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Modeling of Pleistocene European Ice Sheets: Some Experiments with Simple Mass-Balance Parameterizations

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A vertically integrated ice-flow model suitable for use in climate studies is formulated. Large continental ice sheets may be characterized by two fundamental quantities: the height-to-width ratio, and the steepness of the edge. So it is natural to develop a model containing two parameters that can be chosen to give the right values of those characteristic quantities. The result is a model that is close to M. A. W. Mahaffy's (*Journal of Geophysical Research* 81, 1059–1066 (1976)). The model is used to study glaciation in Europe. Dropping the level of zero mass balance creates small stable ice caps in the Alps and the Scandinavian mountains. If the drop exceeds 600 m (with respect to present-day conditions), the feedback between ice-sheet height and mass balance becomes dominating and the Fennoscandian Ice Sheet keeps growing. It does not reach an equilibrium state within 60,000 yr. An experiment simulating rapid onset of a glacial cycle shows that the growth of ice volume in Europe is smaller than that in northern America (J. T. Andrews and M. A. W. Mahaffy, *Quaternary Research* 6, 167–183 (1976)). After 10,000 yr, the volume of the Fennoscandian Ice Sheet (2×10^{15} m^3) is about half the volume of the Laurentide Ice Sheet. This leaves the "observed" sea-level lowering in the period 125,000–115,000 yr B.P. (estimates center around 50 m) unexplained.

INTRODUCTION

During the last decade, evidence has been gathered that changes in the parameters of the earth's orbit have been significantly correlated with total ice volume on earth. Up to now, the most comprehensive statistical study (as far as data permit) is that of Hays *et al.* (1976). Their results lend considerable support to the astronomical theory of the Pleistocene ice ages, more than a century after its first appearance in scientific literature. Although the problem of explaining the ice ages may be considered as partly solved in a statistical sense, the physics that apparently lead from small insolation variations to enormous changes in the size of continental ice sheets are *by no means understood*. The central question was and is: Which processes in the climate system amplify the effect of small insolation variations?

Energy-balance climate models fail to give a response in the temperature field which has the same order of magnitude as Pleistocene temperature variations derived from proxy data. Even if the seasonal cycle is included (North and Coakley, 1979) or energy budgets are separately computed for oceanic and continental regions, thus allowing zonal asymmetry (Oerlemans, 1980), the sensitivity of the model climate is too small. Apparently, positive feedback mechanisms must exist that are not incorporated in those simple climate models. It is in this background that the present study was undertaken. As was first indicated by Bodvarsson (1955), the strong dependence of the annual mass budget G on height may provide strong positive feedback between ice-sheet size and G. This point was investigated in more detail by Weertman (1961, 1976) who concluded that the extreme sensitivity of ice sheets (as a result of the feedback mechanism just mentioned) may explain the occurrence of Pleistocene ice ages.

If continental ice sheets are really so sen-

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sitive, their modeling should be done carefully. The fact that the strong coupling between G and height strongly enhances the sensitivity demonstrates the need for incorporating orography in model studies. In this respect Scandinavia is particularly interesting. In the following section a relatively simple two-dimensional ice-sheet model will be formulated that makes it possible to study the broad characteristics of ice-sheet evolution (and decay) in Europe. In the subsequent sections we will discuss some integrations with simple parameterizations of G.

FORMULATION OF THE MODEL

Large continental ice sheets may be described by two characteristic quantities. First, some idea exists on the height-to-width ratio; for a large sheet this ratio has order of magnitude 10^{-3}. Second, it has been observed that ice sheets have rather steep edges, which is partly due to the nonlinear character of the flow law for ice. These two properties are probably so universal that continental ice sheets in the past also showed them. Thus, from a practical point of view, one should start with *a flow law that provides not more than two degrees of freedom for the model.* Computation of the temperature field within the ice does not make much sense, because the boundary conditions are not known (in a later stage it may turn out to be important, however, to explain rapid deglaciations as the consequence of extensive basal sliding). This line of reasoning also implies that the model should essentially describe a vertically averaged state.

With this philosophy in mind, the formulation of the ice-sheet model is now set up. Since large sheets are considered, the temperature with respect to the melting point will be highest near the bottom. This implies that most of the velocity shear in the vertical takes place in the lower layers. With regard to vertical mean discharge of ice mass, there is thus not much difference between basal sliding and internal deformation. This consideration led Nye (1959) to propose the use of a generalized flow law that accounts for both internal deformation and basal sliding. It reads:

$$u = A\,\tau_b{}^m, \qquad (1)$$

where u is the vertical mean (= from the bottom to the top of the ice sheet) velocity, τ_b the shear stress at the bottom, and A and m constants. Note that A and m introduce the two degrees of freedom needed to obtain a realistic shape of the model ice sheet!

For the present purpose a two-dimensional form of Eq. (1) is desired. Therefore, the basal shear stress is expressed as

$$\tau_b \propto H\nabla(H + h). \qquad (2)$$

Here H is the ice thickness and h represents the elevation of the bedrock above some level of constant geopotential. In the following, density is taken to be *constant,* so mass and volume become completely equivalent concepts. In a Cartesian coordinate system $\nabla = (\partial/\partial x,\ \partial/\partial y)$. The generalization of Eq. (1) now becomes

$$\mathbf{M} = BH^{m+1}\left[\nabla(H + h)\cdot\nabla(H + h)\right]^{(m-1)/2}\nabla(H + h). \qquad (3)$$

$\mathbf{M}$ represents the vertically integrated mass-flow vector or ice-volume vector and B is some constant. The evolution of the sheet is determined by the conservation of ice mass (or volume). If the mass balance is denoted by G (unit: length/time), one has

$$\frac{\partial H}{\partial t} = \nabla\cdot\mathbf{M} + G. \qquad (4)$$

Equations (4) and (5), together with the condition that $H \geq 0$, completely formulate the model.

The term $\nabla\cdot\mathbf{M}$ may be interpreted as a nonlinear diffusion term $[\nabla\cdot D\nabla(H + h)]$. The diffusivity D increases with both the thickness and the slope of the ice surface according to

$$D = BH^{m+1}\left[\left\{\frac{\partial(H + h)}{\partial x}\right\}^2 + \left\{\frac{\partial(H + h)}{\partial y}\right\}^2\right]^{(m-1)/2}. \qquad (5)$$

At this point the model should be compared to that developed by Mahaffy (1976).

Mahaffy formulated the model by starting from Nye's generalized form of Glen's law (Nye, 1957) and introducing some assumptions that are generally valid for large ice sheets. In her model, D depends somewhat more strongly on ice thickness and somewhat more weakly on surface slope, and the basal sliding appears as a boundary condition. Nevertheless, the resemblance between the present model and that of Mahaffy is large. The reason for this is that generalization of Eq. (1) to the two-dimensional case, i.e., Eq. (2), is implicitly based on the same assumptions as Mahaffy used.

Since Eqs. (4) and (5) are only valid if H is at least a few hundred meters, near the edge problems may be expected. At the edge $H = 0$, which implies that $D = 0$, so the ice-sheet edge cannot advance if $G \leq 0$! Obviously, this is unrealistic. The problem can be solved by setting a minimum value for D, or setting D at a certain grid point where $H = 0$ equal to the value of D in the grid point nearest to it with $H \neq 0$. Experiments showed that the formulation is far from crucial to the behavior of the ice sheet.

METHOD OF SOLUTION

Equation (4) is numerically solved on a rectangular grid. The part of Europe covered by the grid is shown in Figure 1. The grid consists of 43 × 31 points, laid out over a stereographic projection. The grid size is 100 km at 60°N. A map factor has been incorporated in the difference scheme to account for the distortion of distances by the projection (for a discussion of map factors, see Haltiner, 1971). Orography has been included by specifying h in each grid point. If in any grid point $h < -200$ m (edge of the continental shelf), the ocean is assumed to act as an infinite sink for ice, so here $H(t) = 0$.

Spatial derivatives were approximated by central differences. The formulation of the difference equations is straightforward, so it will not be carried out here. An explicit forward time scheme is used for the integration in time. The main reason for this is its simplicity in coding and its flexibility with regard to all kinds of boundary conditions. For the linear, two-dimensional diffusion equation ($\partial H/\partial t = D\nabla^2 H$), the stability criterion for an explicit forward scheme is given by (e.g., Smith, 1978)

$$\delta t \leq \frac{(\delta l)^2}{4D}. \quad (6)$$

Here δl is the grid distance and δt the time step. A characteristic value of D is

$$D = L^{*2}/T^*, \quad (7)$$

where L^* and T^* are characteristic length and time scales of a large ice sheet. Thus, with $L^* \simeq 10^6$ m and $T^* \simeq 10^4$ yr yields $D \simeq 10^8$ m²/yr. With $\delta l = 10^5$ m, Eq. (6) then shows that the time step should be smaller than 25 yr to guarantee stability. It is very difficult to derive a stability criterion for the nonlinear case. Extensive experiments showed that $\delta t = 10$ yr gives absolute stability for all experiments of practical interest.

Since in many experiments the initial condition is $H(x,y) = 0$, reduction of computational times may be obtained by using a grid of variable size. This is done by checking at each grid point whether $H = 0$ and $G = 0$, and $H = 0$ in the surrounding

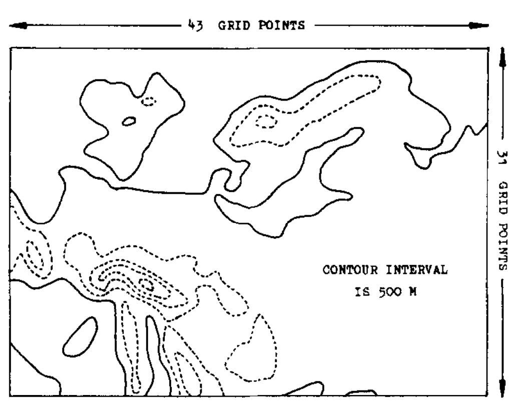

FIG. 1. Outline of the model domain. Europe is covered by a grid of 1333 equally spaced points; the spacing is 100 km. The coastline and the orography, as resolved by this grid, are indicated by solid and dashed lines, respectively.

points. If so, this grid point is dropped until the next check. In this way the grid grows with the ice sheet.

An alternative method of integration frequently used in nonlinear diffusion problems is the ADI method (alternating-direction implicit method; e.g., Smith, 1978). For the present model it seems questionable whether the ADI method is advantageous, because the model domain is very irregular. In addition, the ADI method is difficult to combine with a variable grid size. Based on those considerations and the fact that the explicit scheme appeared to be very stable, no ADI method was tried.

FORMULATION OF THE MASS BALANCE

The input of the ice-sheet model is the mass budget G. Its parameterization is a very difficult task. Conditions over the Fennoscandian Ice Sheet during the Quaternary ice ages are badly known and probably very poorly reflected by present-day conditions in high mountain regions, or those over the Antarctic and Greenland ice sheets. Consequently one should accept a very crude specification of the mass budget, which is hardly based on sound observational evidence.

The dependence of G on height is formulated relative to the "equilibrium-plane altitude" (EPA), according to

$$G = a\,(h + H - \mathrm{EPA}) - b\,(h + H - \mathrm{EPA})^2. \quad (8)$$

If $a = 0.732 \times 10^{-3}$ yr^{-1} and $b = 0.268 \times 10^{-6}$ (m-yr)$^{-1}$, the top of this parabola is at $(h + H - \mathrm{EPA}) = 1500$ m (Fig. 2). The corresponding value of G is 0.5 m/yr. If $(h + H - \mathrm{EPA}) > 1500$ m, G is always equal to 0.5 m/yr. The shape of the curve in Figure 2 is in approximate agreement with that used by Andrews and Mahaffy (1976), but the maximum possible accumulation rate is smaller.

The parameterization of the mass budget may now be completed by specifying the EPA as a function of geographical position.

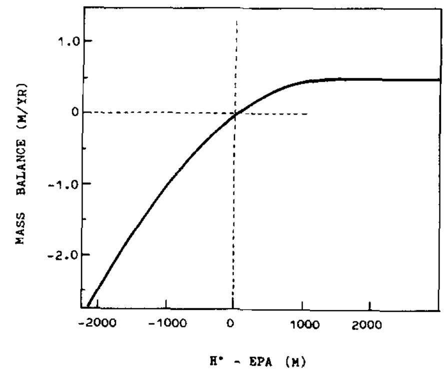

FIG. 2. Height dependence of the mass balance G. The mass balance is formulated in terms of surface height ($H^* = H + h$) with respect to the height of the equilibrium plane (EPA).

In the simplest case it decreases linearly with latitude, i.e.,

$$\mathrm{EPA} = \mathrm{K} + \alpha\,(70^\circ - \Psi), \quad (9)$$

where ψ is latitude in °N. The north–south slope of the equilibrium plane is given by α. From data presented by Charlesworth (1957) one may estimate α to have an *order of magnitude* of 50 m/°N. K is some constant and denotes the height of the equilibrium plane at the northern edge of Scandinavia.

LOWERING OF THE EQUILIBRIUM PLANE

I will now discuss experiments in which the equilibrium plane is lower than today over the whole model domain, i.e., the value of K in Eq. (9) is reduced. In each experiment $B = 3$ m$^{-3/2}$ yr^{-1} and $m = 2.5$. Unless stated otherwise, $\alpha = 50$ m/°N.

Initial conditions are $H(x,y) = 0$, and at the boundary of the model domain the condition $H(t) = 0$ is applied. The latter is relevant at the eastern side of the domain, because the other sides are bounded by deep ocean. The boundary condition can influence the eastward growth of the Fennoscandian Ice Sheet in an unrealistic way, but in the present experiments this effect will be very small (compare Fig. 6).

Figure 3 shows the growth of the ice sheets (in general, two ice sheets form: one

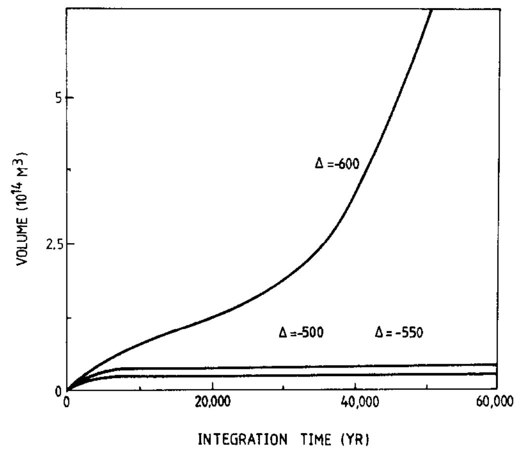

FIG. 3. Evolution of the ice-sheet volume in Europe for three experiments, in which the EPA is set to low values. The change in the EPA is denoted by Δ and measured in meters with respect to present conditions as given by Charlesworth (1957).

in the Alps and one in the Scandinavian mountains) in terms of total ice volume, for three values of K. Those values correspond to a drop in the EPA (denoted by Δ) of 500, 550, and 600 m, that is, with regard to the present-day EPA given by Charlesworth (1957). For $\Delta = -500$, -550 m, after 8000 yr a steady state settles down. In this case very small ice caps are located in the Alps and the Scandinavian mountains. For $\Delta = -600$ m, however, the picture changes drastically: the ice sheet over Scandinavia keeps growing and does not approach an equilibrium state within 60,000 yr. Here, the sheet height–mass balance feedback becomes dominating and increases the characteristic time scale T^* considerably.

The appearance of a critical value of Δ, somewhere between -550 and -600 m, is due to the presence of mountains. In the case of a small ice cap over a mountain range, the sloping bedrock allows an ice-mass discharge to the lower surrounding region large enough to cancel the effect of the height–mass budget feedback. Runs without orography ($h = 0$), which are not shown here, proved this explanation to be correct.

The height–mass budget feedback is determined by the constants a, b, and α. Consequently, T^* depends on these constants. In particular, α plays a very important role—the smaller α, the larger T^*. For values of the model parameters within their realistic range, the experiments showed that T^* is mainly determined by the height–mass balance feedback and only secondarily by the mechanical properties of the ice sheet. To illustrate this point, Figure 4 displays the result of a run in which α was set to a very large value, which may be imagined as putting the equilibrium plane in a vertical direction. In this run the equilibrium plane was situated at 60°N. Also shown in Figure 4 is a run with $\alpha = 50$ m/°N and $\Delta = -700$ m. The differences in the behavior of the ice sheet are evident. In the case of large α, a steady state is reached within 25,000 yr.

A similar experiment is to change the height dependence of the mass balance. The lower curve in Figure 4 (labeled C) shows the result of a run in which a and b in

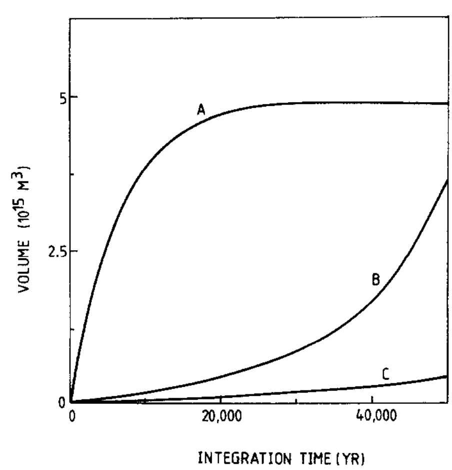

FIG. 4. The dependence of ice-sheet growth on the parameterization of the mass balance G. Curve A shows a run in which the equilibrium plane is vertical (α very large) and located at 60°N. Curve B corresponds to a run with $\alpha = 50$ m/°N and $\Delta = -700$ m. The third run was also carried out with those values of α and Δ, but now a and b in Eq. (8) were replaced by $0.5a$ and $0.5b$, respectively, which corresponds to reducing the vertical mass-balance gradient by 50%. Curve C shows the result.

Eq. (8) are halved. Further, the same constants were used as for curve B. Apparently, the ice sheet grows more slowly if the vertical gradient in the mass balance is smaller.

To avoid confusion, it should be stated that T^* is of course indirectly affected by the mechanical properties of ice because they determine the height and therefore the height–mass balance feedback. However, the speed at which ice mass can be transported toward regions with a deficit ($G < 0$) is so large that it does not slow down the evolution of the ice sheet. Or, in other words, the response time of the ice sheet is much smaller than its growth time.

SIMULATION OF RAPID GLACIATION

The experiment to be discussed now is closely related to that of Andrews and Mahaffy (1976). They estimated the maximum possible growth rate of the Laurentide Ice Sheet, in order to find a quantitative explanation for the rapid sea-level lowering that appeared between 125,000 and 115,000 yr B.P. A typical value of this lowering is 50 m (e.g., Steinen *et al.*, 1973; Bloom *et al.*, 1974). Andrews and Mahaffy concluded that the possible growth rate of the Laurentide Ice Sheet is too small by at least a factor of two to explain this substantial drop in sea level, even if conditions are very favorable for rapid glaciation (e.g., they used an accumulation rate of 1 m/yr for elevations of more than 1000 m above the equilibrium plane).

In their first experiment (reproduced in Fig. 5 by the dotted line) they found a sea-level lowering of 7.5 m in 10,000 yr. In their second experiment (reproduced in Fig. 5 by the dashed line) some feedback between climatic conditions and ice-sheet formation was included by an additional drop in the EPA of 150 m after 2000 yr of integration. In this case a 20-m drop in sea level could be obtained, which is still far from the 50 m that presumably occurred. Therefore, Andrews and Mahaffy concluded that during the same time ice volume must have been

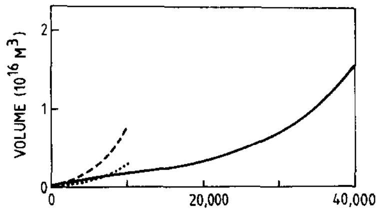

FIG. 5. Growth of the European ice-sheet volume for extreme-cold climatic conditions. Dashed and dotted lines indicate growth of the Laurentide Ice Sheet as computed by Andrews and Mahaffy (1976) in a comparable experiment.

built up in other parts of the world. In this background the following experiment was carried out.

To get an idea of how rapidly glaciation in Europe could take place, an experiment was carried out in which the EPA is prescribed as

$$\text{EPA} = 70° \times (\Psi - 70°) + 25 \times \lambda - 300 \text{ m}. \qquad (10)$$

Here, ψ and λ are latitude and longitude expressed in °N and °E, respectively. Thus, besides the meridional gradient in G, a zonal gradient is now also included. This has been done to mimic higher accumulation rates in the western part of Europe, which presumably prevail during the first stage of a glacial period. Using Eq. (10) corresponds to dropping the EPA by roughly 1000 m with respect to present-day conditions. Such a drop is quite large and may be regarded as a *definite* bound to conditions that create rapid glaciation.

Figure 5 shows the ice volume as a function of integration time (solid line). The dashed and dotted lines show the experiments of Andrews and Mahaffy mentioned above. After 5000 yr, the volume of the European ice sheets is comparable to that of the Laurentide Ice Sheet, but we should remember that the change in climatic conditions in the experiments is not completely similar. After 37,000 yr, the ice volume is 1.3×10^{16} m^3, which is the value given by Flint (1971) as being characteristic for the

Fennoscandian Ice Sheet during a glacial maximum.

After 10,000 yr, the growth of the Laurentide Ice Sheet is accelerating, and it grows much faster than the Fennoscandian one. This probably has to do with differences in lateral (i.e. east–west) ice-mass discharge.

In the beginning, the Fennoscandian Ice Sheet grows slightly faster because of the high elevation of vast regions. After some time, however, a large amount of ice is continuously carried toward the Norwegian Sea, which damps the increase of ice-sheet height. The consequent decrease of the height–mass balance feedback restricts the growth rate of the ice sheet. In the case of the Laurentide Ice Sheet, this mechanism becomes effective in a much later stage.

Figure 6 shows the evolution of the ice sheets in more detail. After 5000 yr, the Alps and the higher part of Scotland are covered with stable ice caps, while in Scandinavia the ice penetrates to lower elevations. We see that after about 30,000 yr the Scottish and Scandinavian ice sheets merge.

Observations have shown that during a glacial maximum the ice extent in Europe is somewhere between those shown in the two lower pictures of Figure 6. According to Figure 5, this corresponds to an ice volume of about 0.7×10^{16} m^3, which is less than the value of 1.3×10^{16} m^3 given by Flint (1971). The volume of the model ice sheet could be made larger by decreasing the value of B, which would result in a larger mean thickness of the sheet. However, the height-to-width ratio appearing in Figure 6 seems reasonable as compared to present-day ice sheets. No sound evidence exists that this ratio was very different for the Fennoscandian Ice Sheet.

DISCUSSION

The present study is no more than a first step in the dynamic modeling of Pleistocene European ice sheets. The results nevertheless give an impression of the "glacial sensitivity" of Europe.

It was found that a 600-m lowering of the equilibrium plane is sufficient to set the sheet height–mass balance feedback in motion. In this case the Fennoscandian Ice

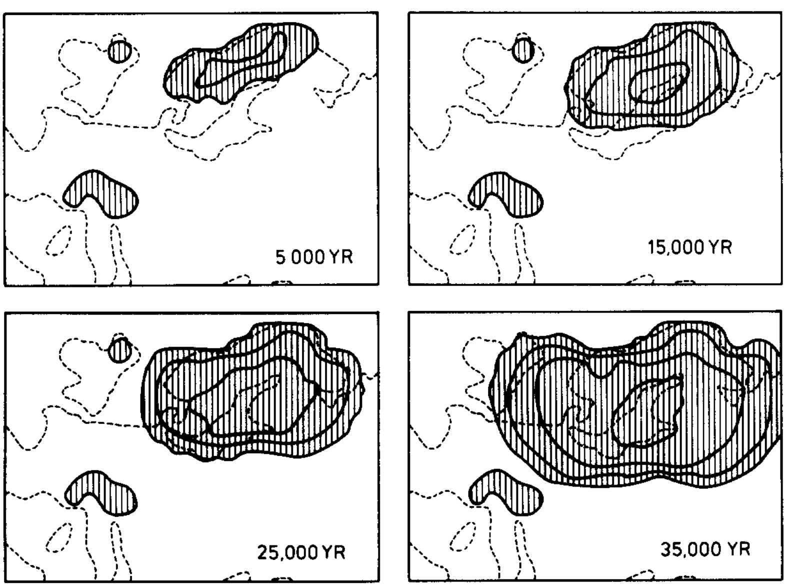

FIG. 6. Areal extent and thickness of the European ice sheets for the experiment shown in Figure 5. Contours of ice thickness (solid lines) are spaced at 1000 m. Integration time is indicated.

Sheet does not reach an equilibrium size within 60,000 yr. The impression may even exist that an equilibrium size does not exist at all and that the sheet would grow to infinity. Experiments with a simplified model were carried out to investigate this point. The simplification consisted of prescribing the ice-sheet profile in the east–west direction, which reduces computational time by a factor of 30. The experiments brought to light that if the north–south slope of the equilibrium plane is positive, an equilibrium solution exists. For small values of this slope, however, the equilibrium size becomes very large and the time needed to reach it becomes very long.

It appeared that the relation between the slope of the equilibrium plane (γ) and the ice-sheet size in the meridional direction (L) can approximately be described by γL = constant, where the constant is of the order of 2000 m if L is measured in meters (γ is dimensionless). Since the slope of the equilibrium plane used in this study was roughly 5×10^{-4}, the corresponding equilibrium size is 4000 km! This means that the model ice sheet would grow until it reaches the Mediterranean.

Observational evidence excludes the possibility that during the Pleistocene the Fennoscandian Ice Sheet ever reached southern Europe. To explain the discrepancy, the following points should be mentioned. First, the parameterization of the mass balance is very crude. In particular, the uncertainty in the meridional slope of the equilibrium plane is very large. If this slope would be twice at large as we used (a possibility that cannot be excluded), the corresponding equilibrium value of L would be about 2000 km. Second, it is unknown whether the Fennoscandian Ice Sheet has ever been close to equilibrium. The large time scale of ice-sheet growth appearing in our experiments suggests that in reality complex interference with insolation variations probably occurred (for a discussion on this point, see Birchfield, 1977).

The experiment on rapid glaciation indicates that during the first stage of a glacial cycle the European ice sheets contribute less to sea-level lowering than the Laurentide Ice Sheet. Together, the model ice sheets may account for a sea-level drop of about 25 m within 10,000 yr, which is still considerably less than the drop inferred from proxy data.

In this study, no attention has been given to the effect of bedrock sinking and the interaction between the ice sheet and the atmospheric circulation. In particular the effect of upslope precipitation could lead to a more rapid growth of the ice sheet. In a forthcoming study, these points will be dealt with.

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