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# The logic of enactment

Lambèr Royakkers\*

Frank Dignum†

## Abstract

## 1 Introduction

In legal discourse reference is made to enacted norms and applicable norms. Not all norms that are enacted are applicable or are only applicable in certain circumstances. In this scope, we are concerned with two concepts of validity: membership and applicability. Both of them play a central role in law and in legal theories (cf. Bulygin, 1982). A norm can be said to be valid in the sense that it belongs to or is a member of a legal system: membership. A norm is often also said to be valid in the sense that it is obligatory or has a ‘binding force’: applicability.

In most legal support/expert systems only the applicable norms are considered. It is impossible in these systems to talk about the rules that select applicable norms explicitly. Neither is it possible to reason about enacted norms that are not applicable. In this paper we give a theory to describe enacted norms and applicable norms separately. We show how we can reason consistently with enacted norms without requiring all enacted norms to be *normative consistent*. The ‘classical’ notion of normative inconsistency is that two authorities promulgated (enacted) two contradictory or conflicting norms, which is a frequently phenomenon, at least in certain areas like law (see Alchourrón and Bulygin, 1981). The conflict arises when the norms become members of the same normative system, not if they belong to different systems. Such a system loses its meaning in a logical sense in the case of inconsistency: everything can be deduced and, in particular, all obligations, permissions, etc., are deducible (*ex falso sequitur quodlibet*). There are two specific types of normative conflicts (cf. Lindahl, 1992):

- *disaffirmation conflicts*: these conflicts describe the same behaviour, but the deontic modalities contradict. For example, *it is forbidden to turn left* and *it is not forbidden (permitted) to turn left*.
- *compliance conflicts*: these conflicts occur when the actions which are obliged by different norms are incompatible. For example: *it is obligated to turn left* and *it is obligated to turn right*. Thus, these conflicts describe the same deontic modalities, but the actions conflict (are incompatible).

The normative conflicts are caused, for instance, by the dynamics of the legal system (by the enactment of new norms), by the uncertainty concerning the content of the legal sources (regulations can be vague or ambiguous), etc. (cf. Sartor, 1992).

One way to deal with these type of conflicts is to add sets of authorities enacting the norms to standard deontic logic. Here we use the term authority in an abstract way. They can be seen as real authorities (e.g. government and city council), but also as source of norms as in the Penal Code, Traffic Regulations, etc. We treat enactment as a variant of the belief theory. However,

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\*Eindhoven University of Technology, Dept. of Technology Management, P.O.box 513, 5600 MB Eindhoven, The Netherlands, tel.+31-40-2474693, email: L.M.M.Royakkers@tm.tue.nl

†Eindhoven University of Technology, Dept. of Mathematics and Computer Science, P.O.box 513, 5600 MB Eindhoven, The Netherlands, tel.+31-40-2473705, email: dignum@win.tue.nl

in this theory we cannot adequately express normative inconsistencies. Therefore, we propose an extension of these theories, based on the theory of ‘local reasoning’ (Fagin and Halpern, 1988). The basic idea behind this treatment is that authorities may enact several (inconsistent) norms, depending on the frame of reference. In a particular situation, norms may not be applicable, because the applicability depends on the situation, on the relative importance of the norms, etc. For example, according to article 15.1 of the Dutch Traffic Regulation vehicles give right of way to vehicles from the right, and according to article 15.2 bicycles must give right of way to cars. These two norms can conflict, therefore they cannot both be applicable in any situation. We make sets of applicable norms for each situation. E.g. in the situation that a car and a bicycle are on a junction and that the bicycle is coming from the right, we choose a set of norms with article 15.2 and other applicable norms (e.g. concerning the maximum speed limit) and not a set of norms consisting article 15.1, whereas in a situation of two cars we do not choose a set of norms consisting article 15.2. We can view the enacted norms as a society of possible applicable norms, each with its own set (or *cluster*) of applicable norms. which may contradict each other.

The organisation of this article is as follows: in section 2 we briefly discuss standard deontic logic and the theory of Bailhache. The logic of enactment, based on the logic of knowledge and belief, is presented in section 3. A problem that arises from this theory is the problem of inconsistent enacted norms, which is discussed in section 4. Section 5 discusses the theory of local reasoning for enactment. In the last section, we give some conclusions and suggestions for further research.

## 2 Deontic logic

In this article we will formalise the enactment of norms. To describe the norms we use deontic logic. Deontic logic is a branch of philosophical logic concerning reasoning about norms. It is the logic of obligations, prohibitions and permissions. As such, it is relevant for the foundations of ethics and law. Deontic logic has been used to analyse the structure of normative law and normative reasoning in law<sup>1</sup> and forms the basis of several legal expert systems (e.g. Den Haan, 1996). Therefore, we start with a brief explanation of deontic logic.

In deontic logic, three deontic operators are used: ‘*O*’ (obligatory), ‘*F*’ (forbidden) and ‘*P*’ (permitted). By connecting propositions to these operators as arguments, well-formed formulas of the system originate from which, by interpretation of the propositions, normative judgements can be formed. E.g.,  $O(p)$  means ‘it is obligatory that  $p$ ’. The deontic operators can be defined in terms of one another. If we take ‘*O*’ as a primitive, then the other operators can be defined as follows:  $F(p) =_{def} O(\neg p)$  and  $P(p) =_{def} \neg O(\neg p)$ . Thus, ‘it is forbidden that  $p$ ’ is defined as ‘it is obligatory that not- $p$ ’, and ‘it is permitted that  $p$ ’ is defined as ‘it is not obligatory that not- $p$ ’.

In this article we use the standard deontic logic, a modal (Kripke-style) version of the now so-called ‘Old System’ of Von Wright (1951), we mean the system  $D^*$ <sup>2</sup> based on propositional logic and axiomatised by the following axiom schemata:<sup>3</sup>

$$\begin{array}{ll} (OC) & (O(p) \wedge O(q)) \rightarrow O(p \wedge q) \\ (ON) & O(p \vee \neg p) \\ (OD) & \neg O(p \wedge \neg p) \end{array}$$

together with the rule of inference:

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<sup>1</sup>However, as so many subjects in philosophical logic and philosophy in general, the subject was also picked up by computer scientists and AI (artificial intelligence) researchers. Deontic logic proved to be relevant as well for such prosaic matters as authorisation mechanisms, decision support systems, database security rules, fault-tolerant software and database integrity constraints; thus, outside the area of legal analysis and legal automation. A survey of applications can be found in Meyer and Wieringa (1991).

<sup>2</sup>System  $D^*$  is the smallest normal  $KD$ -system of modal logic (cf. Chellas, 1980).

<sup>3</sup>Axiom (ON) was rejected by Von Wright (1951, p. 11), since he developed the *principle of deontic contingency*: ‘A tautologous act is not necessarily obligatory, and a contradictory act is not necessarily forbidden’. We have to commit ourselves to this axiom, since otherwise we cannot view deontic logic as a branch of Kripke-style normal modal logic.

$$(ROM) \quad \frac{p \rightarrow q}{O(p) \rightarrow O(q)}$$

The semantics of this system can be given using the following Kripke model structure  $M = (W, \mathcal{R}, V)$  consisting of three elements: the set of possible worlds  $W = \{w_1, w_2, \dots\}$ ; the accessibility function  $R \in \mathcal{R}$ , which takes a world and returns a subset of  $W$ :  $R : W \rightarrow 2^W$  and a valuation function  $V$ , which assigns the values ‘true’ or ‘false’ to a proposition at a world in  $W$ . The intuition behind the function  $R$  is that it yields the *deontically ideal worlds* relative to a given world. The truth conditions for  $O$  and  $P$  can now be defined as follows, where  $M, w \models \theta$  is read as ‘ $\theta$  is true in world  $w$  of structure  $M$ ’:

$$\begin{array}{lll} M, w \models O(p) & \text{iff} & R(w) \subseteq \llbracket p \rrbracket \\ M, w \models P(p) & \text{iff} & R(w) \cap \llbracket p \rrbracket \neq \emptyset \\ M, w \models \neg \theta & \text{iff} & M, w \not\models \theta \\ M, w \models \theta_1 \wedge \theta_2 & \text{iff} & M, w \models \theta_1 \text{ and } M, w \models \theta_2, \end{array}$$

with the function  $\llbracket \ ] \in L \rightarrow 2^W$  and  $L$  the set of well-formed formulas of the propositional calculus.<sup>4</sup> Thus,  $O(p)$  holds in  $w$  if and only if  $p$  is true in all ideal worlds with respect to  $w$ , and  $P(p)$  holds in  $w$  if and only if there is at least one ideal world with respect to  $w$  in which  $p$  is true. The constraint

$$R(w) \neq \emptyset \text{ for all } w \in W$$

will be added to validate the schema  $(OD)$ . At first glance, this axiom does not seem to be controversial, since it merely denies the existence of impossible obligations. However, with the help of this axiom we can derive the formula  $\neg(O(p) \wedge O(\neg p))$ , which is controversial nowadays (see, eg. Alchourrón; 1969, Prakken, 1996; Meyer, Dignum and Wieringa, 1996). The main objection to this formula is that it states that there is no conflict of duties, which is clearly not in line with situations in daily life.

In standard deontic logic there is no explicit indication to the authorities that enacted the norms (they are God given). Bailhache (1981, 1991) was the first one, who added sets of authorities to standard deontic logic to obtain a deontic coherent system.<sup>5</sup> To add authorities to system  $SDL$ , Bailhache modified the norms by sets of authorities. Sets of authorities are needed to determine the consequences of obligations, enacted by a combination of individual authorities. This leads to an extension of standard deontic logic, where the norms are enacted by sets of authorities. Let  $A$  be a subset of the set  $NA$  of normative authorities, then  $O^A(p)$  will be read as ‘ $A$  makes it obligatory that  $p$ ’.

Deontic (or normative) coherence makes it necessary that if sets of authorities enacted that  $p$  and  $q$  are obligatory, then every set of authorities has to respect this and *should permit* that  $p \wedge q$  for that set of individuals. The question arises how we have to interpret the words ‘should permit’. It would be very strange to interpret this as ‘enacted the permission’, since this is counter-intuitive with the fact that there is a normative agreement between all the sets of authorities. Consider, for example, the formula

$$O^A(p) \wedge P^B(\neg p) \tag{1}$$

and  $A \not\subseteq B$ . This formula is not contradictory in his system. However, if we interpret  $P^B(\neg p)$  as ‘ $B$  enacted that it is permitted that  $\neg p$ ’, then formula (1) is counter-intuitive for a deontic coherent system. It expresses that  $A$  enacted that  $O(p)$  and  $B$  enacted that  $P(\neg p)$ . Now, there is no normative agreement between these two sets of authorities, since the enacted norms are conflicting in standard deontic logic:

$$O(p) \wedge P(\neg p) \equiv \neg P(\neg p) \wedge P(\neg p). \tag{2}$$

<sup>4</sup> $\llbracket p \rrbracket = \{w \mid V(w, p) = \text{true}\}$ . It is easy to see that the following properties hold:  $\llbracket p \vee q \rrbracket = \llbracket p \rrbracket \cup \llbracket q \rrbracket$ ,  $\llbracket p \wedge q \rrbracket = \llbracket p \rrbracket \cap \llbracket q \rrbracket$  and  $\llbracket \neg p \rrbracket = \overline{\llbracket p \rrbracket}$ .

<sup>5</sup>I.e., a normative agreement between all sets of authorities. This is accomplished by avoiding any conflict between the obligations enacted by the sets of authorities. It is necessary and sufficient for each set of authorities not to forbid - in other words to permit - what a set of authorities makes obligatory.

We have to interpret ‘should permit’ as ‘did not enact that it is forbidden’, which also follows from the fact that  $P^A(p)$  is the dual of  $O^A(p)$ . Now the formula (1) is not counter-intuitive. It only expresses that  $A$  enacted  $O(p)$  and that  $B$  did not enact  $F(\neg p)$ , i.e.,  $O(p)$ . Thus,  $P^A(p)$  has to be read as ‘ $A$  did not enact that it is forbidden that  $p$ ’.

At first glance, this theory based on Bailhache (1981, 1991), seems a good approach for a coherent deontic system, i.e., a normative agreement between all the sets of authorities. However, this approach has two serious drawbacks.<sup>6</sup>

1. We cannot express the explicit permission;
2. We cannot express that a set of authorities does not enact an obligation or a prohibition.

To overcome these drawbacks, we choose for another approach. Instead of  $A$  modifying  $O(p)$ , we choose to treat  $A$  as a modal operator. Writing  $N_A : O(p)$  instead of  $\neg O^A(p)$ . Now we have two places to write a negation:  $\neg N_A : \theta$  and  $N_A : \neg\theta$ . The first means ‘ $A$  did not enact  $\theta$ ’ and the second ‘ $A$  enacted  $\neg\theta$ , with  $\theta$  a formula (norm) in standard deontic logic. The language of this system consists of assertions concerning the norms and the enactment of norms by sets of authorities. For a formal approach, we refer to Royakkers (1996, chapter 5). This seemingly small change has large consequences. The system becomes more powerful, because we can express

- that a set  $A$  of authorities does not enact a norm, for example,  $\neg(N_A : O(p))$ ;
- that a set  $A$  of authorities enacted a combination of norms, for example,  $N_A : (O(p) \vee F(q))$ ;
- normative conflicts, for example:  $N_A : O(p) \wedge N_B : F(p)$ .

Thus, this new approach acquires new meanings, not expressible in the theory of Bailhache.

### 3 The logic of enactment

As we already mentioned, we treated the operator  $N_A$  in  $N_A : \theta$ , with  $\theta$  a norm, as a modal operator, just as in the systems of *knowledge* and *belief*. We choose for the belief theory, since belief and enactment correspond. For instance, just as a belief is not necessarily true, an enacted norm is not necessarily applicable.

In this section we briefly review possible-worlds semantics for enactment, corresponding with the semantics for belief and knowledge (cf. Chellas, 1980; Halpern and Moses, 1985). The intuitive idea behind the possible-worlds model is that, besides the true state of affairs, there are a number of other possible states of affairs, or possible worlds.

By augmenting the language with modal operators  $L_1, \dots, L_n$ , we can represent belief. A formula  $L_i\phi$  is read as ‘agent  $i$  believes  $\phi$ ’. The notion of belief can be completely characterized by the following *sound* and *complete* axiom system, traditionally called *weak S5* or *KD45* (Chellas, 1980):

$$\text{All tautologies of the propositional calculus.} \tag{A1}$$

$$L_i\phi_1 \wedge L_i(\phi_1 \rightarrow \phi_2) \rightarrow L_i\phi_2 \text{ for } i = 1, 2, \dots, n \tag{A2}$$

$$\neg L_i(\textit{false}) \tag{A3}$$

$$L_i\phi \rightarrow L_iL_i\phi \tag{A4}$$

$$\neg L_i\phi \rightarrow L_i\neg L_i\phi \tag{A5}$$

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<sup>6</sup> Another drawback - although, this drawback is inherent to the developed theory - is that we cannot express normative conflicts, i.e., inconsistencies between enacted norms. Further, the theory of Bailhache is not able to deal with combinations of atomic obligations, e.g.,  $O(p) \vee O(q)$ . The sets of authorities can only enact an atomic obligation ( $O(p)$ ).

$$\frac{\phi_1 \rightarrow \phi_2, \phi_1}{\phi_2} \text{ (modus ponens)} \quad (\text{R1})$$

$$\frac{\phi}{N_{A_i}:\phi} \text{ (} i = 1, 2, \dots, n \text{) (necessitation)} \quad (\text{R2})$$

The first axiom (A1) and rule (R1) are holdovers from propositional calculus. The second axiom says that belief is closed under implication. Note that (A2) is equivalent to  $(L_i\phi_1 \rightarrow \phi_2) \rightarrow (L_i\phi_1 \rightarrow L_i\phi_2)$ , which is sometimes given as an alternative axiom. (A3) says that an agent cannot believe in falsehood. The axioms of introspection, (A4) and (A5), say that each agent has complete knowledge about his sets of belief: ‘An agent believes that he believes something’ (A4) and ‘an agent believes he does not believe something’ (A5). The rule (R2) states that an agent believes every tautology.<sup>7</sup>

The notion of enactment is a restricted version of the system *S5* or *KD45* (Chellas, 1980), in the sense that we are not dealing with nested enactment. A formula  $N_{A_i} : (N_{A_j} : \theta)$ , reading that ‘ $A_i$  enacted that  $A_j$  enacted  $\theta$ ’, is meaningless. Thus, we restrict the language, so that no  $N$  appears within the scope of another. Thus, if  $\theta$  is a deontic formula, then  $N_A : \theta$  is also a formula, with  $A$  a set of normative authorities. The consequence of this is that the axioms (A4) and (A5) are inappropriate in our system. On the other hand, we have to add a rule to the system to express the relation between the sets of authorities. If a set of authorities enacted a norm, then every superset of authorities of that set also enacted that norm. In stead of  $n$  agents we now consider  $2^n - 1$  sets of authorities, with  $NA = \{a_1, \dots, a_n\}$  as the set of normative authorities. We now present an axiomatic system **Ent** for enactment, based on the systems of knowledge and belief, with respect to a set  $NA$  of normative authorities.

$$\text{All tautologies of the propositional calculus.} \quad (\text{A1})$$

$$N_{A_i} : \theta_1 \wedge N_{A_i} : (\theta_1 \rightarrow \theta_2) \rightarrow N_{A_i} : \theta_2 \text{ for } i = 1, 2, \dots, 2^n - 1 \quad (\text{A2})$$

$$\neg N_{A_i} : \theta \quad (\text{A3})$$

$$\frac{\Theta_1 \rightarrow \Theta_2, \Theta_1}{\Theta_2} \quad (\text{R1})$$

$$\frac{\theta}{N_{A_i}:\theta} \quad (\text{R2})$$

$$\frac{A_i \subseteq A_j}{N_{A_i}:\theta \rightarrow N_{A_j}:\theta} \quad (\text{R3})$$

Halpern and Moses (1985) give as semantics a Kripke structure  $(W, V, \mathcal{R}, \mathcal{B}_{A_1}, \dots, \mathcal{B}_{A_m}, NA)$ , where  $\mathcal{B}_{A_i}$  ( $i = 1, \dots, m$  and  $m = 2^n - 1$ ) is a binary relation on  $W$  which is serial<sup>8</sup> and for which it holds that if  $A_i \subseteq A_j$ , then  $\mathcal{B}_i \subseteq \mathcal{B}_j$ <sup>9</sup>, for  $A_i, A_j \subseteq NA$ . Intuitively,  $(w, w') \in \mathcal{B}_{A_i}$  if in a world  $w$ , the set  $A_i$  of authorities considers world  $w'$  possible, i.e., the set  $A_i$  considers his enacted norms in  $w$  applicable in  $w'$  as possible. Thus  $w'$  would be considered a possible way to have the enacted norms apply. The fact that  $\mathcal{B}_{A_i}$  is serial means that in all worlds, the enacted norms can be applied some way; from this it follows that falsehood can not be enacted.

The semantics of formulas is as usual, only now we have to add a clause for  $N_{A_i} : \theta$ :

$$M, w \models N_{A_i} : \theta \text{ iff } M, w' \models \theta \text{ for all } w' \text{ such that } (w, w') \in \mathcal{B}_{A_i}.$$

The clause is designed to capture the intuition that  $\theta$  is enacted by  $A_i$  exactly if  $\theta$  is true in all the worlds conform to the enactment of  $A_i$ . Thus, the sentence ‘ $\theta$  is enacted by  $A_i$ ’ does not say

<sup>7</sup>The name of rule (R2), necessitation, stems from the general modal framework, in which  $L$  (or denoted usually by  $\Box$ ) has the meaning of necessity.

<sup>8</sup>A relation  $R$  is serial if for each  $w \in W$  there is some  $w' \in W$  such that  $(w, w') \in R$ . We do not have the assumptions that  $\mathcal{B}_{A_i}$  is transitive and Euclidean, as in *KD45* (cf. Chellas, 1980), since we are not dealing with nested enactments.

<sup>9</sup>This clause validates rule (R3).

that  $\theta$  is applicable, instead, it says that  $\theta$  is applicable in a world which is ideal conform to the enactment by  $A_i$ . Thus, the statement ‘ $\theta$  is enacted’ describes some idealised world and *not* the actual world, since  $\theta$  can be ineffective (see Kelsen, 1960) or can be overruled by, for example, a norm enacted by a superior authority or a norm enacted at a later point in time.

## 4 The problem of inconsistent enacted norms

In the theory of knowledge and belief there appears the natural question of whether these notions are captured adequately and realistic. The well-known problem of both theories is the problem of *logical omniscience*. This problem pertains to a notion of knowledge and belief that is too idealistic: these notions are closed under logical consequences. ‘Especially for a notion of belief, which should be more fallible if human everyday beliefs are to be captured, this property is obviously not true’ (Meyer and Van der Hoek, 1995). Logical omniscience is not a problem for the notion of enactment. If a norm follows from other enacted norms and not conflicting with another norm, this norm is applicable, even if that norm was not thought of by members of government or parliament when the law was created.

However, there is another property that is very unrealistic, which nevertheless holds in **Ent**:  $N_A : \theta \rightarrow \neg N_A : \neg\theta$ ; *consistency of enactments*. As we already mentioned in the introduction, it is a frequently phenomenon that a set of authorities enacted conflicting norms. A first and perhaps slightly naive attempt to overcome this problem is to drop the axiom (A3). By dropping this axiom we may now represent inconsistent enactment:  $N_A : \theta \wedge N_A : \neg\theta$ , however there still remains the following modified problem concerning inconsistent enacted norms:  $N_A : \theta \wedge N_A : \neg\theta \rightarrow N_A : \text{false}$ , which is already a theorem in the system<sup>10</sup>, stating that, if a norm and its negation are both enacted, every assertion has to be enacted. This problem cannot be solved within the standard modal framework using Kripke-style modal semantics.

Another and perhaps also naive attempt is to solve the problem by distinguishing *implicit* and *explicit* enactment.<sup>11</sup> We have to use an extra modal operator  $N_A^e$ , standing for explicit enactment. We define the implicit enacted norms by  $A$  as the norms that are the logical consequences of the explicit enacted norms by  $A$ . Explicit enactment is defined as follows:

$$N_A^e : \theta := N_A : \theta \wedge \theta \in E_A,$$

with  $E_A$  the set of explicit enacted norms by the set  $A$  of normative authorities. Implicit enactment is the enactment defined in the previous section:

$$N_A^i : \theta := N_A : \theta.$$

Now axiom (A2) does not hold for the explicit enactment. Thus, we can formalise consistently inconsistent enacted norms:  $N_A^e : \theta \wedge N_A^e : \neg\theta$ . This presentation of the explicit and implicit enactment suffers from a serious drawback if the set of explicitly enacted norms is inconsistent: namely, it deals with only the explicitly enacted norms. A viable logic of enactment should be able to capture - within the logic - meta-reasoning about the authority’s enacted norms, since one has to reason about the enacted norms that one has and needs to acquire. The notion of implicit enactment still suffers from the problem that now everything is (implicitly) enacted. Only if the set of explicitly enacted norms is consistent, then the notion of implicit enactment has its value.

Thus, in order to try to solve this problem of inconsistent enacted norms, we have to do something different, and deviate from the standard modal approach even further, by making of the logic of local reasoning presented by Fagin and Halpern (1988).

<sup>10</sup>  $N_A : \theta_1 \wedge N_A : \theta_2 \rightarrow N_A : (\theta_1 \wedge \theta_2)$

<sup>11</sup> The notion of explicit and implicit enactment does not correspond to the notions of explicit and implicit belief by Levesque (1984).

## 5 Local reasoning

In the logic of local reasoning, there is not necessarily one set of worlds that a set of authorities thinks possible, but rather a number of sets, each one corresponding to a different cluster of enacted norms. In a given situation, we specify a cluster as a *maximal consistent set* of the set  $E_A$  of explicitly enacted norms by a set  $A$  of authorities. The basic idea is that a set of authorities may enact inconsistent norms, but these conflicting norms are not be applicable at the same time (depending on the situation). In a particular situation, a cluster (a maximal consistent set of norms) represents the applicable norms.

Suppose that a set of authorities enacted the following three norms ‘vehicles give right of way to vehicles from the right’, ‘bicycles must give right of way to cars’ and ‘it is forbidden to drive faster than 50 km/h’. In the situation that a car and a bicycle are on a junction and that the bicycle is coming from the right, the first two norms cannot both be applicable, since they are conflicting. For this situation there are two clusters: the set consisting of the norms ‘vehicles give right of way to vehicles from the right’ and ‘it is forbidden to drive faster than 50 km/h’, and the set consisting of the norms ‘bicycles must give right of way to cars’ and ‘it is forbidden to drive faster than 50 km/h’. In the situation that the car is coming from the right, there is only one cluster: the set consisting of all the three norms. Since, in that situation the two norms concerning ‘right of way’ do not conflict.

We can view a cluster as representing the worlds the set of authorities thinks are possible in a given frame of reference, when he is focusing on a certain set of issues. More formally, a Kripke structure for local reasoning is a tuple  $M = (W, V, \mathcal{R}, \mathcal{C}_{A_1}, \dots, \mathcal{C}_{A_m}, NA)$ , where  $\mathcal{C}_{A_i}(w)$  is a non-empty set of non-empty subsets of  $W$ . Intuitively, if  $\mathcal{C}_{A_i}(w) = \{T_1, \dots, T_l\}$ , then in world  $w$  (a certain situation) the set  $A_i$  of authorities sometimes thinks that the set of possible worlds is precisely  $T_1$  (a cluster); sometimes the set of possible worlds is precisely  $T_2$ , etc. The set  $\mathcal{C}_{A_i}(w)$  corresponds to the maximal consistent sets following from the set  $E_{A_i}$  of explicitly enacted norms by  $A$  in world  $w$ . Thus,  $\mathcal{C}_{A_i}(w)$  indicates which clusters (frames of reference) are considered by the set  $A_i$  of authorities. We may now distinguish *weak* and *strong* enactment:

- $N_{A_i}^w : \theta$ : the set  $A_i$  of authorities enacted  $\theta$  in a weak sense, i.e. within *some* cluster;
- $N_{A_i}^s : \theta$ : the set  $A_i$  of authorities enacted  $\theta$  in a strong sense, i.e. within *all* clusters.

It is easy to see that strong enactment implies weak enactment. We formally define  $\models$  for these structures as follows:

$$\begin{aligned} M, w \models N_{A_i}^w : \theta & \text{ iff } & \exists T \in \mathcal{C}_{A_i}(w) \forall w' \in T M, w' \models \theta \\ M, w \models N_{A_i}^s : \theta & \text{ iff } & \forall T \in \mathcal{C}_{A_i}(w) \forall w' \in T M, w' \models \theta \end{aligned}$$

It is easy to see from the semantic definitions given that weak enactment is not closed under logical consequences and that strong enactment is closed under logical consequences. More importantly for our purposes is that now a set of authorities may enact inconsistent norms:  $N_{A_i}^w : \theta \wedge N_{A_i}^w : \neg\theta$  is satisfiable, since in one cluster the set  $A_i$  considers that  $\theta$  is applicable, while in another the set considers  $\neg\theta$  applicable. On the other hand,  $N_{A_i}^w : (\theta \wedge \neg\theta)$  is impossible: a set of authorities cannot weakly enact falsehood. It is easy to see that  $N_{A_i}^w$  does not satisfy axiom (A2), although  $N_{A_i}^w$  satisfies all the others axiom and rules of **Ent**.

$N_{A_i}^s : \theta$  represents that the set  $A_i$  of authorities considers that  $\theta$  is applicable in any frame of reference. This means that  $\theta$  is not conflicting with any other norm or set of norms enacted by the set  $A_i$  of authorities. However, the norm can be conflicting with a norm enacted by a superset  $A_j$  of  $A_i$  ( $A_i \subset A_j$ ). Suppose that the set  $A_j$  explicitly enacted  $\neg\theta$ , then the norm  $\theta$  does not appear in all the maximal  $N$ -consistent sets of  $E_{A_j}$ , because  $\neg\theta$  is also an element of some maximal consistent set (cluster) of  $E_{A_j}$ . Thus,  $N_{A_i}^s$  does not satisfy rule (R3), although all the axioms and other rules of **Ent** hold. As above-mentioned, it holds that  $N_{A_i}^s : \theta \rightarrow N_{A_i}^w : \theta$ .

Further, we can formalise with the help of explicit enactment in the previous section the explicit strong enactment. The explicit strong enactment  $N_{A_i}^{se}$  is defined as  $N_{A_i}^{se} : \theta := N_{A_i}^s : \theta \wedge \theta \in E_{A_i}$ , meaning that  $\theta$  is explicitly enacted by the set  $A_i$  of authorities and  $\theta$  does not conflict with any

norm or set of norms enacted by the set  $A_i$ .<sup>12</sup>

A particularly interesting special case we can capture is the strong enactment of norms by the set  $NA$ , the set of all the authorities. These norms are strongly enacted by the set of all the authorities, and all these norms are not conflicting with other norms enacted by the set  $NA$ . So, these norms always are applicable, in other words, these norms are elements of all clusters of the explicit enacted norms by the set  $NA$ . We can formalise this as follows

$$N_{NA}^s : \theta \rightarrow \theta.$$

Thus, explicit strong enacted norm by the set  $NA$  are applicable. This looks like the axiom  $K_i\phi \rightarrow \phi$  of the theory for Knowledge: If an agent  $i$  knows an assertion, then that assertion is true, i.e., known facts are true. However, an explicit strong enacted norm is not necessarily applicable (true), only if such a norm is explicit strong enacted by the set  $NA$ .

We can extend our semantics with another model operator  $D_{A_i}$ , to indicate applicability. The formula  $D_{A_i} : \theta$  means that  $\theta$  is applicable according to  $A_i$  in a particular cluster  $T$ . The clause for  $D_{A_i} : \theta$  is

$$M, w, T \models D_{A_i} : \theta \text{ iff } \forall w' \in TM, w' \models \theta, \text{ for } T \in \mathcal{C}_{A_i}.$$

Evidently, from the semantics it follows that  $D_A : \theta \rightarrow N_A^w : \theta$ , i.e. if is  $\theta$  applicable according to  $A_i$  in a particular cluster, then  $\theta$  is weakly enacted by the set  $A_i$ . Now we can consistently reason within a cluster, since the set of norms in a cluster is consistent. Within a cluster the property holds that the enacted norms are closed under logical consequences:

$$\text{if } M, w, T \models D_{A_i} : \theta_1 \text{ and } M, w, T \models D_{A_i} : (\theta_1 \rightarrow \theta_2), \text{ then } M, w, T \models D_{A_i} : \theta_2.$$

## 6 Conclusions

Normative conflicts can found when all the norms in a regulation are applied to a particular situation. These conflicts happen because normative rules are organised in some kind of hierarchy among the norms: some regarded as more basic than others.

It may be determined in part by considerations arising from the text of the regulations themselves, such as the existence of cross-references from one to another; and it may also be determined in part by factors of a more extrinsic kind, such as the powers and competences of the issuing bodies, dates of promulgation and amendment, and the degree of specificity or generality of the regulations made. (Alchourrón and Makinson, 1981, p. 125)

In this article we have shown how we can determine the applicability of norms in a particular situation by local reasoning. The idea is that in spite of the normative conflicts lawyers make use of a consistent set of rules. The choice of such a set depends on the frame of reference, i.e. for instance, a particular situation.

In this article we choose for a cluster a maximal consistent set. However, this is not necessary for reasoning with inconsistencies, since from the natural semantics of enactment it follows that the formula  $N_{A_i}^w : \theta \wedge N_{A_i}^w : \neg\theta$  is satisfiable.<sup>13</sup> From the natural semantics of  $N_{A_i}^w$  it also follows that the formula  $N_{A_i}^w : \theta \wedge N_{A_i}^w : (\theta \rightarrow \phi) \wedge \neg N_{A_i}^w : \phi$  is satisfiable, simply because in one cluster the set  $A_i$  of authorities considers that  $\theta$  is applicable and in another the set  $A_i$  considers that  $\theta \rightarrow \phi$  is applicable, but the set  $A_i$  never be in a cluster where the set  $A_i$  puts these norms together to conclude  $\phi$ . However, this formula is not satisfiable if we consider a cluster as a maximal consistent set. However, the property - a cluster is a maximal consistent set - has main advantages, such as the determination of norms which never conflict: the strong enacted norms.

We saw that in a particular situation several clusters can be considered. The choice which cluster, set of norms, is applicable, is an open issue and it is interesting to develop a theory to

<sup>12</sup>Note that the explicit weak enactment corresponds with the weak enactment.

<sup>13</sup>A formula  $\Theta$  is satisfiable in  $M$  if  $M, w \models \Theta$  for some  $w$  in  $M$ .

classify these clusters on the basis of a hierarchical structure of the norms. Since we are dealing with enactment by normative authorities, it is maybe possible to indicate the rank of authority between sets of authorities.

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