

A PROJECTION OF FUTURE SEA LEVEL

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Abstract. Evidence is reviewed that suggests faster sea-level rise when climate gets warmer. Four processes appear as dominating on a time scale of decades to centuries: melting of mountain glaciers and small ice caps, changes in the mass balance of the large polar ice sheets (Greenland, Antarctica), possible ice-flow instabilities (in particular on the West Antarctic Ice Sheet), and thermal expansion of ocean water.

For a given temperature scenario, an attempt is made to estimate the different contributions. The calculation yields a figure of 9.5 cm of sea-level rise since 1850 AD, which is within the uncertainty range of estimates of the 'observed' rise.

A further 33 cm rise is found as most likely for the year 2050, but the uncertainty is very large ($\sigma = 32$ cm). The contribution from melting of land ice is of the same order of magnitude as thermal expansion. The mass-balance effects of the major ice sheets tend to cancel to some extent (increasing accumulation on Antarctica, increasing ablation on Greenland). For the year 2100 a value of 66 cm above the present-day stand is found ($\sigma = 57$ cm). The estimates of the standard deviation include uncertainty in the temperature scenario, as presented elsewhere in this volume.

1. Introduction

Changes in global sea level are a potentially very important aspect of the greenhouse problem. Since many heavily populated areas in the world are close to sea level, even changes less than 1 m can have dramatic consequences. In particular since the appearance of Mercer's (1978) paper on possible instability of the West Antarctic Ice Sheet, a great deal of public attention has been focussed on sea-level rise. That the polar ice sheets will melt when the greenhouse warming comes, seems to be obvious to anyone.

However, as soon as more precise questions are asked, we (the few scientists studying the issue) are in trouble. We are able to identify a large number of mechanisms contributing to changes in sea level when climate changes, but a reliable figure of the total effect is hard to get. Our knowledge of the major ice sheets is particularly poor, which is understandable. There have hardly been any other than purely scientific reasons to do geophysical investigations in those remote and poorly accessible regions. Still, the question is asked frequently: *How fast will sea level rise in the near future?*

Before dealing with this, a few other questions can be posed: Why did sea level rise over the last century? Is the present warming trend associated with increasing concentration of trace gases in the atmosphere or just a recovery from the little ice age? What would happen to the major ice sheets when climate would remain perfectly constant? Or to the deep ocean? One would like to find answers to these questions before embarking on a projection of *future* sea level. Unfortunately, we will not have the time and are thus forced to make the best of it. This could (and will) imply that the uncertainty in the projection will be of the same order of magnitude as the 'signal' itself.

Even the estimates of recent sea-level rise differ considerably (Barnett, 1983, 1984; Gornitz *et al.*, 1982; 1987). Much depends on how local effects are taken into account. Most workers put the rate of sea-level change over the last century between +9 and +15 cm.

Although the choice of a realistic temperature scenario is a crucial step in predicting changes in sea level, this point will be postponed to the end of this section. We will first identify and discuss the processes that are able to give a major contribution to world-wide sea level change.

It is obvious that changing circulation patterns in ocean and atmosphere will cause changes in sea level. First of all, the topography of the sea surface is in a dynamic balance with the currents: an intensified North Atlantic gyre, for instance, would lead to a larger difference in sea level between the central Atlantic and the coasts. Also, changes in mean atmospheric pressure may make the sea rise or sink. In shallow coastal waters, a change in mean wind stress will lead to a change in the 'mean' component of the wind set-up. Still, these processes are of a local character and nearly balance when averaged over the entire globe. The order-of-magnitude of such local changes in sea level is not impressive: maybe 10 cm at most for changes in atmospheric circulation that could occur as a consequence of the enhanced greenhouse forcing.

In fact, since we are interested in changes on a time scale of decades to a few centuries, it appears that only a few processes are capable of producing substantial changes in sea level when climate really gets warmer. These are thermal expansion of ocean water and changes in land ice volume. It is useful to split the latter in contributions from mountain glaciers, from the large polar ice sheets (Greenland, Antarctica), and from possible ice-flow instabilities [most importantly, but not exclusively, the West Antarctic Ice Sheet (WAIS)], see Figure 1. In general, the change in relative sea level is determined by the change in ocean volume and the response of the solid earth. Any change in loading will lead to an elastic as well as a viscous response, which will vary from place to place. It is now well accepted that a proper interpretation of proxy sea-level data is not possible without the use of sophisticated earth models (e.g. Clark, 1980; Peltier, 1988). However, for the present purpose a treatment of the geodynamical response is not necessary.

Some general characteristics of land ice are given in Table I. The Antarctic Ice Sheet is by far the largest ice mass: its volume is about ten times that of the Green-

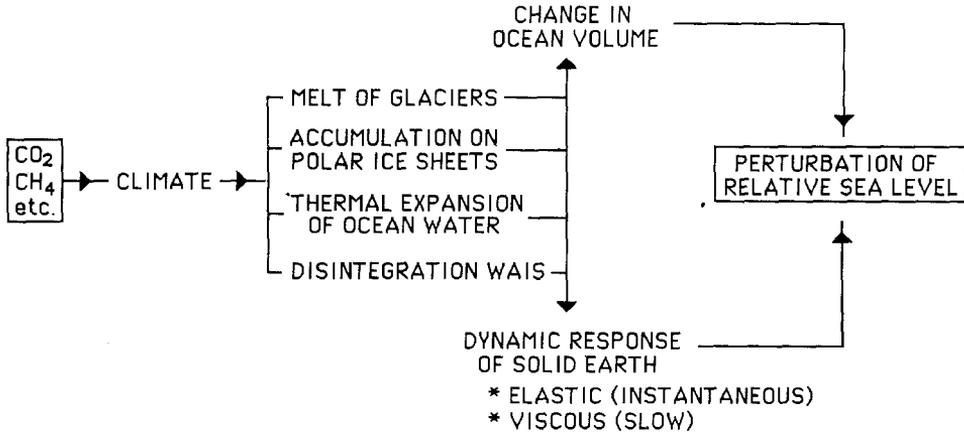


Fig. 1. The most important processes that determine a change of relative sea level due to the greenhouse warming [WAIS = West Antarctic Ice Sheet].

land Ice Sheet. Small ice caps and glaciers seem to represent a negligible fraction of total ice mass. Still, this does not mean that they are unimportant with regard to short-term changes in sea level. The volume of the Antarctic Ice Sheet corresponds to about 80 meters of equivalent sea-level. However, if it would melt, actual sea-level rise would be somewhat less because part of the grounded ice is below sea level. Also, isostatic rebound would modify the picture.

A noticeable difference between the Antarctic and Greenland ice sheets is the character of the mass budget. Ice loss on Greenland is by calving and melting/runoff (roughly equally important), whereas in Antarctica calving dominates. The

TABLE I: Some data on the land ice masses of the earth. Data from Wilhelm (1975), Flint (1971), Ambach (1980), Radok (1982), Drewry (1983), Orheim (1985). Uncertainty generally within 10%, but * indicates an uncertainty of the order of 15%, ** 30%!

	Antarctica	Greenland	Glaciers & small ice caps
Area (10 ⁶ km ²)	11.97	1.80	0.60*
Volume (10 ⁶ km ³)	29.33	3.0	0.18**
Mean thickness (m)	2488	1667	300**
Largest thickness (m)	4700	3400	—
Mean elevation (m)	2000	2080	—
Eq. sea level (m)	82.5	8.3	0.50**
Act. sea level (m)	65*	7.5*	0.50**
Accumulation (km ³ yr ⁻¹)	2200*	500*	—
Ablation (km ³ yr ⁻¹)	1*	290*	—
Calving	2200**	210**	—
Equilibrium-line altitude (m)	-200*	1300	—
Turnover time (yr)	13,331	6000	50-1000

difference in climatic environment is also reflected by the position of the equilibrium line (= zero annual mass-balance at the surface). In Greenland its mean altitude is about 1300 m, in Antarctica below sea level. An implication is that in the present climate the Greenland Ice Sheet would not form: it is due to the altitude-mass balance feedback that it can survive the warm present climate. The situation is illustrated in Figure 2. For an extensive discussion of the nonlinear response of ice sheets to climatic change, see Oerlemans and Van der Veen (1984).

In the last line of Table I turnover times are given. These are defined as volume divided by accumulation, and should not directly be identified with *response* times. The response time of an ice sheet depends on the type of forcing. A temperature perturbation imposed at the surface affects ice flow very slowly (order-of-magnitude 100,000 years for a large ice sheet, e.g. Huybrechts and Oerlemans, 1989), while changes in the mass balance may lead to much quicker reactions (a few thousands of years).

No reference has yet been made to the groundwater reservoir. It probably contains an amount of fresh water that is comparable to the volume of the polar ice caps (SMIC-report, 1971), and it is unclear how this reservoir is affected by climatic change. Most workers seem to expect very marginal changes in case of a warming of a few degrees K. However, this is not based on any calculations with physics-based models. See also Robin (1986) on this point.

A particular problem in constructing a projection of future sea level is the treatment of the *basic trend*. First of all, there may be a local trend associated with crustal movements of isostatic or tectonic origin, or with compaction of sediments. These processes are slow, and, for the present purpose, can be assumed to continue at a constant pace. Secondly, the sea-level rise of 9 to 15 cm during the last century has not been fully explained. One possibility to take this into account in a projection of future sea level would be to assume that the trend continues, and to add

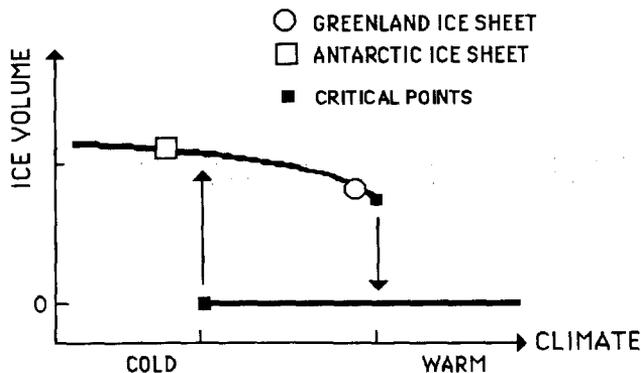


Fig. 2. A general representation of the steady states of an ice sheet on a bounded continent, in dependence of climatic conditions. The Greenland and Antarctic ice sheets can be placed in this diagram. When the Greenland ice sheet would melt, much colder conditions are required to build it up again. The nonlinear behaviour is due to the altitude - mass balance feedback.

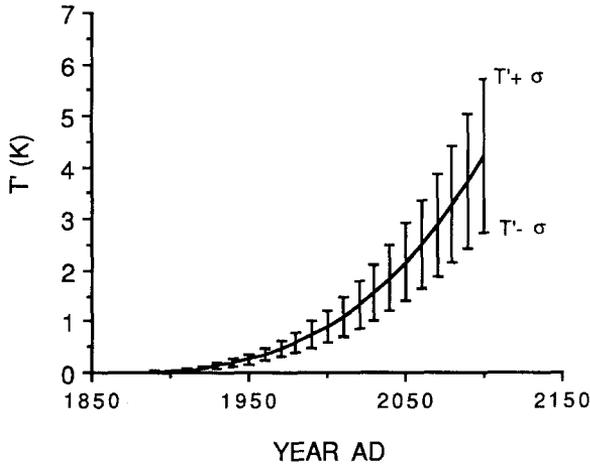


Fig. 3. Temperature scenario used in the present estimate of sea-level changes. Vertical bars indicate the standard deviation, so the probability that the real evolution will be within the bars is roughly 2/3.

estimates of ice melt and thermal expansion relative to the present. Another approach could be to consider the 1850 AD state as unperturbed by the greenhouse warming, try to estimate (calculate) the 1850–1985 AD response in sea level, and take the *unexplained part* of the observed sea-level rise as basic trend for the future.

It is not the intention of this paper to discuss again the scenarios for the greenhouse warming. I simply take a fit to the outcome of the Villach II discussion (see Jaeger, 1989), namely

$$T' = \eta(t - 1850)^3, \quad (1)$$

where time t is in year AD, and $\eta = 27 \times 10^{-8} \text{ K yr}^{-3}$. T' is the temperature perturbation in degree K due to the greenhouse warming and relative to 1850 AD. The standard deviation is set to 35% of the mean. This scenario is illustrated in Figure 3, and will serve as input for the estimate of sea-level changes discussed further on.

2. Mountain Glaciers and Small Ice Caps

As mentioned above, mountain glaciers and small ice caps contain a small part of the total ice mass on earth. However, they are located in warmer climates than the polar ice sheets, with larger accumulation (above the equilibrium line) and larger ablation rates (below the equilibrium line). This implies that those ice masses are more active and react more quickly to climatic change. Records of historic glacier variations provide direct evidence for this.

In Figure 4, a few long records are plotted together. Shown are variations in glacier length. Although ice volume is not necessarily proportional to glacier length, on longer time scales the correlation is strong. For the measured glaciers in

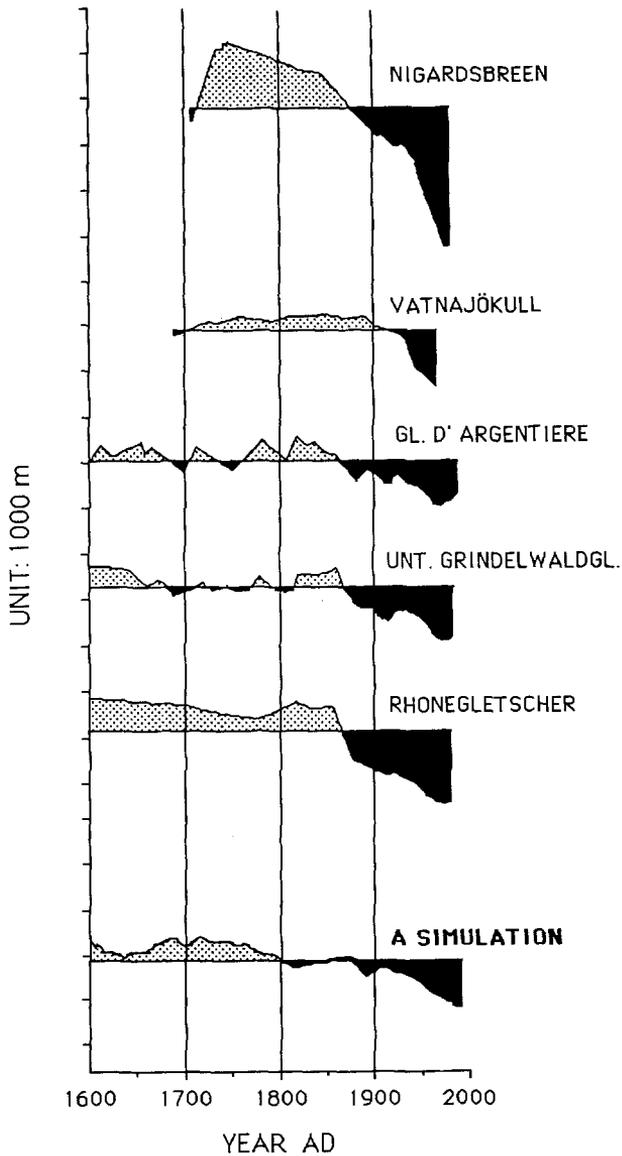


Fig. 4. Historic variations in the length of some glaciers. Nigardsbreen is in Norway, Vatnajökull in Iceland, Glacier d'Argentière in France, Rhonegletscher and Untere Grindelwaldgletscher in Switzerland. The lower curve shows a simulation with a simple climate-glacier model, driven by volcanic activity and greenhouse forcing. See Oerlemans (1988) for further details and data sources.

the Alps, for instance, the correlation coefficient between volume and length is 0.86 (based on a sample of 298 glaciers taken from the World Glacier Inventory, Haeberli, 1985). The most sensitive glaciers are those with a wide accumulation area and a narrow tongue, like Nigardsbreen (Norway). When the glacier variations

are scaled to account for geometric effects, the amplitudes become comparable (see Oerlemans, 1988, for a further discussion and data sources).

The general picture emerging from Figure 4 is that in the period 1600–1850 AD variations were within a limited range, while after 1850 AD a steady retreat began. Long records are only available from European glaciers, but from geomorphological evidence it is now well established that this retreat occurred in all parts of the world (New Zealand, Himalaya, in the tropics, northern Canada, western Greenland). The most comprehensive study of glacier retreat during the last 100 years has been made by Meier (1984). He arrived at the conclusion that this retreat accounts for about 2 to 4 cm of the observed sea-level rise.

Attempts to simulate historic glacier variations for the last four centuries by using *local* proxy data (tree-ring width, eventually combined with series of precipitation and temperature) have not been very successful (Nigardsbreen: Oerlemans, 1986a; Glacier d'Argentiere: Huybrechts *et al.*, 1988; Rhonegletscher: Stroeven *et al.*, 1988). In particular, the retreat during the last 100 years did not show up. Interestingly, experiments with a *global* forcing function derived from a very simple energy-balance climate model, driven by a volcanic activity index and greenhouse forcing, yield much better results. Such a simulation is shown at the bottom of Figure 4 (from Oerlemans, 1988). This suggests that variations in the world-wide radiation balance are a very important factor with regard to glacier variations. (On physical grounds, this is not unexpected. A melting glacier surface cannot raise its temperature and thus not increase outgoing longwave radiation; any surplus of radiation is thus used for additional melting).

Based on these considerations, a substantial further retreat of glaciers and small ice caps must be anticipated for even a moderate warming. This is difficult to quantify, however. There are many glaciers of which geometry and mass balance are poorly known. The result of an experiment with the glacier model referred to above (simulation of lower curve in Figure 4), with greenhouse forcing only, is shown in the upper panel of Figure 5. Normalized volume (1850 AD = 10) shows a dramatic decrease in the coming century. It is difficult, however, to project such a result on the total mass of glaciers and small ice caps. First of all, the character of the mass balance and its sensitivity to temperature changes varies enormously from place to place. Secondly, glaciers span a wide altitude range which should somehow be taken into account.

To arrive at a schematic representation of the bulk effect, the following assumptions are made:

- (i) The equilibrium ice volume drops off exponentially with surface temperature;
- (ii) The rate of melting is proportional to remaining ice volume relative to the equilibrium volume (because this is assumed to be roughly proportional to glacier area) and temperature perturbation.
- (iii) Response time of the glaciers can be characterized by a single number.

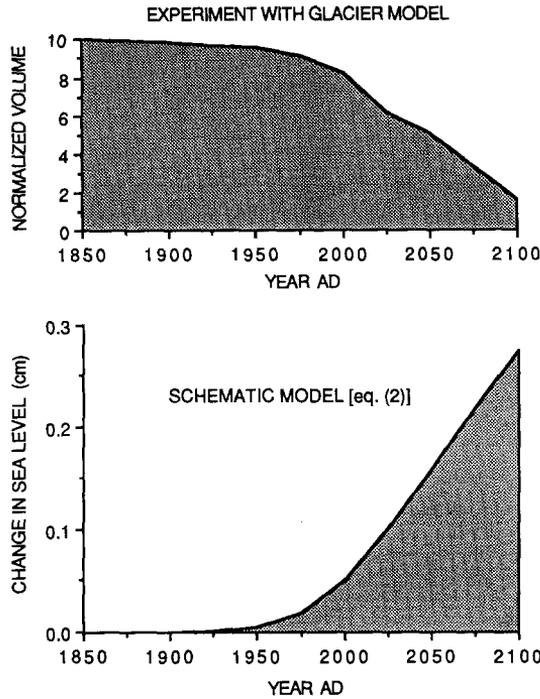


Fig. 5. Illustration of the estimate of the glacier contribution to sea-level change. The upper panel shows an experiment with a glacier model, with imposed greenhouse forcing (the model described in Oerlemans, 1988). The lower panel is based on Equation (2).

This naturally leads to the following equation describing the volume of small ice caps and glaciers:

$$\frac{dV}{dt} = \alpha(T - T_R)[V - V_R \exp\{-(T - T_R)/\beta\}]. \quad (2)$$

Here V is ice volume, and the initial (here 1850 AD) state is defined by initial ice volume V_R and global mean surface temperature T_R . T is the actual (i.e. time-dependent) temperature and α a constant of proportionality involving a characteristic response time. The constant β determines the temperature increase for which the equilibrium ice volume becomes $1/e$ of the undisturbed value. Values used here are $V_R = 0.45$ m sea-level equivalent, $\alpha = 0.05$ (yr K) $^{-1}$ and $\beta = 4.5$ K. The lower panel of Figure 5 shows the resulting contribution to sea level for the temperature scenario employed here. The curve is not a direct reflection of normalized volume as produced by the experiment with the glacier model for reasons explained above (assumption (i)).

Again, the use of Equation (2) clearly is a substantial simplification of reality. There are many types of glaciers, and response times differ widely. Even the greenhouse forcing will be dependent on the background climate in a significant way.

Unfortunately, at present we are not able to incorporate such factors in the analysis. It would require knowledge of how response time is distributed over ice volume, and how ice volume is distributed over continental versus maritime glaciers. Such information is not available at the moment. It would thus not make sense to use a more sophisticated treatment than that described by Equation (2). In spite of its simplicity, the present approach incorporates some features absent in earlier estimates (Meier, 1984; Robin, 1986), namely: melting rates are proportional to the temperature perturbation *and* remaining glacier volume; the equilibrium glacier volume drops off gradually (allowing for a wide range over glacier elevations); a time scale is included.

3. The Antarctic and Greenland Ice Sheets — State of Balance

As noted in the Introduction, it is not at all clear whether the ice sheets of Greenland and Antarctica are in close equilibrium with the present climate. The coupling of ice flow and temperature field (the hardness parameter depends strongly on ice temperature) and the slow response of the earth's crust to changing ice thickness make the approach to equilibrium slow. In view of this, it is very likely that adjustment to the 15,000 BP glacial-interglacial transition has not yet been accomplished. Recent modelling studies confirm this (Greenland: Dahl-Jensen and Johnson, 1986; Antarctica: Huybrechts and Oerlemans, 1989).

Analysis of the Vostok deep ice core has produced strong evidence that the East Antarctic Ice Sheet (see Figure 6) has undergone only minor changes during the last glacial cycle (Lorius *et al.*, 1984). A numerical calculation for the Vostok flow line, including the full thermo-mechanical coupling and bedrock adjustment, has shown that in an unaltered climate ice thickness at Vostok will change in the future.

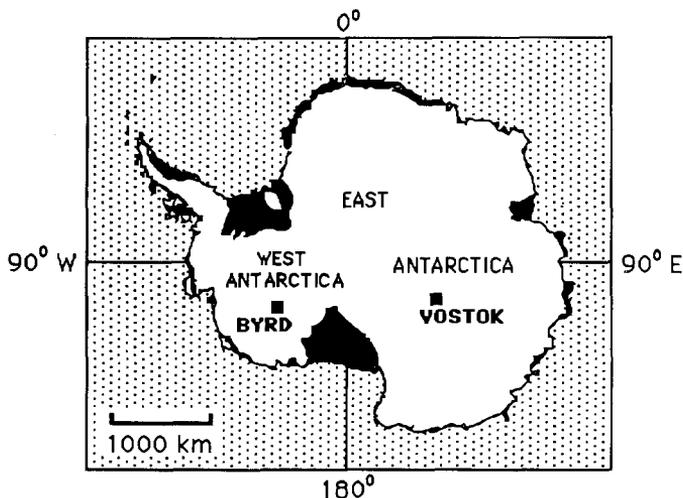


Fig. 6. A map of Antarctica. The locations where the deep ice cores discussed in the text were retrieved are shown (Byrd and Vostok). Black areas represent the major ice shelves.

The effects are small, however: a 12 m increase in ice thickness during the coming 10,000 years, and a 39 m decrease afterwards (this takes 100,000 years!). Assuming that the Vostok flow line is representative for the entire Antarctic Ice Sheet, it is easily seen that the associated changes in sea level are very small (order-of-magnitude: -0.1 mm yr^{-1} as compared to the present rate of $+1$ to $+1.5 \text{ mm yr}^{-1}$).

For the West Antarctic Ice Sheet the situation is more complex. Many workers have the opinion that during the last glacial maximum large parts of the Ross and Weddell seas, now covered by floating ice shelves, supported grounded ice sheets. The ice thickness was probably not very large, and grounding-line retreat was initiated by sea-level rise (due to melting of the Northern Hemisphere ice sheets), e.g. Thomas and Bentley (1978), Denton *et al.* (1986), Robin (1986). Recently, Peltier (1988) has provided an independent check of the deglaciation history of West Antarctica by making a sophisticated interpretation of proxy sea-level data available from almost 400 sites. He carried out experiments with a numerical earth model, varying the imposed deglaciation history of West Antarctica. His major conclusion was that deglaciation started later than assumed so far (i.e. around 13,000 BP). Still, this does not make clear whether grounding line retreat is continuing at present. It has also been suggested that isostatic rebound in the Ross Sea area causes grounding line *advance*. (Thomas, 1976; Greischar and Bentley, 1980.)

From a study of radio-echo layers Whillans (1976) inferred that in the central part of West Antarctica there has been little change over the last 30,000 years. Analysis of the Byrd ice core (see Figure 6 for its location) has confirmed this (Raynaud and Whillans, 1982). So the suggestion of a thin but grounded glacial ice sheet in the Ross and Weddell seas appears as reasonable, and the present-day situation could be close to equilibrium. Still, the dynamics of marine ice sheets are extremely complicated, and because of all the feedback loops involved one cannot rule out the possibility of transient behaviour even without changes in the climatic environment.

The direct way to see how far the Antarctic Ice Sheet is out of balance is to try to evaluate all contributions to the mass balance. Measurements of the surface mass balance and ice velocities can be used to see whether a balance exists between accumulation and ice-mass discharge. In this way, Budd and Smith (1985) have tried to make an assessment of the state of balance. Unfortunately, the field data are so scarce that it can only be stated that the deviation from a balance is not more than 20%.

In summary, there is no evidence that at present the Antarctic Ice Sheet is grossly out of balance. Still a 20% uncertainty must be accepted due to poor data coverage. This corresponds to a rate of sea-level change of 1.2 mm yr^{-1} .

Concerning the Greenland Ice Sheet, the same lack of data is apparent. For an overview, see Radok *et al.* (1982). A few attempts have been made to assess the state of balance of the Greenland ice sheet. In NAP (1985), pp. 25–28 and 155–171 (contribution by N. Reeh), a slight thickening of the ice cap in the interior parts and a slight thinning of the ice margin (ablation zone) is put forward as the general

picture. This is mainly based on measurements along the EGIG line (central Greenland) and historic observations on outlet glaciers (e.g. Weidick, 1967). Kostecka and Whillans (1988) have re-analyzed data from the EGIG-line. By including some ice-velocity measurements, they were able to compare surface accumulation with ice-mass discharge. Both the data from the EGIG-line and from the OSU-line (Oregon State University-line, in south Greenland from Dye 3) show that at present at least the inner part of the ice cap is close to equilibrium.

A detailed modelling study of the temperature profile from a deep bore hole in central Greenland has been carried out by Dahl-Jensen and Johnson (1986). They found that the deep ice in central Greenland is 5 K below the equilibrium value to be expected for present-day boundary conditions. So the glacial cold is still present in the lower layers. The warming up of those layers will lead to a few % of thinning of the ice sheet. However, the time scale involved is large, and the process is insignificant in the present context. This also applies to the removal of so-called soft Wisconsin ice, which, for some reason, is more easily deformable. It is found only in the lower few tens of meters, and a slight thickening of the ice sheet has to be expected when it is replaced by stiffer ice.

A point of concern is the retreat of the outlet glaciers. Most outlet glaciers for which observations exist (this is mainly in central and southern part of the west coast of Greenland) have retreated strongly over the last century (Weidick, 1967). Increased ablation rates must be responsible for this. However, the large ablation zones of the inland ice must have suffered from this too. Although a quantification is hard to make, one should keep in mind that the amount of mass involved can be quite large, and could have contributed to sea-level rise in a significant way!

As for the Antarctic Ice Sheet, there is no direct evidence that the Greenland Ice Sheet is far from its equilibrium state. If it is, the most likely pattern seems to be a thinning of the margins, with little change in the interior parts.

4. The Antarctic and Greenland Ice Sheets — Changes in the Surface Mass Balance

As shown in Table I, there are large differences in the mass budget of the Greenland and Antarctic ice sheets. On the latter, the amount of melting is negligibly small – all loss of ice is by calving. For the Greenland ice sheet, mass loss by melting/runoff and calving are about equally large. The fact that the Antarctic Ice Sheet is in a much colder climate has important consequences concerning the response to a climatic warming. With regard to snow accumulation, air temperature is the limiting factor (in the interior part of the ice sheet, annual accumulation is less than 0.05 m water equivalent per year). When temperature goes up, so does the saturation vapour pressure, and an increase in accumulation has to be expected. In contrast, accumulation on Greenland will decrease in case of a warming, because melting rates will go up sharply.

The point is further illustrated by the generalized mass balance field shown in

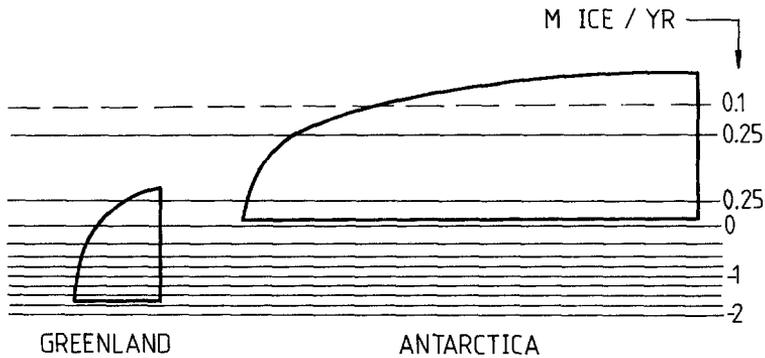


Fig. 7. Generalized mass balance field in which the ice sheets of Antarctica and Greenland are projected. The mass balance generally increases with altitude, reaches a maximum value, and then drops because of low water-vapour content of the atmosphere (low temperatures). A climatic warming, implying an upward shift of the mass-balance field, will increase the total mass balance of Antarctica and decrease that of Greenland.

Figure 7. In general, the mass balance increases with elevation until a maximum is reached. Then there is a steady decrease, more or less in proportion to the saturation water vapour pressure. The ice sheets of Greenland and Antarctica are projected in the figure. Realizing that, to first order, a warmer climate can be represented by an upward shift of the mass balance field, it is seen that the total surface balance of Antarctica will increase, while that of Greenland will decrease.

Several attempts have been made to estimate possible changes for a moderate warming. Based on measurements along the EGIG-line (obtained in the summers of 1959 and 1967), Ambach (1980) derived a relation between altitude of the equilibrium line (zero mass balance) and summer temperature. Assuming that this relation is valid for the entire ice sheet, he estimated a change in mass balance per degree K warming corresponding to a sea-level rise of 0.4 mm yr^{-1} . A recent re-analysis of these data yielded a value of 0.34 mm yr^{-1} (Ambach and Kuhn, 1989).

Bindschadler (1985) took a somewhat different approach. He tried to take into account possible retreat of the ice margin by using a simple so-called perfectly plastic ice-sheet model. The contribution to sea-level rise was calculated for two scenarios: 1.3 mm yr^{-1} for a 3.25 K warming and 3.5 mm yr^{-1} for a 6.5 K warming. This corresponds to a 0.4 and 0.54 mm yr^{-1} per degree K, respectively. Bindschadler also provides an estimate of how the calving rate of the outlet glaciers could be affected by increasing water discharge through the glacier system. The net effect would be an additional 30% increase in sea-level rise, at least for short time scales (decades) and a moderate temperature increase (a few K).

In summary, the Greenland contribution to sea-level change can be estimated as $+0.5 \text{ mm yr}^{-1}$ per degree K warming. Unfortunately, a number of factors lead to a large uncertainty: estimates refer to changes in summer temperature only; it is not clear how representative data from a few spots are for the entire ice sheet; possible

changes in accumulation rate are not considered; possible changes in calving rates are difficult to estimate. *A 50% uncertainty has to be accepted.*

In Oerlemans (1982) a study for the Antarctic Ice Sheet was carried out. Here a numerical ice-flow model, taking into account bed geometry, was used to simulate the evolution of the ice sheet for various scenarios of climatic change. In the mass-balance parameterization, factors like surface slope, elevation, temperature and continentality were taken into account. Melting is parameterized by using measurements from the Greenland Ice Sheet (Ambach, 1979), and assuming no change in the annual cycle under climatic change. When starting from present-day conditions, the model drifts away and ultimately produces an ice sheet with larger volume. There are several reasons for this, which will not be explained here. In any case, useful results can only be obtained by comparing perturbation experiments with a control run.

In Figure 8, some results are presented from such experiments. The change in antarctic ice volume is shown as a function of time. In all runs the forcing was a linearly increasing function of time for the first 100 years, and constant afterwards. The labels on the curves give the perturbations in temperature and accumulation

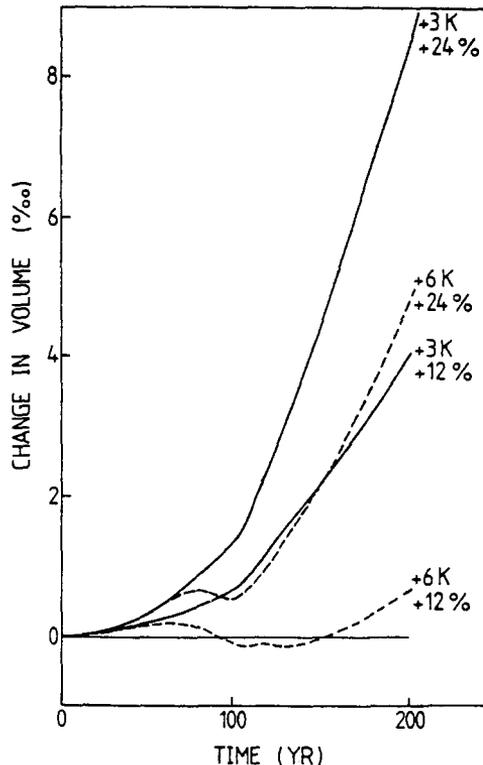


Fig. 8. Change in Antarctic ice volume for greenhouse warming experiments. Labels refer to temperature perturbation and increase in accumulation (linear in time for the first 100 years, constant thereafter). From Oerlemans (1982).

after 100 years. The (+3 K, +12%)-run corresponds to the scenario suggested by the GCM experiment of Manabe and Stouffer (1980), in which the atmospheric CO₂ concentration was doubled.

The irregular behaviour of ice volume for the runs with a +6 K temperature perturbation is an interesting result. Ice volume first increases (increase in accumulation exceeds that in melting), then decreases slightly (increase in melting starts to dominate) and finally goes up again (the marginal zone of the ice sheet is thickening and the mass balance increases). The thickening is due to an increase in ice flux, forced by the larger accumulation inland and melting in the margin, i.e. by an enlarged surface slope. However, for the more realistic scenarios of a 3 K warming in 100 years, the response is fairly regular. Then the results can reasonably well be translated into sea level rise per degree warming. In terms of sea-level equivalent, the (+3 K, +12%)-experiment yields a basic sensitivity of about -0.5 mm yr^{-1} per degree warming.

It is interesting to compare this figure with an estimate directly based on the assumption that the mass balance is proportional to the saturation vapour pressure, as mentioned earlier. Averaged over the entire ice sheet, the mass balance would increase by about 7% per degree warming (Fortuin, personal communication). This corresponds to a -0.43 mm yr^{-1} change in sea level, which is close to the figure given above.

5. Possible Disintegration of the West Antarctic Ice Sheet

Several authors have argued that parts of ice sheets grounded far below sea level may be extremely sensitive to small changes in sea-level or melting rates of the buttressing ice shelves (Mercer, 1978; Thomas *et al.*, 1979; Hughes, 1981; Lingle, 1985; Van der Veen, 1986). This would apply in particular to the West Antarctic Ice Sheet. A schematic overall view of the mechanisms at work is shown in Figure 9. Where grounded ice starts to float an ice shelf forms (several hundreds of meters thick) which runs aground again in several places to form ice rises. These act to produce a 'back stress' on the main grounded body of ice. It is conceivable that disappearance of ice rises reduces the back stress and leads to enhanced ice velocities, with subsequent thinning of the grounded ice and grounding-line retreat.

In qualitative terms the picture is clear, but it is very difficult to make quantitative statements. A number of workers have made attempts to model the ice sheet-shelf system and to study its sensitivity (Thomas *et al.*, 1979; Lingle, 1985; Van der Veen, 1987; Budd *et al.*, 1987). A major problem in modelling of the ice sheet-ice shelf system is to deal in a proper way with the stress field. The simple flow model discussed in Section 1 cannot be applied. In fact, in the vicinity of the grounding line all components of the stress field can become of comparable magnitude, making it impossible to use rigorous simplifications (see Herterich (1987) for an interesting discussion). Even a flow-line model (i.e. a one-dimensional model) is already quite complicated. Here we mention an extensive study of Van der Veen

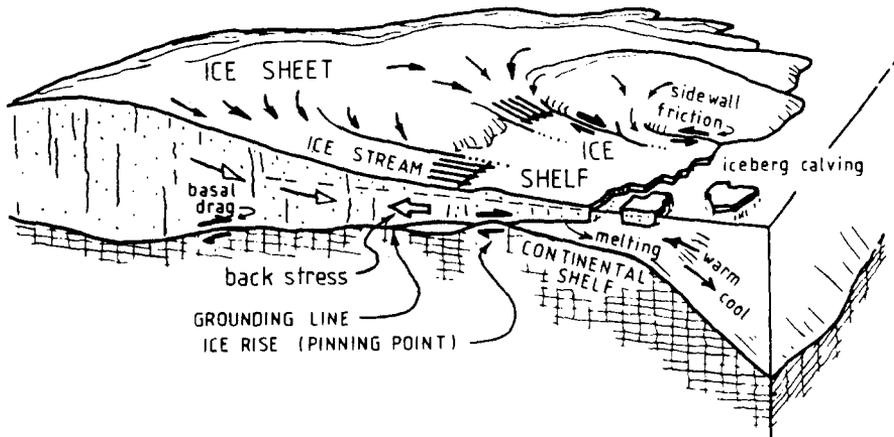


Fig. 9. Processes affecting the stability of the West Antarctic Ice Sheet. From NAP (1985).

(1986). He arrives at the conclusion that earlier estimates of the sensitivity of West Antarctica were too large. His model experiments suggest that a number of feedback processes, omitted in earlier work, tend to stabilize the position of the grounding line. Budd *et al.* (1987) give a fairly detailed discussion on the response of the West Antarctic Ice Sheet to a climatic warming. Their considerations are based on a large number of numerical experiments with flow-band models.

Starting from an initial profile, Budd *et al.* forced the model by prescribing thinning rates for the ice shelves. The larger the thinning rate, the quicker the retreat. Although the model probably gives a reasonable description of the physics of the ice sheet-ice shelf system, the problem is shifted: it is crucial now how the thinning rates will change when climate gets warmer. It appears that very large thinning rates (10 to 100 times present values) would be required to cause rapid disintegration of the West Antarctic Ice Sheet. However, for a probably more realistic situation of a 50% increase in thinning rate for a one-degree warming (order-of-magnitude estimate), the associated sea-level rise would be about 0.1 mm yr^{-1} for the coming decades. This is a small number, but it should be realized that uncertainties are very large. In particular, it has recently become clear that we have a long way to go before we can properly model the marine ice-sheet dynamics. The fast flowing ice streams, which determine the discharge, seem to be connected to till layers which are easily deformable (Bentley, 1987). Attempts are now being made to construct models dealing with this aspect (e.g. Alley *et al.*, 1987). In any case, recent results suggest that earlier flowline/flowband models of West Antarctica may need substantial revision.

5. Thermal Expansion of the Ocean

Thermal expansion of ocean water has been recognized as a major factor contributing to sea level rise. The expansion coefficient of sea water is determined by its

temperature and salinity, but variations of the latter are less important in the present context. The temperature dependence is strong, however. A 100 m thick layer of sea water at a temperature of 25 C will expand approximately 3 cm per degree warming; a layer at a temperature of 0 C only about 0.5 cm.

Recently, a careful analysis of past thermal expansion effects and possible further contributions to future sea-level changes has been presented by Wigley and Raper (1987), where a review of earlier literature on the subject can also be found. They used an upwelling-diffusion model to calculate the penetration of the temperature signal into the ocean. Their results will be referred to later.

A critical factor in the calculation of thermal expansion is the speed at which a temperature signal penetrates downward in the ocean, and how this varies from place to place. A properly designed coupled ocean-atmosphere GCM appears as the best tool to investigate this, but with such a model only a few long runs can be made (Schlesinger, 1986). A more flexible approach, which can be tuned to GCM results and run for any desired greenhouse scenario, is to use a simple upwelling-diffusion (e.g. Wigley and Raper, 1987) or purely diffusion model (e.g. Hoffman et al., 1983). Wigley and Raper showed that the results between those models start to differ after one or two centuries.

In the present attempt to calculate all contributions to sea-level change for any desired temperature scenario, a simple diffusion model was used again:

$$\frac{\partial \theta'}{\partial t} = \frac{\partial}{\partial z} \left(K \frac{\partial \theta'}{\partial z} \right). \quad (3)$$

Here θ' is the temperature perturbation, t time, z depth, and K the eddy diffusivity. It is assumed that K varies with depth according to $K = K_0 \exp(-z/L)$. K_0 and L can be chosen such that the penetration speed of a temperature signal broadly matches that in an ocean GCM experiment (Gates *et al.*, 1985).

Here a single column is used to represent the entire ocean. Part of the justification lies in the fact that the smaller expansion coefficient at higher latitudes is compensated by a faster penetration of the temperature signal (larger K). Also the greenhouse signal increases with latitude in most model calculations. An actual comparison of a 'cold' and a 'warm' column was done by the author (Oerlemans, 1986b) and the results in terms of total expansion were very similar (difference < 8% at all times). Values of the mixing parameters used here are: $K_0 = 0.00025 \text{ m}^2 \text{ s}^{-1}$ and $L = 500 \text{ m}$. The reference temperature profile was taken as $\theta_0 = 25 \exp(-z/900)$, where θ_0 is in °C and z in meters. In the calculation discussed later, the diffusion equation is numerically solved for an ocean of 50 layers, each 75 m thick.

6. Synthesis

In Figure 10 the various contributions to sea level, for the scenario given by Equation (1), are plotted as a function of time. These have to be considered as most likely contributions, and the uncertainties involved will be treated shortly. In the calcu-

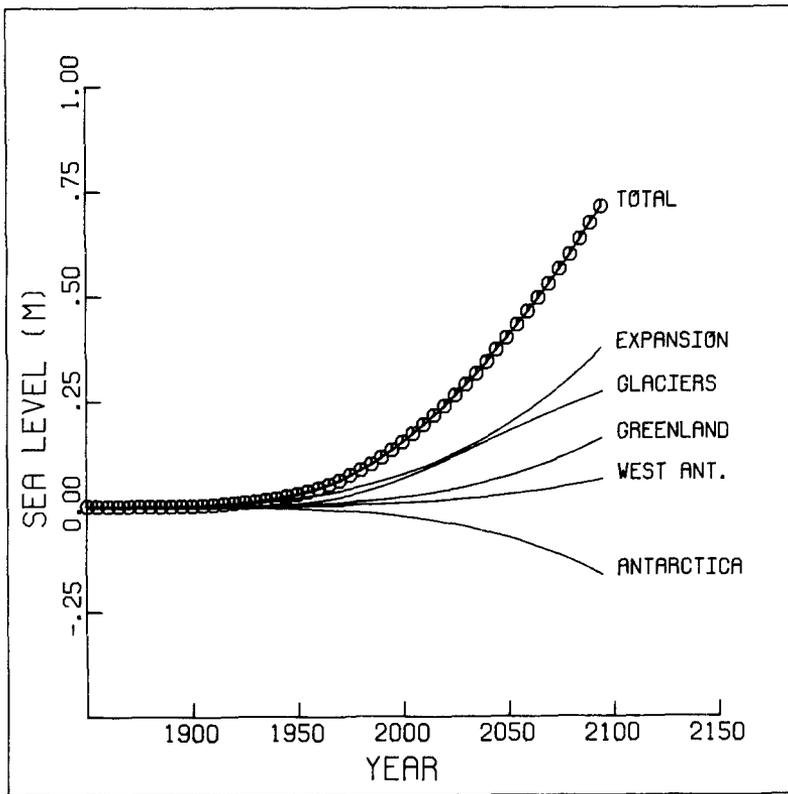


Fig. 10. Estimated contributions to changing sea level for the temperature scenario of Figure 3.

lation, the *dynamic* response of the polar ice sheets has been ignored – only changes in surface mass balance contribute, as discussed in Section 4.

First of all it appears that about 9.5 cm of sea-level rise over the last 150 years are 'explained' by the present model, the major contributors being expansion (5 cm) and glaciers (3.5 cm). This amount is not much less than the observed sea-level rise, which is generally assumed to have been 9–15 cm over the last hundred years. Wigley and Raper (1987) give a range of 2–5 cm for the thermal expansion effect over the last hundred years; the value found here thus is at their upper limit. The reason is a slightly larger diffusion coefficient in the *upper* layers of the ocean, making the initial penetration of the warming somewhat quicker. Meier (1984) estimates the contribution of glacier retreat to sea-level rise to be in the 1.5 to 4 cm range over the last hundred years. The value found here (3.5 cm since 1850 AD) is in agreement with this.

Figure 10 shows that in the next hundred years the contributions from glaciers and thermal expansion are quite comparable. After that glacier wastage diminishes. The contributions from Antarctica and Greenland cancel, not implying that they are unimportant. Errors on these estimates are large and independent, and could

thus work in the same direction. Values of total predicted sea-level rise *relative to 1985* are: for 2000: 6.2 cm; for 2025: 20.5 cm; for 2050: 33.0 cm; for 2100: 65.6 cm.

An important aspect is the uncertainty of those estimates. Ideally, one would like to run sophisticated models in a Monte-Carlo type of experiment, in which model parameters as well as input data are varied. However, such an approach is not yet feasible (but it should be undertaken in the future). Instead, I assume that the uncertainty in the various contributions can be characterized by a normal probability distribution with time-dependent parameters (mean value $M(t)$ and variance $\sigma^2(t)$), and that the distributions are independent. The sum of all contributions then also has a normal distribution (e.g. Parzen, 1962), with parameters:

$$M(t) = M_{\text{glac}} + M_{\text{ant}} + M_{\text{green}} + M_{\text{wais}} + M_{\text{expa}} + TR \quad (4)$$

$$\sigma^2(t) = \sigma_{\text{glac}}^2 + \sigma_{\text{ant}}^2 + \sigma_{\text{green}}^2 + \sigma_{\text{wais}}^2 + \sigma_{\text{expa}}^2 + IV. \quad (5)$$

Here the various subscripts refer to the contributions from, respectively glaciers, the Antarctic Ice Sheet, the Greenland Ice Sheet, instability of the West Antarctic Ice Sheet, and thermal expansion. TR represents a trend, which may consist of the sum of an unexplained part of sea-level rise over the last century plus local effects like isostatic/tectonic movements, compaction, etc. In Equation (5), IV is a standard deviation associated with internal variability in the climate system (see Hasselmann, 1976). As discussed in Oerlemans (1981), internal variability may be generated by short-term variability in precipitation rates over Antarctica, which is 'integrated' by the ice sheet to produce long-term changes in its volume. These can be of the order of a few cm per century.

A further step is to take into account the uncertainty in the temperature scenario. Using a normal distribution again, it follows that the new variance becomes

$$\underline{\sigma}^2(t) = \sigma_i^2 + \{ \sigma_{\text{temp}} M_i / M_{\text{temp}} \}^2 \quad (6)$$

for any contribution i . The mean is not affected, of course.

Assigning standard deviations to the various contributions is a difficult task. The experiments with an Antarctic ice-sheet model referred to earlier (Oerlemans, 1982) suggest 50% as an order of magnitude. This also seems reasonable for the changes expected on the Greenland ice sheet. The calculation of glacier melt would probably be more accurate when the present-day volume was known well. This is not the case (error: 30%), so here a standard deviation of 50% of the mean should also be accepted. The calculation of thermal expansion appears as somewhat more accurate. The range given by Wigley and Raper (1987) is 2 to 5 cm for the last hundred years, but the uncertainty is to a large extent due to uncertainties in the forcing (temperature perturbation). Here a value of 30% is used.

Finally, we have the growing variance due to internal variability. In case of statistical equilibrium, it can be shown that for a large ice sheet the variance of mean ice thickness H and accumulation A_c are related according to

$$\sigma_H^2 = t^* T_R \sigma_{Ac}^2. \quad (7)$$

Here t^* is the averaging time for the mass balance and T_R is the response time of the ice sheet. Variations in accumulation can be estimated from some ice cores, as has been done by Reeh *et al.* (1978). With $t^* = 30$ yr, σ_{Ac} is about 4% of the long-term mean value. With an appropriate 'accumulation response time' for the Antarctic Ice Sheet (a few thousands of years) this turns out to give $\sigma_H = 1.4$ m (equivalent to 4.6 cm of sea level). For the present calculation it should be known how the variance grows in time. Following Hasselmann (1976) this follows from:

$$\sigma^2(t) = \sigma_\infty^2 [1 - \exp(-t/T_R)]. \quad (8)$$

In this equation σ_∞^2 is the equilibrium variance referred to above. For times much smaller than T_R , the growth of variance is thus linear in time. Equation (8) is actually used here, with $T_R = 2100$ yr and $\sigma = 1.4$ m.

The result of the calculation is shown in Figure 11. At this stage no additional trends have been included ($TR = 0$). In broad sense, about 40% of the uncertainty is due to our poor knowledge of future temperature and about 60% to insufficient data and lack of adequate models to calculate the response of ice masses and ocean.

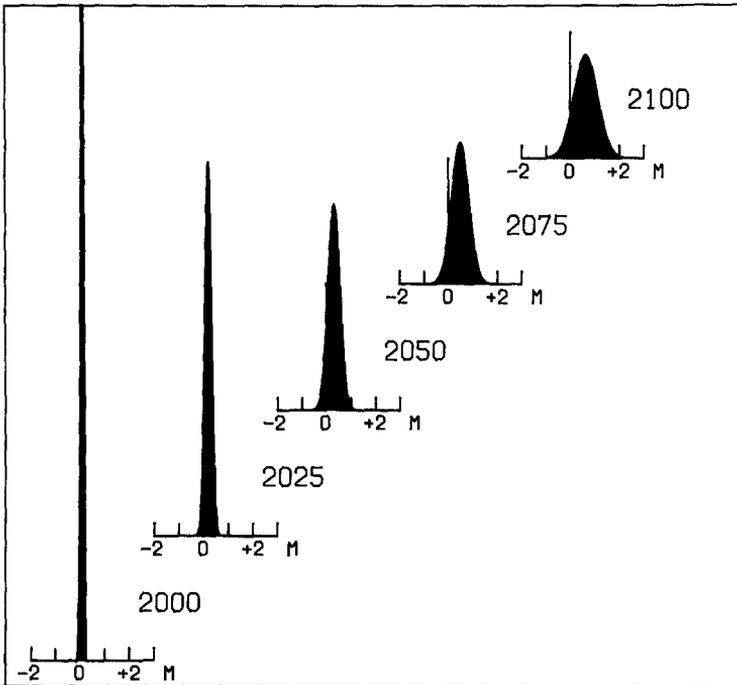


Fig. 11. Probability densities of sea-level stand relative to 1985, for some selected times. There is no basic trend included.

TABLE II: Threshold probabilities (%) of a sea-level stand above some selected values (in cm relative to 1985 AD)

Year	2025	2050	2075	2100
trend = 0				
-50	>>99	>>99	>99	99
-20	>99	99	98	96
-10	99	96	95	94
0	94	91	93	90
+10	78	83	87	87
+20	51	70	81	82
+30	24	55	72	77
+40	7	39	62	70
+50	2	24	51	62
+80	<1	3	21	38
+100	<<1	<1	8	24
+150	<<1	<<1	<1	4
trend = 10 cm 100 yr ⁻¹				
-50	>>99	>>99	>99	99
-20	>99	>99	98	98
-10	99	98	97	96
0	96	96	95	94
+10	87	89	92	91
+20	63	79	87	88
+30	34	65	80	83
+40	12	49	71	78
+50	3	36	61	71
+80	<1	6	28	48
+100	<<1	<1	13	32
+150	<<1	<<1	<1	7

Threshold probabilities are easily found by integrating the probability density. Table II gives results for two values of the basic trend ($TR = 0$ and $+10$ cm/century).

The value of the formal error analysis described above should not be overestimated. The assumption of independence between the factors contributing to the uncertainty is probably justified, but the individual errors are not much more than best guesses. Still, Figure 11 gives an indication of how rapid the uncertainty of the present projection grows in time.

7. Discussion

Several projections of future sea level have been published earlier, so a comparison is in order. Table III summarizes some estimates. Values from Hoffman *et al.* (1983) are the 'mid-scenario' values. Those referred to as Robin (1986) are actually calculated from the sea level - temperature regression used by Robin to estimate sea-level rise for a 3.5 K warming.

TABLE III. A comparison of estimates of future sea-level rise (in cm relative to the present). The comparison is not entirely straightforward, since input scenarios differ and transient effects are not always taken into account. Some values are interpolated to the years indicated. The last line gives the standard deviation for the present estimate

	2025	2050	2075	2100
Revelle (1983)			63.9	
Hoffman <i>et al.</i> (1983)	22.6	43.6	73.6	114.0
Robin (1986)	25.1	52.6	70.8	89.1
this study (mean)	20.5	33.0	50.5	65.6
this study (σ)	21.1	32.0	43.2	56.5

Apparently, the present calculation has yielded values that are lower. There are several reasons for this. Revelle (1983), for instance, does not take into account increasing accumulation on the Antarctic Ice Sheet. The high values in Hoffman *et al.* (1983) are mainly due to the much larger effect of thermal expansion. As pointed out by Van der Veen (1986), this is due to the use of a thermocline temperature that is too high. Since the thermal expansion coefficient increases strongly with temperature, the contribution is thus overestimated. Wigley and Raper (1987) give the most careful analysis of thermal expansion. For the period 1985 to 2025 they give an expansion effect of 4–8 cm. This is in very good agreement with the 6.5 cm found here.

The middle line in Table III, referred to as Robin (1986), is actually derived from the linear relationship between global mean surface temperature and sea level (also discussed by Gornitz *et al.*, 1982). Robin has suggested using such a linear relation to make a projection of future sea level, because it is ‘the best one can do’. However, there are two major drawbacks in such an approach: (i) a small error in the input data for the regression analysis may lead to very large errors in predicted sea level, and (ii) it is doubtful whether a relation derived from data in a 0.5 K temperature range can be applied to temperature perturbations of up to 3 or 4 K. I therefore preferred to estimate the various contributions separately, in spite of the fact that our knowledge is inadequate in many respects.

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