Chapter 2

Social Support between Heterogeneous Partners[*]

Abstract

This chapter derives hypotheses on how dyadic social support is affected by heterogeneity between the actors. We distinguish heterogeneity with respect to three parameters: the likelihood of needing support, the benefits from receiving support relative to the costs of providing support, and the time preferences. The hypotheses are based on a game-theoretic analysis of an iterated Support Game. We predict that, given homogeneity in two of these parameters, the prospect for mutual support is optimal if actors are homogeneous with respect to the third parameter as well. Second, under heterogeneity with respect to two of the parameters, support is most likely if there is a specific heterogeneous distribution with respect to the other parameter that 'compensates' for the original heterogeneity. Third, under weak conditions, the overall optimal condition for mutual support is full homogeneity of the actors.

2.1 Introduction

This chapter seeks to improve the understanding of cooperation in *asymmetric* social dilemmas. We analyze the conditions under which asymmetry with respect to individual properties hampers or facilitates cooperation. The theoretical and experimental literature on social dilemmas predominantly analyzes cooperation in the *symmetric* Prisoner's Dilemmas and some related social dilemma games. A Prisoner's Dilemma is called symmetric if the payoffs of the actors are the same (up to an increasing affine transformation). Asymmetry studied in the literature so far mainly concerns asymmetry in information, i.e., differences between the actors' information about each other's payoffs, the rules of the games, etc. Asymmetry in the sense of 'actor heterogeneity' with respect to other aspects of interactions has received much less attention. Actor heterogeneity refers to differences in preferences, interests, resources, etc. between the actors involved. This chapter focuses on the consequences of actor heterogeneity on cooperation in games that are a variant of the Prisoner's Dilemma. Asymmetries that do not directly reflect differences in properties of actors, such as differences in positions in social networks or differential institutional treatment of actors, will not be addressed in this chapter.

We argue that actor heterogeneity likely has different effects on individual behavior and on collective outcomes in different types of social dilemma situations.[1] Our distinction resembles the 'production function' analysis of collective action (Oliver et al. 1985, Heckathorn 1992). In the first type, exemplified by the *Prisoner's Dilemma*, Pareto optimal outcomes ('cooperation') require that *each* actor contributes (Rapoport 1974, Dawes 1980), while *all* actors have incentives to deviate. If actors do indeed deviate, the resulting outcome is suboptimal (Kreps 1990, Fudenberg and Tirole 1992). A common example is environmental protection. Since time and attention are scarce recourses, everyone has incentives not to separate trash into paper, plastics, metals, glass etc., although everyone may agree that garbage separation is important to protect the environment. Everyone appreciates other people separating their trash, but they may abstain from such costly behavior themselves ('free riding'). If everyone does this, however, the resulting environment will be suboptimal. In the second class of problems, exemplified by the *Volunteer's Dilemma*, the production of the collective good requires contributions by a *sufficient number* of actors. An example is the rescue of a child who has wandered too far out to sea. Bystanders have the same aim – the child needs to be rescued. However, it is not necessary that all bystanders jump into the water to rescue the child. It is sufficient if one actor jumps into the water and rescues the child. This would be a Pareto optimal outcome (Diekmann 1993, Weesie 1993). If more than one actor were to jump into the water, the extra efforts would be wasted, and the outcome would be Pareto suboptimal. Olson and Zeckhauser (1966) illustrated this effect with contributions to the NATO alliance during the 1960's.

---

[1] Since we only focus on actor heterogeneity in this study, 'heterogeneity' refers in the following to 'actor heterogeneity'.

In a dilemma situation of the Prisoner's Dilemma type, the problem is the inducement of the actor(s) with the *least* interest in the production of the collective good to make a contribution. In terms of a Prisoner's Dilemma, the problem is the inducement of the actor(s) with the highest incentive to cheat. We want to indicate an analogy with the strength of a chain in terms of the strength of the links: to make a chain stronger, the *weakest link* has to be strengthened. With unlimited material, we can make all links stronger. With scarce resources, we can make the chain stronger by shifting material from the stronger links to the weakest link. In a chain of maximal strength, all links are equally strong, i.e., homogeneity is optimal. To the extent that this analogy is valid, we would expect that actor homogeneity offers optimal prospects for cooperation in a social dilemma of a Prisoner's Dilemma type.

The effect of heterogeneity on efficiency is different in the second class of social dilemmas, exemplified by the Volunteer's Dilemma. In this class of social dilemmas, an outcome, in which all actors contribute, is suboptimal. The production of the collective good requires the contribution of one actor only. Various theoretical analyses suggest that the actor who gains most from the collective good relative to the costs of producing it is the most likely volunteer. Therefore, the production of the collective good depends on the *strongest link*. Heterogeneity between the actors is likely to make the strongest link stronger, and thus facilitates cooperation. Different reasons have been suggested for the strong impact of the strongest link. Diekmann (1993) derives this prediction by using the theory of risk dominance, one of the prominent equilibrium selection theories of game theory. Weesie (1993) derives a similar prediction, using subgame perfection only, if actors are able to wait and see whether other actors made a contribution. A related argument can be found in mobilization and threshold theories of collective action. Olson (1965) argues that the actors who are more interested in the common good will provide the collective good, regardless of the actions of the group's less interested actors. The 'weak' can thus exploit the 'strong' because everyone knows that the 'strong' will produce the collective good anyway (Olson 1965: 29). Oliver et al. (1993) give a similar argument in their analysis of the critical mass of heterogeneous actors and collective action. Oliver et al. (1985) argue that heterogeneity of interests or resources facilitates collective action because it increases the likelihood that a 'critical mass' of highly motivated contributors will emerge to initiate the collective action (see for a similar argument, Hardin 1982). Coleman (1990: Chapter 9) argues that organizing collective action is more likely in a group that is heterogeneous with respect to normative constraints than in a group with homogeneous normative constraints. The modeling in this area has become increasingly sophisticated, but the main underlying argument remains the same (e.g., Granovetter 1978, 1985, Rapoport 1988, Rapoport et al. 1989, Heckathorn 1992). What all of the 'critical mass' theories or analyses of the Volunteer's Dilemma, like collective good problems, have in common is that the argument of the 'weakest link' is replaced by a 'strongest link' argument. Since these kinds of collective goods can be Pareto optimally produced by a sufficient number of actors, the best prospect for producing the good is to make sure that only a sufficient number of actors is maximally interested in producing the good.

Roughly, we conclude that negative effects of heterogeneity are to be expected in social dilemmas of the Prisoner's Dilemma type and that positive effects of heterogeneity are to be expected in social dilemmas of the Volunteer's Dilemma type. While these predictions are already theoretically well elaborated for the second class of dilemmas, this is not the case for the first class. Thus, the theoretical contributions of this book are concerned primarily with social dilemmas of the Prisoner's Dilemma type, namely Support Games.

2.2 Social Support between Heterogeneous Actors
Social support takes place in various situations and among various people. Common examples are persons helping a neighbor in the garden, children playing with other children's toys, women exchanging cooking books, and colleagues offering advice or helping out to meet deadlines (Homans 1958, 1961, Blau 1968). Why would people be willing to support other actors, i.e., use their resources for the aims of others rather than for their own aims? Following social exchange theorists such as Homans and Blau, we assume that social support, like other human actions, is based on rewards and costs. Supporting a stranger is likely not motivated by rewards obtained from the receiving actor in return. To explain why support is given to a stranger we would have to point to factors such as social norms or psychological processes such as upholding self-esteem etc. (e.g., Frank 1989, Baurmann 2002, Brennan and Pettit 2004). Typically, however, social support occurs in the context of *durable pairwise* relationships (Emerson 1976). In a durable relationship, it can be individually rational to provide support, because the actors on the receiving end may repay with support in the future. If Alter does not help Ego today, Ego might not help Alter next time either. In this sense, the long-term benefits of supporting each other can be higher than the short-term benefits of refusing support. Thus, support in durable relationships is based on the possibility that actors can threaten each other with the refusal of support and actors can promise each other rewards for providing support. This mechanism is known as *reciprocity* (Axelrod 1984). We do not consider 'generalized reciprocity', i.e., mutual support amongst more than two actors (e.g., Gouldner 1960, Yamagishi and Cook 1993).

Various factors influence these long-term benefits: first, there is a 'time-lag' between providing and receiving social support, creating a trust problem (Coleman 1990: Chapter 6). Second, people tend to value future rewards less than immediate rewards (Loewenstein and Elster 1992). Finally, the rewards in social support are to some extent unspecified (Blau 1964: 93). Alter can never be sure what his or her future reward will be. For instance, if Alter takes time to explain to Ego how to use a new computer program, Ego might reward Alter later with an invitation for dinner of unspecified quality.

The interaction structure of social support is similar to a (repeated) Prisoner's Dilemma (Weesie 1988: 114). Each actor has an incentive not to provide support, but prefers to receive the benefits from being supported. Mutual support is efficient if the benefits from received support exceed the costs of providing support. The payoffs of the Prisoner's

Dilemma may be asymmetric, but the Prisoner's Dilemma by itself does not provide a simple interpretation of how asymmetric payoffs may arise from individual differences in preferences or resources. The social support interpretation makes it possible to link individual characteristics to the payoffs. The relevant characteristics of actors are the costs of providing support (c_i), the benefits from receiving support (b_i), and the likelihood of needing support (π_i) (Weesie 1988, Hegselmann 1994a, 1994b). We call the likelihood of needing support from now on 'neediness'. In the social support variant of social dilemmas, we can conceptualize the heterogeneity of actors as the dissimilarity of the actors with respect to these characteristics. We can now address how the individual characteristics affect whether support is being given. We can also analyze how heterogeneity at the individual level, i.e., heterogeneity in individual characteristics, affects the outcome at the dyadic level, i.e., social support between two actors.

The comparison of social support between heterogeneous actors with social support between homogeneous actors, however, is not straightforward. If we want to determine whether heterogeneity facilitates or hampers social support, we need to devise a method to link up support between homogeneous and heterogeneous actors; with which homogenous actors do we want to compare heterogeneous actors? If all homogenous actors have lower costs of providing support than the heterogeneous actors, it will come as no surprise that mutual support is more likely under homogeneity. But such a comparison is not very meaningful, as it mixes up two aspects: the 'size' of the pie and the 'distribution' of the pie. The 'size of the pie' refers to the sum of the probabilities that actors need support and to the sum of the incentives. The 'distribution of the pie' refers to the distribution of the total likelihood of needing support over the actors and the distribution of the incentives over the actors. The analysis of the consequences of heterogeneity is only meaningful if we keep the 'size of the pie' fixed. Thus, we keep the 'total amount' of the costs of providing support, the benefits from receiving support, and the likelihood of needing support fixed and study the consequences of varying the distribution of the parameters between the actors. This resembles the distribution of a fixed amount of tangible resources between the actors. Thus, we interpret the parameters $c, b,$ and π as functions of resources. Similar approaches to study effects of 'inequality' can be found, for instance, in the literature on income inequality (e.g., Atkinson 1970, Sen 1997) and on insurance and uncertainty (Arrow 1951).

Social support is intensively studied, both theoretically (Homans 1958, 1961, Kelley and Thibaut 1959, Blau 1964) and empirically (Ben-Porath 1980, Kirmeyer et al. 1987, Robertson et al. 1991, Engelmann and Fischbacher 2002). This literature proposes many hypotheses that are related to our research. For instance, mutual support in a durable relationship is more likely if the benefits from receiving support are higher, if the costs are lower, and if the relation is more durable (Brown 1986, Smith and Mackie 1995: Chapter 12). Such hypotheses address individual characteristics. However, these are not the focus of our study. The degree of heterogeneity of actors is a dyadic characteristic, and this is intricately related to the (dis)similarity of actors. It is often found in the literature that social support is

more frequently found between similar actors. The interpretation of this finding from the perspective of our model and from our results is discussed in Section 2.5.

2.3 A Model of Support
2.3.1 Description of the Model

We consider an interaction with two actors (Figure 2.1). At any time point, actors may need the support of the other actor. More precisely, at each time t one out of four events may occur: (1) actor A needs support, but actor B does not; (2) actor B needs support, but actor A does not; (3) neither actor A nor actor B needs support, and finally (4) both actors need support. If only one actor needs support, the other actor has two behavioral alternatives, namely to provide support or not to provide support. Providing support costs resources $c_i > 0$, $i = $ A, B; receiving support is beneficial $b_i > 0$. The parameters bi and c_i are utility *differences*, not utilities. The benefits b_i are the differences between i's utility, if i needs and receives support ($x_i + b_i$), and i's utility if i needs but does not receive support (x_i). The costs c_i are the difference between i's utility, if i does not need support and does not give support (y_i), and i's utility if i does not need but does give support ($y_i - c_i$). We assume that the benefits from receiving support are larger than the costs of providing support, $b_i > c_i > 0$. This assumption reflects the situation that social support comprises the use of one actor's time to assist another actor in reaching his or her goals (Coleman 1988). Of the remaining events, one is simple: if neither of the actors needs support, no support has to be given. This event is valued by i at y_i. If both actors need support at the same time, we assume that no support can be given; this event is valued by i at x_i. Our results only depend on b_i and c_i. The behavioral choices of i are irrespective of x_i and y_i. Without loss of generality, we can simplify the presentation of our model and write $x_i = y_i = 0$. We assume $b_i > c_i > 0$.

Consider the example of two colleagues at work on Friday afternoon. Alter needs help putting 1400 questionnaires in envelopes in the next four hours, before the post office closes. Alter's colleague, Ego, has no pressing duties. In this situation, exactly one actor needs help and the other can decide whether to help or not. If Alter has to put the questionnaires in the envelopes alone, Alter may not finish the work in time. If Ego helps Alter, Alter will finish the job in time. The situation is different if both Ego and Alter have to put 1400 questionnaires in envelopes in the next four hours. Now they cannot help each other. Obviously, if neither Alter nor Ego has to put questionnaires in envelopes, neither of them needs help.

In our model it is determined at random which of the four events occurs, denoted by π_i, the probability that only actor i needs support at t, by π_0 that no actor needs support at t, and by π_{AB} that both actors need support at t. Thus,

$$\pi_A + \pi_B + \pi_0 + \pi_{AB} = 1, \text{ with } \pi_A, \pi_B > 0.$$

With probability $\pi_A + \pi_B = \pi_+ > 0$ exactly one actor needs support, while the other actor has to make a decision whether or not he or she provides support. How often support is needed and could also be given depends on different aspects of the interdependency situation. For instance, if an advertising agency has acquired a major assignment, all employees will be involved and might need some help from colleagues to get their part done in time. In this case, the probability π_{AB} that Alter and Ego need help at the same time will be relatively high. However, if Alter is doing the creative part of the work in the early phase of the project, while Ego has to present the results to the client later on, Alter and Ego will not need help at the same time. At the beginning of the project the probability π_A that Alter needs support is high, and at the end of the project the probability π_B that Ego needs help is high; the probability that they need support at the same time (π_{AB}) is low. During the summer holidays, the workload is light and the probability π_0 that no one needs help is relatively high.

The formulation above with four different events encompasses the models proposed by Weesie (1988) and Hegselmann (1994a, 1994b). In our model, as in Weesie (1988), always exactly one actor needs support. Hegselmann assumes that the actors need support independently in the sense of probability theory. As an example of such independence, consider the need for emotional support between friends. Ego and Alter require emotional support because of events that happen in their respective lives, at work, in relationships with other people, etc. Such events will be only loosely coupled; the occurrence of events for Ego will typically not depend much on what happens in the life of Alter. However, the likelihood of needing support can include a degree of dependency between actors. The independence assumption, however, is not fully innocent. As an example with dependency in the need of support, consider two neighboring farmers. After a long period of bad weather, a farmer will need the material support of other people to survive. However, the two neighbors tend to need support at the same time, and they will probably not be able to provide support. Thus, the dependencies of events affect the probabilities π_A and π_B that an actor needs support while at the same time the other actor is able to provide support. Informal support is not very efficient for 'positively' correlated risks; formal insurance schemes are able to pool risks that are geographically more dispersed. These risks are less correlated, and so in this case a formal support mechanism has an efficiency edge over informal support.

Figure 2.1 shows the extensive form of the Support Game (SG). Payoffs represent 'utilities' that correspond to the outcomes of the game. We assume that actors know the game, know that the other actor also knows the structure of the game they are playing, etc; technically, the structure of the game is common knowledge (Rasmusen 2001a, 2001b). The game is played non-cooperatively in the standard game-theoretic sense that actors are unable to make enforceable agreements or commitments.
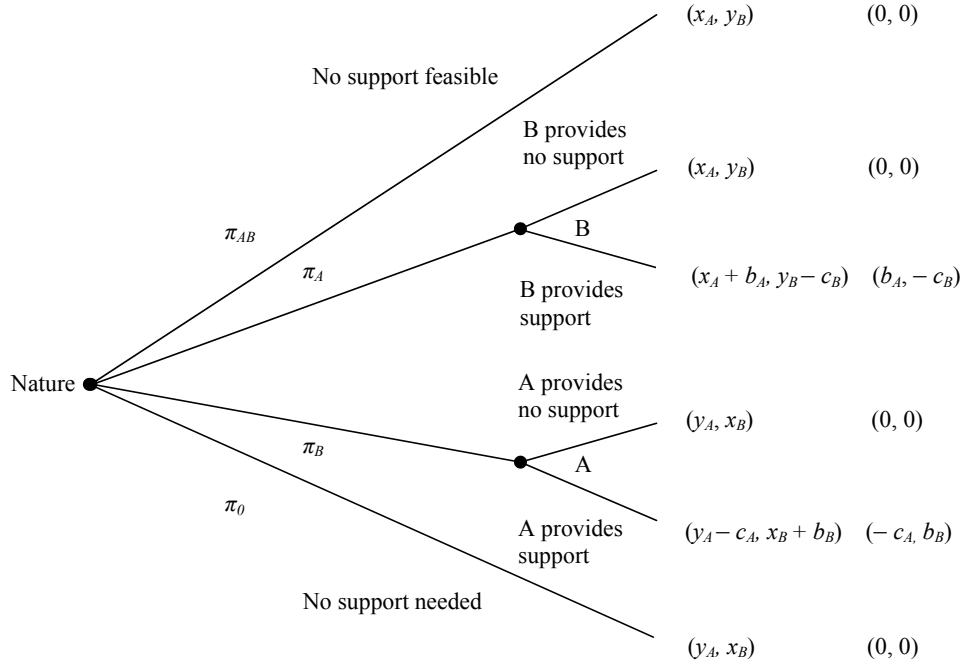
Figure 2.1 Extensive form of the Support Game (SG), with $b_i > c_i > 0$. The equilibrium behavior does not depend on (x_i, y_j). Without loss of generality we assume $x_i = y_i = 0$. The ISG is an iterated version of this game over time, with independent moves of Nature.

We want to study necessary and sufficient conditions for support in SGs: Under what conditions is it individually rational for the actors to provide support? The results for SG are actually trivial: There is a unique (subgame perfect) Nash equilibrium which states that neither actor provides support. This follows from the assumption that $c_i > 0$. This equilibrium is Pareto inferior compared to the situation in which *both* actors provide support in the events that support is needed by exactly one actor, if and only if the expected gains from received support exceed the expected costs of providing support,

$$\pi_i b_i - \pi_j c_i > 0 \text{ for } i = \text{A, B}, i \neq j. \tag{2.1}$$

Throughout this Chapter (2.1) is assumed. If (2.1) is met, individually rational behavior leads to a collectively irrational outcome (Rapoport 1974).[2] Under (2.1), the SG is a variant of the Prisoner's Dilemma game. It is a Pareto improvement for both actors to provide support instead of not to provide support, but it is individually rational not to provide support.

---

[2] It can be shown that, even if (2.1) does not hold, there is always a Pareto improvement in which at least one actor j always provides support $\alpha_j = 1$, while the other actor i provides support with some probability α_i, $0 < \alpha_i \leq 1$.

As discussed in Section 2.2, social support in a durable relationship is based on the principle of reciprocity. Hence, social support has to be studied not in a 'one-shot' game, but in the context of durable relationships: an Iterated Support Game (ISG). The example of the two colleagues provides some intuition why social support has to be modeled as an ISG. The giving of help and advice between colleagues is not controlled or organized by formal contracts between the colleagues. The underlying mechanism of mutual support is reciprocity; a colleague who provides support loses leisure time or might run into a delay with his or her own work. However, the long-term benefits can overcome Alter's short-term incentive not to offer his own working time to help Ego. Next time, when Alter needs help, Alter will be better off if Ego rewards Alter for being helped before instead of refusing help. 'Iterated' games are games in which actors have to make similar choices repeatedly and they can take into account what has happened in previous periods of play (Fudenberg and Tirole 1991). It is well known that if an one-shot game with an inefficient outcome is repeated an indefinite number of times, it can be individually rational to exhibit efficient behavior, provided that the future is 'important enough' and deviations from Pareto optimal behavior are appropriately sanctioned (Friedman 1986: 103). In our application, an efficient outcome means that support is provided whenever support is needed and when giving support is possible.

Let us now address the issue of the importance of the future. In the theory of repeated games it is usually assumed that actors value the payoffs from the present game higher than the payoffs from future games. There are two reasons for the preferential treatment of the present. The first reason is negative time preferences: people prefer to benefit immediately, rather than later (Loewenstein and Elster 1992). A natural way to take this into account is to discount payoffs over time such that the payoffs from the next round are worth less than the payoff from the current round. The 'weight' that actor i assigns to the next round relative to the present one is called i's *discount parameter*

θ_i, with $0 < \theta_i < 1$.

Empirical evidence suggests sizable interpersonal differences in discounting that are, at least partly, socially produced (Gattig 2002). The second reason to down weight the future is that there is no assurance that future games will be played, because relationships may by then have ended. We thus distinguish the time discount parameter θ_i from the probability that after each round another round will be played. The continuation probability then is

w, with $0 < w < 1$.

We stress that the 'objective' continuation probability w is the same for both actors, while the 'subjective' individual discount parameter θ_i may well differ between the actors.

The payoffs of the repeated game can now be defined as follows: in every time period t, actor i obtains a payoff u_{it}. Here, $t = 0, 1, 2, \ldots$ is a discrete time parameter, and $i = A, B$ indexes the actors. At each period t in which actor i does not play, he or she obtains a zero payoff, $u_{it} = 0$. The expected discounted payoff is the total payoff of an actor i associated with the infinite stream of payoffs $(u_{i0}, u_{i1}, u_{i2}, \ldots)$

$$\sum_{t=0}^{\infty} \theta_i^t Eu_{it},$$

where Eu_{it} is the expected utility for actor i at time t. Eu_{it} depends on the continuation parameter w, the strategies used by the players, and the benefits from receiving support, costs of providing support, and likelihood of needing support. For instance, if both players always provide support, the expected payoff attained at time t equals

$$Eu_{it} = w^t \left(\pi_i b_i - \pi_j c_i \right) + (1 - w^t)0 = w^t \left(\pi_i b_i - \pi_j c_i \right).$$

Here w^t is the probability that the relationship has continued up to time t. Thus, the expected discounted payoff obtained over the full interaction if both actors 'always provide support' is

$$\sum_{t=0}^{\infty} \theta_i^t Eu_{it} = \sum_{t=1}^{\infty} \theta_i^t w^t (\pi_i b_i - \pi_j c_i) = \frac{\pi_i b_i - \pi_j c_i}{1 - w\theta_i}. \qquad (2.2)$$

The expected discounted payoff if both actors never provide support is 0. Expected discounted payoffs for other strategies can be derived in similar ways. Observe that the discount rate commonly included in repeated game models is now the product $w\theta_i$. The distinction between the discount parameter θ_i and the continuation probability w allows us to address how support in the ISG depends on the 'stability of the relation' indicated by the continuation probability w as well as the individual discount parameters θ_i.

The ISG is similar to games studied in the literature (Weesie 1988, Hegselmann 1994a, 1994b). In Weesie (1988), the focus is, as in this chapter, on the dyadic level and so on support generated by reciprocity. Differences in the discount parameters, however, are not taken into account. Hegselmann (1994a, 1994b) differentiates actors only by the likelihood of needing support, but they are able to search for a reasonable partner to exchange support with. Thus, in Hegselmann (1994a, 1994b) support is the consequence of market-like social processes.

2.3.2 Analysis of the Model

Now we can derive the condition for mutual support in the ISG. We focus on conditions under which actors are expected to support each other fully and at all time points. Thus, as the term is used here, 'mutual social support' means that both actors always provide each other with support. However, support will be given only if the long-term benefits of mutually providing support are higher than the short-term benefits of refusing support. The costs of providing support in the long run depend on the strategies used by the actors. We restrict our attention to trigger strategies. Trigger strategies are a particular implementation of reciprocity. A trigger strategy is a *totally unforgiving* strategy that employs *permanent retaliation*. A trigger strategy is never the first to refuse support, but if the other refuses support even once, a trigger strategy refuses support from that time onwards. Even after an own 'unintended' refusal of support, trigger strategies refuse support from then on. This is needed to ensure subgame perfection (Kreps 1990, Binmore 1998).

We do not claim that people in fact use trigger strategies. Furthermore, trigger strategies are not very suitable templates for the study of the dynamics of social strategies. Even though, the analytical treatment of trigger strategies is relatively easy; they do serve an important theoretical purpose. Trigger strategies are theoretically interesting for the study of the preconditions of cooperation. If mutual social support is not individually rational between trigger strategies, then mutual social support is not individually rational between other strategies either. This is the case because in this game having a permanent retaliation is the most severe punishment (Abreu 1988). The following lemma states the necessary and sufficient condition for an equilibrium in trigger strategies.

Lemma 2.1 A pair or trigger strategies (τ_A, τ_B) is a subgame-perfect equilibrium if and only if

$$\zeta^* = \max(\zeta_A, \zeta_B) \leq w, \tag{2.3}$$

where

$$\zeta_i = \frac{1}{\theta_i} \frac{c_i}{\pi_i b_i + (1-\pi_j)c_i} = \frac{1}{\theta_i} \frac{1}{\pi_i \eta_i + (1-\pi_j)}, \text{ with } \eta_i = \frac{b_i}{c_i} > 1.$$

For a proof of this lemma and the subsequent theorems we refer to Appendix A.

We observe that the equilibrium depends on the costs and benefits via the benefit-cost ratios denoted by $\eta_i = \frac{b_i}{c_i} > 1$. The benefit-cost ratio is a ratio of two utility differences, and so it is a scalar scale free quantity. Thus, interpersonal comparison of the preference parameter η_A and η_B is possible without indulging in the intricate problems of the interpersonal comparison of utility (Harsanyi 1977: Chapter 4, Coleman 1990: Chapter 29, Sen 1997). The equilibrium condition shows that mutual support depends on the dyadic continuation probability (*w*) and on all individual parameters: the benefit-cost ratios (η_i), the time-preferences (θ_i), and neediness (π_i). We will use this condition to predict the frequency of support under homogeneous and heterogeneous distributions of the parameters.

An important analytical result of the theory of repeated games is that cooperation is consistent with individually rational behavior in a repeated game if the continuation probability *w* is sufficiently large (Kreps 1990: Chapter 14). This result is replicated here. According to (2.3), it is individually rational to provide support if the continuation probability *w* exceeds the *dyadic threshold* ζ^*. If ζ^* decreases or *w* increases, the equilibrium condition 'is more easily met'. We will interpret this to mean that 'support is more likely' if ζ^* decreases or *w* increases. Keeping *w* fixed, ζ^* will then be treated as an indicator for 'how likely mutual support' is between the actors. The dyadic threshold ζ^* depends on the *individual thresholds* ζ_i. The individual threshold ζ_i increases in the benefit-cost ratio's η_i (i.e., increases in the costs and decreases in the benefits), and in neediness π_i, but decreases

in the time preference θ_i. If ζ_A or ζ_B increases, ζ^* increases as well. Analogously, ζ^* decreases if ζ_A and ζ_B decrease. Therefore, $\zeta^* = \max(\zeta_A, \zeta_B)$ increases in η_i, and decreases in π_i and θ_i for both actors. This result is in accordance with the empirical findings on social support mentioned in Section 2.2.

2.3.3 Heterogeneity between the Actors

We model heterogeneity of the actors as interpersonal differences in the benefit-cost ratios (η_i), in neediness (π_i), or in the time preferences (θ_i). Actors are called *homogeneous* if they do not differ with respect to the individual parameters,

$$\eta_A = \eta_B, \ \pi_A = \pi_B, \text{ and } \theta_A = \theta_B,$$

otherwise the actors are said to be *heterogeneous*. The continuation probability w is always the same for both actors. For instance, if Alter stops working at a company, the work relation between the colleagues Ego and Alter ends for both of them.

We argue that a comparison of social support between homogeneous actors with social support between heterogeneous actors is meaningful if the 'total amounts' of the homogeneous and heterogeneous distributed parameters are fixed

$$\eta_A + \eta_B = \eta_+, \ \pi_A + \pi_B = \pi_+, \text{ and } \theta_A + \theta_B = \theta_+. \tag{2.4}$$

This is our *constant-sum condition* or *budget constraint*. Under this condition, we cannot reduce the individual parameters of both actors at the same time. If we decrease one individual parameter, for instance, the benefits of actor A, we have to increase the benefits of actor B so that the sum of the benefit-cost ratio is not affected ($\eta_A + \eta_B = \eta_+$). To keep the sums η_+, π_+ and θ_+ fixed resembles the distribution of a fixed amount of a tangible resource between the actors. The interpretation of a distribution of a tangible good seems appropriate for the benefit-cost ratios and for neediness. Differences in costs and benefits and in neediness reflect differences in resources. The interpretation is admittedly less compelling for the psychological discount parameters. To give a numerical example, we assume a homogeneous distribution of all parameters, except of the distribution of the costs, which is heterogeneous. If we assume that $\eta_A + \eta_B = 6$, we can compare $\eta_A = \eta_B = 3$, with $\eta_A = 4$, $\eta_B = 2$ or $\eta_A = 5$, $\eta_B = 1$. Thus, we can study the effects of an increase of heterogeneity in the benefits-costs ratio. Heterogeneity is conceived as an unequal distribution of at least one parameter between the actors. The equilibrium condition is least restrictive if ζ^* is minimal under every possible distribution of the individual level parameters, given the constant-sum condition (2.4). We use the phrase 'distribution' of a parameter to stress that changes at the individual level should not violate the constant-sum condition that ensures that a meaningful comparison of social support between homogeneous and heterogeneous actors. Now, the minimization of

ζ^* is not equivalent to minimizing ζ_A and ζ_B at the same time. Given the budget constraint, if ζ_A decreases, ζ_B necessarily increases.

2.3.4 Theorems

To illustrate how social support is affected by different distributions of the parameters between the actors, we consider a numerical example with homogeneity with respect to the benefit-cost ratio η and the time discounting parameter θ,

$$\eta_A = \eta_B = 3, \text{ and } \theta_A = \theta_B = 1.$$

First, we consider homogeneity with respect to the need of support, $\pi_A = \pi_B = \frac{1}{2}$. According to Lemma 2.1 support can be expected if the continuation probability w is larger than or equal to the dyadic threshold

$$\zeta^* = \max\left(\frac{1}{1}\frac{1}{\frac{1}{2}\frac{9}{3}+(1-\frac{1}{2})}, \frac{1}{1}\frac{1}{\frac{1}{2}\frac{9}{3}+(1-\frac{1}{2})}\right) = \max(0.5, 0.5) = 0.5.$$

Next, we consider heterogeneity with respect to neediness, $\pi_A = \frac{1}{3}$ and $\pi_B = \frac{2}{3}$. In this case the dyadic threshold ζ^* for individual rational mutual support is

$$\zeta^* = \max\left(\frac{1}{\frac{1}{3}\frac{9}{3}+(1-\frac{2}{3})}, \frac{1}{\frac{2}{3}\frac{9}{3}+(1-\frac{1}{3})}\right) = \max(0.75, 0.37) = 0.75.$$

Hence, the equilibrium condition is more restrictive in the heterogeneous case than in the comparable homogenous case. Figure 2.2 shows how the individual thresholds ζ_A and ζ_B, and the dyadic threshold ζ^* vary with π_i. The figure shows that ζ^* is minimal if the individually thresholds are equal, $\pi_A = \pi_B = \frac{1}{2}$, i.e., social support is 'most likely' under homogeneity.
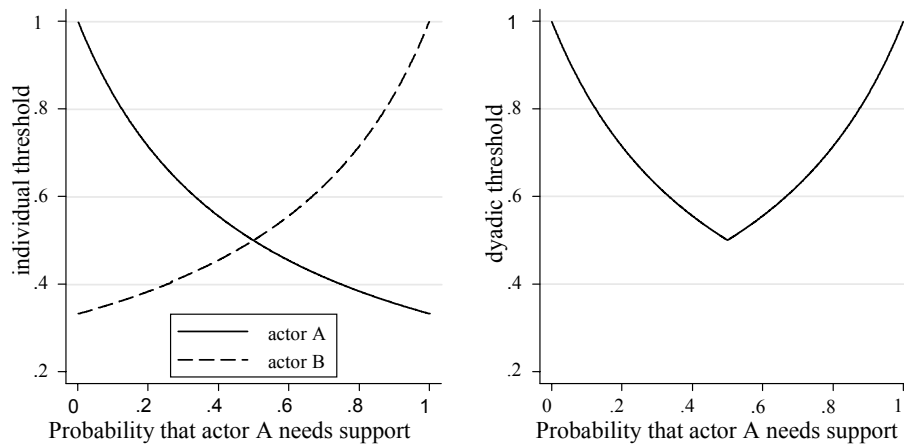

Figure 2.2 Numerical example of Theorem 2.1 (heterogeneity in one parameter).

To provide some intuition why homogeneity facilitates social support, we consider the analogy discussed in the introduction. We want to make a chain as strong as possible, using the available iron and carbon. With two links and equal amounts of iron already dedicated to produce each of the links, how then should the carbon be allocated? The strength of the chain is that of the weakest link, as the weakest link is the first to break under pressure. Clearly, we should make both links equally strong. An unequal distribution of carbon would make one link stronger than the other. Since the chain is as strong as its weakest link, the extra carbon used in the stronger link is wasted; the chain is not optimal. The chain would become stronger if some carbon is shifted from the more affluent link higher in carbon to the link poorer in carbon. In an optimal chain, assuming that the iron was distributed equally, the carbon has to be distributed equally as well.

The following theorem states that the example above generalizes beyond the arbitrary numbers of the example, and also applies to heterogeneity with respect to the other individual level parameters.

Theorem 2.1 (heterogeneity in one parameter): *Consider an ISG with heterogeneity in one parameter $\mu \in \{\eta, \pi, \theta\}$ and homogeneity in the remaining two parameters. In this case ζ^* is minimal if μ is distributed equally between the actors, $\mu_A = \mu_B$, and so $\zeta_A = \zeta_B = \zeta^*$.*

Homogeneity is best if we allow heterogeneity only in a single dimension. Assume that Alter and Ego are colleagues. From time to time Ego and Alter have to run experiments. To do this they require each others' assistance - if they were to conduct the experiments at the same time, they could not assist each other. If Alter assists Ego, it costs Alter time, if Ego assists Alter the situation is the same vice versa. If both actors require help from each other equally often the prospects for mutual support are optimal.

In our first example, we illustrated the optimal adjustment of one parameter if the other two parameters are homogenous. Now we consider an example with heterogeneity with respect to the costs and benefits and homogeneity with respect to time discounting.

$$\eta_A = \tfrac{12}{3}, \ \eta_B = \tfrac{6}{3}, \text{ and } \theta_A = \theta_B = 1.$$

We want to study whether or not homogeneity, with respect to neediness, is still optimal. First, we consider again the homogeneous case, $\pi_A = \pi_B = \tfrac{1}{2}$. The dyadic threshold is

$$\zeta^* = \max\left(\frac{1}{1}\frac{1}{\frac{1}{2}\frac{12}{3}+(1-\frac{1}{2})}, \frac{1}{1}\frac{1}{\frac{1}{2}\frac{6}{3}+(1-\frac{1}{2})}\right) = \max(0.4, 0.7) = 0.7.$$

Next, we consider a particular heterogeneous distribution of neediness, $\pi_A = \tfrac{1}{3}$ and $\pi_B = \tfrac{2}{3}$. In this case we have

$$\zeta^* = \max\left(\frac{1}{1}\frac{1}{\frac{1}{3}\frac{12}{3}+(1-\frac{2}{3})}, \frac{1}{1}\frac{1}{\frac{2}{3}\frac{6}{3}+(1-\frac{1}{3})}\right) = \max(0.6, 0.5) = 0.6.$$

Thus, we see that in this example homogeneity in neediness does *not* lead to the minimum of ζ^*, i.e., equal need of support is not the most favorable condition for mutual support. The particular heterogeneous distribution of the π's that we looked at leads to a smaller dyadic threshold than a homogeneous distribution of the π's. It is, however, not the heterogeneity of the π's per se that facilitates mutual support. If the distribution of neediness is oppositely skewed, namely $\pi_A = \frac{2}{3}$ and $\pi_B = \frac{1}{3}$, ζ^* would be even larger than under homogeneity:

$$\zeta^* = \max \left(\frac{1}{1} \frac{1}{\frac{2}{3}\frac{12}{3} + (1-\frac{1}{3})}, \frac{1}{1} \frac{1}{\frac{1}{3}\frac{6}{3} + (1-\frac{2}{3})} \right) = \max (0.3, 1) = 1.$$

Since $0 < w < 1$, mutual social support is not individually rational under this condition, no matter how durable the relationship is. If $\pi_A = \frac{1}{3}$ and $\pi_B = \frac{2}{3}$, there is an *interaction effect* between the parameters; heterogeneity in one parameter (η) is *compensated* by heterogeneity in another parameter (π). Figure 2.3 displays the individual thresholds (ζ_i) and the dyadic threshold (ζ^*) for varying π_i. We see that mutual support is 'most likely' under a heterogeneous distribution of neediness. If $\pi_A = \frac{1}{3}$ and $\pi_B = \frac{2}{3}$, the individual thresholds are equal and the dyadic threshold has the smallest value. If $\pi_A = \pi_B = \frac{1}{2}$, ζ^* is larger, and furthermore we see that it is not the heterogeneity of the π's per se that facilitates mutual support, because ζ^* is even larger if $\pi_A = \frac{2}{3}$ and $\pi_B = \frac{1}{3}$. In this latter case, the heterogeneity in the two parameters re-enforce each other.
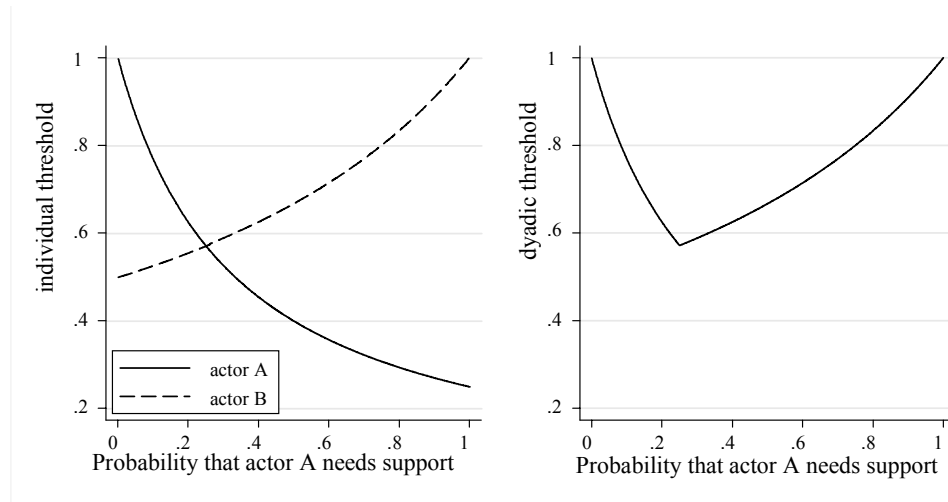
Figure 2.3 Numerical example of Theorem 2.2 (compensation).

We conclude that homogeneity in a given parameter is not necessarily 'optimal', irrespective of the distributions of the other parameters. If there is heterogeneity in one parameter, it is usually the case that better prospects for social support are possible if there is heterogeneity in the other dimension as well, provided that these heterogeneities are well aligned.

Theorem 2.2 (compensation): *Consider an ISG with heterogeneity in η, π, and θ. ζ^* is minimal in one of the parameters $\mu \in \{\eta, \pi, \theta\}$ if μ_A and μ_B are adjusted so that $\zeta_A = \zeta_B = \zeta^*$.*

While generally it is not true that homogeneity with respect to π, or any other parameter for that matter, favors mutual support, some form of homogeneity is beneficial, namely the *theoretical homogeneity* in terms of the individual thresholds ζ_i. These are the theoretical analogues to the 'strength of the link'. Considering the numerical example, but now with the general probabilities π_A and $\pi_B = 1 - \pi_A$, we have

$$\zeta_A = \frac{1}{1(\frac{12}{3}\pi_A + (1-\pi_B))} = \frac{1}{5\pi_A}, \text{ and } \zeta_B = \frac{1}{1(\frac{6}{3}\pi_B + (1-\pi_A))} = \frac{1}{3(1-\pi_A)}.$$

According to Theorem 2.2, the optimal ζ^* is characterized by $\zeta_A = \zeta_B$. Solving for π_A is $\pi_A = \frac{3}{8}$, and so the optimal dyadic threshold is $\zeta^* = \frac{8}{15}$.

We consider again the example of the two colleagues who assist each other during their experiments. We assume that Alter and Ego have equal benefits from receiving support. However, Ego runs experiments more often than Alter does. The optimal prospect for mutual support would then be that Ego's experiments are shorter than Alter's experiments. Thus, Alter has lower costs of providing support than Ego. The low costs of providing support interact positively with the high frequencies of running the experiments (compensation effect).

So far, we have analyzed how *one* parameter (μ) is optimally distributed, given how the other two parameters are distributed. We now analyze variations in all three parameters simultaneously, subject to the constant-sum condition (2.4). Each parameter can be distributed homogeneously or heterogeneously. How should the parameters be *simultaneously* distributed in order to make the condition for mutual support least restrictive? The question is answered in the next theorem.

Theorem 2.3 (homogeneity is globally best): Let $\pi_+ = 1$.[3] *The minimum of ζ^* subject to the constant-sum condition is attained by homogeneity with respect to each of the parameters, i.e., $\eta_A = \eta_B = \frac{1}{2}\eta_+$, $\pi_A = \pi_B = \frac{1}{2}$, and $\theta_A = \theta_B = \frac{1}{2}\theta_+$.*

In the chain analogy, all material should be distributed equally between all links to make the chain as strong as possible. This is what happens if a chain out of steel will be produced: given some amounts of iron, zinc and copper, and no constraints on how the elements should be distributed to produce the chain, every link of the chain will have the same amount of iron, zinc, and copper. With respect to the example of the two colleagues, the optimal prospect for mutual support would be that both receive the same costs for assistance in conducting

---

[3] For a discussion of the technical assumption that $\pi_+ = 1$, see the proof of Theorem 2.3 in Appendix A.

experiments and the same benefits when receiving support with the own experiment, and that both need support equally often, i.e., both run experiments equally often.

2.4 Conclusion and Discussion

In this chapter we have argued that the effects of asymmetry in social dilemmas can be conveniently studied in terms of social support with asymmetry conceptualized as *actor heterogeneity* with respect to the costs of providing support, the benefits from receiving support, the likelihood of needing support, and how much actors value future rewards. Our analyses yielded a series of theorems. First, if there is homogeneity in all but one parameter, homogeneity in the remaining parameter leads to the optimal condition for social support. Second, if there is heterogeneity in at least one parameter, mutual support is most likely if there is a specific heterogeneous distribution of the other parameters that compensates for the original heterogeneity. Thus, homogeneity in a single dimension does *not* necessarily lead to the optimal condition of mutual support. Third, if all parameters are varied 'simultaneously', mutual support is most likely if all parameters are distributed equally between the actors. Thus, homogeneity in all dimensions is the globally best outcome.

A good intuition for our results can be obtained from the analogy of how to make a multiple-link chain as strong as possible. This is achieved by making the weakest link as strong as possible, and so we should make all links equally strong. There are however many ways to do this. Intuitively it is not implausible that the chain is indeed strongest if all links are the same. A sufficient reason for the optimality of homogeneity is *convexity* of the strength of links in terms of the amounts of different materials used, and so the different materials need to be *complementary*.

In the social support case, *all* actors need to be willing to stick to the implicit cooperative agreement enforced by trigger strategies. Thus, to facilitate cooperation, the individual thresholds of *both* actors should be as small as possible. We conclude that the individual thresholds have a similar role as the inverse of the strength of the links. Both individual thresholds are related due to the 'fixed mean' of the parameters: a decrease of Alter's individual threshold can be accomplished only by an increase of Ego's individual threshold and vice versa. Thus, making both individual thresholds as small as possible implies that the individual thresholds should be equal. To obtain the result that optimal individual thresholds involve homogeneity with respect to the parameters, however, requires additional arguments, just as we needed for the chain analogy. In the proof of Theorem 2.3 (see Appendix A), we show that the individual thresholds are indeed convex in the parameters. One can think of this as a positive interaction effect between any two of the parameters.

We want to emphasize that 'homogeneity is globally best' does not mean that 'less heterogeneity' between actors necessarily yields more cooperation than 'more heterogeneity'. Heterogeneity decreases if the distribution of any of the parameters is made more equal. However, if such a reduction in heterogeneity is performed in a parameter that was skewed to

compensate for heterogeneity in another parameter, the compensation effect will become smaller, and so cooperation will be hampered, not facilitated. Numerical examples of this phenomenon were presented in Section 2.3. We turn again to our analogy for illustration. Evening the distribution of the construction materials out over the links does not necessarily make the chain stronger. Consider the example of a chain made out of copper and iron. The iron is distributed equally among all the links of the chain, except for one link that has only half the amount of iron. If we employ the copper equally, the link with less iron will still be the weakest link. If we put more copper in the link that is low in iron than in the other links, we will be using the materials less homogenously and this improves the strength of the chain.

This discussion may shed new light on the research of similarity in social support theory. Theories of social support often argue that 'similarity' between actors positively influences support behavior. The general idea is that in deciding to help or support, people take into account how similar the person in need is to them (e.g., Dovidio et al. 1991, Smith and Mackie 1995). The theories claim that similarity generally enhances cooperative tendencies such as conflict resolutions, support behavior etc.: "similarities in beliefs, attitudes, and values […] are usually conducive to […] cooperative resolutions of conflicts" (Deutsch 1973: 374). These theories generally study the effects of heterogeneity and homogeneity on social support in variables such as age, gender, religion, social status, or education. It is very seldom that the effects of *different degrees* of homogeneity and heterogeneity on support *between* several independent variables such as age, gender, etc. are compared with each other.

As an example of an article that uses homogeneity additively, we discuss Louch's article on network integration (Louch 2000). The article combines studies of transitivity and homophily (homogeneity in our terms) in an empirical analysis of personal network integration. For the purpose of our demonstration we focus only on the homophily hypothesis and neglect the other hypotheses. The hypothesis states that homophily improves the probability of integration in personal networks (triads). Louch uses race, gender, education, age, and religion to test this hypothesis. Using a logistic regression model, Louch analyses the simultaneous effects of all variables on the likelihood of a connection existing between triads. Louch uses the homophily variables additively in this model. The effect of each homophily variable is studied by keeping the other homophily variables constant. The effect of age is, for instance, studied by keeping education, religion, gender, etc. constant. Given the third theorem on compensation, we would expect an interaction effect between homophily variables. Since the difference in homogeneity and heterogeneity *between* the variables *per se* has an effect on support behavior, a cumulative analysis of homophily variables is misleading. The compensation theorem states a complex interaction effect between heterogeneous variables. Take as an example two colleagues assisting each other by running experiments. It may be the case that Alter runs experiments more often than Ego. This means that Ego has to assist Alter more often than Alter assists Ego. If the experiments of Ego and Alter take equally long, then we have a situation where in Ego has to help Alter more often and each time Ego helps it costs a lot of time. It is easy to imagine that Ego will not be willing to assist

Alter all the time. However, if Ego's experiments take much longer than Alter's experiments, then the situation is different. Now, Ego still has to help Alter more often, but since Alter always assists Ego for longer than Ego assists Alter, this situation might be more equal than the first situation. Only looking at differences in each of the variables separately, one would expect that heterogeneity in frequencies as well as in the length of the experiments make helping each other more problematic. However, it is clear for this example that the main predictor for supportive behavior should be the total time each person needs support, which is given by the interaction between frequency and length of the experiments. In other words, if someone needs support more often it is less costly for the others to support, if the heterogeneity in one dimension compensates the other. However, this interaction between the variables is neglected in the literature on homophily. The implicit assumption of most of the homophily analyses that it is generally better to have *more* dimensions that are homogeneous is not necessarily true. Given heterogeneity in one dimension, it can be even better for mutual support to have even *more* heterogeneous dimensions. Heterogeneity in one dimension can namely be compensated by heterogeneity in another dimension. However, the positive side of heterogeneity, namely compensation, is ignored in the literature on similarity or homophily.

We like to stress that *compensation* has nothing to do with *complementarity*. Complementarity may also influence the parameters of the game. Kelley and Thibaut (1959) emphasize that similar actors may not be *able* to provide each other with support. Complementarity of the actors is an essential precondition for mutual support. This issue was also apparent in the example of neighboring farmers; actors facing highly positively correlated risks are not complementary. Complementarity of actors allows them to provide support, because they are not in trouble themselves.

Finally, we want to discuss limitations of our analysis and possible remedies to overcome these shortcomings. We focus on all-or-nothing trigger strategies. Actors either provide full support, backed up by the threat to cancel all future support after any misdeed, or actors do not provide support at all. The restrictions of these two equilibria seem to some extent unreasonable. We are interested in heterogeneity between actors, but we study equilibria in which all actors use the same strategies. Maybe we should consider asymmetric equilibria that typically exist in iterated games (see 'folk theorem' results in, e.g., Friedman 1971, Kreps 1990). These asymmetric equilibria may be Pareto ordered, but may also be Pareto incomparable, leaving a bargaining problem. Who gains how much? To answer the question 'who gains how much', we suggest a study of the 'terms of trade' which can conveniently take the form of fractional support. In the model with fractional support, actors make decisions with respect to the degree α_i to which they give support, $0 \leq \alpha_i \leq 1$, with costs c_i and benefits b_i proportional to α_i (for an analysis of such a 'continuous game', see Nowak et al. 1989, 1999). For instance, either Alter helps Ego to search for participants for an experiment, or Alter only helps to analyze the results, or Alter helps with both, or Alter does nothing. In our analysis, we fixed $\alpha_i = 1$. Do we have any reasons to fix the α's in a certain way? If not, we face a selection problem of the α's. We need to address the bargaining

problem and study the situation with a bargaining model (Nash 1950, Kalai-Smorodinsky 1975, Rubinstein 1982, 1990). However, in our case bargaining theories have a special disadvantage: the cooperation problem is solved 'by assumption', because bargaining theories assume Pareto-efficiency from the outset. As a solution, we may further use an evolutionary approach (Holland 1975, Axelrod 1984, Weibull 1995, Hofbauer and Sigmund 1998). By using an evolutionary approach, we provide an answer on (a) how likely social support is between heterogeneous actors and on (b) to what extent which of the heterogeneous actors provides support.