen, one could still argue that MIMF seem not necessarily engaged with a realistic (representational or not) stance and thus can evade the problem simply by noticing that quantum possibility is just a theoretical device to account for the measurement outcomes. The idea which van Fraassen sustains is that: *modalities are in our theories, not in the world.* So in this sense, van Fraassen's agnosticism regarding the interpretation of the formalism keeps him safe of the MKS theorem. From a realist stance one could still use a classical notion of possibility at the price of clearly justifying the relation in between the quantum and the classical realms. Until today, this reductionistic strategy has not been able to provide convincing arguments which would allow us to explain and justify such relation. Decoherence, which seems to be the best hope of this strategy, seems incapable of providing the foundational grounds to account for the quantum to classical limit.[60] However, from a representational realist perspective the problem cannot be bypassed, for it is the quest of such stance to provide a closed conceptual and formal scheme in which each of the notions used in the theory have a clear explanation from *within* the theory itself.

For the case of MIMP, which stand on classical realistic grounds and intend to restore a non-contextual classical notion of possibility —avoiding in this way the two levels of description found in the quantum wave function—, the theorem has also a direct consequence. If MIMP attempt to provide a justification of the concepts used within the theory, it seems that *MIMP are forced to find a new formalism —different from the orthodox— which provides the same empirical adequacy, but allows the introduction of a classical notion of possibility.* This is the strategy followed by Bohm, who is able in his new theory to provide a description in classical terms (we shall come back to this in section 14.2).[61] From a representational realist stance this strategy seems to change the subject of the discussion, for by changing the formalism it is the theory itself which cannot be considered to be the same. This means the problem would not be to interpret quantum mechanics but rather to find a new theory. It might seem at first sight that "looking for new theories" would be a more revolutionary quest that keeping the already old formalism of quantum mechanics and, despite of all the problems encountered, keep trying to interpret it in a way it convincingly makes sense of what the theory is talking about. We might remark that the main force which seems to guide this approach can be also regarded as deeply conservative, for it is grounded on the desire to restore, at least some part, of our classical metaphysical conception and understanding of the world. If MIMP found in the end such a formalism, it is still not at all clear how they will be able, for example, to recover the holistic nature of quantum mechanics —that which van Fraassen ([255], p. 73) has called "the most striking feature of quantum theory"— as well as many of the emergent properties present in the quantum theory.[62]

Due to the enormous distinctions between both MIMF and MIMP, it is not at all obvious how to designate a general characterization of modal interpretations. We believe that

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[60] As a matter of fact, today, even those who accept the importance of decoherence within the interpretational discussions, share the position that decoherence does not provide an answer to the so called "quantum to classical limit". For discussion see [24].

[61] As remarked by Albert Solé ([243], p. 1) in his Dissertation about the interpretation of Bohmian mechanics: "A major part of the formal and mathematical structure of Bohmian mechanics is completely foreign to quantum mechanics. We think, for example, of the equation which allows to account for the trajectories of Bohmian particles in terms of the quantum field and the theory of attribution of properties to such particles, always well defined. Thus, taking into account such profound formal differences, it seems more adequate to claim that Bohmian mechanics is a different theory than quantum mechanics, though empirically equivalent, rather than an interpretation of the latter."

[62] An attempt in this direction has been made by Dieks [96] in relation to the atomic version.

for a clear discussion in relation to modal interpretations this distinction should be clearly addressed. MIMF and MIMP are modal in a very different way. We will come back to this point later on chapter 14.

The Contextual Character of Quantum Correlations

The Ithaca interpretation of quantum mechanics (IIQM) was proposed by David Mermin in a series of papers in which he discussed the foundational problems of quantum theory [187; 188; 189]. Mermin's main idea was to put forward a relational interpretation of quantum mechanics in which *correlations* are considered as local realistic elements of physical reality. The IIQM finds its solid ground in the so called SSC theorem which states that subsystem correlations (for any resolution of the system into subsystems) are enough to determine the state of the entire system uniquely. There are however in the literature two no-go theorems, namely, Cabello's theorem and Seevinck's theorem, which block the possibility of interpreting correlations as actual elements of physical reality. In this chapter we will address the tension which appears from the mutual co-existence of the SSC theorem, on the one hand, and Cabello's and Seevinck's theorems, on the other. We will show that there is a valuable lesson to be learnt with respect to correlations in quantum mechanics and their contextual character.

12.1 Correlations as Elements of Physical Reality

Mermin's main idea can be summarized in the following dictum:

> "Fields in empty space have physical reality; the medium that supports them does not. Correlations have physical reality; that which they correlate does not."

The IIQM is based on six desiderata:

1. The theory should describe an objective reality independent of observers and their knowledge.
2. The concept of measurement should play no fundamental role.
3. The theory should describe individual systems -not just ensembles.
4. The theory should describe small isolated systems without having to invoke interactions with anything external.
5. Objectively real internal properties of an isolated individual system should not change when something is done to another non-interacting system.

6. It suffices (for now) to base the interpretation of quantum mechanics on the (yet to be supplied) interpretation of objective probability.

After this set of desiderata for an acceptable theory Mermin presents the pillars of the IIQM: the first one is that *i) correlations are the only fundamental and objective properties of the world.* The second pillar is that *ii) a density matrix of a system is a fundamental objective property of that system whether or not it is a one dim projection operator (i.e. mixed states are as fundamental as pure states).* Mermin argues that the quantum state is an encapsulation of internal correlations. According to Mermin, the "no-hidden-variables theorems" require that if all correlations have simultaneous physical reality, then whatever they correlate (correlated quantities) cannot. It is only when one tries to go beyond their inter-subsystem correlations to actual correlata (particular values for the subsystem observables) that non-commuting observables are incapable of sharing simultaneous physical reality. In this respect Mermin's Ithaca interpretation proposes an interesting methodology:

> "Einstein used this supposition [to hold fast to the real factual situation of the system S_1 is independent of what is done with the system S_1], together with his intuitions about what constituted a real factual situation, to conclude that quantum mechanics offers an incomplete description of physical reality. I propose to explore the converse approach: assume that quantum mechanics does provide a complete description of physical reality, insist on generalized Einstein-locality, and see how this constrains what can be considered physically real." D. Mermin (Quoted from [187], p. 552)

The idea of physical reality which Mermin has in mind is clear: we should be able to discuss about a preexistent world independently of our choices regarding measurements. This fits the metaphysical ideal in terms of the notion of an *actual preexistent state of affairs.* In order to construe a physical theory which is about an actual state of affairs the notion of measurement in quantum mechanics should not be taken as primitive in the theory, nor should it determine the mode of existence of the properties being measured or not.

> "There is a world out there, whether or not we choose to poke at it, and it ought to be possible to make unambiguous statements about the character of that world that make no reference to such probes. A satisfactory interpretation of quantum mechanics ought to make it clear why 'measurement' keeps getting in the way of straight talk about the natural world; 'measurement' ought not to be a part of that straight talk. Measurement should acquire meaning from the theory —not vice versa... Physics ought to describe the unobserved unprepared world. 'We' shouldn't have to be there at all." D. Mermin (Quoted from [187], p. 551)

In order to escape the measurement problem "right from the start" and provide an objective account of physical reality, Mermin [187] attempts to leave aside the notion of measurement, "if correlations constitute the full content of physical reality, then the fundamental role probability plays in Q.M. has nothing to do with ignorance... [...] The very much broader concept of correlation ought to replace measurement in a serious formulation of what Q.M. is all about..." Mermin argues that to free quantum mechanics from measurement the key is to note that measurement consists of a particular kind of correlation between two particular kinds of subsystems, and to insist that everything that can be said about the physical reality of the correlations applies equally well to the correlations among any two subsystems of a quantum system. Mermin also expects to eliminate the strange relationship between the quantum and the classical realm.

> "The classical domain plays a central role only if one restricts the correlations one is willing to call physically real, to those between specimens and apparatuses [...] The Quantum theory allows us to contemplate together all the correlations among arbitrary subsystems, and it is simply a bad habit not to grant micro-micro... correlations as much objective reality as the traditional emphasis on measurements has granted to micro-macro correlations..." D. Mermin (Quoted from [188], p. 757)

Similar to Mermin's proposal, Carlo Rovelli [230] developed a relational interpretation of quantum mechanics based on the notion of *information*, which shares many of the problems and discussions presented in the IIQM. It is interesting to notice that the difference between the view of Mermin with respect to Rovelli's relational account is an ontological one. While Rovelli seems to remain agnostic regarding the ontological character of information, Mermin takes a clear position claiming that correlations are the *elements of physical reality of the theory.* This means that the quantum world is built up of correlations in a fundamental manner. In order to sustain this strong claim regarding correlations Mermin founds his interpretation on the solid ground of what he calls the subsystem correlation (SSC) theorem:

Definition 12.1.1. SSC Theorem: *Subsystem correlations (for any resolution of the system into subsystems) are enough to determine the state of the entire system uniquely; i.e. the density matrix of a composite system determines all the correlations among the subsystems that make it up and, conversely, the correlations among all the subsystems completely determine the density matrix for the composite system they make up.*

According to this theorem, the correlations among subsystems completely determine the density matrix (i.e. the physical state) for the composite system they make up and thus, "anything you can say in terms of quantum states can be translated into a statement about subsystem correlations, i.e., about joint distributions." Motivated by this result, the IIQM proposes the following ontological picture of the quantum world: *physical reality consists of correlations.*

In the following two sections we shall present two no-go theorems that show that this representation of physical reality, namely, that quantum correlations are the local realistic properties that (pre)exist in the world faces serious problems.

12.2 Cabello's Kochen-Specker Type Theorems

Cabello was able to develop in [62; 63] two no go theorems by taking into account the following assumptions of Mermin's proposal:

1. **Density matrices:** describe isolated individual systems —not just ensembles. Density matrices fully describe all the internal-correlations of an isolated individual system.
2. **Reality:** All correlations between subsystems of an isolated composed system are real objective internal properties of the composed system.

3. **Locality:** Real objective internal properties of an isolated system "cannot change in immediate response to what is done to a far-away system that may be correlated but does not interact with the first."

Cabello [62] was able to prove that if one assumes (a) and (b), then assumption (c) cannot be correct. More specifically, Cabello shows that "the assumption that correlations between subsystems of an individual isolated composite quantum system are real, objective local properties of that system is inconsistent." To do so, he considers the following situation.

Given two sources, each of which emits a single pair of spin particles in the singlet state, the initial state of the four particles is given by

$$|\Psi\rangle = \frac{1}{2}(|+\rangle_1 \otimes |-\rangle_2 - |-\rangle_1 \otimes |+\rangle_2) \otimes (|+\rangle_3 \otimes |-\rangle_4 - |-\rangle_3 \otimes |+\rangle_4) \tag{12.1}$$

Cabello then considers the following two experiments:

Experiment 1: On particles 2 and 3, we perform a measurement of component z of the spin of each particle. This measurement projects the combined state of a single pair of particles 2 and 3 onto one of the following four factorizable pure states:

$$|+\rangle_2 \otimes |+\rangle_3, |+\rangle_2 \otimes |-\rangle_3, |-\rangle_2 \otimes |+\rangle_3, |-\rangle_2 \otimes |-\rangle_3 \tag{12.2}$$

This measurement on particles 2 and 3 also projects the combined state of the corresponding single pair of particles 1 and 4 onto, respectively, one of the following factorizable pure states:

$$|-\rangle_1 \otimes |-\rangle_4, |-\rangle_1 \otimes |+\rangle_4, |+\rangle_1 \otimes |-\rangle_4, |+\rangle_1 \otimes |+\rangle_4 \tag{12.3}$$

There is then a one-to-one correspondence between the four states (2) and the four states of (3).

Experiment 2: Instead of a spin measurement on each of particles 2 and 3, we perform a measurement of the Bell operator on particles 2 and 3. This measurement projects the combined state of a single pair of particles 2 and 3 onto one of the four Bell states:

$$|\Psi^{\pm}\rangle_{23} = \frac{1}{2}(|+\rangle_2 \otimes |-\rangle_3 \pm |-\rangle_2 \otimes |+\rangle_3) \tag{12.4}$$

$$|\Phi^{\pm}\rangle_{23} = \frac{1}{2}(|+\rangle_2 \otimes |+\rangle_3 \pm |-\rangle_2 \otimes |-\rangle_3) \tag{12.5}$$

which form a complete basis for the combined system of particles 2 and 3. This measurement on particles 2 and 3 also projects the combined state of the corresponding single pair of particles 1 and 4 onto, respectively, one of the Bell states.

$$|\Psi^{+}\rangle_{14},\ |\Psi^{-}\rangle_{14}, |\Phi^{+}\rangle_{14}, |\Phi^{-}\rangle_{14} \tag{12.6}$$

Before any of the alternative experiments there is no correlation between any of the particles 1 and 2 and any of the particles 3 and 4. On the other hand, before any of the experiments, particles 2 and 3 form a dynamically isolated subsystem; i.e., they have no external interactions. After any of the experiments, particles 2 and 3 do not form a dynamically isolated system since they have interacted with the measuring apparatus. If particles 1 and 4 are space-like separated from the experiment performed on particles 2 and 3, then particles 1 and 4 cannot interact with the measuring apparatus. Therefore, particles 1 and 4 form a dynamically isolated system before and after any of the experiments.

Mermin's interpretation assumes physical locality, defined as "the fact that the internal correlations of a dynamically isolated system do not depend on any interactions experienced by other systems external to it". However, while any of the four possible states of particles 1 and 4 after an experiment of the first type given in Eq. (13.3) are factorizable, any of the four possible states after an experiment of the second type given in Eq. (13.6) are maximally entangled. This means that while after an experiment of the first kind (regardless of the result), particles 1 and 4 have their spins correlated only in the z direction, after an experiment of the second kind (irrespective of the result), particles 1 and 4 are highly correlated: every component of spin of particle 1 is correlated with other component of spin of particle 4, and vice versa. Therefore, the internal correlations between particles 1 and 4 are completely different depending on the interaction between particles 2 and 3 and an external system.

> "Accepting assumptions (a) and (b) means, in this example, the violation of physical locality as defined by Mermin. By this violation of physical locality I do not mean that the internal correlations between particles 1 and 4 "change" after a spacelike separated experiment (this does not happen in the sense that no new internal correlations are "created" that were not "present" in the reduced density matrix for the system 1 and 4 before any interaction), but that the type of internal correlations (and therefore, according to Mermin, the reality) of an individual isolated system can be chosen at a distance." A. Cabello (Quoted from [62], p. 2)

After this paper, Cabello [63] went even further and showed explicitly that quantum mechanics is incompatible with the assumption that: "all possible correlations between subsystems of an individual isolated composite quantum system are contained in the initial quantum state of the whole system, although just a subset of them is revealed by the actual experiment." In other words, one gets into contradictions if one assumes that correlations are objective properties of the quantum wave function, properties which can be un-veiled through measurement.

> "In (1998) I wrote "I do not mean internal correlations 'change'... no new correlations are 'created' that were not 'present' in the reduced density matrix... but that the internal correlations of an individual isolated system can be chosen at a distance." So implicitly I admitted that all such possible correlations between two parts were present somehow in the initial quantum state of the whole system, although just a subset of them is revealed by the actual experiment. The aim of this note is to show that even this innocuous-looking assumption is incompatible with quantum mechanics" A. Cabello (Quoted from [63], p. 1)

Cabello was able to show, through a GHZ like proof [133] and a Hardy like proof [136] that quantum mechanics does not contain all the correlations in the initial state to which one can simultaneously ascribe definite values. Take three pairs of spin-$\frac{1}{2}$ particles labeled from 1 to 6. The Hilbert space in which we describe the spin state of this system has dimension sixty four. I will call it H_{64}. Let A_{ij} be the non-degenerate operator acting on the four-dimensional subspace of particles i and j, defined as:

$$A_{ij} = 2\alpha_{ij}^{++} + \alpha_{ij}^{+-} - \alpha_{ij}^{-+} - 2\alpha_{ij}-- \tag{12.7}$$

Were α_{ij}^{+-} is the projection operator onto the state $|\alpha_{ij}^{+-}\rangle = |+\rangle_i \otimes |-\rangle_j$, etc. Let B_{ij} the non degenerate Bell operator defined as:

$$B_{ij} = 2\Phi_{ij}^{+} + \Psi_{ij}^{+} - \Psi ij^{-} - 2\Phi_{ij}^{-} \tag{12.8}$$

Where Φ_{ij}^{+} is the projection operator onto the state $|\Phi_{ij}^{+}\rangle_{ij}$. If we consider the operators acting in H_{64} and let one of these common eigenvectors, defined by the following equations, $|\mu_i\rangle$ be the initial state of the six-particle system:

$$A_{12}A_{34}B_{56}|\mu_i\rangle = |\mu_i\rangle \tag{12.9}$$

$$A_{12}B_{34}A_{56}|\mu_i\rangle = |\mu_i\rangle \tag{12.10}$$

$$B_{12}A_{34}A_{56}|\mu_i\rangle = |\mu_i\rangle \tag{12.11}$$

$$B_{12}B_{34}B_{56}|\mu_i\rangle = -|\mu_i\rangle \tag{12.12}$$

Assume now that the correlations between subsystems of the composed system are real objective internal local properties of such subsystems. In particular, consider three subsystems: the first is composed by particles 1 and 2, the second by particles 3 and 4, and the third by particles 5 and 6. We will assume that all possible correlations between particles 1 and 2 (for instance) are encoded in the initial state for the whole system, and they do not depend on any interaction experienced by the other subsystems, so they cannot change (in particular, they cannot be created) as a result of any experiment performed on particles 3 to 6 (supposed to be spacelike separated from particles 1 and 2).

Now consider three observers, each having access to one pair of particles. On each pair, they may measure either A_{ij} or B_{ij}, without disturbing the other pairs. The results of these measurements will be called a_{ij} or b_{ij}, respectively.

$$a_{12}a_{34}b_{56} = 1 \tag{12.13}$$

$$a_{12}b_{34}a_{56} = 1 \tag{12.14}$$

$$b_{12}a_{34}a_{56} = 1 \tag{12.15}$$

$$b_{12}b_{34}b_{56} = -1 \tag{12.16}$$

We can associate each one of the eigenvalues a_{ij} and b_{ij} with a type of correlation between particles i and j initially hidden in the original state of the system, but “revealed” by performing measurements on the two other distant pairs.

Following the idea that correlations (pre)exist, if B_{12} and B_{34} are measured and their results are both 1, then one can predict with certainty that particles 5 and 6 are in the singlet state, and since arriving to this conclusion does not require any real interaction on particles 5 and 6, then we assume that the spins of particles 5 and 6 were initially correlated in the singlet state (i. e., the same spin component of particles 5 and 6 would have opposite signs), so we assign the value -1 for the observable B_{56} to the initial state $|\mu_i\rangle$. Analogously, we can do the same for the other correlations. Such predictions with certainty and without interaction would lead us to assign values to the six types of correlations given by A_{12}, B_{12}, A_{34}, B_{34}, A_{56}, and B_{56}. However, such an assignment cannot be consistent with the rules of quantum mechanics because the four equations (13.13-13.16) cannot be satisfied simultaneously, since the product of their left-hand sides is a positive number (because each value appears twice), while the product of the right-hand sides is -1. Therefore, the whole information on the correlations between the particles of the three pairs cannot be encoded in the initial state as we assumed.

The conclusion to which Cabello arrives extends the result, already present in the KS theorem, that it is not possible to give definite values to properties which pertain to different contexts, to the following: it is not possible to give definite values to the correlations between properties which pertain to different contexts. It is important to stress at this point that, within his reading of Mermin's interpretation, Cabello assumed that correlations are given by *definite values* of correlations among the *properties*. Thus, in analogous manner to our definition of *actual preexistent property* (definition 5.3.1) we can define:

Definition 12.2.1. Actual Preexistent Correlations: *the definite values of correlations among the preexistent properties of a state at a definite time.*

The moral which must be drawn form this interesting set of papers seems to me a valuable one: not even correlations (understood in the sense of Cabello) are exempt of respecting the contextual character of quantum mechanics. Quantum correlations are contextual and thus, cannot be considered as preexistent elements of physical reality. In the following section we shall discuss if such a notion of correlation is well suited for the IIQM or if Mermin can escape the criticisms of Cabello by defining a new idea of correlation.

12.3 Seevinck's Bell Type Inequality

So still, in order to escape Cabello's theorem, we could argue together with Birman [41], that what Mermin considers to be correlations are not defined in terms of definite values of properties but rather, in terms of joint probability distributions. In a manner analogous to our definition of *statistical property* (definition 5.3.4) we can define now actual observed statistical correlations.

Definition 12.3.1. Actual Observed Statistical Correlations: *the correlations among subsystems are the mean values, at a time, of the system's observables that are products of individual subsystem observables.*

However, even if we bite the pill and assume, regardless of the problems just mentioned, that this notion of correlation is the one we need to sustain, (once again!) contextuality pops up and precludes the possibility of considering (also) such statistical correlations as making reference to an actual (non-contextual) state of affairs.

In [237] it was discussed whether the quantum world could be considered to be built up from local statistical correlations. Through the derivation of a Bell type inequality Seevinck was able to answer this question negatively. In order to do so, Seevinck considers the correlations between two spatially separated parties I and II which have, each of them, a bi-partite system. Assuming that each party determines the correlations (the joint probability distributions $P^{I}_{AB}(ab)$ and $P^{II}_{CD}(cd)$ of the bi-partite system at his side) and assuming local realism for the correlations, the joint probability distribution over the four possible outcomes factorises:

$$P_{AB,CD}(ab,cd) = \int P^{I}_{AB}(ab|\lambda)P^{I}_{AB}(ab|\lambda)\rho(\lambda)d\lambda \tag{12.17}$$

Suppose now that we deal with dichotomic quantities A, B, C, D with possible outcomes $a, b, c, d, -1, 1$. The mean value of the product of two correlations is given by

$$E(AB, CD) = \sum abcdP_{AB,CD}(ab, cd) \tag{12.18}$$

And because of the factorisability of Eq. 12.17 one can derive the following Bell inequality in the CHSH form:

$$|E(AB, CD) + E(AB, (CD)') + E((AB)', CD) - E((AB)', (CD)')| \leq 2 \tag{12.19}$$

Where $AB, (AB)'$ denote two sets of quantities that give rise to two different joint probabilities at party I, and equivalently for CD. With this Bell inequality in terms of correlations Seevinck is able to provide the quantum mechanical version of this Bell inequality:

$$|E_{Wo}(\text{ß})| = |E_{Wo}(AB, CD) + E_{Wo}(AB, (CD)') + E_{Wo}((AB)', CD) - E_{Wo}((AB)', (CD)')| \leq 2 \tag{12.20}$$

Where $E_{Wo}(AB, CD) = Tr[WoA \otimes B \otimes C \otimes D]$, and is the so called Bell operator.

Seevinck provides then an example of a violation with the following quantum experiment. Consider two sets of two dichotomic observables represented by self-adjoint operators a, a' and b, b' for party I or II respectively. Each observables acts on the subspace $H = C_2 \otimes C_2$ of the bi-partite system held by the respective party I or II. These observables are chosen to be dichotomous, i.e. to have possible outcomes in $-1, 1$. They are furthermore chosen to be sums of projection operators and thus give rise to unique joint probability distributions on the set of quantum states. Measuring these observables thus implies determining some quantum correlations. For these observables a, a' and b, b' the Bell operators on $H = C_2 \otimes C_2 \otimes C_2 \otimes C_2$ becomes $\text{ß} = a \otimes b + a \otimes b' + a' \otimes b + a' \otimes b'$. The observables of the following form.

$$a = P_{\Psi^+} + P_{\Phi^+} - P_{\Psi^-} - P_{\Phi^-} \tag{12.21}$$

$$a' = P_{|\uparrow\uparrow\rangle} + P_{|\uparrow\downarrow\rangle} - P_{|\downarrow\uparrow\rangle} - P_{|\downarrow\downarrow\rangle} \tag{12.22}$$

$$b = P_{|\uparrow\uparrow\rangle} + P_{|b+\rangle} - P_{b-} - P_{|\downarrow\downarrow\rangle} \tag{12.23}$$

$$b' = P_{|\downarrow\downarrow\rangle} + P_{|b'-\rangle} + P_{|\uparrow\downarrow\rangle} - P_{|\uparrow\uparrow\rangle} \tag{12.24}$$

where $|b^{\pm}\rangle = C^{\pm}(|\uparrow\downarrow\rangle + (1\sqrt{2})|\downarrow\uparrow\rangle)$ and $|b'\rangle = C^{\pm}(|\uparrow\downarrow\rangle + (-1\sqrt{2})|\downarrow\uparrow\rangle)$, with normalization coefficients $C\pm = (4 \pm 2\sqrt{2})^{-6}$. Then, the mean value of the Bell operators for the above choice of a, a', b, b' in the state $|\Psi\rangle$ is equal to

$$|Tr[\text{ß}|\Psi\rangle\langle\Psi|]| = 2\sqrt{2} \tag{12.25}$$

Which gives a violation of Bell inequality of Eq. 12.20 by a factor $\sqrt{2}$. This violation proves that quantum correlations considered as joint probability distributions can neither be considered to be local realistic elements of physical reality.

12.4 Quantum Correlations and the SSC Theorem Revisited

All of this sounds very strange if we recall the SSC theorem, which states that subsystem correlations (for any resolution of the system into subsystems) are enough to determine the state of the entire system uniquely. How is it possible to reconcile this theorem with Cabello's and Seevinck's constraints? In order to consider this question we must first of all understand the presuppositions involved in the SSC theorem.

The SSC theorem relies on three mathematical facts: Firstly, the mean values of all observables for the entire system determine its state. Secondly, the set of all products over subsystem of subsystem observables from a basis for the algebra of all such system-wide observables. Thirdly, the algorithm that supplies observables with their mean value is linear on the algebra of observables. Supposing that the system can be decomposed into subsystems, the question is then whether we can assume that the global state of the system can be completely determined by specifying the joint probability distributions, this is called the global state assumption. As noted by Seevinck [237], this assumption holds for classical probability theory and for quantum mechanics on a complex Hilbert space but it need not be satisfied in an arbitrary theory, which shows that the SSC theorem is not trivial. For example, Wootters [276] has shown that for quantum mechanics on a real Hilbert space the assumption does not hold because the correlations between subsystem do not suffice to build up the total state. The first important aspect to consider is the fact that, given a particular context (i.e. a cut of the system into subsystems), the subsystem correlations are enough to determine the system uniquely. Thus, for each context, the SSC theorem provides, in terms of joint probability distributions, a one-to-one relation in between the system and the (correlations between) subsystems. The problem is that the SSC theorem says nothing about the multiple sets of correlations which arise in each context. The SSC theorem remains silent about the contextual character of these correlations. But it is exactly this aspect which needs to be considered when attempting to interpret correlations as preexistent elements of physical reality. If Mermin attempts to discuss about a description of a world independent of consciousness and decisions, there has to be a sense in which such correlations exist independently of the particular choice of the context. In there is a world out there built up from correlations, one could imagine that that all such correlations should be compatible with a preexistent state of affairs. It is exactly this question which is asked by Michiel Seevinck in his paper.

For a classical system, the multiple sets of joint probability distributions arising from different cuts of the system into subsystems will provide a compatible set of joint probability distributions. But, do to contextuality this is not obvious in the quantum case. The theorem does not *necessarily* consider correlations as being derived from (local realistic) *actual values* of properties and this leaves the door open for contextuality to pop in once again. The paradox is resolved when we recognize that the SSC theorem is talking about *correlations* in a different manner from that of Cabello and Seevinck. The SSC theorem speaks about *statistical correlations*, but *there is no reference whatsoever to a preexistent set of actual properties from which such correlations are discovered.* Unlike classical statistical mechanics there is no presupposition of a definite state of affairs represented by a set of definite valued properties. There is no question about the preexistence of an actual state of affairs upon which the statistical correlations, which are given through the quantum state, supervene. Cabello talks about non-contextual *actual preexistent correlations*, correlations which imply an actual preexistent state of affairs. Mermin escape route is to say that one should consider, instead of actual preexistent correlations between properties, joint probability distributions. However, as Seevinck has shown, once such *actual observed statistical correlations* are thought to be supervenient on a, maybe unknown, but still preexistent (non-contextual) actual preexistent state of affairs —an idea which one needs to consider if one wants to recover the realist ideal of a world independent of our consciousness and existence as human beings— one can derive a Bell type inequality which shows that quantum statistical correlations are contextual and cannot be thought in terms of *actual preexistent statistical correlations.*

Definition 12.4.1. Actual Preexistent Statistical Correlations: *the non-contextual correlations among subsystems are the mean values, at a time, of all the system's observables that consist of products over subsystems of individual subsystem observables and which supervene on a actual preexistent state of affairs.*

Recalling our earlier analysis (see section 5.3), we must conclude there is a great difference in quantum theory when making reference to statistical or average values of properties on the one hand, and to actual values of properties on the other. In quantum mechanics, although we have well defined average values for every observable, when we want to go to the "actual preexistent level", the formal structure precludes this move. Contrary to classical statistical mechanics, the statistical findings of quantum mechanics cannot be referred to as providing information about an unknown *actual preexistent state of affairs.* In the literature this is related to the impossibility to interpret mixtures and quantum probabilistic statements in terms of ignorance. In other words, if quantum mechanics would be *only* talking about ensembles there would be no problem whatsoever, simply because the mean values are well defined for every and each observable. The problem only arises when we attempt to go to the level of preexistent actuality. Thus, while Cabello and Seevinck are making reference to an "actual preexistent state of affairs" the SSC theorem makes reference to a system described in statistical terms, a "statistical system" which does not necessarily supervene on an actual preexistent state of affairs. There is no metaphysical constraint implied by the SSC theorem with respect to the supervenient realm for which such statistical correlations arise and thus, it is not committed to non-contextuality.

The introduction of the distinction between *actual preexistent correlations*, to which Cabello refers, *actual observed statistical correlations*, to which the SSC theorem refers, and the *actual preexistent statistical correlations*, which is what Seevinck refers to, makes clear that within interpretational issues it is of outmost importance to carefully consider the presuppositions involved in our statements. We have recovered through this analysis an old lesson of quantum mechanics which says that definite values of properties relate in a contextual manner, now in a new light. The new lesson to be learnt is that also *quantum correlations* behave in a contextual manner. Mermin overinterpreted the SSC theorem by assuming that actual observed statistical correlations imply actual preexistent statistical correlations. This idea has lead him to mistaken statements regarding the possible interpretation of quantum correlations; i.e. that such correlations can be thought in terms of an independent (non-contextual) preexistent account of the world. Seevinck has shown that such presupposition is far from obvious. The contextual character of quantum mechanics cannot be bypassed by simply changing the discourse to a statistical level. Of course the subtleties involved are not easy to tackle. Independently of the criticisms we have exposed, we consider that the attempt of Mermin remains deeply interesting and original. Above all, the IIQM has led us to a lot of new understanding regarding the possible interpretation of the quantum formalism in terms of relations. Through the work of Mermin and others we have learnt that correlations in quantum mechanics are contextual.

V

Modality Revisited

The Metaphysics of Actuality and Potentiality

As we have argued throughout this dissertation, our interest is mainly concerned with the possibility to develop a constructive metaphysical scheme for quantum mechanics. Therefore, we have called attention to the importance of considering the notions of possibility and actuality in quantum mechanics as related to metaphysical schemes. In this sense, we believe we should remind ourselves of Alfred North Whitehead's [273] remark in relation to occidental philosophy that: "The safest general characterization of the European philosophical tradition is that it consists of a series of footnotes to Plato." We could also add: "and Aristotle!", for it was he who developed in a formal scheme the basis of present classical logic, and with it, our basic formal representation and understanding of the world. In this respect, as noticed by Giorgio Agamben [13]: "The concept of potentiality has a long history in Western philosophy, in which it has occupied a central position at least since Aristotle. In both his metaphysics and physics, Aristotle opposed potentiality to actuality, *dynamis* to *energeia*, and bequeathed this opposition to Western philosophy and science." Our constructive metaphysical scheme calls for an ontological consideration of the concepts involved within physical theories. An obvious candidate to interpret possibility in ontological terms is the Aristotelian notion of potentiality. This path has been developed in the past, in relation to quantum mechanics, by authors like Heisenberg, Margenau and Popper. This development, although marginal, has continued to our days. Today, there is an ongoing work in philosophy of science which has recovered the Aristotelian metaphysics of causal capacities and attempts to interpret physics in general, and quantum theory in particular, following these lines of thought [198; 199; 2; 67; 68; 248; 109; 112; 116]. However, these attempts rely on a one sided interpretation of potentiality which relates to what Aristotle calls *irrational potentiality.* In the following chapter we attempt to go back to Aristotle's original distinction between *rational* and *irrational* potentiality. We shall discuss the metaphysical configuration of both actuality and potentiality in Aristotelian metaphysics in order to propose, in the following chapter, a different strategy to interpret quantum mechanics based on the notion of *rational potentiality.* We believe that this concept could provide new insights for thinking about the metaphysical questions posed by quantum theory. This chapter will also allow us to clearly distinguish the very different metaphysical interpretations of quantum possibility.

13.1 The Actual and Potential Realms in Aristotelian Metaphysics

The debate in Pre-Socratic philosophy is traditionally understood as the contraposition of the Heraclitean and the Eleatic school of thought [232]. Heraclitus was considered as defending the theory of flux, a doctrine of permanent motion and unstability in the world. He stated that the ever ongoing change or motion characterizes this world and its phenomena. This doctrine precluded, as both Plato and Aristotle stressed repeatedly, the possibility to develop certain knowledge about the world. An ever changing object does not allow even to be named, let alone to be known. “This is so because Being, over a lapse of time, has no stability. Everything that it is at this moment changes at the same time, therefore it is not. This coming together of Being and non-Being at one instant is known as the principle of coincidence of opposites.” ([262], p. 2) In contraposition to the Heraclitean school we find Parmenides as the main character of the Eleatic school. Parmenides, as interpreted also by Plato and Aristotle, taught the non-existence of motion and change in reality, reality being absolutely One, and being absolutely Being. In his famous poem Parmenides stated maybe the earliest intuitive exposition of the *principle of non-contradiction*; i.e. that which *is* can only *be*, that which *is not, cannot be.* Aristotle developed a metaphysical scheme in which, through the notions of *actuality* and *potentiality*, he was able to articulate both the Heraclitean and the Eleatic school. The well known phrase of Aristotle: “Being is said in different ways” refers to the modes of being in which Being itself can be thought to exist either in the realm of actuality or in the realm of potentiality.

In relation to the stable non-contradictory realm of actual existence *hic et nunc*, Aristotle developed —in order to solve the problem of movement— a logical scheme in which the principles of *existence*, *non-contradiction* and *identity* would constitute the concept of entity.[63] Through these principles the notion of entity is capable of unifying, of totalizing in terms of a “sameness” creating certain stability for knowledge to be possible. This representation or transcendent description of the world is considered by many the origin itself of metaphysical thought.[64] Actuality is then linked directly to metaphysical representation and understood as

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[63] There are three main principles which determine classical (Aristotelian) logic, namely, the existence of objects of knowledge, the principle of non-contradiction and the principle of identity. As noticed by Verelst and Coecke, these principles are “exemplified in the three possible usages of the verb ‘to be’: existential, predicative, and identical. The Aristotelian syllogism always starts with the affirmation of existence: something is. The principle of contradiction then concerns the way one can speak (predicate) validly about this existing object, i.e. about the true and falsehood of its having properties, not about its being in existence. The principle of identity states that the entity is identical to itself at any moment (a=a), thus granting the stability necessary to name (identify) it.” K. Verlest and B. Coecke (Quoted from [262], p. 167)

[64] As noticed by Verelst and Coecke: “the ‘contradiction’ seen by classical philosophy between Heraclitus and Parmenides is not necessarily a correct understanding of the earlier ‘philosophies’. One could as well infer that Heraclitus and Parmenides do articulate the same world-experience, the former as the experience of reality over a lapse of time, the latter as the experience of the absolute reality of this moment (to understand better what this means, try to deny by yourself you are experiencing yourself as existing at this moment). This has nothing to do with the intellectual question what it means to exist, or whether our existence is ‘real’ or not. These questions concern things ‘as such’, objects, and their identity in past and future. But this type of interpretation —which is the interpretation of classical philosophy and of science, and which entails a representation of reality outside of its actual and momentaneous experience doesn’t make sense, because for Heraclitus no things ‘as such’ do exist, and for Parmenides there is no motion, which implies that there is no

characterizing a mode of existence independent of observation and the *hic et nunc*. This is the way through which metaphysical thought was able to evade the 'I' and go into a 'third person scheme'. A world beyond the world, a world of concepts. But apart from considering the realm of actuality as part of his metaphysical scheme, Aristotle also characterized the realm of potentiality. Furthermore, in the book Θ of *Metaphysics*, Aristotle (1046b5-1046b24) remarks there are two types of potentiality: "[...] some potentialities will be non-rational and some will be accompanied by reason." For obvious reasons Aristotle calls these two potentialities rational and irrational. In the following we shall discuss these two distinct notions.

13.1.1 Irrational Potentiality

In his book, *Potentialities*, Giorgio Agamben discusses the meaning of irrational potentiality in Aristotle's metaphysics: "There is a generic potentiality, and this is the one that is meant when we say, for example, that a child has the potential to know, or that he or she can potentially become the head of the State." The child has the potentiality to become something else than what he is in actuality. Irrational potentiality implies a realm of 'indefiniteness', a realm of 'incompleteness' and 'lack'. It is then, only when turning into actuality, that the potential is fulfilled, completed. The child becomes then a man, the seed can transform into a tree.

> "The word 'actuality', which we connect with fulfillment, has, strictly speaking, been extended from movements to other things; for actuality in the strict sense is identified with movement. And so people do not assign movement to non-existent things, though they do assign some other predicates. E.g. they say that non-existent things are objects of thought and desire, but not that they are moved; and this because, while they do not actually exist, they would have to exist actually if they were moved. For of non-existent things some exist potentially; but they do not exist, because they do not exist in fulfillment." Aristotle (1047b3-1047b14)

The path from irrational potentiality into actualization may be related to the *process* through which *matter* turns into *form*. The matter of a substance being the stuff it is composed of; the form, the way that stuff is put together so that the whole it constitutes can perform its characteristic functions. Through this passage substance becomes more perfect and, in this way, closer to God, *pure acto* (1051a4-1051a17).[65] Because of this it makes no sense to consider the realm of irrational potentiality independently of actuality. Causality plays a major role in this sense, for it is the link which allows to close the gap in between both realms. As noticed by Smets in [242] the idea of irrational potentiality is directly linked to Aristotle's theory

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time. It is our conviction that, rather than revealing the contradiction between the 'thought-systems' of the two pre-Socratic 'philosophers', Plato's interpretation reveals the difference between their world-experience and what we think to be ours, constructed on the rational base laid down by classical philosophy. The non-existence of metaphysical worldviews in the pre-Socratic period is then due to a different kind of awareness of one's being-in-the-world that characterized the transition from mythical awareness to rational self-consciousnes." See also references therein.

[65] As noticed by Verelst and Coecke ([262], p. 168): "change and motion are intrinsically not provided for in this [Aristotelian logical] framework; therefore the ontology underlying the logical system of knowledge is essentially static, and requires the introduction of a First Mover with a proper ontological status beyond the phenomena for whose change and motion he must account for." This first mover is God, *pure acto*, pure definiteness and form without the contradiction and evil present in the potential matter.

of teleological causality: “the transition from being [irrational] potential to actual has to be placed within the context of [Aristotle] theory of movement and change, which is embedded in his teleological conception of causality (1050a7).” It is this teleological aspect which shows the extreme delimitation of the realm of irrational potentiality with respect to actuality. Irrational potentiality can be only thought in terms of its actualization, in terms of its passage into the actual.

Although Aristotle first argues that both actuality and potentiality must be considered as independent ontological modes of existence it becomes clear that very soon he chooses the actual realm as superior to the potential realm (see [75], section 12). However, and independently of this choice, according to Agamben ([13], p. 179), it is not this potentiality which seems to interests Aristotle, rather, it is “the one that belongs to someone who, for example, has knowledge or ability. In this sense, we say of the architect that he or she has the *potential* to build, of the poet that he or she has the *potential* to write poems. It is clear that this *existing* potentiality differs from the *generic* potentiality of the child.” We shall now turn our attention to this second kind of potentiality which, we believe, can allow us to develop a notion of potentiality independent of the actual realm and actualization —evading at the same time causal considerations.

13.1.2 Rational Potentiality

Rational potentiality is characterized by Aristotle as related to the problem of possessing a capability, a faculty (1046b5-1046b24), to what I mean when I say: “I can”, “I cannot”. As explicitly noticed by Aristotle, potentiality implies a mode of existence which must be considered as real as actuality. In chapter 3 of book Θ of *Metaphysics* Aristotle introduces the notion of rational potentiality. Aristotle goes against the Megarians who considered actuality as the only mode of existence:

> “There are some who say, as the Megaric school does, that a thing can act only when it is acting, and when it is not acting it cannot act, e.g. he who is not building cannot build, but only he who is building, when he is building; and so in all other cases. It is not hard to see the absurdities that attend this view. For it is clear that on this view a man will not be a builder unless he is building (for to be a builder is to be able to build), and so with the other arts. If, then, it is impossible to have such arts if one has not at some time learnt and acquired them, and it is then impossible not to have them if one has not sometime lost them (either by forgetfulness or by some accident or by time; for it cannot be by the destruction of the object itself, for that lasts for ever), a man will not have the art when he has ceased to use it, and yet he may immediately build again; how then will he have got the art?” Aristotle (1046b29 - 1047a10)

Aristotle then continues:

> “[...] evidently potentiality and actuality are different; but these views make potentiality and actuality the same, so that it is no small thing they are seeking to annihilate. [...] Therefore it is possible that a thing may be capable of being and not be, and capable of not being and yet be, and similarly with the other kinds of predicate; it may be capable of walking and yet not walk, or capable of not walking and yet walk.” Aristotle (1046b29 - 1047a10)

That which exists within rational potentiality is then characterized as being capable of both contrary effects:

> "Since that which is capable is capable of something and at some time and in some way —with all the other qualifications which must be present in the definition—, and since some things can work according to a rational formula and their potentialities involve a formula, while other things are non-rational and their potentialities are non-rational, and the former potentialities must be in a living thing, while the latter can be both in the living and in the lifeless; as regards potentialities of the latter kind, when the agent and the patient meet in the way appropriate to the potentiality in question, the one must act and the other be acted on, but with the former kind this is not necessary. For the non-rational potentialities are all productive of one effect each, but the rational produce contrary effects, so that they would produce contrary effects at the same time; but this is impossible. That which decides, then, must be something else; I mean by this, desire or choice." Aristotle (1048a1-1048a24)

This also means that potentiality is capable of being and not being at one and the same time.

> "Every potentiality is at one and the same time a potentiality for the opposite; for, while that which is not capable of being present in a subject cannot be present, everything that is capable of being may possibly not be actual. That, then, which is capable of being may either be or not be; the same thing, then, is capable both of being and of not being." Aristotle (1050b7-1050b28)

The contradiction of being and not being present in rational potentiality is only dissolved when, considering the actual realm, one of the terms is effectuated. Contrary to the case of irrational potentiality, where a teleological cause places the end in actuality, rational potentiality might be interpreted, following Agamben [13], as a realm independent of actuality. According to Agamben: "the key figure of potentiality, the mode of existence as potentiality [...] is a potentiality that is not simply the potential to do this or that thing but potential to not-do, potential not to pass into actuality. This is why Aristotle criticizes the position of the Megarians, who maintain that all potentiality exists in actuality. What Aristotle wants to posit is the existence of potentiality: that there is a presence and a face of potentiality." It is not clear if such an interpretation lies beyond the limits of the Aristotelian scheme.

Aristotle calls attention to the fact that such rational potentiality can become actual only when the state of affairs allows it. Thus, "[...] everything which has a rational potentiality, when it desires that for which it has a potentiality and in the circumstances in which it has it, must do this. And it has the potentiality in question when the passive object is present and is in a certain state; if not it will not be able to act." (1048a1-1048a24) This opens the question of the existence of such potentiality independent or not of the actual state of affairs. While irrational potentialities are automatically triggered when active and passive potentialities come together, this is not the case with rational potentialities, as a rational agent can choose to withhold the realization of the potentiality even though it can be realized. This might allow us to think a realm of potentiality completely independent of actuality, a non-causal realm of potentiality (which we will consider in section 14.1.6). Contrary to this idea, it seems Aristotle choose once again to limit the expressivity of potentiality within the gates of the actual realm.

> "To add the qualification 'if nothing external prevents it' is not further necessary; for it has the potentiality in so far as this is a potentiality of acting, and it is this not in all circumstances but on certain conditions, among which will be the exclusion of external hindrances; for these are barred by some of the positive qualifications. And so even if one has a rational wish, or an appetite, to do two things or contrary things at the same time, one cannot do them; for it is not on these terms that one has the potentiality for them, nor is it a potentiality for doing both at the same time, since one will do just the things which it is a potentiality for doing." Aristotle (1048a25-1048b9)

From chapter 6 of book Θ, Aristotle concentrates on the relation between potentiality and actuality, placing actuality in central place of his architectonic and relegating potentiality to a mere supplementary element: "We have distinguished the various senses of 'prior', and it is clear that actuality is prior to potentiality. [...] For the action is the end, and the actuality is the action. Therefore even the word 'actuality' is derived from 'action', and points to the fulfillment." (1050a17-1050a23) Aristotle then continues to provide arguments in this line which show "that the good actuality is better and more valuable than the good potentiality is evident" Aristotle (1051a4-1051a17) (see [75], section 12).

Independently of the Aristotelian architectonic, we argue that the idea of rational potentiality could be used to developed a mode of existence completely independent of actuality. As recognized by Pauli:

> "Aristotle [...] created the important concept of *potential being* and applied it to *hyle.* [...] This is where an important differentiation in scientific thinking came in. Aristotle's further statements on matter cannot really be applied in physics, and it seems to me that much of the confusion in Aristotle stems from the fact that being by far the less able thinker, he was completely overwhelmed by Plato. He was not able to fully carry out his intention to grasp the *potential*, and his endeavors became bogged down in early stages." W. Pauli (Quoted from [194], p. 93)

We believe it was Pauli who had most clearly seen this path in relation to the development of quantum mechanics itself. As noted in a letter to C. G. Jung dated 27 February 1953:

> "Science today has now, I believe, arrived at a stage were it can proceed (albeit in a way as yet not at all clear) along the path laid down by Aristotle. The complementarity characteristics of the electron (and the atom) (wave and particle) are in fact 'potential being,' but one of them is always 'actual nonbeing.' *That is why one can say that science, being no longer classical, is for the first time a genuine theory of becoming and no longer Platonic.*" W. Pauli (Quoted from [194], p. 93, emphasis added)

13.2 The Actual Realm and Classical Physics

The importance of potentiality, which was first placed by Aristotle on equal footing to actuality as a mode of existence, was soon diminished in the history of Western thought. As we have seen above, it could be argued that the seed of this move was already present in the Aristotelian architectonic, whose focus was clearly placed in the actual realm. The realm of potentiality, as a different (ontological) mode of the being was neglected becoming not more than mere (logical) *possibility*, a process of fulfillment.[66] In relation to the development of physics, the focus and preeminence was also given to actuality. The XVII century division between 'res cogitans' and 'res extensa' played in this respect an important role separating also the realms of actuality and potentiality. The philosophy which was developed after Descartes kept 'res cogitans' (thought) and 'res extensa' (entities as acquired by the senses) as separated realms.[67]

> "Descartes knew the undisputable necessity of the connection, but philosophy and natural science in the following period developed on the basis of the polarity between the 'res cogitans' and the 'res extensa', and natural science concentrated its interest on the 'res extensa'. The influence of the Cartesian division on human thought in the following centuries can hardly be overestimated, but it is just this division which we have to criticize later from the development of physics in our time." W. Heisenberg (Quoted from [141], p. 73)

This materialistic conception of science based itself on the main idea that extended things exist as being definite, that is, in the actual realm of existence. Paradoxically as it might seem, the division produced in the XVII century between 'res cogitans' and 'res extensa' together with the subsequent preeminence of "extended things" could be understood as the triumph of the actualist Megarian path. It is also true that the transformation from medieval to modern science coincides with the abolition of Aristotelian metaphysics as the foundation of knowledge. However, the basic structure of his metaphysical scheme and his logic still remained the basis for correct reasoning, the principle of non-contradiction —as Kant, Leibniz and many others proclaimed— the most certain of all principles.[68] It was Isaac Newton who was able to translate into a closed mathematical formalism both, the ontological presuppositions present in Aristotelian logic and the materialistic ideal of 'res extensa' together with actuality as its mode of existence. In classical physics, every physical system may be described exclusively by means of its actual properties. A point in phase space is related to the set of values of properties that characterize the system. In fact, an actual property can be made to correspond to the set of states (points in phase space) for which this property is actual. The change of the system may be described by the change of its actual properties. Potential or possible properties are considered as the points to which the system might arrive in a future instant of time. These properties can be thought in terms of irrational potentiality, as properties which

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[66] In order to properly consider the development of the potential and the actual in Western thought we refer to [238].

[67] While 'res cogitans', the soul, was related to the *indefinite* realm of potentiality and is discussed by Aristotle in *De Anima*, 'res extensa', the entities as characterized by the principles of logic gave place to the actual considered in terms of *definiteness*.

[68] As noticed by Verlest and Coecke (Quoted from [262], p. 7): "Dropping Aristotelian metaphysics, while at the same time continuing to use Aristotelian logic as an empty 'reasoning apparatus' implies therefore loosing the possibility to account for change and motion in whatever description of the world that is based on it. The fact that Aristotelian logic transformed during the twentieth century into different formal, axiomatic logical systems used in today's philosophy and science doesn't really matter, because the fundamental principle, and therefore the fundamental ontology, remained the same ([40], p. xix). This 'emptied' logic actually contains an Eleatic ontology, that allows only for static descriptions of the world."

might become actual. As also noted by Dieks: "In classical physics the most fundamental description of a physical system (a point in phase space) reflects only the actual, and nothing that is merely possible. It is true that sometimes states involving probabilities occur in classical physics: think of the probability distributions ρ in statistical mechanics. But the occurrence of possibilities in such cases merely reflects our ignorance about what is actual. The statistical states do not correspond to features of the actual system (unlike the case of the quantum mechanical superpositions), but quantify our lack of knowledge of those actual features." Classical mechanics tells us via the equation of motion how the state of the system moves along the curve determined by initial conditions in the phase space and thus, as any mechanical property may be expressed in terms of phase space variables. Useless to say, in the classical realm the measurement process plays no distinctive role and actual properties fit the definition of *elements of physical reality* in the sense of the EPR paper [113]. Moreover, the structure in which actual properties may be organized is the (Boolean) algebra of classical logic.

13.3 Heisenberg and the Recovery of the Potential Realm

The mechanical description of the world provided by Newton can be sketched in terms of static pictures which provide at each instant of time the set of definite actual properties within a given state of affairs (see [163], p. 609). Obviously there is in this description a big debt to the Aristotelian metaphysical scheme. However, the description of motion is then given, not *via* the path from the irrational potential to the actual, from *matter* into *form*, but rather *via* the successions of actual states of affairs; i.e., "pictures" constituted by sets of actual properties with definite values. As we discussed above, potentiality becomes then superfluous. With the advenment of modern science and the introduction of mathematical schemes, physics seemed capable of reproducing the evolution of the universe. The idea of an actual state of affairs (i.e. the set of actual properties which characterize a system) supplemented by the dynamics allowed then to imagine a Demon such as that of Laplace capable of knowing the past and future states of the universe. If we could know the actual values at the definite instant of time we could also derive the actual set of properties in the future and the past. As Heisenberg explains, this materialistic conception of science chose actuality as the main aspect of existence:

> "In the philosophy of Aristotle, matter was thought of in the relation between form and matter. All that we perceive in the world of phenomena around us is formed matter. Matter is in itself not a reality but only a possibility, a 'potentia'; it exists only by means of form. In the natural process the 'essence,' as Aristotle calls it, passes over from mere possibility through form into actuality. [...] Then, much later, starting from the philosophy of Descartes, matter was primarily thought of as opposed to mind. There were the two complementary aspects of the world, 'matter' and 'mind,' or, as Descartes put it, the 'res extensa' and the 'res cogitans.' Since the new methodical principles of natural science, especially of mechanics, excluded all tracing of corporeal phenomena back to spiritual forces, matter could be considered as a reality of its own independent of the mind and of any supernatural powers. The 'matter' of this period is 'formed matter,' the process of formation being interpreted as a causal chain of mechanical interactions; it has lost its

> connection with the vegetative soul of Aristotelian philosophy, and therefore the dualism between matter and form [potential and actual] is no longer relevant. It is this concept of matter which constitutes by far the strongest component in our present use of the word 'matter'." W. Heisenberg (Quoted from [141], p. 129)

As mentioned above, in classical mechanics the mathematical description of the behavior of a system may be formulated in terms of the set of actual properties. The same treatment can be applied to quantum mechanics. However, the different structure of the physical properties of the system in the new theory determines a change of nature regarding the meaning of possibility and potentiality. As we argued on chapter 7, quantum mechanics was related to modality since Born's interpretation of the quantum wave function Ψ in terms of a density of probability. But it was clear from the very beginning that this new quantum possibility was something completely different from that considered in classical theories. "[The] concept of the probability wave [in quantum mechanics] was something entirely new in theoretical physics since Newton. Probability in mathematics or in statistical mechanics means a statement about our degree of knowledge of the actual situation. In throwing dice we do not know the fine details of the motion of our hands which determine the fall of the dice and therefore we say that the probability for throwing a special number is just one in six. The probability wave function, however, meant more than that; it meant a tendency for something." ([141, p. 42]) It was Heisenberg who went a step further and tried to interpret the wave function in terms of the Aristotelian notion of potentia. Heisenberg argued that the concept of probability wave "was a quantitative version of the old concept of 'potentia' in Aristotelian philosophy. It introduced something standing in the middle between the idea of an event and the actual event, a strange kind of physical reality just in the middle between possibility and reality." According to Heisenberg, the concept of potentiality as a mode of existence has been used implicitly or explicitly in the development of quantum mechanics:

> "I believe that the language actually used by physicists when they speak about atomic events produces in their minds similar notions as the concept of 'potentia'. So physicists have gradually become accustomed to considering the electronic orbits, etc., not as reality but rather as a kind of 'potentia'." W. Heisenberg (Quoted from [141], p. 156)

In this respect, one of the most interesting examples of an implicit use of these ideas has been provided by Richard Feynmann in his path integral approach [120]. Even though Feynman talks about calculating probabilities, he thinks in terms of existent potentialities. Why, if not, should we take into account the mutually incompatible paths of the electron in the double-slit experiment? His approach takes into account every path as existent in the mode of being of potentiality, there where the constraints of actuality cannot be applied. But as we discussed above, Heisenberg's attempt to interpret quantum mechanics with a non-classical conceptual scheme might have been highly compromised by Bohr's own agenda. In any case, we must admit that apart from some few remarks and analogies, Heisenberg's interpretation remained not only incomplete but also unclear in many aspects.

An important aspect which must be explicitly noticed is the fact that, although Heisenberg criticized the abolition of the potential realm in science and attempted to go back to this realm in order to overcome the interpretational problems of quantum mechanics, he restricted potentiality to the consideration of *irrational potentiality* only, neglecting completely *rational potentiality*. But, as we have seen above, irrational potentiality is subsidiary —through its teleological relation to actuality— to the actual realm. Thus, by restricting his analysis to irrational potentiality Heisenberg was also trapped within an actualist account of reality.[69]

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[69] In this sense it is interesting to take into account the question posed by Heisenberg to Henry Stapp regarding the ontological meaning of ideas: "When you speak about the ideas (especially in [Section 3.4]), you always speak about human ideas, and the question arises, do these ideas 'exist' outside of the human mind or only in the human mind? In other words: Have these ideas existed at a time when no human mind existed in the world. (Heisenberg, 1972)" (Quoted from [244]).

Quantum Possibility and the Interpretation of Superpositions

As we discussed in chapter 7, the formal difference of using vectors in $\mathcal{H}$ instead of points in Γ implies that in quantum mechanics —apart from the realm of possibility which is encountered in classical mechanics— there is another, different realm which must be necessarily considered and refers, at each instant of time, to *indefinite properties.* To see this, consider the following example: given a spin 1/2 system whose state is $\mid \uparrow_z>$, we let it interact with a magnetic field in the z direction. All outcomes that will become actual in the future are irrational potential properties of the system, in an analogous manner as all possible reachable positions of a pendulum in the classical case. But at each instant of time, for example at the initial instant, if we consider the z direction and the projection operator $\mid \uparrow_z><\uparrow_z \mid$ as representing a preexistent actual property, there are other incompatible properties, namely, spin projections in other directions. For example, in the x direction, the projection operators $\mid \uparrow_x><\uparrow_x \mid$ and $\mid \downarrow_x><\downarrow_x \mid$ do not commute with $\mid \uparrow_z><\uparrow_z \mid$ and thus, cannot be considered to possess definite values simultaneously. In view of the Born interpretation of the wave function, these properties are usually considered as *possible.* However, this possibility is essentially different from the idea of possibility discussed in classical physics which relates to irrational potentiality or the idea of a process. If we consider that the formalism of quantum mechanics provides a description of the world, a representation of *what there is* —and does not merely make reference to measurement outcomes—, at each instant of time all properties (e.g. $\mid \uparrow_z><\uparrow_z \mid$, $\mid \uparrow_x><\uparrow_x \mid$ and $\mid \downarrow_x><\downarrow_x \mid$) must be considered in terms of their mode of existence. Or in other words, in what sense do these properties exist according to quantum mechanics?

In the quantum logic approach one of the properties, namely, the one in which we can write the state of affairs as a single term is considered as *actual* while the others as *potential* —these properties can become actual in a future instant of time according to the probability given by the numbers in square modulus accompanying the states. These properties, e.g. $\mid \uparrow_x><\uparrow_x \mid$, $\mid \downarrow_x><\downarrow_x \mid$, $\mid \uparrow_y><\uparrow_y \mid$ and $\mid \downarrow_y><\downarrow_y \mid$ in our example, are always part of superpositions with more than one term. However, from a mathematical perspective, independently of their mode of existence, both potential and actual properties are placed at the same level in the algebraic frame which describes the state of affairs according to quantum mechanics: the projections of the spin in all directions are atoms of the lattice and there is no formal priority of the actual properties over the potential properties. The difference only appears with the choice of the Boolean subalgebra of the context, it is only then that we can distinguish between actual (e.g. $\mid \uparrow_z >< \uparrow_z \mid$) and potential (e.g. $\mid \uparrow_x >< \uparrow_x \mid$ and $\mid \downarrow_x >< \downarrow_x \mid$) properties. In the laboratory, it is precisely this potential realm which the experimentalist needs to consider in the developments which are taking place today regarding the processing of quantum information (think of quantum computing and quantum communication) [178; 192]. This seems to point in the direction that these properties have an existence which cannot be reduced to their becoming actual at a future instant of time.

In the following chapter we shall review some of the different interpretations of possibility present in the literature, we shall also investigate how these different notions relate to the specific meaning of superpositions in quantum mechanics. We shall also discuss the strategy we propose to follow through our constructive metaphysical approach.

14.1 The Different Interpretations of Quantum Possibility

The formal description of quantum mechanics seems to imply a deep departure from the classical notion of possible or probable. However, there are many interpretations which attempt to consider the problem of modality in quantum theory and try to provide a convincing answer to the question. The empiricist position has no problem at all with the meaning of modality, for as van Fraassen claims, modality can be regarded as a theoretical device with no ontological meaning: *modalities are in our theories, not in the world.* The realist position, taken for example by the hidden variable program, considers that quantum mechanics can be understood in terms of classical concepts —such as well defined positions and trajectories— and that the probable and the possible are in no way different from the notions encountered in classical physical theories. For different reasons these two positions seem to restrict the importance of the formalism when the time comes to interpret it as telling us something about the world. Both stances have their own arguments to support their views. There are however other positions which reflect a deep interest in starting their analysis from the formal structure of the theory and deriving metaphysical consequences from it. One of them is the so called many worlds interpretation which claims that possibility can be understood in terms of multiple worlds; what is called *modal realism.* A different position is taken by the Geneva School of quantum logic which —within an operationalistic perspective— considers, apart from the mode of being of *actuality*, a mode of being called *potentiality* —which can be equated to what Aristotle calls irrational potentiality. A related position is that of Suárez, Dorato and Esfeld who call for an interpretation in terms of *propensities* or *dispositions.* These last attempts, rather than pertaining to the classical realist accounts, could be considered under the umbrella of a constructive metaphysical stance.

Closely related to the interpretation of possibility in quantum mechanics stands the meaning of the notion of superposition. For, as we shall argue, each notion of quantum possibility implies as well an interpretation of the meaning of a quantum superposition. Put in a nutshell: Tell me what interpretation of possibility you choose and I will tell you what interpretation of a quantum superposition you have (or *vice versa*). In this respect, it is important to clearly distinguish between the different levels of discourse. In this sense, we accept the remark of Dieks ([89], p. 189) regarding the superposition: “It is the state vector which is in a superposition, not the cat itself. ‘State vector’ and ‘cat’ are two concepts at different levels of discourse.” However, the question still arises from a representational realist stance, whether this formal or mathematical representation given by a superposition, namely equation 7.1 —which allow us to calculate the probability of the possible measurement outcomes—, can be related conceptually to a notion which can allow us to think —independently of measurement outcomes— about the 'superposition of states in Hilbert space' in an analogous manner as we think of a 'point in phase space' (in the formal level) as describing an 'object in space-time' (in the conceptual level). From the stance we have put forward we need to answer the question: What is a mathematical superposition describing in metaphysical terms? Can we create or find adequate concepts which can provide a representational realistic account of a quantum superposition independent of measurement outcomes? To provide an answer to this question is what we consider the representational realist quest. There are different proposals which sustain our view that such interpretation can be developed. In the following we shall discuss the several interpretations of possibility and superpositions in quantum mechanics. Furthermore, in section 14.1.6, we shall propose a new interpretation of quantum possibility based on the notion of rational potentiality.

14.1.1 Quantum Possibility as a Theoretical Device

The idea that the quantum wave function is just a theoretical device with no ontological content goes back to Bohr's interpretation of quantum mechanics.[70] As we discussed earlier, this position, if radically worked out seems to end up in the instrumentalistic account shared implicitly by many and developed explicitly by Fuchs and Peres (see section 2.1). Bas van Fraassen, whom we consider a close follower of Bohr's ideas, has also taken an anti-metaphysical position with respect to the interpretation of the quantum wave function. His justification stands on his empiricist account of possibility. According to van Fraassen ([253], p. 197): "the only believe involved in accepting a scientific theory is belief that it is empirically adequate: all that is *both* actual *and* observable finds a place in some model of the theory. So far as empirical adequacy is concerned, the theory would be just as good if there existed nothing at all that was either unobservable or not actual. Acceptance of the theory does not commit us to belief in the reality of either sort of thing." Modality is then taken as a theoretical structure which allow us to save the phenomena. From an empiricist perspective the formalism only needs to account for the actual measurement result, wether we have or not a metaphysical scheme which allows to configure this actual observation is of a secondary importance. Empiricism is linked by van Fraassen to probability in terms of the frequency interpretation which rests, contrary to the original conception of probability, not on the idea that probability describes

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[70] More explicitly, this question was addressed by Bohr in a letter to Carl Friedrich von Weizsäcker. Von Weizsäcker had wrote an article named: *"Komplementarität und Natuurwissenschaft"* for the 70th birthday of Niels Bohr. In this article he explained the concept of complementarity in two different forms, namely, what he called *parallel complementarity* and *circular complementarity. Parallel complementarity* was defined by von Weizsäcker as the complementary relation which takes place between, for example, *position* and *momentum* or *particles* and *waves.* On the other hand *circular complementarity* was defined by von Weizsäcker as complementarity between *classical concepts* and the description given by the *Schrödinger wave function.* These two definitions were attributed to Bohr himself. The article ends with a rectification of von Weizsäcker in which he explains that he received a letter from Bohr ([270], p. 338) expressing that complementarity can be only defined with respect to phenomena, and as the Schrödinger wave equation is just an *abstract magnitude of calculus* and it does not designate in itself any phenomena, such circular complementarity is by no means possible. According to Bohr, only parallel complementarity as described by Weizsäcker should be taken into account.

an existing state of affairs in terms of ignorance, but rather in terms of the set of empirical results found in a series of repeated measurements. In tune with van Fraassen, Dieks argues in one of his latest papers in favor of a Humean position:

> "The Humean maintains that we need to assume the existence of only one world, namely the ordinary actual one; that the regularities of this world are expressed in our laws and theories; and that we introduce possible other worlds and counterfactual circumstances purely as thought constructions, in order to bring out the peculiarities of the laws we have formulated. Possible worlds are mental tools and not really existing entities. Modalities, like necessity and possibility, are concepts we introduce on the basis of our theories and do not correspond to features of reality that transcend the ordinary description in terms of actual events." D. Dieks ([100], p. 126)

Both van Fraassen and Dieks share an open attitude with respect to what the world is like (or may be like) according to quantum mechanics. However, while van Fraassen remains agnostic regarding whether physical magnitudes have values, Dieks proposes a property ascription which allows to consider one of the properties as actual (independently of measurement). Van Fraassen seems forced to take superpositions as a theoretical device which —alike possibility— allows to predict actual measurement outcomes in a probabilistic manner. It seems one could say with respect to possibility, following van Fraassen's attitude, that *quantum superpositions are in our theories, not in the world.* In this case a metaphysical interpretation of superpositions remains absent. Dieks' move is more subtle for, though he proposes one of the properties to be actually existent (independently of measurement), the superposition —alike van Fraassen— is still understood as an algorithmic device to calculate the probability of *finding* this actual preexistent property and not as a description of how the world is in metaphysical terms. In the case of Dieks, alike in Bohm theory, there is an actual state of affairs and quantum possibility refers to it in the same epistemic manner as classical possibility. Following Dieks, we might be justified to state that: *There is always one actual property with a definite value. Quantum superpositions are in our theories, not in the world.*.

14.1.2 Quantum Possibility as Classical Possibility

As noticed by Bacciagaluppi ([22], p. 74), the hidden variable program attempts to "restore a classical way of thinking about *what there is*." In this sense, Bohm's proposal attempted to interpret quantum mechanics in terms of an *actualist* account of the world, or in other words, to restore the possibility to describe what there is in terms of a set of definite valued properties. In Bohmian mechanics the state of a system is given by the wave function Ψ together with the configuration of particles X. The quantum wave function must be understood in analogy to a classical field that moves the particles in accordance with the functional relation $\frac{dx}{dt} = \nabla S$, where $S = \hbar\delta$ (δ being the phase of ψ). Thus, particles always have a well defined position and the evolution depends on the quantum field. It then follows that, there are no quantum superpositions of states, the superposition is given only at the level of the field and remains as little mysterious as the superposition of classical fields. Given a quantum field $\varphi(x)$ the particle will move according to it. If we change the quantum field by adding another filed $\phi(x)$ such that the new quantum field is now the superposition: $\varphi(x)+\phi(x)$, there is no ontological peculiarity involved for now the particle also has a well defined position and will evolve according to the new field. Presumably, due to the fact that the new field is different from the original one, the particle will move in a different way and will follow a different trajectory compared to the first case. The field does not only have a dynamical character but also determines the epistemic probability of the configuration of particles *via* the usual Born rule. This means that the meaning of probable or possible recovers the classical ideal of discussing, though in terms of ignorance, about *what there is* (see in this respect [243; 65]). Possibility is then understood in epistemic terms and does not contain an ontological aspect. One can consider in this sense the proposals of Bub and Bacciagaluppi and Dickson as continuing the path laid down by the hidden variable program. Dieks might be also related to this scheme, for he also attempts to provide an interpretation in which there is one well defined actual property, however, his idea to retain the orthodox formalism seems to place a substantial difference with such proposals.

14.1.3 Quantum Possibility as Possible Worlds

A very different way to consider possibility has been developed in philosophy of quantum mechanics in terms of modal realism through the so called many worlds interpretation. Many worlds interpretations of quantum mechanics have become one of the most important lines of investigation within the foundations of quantum theory domain. As the modal interpretation, many worlds interpretation is a no-collapse interpretation and respects the orthodox formulation of quantum mechanics. The many worlds interpretation is considered to be a direct conclusion from Everett's first proposal [117] in terms of 'relative states'.[71]

> "[Everett's interpretation of quantum mechanics] denies the existence of a separate classical realm and asserts that it makes sense to talk about a state vector for the whole universe. This state vector never collapses and hence reality as a whole is rigorously deterministic. This reality, which is described jointly by the dynamical variables and the state vector, is not the reality we customarily think of, but is a reality composed of many worlds. By virtue of the temporal development of the dynamical variables the state vector decomposes naturally into orthogonal vectors, reflecting a continual splitting of the universe into a multitude of mutually unobservable but equally real worlds, in each of which every good measurement has yielded a definite result and in most of which the familiar statistical quantum laws hold." De Witt and Graham (Quoted from [84])

Everett's idea was to let quantum mechanics find its own interpretation, doing justice to the symmetries inherent in the Hilbert space formalism in a simple and convincing way [84]. As we mentioned above, the main idea behind many worlds interpretations is that superpositions relate to collections of worlds, in each of which exactly one value of an observable, which corresponds to one of the terms in the superposition, is realized. Apart from being simple, the claim is that it possesses a natural fit to the formalism, respecting its symmetries. The solution proposed to the measurement problem is provided by assuming that each one of the terms in the superposition is *actual* in its own world. Thus, it is not only the single value which we see in 'our world' which gets actualized but rather, that a branching of worlds takes place in every measurement, giving rise to a multiplicity of worlds with their corresponding actual values. The possible splits of the worlds are determined by the laws of quantum mechanics but each world becomes again 'classical'.

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[71] The subtleties involved in this interesting story are discussed in [191].

> “The whole issue of the transition from ‘possible’ to ‘actual’ is taken care of in the theory in a very simple way —there is no such transition, nor is such a transition necessary for the theory to be in accord with our experience. *From the viewpoint of the theory all elements of a superposition (all ‘branches’) are ‘actual’, none any more ‘real’ than the rest.* It is unnecessary to suppose that all but one are somehow destroyed, since all the separate elements of a superposition individually obey the wave equation with complete indifference to the presence or absence (‘actuality’ or not) of any other elements. This total lack of effect of one branch on another also implies that no observer will ever be aware of any ‘splitting’ process.” H. Everett (Quoted from [119], pp. 146-147, emphasis added)

The many worlds interpretations of quantum mechanics can be seen in quite analogous fashion to the proposal of David Lewis who attempts to understand modality itself, also in terms of possible worlds [179]. According to Lewis i) possible worlds exist and are as real as our world, ii) these worlds are the same sort of things as our world, only differing in content but not in kind. Furthermore, iii) the possible worlds are irreducible entities and the term actual is indexical; iv) possible worlds are unified by the spatiotemporal interrelations of their parts; v) every world is spatiotemporally isolated from every other world. And finally, vi) possible worlds are causally isolated from each other. Within this scheme Lewis is able to account for logical modality and provide a direct meaning to the notions of *possibility* and *necessity.* A proposition is *necessary* if it is true in all possible worlds, and *possible* if it is true in at least one.

But apart from his logical analysis Lewis goes one difficult step further into metaphysics and is very eager to defend these ‘possible worlds’ as actually existent ‘real worlds’.

> “When I profess realism about possible worlds, I mean to be taken literally. Possible worlds are what they are, and not some other thing. If asked what sort of thing they are, I cannot give the kind of reply my questioner probably expects: that is, a proposal to reduce possible worlds to something else. I can only ask him to admit that he knows what sort of thing our actual world is, and then explain that possible worlds are more things of that sort, differing not in kind but only in what goes on at them.” D. Lewis (Quoted from [179], p. 85)

Lewis provides some arguments in order to sustain his position regarding possible worlds. For example, regarding its pragmatic use, he claims that, since many abstract mathematical entities are held to exist simply because they are useful, the same should go for possible worlds (see for discussion [239] and [204]). Going back to quantum mechanics and the many worlds interpretations, there are also strong criticisms to the extension of multiple worlds into the realm of reality. For example, regarding the reification of modalities in the form of possible worlds, Dieks states in a recent paper:

> ‘ The added metaphysical burden is enormous here, while the theoretical virtues that should compensate this remain obscure. In particular, although it is true that the notion that all possibilities are equally real and that there is no ontological distinction between the actual and the possible (the central tenet of the now popular many worlds interpretation)

> resonates well with the democracy of the terms making up a quantum mechanical superposition, this same symmetry makes it difficult to explain and even to accommodate the indeterministic character of the theory. If all possibilities are realized in the same way, it appears there can be no room for probability considerations. There have been interesting attempts to introduce such probabilities nevertheless, and even to derive them as a natural consequence of the many worlds scheme (see Wilson, 2006; Wallace, forthcoming). The probabilities in these accounts are introduced as subjective uncertainties about the world one will end up in after a measurement; but uncertainties of this type remain difficult to square with the certainty that all worlds are to be considered as equally real and actual."
> D. Dieks (Quoted from [100], pp. 134-135)

Independently of these (mainly) metaphysical criticisms, in [107], we developed an algebraic framework which allows us to analyze and discuss the modal aspects of many worlds interpretations from a logical perspective. In particular, we can give a formal understanding of why modal interpretations fall pray to KS-type contradictions while many worlds interpretations escape them. In the many worlds interpretations, all possibilities encoded in the wave function take place, but in different worlds. In algebraic terms, when a measurement of a physical magnitude $\mathbf{M}$ is performed and one of its possible outcomes a_1 occurs, then in another world a_2 occurs, and in some other world a_3 occurs, etc. In modal wording, suppose that $\mathbf{M}$ has associated a Boolean sublattice $W_{\mathbf{M}}$ of $\mathcal{L}(\mathcal{H})$. The family $(\mathbf{P}_i)_i$ is identified as elements of $W_{\mathbf{M}}$. If a measurement is performed and its result is a_i, this means that we can establish a Boolean homomorphism

$$v_i : W_{\mathbf{M}} \rightarrow \mathbf{2} \qquad s.t.\ \ v_i(\mathbf{P}_i) = 1$$

In a possible world where $v_i(\mathbf{P}_i) = 1$ we will have classical consequences. We can take an arbitrary modal extension $\mathcal{L}^{\Diamond}$ of $\mathcal{L}(\mathcal{H})$ and consider the set $Cons_{\mathcal{L}^{\Diamond}}(\mathbf{P}_i)$. The modal extension *does not depend* on the valuation over the family $(\mathbf{P}_i)_i$. Thus, it is clear that the modal extension is independent of any possible world. Modal extensions are simple algebraic extensions of an orthomodular structure. By Proposition 3.5 we have that $Cons_{\mathcal{L}^{\Diamond}}(\mathbf{P}_i) = \{x \in \Diamond\mathcal{L}(\mathcal{H}) : \Diamond\mathbf{P}_i \leq x\}$. Thus, for any arbitrary modal extension $\mathcal{L}^{\Diamond}$ of $\mathcal{L}(\mathcal{H})$ in terms of classical consequences, the classical consequences of $v_i(\mathbf{P}_i) = 1$ are exactly the same ones as $\Diamond\mathbf{P}_i$ (independently of any possible splitting). In terms of classical consequences which refer to a property $\mathbf{P}_i$, it is the same to consider the classical consequences in the possible world where $v_i(\mathbf{P}_i) = 1$, as study the classical consequences of $\Diamond\mathbf{P}_i$ before the splitting. Many worlds interpretations *maintains that in each respective i-world,* $v_i(\mathbf{P}_i) = 1$ *for each* i. Thus, a family of valuations $(v_i(\mathbf{P}_i) = 1)_i$ may be simultaneously considered, each member being realized in each different i-world. From an algebraic perspective, this would be equivalent to have a family of pairs $\langle \mathcal{L}(\mathcal{H}), v_i(\mathbf{P}_i) = 1 \rangle_i$, each pair being the orthomodular structure $\mathcal{L}(\mathcal{H})$ with a distinguished Boolean valuation v_i over a spectral sub-algebra containing $\mathbf{P}_i$ such that $v_i(\mathbf{P}_i) = 1$. In [107], we have shown that the $\mathcal{OML}^{\Diamond}$ structure is able to capture this fact in terms of classical consequences. While many worlds interpretations considers a family of pairs $\langle \mathcal{L}(\mathcal{H}), v_i(\mathbf{P}_i) = 1 \rangle_i$ for each possible i-world and the classical consequences of $v_i(\mathbf{P}_i) = 1$ in the i-world, the $\mathcal{OML}^{\Diamond}$ structure, by Proposition [107, prop3.5], considers classical consequences of each $v_i(\mathbf{P}_i) = 1$ coexisting simultaneously in one and the same structure. In fact, as a valuation $v : \Diamond\mathcal{L} \rightarrow \mathbf{2}$ exists such that $v(\Diamond\mathbf{P}_i) = 1$ for each i, each element $x \in \Diamond\mathcal{L}$ such that

$\mathbf{P}_i \leq x$ necessarily satisfies $v(x) = 1$. Thus we see that, besides the wording about possibility that is present in the many worlds interpretations, only actuality plays a role.

There is in this sense a deep difference with respect to the meaning of superpositions in the proposal of Dieks and that of many worlds. While for Dieks the superposition is a theoretical device which allows us to account for the actual property described by the theory in probabilistic terms, in the case of the many worlds interpretation, quantum superpositions are interpreted in a metaphysical fashion. Each superposition expresses the existence of multiple worlds, each of which exists in (its own) actuality. However, there are no superpositions in this, our actual world —each world is restored as a "classical world". The many worlds interpretation seems to be able to recover these islands of classicality at the price of multiplying the 'actual realm'. Modal interpretations on the other hand, by remaining at a distance with respect to the metaphysical interpretation of superpositions, seem to have to pay a much less metaphysical price.

14.1.4 Quantum Possibility as Irrational Potentiality

The Geneva approach to quantum logic attempts to consider quantum physics as related to the realms of actuality and potentiality analogously to classical physics. According to the Geneva school, both in classical and quantum physics measurements will provoke fundamental changes of the state of the system. What is special for a classical system, is that 'observables' can be described by functions on the state space. This is the main reason that a measurement corresponding to such an observable can be left out of the description of the theory 'in case one is not interested in the change of state provoked by the measurement', but 'only interested in the values of the observables'. It is in this respect that the situation is very different for a quantum system. Observables can also be described, as projection valued measures on the Hilbert space, but 'no definite values can be attributed to such a specific observable for a substantial part of the states of the system'. For a quantum system, contrary to a classical system, it is not true that 'either a property or its negation is actual'.[72]

Continuing Heisenberg's considerations in the new physics, Constantin Piron has been one of the leading figures in developing the notion of potentiality within the logical structure of quantum mechanics [198; 200]. As reviewed in [8], the series of pioneer studies in QL searching for an axiomatic theory for quantum mechanics where the Hilbert space structure could be derived from physically plausible axioms [42; 182; 197; 157; 260; 33], was followed by the theorem of Piron that the lattice $\mathcal{L}$ of propositions about a quantum system is the lattice of all 'biorthogonal' subspaces of a vector space over a division ring [197]. Going along with the search for 'good' axioms was also the idea of founding the basic notions for this axiomatics in a physically clear and operational way. This gave rise to operational approaches [124; 214; 215; 216; 217] in which the Geneva and the Brussels approaches are framed [157; 197; 198; 199; 200; 2; 3; 4]. 'Operationality' means that the axioms should be introduced in such a way that they may be related to 'physical operations' that can be performed in a laboratory. In these approaches, any physical system is at each moment in a certain state, knowledge about the system is gathered by means of experiments and the possibilities for certain outcomes can be structured in a probabilistic theory. These are the basic notions that are to be formalized.

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[72] Private discussion with Diederik Aerts, August, 2010.

Following [242], a physical property, never mind whether a classical or quantum one, is specified as what corresponds to a set of definite experimental projects. A *definite experimental project* (DEP) is an experimental procedure (in fact, an equivalence class of experimental procedures) consisting in a list of actions and a rule that specifies in advance what has to be considered as a *positive* result, in correspondence with the *yes* answer to a dichotomic question. Each DEP tests a property. A given DEP is called *certain* (correspondingly, a dichotomic question is called *true*) if it is sure that the positive response would be obtained when the experiment is performed or, more precisely, in case that whenever the system is placed in a measurement situation then it produces certain definite phenomenon to happen. A physical property is called *actual* in case the DEPs which test it are certain and it is called *potential* otherwise. Whether a property is actual or potential depends on the state in which one considers the system to be. Though in this approach both actuality and potentiality are considered as modes of being, actual properties are considered as attributes that *exist*, as elements of (EPR) physical reality, while potential properties are not conceived as existing in the same way as real ones. They are thought as *possibilities* with respect to actualization, because potential properties may be actualized due to some change in the state of the system. In this case the superposition provides a measure —given by the real numbers which appear in the same term as the state— over the irrational potential properties which could become actual in a given situation. Thus, potentiality, as in the classical physical sense, can be regarded as *irrational potentiality*, as referring to a future in which a given property can become actual. The main difference with respect to the modal interpretation seems to be the operationalistic approach embraced by the Geneva scheme. We believe that a more profound and detailed analysis in this respect deserves investigation.

14.1.5 Quantum Possibility as Propensities or Dispositions

Closely related to the development of Heisenberg in terms of potentialities stands the development of Margenau and Popper in terms of latencies, propensities or dispositions. As recalled by Suárez [248], Margenau was the first to introduce in 1954 a dispositional idea in terms of what he called *latencies*. In Margenau's interpretation the probabilities are given an objective reading and understood as describing tendencies of latent observables to take on different values in different contexts [184]. Later, Karl Popper [207], followed by Nicholas Maxwell [185], proposed a propensity interpretation of probability. Quantum reality was then characterized by irreducibly probabilistic real propensity (propensity waves or propensitons).[73] More recently, Mauricio Suárez has put forward a new propensity interpretation in which the quantum propensity is intrinsic to the quantum system and it is only the manifestation of the property that depends on the context [246; 247; 248]. Mauro Dorato has also advanced a dispositional approach towards the GRW theory [109; 110; 111]. The GRW theory after their creators: Ghirardi, Grimmini and Weber [130]; is a dynamical reduction model of non-

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[73] The realist position of Popper seemed in this respect much more radical than the interpretation of Heisenberg in terms of potentia. Heisenberg ([141], 67-69) seemed to remain within a subjectivist definition of such potentia: "Such a probability function [i.e. the statistical algorithm of quantum theory] combines objective and subjective elements. It contains statements on possibilities, or better tendencies ('potentiae' in Aristotelian philosophy), and such statements are completely objective, they don't depend on any observerthe passage from the 'possible' to the real takes place during the act of observation". This was something Popper was clearly against.

relativistic quantum mechanics which modifies the linearity of Schrödinger's equation. As remarked by Dorato ([109], p. 11): "According to this reduction model, the fundamentally stochastic nature of the localization mechanism is not grounded in any categorical property of the quantum system: the theory at present stage is purely 'phenomenological', in the sense that no 'deeper mechanism' is provided to account for the causes of the localization. 'Spontaneous', as referred to the localization process, therefore simply means 'uncaused'."

In a recent paper [109], Dorato discusses the meaning of dispositions and reviews the need of different interpretations of quantum mechanics to account for such intrinsic tendencies within the theory.

> "[...] whether and in what sense QM, in its various interpretations, forces us to accept the existence of ungrounded, irreducible, probabilistic dispositions, i.e. dispositions, that, unlike fragility or permeability, lack any categorical basis to which they can be reduced to. My claim is that the presence of irreducible quantum dispositions in many (but not all) interpretations involves the difficulty of giving a spatiotemporal description to quantum phenomena, and is therefore linked to our lack of understanding of the theory, i.e., of our lack of a clear ontology underpinning the formalism." M. Dorato (Quoted from [109], p. 3)

Dorato explains very clearly the meaning of dispositional properties as well as their relation to categorical properties:

> "Intuitively, a disposition like permeability is not directly observable all the times, as is the property given by the form of an object ('being spherical'), but becomes observable only when the entity possessing it interacts with water or other fluids. [...] From these ordinary language examples, it would seem that the function of dispositional terms in natural languages is to encode useful information about the way objects around us would behave were they subject to causal interactions with other entities (often ourselves). This remark shows that the function of dispositional predicates in ordinary language is essentially predictive. [...] In a word, dispositions express, directly or indirectly, those regularities of the world around us that enable us to predict the future. Such a predictive function of dispositions should be attentively kept in mind when we will discuss the 'dispositional nature' of microsystems before measurement, in particular when their states is not an eigenstate of the relevant observable. In a word, the use of the language of 'dispositions' does not by itself point to a clear ontology underlying the observable phenomena, but, especially when the disposition is irreducible, refers to the predictive regularity that phenomena manifest. Consequently, attributing physical systems irreducible dispositions, even if one were realist about them, may just result in more or less covert instrumentalism." M. Dorato (Quoted from [109], pp. 2-4)

Dorato ([109], p. 5) argues that contextuality, as it emerges in the KS theorem, seems to call for dispositional properties: "Within QM, it seems natural to replace 'dispositional properties' with 'intrinsically indefinite properties', i.e. with properties that before measurement are objectively and actually 'indefinite' (that is, without a precise, possessed value). So the passage from dispositional to non-dispositional is the passage from the indefiniteness to the definiteness of the relevant properties, due to measurements interactions." We can see here the direct relation between Aristotle's metaphysics, his potentiality-actuality scheme conceived in terms of causality, and the dispositional account developed in order to understand quantum mechanics in terms of *causal capacities* [116]. It is also interesting to notice that the joint proposal of Dorato and Esfeld regarding the interpretation of quantum mechanics relies on the a-causal stochastic GRW theory. Going back to dispositions and the remark of Dorato, it is interesting to notice that his idea of 'observability' determines very explicitly the distinction between dispositional and categorical properties. This idea goes against our representational realist stance.[74] But independently of our own considerations regarding observability, it is not at all clear if such dispositions are not simply a black box were we can hide the mystery surrounding quantum mechanics. "It must be granted that introducing irreducible physical dispositions is implicitly admitting that there is something we don't understand. Admitting an in-principle lack of any categorical basis to which dispositions could be reduced, in both the non-collapse views and Bohr's seems a way to surrender to mystery." ([109], p. 9) As very clearly exposed by Dorato:

> "That the distinction between dispositions and categorical properties cannot be so sharp is further confirmed by Mumford's analysis of the problem of the reducibility of dispositions to their so-called 'categorical basis'. According to Mumford (1998), the difference between a dispositional property like fragility and the microscopic property of glass constituting its categorical basis is merely linguistic, and not ontological. Referring to a property by using a dispositional term, or by choosing its categorical-basis terms, depends on whether we want to focus on, respectively, the functional role of the property (the causal network with which it is connected), or the particular way in which that role is implemented or realized.
>
> But notice that if we agree with Mumford's analysis, it follows that it makes little sense to introduce irreducible quantum dispositions as ontological hypotheses. If, by hypothesis, no categorical basis were available, we should admit that we don't not know what we are talking about when we talk the dispositional language in QM, quite unlike the cases in which we refer to 'fragility' or 'transparency', in which the categorical bases are available and well-known. Introducing irreducible quantum dispositions would simply be a black-box way of referring to the functional role of the corresponding property, i.e., to its predictive function in the causal network of events.
>
> In a word, the use of the language of 'dispositions' by itself does not point to a clear ontology underlying the observable phenomena. On the contrary, when the dispositions

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[74] From our representational realist stance it makes no sense to distinguish between 'observable entities' and 'non-observable entities'. There is no pre-theoretical experience of the world. It is only the coherence of the conditions of observability which allow us to conceive an object as existent. Thus, there is no difference in between observing a table or a very small insect in a microscope, provided that the conditions on which such observations arise are closed and coherent as related to the theory which allows such observations. According to our approach we also need a theory to observe 'tables' and 'chairs'! Secondly, according to our stance, properties are always part of singular observations. We never see 'an object being spherical', we only see images (circles at most) which we 'put together' in order to consider in our minds 'an object being spherical'. The notion of 'object' is a presupposition compatible with the physical experience of multiple, interrelated and complicated observations. If we take into account representation as constituting observation, as we do, it makes no sense to talk about 'self evident' observations, such as those of 'tables' and 'chairs'. But even their properties (e.g. 'being spherical') are only observable through certain presupposed 'conditions of observability' for it is only the theory which can tell you what can be observed. Of course objects exist independent of the singular observations, but then it is difficult to say what is the difference between 'fragile' and 'being spherical'. 'Being spherical' also depends on certain conditions, for example, the need of enough light to observe the object, or does the categorical property 'spherical' become a dispositional property in the dark?

> in question are irreducible and their categorical bases are unknown, such a use should be regarded as a shorthand to refer to the regularity that phenomena manifest and that allow for a probabilistic prediction. Consequently, attributing physical systems irreducible dispositions may just result in a more or less covert instrumentalism, unless the process that transforms a dispositional property into a categorically possessed one is explained in sufficient detail." M. Dorato (Quoted from [111], pp. 8-9)

Dispositional proposals need thus to provide descriptions of the selecting physical process which takes place during the path from the indefinite level of dispositional properties to the definite level of actual properties. As argued by Suárez, Dorato and Esfeld the GRW might be in this case a good candidate to consider.

14.1.6 Quantum Possibility as Rational Potentiality

From the orthodox formalism it follows that all terms in the superposition cannot be considered as existing in actuality simultaneously. As we have seen, there are different approaches which attempt to overcome this interpretational difficulty. From the representational realist stance we have proposed the idea that the formalism of a theory needs to be considered in relation to the conceptual structure which allows to discuss, in metaphysical terms, what is this theory telling us about the world. From this perspective, contrary to empiricism, one needs to carefully consider the conditions on which empirical data is constituted. Also, contrary to classical realism, the world need not be necessarily understood in terms of a classical conceptual scheme. Apart from Bohmian mechanics or GRW theory, which change the formalism itself, and the many worlds interpretations, which provides an expensive metaphysical scheme with no phenomenological evidence, we believe there is a different (representational) realist solution to the riddle of quantum possibility and quantum superpositions which deserves consideration.

We claim that it is empirical data which points in the direction of the existence of a potential realm. Though an adequate concept is still missing to characterize quantum superpositions, it is the physical experience done today in laboratories which points in the direction of considering quantum superpositions as ontologically robust. Instead of denying their existence —as it is done in Bohmian mechanics or instrumentalist positions— our proposal is to interpret the quantum superpositions in terms of a mode of being that is different and independent from actuality. Such mode of existence —where an adequate concept which relates to superpositions would rest— could be thought and developed by considering the Aristotelian notion of *rational potentiality* [226]. We believe that the notion of rational potentiality can allow us to develop a conceptual scheme that interprets the quantum superposition in a new light, providing at the same time new insights to account for the non-actualized possible level. The independence of both realms does not imply a complete denial of the relation between the potential and the actual, but rather, the possibility to turn upsidedown the gnoseological relation between the actual and the potential. In our scheme it is not the potential which needs to explain the actual, but the actual which can explain the potential. In fact, the main elements which seem to constitute the formal structure of a superposition, namely, *its existence regardless of the effectuation of one of its terms*, as shown by the interference of different possibilities in *welcher-weg* type experiments, *its reference to contradictory properties*, as in the Schrödinger cat state, and *its non-standard route to actuality*, as explicitly shown by the

MKS theorem, can be considered in analogous manner to the non-causal powers which exist in the mode of being of rational potentiality. We use the term ‘non-causal’ in order to make explicit our denial of the ontological superiority of the actual over the potential as well as the independence of the latter.

A power in rational potentiality can be thought in terms of a sense, a capability. As Aristotle explains in *De Anima*:

> “Every sense seems to be concerned with a single pair of contraries, white and black for sight, sharp and flat for hearing, bitter and sweet for taste; but in the field of what is tangible we find several such pairs, hot cold, dry moist, hard soft, etc. This problem finds a solution, when it is recalled that in the case of the other senses more than one pair of contraries are to be met with, e.g. in sound not only sharp and flat but loud and soft, smooth and rough, etc.; there are similar contrasts in the field of colour.” Aristotle (422b17-422b32)

It is not self evident that only animated things have capabilities, for one could think that the glass has the power to break or the river to flow. As Aristotle remarks, “the non-rational potentialities are all productive of one effect each, but the rational produce contrary effects, so that they would produce contrary effects at the same time; but this is impossible.” (1048a1-1048a24) However, in privation, when the power does not pass to the actual, rational potentiality allows for contradiction.

> “privation is a kind of contradiction; for what suffers privation, either in general or in some determinate way, is either that which is quite incapable of having some attribute or that which, being of such a nature as to have it, has it not; here we have already a variety of meanings, which have been distinguished elsewhere. Privation, therefore, is a contradiction or incapacity which is determinate or taken along with the receptive material. This is the reason why, while contradiction does not admit of an intermediate, privation sometimes does. If, then, the changes which happen to the matter start from the contraries, and proceed either from the form and the possession of the form or from a privation of the form or shape, clearly all contrariety is a privation.” Aristotle (1055b17-1667)

A power can be thought to exist within the realm of rational potentiality, completely independent of the actual realm. Furthermore, as it is a mode of existence, it seems one could think of “things happening”, within this realm, independently of actualization. This idea breaks the causal relation between the ontological potential and the actual, for the effectuation of a power is not understood as a *fulfillment* of the power itself. A power can rest as privation and exist independently of its effectuation. This idea opens the door to the consideration of something that exists which is not necessarily actual, an ontology of powers which needs to be considered on equal footing to the ontology of entities in classical physics. This idea goes in line with our proposal to continue the path of Heisenberg regarding closed theories as independent interconnected set of concepts linked to a mathematical structure. Contrary to irrational potentiality, where there is a *causal teleological end* which guides the path from the potential to the actual, and where the actual is implicitly considered as “more real”, within rational potentiality the particular actualization of a power does not change in any way the power itself which, independently of its particular actual expression, remains exactly the same.

It is through the actual that we learn about the potential and not the potential which needs to explain the actual. These aspects might reminds us of some main features of possibility within the modal interpretation itself, now read from an ontological perspective.

Modal interpretations are non-collapse interpretations (see section 7.1) and do not consider the collapse of the quantum wave function as a physical interaction.[75] As mentioned above, in quantum mechanics there are two types of evolution. Firstly, the *deterministic* evolution commanded by the Schrödinger equation and, secondly, the *indeterministic* evolution given by the collapse of the superposition into the term that corresponds to the actual measurement outcome observed in the apparatus. Modal interpretations keep the indeterministic non-causal relation in between the quantum superposition and the effectuation of one of its terms. As we discussed above, the fact that quantum mechanics admits superpositions as states of the system is central when dealing with the measurement process, where the various terms associated with the possible outcomes of a measurement must be assumed to be present together in the description at the possible level. Contrary to the orthodox interpretation, modal interpretations keep the complete superposition in the level of possibility independently of the particular actualization. “In modal interpretations the state is not updated if a certain state of affairs becomes actual. The non-actualized possibilities are not removed from the description of a system and this state therefor codifies not only what is presently actual but also what was presently possible. These non-actualized possibilities can, as a consequence, in principle still affect the course of later events.” ([266] p. 295) To neglect the collapse might be regarded as a way out of the direct causal relation in between the possible and the actual. At the same time, it makes a cut in between two levels of description: the possible and the actual.[76] It is this distinction which opens the door to take into account the possible in an ontological fashion. Furthermore, if we consider the level of possibility as describing an ontological aspect of reality, it seems there is also place for an ontological account of superpositions independent of actuality. This seems to go in line, not only with our notion of ontological potentiality, but also with the experiments done today, related to quantum superpositions and quantum entanglement, where that which happens does not seem to happen *in actuality*.[77]

We claim that our notion of ontological potentiality can allow us to think anew the phenomenology of quantum mechanics as related to the world. This does not mean necessarily to go beyond the actual, but rather to understand the actual in a different way. Breaking the direct causal relation in between the potential and the actual realms means to place the potential in a completely different ontological ground. Non-collapse modal interpretations have already investigated the suppression of this causal relation. In the modal interpretation observation does not discover the state, for if we take out the collapse of the quantum wave function, and assume that the quantum superposition describes reality, the discovery of a sin-

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[75] Van Fraassen discusses the problems of the collapse of the quantum wave function in [255], section 7.3. See also [85].

[76] These levels are explicitly formally accounted for in both van Fraasen and Dieks modal interpretations. While van Fraassen distinguishes between the ‘dynamical states’ and the ‘value states’, Dieks and Vermaas consider a distinction between ‘physical states’ and ‘mathematical states’.

[77] This could be also thought in terms of the *quantum information flow capabilities* explained by Bob Coecke in [74].

gle term is always different to the quantum superposition itself. There is thus no one-to-one relation in between what is observed and the representation given by the theory. As noticed by Dieks ([97], p. 403): "there is not a one-to-one correspondence between the quantum state and physical reality. Rather, the quantum state will determine what *may* be the case, what the possible physical situations are." Our interpretational move is to turn upside down van Fraassen's empiricist proposal and consider the actual set of observations as describing potentiality in an ontological fashion. According to our proposal, superpositions are mathematical expressions of an existent element of the theory —to be explained in terms of an adequate concept— which needs to be understood in terms of ontological potentiality. Just like actuality, potentiality is a mode of existence of the world. The actual can be understood now as bringing into stage a particular expression of the potential. Each particular effectuation teach us something about our potential existent, but contrary to dispositional properties it is not the property itself which interests us, rather, it is the quantum superposition —lying in the ontological potential realm— which needs to be understood. Of course, to what extent we learn about this potential level, still depends on the set of actual observations. Our metaphysical scheme attempts to consider what could be thought in terms of this set —the notion which relates to superpositions— as ontologically existent.

As we discussed above, the potential realm also allows for the consideration of contradictory elements. This main characteristic does not find a place in other interpretations of quantum mechanics and is a key aspect we would like to investigate —in this, our world [77]. Could it be possible to consider contradictory elements that describe a potential existent? If such is the case, then our metaphysical approach will not be superfluous, but will allow us to consider superpositions as bringing into stage a concept —clearly not the notion 'cat'— which can be considered as ontologically existent in an analogous fashion as we consider the existence of objects in classical physics. In turn this might allow us to think new physical experience. Schrödinger showed that such quantum concept cannot be considered in terms of a 'classical object' but this does not mean necessarily that it cannot be considered in terms of a different, yet to be developed, new concept. For as Heisenberg ([143], p. 264) remarked: "The history of physics is not only a sequence of experimental discoveries and observations, followed by their mathematical description; it is also a history of concepts. For an understanding of the phenomena the first condition is the introduction of adequate concepts. Only with the help of correct concepts can we really know what has been observed."

14.2 Modality within the Modal Interpretation Revisited

As we have discussed in the previous sections there are different possible strategies to account for the interpretation of modality within quantum mechanics. These very different strategies make it difficult to distinguish exactly what is a 'modal interpretation' from what it is not. As we have seen, within van Fraassen's Copenhagen variant the empiricist strategy seems to keep a secure distance with respect to the metaphysical commitments involved within an interpretation of quantum mechanics. Contrary to this stance but still within what is considered to be the modal scheme, the proposals of Bub, Bacciagaluppi and Dickson stand on the hidden variable program which attempts to restore a classical way of thinking about what there is. Dieks strategy might be considered half way in between van Fraassen empiricist approach and the realist hidden variable program, for though he interprets quantum mechanics as describing actual properties in the same way as Bohm —as preexistent to measurement, going beyond the discourse about actual observations— he is not ready to leave aside the orthodox formulation of quantum mechanics. Furthermore, Dieks seems to remain open to the investigation of different metaphysical interpretations. In this sense, one might even leave open the possibility to interpret modality following the proposal of Lewis in terms of real existent worlds. Our proposal to develop modal interpretations by considering the possible realm as ontologically significant takes a distance from the latter approaches in important points. The consideration of a realm independent of actuality opens the door to understand quantum superpositions in a new light, independent of the actual realm. In this respect it is important to point out that the origin of modal interpretations was related directly to the explicit criticism by van Fraassen on the relation between the quantum state and the single result obtained after a measurement —known in the literature as the *eigenstate-eigenvalue link*. We believe that this same criticism can open the doors to consider observability in a new light.

In line with realism we believe that explanation relates directly to conceptual structures capable of providing an account of physical experience. Our stance, based on what we call representational realism, attempts to provide an interpretation of the orthodox formalism of quantum mechanics independently of measurement outcomes. According to our stance, the problem of a representational realist remains to build up a representational conceptual scheme which would allow us to relate coherently the quantum formalism to the empirical structure predicted by the theory in a closed manner. The concepts must be internally defined by the theory itself and not presupposed. This means, for example, that it is not self evident to us that the notion of possibility used in classical physics is, or should be considered as, the same notion of possibility that one uses in quantum theory. The concepts which allow to provide a coherent account of quantum phenomena must be able to provide a story about how the world is according to quantum mechanics. In turn, these same concepts might be even capable of developing new physical experience. In this sense, we also need to provide an answer to the question: what is the adequate concept with which we can relate to a quantum superposition?

Concluding Remarks: The Constructive Metaphysical Path

In this dissertation we have characterized and analyzed the interpretational map of quantum mechanics in terms of metaphysical and anti-metaphysical stances. In particular, we have centered our attention on the so called modal interpretations characterizing them in terms of their methodology and agenda (as MIMP and MIMF). In this context, we have put forward the representational realist quest which considers physical theories as closed sets of interconnected mathematical expressions and concepts which allow us to think about physical experience. From this perspective, starting from the orthodox formalism, the problem of quantum mechanics is to find the adequate concepts which would allow us to account, in a coherent manner, for the quantum phenomena. This constructive metaphysical path has of course a long way to go.

Following our representational realist stance we have investigated formally and conceptually the meaning of two key concepts: contextuality and quantum possibility. We have discussed the meaning of contextulaity in relation to physical representation and argued that its understanding depends explicitly on the metaphysical or anti-metaphysical stance taken by the different interpretations of quantum mechanics. In this sense, we have characterized the very different strategies involving the relation between contextuality and quantum theory. We have also investigated the meaning of possibility as related to the orthodox quantum formalism, as well as its relation to actuality in the modal interpretation. From a conceptual perspective, we have discussed the meaning of the MKS theorem. Also from a conceptual perspective, we have characterized several different interpretations of possibility found in the philosophy of quantum mechanics and proposed an interpretation of possibility in terms of a development of the Aristotelian notion of rational potentiality. We have argued that this path might allow us to think of an ontological realm independent of actuality. In relation to the discussion about the meaning of quantum possibility we have linked its interpretation to that of the quantum superposition. The problem we have posed, in line with our representational realist stance, is that we still need to provide a concept which allows us to think about 'quantum superpositions' in an analogous manner as we can think of 'a point in phase space' as making reference to 'an object in space-time'. A main methodology which we have proposed in order to accomplish this task is to go back again to the quantum formalism in order to learn more about its intrinsic features and let quantum mechanics find its own interpretation.

15.1 In Favor of Representational Realism

Independently of the capacity of physical sciences to account for the world and phenomena, we have concentrated on the justification one can give to account for such relation. This justification can be grounded on political, ideological or even moral ideas.

> "It seems to me that the attack on realism, although intellectually interesting and important, is quite unacceptable, especially after two world wars and the real suffering — avoidable suffering— that was wantonly produced by them; and that any argument against realism which is based on modern atomic theory —on quantum mechanics— ought to be silenced by the memory of the reality of the events of Hiroshima and Nagasaki." K. R. Popper (Quoted from [206], p. 2)

Our interest in considering the question of representation as a *constitutive* element of physical theories which cannot be bypassed relates to the need of providing a metaphysical justification through which we are able to make sense of physical experience in relation to the world. Scientific realists stand on a dogmatic and uncritical idea: that science provides true knowledge about the world. If scientific realists continue to consider physical experience as an objective, clean and open evidence which is simply presented to scientists, maybe its time they step inside a laboratory. It is, we believe, this dogmatism —very severely criticized by van Fraassen— which has neglected and obturated a lot of interesting questions which we, as philosophers of science, should start to reconsider anew. Independently of any argumentation, the question that divide us is: "Do we believe we really understand the world around us?" The problem continues to be the justification of scientific experience as related to the world:

> "The central issue here is realism. That is to say, the reality of the physical world we live in: the fact that this world exists independently of ourselves; that it existed before life existed, according to our best hypotheses; and that it will continue to exist, for all we know, long after we have all been swept away." K. R. Popper (Quoted from [206], p. 2)

From our perspective, the basic problem of scientific realists is that they believe they already know what the world is like. Such dogmatic metaphysical idea remains at the heart of their considerations and is responsible for the closure regarding any type of fundamental conceptual development of physical theories. It is quantum mechanics which remains an insistent proof of the incapability of scientific realism to account for physical theories. It is scientific realism which cannot account for what they themselves consider: "the best theory we have".

The approach we have proposed evades many problems encountered by scientific realism such as the reductionistic relation between theories, the metaphysical unjustified presupposition of the existence of objects as self evident features of reality with no theoretical content, the mixture of concepts of different theories and levels of discourse without a clear analysis of their limits of applicability. Our constructive metaphysical approach calls for the development of new concepts, new structures of thought which can allow us, not only to understand quantum mechanics, but also to develop and consider new physical experience. It also calls for the analysis and meaning of representation in physical theories; something which is fortunately today more and more considered by philosophers of science [245; 257]. We have argued that it is of the utmost importance for the discussions regarding the possible interpretation of quantum mechanics to expose the presuppositions involved within the different strategies. Our approach goes also together with the idea of ‘closed theories’ and the call for *coherency* with respect to the concepts involved within each particular physical theory. This coherency, as we have remarked, deals with a closed interpretation relating concepts and mathematical expressions which can allow us to think about phenomena. Contrary to scientific realism, our approach does not rely on a classical conception of the world (i.e. a world built up from objects such as ‘tables’ and ‘chairs’). Rather, it considers the classical account of the world as a particular metaphysical construction capable of outstanding phenomenological predictions. In this sense, our position implies a distance from the idea of observability as an action which *presents* the world to the observer. It is in quantum mechanics that we believe, it is of outmost importance to reconsider the meaning itself of a ‘measurement outcome’. As Diederik Aerts reflected: “Perhaps in the future one shall have to rethink what it really means to get at the end of an experiment a number, that one has to compare with the number predicted by the theory” (see also [257], chapter 7). From this perspective the meaning of understanding a theory is very direct: *a physical theory is understood if it is capable of providing a coherent relation in between its formalism and concepts such that it is able to account for the phenomena it talks about.* In this same sense, we regard phrases like “quantum mechanics talks about elementary particles” as completely meaningless for, we do not know, still today, what “elementary particles” are.

15.2 Our Strategy: How to Continue?

Our main attempt is to provide quantum mechanics with a closed interpretation which relates the mathematical expressions of the theory to a conceptual structure which allows us to provide an understanding of quantum experience beyond the measurement outcomes. This development is important in itself not only for the clarification of already known phenomena but also, maybe even more importantly, for the possibility to think new physical experience. We believe that this theoretical development must be related to the present ongoing physical experience in the laboratory and the new formal mathematical schemes and structures which can allow us to radicalize —instead of overcome— the quantum features and consider them “right from the start”. Our argument is that, if we understand better the formal structure of the theory, we might then be able to know what are the conditions, adequate concepts must accomplish, in order to account for such internal formal relations. In our ongoing investigation with Graciela Domenech and Hector Freytes we attempt to continue this analysis; at present such research is focused on the meaning of possibility in orthomodular structures [125]. Together with Newton da Costa and Décio Krause, we have started an analysis regarding the formalization of quantum superpositions through para-consistent logic [77]. Regarding the notion of ontological potentiality we proposed in section 14.1.6, we attempt to continue our analysis comparing our non-causal power approach to the to the causal capacities discussed by Nancy Cartwright, Mauro Dorato and Michael Esfeld [67; 68; 110; 112; 116].

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Samenvatting

De voorliggend dissertatie is gebaseerd op het werk dat ik de laatste jaren heb verricht. Ik verwijs daarbij graag naar de papers die ik schreef samen Diederik Aerts, Bart D'Hooghe, Graciela Domenech, Hector Freytes, Federico Holik, Wim Christiaens en Vincent Bontems. Het gemeenschappelijke thema van deze teksten is de zoektocht naar een beter begrip van de formele structuur van de kwantummechanica —haar mogelijkheden en onmogelijkheden.

Ik kwam voor het eerst in contact met de modale interpretatie van de kwantummechanica in Utrecht in 2002. Vrij vlug geraakte ik ervan overtuigd dat de geest van deze aanpak in overeenstemming was met die van de grondleggers van de kwantummechanica. Eén van de interessantste aspecten van de modale interpretatie is, wat mij betreft, het idee dat men zo dicht mogelijk bij het standaardformalisme blijft. In die jaren presenteerde professor Dieks, samen met Gyula Bene, een paper met als titel *A Perspectival Version of the Modal Interpretation*. Daarin ontwikkelden ze een relationele theorie die de typische problemen van de modale interpretatie met de bepaaldheid van eigenschappen ontweek. Dankzij deze inzichten, en dankzij de modale interpretatie in haar geheel, slaagde ik erin een studie te maken van de *modaliteiten ('modes') van het zijn* voor eigenschappen die worden beheerst door de formele structuur van de kwantummechanica.

In 2004 ging ik naar Brussel, waar ik mijn onderzoek voortzette binnen de school van Genève (de aanpak van de kwantummechanica van Josef-Maria Jauch en Constantin Piron). Ik geraakte vertrouwd met de logische aspecten van zowel de klassieke als de kwantumtheorie. Daarenboven was de realistische inslag van Piron en Aerts voor mij een nieuwe invalshoek om na te denken over 'elements of reality'. Binnen deze aanpak kregen de noties van mogelijkheid en potentialiteit, die ook in de modale interpretatie een centrale rol spelen, een heel specifieke invulling.

Terug in Buenos Aires dacht ik, samen met Graciela Domenech en Hector Freytes, verder na over contextualiteit en mogelijkheid. De kwantummechanica heeft natuurlijk altijd wel iets te maken gehad met mogelijkheid —al was het maar via het begrip waarschijnlijkheid—, maar het theorema van Kochen en Specker beperkt alleen actuele eigenschappen in het hebben van welbepaalde waardenben. Het theorema zegt niets over mogelijkheid, spreekt zich alleen uit over de actualiteit van bepaalde eigenschappen. Eigenlijk stelden we ons een vrij voor de hand liggende vraag: zegt het KS-theorema ons iets over mogelijke eigenschappen? Het is deze vraag die verder wordt uitgediept in de voorliggende dissertatie.

Meer algemeen kunnen we zeggen dat onze onderliggende vraag er één is over 'interpretatie' als zodanig. Wat zijn de voorwaarden of beperkingen van eender welke interpretatie van de kwantummechanica, gegeven een klassieke visie op natuurkunde? Welke invulling kunnen we geven aan 'de klassieke visie op natuurkunde'? Welke elementen laten ons toe een coherente interpretatie te construeren van de kwantummechanica?

Het probleem aangevat

Dit proefschrift bevat vier belangrijke elementen.

1. We presenteren een **overzichtskaart**, van de interpretaties van de kwantummechanica in het algemeen, en van de modale interpretatie in het bijzonder —uitgaande van metafysische en anti-metafysische stellingnames ('stances')
2. We zetten ons af tegen anti-metafysische en klassieke stellingnames: onze argumentatie wijst in de richting van een **constructieve metafysische aanpak**.

3. We analyseren de betekenis van **contextualiteit** en **mogelijkheid** —in de kwantummechanica in het algemeen, in de modale interpretatie in het bijzonder.
4. We eindigen ten slotte met een voorstel voor de ontwikkeling van een mogelijk constructief fysisch schema gebaseerd op de **notie van potentialiteit**.

Door onze analyse van de betrokken concepten en van hun relatie tot het formalisme, werpen we een nieuw licht op problemen die men ondertussen goed kent uit de bestaande literatuur. We proberen de spanning die bestaat tussen de theoretische voorwaarden en de conceptuele structuur van de theorie, expliciet te maken, ten einde de metafysische ideeën inherent aan de verschillende problemen van de interpretatie bloot te leggen en te bediscussiëren. Een dergelijke analyse kan ons helpen begrijpen wat de impliciete achtergrond is van deze theoretische voorwaarden.

In deze dissertatie concentreren we ons op het modale en contextuele karakter van de kwantummechanica. Aan de ene kant, behandelen we het contextuele karakter van kwantummogelijkheid op basis van de modale versie van het theorema van Kochen-Specker; aan de andere kant, analyseren we het contextuele karakter van de kwantumcorrelaties door het onderscheid van eigenschappen. In het laatste deel volgt een discussie over meer speculatieve ideeën die betrekking hebben op de mogelijke metafysische ontwikkeling van de kwantummechanica gebaseerd op potentialiteit.

Overzicht

Deze thesis is opgedeeld in vijf stukken. **Deel I**, de INTRODUCTIE, presenteert de basisideeën. In **hoofdstuk 1** stellen we het algemene schema voor dat we volgen; we bediscussiëren ook de belangrijkste problemen die we behandelen. In **hoofdstuk 2** volgen we van Fraassen wanneer hij zegt dat filosofie in de eerste plaats een 'stance' is (eventueel te vertalen als: stellingname): vooral een existentiële onderneming. We presenteren ook, in het kort, het centrale aspect van onze stellingname: naast de empirische elementen en de theoretische structuur, moet juist bijzondere aandacht worden gegeven aan de representatie-gerelateerde conceptuele structuur van de theorie.

Deel II, KUANTUMMECHANICA EN DE INTERPRETATIE VAN DE FYSISCHE WERKELIJKHEID, configureert de drie stellingnames die we in kaart zullen brengen: de anti-metafysische, de klassieke metafysische en de constructief-metafysische stellingname. **Hoofdstuk 3** gaat in op de hedendaagse analytische filosofie van de fysica en van de kwantummechanica in het bijzonder. Binnen deze context bediscussiëren we de noodzaak om in te gaan op het probleem van de representatie van de werkelijkheid in fysische theorieën. In **hoofdstuk 4** argumenteren we dat Mach's positivistische filosofie en zijn kritiek op het Kantiaanse a priori schema, een sleutelelement was van de kwantumrevolutie. We bediscussiëren opvattingen van Einstein, Heisenberg en Pauli, zowel wat betreft hun houding t.o.v. fysica als metafysica. We analyseren Heisenbergs principe van onbepaaldheid en Bohrs complementariteitsaanpak. Met Bohrs interpretatie van de fysica begint ook voor de filosofie van de fysica de zogenaamde 'linguistic turn'. **Hoofdstuk 5** handelt over de metafysische voorwaarden van het kwantummechanische formalisme die impliciet worden opgelegd bij het tot stand komen van twee paradigmatische problemen van de interpretatie: de Bell ongelijkheden en het theorema van Kochen-Specker. We onderscheiden *actuele observable, actueel voorbestaande* (preexistant), *actueel statistische observable en actueel statistisch voorbestaande.* Coherent met onze stellingname argumenteren we in **hoofdstuk 6** voor een constructieve metafysische positie.

Deel III, MODALITEIT, handelt over de noties mogelijkheid en actualiteit binnen de formele structuur van de kwantummechanica. We presenteren ons methodologisch onderscheid en situeren onze argumenten voor een constructieve metafysische stellingname. In **hoofdstuk 7** gaan we in op de modale interpretatie van de kwantummechanica en bediscussiëren we enkele van haar varianten. We onderscheiden de *modale interpretatie die vertrekt van metafysische condities* (Modal interpretation that starts from metaphysical conditions: MIMP) en de *modale interpretatie die vertrekt van het* *wiskundige formalisme* (MIMF). In **hoofdstuk 8** hebben we het over de limieten die actuele eigenschappen opleggen binnen de modale interpretatie in verschillende theorema's van het type KS, zoals deze van Bacciagaluppi of Clifton, wat samenhangt met het probleem van de geprefereerde factorizatie. In **hoofdstuk 9** analyseren we de 'metafysische houdbare condities' die de verschillende versies ons opleggen en stellen we onze eigen strategie voor om de kwantummechanica te interpreteren, een strategie die gebaseerd is op onze constructief-metafysische stellingname.

Deel IV, CONTEXTUALITEIT, analyseert de betekenis van het contextuele karakter van de formele structuur van de kwantummechanica. We voeren ook een discussie over het contextuele karakter van de concepten *kwantummogelijkheid* en *kwantumcorrelatie*. In **hoofdstuk 10** hebben we het over de betekenis van het probleem hoe de fysische werkelijkheid te representeren. We relateren dit aan de discussie over de klassieke en constructief-metafysische stellingnames. In **hoofdstuk 11** gaat de discussie, via het oorspronkelijke modale KS-theorema, over het contextuele karakter van *kwantummogelijkheid*. We baseren ons op dit theorema om te argumenteren dat de notie van mogelijkheid die opduikt in de orthodoxe structuur van de kwantummechanica, niet klassiek is; en om te argumenteren dat, vanuit een representatie-realistisch perspectief, modale interpretaties hier rekening mee moeten houden wanneer ze het hebben over de ontwikkeling van de theorie met betrekking tot haar interpretatie —in het bijzonder in relatie tot MIMP en MIMF. **Hoofdstuk 12** analyseert het contextuele karakter van *kwantumcorrelaties*; onder meer Mermins voorstel om correlaties als elementen van de werkelijkheid te beschouwen. We helderen schijnbare contradicties op —zoals het tegelijk bestaan van het theorema van de subsysteem-correlatie (SSC) aan de ene kant en Cabello's and Seevincks theorema's aan de andere kant— met het onderscheid tussen eigenschappen dat we eerder maakten.

Deel V, MODALITY REVISITED, analyseert de concepten mogelijkheid en actualiteit in formele en metafysische termen. We bediscussiren ook een voorstel om, binnen de modale interpretatie, kwantummogelijkheid te interpreteren in termen van rationele potentialiteit. In **hoofdstuk 13** herinneren we aan de zijnswijzen van actualiteit en potentialiteit in de metafysica van Aristoteles. We analyseren beide concepten binnen de klassieke fysica; het is belangrijk dat Heisenbergs interpretatie rekening houdt met de notie van *potentia* in de kwantummechanica. In **hoofdstuk 14** overlopen we de verschillende betekenissen van kwantummogelijkheid in de kwantummechanica. Deze hebben te maken met de verschillende interpretaties van superpositietoestanden. Om af te sluiten geven we in **hoofdstuk 15** enkele opmerkingen over ons representatie-realistisch voorstel en de constructief-metafysische strategie bij het ontwikkelen van een coherente interpretatie van de kwantummechanica.

Related Publications by C. de Ronde

1. Aerts, D., de Ronde, C. and D'Hooghe B., 2011, "Compatibility and Separability for Classical and Quantum Entanglement", In *Worldviews, Science and Us: Bridging knowledge and its implications for our perspectives of the world*, D. Aerts, B. D'Hooghe and N. Note (Eds.), World Scientific, Singapore, forthcoming.
2. Christiaens, W. and de Ronde, C, 2009, *Fysica & wiskunde*, In E. Romein, M. Schuilenburg, S. van Tuinen (Eds.), 328-344, Boom, Amsterdam.
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Curriculum Vitae

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