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Trapping a single atom with a fraction of a photon using a photonic crystal nanocavity

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We consider the interaction between a single rubidium atom and a photonic crystal nanocavity. Because of the ultrasmall mode volume of the nanocavity, an extremely strong coupling regime can be achieved in which the atom can shift the cavity resonance by many cavity linewidths. We show that this shift can be exploited to trap a single atom above the cavity. The atom is trapped by light that is only inside the cavity as a result of the shift of the cavity resonance caused by the proximity of the atom itself. This atom trap requires only a fraction of a photon inside the cavity. Surprisingly, the damping of the cavity plays a pivotal role in this trapping mechanism.

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The interaction between single atoms and macroscopic high-finesse optical cavities has led to great advances in the field of cavity quantum electrodynamics [1–5]. In the pioneering work by Pinkse *et al*. it was shown that a single atom can be trapped by the light field of a single photon inside a high-finesse optical cavity [6,7]. It was recently suggested that nanoscale optical resonators such as photonic crystal nanocavities could also be used for this purpose [8]. Interestingly, even though the linewidths of such nanocavities are large compared to those of their macroscopic counterparts, a much stronger coupling can still be achieved using nanocavities. This is because, whereas the mode volume of state-of-the-art macroscopic cavities is on the order of 10^5 μm^3, a photonic crystal nanocavity typically has a mode volume of a cubic wavelength. Because of this small mode volume, a photonic crystal nanocavity can easily be tuned by bringing a nano-object into the near field of the cavity [10–12]. For experiments in cavity quantum electrodynamics [13–15], a single emitter needs to be brought into the near field of the cavity. Although a lot of progress was made in this field [16–18], accurate placement of exactly one single emitter in the near field of the cavity remains extremely challenging. In experiments with single atoms, rather than the solid state-based emitters used so far, the atom has to be trapped above the nanocavity by some optical or magnetic means. The challenge is thus to realize a trapping potential deep enough inside the evanescent field of the nanocavity and to make this potential sufficiently tight and accurately aligned to the optical mode of the cavity.

In this Rapid Communication, we propose to use the optical field of the nanocavity itself to trap a single rubidium atom. Using parameters from state-of-the-art nanocavities, we calculate, first classically and subsequently quantum mechanically, the optical potential an atom experiences when it approaches the cavity. We find that the atom-cavity interaction can be exploited to create a stable optical trapping potential at nonzero atom-cavity separation and that this separation can be tuned by changing the detuning between the cavity and the laser. This trapping mechanism is the atomic equivalent of the self-induced back-action mechanism recently used to trap classical nanoparticles [19]. Surprisingly, a cavity energy corresponding to only a fraction of a photon is enough to create a trapping potential with a depth of a few mK.

The proposed experimental setup is illustrated in Fig. 1. The cavity a waveguide coupled photonic crystal nanocavity, which is assumed to have a quality factor of $Q \sim 10^5$ and a mode volume of $\mathcal{V}_c \sim \lambda_c^3$, where $\lambda_c = 780$ nm. A design for a comparable cavity was recently proposed [20]. For this design the mode volume $\mathcal{V}_c \approx 5.9 \times 10^{-20}$m^3.

The near-resonant polarizability of the atom can semiclassically be written as $\alpha(\omega_l) = \frac{3\varepsilon_0\lambda_a^3}{8\pi^2}\frac{\delta_a/\gamma_a}{1+4(\delta_a/\gamma_a)^2}$, where $\delta_a = \omega_a - \omega_l$ is the detuning of the atomic transition frequency (ω_a) with respect to the frequency of the light (ω_l) and γ_a the natural linewidth of the atomic transition [21]. When driven below resonance, the atom has a positive polarizability. Thus, when the atom is inside the evanescent field of a cavity, it effectively increases the cavity mode volume and therefore causes the cavity resonance to red shift. The maximum shift in the resonance frequency is given by $\Delta\omega_{\max} = -\frac{\alpha(\omega_l)\omega_{c0}}{2\varepsilon_0\mathcal{V}_c}$, where ω_{c0} is the unperturbed cavity resonance and $\mathcal{V}_c$ is the cavity mode volume [10]. The maximum of the polarizability of a rubidium atom, obtained near the atomic resonance, is approximately $4.5 \times 10^{-21}\varepsilon_0$ m^3. As this corresponds to almost 10% of the cavity mode volume, a single atom can significantly shift the cavity resonance. The actual shift depends on the separation z between the cavity and the atom. For simplicity, the evanescent field above the cavity is assumed to have an exponential decay, such that $\Delta\omega(z) = \Delta\omega_{\max}e^{-z/l_z}$, although in general the decay above a photonic crystal structure can be more complicated [22]. An optical tweezer keeps the atoms on-axis above the center of the cavity. In Fig. 2(a), the laser-cavity detuning is shown as a function of atom-cavity separation. The proximity of the atom clearly shifts the cavity resonance. Figure 2(b) shows the cavity energy when the cavity is continuously illuminated. The cavity resonance is described by a Lorenztian line shape with a width $\gamma_c = \omega_c/Q$. The cavity energy reaches a maximum when the atom-cavity separation is such that the atom pushes the cavity into resonance with the pump light.

The red shift of the cavity resonance is caused by the fact that the field inside the cavity polarizes the atom. This same polarization causes a force on the atom toward the point with the highest light intensity [21]. As the cavity energy depends on the atom-cavity separation, the atom experiences the highest intensity at a nonzero height above the cavity, as illustrated in Fig. 2(c). In short, the atom perturbs the cavity and is trapped

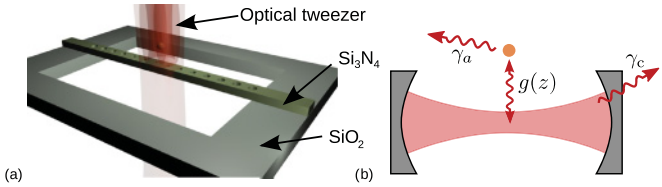

FIG. 1. (Color online) (a) Illustration of the proposed experimental setup. A photonic crystal cavity embedded in an air-bridge waveguide is used to trap light. A rubidium atom is brought into the evanescent field of the cavity and can strongly perturb the cavity mode. For suitable parameters, the atom is trapped at a nonzero height above the cavity. The optical tweezer (vertical red beam) is used to transfer atoms to the cavity and to keep them on-axis above the cavity center. (b) Schematic representation of the atom-cavity system. The atom and the cavity are coupled by a vacuum Rabi frequency $g(z)$ which depends on the atom-cavity separation z. The maximum vacuum Rabi frequency $g_{\max} \approx 2\pi \times 18.5$ GHz. The spectral linewidth of the cavity is $\gamma_c \approx 2\pi \times 3.85$ GHz and of the atom $\gamma_a \approx 2\pi \times 5.89$ MHz.

by the back-action of the cavity on the atom. Note that this is not an active feedback mechanism, as the pump laser frequency is kept constant during the experiment. For these parameters, the atom experiences a trap depth of 50 μK. And according to Fig. 2(b), this trapping potential can be achieved with very little energy inside the cavity. In fact, when we compare the cavity energy to the photon energy (≈1.59 eV), we see that a only fraction of the energy of a single photon is required.

In the classical picture above, the atom was described as an effective polarizability and not as the quantum mechanical single emitter it actually is. Furthermore, as the relevant cavity energies correspond to only a fraction of a photon, the atom-cavity system should in fact be treated quantum mechanically. The starting point for that treatment is the well known Jaynes-Cummings model [23]

$$H = \hbar\omega_c a^\dagger a + \hbar\omega_a |e\rangle\langle e| + \hbar g(z)(a^\dagger |g\rangle\langle e| + a|e\rangle\langle g|), \qquad (1)$$

which couples the occupation of the cavity modes with the atomic transition. Here, ω_c is the cavity resonance frequency, ω_a is the atomic transition frequency, and $g(z) = g_{\max}\exp(-z/l_z)$ the vacuum Rabi frequency as a function of the atom-cavity separation z. The maximum vacuum Rabi frequency is given by $\hbar g_{\max} = -\mu_{eg}\sqrt{\hbar\omega_c/2\varepsilon_0 V_c}$, with μ_{eg} as the atomic transition dipole moment. For the cavity discussed above, the maximum vacuum Rabi frequency is $g_{\max} = 2\pi \times 18.5$ GHz.

The Jaynes-Cummings Hamiltonian only couples states in pairs $|n,e\rangle \leftrightarrow |n+1,g\rangle$, where n is the number of photons in the cavity mode. As we are interested in low-light levels, only the excited states $|0,e\rangle, |1,g\rangle$ will be considered. To calculate the trapping potential, the effect of these levels on the ground state $|0,g\rangle$ has to be calculated. For that reason, the atom-cavity system is pumped by introducing a classical Rabi coupling:

$$H_{\text{pump}} = -\tfrac{1}{2}\hbar\Omega_c a^\dagger - \tfrac{1}{2}\hbar\Omega_a |e\rangle\langle g| + \text{H.c.}, \qquad (2)$$

where the Rabi frequencies Ω_c and Ω_a depend on the details of the pumping scheme. For the waveguide coupled cavity discussed here, the direct coupling of light in the access waveguide to the atom can be neglected, i.e., $\Omega_a = 0$. For the cavity, a power of 2.5 pW impinging through the access waveguide corresponds to a Rabi frequency of $\Omega_c \sim 200$ MHz.

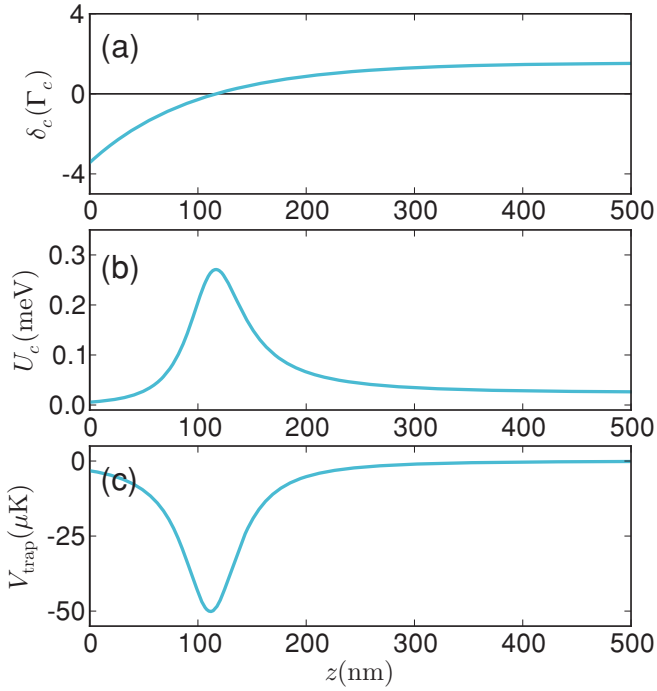

FIG. 2. (Color online) Atom-cavity interaction. In all plots, the unperturbed cavity resonance coincides with the atomic resonance. The cavity-laser detuning is chosen to be $10^3\gamma_a$ (≈$1.6\gamma_c$). The evanescent field extending above the cavity is assumed to have an exponential decay with a decay constant $l_z = 100$ nm. (a) Cavity-laser detuning as a function of atom-cavity separation. As the atom approaches the cavity, the cavity resonance is shifted toward the laser frequency. At an atom-cavity separation of ≈100 nm, the resonance crosses the laser frequency. (b) Optical energy inside the cavity under continuous pumping conditions. Around $z = 100$ nm, the cavity is in resonance, thus the cavity energy peaks. (c) Resulting atomic trapping potential expressed in μK. The atom is pulled toward the point where it experiences the highest light intensity. Due to the atom-cavity interaction, this point is not on the cavity surface, but on the point above the cavity where the cavity is resonant with the laser light.

To account for the linewidth of the atomic transition and the cavity resonance, damping is introduced by adding a small imaginary component to the uncoupled cavity resonance and atomic transition frequencies, i.e., $\omega_c \to \omega_c + i\gamma_c/2$ and $\omega_a \to \omega_a + i\gamma_a/2$, where γ_c and γ_a are the linewidths of the cavity and the atom, respectively.

The Hamiltonian can finally be written as a 3×3 matrix

$$\frac{H}{\hbar} = \begin{pmatrix} 0 & 0 & -\Omega_c/2 \\ 0 & \delta_a + i\gamma_a/2 & g(z) \\ -\Omega_c^*/2 & g(z) & \delta_c + i\gamma_c/2 \end{pmatrix}, \qquad (3)$$

where $\delta_a = \omega_a - \omega_p$ is the detuning of the pump with respect to the atomic resonance and $\delta_c = \omega_c - \omega_p$ is the detuning with respect to the cavity resonance.

The energies and lifetimes of the coupled energy levels are then determined by the poles of the Greens function [24]:

$$G(\delta) = \lim_{\eta\downarrow 0}[\hbar\delta - H + i\eta]^{-1}. \qquad (4)$$

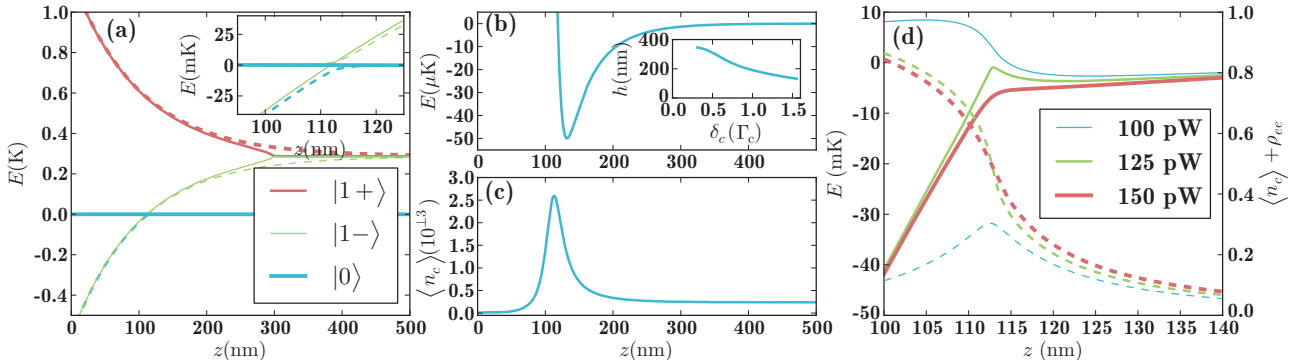

FIG. 3. (Color online) (a) Energies of the lowest three levels in the atom-cavity system. The solid lines take into account damping, the dashed lines are obtained with damping switched off. The red, blue, and lines show the $|1+\rangle$, $|1-\rangle$, and $|0\rangle$ states, respectively. Due to the damping, the vacuum Rabi splitting between the $|1+\rangle$ and $|1-\rangle$ states is suppressed until an atom-cavity separation $z < 300$ nm, because only at this close proximity, the vacuum Rabi frequency $g(z)$ becomes large enough to compete with the damping of the cavity. The inset shows that without damping (inset, dashed lines) an avoided crossing occurs between the $|1-\rangle$ state and the $|0\rangle$ state. This avoided crossing disappears when damping is switched on (solid lines). (b) Ground-state energy of the atom-cavity system as a function of cavity atom separation. This figure is a zoom in of (a). As can be seen the atom experiences a Lennard-Jones–like potential with a depth of around 50 μK and a sharply repulsive core. The inset shows the position of the minimum of the trap as a function of the cavity-laser detuning. (c) Average photon number inside the cavity corresponding to the trapping potential in (b). An occupation of less than 2.5×10^{-3} photons is thus required to trap the atom. (d) Trapping potential (solid lines, left axis) and total excitation (dashed lines, right axis) for different powers.

In practice, these poles are obtained by determining the complex eigenvalues of the Hamiltonian. The real parts of these eigenvalues correspond to the energy levels and the imaginary parts correspond to the linewidth of the levels.

The lowest three eigenstates of the atom-cavity system are denoted by $|0\rangle$, $|1-\rangle$, and $|1+\rangle$. The result of the calculation is shown in Fig. 3(a). The dashed lines show the result in the case without damping. In the inset, an avoided crossing can be seen between the $|0\rangle$ and the $|1-\rangle$ branch. This means that when the atom approaches the cavity, the coupling between the atom and the light will adiabatically take the atom-cavity system from the ground state to the first excited state. In other words, by moving the atom into the evanescent field of the cavity, a photon is brought into the cavity mode. However, as shown in the inset, when damping is switched on (the solid lines), the avoided crossing disappears, as the Rabi frequency then is too small to cause a Rabi oscillation within the lifetime of the $|1-\rangle$ state. Therefore, on the scale of Fig. 3(a), the solid blue line at first glance appears to be flat.

Importantly, the crossing between the $|1-\rangle$ and the $|0\rangle$ states does cause a significant change in the energy of the ground state. This becomes apparent by closer inspection of the energy of the $|0\rangle$ state [Fig. 3(b), note the different vertical scaling]. Due to the crossing, a Lennard-Jones–like potential arises with a potential minimum at an atom-cavity separation of 130 nm, a depth of roughly 50 μK, and a strongly repulsive barrier with a height of roughly 200 μK. As shown in the inset, the location of the trap bottom can be varied by changing the detuning of the pump. This opens up a fascinating possibility. As the vacuum Rabi splitting g depends on the atom-cavity separation, by changing the detuning of the pump one can tune the vacuum Rabi splitting on demand. By analyzing the eigenstates of the atom-cavity system, the average photon number inside the cavity can be determined. The result of this calculation is shown in Fig. 3(c). In accordance with the classical estimate, an occupation of less than three thousandths of a photon is necessary to obtain a trap depth of 50 μK. The power required in the access waveguide to realize this situation is around 3 pW. When the power is increased to 120 pW, the depth of the potential increases to 3 mK. When the power is increased further, the trap becomes unstable, as shown in Fig. 3(d).

There are clear differences between the classical and the quantum results. For instance, a different position for the bottom of the trap is found when comparing the classical and the quantum result ($\approx$112 nm as opposed to $\approx$ 132 nm). A more profound difference is in the potential energy the atom would experience on the cavity surface ($z = 0$). The classically calculated potential energy at the cavity surface [Fig. 2(c)] is slightly lower than the energy in infinity. In the quantum case [Fig. 3(b)] the surface of the cavity is protected by a sharply repulsive core. Experimentally, this repulsive core reduces the probability that the atom hits and is absorbed by the cavity surface. The most striking difference is that in the quantum calculation, the potential minimum at nonzero atom-cavity separation disappears when the classical Rabi frequency becomes larger than the damping, whereas in the classical calculation, increasing the applied light intensity will just scale the depth of the potential. It is natural that these details are different: the classical calculation is the result of perturbation theory whereas the quantum calculation shows that the atom cannot be considered as a small perturbation to the cavity mode.

In conclusion, we have proposed a novel atom trap geometry using a photonic crystal nanocavity. The trap works radically different from a normal optical dipole trap. In a standard dipole trap, the presence of the atom has a negligible influence on the trapping light. In contrast, the trap discussed in this work only exists because of the very influence that the atom exerts on the light, as without the atom, there would be almost no light inside the cavity. Trapping in a nanocavity is

also very different from trapping in a conventional cavity. In conventional cavities, atoms are trapped at the antinodes of the cavity mode. In our proposed trapping mechanism, the position of the trapping minimum can be changed during the experiment. As a result, the vacuum Rabi splitting g can be controlled on demand. It is important to note that although the trap is conservative, damping of the cavity is crucial in this trapping mechanism. This use of a photonic crystal nanocavity is fascinating, because extremely strong coupling regime one can achieve with current state-of-the-art nanocavities is out of reach of conventional cavities. And this unique parameter regime leads to surprising results, such as the novel trapping mechanism presented in this work. This trapping mechanism allows one to trap an atom with a light field inside the cavity that has an energy equivalent to a small fraction of a photon energy. In fact, the calculation shows that the trapping minimum only exists for such a low photon number and that when the photon number is too high, the trap minimum disappears. An interesting open question is what kind of cooling mechanisms can be used in the trap. Because of the tight confinement, it is expected that stimulated Raman sideband cooling can be employed [25]. Heating in the trap is expected to be small due to the low-excitation probability of the atom and because the atom is strongly in the Lamb-Dicke regime [26], which means momentum diffusion [27] is strongly suppressed. Experimentally, multiple coupled cavities can also be readily fabricated on the same device [28]. One can then study the interaction between many coupled atom-cavity systems. We expect that this will result in a new type of slowed light. The proposed trap thus opens up a novel path for experiments in quantum optics and ultracold atoms using photonic crystals and integrated optics, as well as a completely novel way of controlling the propagation properties of light.

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