

Internal Josephson Oscillations for Distinct Momenta Bose-Einstein Condensates

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The internal Josephson oscillations between an atomic Bose-Einstein condensate (BEC) and a molecular one are studied for atoms in a square optical lattice subjected to a staggered gauge field. The system is described by a Bose-Hubbard model with complex and anisotropic hopping parameters, which are different for each species, atoms and molecules. When the flux per plaquette for each species is small, the system oscillates between two conventional zero-momentum condensates. However, there is a regime of parameters where Josephson oscillations between a vortex-carrying atomic condensate (finite momentum BEC) and a conventional zero-momentum molecular condensate may be realized. The experimental observation of the oscillations between these qualitatively distinct BEC's is possible with state of the art Ramsey interference techniques.

Introduction - Quantum coherence is a subject of fundamental importance and practical interest, especially concerning the construction of quantum logic devices. The Josephson effect, historically proposed to occur in superconductors, is actually a very useful tool to measure the degree of coherence between different quantum states. In cold atoms, a quantum superposition between two chemically different species (atoms and molecules), which is yet another ramification of the same effect, has been observed by means of Ramsey-like interference experiments for Bose-Einstein condensates (BECs) in a trap [1–3]. Later, experiments and theoretical studies included an optical lattice [4–6]. However, the studies were restricted to relatively simple quantum systems.

The recent realization of artificial gauge fields for cold atoms has proven to be very fruitful to emulate more *complex* quantum systems. The possibility of creating vortex lattices with synthetic gauge fields [7] provides an exciting prospect of reaching the fractional quantum Hall regime. Besides, the generation of a staggered magnetic flux in a driven 2D optical lattice holds promises of unprecedented simplicity [8, 9]. By loading a staggered-flux optical lattice with cold bosonic atoms, distinct superfluids can form, depending on the value of the flux ϕ per plaquette. For $\phi < \pi$, the bosons condense at zero momentum, whereas for $\phi > \pi$ a finite momentum BEC is realized, which carries a vortex-antivortex lattice [9]. In addition, when manipulating the interactions in the system by means of a Feshbach resonance, a bound state of two bosonic atoms (a molecule) can occur, thus raising even further the parameter space for the realization of different BECs: indeed, each type of particles, atoms and molecules can, in principle, condense either at zero or at finite momentum.

Here, we first study the two-body problem of the 2D staggered-flux lattice for cold atoms. The staggered flux splits the lattice into \mathcal{A} and \mathcal{B} sublattices, thus introducing a pseudospin degree of freedom into the problem. Due to the breaking of time-reversal symmetry and the presence of the pseudospin, bound states always ap-

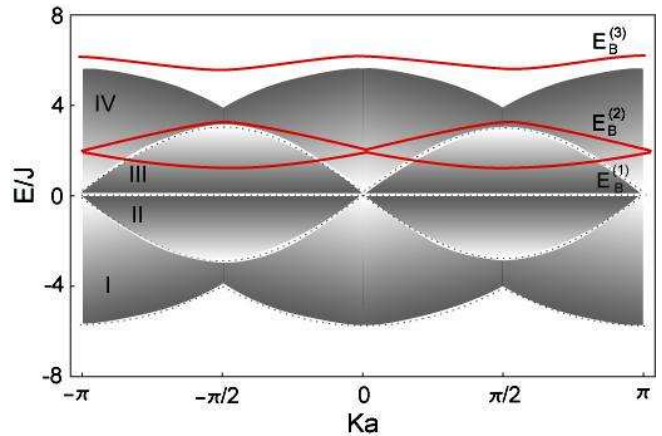


FIG. 1. (color online) Energy bands E of the scattering continua for the four pseudospin components as a function of the center-of-mass quasimomentum \mathbf{K} for $U/J = 1.818$. Also shown are the bound state energy dispersions (full curves), which lie above their respective scattering continuum.

pear, irrespective of the repulsive interaction strength, see Fig. 1. This surprising finding provides a unique opportunity for realizing cold atoms experiments where pseudospin degrees of freedom play an important role in scattering processes, and may shed light on the corresponding fermionic many-body problem in the context of high- T_c cuprates. Indeed, a staggered- π -flux phase was first proposed by Marston and Affleck [10] to describe the pseudogap regime of high- T_c cuprates, and has been advocated by many to be the hidden-order behind high- T_c superconductivity [11].

Second, by taking into account the molecular formation, we use Bogoliubov theory to study the collective behavior of the generalized Bose-Hubbard model. The interplay between species and pseudospin degrees of freedom results in an effective four-band description that supports various out-of-phase collective modes, also known as Leggett modes [12]. In particular, we find a regime of parameters where coherent oscillations be-

tween two qualitatively distinct BECs may be realized: When the flux per plaquette for each species $\phi_{1,2} < \pi$ and the hopping amplitudes $J_{1,2} > 0$, the system oscillates between two conventional zero-momentum condensates. However, for $\pi < \phi_1 < 3\pi$, $3\pi < \phi_2 < 4\pi$, and $J_2 < 0$, a coherent oscillation between a conventional zero-momentum molecular BEC and a vortex-antivortex carrying atomic BEC may occur. The latter describes internal Josephson oscillations between two macroscopic groundstates carrying different quantum numbers.

The Model - We consider the 2D Hubbard model for bosonic atoms and molecules in the presence of an external staggered flux,

$$H_{\sigma} = J_{\sigma} \sum_{\langle i,j \rangle, l} e^{i\phi_{\sigma}(-1)^l/4} a_{i,\sigma}^{\dagger} b_{j,\sigma} + \frac{U_{\sigma}}{2} \sum_{i \in A} a_{i,\sigma}^{\dagger} a_{i,\sigma}^{\dagger} a_{i,\sigma} a_{i,\sigma} + \frac{U_{\sigma}}{2} \sum_{j \in B} b_{j,\sigma}^{\dagger} b_{j,\sigma}^{\dagger} b_{j,\sigma} b_{j,\sigma}. \quad (1)$$

Here, J_{σ} is the hopping amplitude between nearest neighbor sites and ϕ_{σ} is the flux per plaquette which alternates in sign between neighbouring plaquettes. Both parameters depend on the species index $\sigma = 1$ for atoms and $\sigma = 2$ for molecules because the masses are different, $m_2 = 2m_1$. The operators $a_{i,\sigma}^{\dagger}$ ($a_{i,\sigma}$) and $b_{i,\sigma}^{\dagger}$ ($b_{i,\sigma}$) create (annihilate) particles of species σ at site i in the sublattices \mathcal{A} and \mathcal{B} , respectively, with a sublattice constant $a = \lambda/\sqrt{2}$ given in terms of the laser wavelength λ . The parameter U_{σ} denotes the on-site Hubbard interaction, and $l = 1, 2, 3$, and 4 numbers the four bonds emanating from site i (see Ref. [9] for more details).

Two-atom scattering - Let us start by considering the scattering of two atoms at positions \mathbf{r}_1 and \mathbf{r}_2 . The Schrödinger equation related to the Hamiltonian (1) reads

$$\begin{pmatrix} U\delta_{\mathbf{r}_1, \mathbf{r}_2} & \Delta_{\mathbf{r}_2} & \Delta_{\mathbf{r}_1} & 0 \\ \Delta_{\mathbf{r}_2}^* & U\delta_{\mathbf{r}_1, \mathbf{r}_2} & 0 & \Delta_{\mathbf{r}_1} \\ \Delta_{\mathbf{r}_1}^* & 0 & U\delta_{\mathbf{r}_1, \mathbf{r}_2} & \Delta_{\mathbf{r}_2} \\ 0 & \Delta_{\mathbf{r}_1}^* & \Delta_{\mathbf{r}_2}^* & U\delta_{\mathbf{r}_1, \mathbf{r}_2} \end{pmatrix} \vec{\Psi} = E\vec{\Psi}, \quad (2)$$

with the spinorial two-atoms wavefunction $\vec{\Psi}^T \equiv (\Psi_{\mathcal{A}\mathcal{A}}(\mathbf{r}_1\mathbf{r}_2), \Psi_{\mathcal{A}\mathcal{B}}(\mathbf{r}_1\mathbf{r}_2), \Psi_{\mathcal{B}\mathcal{A}}(\mathbf{r}_1\mathbf{r}_2), \Psi_{\mathcal{B}\mathcal{B}}(\mathbf{r}_1\mathbf{r}_2))$ that arises due to the sublattice degrees of freedom. The displacement operator is defined as $\Delta_{\mathbf{r}} f(\mathbf{r}) = -J\{e^{i\phi/4}[f(\mathbf{r} + e_x) + f(\mathbf{r} - e_x)] + e^{-i\phi/4}[f(\mathbf{r} + e_y) + f(\mathbf{r} - e_y)]\}$, with e_x and e_y denoting unit lattice displacements in the x - and y -directions, respectively (from $\mathcal{A} \rightarrow \mathcal{B}$ sites), and the species label $\sigma = 1$ has been dropped here. By using a unitary matrix \hat{S} , it is possible to express Eq. (2) in terms of the center of mass $\mathbf{R} = (\mathbf{r}_1 + \mathbf{r}_2)/2$ and relative $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$ coordinates [13]. Using plane-wave states, $\hat{S}\vec{\Psi}(\mathbf{r}_1\mathbf{r}_2) = e^{i\mathbf{K}\cdot\mathbf{R}} e^{i\mathbf{k}\mathbf{r}} \vec{\Psi}_{\mathbf{K},\mathbf{k}}$, we rewrite the non-interacting Schrödinger equation, which may then be diagonalized by using another unitary matrix \hat{S}' , to yield

$$(E - \lambda_{\mathbf{K},\mathbf{k}}^{(i)})\Phi_{0,\mathbf{K},\mathbf{k}}^{(i)} = 0, \quad \text{for } i = 1, 2, 3, 4, \quad (3)$$

where $\lambda_{\mathbf{K},\mathbf{k}}^{(i)}$ are the eigenenergies corresponding to the ‘‘pseudospin’’ vector $\vec{\Phi}_{0,\mathbf{K},\mathbf{k}} = \hat{S}'\vec{\Psi}_{\mathbf{K},\mathbf{k}}$. Since the interaction matrix $\langle \mathbf{k}|\hat{U}|\mathbf{k}' \rangle = U\mathbb{I}$ is diagonal and momentum independent, the Lippman-Schwinger equation for the scattering problem

$$|\vec{\Phi}_{\mathbf{K},\mathbf{k}} \rangle = |\vec{\Phi}_{0,\mathbf{K},\mathbf{k}} \rangle + \hat{G}(E_{\mathbf{K},\mathbf{k}})\hat{U}|\vec{\Phi}_{\mathbf{K},\mathbf{k}} \rangle$$

can be resummed to all orders in \hat{U} in the new basis $\vec{\Phi}_{0,\mathbf{K},\mathbf{k}}(\mathbf{r})$ for each pseudospin component to yield

$$\Phi_{\mathbf{K},\mathbf{k}}^{(i)}(\mathbf{r}) = \Phi_{0,\mathbf{K},\mathbf{k}}^{(i)}(\mathbf{r}) + \frac{UG^{(i)}(E_{\mathbf{K},\mathbf{k}}, \mathbf{r})}{1 - UG^{(i)}(E_{\mathbf{K},\mathbf{k}}, 0)}, \quad (4)$$

for $i = 1, 2, 3, 4$. Here, $G^{(i)}(E, \mathbf{r})$ is the Fourier transform of the non-interacting Green’s function $\tilde{G}^{(i)}(E, \mathbf{k}') = 1/(E - \lambda_{\mathbf{K},\mathbf{k}'}^{(i)} + i0)$. We see that in contrast to the usual atomic scattering problem, the pseudospin-carrying particles give rise to four independent scattering continua, see Fig. 1. Each scattering continuum, in addition, may support a bound state lying above it, which occurs at the diverging scattering amplitude, i.e., at the position of the pole with energy $E_B^{(i)}$,

$$\int_{1BZ} \frac{d^2\mathbf{k}'}{(2\pi)^2} \frac{1}{E_B^{(i)} - \lambda_{\mathbf{K},\mathbf{k}'}^{(i)}} = \frac{J}{U}. \quad (5)$$

Since the energy of the bound state belonging to one of the pseudospin components can lie in the scattering continuum of another pseudospin component, the model thus supports both scattering and on-resonance processes simultaneously at a specific energy. Furthermore, due to the breaking of time-reversal invariance, the existence of two flat bands $\lambda_{\mathbf{K}=0,\mathbf{k}}^{(i)} = 0$ as a function of the relative quasimomentum \mathbf{k} , at zero center-of-mass quasimomentum $\mathbf{K} = 0$, is topologically protected. In turn, it implies that a bound state solution *always* exists, with energy $E_B = U/J$, even for arbitrarily weak repulsive interaction strength U at $\mathbf{K} = 0$. This is the first central result of the current work: a bound state always exists due to the topologically protected flat bands. This is in stark contrast to the usual Hubbard model in a simple cubic lattice, where a critical repulsive interaction strength is required for a bound state solution [14].

Effective model - We now determine the collective mode associated with the bosonic system, when molecular formation is taken into account. The coherent conversion of atoms into molecules is described by the Hamiltonian

$$H_{\text{int}} = g \sum_{i \in A} a_{i,2}^{\dagger} a_{i,1} a_{i,1} + g \sum_{j \in B} b_{j,2}^{\dagger} b_{j,1} b_{j,1} + \text{h.c.}, \quad (6)$$

with an effective coupling strength g . The significant development in the understanding of molecular formation in an optical lattice, both theoretically and experimentally, by using the technique of Feshbach resonance allows for an accurate control of this term [5, 15–17]. The full

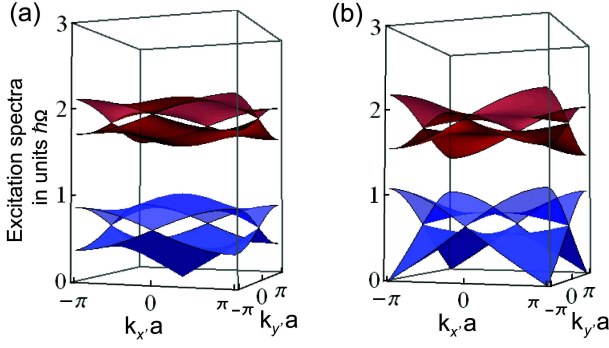


FIG. 2. (color online) Collective modes of the atom-molecule system subjected to a staggered flux: (a) scenario (I) with $J_1 = 0.11$, $J_2 = 0.08$, $\phi_{1,2} = \pi/5$, and (b) scenario (II) with $J_1 = -0.11$, $J_2 = -0.08$, $\phi_1 = \pi/3$, $\phi_2 = \pi/5$. The lattice site represents a sufficiently deep harmonic trap with frequency $\hbar\Omega = 10\text{kHz}$, in term of which all energy units are based. The other parameters are chosen as: $g = 0.8$, $U_1 = 0.2$, $\epsilon_1 = 1.5$, $\epsilon_2 = 2$, and $n_1 = 0.5$.

Hamiltonian $H = H_1 + H_2 + H_{\text{int}}$, describes a superfluid-Mott-insulator transition and an Ising transition within the superfluid phase [6, 18, 19]. The latter is due to an additional Z_2 symmetry associated with the atomic field, resulting in a phase boundary that separates a phase with a molecular condensate (MC) and normal atoms with an Ising symmetry, and a phase where both molecules and atoms (MC+AC) are condensed, see inset of Fig. 3.

To study the dynamics of the superfluid phases,

we first consider the regime where both atoms and molecules are condensed. Since the atomic and molecular fields are subjected to different flux values ϕ_σ , BEC states with distinct momenta can concurrently form. By performing a canonical transformation to the band representation, we find that the Hamiltonian is described by four distinct bands $\alpha_{\mathbf{k},\sigma}^\pm$ with energy dispersions $E_{\mathbf{k},\sigma}^\pm = \pm 2J_\sigma \{ \cos^2[k^+a] + \cos^2[k^-a] + 2 \cos[\phi_\sigma/2] \cos[k^+a] \cos[k^-a] \}^{1/2}$, with $k^\pm = (k_{x'} \pm k_{y'})/2$. The on-site interaction and atom-molecule coupling terms take the general inter-/intra-band coupling form. We then consider the formation of BECs, where the lowest energy operators for the atoms and for the molecules, both acquire a non-zero expectation value at the respective condensation points k_0 , with condensate number $N_{0,\sigma}$. The Bogoliubov approximation amounts to substituting $\alpha_{k_0,\sigma}^- \rightarrow \sqrt{N_{0,\sigma}} + \alpha_{k_0,\sigma}^-$ and keeping the fluctuations up to the quadratic order. Furthermore, it is justifiable to take $U_2 = 0$ for sufficiently low molecular condensate fraction $n_2 \ll 1$. The grand canonical mean-field Hamiltonian then reads

$$H = \frac{1}{2} \sum_{i\omega_n, \mathbf{k}} \hat{\psi}_{\mathbf{k}}^\dagger \hat{G}_{\mathbf{k}} \hat{\psi}_{\mathbf{k}}, \quad \hat{G}_{\mathbf{k}} = \begin{bmatrix} \hat{M}_{\mathbf{k},1}(g) & \hat{Q}_{\mathbf{k}} \\ \hat{Q}_{\mathbf{k}}^\dagger & \hat{M}_{\mathbf{k},2}(0) \end{bmatrix}, \quad (7)$$

where the matrices

$$\hat{Q}_{\mathbf{k}} = g \begin{bmatrix} \sqrt{n_1}(s_k^-)^* & 0 & \sqrt{n_1}(s_k^+)^* & 0 \\ 0 & \sqrt{n_1}s_{-k}^- & 0 & \sqrt{n_1}s_{-k}^+ \\ \sqrt{n_1}(s_k^+)^* & 0 & \sqrt{n_1}(s_k^-)^* & 0 \\ 0 & \sqrt{n_1}s_{-k}^+ & 0 & \sqrt{n_1}s_{-k}^- \end{bmatrix},$$

$$\hat{M}_{k,\sigma}(\kappa) = \begin{bmatrix} \varepsilon_{k,\sigma}^+ & \frac{1}{2}U_\sigma n_1 u_k^+ + \kappa\sqrt{n_2}(v_k^-)^* & 0 & \frac{1}{2}U_\sigma n_1 u_k^- + \kappa\sqrt{n_2}(v_k^+)^* \\ \frac{1}{2}U_\sigma n_1 (u_k^+)^* + \kappa\sqrt{n_2}v_k^- & (\varepsilon_{k,\sigma}^+)^* & \frac{1}{2}U_\sigma n_1 (u_{-k}^-)^* + \kappa\sqrt{n_2}v_{-k}^+ & 0 \\ 0 & \frac{1}{2}U_\sigma n_1 u_{-k}^- + \kappa\sqrt{n_2}(v_{-k}^+)^* & \varepsilon_{k,\sigma}^- & \frac{1}{2}U_\sigma n_1 u_k^+ + \kappa\sqrt{n_2}(v_k^-)^* \\ \frac{1}{2}U_\sigma n_1 (u_k^-)^* + \kappa\sqrt{n_2}v_k^+ & 0 & \frac{1}{2}U_\sigma n_1 (u_k^+)^* + \kappa\sqrt{n_2}v_k^- & (\varepsilon_{k,\sigma}^-)^* \end{bmatrix},$$

with the Nambu spinor defined as $\hat{\psi}_{\mathbf{k}}^\dagger = (\alpha_{k,1}^{\dagger}, \alpha_{-k,1}^{\dagger}, \alpha_{k,1}^{\dagger}, \alpha_{-k,1}^{\dagger}, \alpha_{k+k_0,2}^{\dagger}, \alpha_{-k+k_0,2}^{\dagger}, \alpha_{k+k_0,2}^{\dagger}, \alpha_{-k+k_0,2}^{\dagger})$. Here, $n_1(n_2)$ denotes the atomic (molecular) condensate fraction, $\varepsilon_{k,\sigma}^\pm = i\hbar\omega + E_{k,\sigma}^\pm + \epsilon_\sigma - \sigma\mu + 2U_\sigma n_\sigma$, where $i\omega_n$ is the Matsubara frequency, $\epsilon_{\sigma=1(2)}$ is the on-site energy for atoms (molecules) and μ is the chemical potential. The transformation coefficients are given as: $u_k^\pm = 1 \pm \eta_{1,k}^* \eta_{1,-k} \eta_{1,k_0} \eta_{1,k_0}$, $v_k^\pm = 1 \pm \eta_{2,0}^* \eta_{1,k} \eta_{1,-k}$, $s_k^\pm = 1 \pm \eta_{2,k+k_0}^* \eta_{1,k_0} \eta_{1,k}$, where $\eta_{\sigma,k} = \text{Arg}\{e^{i\phi_\sigma/4} \cos[(k_{x'} + k_{y'})/2a] + e^{-i\phi_\sigma/4} \cos[(k_{x'} - k_{y'})/2a]\}$.

Due to momentum conservation and the self-consistency condition (by demanding terms which are linear in the fluctuations to vanish), the two-species system is only allowed to have two condensation scenarios: (I) for $J_{1,2} > 0$, $0 \leq \phi_{1,2} < \pi$ and (II) for $J_2 < 0$, $\pi < \phi_1 < 3\pi$, $3\pi < \phi_2 < 4\pi$. In scenario (I), both atoms and molecules condense at the same momentum $\mathbf{k}_0 = 0$, whereas in scenario (II), while the molecules condense at $\mathbf{k}_0 = 0$, the atoms condense at a different momentum $\mathbf{k}_0 = (\pi/a, \pi/a)$. Outside these regimes, the system is

unstable due to the presence of linear fluctuations.

The excitation spectrum for both scenarios, obtained by solving the mean-field Hamiltonian (7), consists of four collective modes (keeping in mind the \mathcal{A} - \mathcal{B} sublattice and atom-molecule species degrees of freedom), see Fig. 2. The lowest branch is the Goldstone mode associated with \mathcal{A} - \mathcal{B} sublattice and atom-molecule in-phase density fluctuations. The next lowest branch corresponds to the \mathcal{A} - \mathcal{B} out-of-phase but atom-molecule in-phase oscillation mode. The third and fourth branches, separated

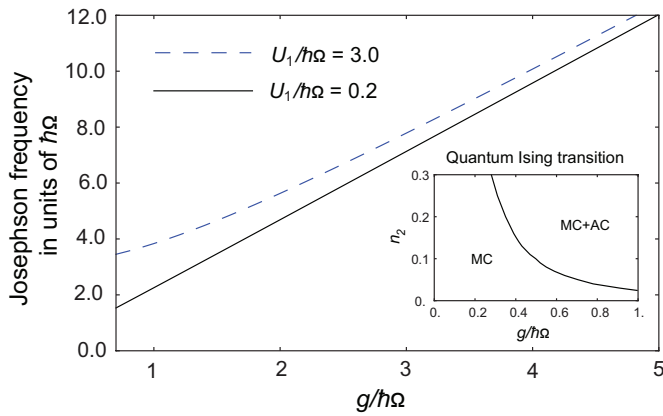


FIG. 3. (color online) Josephson frequency of two BECs with distinct momenta as a function of coupling strength for different on-site interaction strength, with $\hbar\Omega = 10\text{kHz}$. The inset shows the phase boundary of the quantum Ising transition plotted as a function of coupling strength. The latter is determined by the development of an instability in the atomic spectrum in the phase with only a molecular condensate.

by a gap from the two lowest branches, describe collective modes where atoms and molecules oscillate out-of-phase. In fact, the long wavelength physics of the third branch is equivalent to a coherent oscillation of density number of the two species in real-time. It thus describes an internal Josephson effect of an atomic and a molecular condensate, which can have distinct quantum numbers, with a frequency that is given by the energy gap. The dependence of the gap on the coupling strength g for scenario (II) is shown in Fig. 3 for weak and strong on-site interactions. This result provides a promising experimental opportunity to study the coherent dynamics of two *distinct* macroscopic ground states in a cold atomic system.

Experimental observation.— Bound states with pseudospin components can be observed experimentally, since the binding energy E_B^i can be inferred by radiofrequency spectroscopy [20, 21]. The lifetime of the bound state can also be probed through the techniques used in Ref. [14] via purification pulses. More importantly, a coherent oscillation between distinct macroscopic ground states can be experimentally observed by performing double-pulse Ramsey experiments [1, 4] for bosonic atoms in the optical lattice setup described in Ref. [8]. The regime of negative hopping parameters $J < 0$ for the staggered flux optical lattice may be realized experimentally, as shown in Ref. [22]. Recent experiments in optical lattices without flux have probed the negative hopping regime by shaking the lattice with a periodic perturbation [23], similar to the one occurring in Ref. [8]. The results can then be probed by combining state of the art techniques.

In conclusion, we consider a generalized Hubbard model subjected to a staggered flux with molecular for-

mation. The fact that the atoms carry a pseudospin gives rise to new scattering properties that are not present in conventional cold atomic systems. We propose a way to realize a quantum superposition of different species with different momenta, zero for the atomic condensate and finite for the molecular one. This work opens up new perspectives in the realization of more complex BECs and in the precise control of their dynamics.

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