Band 21 (1985), S. 291–299

ZEITSCHRIFT FÜR
GLETSCHERKUNDE
UND GLAZIALGEOLOGIE

THE HEAT BUDGET OF THE ANTARCTIC ICE SHEET

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With 2 figures

ABSTRACT

Averaged over an entire drainage basin of a polar ice sheet, the thermodynamic equation takes a simple form. In particular, dissipative heating can be obtained directly from the release of gravitational energy. When ice-accumulation rate, surface temperature, elevation and ice thickness are known, the mean temperature of the ice leaving the drainage basin can be calculated in the case of equilibrium.

We have applied this procedure to the drainage basins of the Antarctic Ice Sheet. Mean basal outlet temperatures appear to vary between −21.3 and −8.3° C. The latter value was found for the basin that feeds the Ross Ice Shelf. Drainage basins with higher surface elevation generally have lower outlet temperatures, in spite of the fact that ice thickness is generally greater.

Assigning a characteristic length scale to a drainage basin makes it possible to estimate the typical base stress, and by assuming a balance between discharge and accumulation, the global flow parameter. Sensitivity to changes in mass balance can be studied, including the temperature feedback on ice flow. The procedure is applied to one drainage basin, which shows that temperature feedback almost doubles the sensitivity of mean ice thickness to changes in accumulation.

DIE WÄRMEBILANZ DES ANTARKTISCHEN EISSCHILDS

ZUSAMMENFASSUNG

Die mittlere Wärmehaushaltsgleichung eines ganzen Einzugsgebiets in einem polaren Eisschild hat eine einfache Form, bei der vor allem die Wärmedissipation direkt aus der frei werdenden potentiellen Energie berechnet werden kann. Wenn die Akkumulation, Oberflächentemperatur, Höhe und Eisdicke bekannt sind, kann die Mitteltemperatur des Eises beim Austritt aus dem Becken bei Gleichgewichtsbedingungen berechnet werden.

Wir haben diese Methode auf die Einzugsgebiete des antarktischen Eisschilds angewendet. Mittlere basale Temperaturen beim Verlassen des Beckens liegen zwischen −21,3 und, im Einzugsgebiet des Ross-Eisschelfs, −8,3° C. Becken mit höherer Oberfläche haben im allgemeinen trotz größerer Eisdicke tiefere Auslaßtemperaturen.

Mit einer charakteristischen Länge kann die basale Schubspannung eines Beckens geschätzt werden und, wenn Gleichgewicht von Akkumulation und Abfluß angenommen wird, auch der globale Fließparameter. Die Empfindlichkeit der Reaktion auf Massenbilanzänderungen einschließlich der Rückkoppelung zwischen Temperatur und Eisfluß kann getestet werden. Anwendung auf ein Einzugsgebiet zeigt, daß die Temperaturrückkoppelung die Empfindlichkeit der mittleren Eisdecke auf Änderungen in der Akkumulation fast verdoppelt.

1. INTRODUCTION

It is well known that the temperature distribution in an ice sheet has a large effect on the ice-mass discharge (e. g. Paterson 1981). For a given stress, strain rates vary by a factor of 100 for a 20° C temperature difference. In addition, substantial basal sliding is only possible when basal ice is at melting point. A proper treatment of ice flow, therefore, requires full consideration of the thermomechanical system, that is, simultaneous integration of the thermodynamic and flow equations. Some attempts have been made in this direction (e. g. Budd et al. 1984, Jenssen 1977, Young 1981, Oerlemans 1982, Oerlemans and Van der Veen 1984).

It seems useful, however, to support the numerical integration method by a less complicated analysis, in which the total heat budget is considered. As shown by Van der Veen and Oerlemans (1984), the thermodynamic equation for ice flow takes a simple form when integrated over an entire drainage basin. In particular dissipation is easily calculated from the release of gravitational energy.

In this paper the "total heat budget approach" is applied to the Antarctic Ice Sheet. Assuming that the drainage basins are in equilibrium, the net heat budgets are zero. The mean temperature of the ice leaving the drainage basin can then be calculated if surface temperature and accumulation, ice thickness, bedrock topography and geothermal heat flux are known. This procedure will be applied to 11 Antarctic drainage systems. Since different drainage basins have different outlet temperatures, the results can be used to find out, for instance, how the importance of the geothermal input varies with the accumulation rate.

The integrated thermodynamic equation may, with some limitations, also be used to investigate how a drainage basin will react to changes in the mass balance, surface temperature, etc. An example will be given for one drainage basin. The results indicate that the mass balance is extremely important, particularly in cold conditions where accumulation is very small.

2. PROCEDURE AND DATA

The total internal energy of an amount of ice (of constant density ρ) is defined as $I = \rho c V T_m$, where c is heat capacity, V total volume, and T_m mean ice temperature. In case of equilibrium, I is constant and the total of all internal heat fluxes should be zero. These fluxes are: geothermal heat input, mechanical dissipation, ice accumulation at the surface, and loss by calving. Diffusive transport at the surface is disregarded, because in ice sheets without surface melting vertical temperature gradients in the upper layers are very small (e. g. Paterson 1981).

Let S be the area of a drainage basin and G the geothermal heat flux. The geothermal input then simply equals GS. The net change in internal energy due to accumulation and calving equals:

$$\rho c \int_S M T_s \, ds - \rho c T_e \int_S M \, ds. \qquad (1)$$

In this expression M is the mass balance (in ice depth/yr). T_s surface temperature, and T_e the mean temperature of the ice leaving the drainage basin (at the grounding line). Note that if M does not vary over the drainage basin, the change in T_m due to accumulation and calving (termed "advection" in the following) would be proportional to

$\overline{T}_s - T_e$. In reality, however, M generally increases towards the coast. In this study the annual surface temperature T_s is calculated from

$$T_s = T_{so} - \gamma h, \tag{2}$$

where T_{so} is annual sea-level temperature, γ the lapse rate along the ice-sheet surface, and h surface elevation. Different values of T_{so} and γ will be used for the different drainage basins.

The dissipative heating is obtained from the release of potential energy. This is possible because the kinetic energy of the ice flow is negligibly small. When h_e denotes the mean elevation (relative to sea level) of the ice mass leaving a drainage system, the dissipation equals

$$\rho g \left[\int_S M h \, ds - h_e \int_S M \, ds\right]. \tag{3}$$

Summing all these contributions to the heat budget and setting the result to zero yields an equation from which T_e can be found. The result is

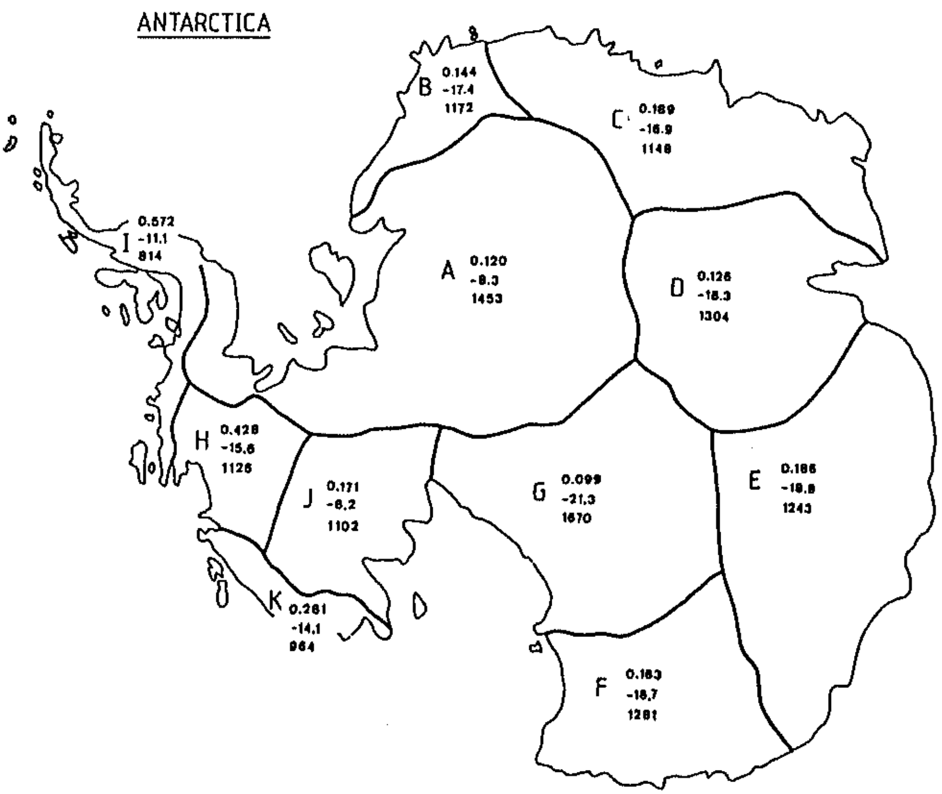

Fig. 1: Antarctic drainage basins for which the heat budget calculation was performed. The numbers give mean annual accumulation (m water equivalent), basal outlet temperature (° C) and depth below sea level (m) where basal melting is expected to occur

$$T_e = T_{so} - \frac{g}{c} h_e + \left(\frac{g}{c} - \gamma\right) \overline{M h}/\overline{M} + G/(\rho c \overline{M}). \quad (4)$$

From this equation a few useful conclusions can immediately be drawn.

Firstly we note that $g/c = 0.00465°$ C/m. Since in reality γ is always substantially larger than this value (typically 0.01° C/m), the third term on the right-hand side of (4) is always negative. Higher mean surface elevation therefore implies a lower value of the outlet temperature T_e. Apparently, the enhanced dissipation is smaller than the cooling due to lower surface temperatures.

When the spatial distribution of M is more or less fixed, $\overline{M h}/\overline{M}$ will be almost independent of $\overline{M}$. The effect of a varying mass balance, irrespective of associated changes in ice thickness, is thus determined by the last term in (4): a smaller mass balance obviously leads to higher outlet temperatures.

The data that we used was derived from Atlas Antarktiki (1966) and Drewry (1983). The values needed to calculate T_e are surface elevation h, ice thickness H, mass balance M, and, typical for each drainage system, annual surface temperature T_{so} and lapse rate γ. The maps in these references were read out on a 50 km grid. Although this implies a certain lack of resolution, the accuracy should be sufficient for the present purpose. It is doubtful anyway whether a higher resolution should be compatible with the accuracy of the initial observations.

The partition of the Antarctic Ice Sheet into drainage systems which we used is the classical one, with some small modifications. It is shown in fig. 1. The drainage systems, labelled A to K, have negligible flow of ice mass through the boundaries (except at the grounding line, of course). Basic input quantities are given in table 1.

Table 1: Input quantities for the calculation of ice temperature at the edge of the drainage systems. The mean mass balance is given in terms of water equivalent per year. In the calculations, standard ice density is used to convert this to ice depth

drainage basin	$\overline{M}$ (m/yr)	$\overline{h}$ (m)	$\overline{H}$ (m)	$\overline{M h}/\overline{M}$ (m)	h_e (m)	T_{so} (° C)	γ (° C/m)
A	0.120	2530	2160	1858	−480	−20	0.010
B	0.144	2170	1120	1840	−140	−16	0.011
C	0.189	2180	1400	1873	−140	−12	0.012
D	0.126	2800	1750	2413	−180	−12	0.012
E	0.186	2370	2510	2011	−140	−13	0.012
F	0.183	2090	1900	1934	−160	−16	0.011
G	0.099	2850	2330	2545	−280	−22	0.010
H	0.428	1200	1800	1030	−160	−20	0.007
I	0.572	995	619	961	−120	−14	0.007
J	0.171	1220	1650	1234	−320	−20	0.007
K	0.281	1210	1080	1189	−120	−18	0.007

According to Atlas Antarktiki (1966), annual sea-surface temperatures in the coastal areas are low for the West Antarctic Ice Sheet and higher for the East Antarctic Ice Sheet. The mean lapse rate along the ice-sheet surface, on the other hand, is largest for East Antarctica. Accumulation generally decreases with surface elevation. This is reflected by the fact that for all drainage systems, except J, $\overline{h} > \overline{M h}/\overline{M}$.

3. HEAT BUDGET OF THE ANTARCTIC DRAINAGE BASINS

The results of the temperature calculation are shown in table 2 and fig. 1. The first column in the table gives the equilibrium solution for T_e. Values are typically in the −15 to −25° C range. It is interesting to note that, in spite of higher values of T_{so}, the East Antarctic drainage basins have the lowest outlet temperatures. This is due to the large surface elevation and associated low temperatures of the accumulated ice mass.

Table 2: Results of the temperature calculation for the various drainage systems. The three contributions to the heat budget (dissipation, geothermal input and advection) are given in terms of annual heating rate

drainage basin	T_e (° C)	diss.	geoth. (0.0001° C/yr)	advec.	diss./ geoth.
A	−20.3	6.59	4.12	−10.71	1.60
B	−20.9	12.90	7.95	−20.85	1.62
C	−20.4	13.78	6.36	−20.14	2.17
D	−21.8	9.46	5.09	−14.55	2.66
E	−22.3	8.08	3.55	−11.63	2.28
F	−22.7	10.21	4.68	−14.89	2.18
G	−25.3	6.08	3.82	− 9.90	1.59
H	−19.6	14.34	4.95	−19.29	2.90
I	−14.1	50.57	14.38	−64.95	3.52
J	−16.2	8.18	5.39	−13.57	1.28
K	−17.1	17.28	8.24	−25.52	2.10

Observations on the temperature distribution in the Antarctic Ice Sheet are very scarce, and a comparison of the present results with reality is difficult. For drainage basin J, however, some information is available. Young (1981) has calculated the temperature distribution along a flow line in this drainage basin. His results fit well with temperature-depth observations at stations J9 and Q 13 (on the Ross Ice Shelf): At the grounding line, his model predicts a vertical mean ice temperature of −16.0° C, which is very close to the value found here (−16.1° C).

The various contributions to the heat budget (dissipation, geothermal input, advection) are also given in table 2. To calculate the geothermal contribution, a value of 54 mW m^{-2} was used (Turcotte and Schubert 1983). The numbers give the heating rate in ° C/yr. It is of interest to see how important the geothermal heat flux is. In the fifth column the ratio of dissipative heating to geothermal input is listed. Apparently, dissipation is typically twice as large. The geothermal heat flux generally becomes less important when the rate of accumulation increases.

Probably the most relevant factor to ice-mass discharge is the mean *basal* temperature at the grounding line. Although this quantity cannot be obtained directly because vertical velocity and temperature profiles are not available from the present analysis, a crude estimate can be made assuming that T_e equals the ice temperature halfway between the base and the surface. Extrapolating downwards, using a constant temperature gradient of 2° C yields a value for the basal temperature T_{eb}. Results of this procedure are shown in fig. 1. High values of T_{eb} are found for basins A and J, due to the fact that the grounding line is generally found at great depth there. In a similar way it is possible to calculate the depth at which basal melting would occur. This critical depth h_{cr} is also shown in fig. 1; a typical value is 1000 m below sea level.

Although these estimates of T_{cb} and h_{cr} are admittedly crude, they probably provide a fair comparison between the various drainage basins.

4. FURTHER INFERENCES CONCERNING DYNAMICS

It is tempting to relate the basal outlet temperature to the global dynamics of the drainage basins. A direct comparison of outlet temperature and mean ice thickness, however, is not appropriate, because the mean surface slope may vary from basin to basin. Instead, we include schematic ice-flow mechanics.

For large-scale ice flow, a typical ice velocity U can be estimated from (Nye 1959)

$$U = Q\,H\,\tau^3. \tag{5}$$

Here τ is a characteristic basal stress, H mean ice thickness, and Q is a global flow parameter. In the equilibrium situation, the ice-mass discharge should equal the total accumulation in the drainage basin. So

$$\overline{MS} = U\,H\,R = Q\,H^2 R\,\tau^3. \tag{6}$$

Here R is the length of the grounding line segment of the drainage basin under consideration.

It should be noted that H, the *mean* ice thickness, may not be the most appropriate quantity to scale the ice thickness. The ice thickness relevant to estimating the discharge (and the characteristic base stress), however, is assumed to be proportional to H. The constant of proportionality, which may be different for the various drainage basins, can then be absorbed in the global flow parameter Q. The same applies to the quantity S/R.

So we now estimate the base stress to be:

$$\tau \doteqdot H\,h/L, \tag{7}$$

where L is a characteristic length scale for the drainage basin. Combining (6) and (7) then yields an equation for the mean ice thickness:

$$M = Q_0\,H^5 (H + b)^3 / L^3. \tag{8}$$

Surface elevation has been replaced by ice thickness plus mean bed elevation b. By inserting observed values for H, $\overline{M}$, b and L for a particular drainage basin, Q_0 can be solved from (8). Once the appropriate value of Q_0 has been obtained, further investigation can be made of how the ice thickness reacts to changes in mass balance. Since we are interested in comparatively small changes in the present state, a further possible simplification is to linearise (6) around this state (characterised by H_0, b_0, M, Q_0). Perturbation quantities are denoted by primes. We find

$$\begin{aligned} M' = \frac{Q_0}{L^3}[H'(8\,H_0^7 + 5\,b_0^3 H_0^4 + 21\,b_0 H_0^6 + 18\,b_0^2 H_0^5) \\ - H'(3\,b_0^2 H_0^5 + 3\,H_0^7 + 6\,b_0 H_0^6)] + \frac{Q'}{L^3} H_0^5 (H_0 + b_0)^3. \end{aligned} \tag{9}$$

In deriving this equation it has been assumed that $b' = -\alpha H'$, where α is the ratio of ice density to rock density, thus an isostatic balance is supposed to prevail. To keep the analysis simple, L is not allowed to vary, so the present procedure can only be applied to drainage basins of more or less fixed size.

As an example, we consider drainage basin E. The relevant quantities are: $\overline{M}=0.203$ m/yr, $H_0=2510$ m, $b_0=-140$ m, $L=600$ km. From (8), with $\alpha=1/3$, it is then found that $Q_0=3.30 \times 10^{-11}$ yr^{-1} m^{-4}. Omitting the Q'-term in (7) for the moment, it follows that $H'=1488\,M'$ (H in m, M in m ice depth/yr). See fig. 2. If the mass balance and ice thickness increase, however, the mean outlet temperature will change. This will affect the global flow parameter, and a further change in H occurs.

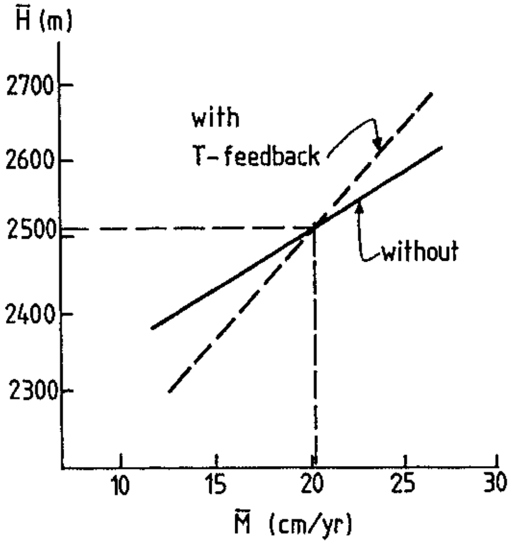

Fig. 2: Ice thickness vs. mass balance for drainage basin E, according to a linear analysis around the present state

To investigate this we first estimate the perturbation outlet temperature from (4).

$$T'_e=\left(\frac{g}{c}-\gamma\right)\frac{M h_0}{\overline{M}}\frac{1}{h_0}(H'+b')-\frac{G}{\rho c \overline{M}^2}M'. \qquad (10)$$

Inserting relevant quantities again we obtain $T'_e=-0.0042\,H'-31\,M'$ (T_e in °C, H' in m, M' in m ice depth/yr). So, according to these results, an increase in the mass balance of 0.03 m would lead to a 44 m increase in mean ice thickness. The associated drop in outlet temperature would be about 1° C.

Ice deformation depends strongly on ice temperature, and an order of magnitude change in the flow parameter for a 10° C change in ice temperature is typical (e. g. Paterson 1981). For the present schematic analyses it is simply assumed that $Q'=0.1\,Q_0 T'_e$ (T'_e in °C). Eliminating T'_e and Q' yields a new relation: $H'=2764\,M'$. So, obviously, the temperature feedback on the ice flow enhances the sensitivity of ice thickness to mass-balance variations. The result obtained here compares very well with the numerical integrations of a more refined model reported in Oerlemans and Van der Veen (1984). This indicates that the present analysis, although simple, is a powerful method to study the basic sensitivity of drainage basins of polar ice sheets.

5. DISCUSSION

The results of this study can be summarised as follows:

(i) An increase in mean ice thickness, all other things being equal, leads to a drop in mean outlet temperature. In this case, increased dissipation does not compensate for lower surface temperature.

(ii) An increase in the mass balance, all other things being equal, also leads to lower outlet temperature.

(iii) In spite of higher sea-level temperature, the drainage basins of the East Antarctic Ice Sheet have lower outlet temperature than those of the West Antarctic Ice Sheet.

(iv) When the accumulation increases, ice thickness also increases. The resultant cooling of the ice sheet causes an additional increase in ice thickness almost as large as the direct effect.

It should be noted that errors in the heat budget calculations for the various drainage basins are almost entirely due to errors in the input data; the assumption of no conductive heat flux in the upper ice layers should hold very well for the Antarctic Ice Sheet. The relative importance of errors in the input quantities can be obtained directly from (4). The result is shown in table 3. Although errors may accumulate, the estimates in the table suggest that the differences in T_e and T_{eb} between the various drainage basins are significant.

Table 3: Effect of errors in input data on the equilibrium outlet temperature

type of error	ΔT_e (° C)
1° C in T_{s0}	1
100 m in h_e	0.5
100 m in h	0.5
0.001° C/m in γ	2
10 % in G	0.5

A further comment on local basal temperatures is in order. According to fig. 1, a typical depth at which basal melting is expected at the grounding line is 1000 m below sea level. This is an estimate, however, based on the total heat budget of the drainage basin in consideration. If, locally, the ice-mass discharge is larger, this critical depth may be less due to enhanced dissipation. Such local features can not be studied with the present approach, but may substantially affect the discharge from a drainage basin.

Finally, the heat budget calculation performed here is quite simple. As soon as a more accurate data become available, the procedure can be repeated.

REFERENCES

Atlas Antarktiki, 1966: Glavnoie Upravlenie geodezii i Kartografii MG.

Budd, W. F., D. Jenssen, and I. N. Smith, 1984: A three-dimensional time-dependent model of the West Antarctic Ice Sheet. Annals of Glaciology 5: 29—36.

Drewry, D. J., 1983: Antarctica: glaciological and geophysical folio. Scott Polar Research Institute, Cambridge.

Oerlemans, J., 1982: A model of the Antarctic Ice Sheet. Nature 297: 550—553.

Oerlemans, J., and C. J. Van der Veen, 1984: Ice sheets and climate, 217 pp., Reidel, Dordrecht.
Paterson, W. S. B., 1981: The physics of glaciers, 2nd ed., 380 pp., Pergamon, Oxford.
Turcotte, D. L., and G. Schubert, 1982: Geodynamics, 450 pp., John Wiley, New York.
Van der Veen, C. J., and J. Oerlemans, 1984: Global thermodynamics of a polar ice sheet. Tellus 36 A: 228—235.
Young, N. W., 1981: Responses of ice sheets to environmental changes. IAHS Publ. no. 131: 331—360.

Paper given at the Symposium on Climate and Paleoclimate of Lakes, Rivers and Glaciers in Igls, Austria, 4—7 June 1984.

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