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Giving Permission Implies Giving Choice

Abstract

This article presents the formalisation of the weak and strong permission in deontic logic based on the logic of enactment. A permission that follows from the absence of a prohibition, we call a weak permission; this permission is not enacted. A strong permission is always enacted (implicitly or explicitly), and implies a giving choice. The distinction between these two types of permission is a consequent of the universality of a normative system by the closure rule: ‘whatever is not forbidden, is permitted’. Further we present a modification of the logic of enactment with local reasoning. The basic idea behind this treatment is that authorities may enact conflicting normative rules, depending on the frame of reference. The main strength of the presented theory is that we are now able to reason consistently in the presence of inconsistent enacted norms.

1. Introduction

When we want to examine different kinds of forms of acts within the framework of the description of the Dutch criminal law whether an act is permitted or not permitted, we can encounter an difference. On the one hand, it could be the case that a certain act is permitted by a competent normative authority. For example, the Dutch Code of Criminal Procedure teems with such regulations. On the other hand, it could be the case that in the Dutch criminal law a certain act is *weak* permitted without a competent normative authority having enacted that permission. Nevertheless, the act in the last case is also permitted in the Dutch criminal law on the basis of Feuerbach’s legal expression *nullum crimen, nulla poena sine praevia lege poenali*, stated in article 1 sub 1 of the Dutch Penal Code. In jurisprudence this principle is usually labelled as ‘sealing legal principle’, often formulated as ‘whatever is not forbid-

den, is permitted’ (cf. [28]; see also [5]). This regulation ‘does not function here through logical necessity, but because of legislation, in this way making absence of norms within these systems impossible’ ([27]). We call these systems *closed legal systems*, also called universal normative systems. Within these systems there is a positive legal norm - a general closure rule - governing all acts which are not subject to other legal norms ([2]).

A consequent of the postulated universality by ‘sealing legal principle’ is that, if we want to talk about criminal law, we have to take into account the difference between two types of permission. Permissions, which are enacted by competent normative authorities: *strong permissions*, and permissions, which are not enacted, but follow from a ‘sealing legal principle’ (or from the absence of prohibitions): *weak permissions*. When a permission is enacted (a strong permission) to a person it seems in practise that he has always a choice to perform the action or not, without a liability to sanction.

In this article we will investigate the nature of the permission with respect to the universality of the Dutch criminal law, and the formalisation of the strong and weak permission. The formalisation rest on the relation between enacted norms and applicable norms. In legal discourse reference is made to enacted norms and applicable norms. Not all norms that are enacted are applicable or are only applicable in certain circumstances. In this scope, we are concerned with two concepts of validity: membership (enactment) and applicability. A norm can be said to be valid in the sense that it belongs to or is a member of a legal system: membership. A norm is often also said to be valid in the sense that it is obligatory or has a ‘binding force’: applicability. Both of them play a central role in law and in legal theories (cf. [7]).

In most legal support/expert systems only the appli-

cable norms are considered. It is impossible in these systems to talk about the rules that select applicable norms explicitly. This is the main disadvantage of using non-monotonic logic (see [23] and [6]), logics with minimal models (see [8]) and argumentation approaches (cf. [10]; [11]; [22]). In these theories we can reason with inconsistent information, but the level is always of applicable norms. One cannot reason about enacted norms which are not applicable. Any derivation using these norms is blocked through the implicit consistency requirement of the non-monotonic *modus ponens* rule. In this paper we give a theory to describe enacted norms and applicable norms separately. This can be seen as naming the minimal models with sets of authorities. Making this explicit in a modal operator enables us also to reason about enacted norms that are not applicable (within a non-preferred minimal logic).

We show how we can reason consistently with enacted norms without requiring all enacted norms to be *normative consistent*. The ‘classical’ notion of normative inconsistency is that two authorities promulgated (enacted) two contradictory or conflicting norms, which is a frequent phenomenon, at least in certain areas like law (see [3]). The conflict arises when the norms become members of the same normative system, not if they belong to different systems. Such a system loses its meaning in a logical sense in the case of inconsistency: everything can be deduced and, in particular, all obligations, permissions, etc., are deducible (*ex falso sequitur quodlibet*). There are two specific types of normative conflicts (cf. [17]):

- *disaffirmation conflicts*: these conflicts describe the same behaviour, but the deontic modalities contradict. For example, *it is forbidden to turn left* and *it is not forbidden (permitted) to turn left*.
- *compliance conflicts*: these conflicts occur when the actions which are obliged by different norms are incompatible. For example: *it is obligated to turn left* and *it is obligated to turn right*. Thus, these conflicts describe the same deontic modalities, but the actions conflict (are incompatible).

The normative conflicts are caused, for instance, by the dynamics of the legal system (by the enactment of new norms), by the uncertainty concerning the content of the legal sources (regulations can be vague or ambiguous), etc. (cf. [26]).

One way to deal with these type of conflicts is to add sets of authorities enacting the norms to standard deontic logic. Here we use the term authority in an abstract way. They can be seen as real authorities (e.g. government and city council), but also as source of norms as in the Penal Code, Traffic Regulations, etc. We treat

enactment as a variant of the belief theory. However, in this theory we cannot adequately express normative inconsistencies. Therefore, we propose an extension of these theories, based on the theory of ‘local reasoning’ (see [12]). The basic idea behind this treatment is that authorities may enact several (inconsistent) norms, depending on the frame of reference. The applicability of norms depends on the situation, on the relative importance of the norms, etc. For example, according to article 15.1 of the Dutch Traffic Regulation vehicles give right of way to vehicles from the right, and according to article 15.2 bicycles must give right of way to cars. These two norms can conflict, therefore they cannot both be applicable in any situation. We make sets of applicable norms for each situation. E.g. in the situation that a car and a bicycle are on a junction and that the bicycle is coming from the right, we choose a set of norms with article 15.2 and other applicable norms (e.g. concerning the maximum speed limit) and not a set of norms consisting article 15.1, whereas in a situation of two cars we do not choose a set of norms consisting article 15.2. We can view the enacted norms as a society of possible sets of applicable norms (*clusters*), which may contradict each other.

The organisation of this article is as follows: in section 2 we briefly discuss standard deontic logic. Section 3 discusses the strong and weak permission. The logic of enactment - based on the logic of knowledge and belief - to formalise adequately the strong and weak permission is presented in section 4. In section 5 we assume that the set of enacted norms is normatively consistent to discuss the principle ‘all enacted norms are applicable’ and add this principle to our system. In contrast to section 5, we discuss the problem of *inconsistent* enacted norms in section 6. To solve this problem, section 7 presents the theory of local reasoning for enactment. In the last section, we give some conclusions and suggestions for further research.

2. Deontic logic

To describe the norms that are used in the law we use deontic logic. Deontic logic is a branch of philosophical logic concerning reasoning about norms. It is the logic of obligations, prohibitions and permissions. As such, it is relevant for the foundations of ethics and law. Deontic logic has been used to analyze the structure of normative law and normative reasoning in law. However, as so many subjects in philosophical logic and philosophy in general, the subject was also picked up by computer scientists and AI (artificial intelligence) researchers. Deontic logic promises to be relevant as well for such prosaic matters as authoriza-

tion mechanisms, decision support systems, database security rules, fault-tolerant software and database integrity constraints; thus, outside the area of legal analysis and legal automation. A survey of applications can be found in [20]. Deontic logic forms the basis of several legal expert systems (e.g. [9]). Therefore, we start with a brief explanation of deontic logic.

In deontic logic, three deontic operators are used: ‘ O ’ (obligatory), ‘ F ’ (forbidden) and ‘ P ’ (permitted). By connecting propositions to these operators as arguments, well-formed formulas of the system originate from which, by interpretation of the propositions, normative judgements can be formed. E.g., $O(p)$ means ‘it is obligatory that p ’. The deontic operators can be defined in terms of one another. If we take ‘ O ’ as a primitive, then the other operators can be defined as follows: $F(p) := O(\neg p)$ and $P(p) := \neg O(\neg p)$. Thus, ‘it is forbidden that p ’ is defined as ‘it is obligatory that not- p ’, and ‘it is permitted that p ’ is defined as ‘it is not obligatory that not- p ’.

In this article we use the standard deontic logic, a modal (Kripke-style) version of the now so-called ‘Old System’ of Von Wright ([30]). We mean the system D^* (the smallest normal KD -system of modal logic (cf. [8])), based on propositional logic and axiomatised by the following axiom schemata:

$$\begin{aligned} (OC) \quad & (O(p) \wedge O(q)) \rightarrow O(p \wedge q) \\ (ON) \quad & O(p \vee \neg p) \end{aligned}$$

together with the rule of inference:

$$(ROM) \quad \frac{p \rightarrow q}{O(p) \rightarrow O(q)}$$

Axiom (ON) was rejected by Von Wright ([30]), since he developed the *principle of deontic contingency*: ‘A tautologous act is not necessarily obligatory, and a contradictory act is not necessarily forbidden’. We have to commit ourselves to this axiom, since otherwise we cannot view deontic logic as a branch of Kripke-style normal modal logic.

The semantics of this system can be given using the following Kripke model structure $M = (W, \mathcal{R}, V)$ consisting of three elements: the set of possible worlds $W = \{w_1, w_2, \dots\}$; the accessibility function $R \in \mathcal{R}$, which takes a world and returns a subset of W : $R : W \rightarrow 2^W$ and a valuation function V , which assigns the values ‘true’ or ‘false’ to a proposition at a world in W . The intuition behind the function R is that it yields the *deontically ideal worlds* relative to a given world. The truth conditions for O and P can now be defined as follows, where $M, w \models \theta$ is read as ‘ θ is true in world w of structure M ’ with θ a formula of D^* :

$$M, w \models O(p) \quad \text{iff} \quad R(w) \subseteq \llbracket p \rrbracket$$

$$\begin{aligned} M, w \models P(p) & \quad \text{iff} \quad R(w) \cap \llbracket p \rrbracket \neq \emptyset \\ M, w \models \neg \theta & \quad \text{iff} \quad M, w \not\models \theta \\ M, w \models \theta_1 \wedge \theta_2 & \quad \text{iff} \quad M, w \models \theta_1 \text{ and } M, w \models \theta_2, \end{aligned}$$

with the function $\llbracket \cdot \rrbracket \in L \rightarrow 2^W$ and L the set of well-formed formulas of the propositional calculus. $\llbracket p \rrbracket = \{w \mid V(w, p) = \text{true}\}$. It is easy to see that the following properties hold: $\llbracket p \vee q \rrbracket = \llbracket p \rrbracket \cup \llbracket q \rrbracket$, $\llbracket p \wedge q \rrbracket = \llbracket p \rrbracket \cap \llbracket q \rrbracket$ and $\llbracket \neg p \rrbracket = \overline{\llbracket p \rrbracket}$. Thus, $O(p)$ holds in w if and only if p is true in all ideal worlds with respect to w , and $P(p)$ holds in w if and only if there is at least one ideal world with respect to w in which p is true. We do not have the constraint $R(w) \neq \emptyset$ for all $w \in W$, since this would validate OD : $\neg O(p \wedge \neg p)$. At first glance, this axiom does not seem to be controversial, since it merely denies the existence of impossible obligations. However, with the help of this axiom we can derive the formula $\neg(O(p) \wedge O(\neg p))$, which is controversial nowadays (see, eg. [1]; [18]; [21]). The main objection to this formula is that it states that there is no conflict of duties, which is clearly not in line with situations in daily life. A consequence is that the formula $O(p) \wedge F(p)$ can be consistent, however these norms ($O(p)$ and $F(p)$) are obviously conflicting: they are *normatively inconsistent*. According to [17], these conflicts are *compliance conflicts*.

3. The strong and weak permission

In the Dutch Traffic Regulation - part of the Dutch criminal law - a regulation concerning a permission is always an exception of an obligation or a prohibition ([15]). Otherwise, the permission would be superfluous, because of the sealing legal principle ‘whatever is not forbidden, is permitted’. In what way are we to interpret this rule? The obvious thing would be to accept the following principle:

$$\neg F(p) \rightarrow P(p).$$

This principle seems to be in accordance with what is meant, but does not add anything, because it is already incorporated in the deontic system. If interpreted in this way, the rule mentioned does not guarantee universality, for axioms, theorems and definitions have to be applicable to non-universal systems. The principle

‘what is not forbidden is permitted’ addresses the judge and forbids him to extend the whole of legal prohibitions on the grounds of his own or someone else’s political or moral conviction. ([27])

So, we can distinguish two types of permission:

- the permission which is an exception of an obligation or a prohibition: *the strong permission*;
- the permission which follows from the absence of a prohibition: *the weak permission* (cf. [31]).

A consequence of the strong permission is that also the negation of the permitted act is permitted, since the strong permission is an exception to a general article (rule), and one does not break the law by following nevertheless that general article. So, this implies a choice for the norm subjects to perform that act or not, without a liability to sanction.

We will give an example concerning ‘overtaking’ of the Dutch Traffic Regulation:

- Section 11.** 1. Overtaking occurs on the left.
 2. ...
 3. Cyclists and moped-drivers should overtake other cyclists and moped-drivers on the left; They are permitted to overtake other drivers on the right.
 4. Drivers, who are on the right of a block-signposting are permitted to overtake drivers, whom are on the left of that signposting, on the right.
 5. Drivers are permitted to overtake trams on the right.

All the permissions mentioned in section 11 are exceptions of the general rule ‘overtaking occurs on the left’. This rule does not contain any deontic operator. However, it is easy to model this in a regulation model. For instance, ‘overtaking on the right is forbidden’. The permission in the Dutch Traffic Regulation has a higher priority than the obligations and prohibitions which are incompatible with that permission. So, the law is not merely a set of norms, but a hierarchical system. When we consider the norms in a legal system, we can often discern some kind of hierarchy among the norms: some are regarded as more basic than others (cf. [4]). The consequence of the fact that an enacted permission is always an exception of a prohibition or obligation is that the addressees (to whom the permission is directed) always have a choice to perform the permitted act or not. For example, from the permission that drivers are allowed to overtake trams on the right it follows that drivers are also allowed *not* to overtake trams on the right. This does not mean that drivers are allowed to overtake trams on the left. It only means that it is permitted not to overtake a tram on the right in at least one way. Probably, there are many ways of performing the act ‘not to overtake a tram on the right’, some of which are forbidden. In contrast with the permission, the prohibition means that *all ways* of performing an act are forbidden. From the above-mentioned consequence, it is tempting to add the

following formula to a deontic system:

$$P(p) \rightarrow P(\neg p).$$

However, this is unacceptable, since from this it follows that $O(p) \rightarrow F(p)$ - because $(P(p) \rightarrow P(\neg p)) \equiv (\neg O(\neg p) \rightarrow \neg O(p)) \equiv (O(p) \rightarrow F(p))$ - which says that if p is obligatory, then p is also forbidden. We have to make a distinction of this type of permission, the *strong* permission and the *weak* permission in our formalisation, since for the weak permission it does not hold that the negation of a weakly permitted act is permitted. An example of a weak permission is the permission to drive at a speed of 95 km/h on a motorway. According to the Dutch Traffic Regulation a speed limit of 120 km/h holds on motorways. Nothing is said about a speed of 95 km/h. Consequently, because of the sealing legal principle (or because of the absence of a prohibition to that act), this act (to drive at a speed of 95 km/h on a motorway) is permitted: weak permission. However, the prohibition to exceed the speed limit of 120 km/h clearly does not imply that all speeds below the 120 km/h are *by definition* permitted: one has to take other speed prohibitions and orders into account, such as article 25 of the Traffic Act: On the road, it is prohibited to act in such a way that the freedom of traffic is hindered without necessity or that road security is jeopardised or may reasonably be expected to be jeopardised. It would be absurd to stick to this meaning when one considers the following example. There is a traffic jam on the motorway, and a car is driving at a speed of 100 km/h. The driver breaks the law in this situation, and he cannot maintain that it was permitted to drive at a speed of 100 km/h on this section of the road (cf. [24]).

As it has been said, a permission is weak (in the sense here relevant) if the act (proposition) is not forbidden. The permission is strong if the act is not forbidden, though subject to norm (cf. [27]). This means that a permission is strong if and only if it is enacted by a normative authority (explicitly enacted permission) or if the permission is a logical consequence from a norm or set of norms enacted by a normative authority (implicitly enacted permission). Consequently, if an act is strongly permitted, then also the negation of that act, since from the fact that an authority enacted that an act is permitted, which is an exception of a prohibition or obligation, it follows that the authority also enacted (although implicitly) that the negation of that act is permitted. This does not hold for the weak permission. For instance, if no authority enacted that a certain act is forbidden and some authority enacted that that act is obligatory, then that act is weakly permitted and obligatory simultaneously. Consequently, the negation

of that act is *not* permitted, since it is forbidden by the obligation $O(p)$ ($O(p) \equiv F(\neg p)$). So, the weak permission entails no choice, unlike the strong permission.

The legal difference between these two types of permission is quite obvious. The idea is that a later enacted prohibition could be in conflict with a strong permission, but not with a weak permission. If an act is weak permitted then the prohibition of that act can be given with any conflict (cf. [29]). In standard deontic logic, we cannot distinguish the strong and weak permission, since there is no explicit indication to the enactment of norms (they are God given). In the next section we will formalise these two types of permission with the help of the logic of enactment (cf. [25]). The basic idea of the treatment is that a norm enacted by a normative authority is not always applicable, since, for example, the norm can be ineffective ([14]) or the norm can be overruled (derogated) by a norm enacted by a superior authority or by a norm enacted at a later point in time. Thus, a strong permission is not always applicable, but a weak permission is.

4. The logic of enactment

As we already mentioned, a strong permission is always enacted (explicitly or implicitly), and on the contrary a weak permission is not enacted, but follows from the absence of an enacted prohibition. To formalise this distinction between the weak and strong permission we add an operator N_{A_i} to the deontic logic. A formula $N_{A_i} : \theta$, with θ a norm, is read as ‘the set A_i of authorities enacted the norm θ ’. Sets of authorities are needed to determine the consequences of norms enacted by a combination of individual authorities. We define $NA = \{a_1, \dots, a_n\}$ as the set of normative authorities. Since we consider sets of authorities, we have $2^n - 1$ sets of authorities, instead of considering n authorities. We treat N_{A_i} as a *modal* operator, just as in the systems of *knowledge* and *belief*. The theory of enactment is based on an extension of belief theory. The main reason being that belief theories also have to deal with reasoning about inconsistent beliefs (information). So, the similarity concerns mainly this part of reasoning in the present of inconsistencies within a modal operator. Maybe clear that some axioms for belief - like ‘if an agent believes ϕ , then he believes that he believes ϕ ’ - does not make sense for enactment. Furthermore, belief and enactment correspond in the sense that a belief is not necessarily true, just as an enacted norm is not necessarily applicable.

In this section, we briefly review possible-worlds semantics for enactment, corresponding with the semantics for belief and knowledge (cf. [8]; [13]). The in-

tuitive idea behind the possible-worlds model is that, besides the true state of affairs, there are a number of other possible states of affairs, or possible worlds.

On the one hand, the notion of enactment is a restricted version of the system weak *S5* or *KD45*, in the sense that we are not dealing with nested enactment. A formula $N_{A_i} : (N_{A_i} : \theta)$, reading that ‘ A_i enacted that A_i enacted θ ’, is meaningless. Thus, we restrict the language, so that no N appears within the scope of another. Thus, if θ is a deontic formula, then $N_{A_i} : \theta$ is also a formula. On the other hand, the notion of enactment is an extended version of weak *S5* or *KD45*, in the sense that we are adding some extra axioms and rules. We now present an axiomatic system **Ent** for enactment with respect to set NA of normative authorities, where θ , θ_1 and θ_2 are formulas of the system D^* , and Θ_1 and Θ_2 are formulas of system **Ent**:

$$\text{All tautologies of the propositional calculus.} \quad (\text{A1})$$

$$N_{A_i} : \theta_1 \wedge N_{A_i} : (\theta_1 \rightarrow \theta_2) \rightarrow N_{A_i} : \theta_2 \quad (\text{A2})$$

$$\neg N_{A_i} : \text{false} \quad (\text{A3})$$

$$\frac{\Theta_1 \rightarrow \Theta_2, \Theta_1}{\Theta_2} \quad (\text{R1})$$

$$\frac{\theta}{N_{A_i} : \theta} \quad (\text{R2})$$

$$\frac{A_i \subseteq A_j}{N_{A_i} : \theta \rightarrow N_{A_j} : \theta} \quad (\text{R3})$$

The first axiom (A1) and rule (R1) are holdovers from propositional calculus. The second axiom says that enactment is closed under implication. Note that (A2) is equivalent to $N_{A_i} : (\theta_1 \rightarrow \theta_2) \rightarrow (N_{A_i} : \theta_1 \rightarrow N_{A_i} : \theta_2)$, which is sometimes given as an alternative axiom. (A3) says that an authority cannot enact falsehood. The rule (R2) states that every tautology is enacted. The name of this rule: necessitation, stems from the general modal framework, in which N_{A_i} (or denoted usually by \Box) has the meaning of necessity. Rule (R3) expresses the relation between the sets of authorities. If a set of authorities enacted a norm, then every superset of authorities of that set also enacted that norm.

We will now introduce two axioms, which enables us to make a distinction between weak and strong permission.

$$N_{A_i} : P(p) \rightarrow N_{A_i} : P(\neg p) \quad (\text{A4})$$

$$\forall i \in \{1, \dots, 2^n - 1\} \neg(N_{A_i} : F(p)) \rightarrow P(p) \quad (\text{A5})$$

These two axioms are the added axioms in relation to the restricted version of weak *S5* or *KD45*. Axiom

(A4) we call *the axiom of the strong permission* and axiom (A5) we will call *the axiom of the weak permission*. The axiom of the strong permission says that if an authority A_i or a set A_i of authorities enacted that an act is permitted, A_i also enacted that the negation of that act is permitted. Thus, the addressee to whom the enacted permission is enacted, has a choice to perform the permitted act or not, (without a liability to sanction). The axiom of the weak permission says that the absence of a prohibition implies a permission; i.e. set A of authorities enacted that something is forbidden, then it is permitted. The difference between the strong and weak permission reveals itself in these axioms. A strong permission owes his existence to the fact that it is enacted, and a weak permission owes his existence by the absence of a prohibition.

The semantics will be given by a Kripke structure $(W, \mathcal{R}, V, \mathcal{B}_{A_1}, \dots, \mathcal{B}_{A_m}, NA)$, where \mathcal{B}_{A_i} ($i = 1, \dots, m$ and $m = 2^n - 1$) is a binary relation on W which is serial and for which it holds that if $A_i \subseteq A_j$, then $\mathcal{B}_{A_j} \subseteq \mathcal{B}_{A_i}$, which validates rule (R3). A relation R is serial if for each $w \in W$ there is some $w' \in W$ such that $(w, w') \in R$. The fact that \mathcal{B}_{A_i} is serial means that in all worlds, the enacted norms can be applied some way; from this it follows that falsehood cannot be enacted. We do not have the assumptions that \mathcal{B}_{A_i} is transitive and Euclidean, as in *KD45* (cf. [8]), since we are not dealing with nested enactments. Intuitively, $(w, w') \in \mathcal{B}_{A_i}$ if in a world w , the set A_i of authorities considers world w' possible, i.e., A_i considers his enacted norms in w applicable in w' as possible. Thus w' would be considered a possible way to have the enacted norms apply.

The semantics of the formulas is very similar to that of the deontic logic. The clause for the new operator N_{A_i} is as follows, where θ is a formula of D^* :

$$M, w \models N_{A_i} : \theta \text{ iff } M, w' \models \theta$$

for all w' such that $(w, w') \in \mathcal{B}_{A_i}$. The clause is designed to capture the intuition that θ is enacted by A_i exactly if θ is true in all the worlds conform to the norms enacted by A_i . Thus, the sentence ' θ is enacted by A_i ' does not say that θ is applicable, instead, it says that θ is applicable in a world which is ideal conform to the enactment by A_i . Thus, the statement ' θ is enacted' describes some idealised world and *not* the actual world.

From the semantics it follows that the rules and the first three axioms are valid (cf. [8]). To validate the axioms (A4) and (A5) we have to add two clauses. (A4) becomes valid by adding the clause:

$$R(w') = \emptyset \text{ or } R(w') \cap \llbracket p \rrbracket \neq R(w')$$

for all w' such that $(w, w') \in \mathcal{B}_{A_i}$. Axiom (A5) becomes valid by adding the clause

$$\text{if } \forall_{A \in \{1, \dots, n\}} (R(w') \subseteq \llbracket \neg p \rrbracket) \text{ then } R(w) \cap \llbracket p \rrbracket \neq \emptyset$$

for all w' such that $(w, w') \in \mathcal{B}_{A_i}$.

5. The axiom: enacted norms are true

In the theory of knowledge and belief there appears the natural question of whether these notions are captured adequately and realistic. The well-known problem of both theories is the problem of *logical omniscience*. This problem pertains to a notion of knowledge and belief that is too idealistic: these notions are closed under logical consequences. 'Especially for a notion of belief, which should be more fallible if human everyday beliefs are to be captured, this property is obviously not true' ([19]). Logical omniscience is not a problem for the notion of enactment. If a norm follows from other enacted norms and not conflicting with another norm, this norm is applicable, even if that norm was not thought of by members of government or parliament when the law was created.

However, there is another property that is very unrealistic, which nevertheless holds in **Ent**: $N_{A_i} : \theta \rightarrow \neg N_{A_i} : \neg \theta$; *consistency of enactments*. As we already mentioned, it is a frequent phenomenon that normative authorities enact conflicting norms. This is a very serious problem, which we discuss in the next two sections (see also [12] and [25]). For the moment, we assume that the set of enacted norms is normatively consistent, i.e., there are no conflicting norms. From this fact, we can create an actual world that corresponds with the ideal world: all the enacted norms are applicable. This can be axiomatised by the following formula:

$$(N_{A_i} : \theta) \rightarrow \theta \tag{A6}$$

This corresponds with the axiom $K_i \phi \rightarrow \phi$ of the theory for knowledge: if an agent i knows an assertion ϕ , then that assertion is true, i.e., known facts are true. In our terminology, axiom (A6) is read 'enacted norms are true' or more practical 'enacted norms are applicable'. Axiom (A6) becomes valid by adding the claim that the relation \mathcal{B}_{A_i} is reflexive, i.e., if

$$\forall_{w \in W} (w, w) \in \mathcal{B}_{A_i}.$$

Note that the formula $P(p) \wedge O(p)$ is *satisfiable*, in contrast to the formula $N_{A_i} : P(p) \wedge N_{A_i} : O(p)$. We say a formula θ is *satisfiable in M* if $M, w \models \theta$ for some world w in M ; θ is *satisfiable* if it is satisfiable in some Kripke structure. The last formula is inconsistent, since $N_{A_i} : P(p) \wedge N_{A_i} : O(p)$ implies

$N_{A_i} : P(\neg p) \wedge N_{A_i} : \neg P(\neg p)$, and this in its turn implies, according (A6), $P(\neg p) \wedge \neg P(\neg p)$, which is equivalent with falsehood. Conversely, formula $P(p) \wedge O(p)$ can hold: if $P(p)$ follows from the absence of prohibition $F(p)$ (i.e., (A5)) and $O(p)$ follows from (A6).

As we mentioned in section 2, we dropped axiom (OD), i.e. $\neg O(p \wedge \neg p)$ for our deontic system. Suppose we do not drop this axiom, then we can derive $O(p) \rightarrow P(p)$. This would be a menace for our theory, since if an obligation is enacted, then this implies falsehood. Suppose that $N_{A_i} : O(p)$ holds, then consequently $N_{A_i} : \neg P(\neg p)$ and $N_{A_i} : P(p)$. This can easily be proved by the following rule (N_{A_i} -distribution), which is derivable in system **Ent**:

$$\frac{\theta_1 \rightarrow \theta_2}{N_{A_i} : \theta_1 \rightarrow N_{A_i} : \theta_2}.$$

From $N_{A_i} : P(p)$ we can derive $N_{A_i} : P(\neg p)$, thus now with axiom (A6) it follows that $P(\neg p)$ and $\neg P(\neg p)$ both can be derived, which leads to falsehood. However, axiom (OD) is controversial nowadays - as we already mentioned -, so this is not a drawback for our system. Now we can easily check whether a permission is weak or strong. Suppose $P(p)$ is derivable, then this permission is strong if $N_{A_i} : P(p)$ is derivable for some i , and weak if the permission is strong or $N_{A_i} : P(p)$ is not derivable for all i . Thus, the strong permission implies the weak permission.

Axiom (A6) can only be added to a system, if we consider a *normative consistent* set of enacted norms. An advantage of dropping this axiom, is that we can make a distinction between *enacted* and *applicable* norms. Further, this distinction is of great use to determine which of the (conflicting) enacted norms are applicable in a certain situation. The determination of which set of norms is applicable in a certain situation is an open issue and it is interesting to develop a theory to classify these sets of applicable norms. However, there are tools developed, which can help to exploit the progress of this research, for example, non-monotonic logic, defeasible reasoning, local reasoning and logic of preference.

6. The problem of inconsistent enacted norms

A first and perhaps slightly naive attempt to overcome the problem of inconsistent enacted norms is to drop the axiom (A3). By dropping this axiom we may now represent inconsistent enactment: $N_{A_i} : \theta \wedge N_{A_i} : \neg\theta$, however there still remains the following modified problem concerning inconsistent enacted norms: $N_{A_i} : \theta \wedge N_{A_i} : \neg\theta \rightarrow N_{A_i} : \text{false}$, which is already a theorem in the system - since $N_{A_i} : \theta_1 \wedge N_{A_i} : \theta_2 \rightarrow N_{A_i} : (\theta_1 \wedge \theta_2)$

- stating that, if a norm and its negation are both enacted, every assertion has to be enacted. This problem cannot be solved within the standard modal framework using Kripke-style modal semantics.

Another and perhaps also naive attempt is to solve the problem by distinguishing *implicit* and *explicit* enactment. (The notion of explicit and implicit enactment does not correspond to the notions of explicit and implicit belief by [16].) We have to use an extra modal operator $N_{A_i}^e$, standing for explicit enactment. We define the implicit enacted norms by A_i as the norms that are the logical consequences of the explicit enacted norms by A_i . Explicit enactment is defined as follows:

$$N_{A_i}^e : \theta := N_{A_i} : \theta \wedge \theta \in E_{A_i},$$

with E_{A_i} the set of explicit enacted norms by the set A_i of normative authorities. Implicit enactment is the enactment defined in the previous section:

$$N_{A_i}^i : \theta := N_{A_i} : \theta.$$

Now axiom (A2) does not hold for the explicit enactment. Thus, we can formalise consistently inconsistent enacted norms: $N_{A_i}^e : \theta \wedge N_{A_i}^e : \neg\theta$. This presentation of the explicit and implicit enactment suffers from a serious drawback if the set of explicitly enacted norms is inconsistent: namely, it deals with only the explicitly enacted norms. A viable logic of enactment should be able to capture - within the logic - meta-reasoning about the authority's enacted norms, since one has to reason about the enacted norms that one has and needs to acquire. The notion of implicit enactment still suffers from the problem that then everything is (implicitly) enacted. Only if the set of explicitly enacted norms is consistent, then the notion of implicit enactment has its value. Note that explicit enactment implies implicit enactment.

In [8], Chellas argues that an adequate logic of norms should be able to express conflicts of norms, but he also wants to be able to retain the deontic counterparts of (A3). However, one cannot keep (A3) and allow consistent expression of normative conflict (i.e. $O(p) \wedge O(\neg p)$) in a normal modal system. That is why Chellas introduces classical modal systems with minimal models for a revised deontic logic. However, in these systems we cannot specify explicitly the minimal models. Thus, in order to try to solve this problem of inconsistent enacted norms, we have to do something different, and deviate from the standard modal approach even further, by making of the logic of local reasoning presented by [12].

7. Local reasoning

In the logic of local reasoning, there is not necessarily one set of worlds that a set of authorities thinks possible, but rather a number of sets, each one corresponding to a different cluster of enacted norms. In a given situation, we specify a cluster as a *maximal consistent set* of the set E_{A_i} of explicitly enacted norms by a set A_i of authorities. The basic idea is that a set of authorities may enact inconsistent norms, but these conflicting norms are not applicable at the same time (depending on the situation). In a particular situation, a cluster represents the applicable norms.

Suppose that a set of authorities enacted the following three norms ‘vehicles give right of way to vehicles from the right’, ‘bicycles must give right of way to cars’ and ‘it is forbidden to drive faster than 50 km/h’. The first two norms cannot both be applicable, since they may conflict. For instance, in the situation that a car and a bicycle are on a junction and that the bicycle is coming from the right. Thus, there are two clusters: the set consisting of the norms ‘vehicles give right of way to vehicles from the right’ and ‘it is forbidden to drive faster than 50 km/h’, and the set consisting of the norms ‘bicycles must give right of way to cars’ and ‘it is forbidden to drive faster than 50 km/h’.

We view a cluster as representing the worlds the set of authorities thinks are possible in a given frame of reference, when he is focusing on a certain set of issues. More formally, a Kripke structure for local reasoning is a tuple $M = (W, V, \mathcal{R}, \mathcal{C}_{A_1}, \dots, \mathcal{C}_{A_m}, NA)$, where $\mathcal{C}_{A_i}(w)$ is a non-empty set of non-empty subsets of W . Intuitively, if $\mathcal{C}_{A_i}(w) = \{T_1, \dots, T_l\}$, then in world w (a certain situation) the set A_i of authorities sometimes thinks that the set of possible worlds is precisely T_1 (a cluster); sometimes the set of possible worlds is precisely T_2 , etc. The set $\mathcal{C}_{A_i}(w)$ corresponds to the maximal consistent sets following from the set E_{A_i} of explicitly enacted norms by A_i in world w . Thus, $\mathcal{C}_{A_i}(w)$ indicates which clusters (frames of reference) are considered by the set A_i of authorities. We may now distinguish *weak* and *strong* enactment:

$N_{A_i}^w : \theta$: the set A_i of authorities enacted θ in a weak sense, i.e. within *some* cluster considered by A_i ;

$N_{A_i}^s : \theta$: the set A_i of authorities enacted θ in a strong sense, i.e. within *all* clusters considered by A_i .

It is easy to see that strong enactment implies weak enactment. We formally define \models for these structures as follows:

$$\begin{aligned} M, w \models N_{A_i}^w : \theta & \text{ iff } \exists T \in \mathcal{C}_{A_i}(w) \forall w' \in T M, w' \models \theta \\ M, w \models N_{A_i}^s : \theta & \text{ iff } \forall T \in \mathcal{C}_{A_i}(w) \forall w' \in T M, w' \models \theta \end{aligned}$$

It is easy to see from the semantic definitions given that weak enactment is not closed under logical consequences and that strong enactment is closed under

logical consequences. More importantly for our purposes is that now a set of authorities may enact inconsistent norms: $N_{A_i}^w : \theta \wedge N_{A_i}^w : \neg\theta$ is satisfiable, since in one cluster the set A_i considers that θ is applicable, while in another the set considers $\neg\theta$ applicable. On the other hand, $N_{A_i}^w : (\theta \wedge \neg\theta)$ is impossible: a set of authorities cannot weakly enact falsehood. It is easy to see that $N_{A_i}^w$ does not satisfy axiom (A2), although $N_{A_i}^s$ satisfies all the others axiom and rules of **Ent**.

$N_{A_i}^s : \theta$ represents that the set A_i of authorities considers that θ is applicable in any frame of reference. This means that θ is not conflicting with any other norm enacted by the set A_i of authorities. However, the norm can be conflicting with a norm enacted by a superset A_j of A_i ($A_i \subset A_j$). Suppose that the set A_j explicitly enacted $\neg\theta$, then the norm θ does not appear in all clusters of E_{A_j} , because $\neg\theta$ is also an element of some maximal consistent set (cluster) of E_{A_j} . Thus, $N_{A_i}^s$ does not satisfy rule (R3), although all the axioms and other rules of **Ent** hold. As above-mentioned, it holds that $N_{A_i}^s : \theta \rightarrow N_{A_i}^w : \theta$.

Further, we can formalise with the help of explicit enactment in the previous section the explicit strong enactment. The explicit strong enactment $N_{A_i}^{se}$ is defined as $N_{A_i}^{se} : \theta := N_{A_i}^s : \theta \wedge \theta \in E_{A_i}$, meaning that θ is explicit enacted by the set A_i of authorities and θ does not conflict with any norm or set of norms enacted by the set A_i . Note that the explicit weak enactment corresponds with the weak enactment.

A particularly interesting special case we can capture is the strong enactment of norms by the set NA , the set of all the authorities. These norms are strong enacted by the set of all the authorities, and all these norms are not conflicting with other norms enacted by the set NA . So, these norms always are applicable, in other words, these norms are elements of all clusters of the explicit enacted norms by the set NA . We can formalise this as follows

$$N_{NA}^s : \theta \rightarrow \theta.$$

Thus, explicit strong enacted norm by the set NA are applicable, corresponding to the axiom $K_i\phi \rightarrow \phi$ of the theory for knowledge (see section 5). However, an explicit strong enacted norm is not necessarily applicable (true), only if such a norm is explicit strong enacted *by the set NA* .

We can extend our semantics with another model operator D_{A_i} , to indicate applicability. The formula $D_{A_i} : \theta$ means that θ is applicable according to A_i in a particular cluster T . The clause for $D_{A_i} : \theta$ is

$$M, w, T \models D_{A_i} : \theta \text{ iff } \forall w' \in T M, w' \models \theta, \text{ for } T \in \mathcal{C}_{A_i}.$$

Evidently, from the semantics it follows that $D_{A_i} : \theta \rightarrow$

$N_{A_i}^w : \theta$, i.e. if θ is applicable according to A_i in a particular cluster, then θ is weakly enacted by the set A_i . Now we can consistently reason within a cluster, since the set of norms in a cluster is consistent. Within a cluster the property holds that the enacted norms are closed under logical consequences:

$$\begin{aligned} \text{if } M, w, T \models D_{A_i} : \theta_1 \text{ and } M, w, T \models D_{A_i} : (\theta_1 \rightarrow \theta_2), \\ \text{then } M, w, T \models D_{A_i} : \theta_2. \end{aligned}$$

This is a first step to determine which norms are valid. The second step would be the determination which clusters are applicable, but this falls outside the scope of this paper and requires further research.

8. Conclusions

The importance of the distinction between strong and weak permission reveals in the context of postulated universality, realised by the general closure rule: ‘what-ever is not forbidden, is permitted’. In the Dutch criminal law, this rule is stated in article 1 sub 1 of the Dutch Penal Code. The distinction can be expressed by the addition of a modal operator N_{A_i} , which expresses *enactment*, to a deontic system. A strong permission is always enacted (implicitly or explicitly) and a weak permission owes his existence to the absence of a prohibition. Thus, a weak permission is not enacted. Another difference is that the strong permission implies a giving choice to the addressees.

An enacted norm is not necessarily applicable and visa versa. For example, a norm can be enacted, although not applicable, since that norm is overruled by a norm enacted by a superior authority, and an act can be permitted although it is not enacted, since that permission follows from the absence of a prohibition. In this paper we have developed a theory, that can make the difference between enactment and applicability, and so also between strong and weak permission.

In the future, the theory can be used to order the enacted norms implemented in an expert system. Expert systems should contain rules indicating which of the enacted norms are applicable. These rules can be obtained by factors of the powers and competencies of the normative authorities (issuing bodies), dates of promulgation and amendment, the degree of specificity or generality of the regulations, etc. (See [26].)

Further, we have shown how we can determine the applicability of norms in a particular situation by local reasoning. The idea is that in spite of the normative conflicts, lawyers make use of a consistent set of rules. The choice of such a set depends on the frame of reference, i.e. for instance, a particular situation.

In this article a cluster is defined as a maximal consistent set. However, this is not necessary for reasoning with inconsistencies, since from the natural semantics of enactment it follows that the formula $N_{A_i}^w : \theta \wedge N_{A_i}^w : \neg\theta$ is satisfiable.

We saw that in a particular situation several clusters can be considered. On the basis of the applicability of norms per set of authorities ($D_{A_i} : \theta$), it is possible to determine which norms hold in the present world. The choice which clusters, sets of norms, are applicable, is an open issue and it is interesting to develop a theory to classify these clusters. This can be done on the basis of an authority hierarchy, the degree of specificity and generality, etc. Since, we are dealing with enactment by normative authorities, it is maybe possible to indicate the rank of authority between sets of authorities.

In most legal support/expert systems only the applicable norms are considered. It is impossible in these systems to talk about the rules that select applicable norms explicitly. Neither is it possible to reason about enacted norms that are not applicable. We gave a first step to realise the formalisation of enacted and applicable norms in the context of postulated universality and with the help of local reasoning. The theory, that is developed in this paper can be used to order the enacted norms implemented in an expert system. Expert systems should contain rules which of the enacted norms are applicable. These rules can be obtained by the ideas given above. The main advantage is that the expert systems can now reason with the use of normal first-order logic within a cluster in stead of using nonmonotonic logic.

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