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Continental Ice Sheets and the Planetary Radiation Budget

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The interaction between continental ice sheets and the planetary radiation budget is potentially important in climate-sensitivity studies. A simple ice-sheet model incorporated in an energy-balance climate model provides a tool for studying this interaction in a quantitative way. Experiments in which the ice-sheet model is coupled step by step to the climate model show that ice sheets hardly affect the zonal mean radiation balance because the albedo feedback due to sea ice and snow cover is dominating. The model requires a 5% drop in the solar constant to create ice sheets of ice-age size.

If the feedback between surface elevation and ice-mass balance is included (in a very crude way), the ice-sheet size (L, measured southward from 70°N) becomes much more sensitive to in insolation. For a range of normalized solar constants, roughly from 0.98 to 1.02, two stable solutions exist: $L \simeq 0$ and $L \simeq 2000$ km. This result demonstrates that the response of ice sheets to insolation variations is far from linear. It also stresses the need for explicit modeling of the ice-mass balance of ice sheets, particularly its dependence on surface elevation.

INTRODUCTION

Recently, climate models based on a zonally and vertically averaged energy balance (to be abbreviated by EBM's), first introduced by Budyko (1969) and Sellers (1969), have received considerable interest. In those models, the ice cover–albedo–temperature loop is the mechanism of most interest.

Generally, the southern boundary of the ice cover is identified with some isotherm (Budyko, 1969; North, 1975; Gal-Chen and Scheider, 1976; among others). A slight improvement of this approach is to make the fraction of the surface covered with ice a smoother function of temperature, thus allowing the southern edge of the ice cover to vary with longitude (Oerlemans and Van den Dool, 1978). The next natural step is to include continental ice sheets with simple mechanics. Pollard (1978) carried out a study in this direction.

In this paper an attempt is made to quantify the interaction between continental ice sheets and the large-scale radiation budget of the earth. It is frequently stated that ice sheets modify this budget considerably by changing the surface albedo and lowering the effective radiation temperature. A simple model of continental ice sheets together with an EBM should provide the tool for determining the importance of the interaction. It is not the intention to reproduce any paleoclimatic curve, but merely to demonstrate how the equilibrium climatic state is influenced by the presence of ice sheets (as far as the zonal mean radiation budget is concerned). Although pure equilibrium states will never occur in nature, it is worthwhile to investigate them because they provide some insight into the physics of the large-scale climate.

EBM's are very crude climate models. They compute zonal mean radiation budgets (e.g., for 5° latitude belts) which are coupled through energy transports in the south–north direction. Expressing all components of the energy balance in terms of the zonal mean sea-level temperature, T, the resulting equation may be solved for T, given the distribution of incoming solar radiation. In view of the crude approximations involved, it does not make sense to use a sophisticated ice-sheet model. Instead a very simple model is formulated which is compatible with the simplicity of EBM's.

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All mechanisms investigated in this study have been discussed by other workers, mostly in a qualitative sense. The main goal of this paper is to build up a clear picture of ice sheet–radiation feedback by quantifying the various components and putting them in proper perspective.

A list of symbols if given in the Appendix.

A SIMPLE ICE-SHEET MODEL

The distribution of the net gain of ice mass (denoted by g) constitutes the driving force of an ice sheet. Given this distribution, it is possible to derive some characteristics of the ice sheet without looking in detail at the physics of ice flow, that is, the major large-scale characteristics needed for our purpose. We will approach the problem by looking north–south over the major continents of the Northern Hemisphere and ignoring any mass discharge in the east–west direction. In all experiments the Antarctic Ice Sheet is kept fixed.

Weertman (e.g., 1976) uses a step function for g, thus introducing a discontinuity at the equilibrium point. A continuous function of latitude is preferred, however. We know that both surface temperature and incoming radiation decrease steadily with latitude and it is very likely that this was also the case in colder climates. Concerning snowfall, reliable information on colder climates is very scarce. Recent experiments with general circulation models of the atmosphere, in which an ice-age climate is simulated by using ice-age boundary conditions (Williams *et al.*, 1974; Manabe and Hahn, 1977) indicate that precipitation was less during the last ice age, the difference being rather independent of latitude. This result, together with the fact that turbulent heat exchange and incident solar radiation make up the major energy source for melting of ice (e.g., Paterson, 1969), suggests that the following parameterization of the gain of ice mass is a reasonable approximation:

$$g(x) = g_0 + p(x - \lambda). \quad (1)$$

In Eq. (1), g_0 is some constant, p is the rate of increase of ice-mass gain with x, x is the distance along a meridian, positive to the south and with the origin at the edge of the polar sea (approximately at a latitude of 70°N), and λ is some reference point. Equation (1) states that rates of ice-mass gain shift northward and southward according to the ruling climate regime which fixes λ, and that g varies linearly with latitude in each climate. Note that the coupling of ice-sheet height and mass balance is not included in Eq. (1); this point will be discussed later. The position of the equilibrium point is found by equating $g(x)$ to zero, yielding

$$x_{eq} = \lambda - g_0/p. \quad (2)$$

A few considerations of ice sheets are necessary now. First, the northern edge of a sheet coincides with the southern boundary of the Arctic Ocean ($x = 0$), which acts as an infinite sink for ice. Second, the ice sheet is assumed to reach a maximum elevation at its center. Ice-sheet theory (e.g., Nye, 1959; for a review, see Budd and Radok, 1971) and observational evidence justify this assumption. Physically, it means that the ice behaves as an almost perfect plastic substance (the assumption of perfect plasticity has been employed in particular by Weertman, e.g., 1964). Since ice is forced to flow by horizontal pressure gradients caused by the acceleration of gravity acting on the ice mass itself, the transport through the center is zero. This implies that the net gain of ice north of the center must be compensated by the loss in the Arctic Ocean.

Figure 1 illustrates the picture. It shows an ice sheet in equilibrium with the gain of ice mass as prescribed by Eq. (1). Since no mass transport through the center of the ice sheet occurs, the accumulation and ablation zones on the southern half of the sheet have the same extent. Thus, from this simple geometric exercise we find that the equilibrium point is situated a $x = 0.75\,L$ (L is the diameter of the ice sheet).

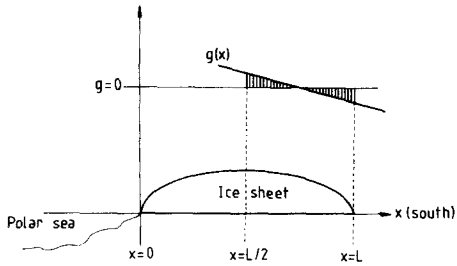

FIG. 1. Illustration of the mass balance of a continental ice sheet in the Northern Hemisphere. See text for explanation.

In order to find the size of an ice cap for a given position of the "climate reference point," λ, the mass balance for its southern half is used. It reads

$$\frac{dV}{dt} = \int_{L/2}^{L} g\,dx = \frac{3}{8}pL^2 + \frac{1}{2}(g_0 - \lambda p)L, \quad (3)$$

where V is the half-volume of the ice sheet. Two equilibrium solutions exist, namely,

$$L = 0,$$
$$L = \frac{4}{3}\frac{p\lambda - g_0}{p}. \quad (4)$$

Before incorporating this ice-sheet model into an EBM, the stability properties of the equilibrium solutions will be discussed.

So far, two potentially important effects have been disregarded. First, no account was made for the diffluence of the meridians (looking in the southward direction). Second, variations in bedrock height may influence the mass balance of the sheet.

The "diffluence effect" could be included by weighting $g(x)$ in Eq. (3) by the cosine of latitude. For example, for an ice sheet extending from 70° to 50°N, the importance of the diffluence effect is approximately given by cos(57.5°)/cos(52.5°) = 0.88. This corresponds to a 6% error in L, which is quite acceptable in view of the uncertainties involved in using Eq. (1). Thus this effect is disregarded for simplicity.

Variations in bedrock height, both spatial and temporal, are very difficult to handle. In this study, no attempt is made to do so. However, bedrock sinking maintaining an isostatic balance does not influence the equilibrium size of the ice sheet. This may be seen as follows. If an ice sheet forms, an isostatic balance is established by sinking of the bedrock. It is easy to show that this sinking is a constant fraction of the ice-sheet height, measured up from the original height of the bedrock. This fraction is determined by the difference in density between the bedrock and ice. Since it is constant, the effect of isostatic sinking is merely an increase in V by the same fraction. From Eq. (3) it is obvious that the equilibrium solutions are unaltered.

ANALYSIS OF INTERNAL STABILITY

In the remainder of this paper, the climate reference point is chosen in such a way that $g_0 = 0$; it thus coincides with the equilibrium point. The second equilibrium solution is now given by $L = 4\lambda/3$.

It is obvious that $L = 0$ represents a stable solution if the equilibrium point is situated in the polar sea ($\lambda < 0$). In this case the second solution is unstable because the whole ice sheet is subject to a net loss of ice. On the other hand, if the equilibrium point is situated on the continent ($\lambda > 0$), $L = 0$ is an unstable solution while the second solution is stable.

A linear stability analysis of the second solution shows that it is stable if $p < 0$, i.e., if the gain of ice mass increases with latitude. This is expected, of course. The relaxation time of the sheet, defined as the reciprocal of the damping coefficient in the linearized equation, is given by

$$T_{\rm rel} = 8\,\beta n L_0^{n-2}/3p. \quad (5)$$

In Eq. (5), n and β relate the half-volume of the ice sheet to its width according to

$$V = \beta L^n. \quad (6)$$

Equation (6) implicitly reflects the shape of the ice sheet. If its height is constant, $n = 1$. A triangular ice sheet, on the other hand, corresponds to $n = 2$. Those cases may be considered as *very extreme*. In between is the perfectly plastic ice sheet with a parabolic profile (height proportional to $L^{1/2}$), in which case $n = 1.5$. It is interesting to investigate to what extent the shape of an ice sheet affects its relaxation time. Again, two "academic" cases are considered. First, in comparing ice sheets of different shape, the volume is kept fixed, so $\beta = VL^{-n}$. Upon substituting this in Eq. (5), $T_{rel} \propto n$. Second, β is kept fixed (the interpretation is somewhat difficult; it involves theory of ice-flow mechanics which will not be discussed here). It now is found that $T_{rel} \propto n(n - 2)$. For large ice sheets perfect plasticity is a good approximation. With the very extreme cases mentioned in mind, it is realistic to assume that n varies between 1.4 and 1.6. Is is easily seen that in this case T_{rel} varies within 15% of its value in the perfect-plasticity case.

Figure 2 displays T_{rel} for the case of perfect plasticity, for two values of the yield stress, τ_0, which determines β. a value of $-5 \cdot 10^{-7}$ yr^{-1} has been used for p; this corresponds to a difference in net gain of ice mass of 0.5 m yr^{-1} over a distance of 1000 km. For large ice sheets, T_{rel} has a typical value of 6000 yr.

An important result of the stability analysis is that the relaxation time decreases with increasing ice-sheet size. This is in contradiction with Weertman's (e.g., 1976) ice-sheet model, which predicts an increasing relaxation time for larger sheets. The reason may be found in the different formulation of the ablation and accumulation regimes. In Weertman's model, the zones of ablation and accumulation are characterized by *constant* ablation and accumulation rates, whereas in this study the net gain of ice mass is prescribed as a linear function of latitude. In this case, therefore, the difference between the net gain in the regions $\frac{1}{2}L < x < \lambda$ and $\lambda < x < L$ increases with increasing ice-sheet size. As a consequence, the flow of ice at the equilibrium point, *scaled with the total volume*, becomes larger, thus increasing the ability of the sheet to restore equilibrium (which corresponds to a smaller T_{rel}). In the case of constant ablation and accumulation rates, however, the flow of ice at the equilibrium point, scaled with V, decreases. This results in increasing T_{rel} for increasing ice-sheet size.

INCORPORATION OF ICE SHEETS IN A SIMPLE CLIMATE MODEL

The continental ice sheets can now be included in a simple climate model. Before doing so, the findings are summarized as follows:

(i) If the net gain of ice mass is a linear function of latitude, the ice-sheet size

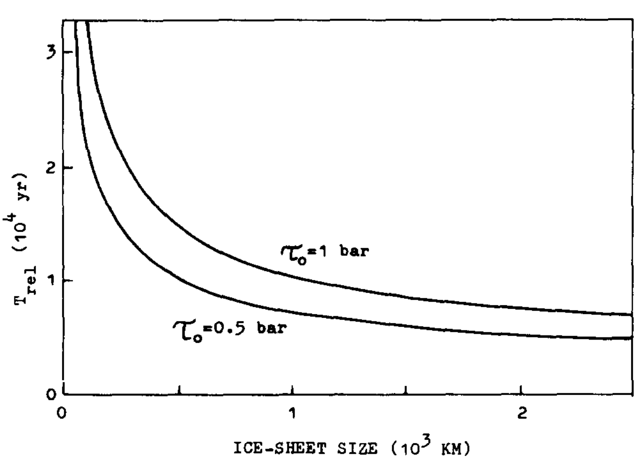

FIG. 2. Relaxation time as a function of ice-sheet size for a perfectly plastic sheet. Results are given for two values of the yield stress, τ_0.

is given by $L = 4\lambda/3$, where λ is the position of the equilibrium point. Thus the value of L does not depend on the absolute ablation and accumulation rates.

(ii) The relaxation time of an ice sheet depends weakly on its shape. It decreases with increasing ice-sheet size.

Result (i) is very convenient. It implies that the equilibrium ice-sheet size can be computed without knowledge of p. It is only necessary to know the position of the equilibrium point ($g = 0$). This result might seem suspicious, but one should realize that it follows directly from the assumption of *linearly* increasing net gain of ice mass. For reasons discussed earlier, this assumption seems to be better than using constant accumulation and ablation rates on both sides of the equilibrium point. Anyway, for the present purpose we may have confidence in working with $L = 4\lambda/3$ in a simple climate model.

The coupling of the ice-sheet model with an EBM is carried out step by step. In this way one can hope to discover how various feedback mechanisms effect the equilibrium state of the climate system. The EBM to be used is a modified version of the model described by Oerlemans and Van den Dool (1978). A brief description of the most relevant aspects only is given here.

The model is based on a zonal, vertical, and annual mean energy balance, which reads

$$Q(x)[1 - \alpha(x)] + D\nabla^2 T(x) = I(x). \quad (7)$$

In Eq. (7), Q is the incoming solar radiation at the "top" of the atmosphere, α the planetary albedo, T the temperature at sea level, D the diffusivity for total energy, and I the net outgoing long-wave radiation. The parameterization of albedo and outgoing radiation takes into account the effects of sea/land distribution, surface elevation, fraction of the ocean covered with sea ice, fraction of land covered with ice, fraction of the surface covered with snow, and the distribution of clouds, prescribed as a function of latitude but fixed with regard to temperature. Clouds appear to have a profound influence on the stability properties of the model (H. M. Van den Dool, personal communication, 1979).

Data on which the most important parameterizations are based are the radiation-budget measurements compiled by Ellis and VonderHaar (1976) and the cloud climatology presented by Berliand and Strokina (1975). Those data seem to form the most complete and reliable input available at present.

Figure 3 shows the performance of the model for present conditions with regard to sea-level temperature. Observed values are from Oort and Rasmusson (1971). The correspondence is very good, except at latitudes higher than 70°N. This may be attributed to the use of a constant diffusivity parameter (D). The computed temperature distribution at high latitudes may be adjusted to the observed one by decreasing D in that region. This point will be discussed later. In the first instance, D is kept constant at 0.7 W m^{-2} °C^{-1}.

The main point of interest concerns the ice cover over land. In the original model it is parameterized by a direct relationship between surface temperature and the fraction of land covered with ice (denoted by δ). More specifically, $\delta = 1$ for $T < -20$°C and $\delta = 0$ for $T > 0$°C, with a gradual transition within the -20° to 0°C range. This repre-

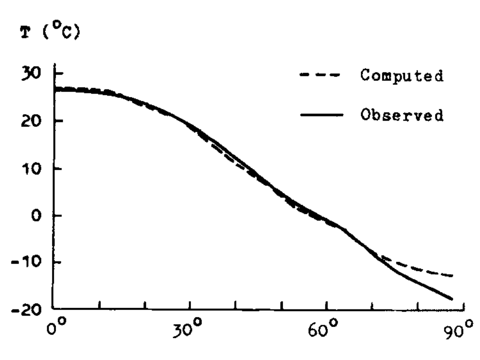

FIG. 3. A comparison between observed zonal mean sea-level temperatures and those computed with the climate model used in this study.

sentation of an ice sheet is now dropped. Instead, $L = 4\lambda/3$ is used and λ, the position of the equilibrium point, is identified *with the latitude where the temperature has a prescribed value*. So the ice-sheet model and the EBM are coupled by the assumption that the equilibrium point moves northward and southward with an isotherm. It is not clear whether sea-level temperature or surface temperature is the most suitable variable to attach λ to. In the EBM they are strongly coupled through a constant lapse rate. Consequently, the choice is not very critical with regard to the experiments in mind. To be consistent with present-day conditions, a sea-level temperature of −8°C was chosen to give the position of the equilibrium point. Corresponding surface temperatures range from −8° to −12°C, depending on latitude.

In addition, Figure 4 shows how sea ice and snow cover are parameterized in terms of temperature. The clear-sky albedos for snow- and ice-covered regions are taken to be 0.61 and 0.56, respectively.

The essential difference between the ice-line concept, introduced by Budyko (1969), and an ice sheet driven by its own weight becomes clear if the southern edge of the ice mass is examined. In the first case, this edge follows an isotherm. In the second case, however, the temperature at the edge depends on the ice-sheet size. A larger ice sheet is associated with a higher temperature at $x = L$. This enhances the albedo−temperature feedback and thus the sensitivity of the earth's climate. Since the tongue of the ice sheet extends into a region with relatively high insolation, this mechanism is potentially important.

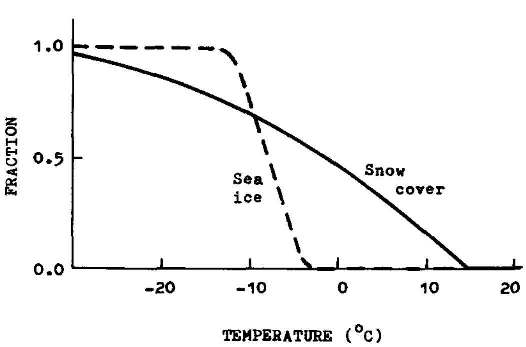

FIG. 4. Parameterization of sea ice and snow cover employed in the climate model. In the case of sea ice, fraction refers to the oceanic part along a latitude circle; in the case of snow cover, to land and frozen ocean.

Another feedback mechanism involving ice sheets is the lowering of the effective radiation temperature. Since the surface temperature is considerably lower if large ice sheets are present, the amount of energy radiated to space will be smaller. This counteracts the albedo effect.

Next the various feedback mechanisms are explored by incorporating them step by step. Since they interact, it is not possible to give a definite measure of their relative importance.

INTERACTION OF ICE SHEETS AND RADIATION BALANCE

Sensitivity experiments are now discussed in which the solar constant S is varied. All experiments refer to climate sensitivity with respect to uniform changes in the incident radiation at the top of the atmosphere. This seems to be the most transparent way of investigating and comparing various feedback mechanisms.

The first experiment involved computing the ice-sheet size as a function of S without any temperature−albedo coupling. At each latitude α was kept fixed at its present value. The result is shown in Figure 5. A drop in S of more than 2% is required to create an ice sheet. Even with a drop of 5%, its size would not be larger than 1000 km and the global mean temperature would be 5°C lower than today.

The next step is to include feedback between snow cover/sea ice and albedo. As shown in Figure 5, this strongly enhances the sensitivity of the climate. Continental ice sheets form more rapidly now, but a 5% drop in S is still needed to obtain ice sheets with an equilibrium size of 2000 km. The sensitivity of the model in terms of the sea-level temperature is about 1.5°C per 1%

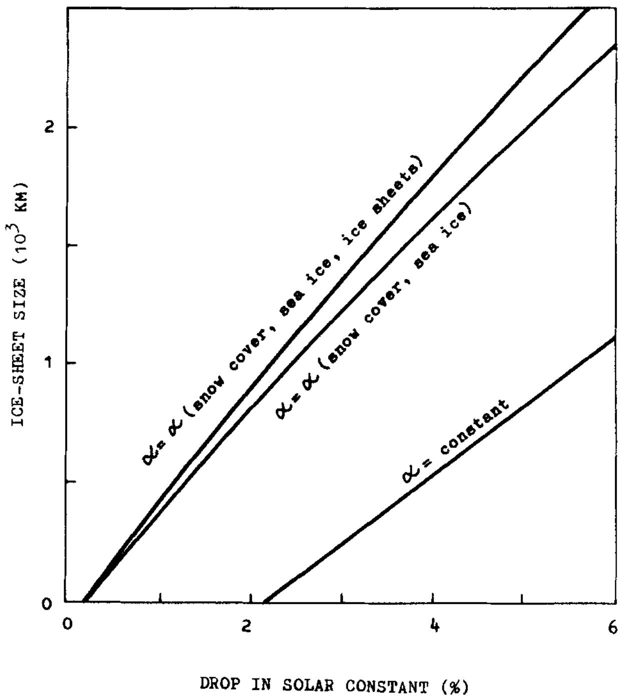

FIG. 5. Ice-sheet size as a function of the drop in S. The kind of albedo feedback used is indicated in the figure.

change in S. This value applies to all latitudes.

The third curve in Figure 5 displays the results if the effect of ice sheets on the planetary albedo is also included in the case where $\delta = 1$, i.e., in zonal direction continental ice sheets run from coast to coast. This value of δ was used to find an upper bound for this feedback mechanism. Later experiments used $\delta = 0.7$ which is more in agreement with the results of CLIMAP (1976). Apparently, the effect is small. This result is a direct consequence of the existence of snow cover if temperatures are so low that an ice sheet forms. The planetary albedo does not 'feel' whether ice is present under the snow cover or not. This demonstrates the fact the various feedback mechanisms cannot be separated; if one starts with the incorporation of ice sheets and includes snow cover in a later stage, the reverse will be found.

So far, the *height* of continental ice sheets has not been taken into account. Two effects may occur, however. The lower surface temperature will cause (i) a decrease in outgoing radiation and (ii) an increase in albedo because snow cover will prevail during a larger part of the year. To investigate (i) and (ii) it was assumed that the ice sheet behaves as a perfect-plastic substance. Its profile is then given by

$$h(x) = \sigma\left[\tfrac{1}{2}L - \left|x - \tfrac{1}{2}L\right|\right]^{1/2}. \qquad (8)$$

The constant σ was chosen in such a way that an ice sheet with $L = 2000$ km has a maximum height of 3500 m. Incorporation of the ice-sheet profile was established by an effective lowering of the zonal mean surface temperature, in accordance with the fraction of ice-covered land along a latitude circle ($\delta = 0.7$ now). The results are shown in Table 1. Values of L are given for three experiments and for three values of the decrease in S. The first row shows results from the experiment described in the last paragraph, neglecting the influence of the height of the sheets. The second row shows what happens if effect (ii) is included: the ice-sheet size increases slightly.

TABLE 1. ICE-SHEET SIZE (KM) FOR SOME VALUES OF THE SOLAR CONSTANT ACCORDING TO THREE MODEL EXPERIMENTS[a]

	Drop in solar constant		
	1%	2%	4%
Reference experiment	453	880	1721
Including (ii)	484	968	1785
Including (i) and (ii)	418	839	1674

[a] In the reference experiment, the height of the ice sheet is not taken into account. In the second experiment ("Including (ii)"), lower surface temperatures due to the ice-sheet height are used in the computation of the albedo. In the third experiment ("Including (i) and (ii)"), both albedo and outgoing radiation are computed with those lower surface temperatures. (i) and (ii) refer to the further explanation given in the text.

In the third row, results are given of a model run in which effect (i) is also taken into account. As expected, it counteracts the albedo effect. Therefore, lower surface temperatures due to the presence of ice sheets have a very small influence on the zonal mean radiation budget.

To complete this set of experiments, a run was carried out with adjusted values of D in the polar region. More precisely, D was gradually decreased from 0.7 at 70°N to 0.3 at the pole. This simulates the present polar climate very well. Varying S now brought to light that this modification of the energy transport hardly effects the sensitivity of the climate. The temperature gradient north of 70°N is slightly enhanced but the continental ice sheets are not much affected.

The insensitivity of the global climate to ice-sheet feedback as revealed by the foregoing experiments is partly due to the *large capacity* of the model climate system to *redistribute energy*. The sensitivity parameter $\partial T/\partial S$ hardly depends on latitude. In addition, it is very important to realize that the interaction between ice sheets and the *zonal mean* radiation budget has been investigated. This corresponds to an infinitely large diffusivity for energy in the west–east direction.

DISCUSSION OF ZONAL ASYMMETRY

Since in reality the redistribution of heat along a latitude circle takes some time, the difference in radiation budget over oceanic and continental regions may lead to a substantial difference in temperature. At least in the way they have been incorporated, continental ice sheets respond to continental temperatures and consequently their size depends on this temperature difference.

From a simple radiative equilibrium model of the atmosphere (e.g., Houghton, 1977) it follows that the time constant of the troposphere for radiative processes is on the order of 5 days. Over the oceans, the time constant of the vertical column used in formulating the energy balance (Eq. (7)) will be considerably larger, but over the continents 5 days will be an appropriate value (although extensive snow and ice melt may obscure the picture). On the other hand, with a mean zonal wind speed of 10 m/sec and a characteristic west–east length scale of 4000 km (i.e., from midocean to midcontinent), a time constant of 4.6 days is found for advective processes. It thus follows that radiative and advective processes are approximately of equal importance, or, in other words, zonal asymmetry created in the temperature field is not immediately destroyed by zonal energy transport. It may well be that the difference between oceanic and continental temperature changes with S, and therefore establishes additional forcing of ice sheets.

Hartmann and Short (1979) investigated zonally asymmetric forcing in a schematic way and concluded that it might be of considerable importance. Zonal asymmetry is strongly coupled to quasi-stationary planetary waves in the atmosphere. It is obvious that large ice sheets contribute strongly to the forcing of those waves. In particular, the orographic effect must have the same order of magnitude as the major Northern Hemisphere mountain ranges have at present. It is thus tempting to postulate that the larger zonal asymmetry during the last ice age (as revealed by CLIMAP, 1976) was mainly due to the presence of large ice sheets. This is a well-known hypothesis. Obviously, a proper investigation of this feedback requires a separate treatment of the radiation budgets over ocean and continent and, to compute the ocean–continent temperature difference, the use of a diffusivity of energy in the west–east direction. In a forthcoming paper, extensive model experiments in this direction will be discussed.

OTHER FEEDBACK EFFECTS

It is not the aim of this paper to list all the feedback loops involved in the interaction between continental ice sheets and climate. In the present context a few additional mechanisms should be mentioned, however.

First, a higher elevation of the surface will not only modify the radiation balance through lower surface temperatures, but will also reduce the effective optical depth of the atmosphere. Although in this case both incoming shortwave and outgoing longwave radiation increase and thus counteract in the radiation balance, the net effect may be important. To study this point a more sophisticated radiation model is needed.

Second, the mass balance of the ice-sheet model used in this study was prescribed as a function of latitude only. This suppresses the sensitivity of the ice sheet. In reality the mass balance increases with height which constitutes a positive feedback loop. The extreme sensitivity of Weertman's (1976) model to changes in incoming radiation is due to this feedback loop (see also Weertman, 1961).

To reveal how dramatically the coupling between height and mass budget may effect the ice-sheet size, this coupling is simulated in the present model. Instead of using the sea-level temperature for determining the position of the equilibrium point, the surface temperature in the continental region (T_s) is now used. The latter is related to the former by $T_s = T - \Gamma h$, where h is the surface elevation (including the ice sheet!) and Γ is the lapse rate in the lower troposphere

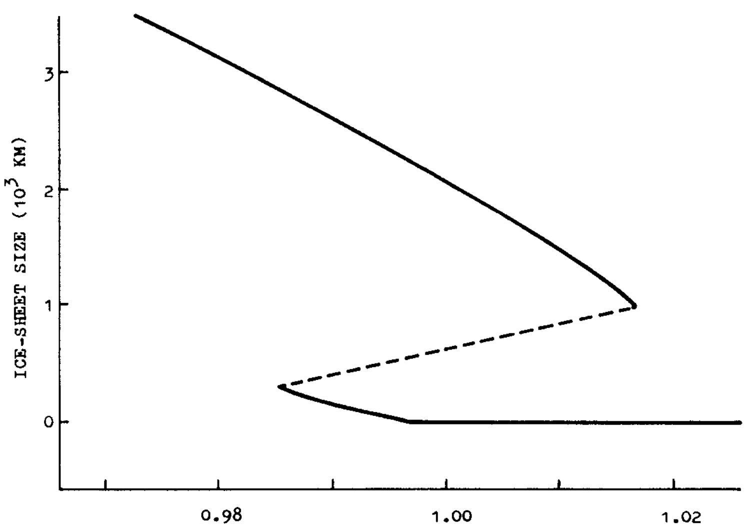

FIG. 6. Solution diagram for the case that the feeback between ice-sheet height and mass balance is incorporated in a crude way. Due to this feedback, a range of solar constants occurs where two stable solutions exist (solid line). In between is an unstable solution (dashed line) which separates the regions of attraction of the stable solutions. It depends on the initial conditions to which stable solution the ice sheet converges.

(6°C/km in this experiment). With this parameterization of the latitude of the equilibrium point, L (ice-sheet size) was computed as a function of S (normalized solar constant), starting with various initial conditions. In the first runs the equilibrium point was set at the latitude where $T_s = -10$°C. Many other runs were then carried out to see whether the results changed substantially when this value of T_s was changed a few degrees, and when the other model parameters were slightly modified. It appeared that the characteristics of the solution diagram (L versus S) do not depend strongly on the precise values of the model parameters. Therefore one representative solution diagram is discussed.

Figure 6 shows such a diagram. The most striking feature is the occurrence of hysteresis, i.e., for a certain range of solar constants two stable solutions exist, with an unstable solution in between (dashed line). It depends on the initial condition to which stable equilibrium solution the ice sheet will converge. Although the present simulation of the feedback between ice-sheet height and mass balance is admittedly very crude, it demonstrates the potential importance of this mechanism. Figure 6 implies that the response of ice sheets to radiation variations may be strongly nonlinear, as has in fact been found by Weertman (1976), Birchfield (1977), and Pollard (1978).

A picture similar to Figure 6 applies to the global mean temperature. For $S \simeq 1.00$, the difference between the two stable solutions in terms of global sea-level temperature is 3° to 4°C. So *in this case,* continental ice sheets have a significant influence on the global radiation budget.

CONCLUSIONS

In this study a simple ice-sheet model has been incorporated in an energy-balance climate model, thus providing an opportunity to quantify the feedback between ice sheets and the radiation budget. Two major conclusions can be drawn:

The influence of continental ice sheets on the zonal mean radiation budget is rather small, that is, if snow cover is already taken into account. Sea ice and snow cover mainly determine the strength of the albedo feedback (Fig. 5). Including the feedback between ice-sheet height and ice-mass budget substantially increases climate sensitivity. In the present study this feedback has been represented in a very crude manner, but the results illustrate once again that explicit modeling of the ice-mass budget is necessary in future studies.

APPENDIX: SYMBOLS

D	diffusivity for energy
g	ice-mass balance
g_0	constant
h	height of ice sheet
I	outgoing longwave radiation
L	diameter of ice sheet along x axis
n	constant
p	rate of change of ice-mass balance with x
Q	insolation at top of atmosphere
S	normalized solar constant
t	time
T	sea-level temperature
T_s	surface temperature
T_{rel}	relaxation time of ice sheet
V	half-volume of ice sheet
x	distance along a meridian, from 70°N latitude southwards
x_{eq}	position of equilibrium point
α	planetary albedo
β	constant
Γ	lapse rate in the lower troposphere
δ	fraction of land along a latitude circle covered with ice
λ	position of climate reference point
σ	constant
τ_0	yield stress of ice

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