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Causal Dynamical Triangulations and the Quest for Quantum Gravity

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Abstract

Quantum Gravity by Causal Dynamical Triangulation has over the last few years emerged as a serious contender for a nonperturbative description of the theory. It is a nonperturbative implementation of the sum-over-histories, which relies on few ingredients and initial assumptions, has few free parameters and – crucially – is amenable to numerical simulations. It is the only approach to have demonstrated that a classical universe can be generated dynamically from Planckian quantum fluctuations. At the same time, it allows for the explicit evaluation of expectation values of invariants characterizing the highly nonclassical, short-distance behaviour of spacetime. As an added bonus, we have learned important lessons on which aspects of spacetime need to be fixed a priori as part of the background structure and which can be expected to emerge dynamically.

1 Quantum gravity - taking a conservative stance

Many fundamental questions about the nature of space, time and gravitational interactions are not answered by the classical theory of general relativity, but lie in the realm of the still searched-for *theory of quantum gravity*: What is the quantum theory underlying general relativity, and what does it say about the quantum origins of space, time and our universe? What is the microstructure of spacetime at the shortest scale usually considered, the Planck scale $\ell_{\mathrm{Pl}} = 10^{-35}m$, and what are the relevant degrees of freedom determining the dynamics there? Are they the geometric dynamical variables of the classical theory (or some short-scale version thereof), or do they also include the topology and/or dimensionality of spacetime, quantities that classically are considered fixed? Can the dynamics of these microscopic degrees of freedom *explain* the observed large-scale structure of our own universe, which resembles a de Sitter universe at late times? Do notions like "space", "time" and "causality" remain meaningful on short scales, or are they merely macroscopically *emergent* from more fundamental, underlying Planck-scale principles?

Despite considerable efforts over the last several decades, it has so far proven difficult to come up with a consistent and quantitative theory of quantum gravity, which would be able to address and answer such questions [Kiefer, 2007]. In the process, researchers in high-energy theory have been led to consider ever more radical possibilities in order to resolve this apparent impasse, from postulating the existence of extra structures unobservable at low energies to invoking ill-defined ensembles of multiverses and anthropic principles [Ellis, 2006]. A grand unified picture has quantum gravity inextricably linked with the quantum dynamics of the three other known fundamental interactions, which requires a new unifying principle. Superstring theory is an example of such a framework, which needs the existence of an as yet unseen symmetry (supersymmetry) and ingredients (strings, branes, fundamental scalar fields). Loop quantum gravity, a non-unified approach, postulates the existence of certain fundamental quantum variables of Wilson loop type. Even more daring souls contemplate – inspired by quantum-gravitational problems – the abandonment of locality [Giddings, 2009] or substituting quantum mechanics by a more fundamental, deterministic theory ['t Hooft, 2007].

In view of the fact that none of these attempts has as yet thrown much light on the questions raised above, and that we have currently neither direct tests of quantum gravity nor experimental facts to guide our theory-building, a more conservative approach may be called for. What we will sketch in the following is an alternative route to quantum gravity, which relies on nothing but standard principles from quantum field theory, and on ingredients and symmetries already contained in general relativity. Its main premise is that *the framework of standard* *quantum field theory is sufficient to construct and understand quantum gravity as a fundamental theory,* if *the dynamical, causal and nonperturbative properties of spacetime are taken into account properly.*

Significant support for this thesis comes from a new candidate theory, *Quantum Gravity from Causal Dynamical Triangulation (CDT)*, whose main ideas and results will be described below. CDT quantum gravity is a nonperturbative implementation of the gravitational path integral, and has already passed a number of nontrivial tests with regard to producing the correct classical limit. Its key underlying idea was conceived more than ten years ago [Ambjørn and Loll, 1998], in an effort to combine the insights of geometry-based nonperturbative canonical quantum gravity with the powerful calculational and numerical methods available in covariant approaches. After several years of modelling and testing both the idea and its implementation in spacetime dimensions two and three, where they give rise to nontrivial dynamical systems of "quantum geometry" [Ambjørn et al., 2000a, Ambjørn et al., 2001], the first results for the physically relevant case of four dimensions were published in 2004 [Ambjørn et al., 2004a, Ambjørn et al., 2005a].

Let us also mention that an independent approach to the quantization of gravity, much in the spirit of our main premise[1] and based on the 30-year-old idea of "asymptotic safety" has been developing over roughly the same time period [Niedermaier and Reuter, 2006, Niedermaier, 2006]. It shares some features (covariance, amenability to numerical computation) as well as some results (on the spectral dimension) with CDT quantum gravity, and may ultimately turn out to be related.

2 What CDT quantum gravity is about

Quantum gravity theory based on causal dynamical triangulations is an explicit, nonperturbative and background-independent realization of the formal *gravitational path integral* (a.k.a. the "sum over histories") on a differential manifold M,

$$Z(G_N,\Lambda) = \int_{\mathcal{G}(\mathrm{M})=\frac{\mathrm{Lor(M)}}{\mathrm{Diff(M)}}} \mathcal{D}g_{\mu\nu}\; \mathrm{e}^{iS^{\mathrm{EH}}[g_{\mu\nu}]}, \quad S^{\mathrm{EH}} = \int d^4x \sqrt{\det g}\left(\frac{1}{G_N}R - 2\Lambda\right), \quad (1)$$

where S^{EH} denotes the four-dimensional Einstein-Hilbert action, G_N is the gravitational or Newton's constant and Λ the cosmological constant, and the path integral is to be taken over all spacetimes (metrics $g_{\mu\nu}$ modulo diffeomorphisms),

[1] although the role of "causality", which enters crucially in CDT quantum gravity, remains unclear in this approach

with specified boundary conditions. The method of causal dynamical triangulation turns (1) into a well-defined finite and regularized expression, which can be evaluated and whose continuum limit (removal of the regulator) can be studied systematically [Ambjørn et al., 2000].

One proceeds in analogy with the path-integral quantization à la Feynman and Hibbs of the nonrelativistic particle. This is defined as the continuum limit of a regularized sum over paths, where the contributing 'virtual' paths are taken from an ensemble of piecewise straight paths, with the length a of the individual segments going to zero in the limit. The corresponding CDT prescription in higher dimensions is to represent the space $\mathcal{G}$ of all Lorentzian spacetimes in terms of a set of triangulated, piecewise flat manifolds[2], as originally introduced in the classical theory as "general relativity without coordinates" [Regge, 1961]. For our purposes, the simplicial approximation $\mathcal{G}_{a,N}$ of $\mathcal{G}$ contains all simplicial manifolds T obtained from gluing together at most N four-dimensional, triangular building blocks of typical edge length a, with a again playing the role of an ultraviolet (UV) cut-off (see Fig. 1). The explicit form of the regularized gravitational path integral in CDT is

$$Z^{\mathrm{CDT}}_{a,N} = \sum_{\substack{\text{triangulated} \\ \text{spacetimes } T\in\mathcal{G}_{a,N}}} \frac{1}{C_T} \mathrm{e}^{iS^{\mathrm{Regge}}[T]}, \tag{2}$$

where S^{Regge} is the Regge version of the Einstein-Hilbert action associated with the simplicial spacetime T, and C_T denotes the order of its automorphism group. The discrete volume N acts as a volume cutoff. We still need to consider a suitable continuum or scaling limit

$$Z^{\mathrm{CDT}} := \lim_{\substack{N\to\infty \\ a\to 0}} Z^{\mathrm{CDT}}_{a,N} \tag{3}$$

of (2), while renormalizing the original bare coupling constants of the model, in order to arrive (if all goes well) at a theory of quantum gravity. The two limits in (3) are usually tied together by keeping a physical four-volume, defined as $V_4 := a^4 N$ fixed. In the limiting process a is taken to zero, $a \to 0$, and the individual discrete building blocks are then literally "shrunk away".

Let us summarize the key features of the construction scheme thus introduced. Unlike what is possible in the continuum theory, the path integral (2) is defined directly on the physical configuration space of *geometries*. It is nonperturbative in the sense of including geometries which are "far away" from any classical solutions, and it is background-independent in the sense of performing the sum

[2] Unlike in the particle case, there is no embedding space; all geometric spacetime data are defined intrinsically, just like in the classical theory.

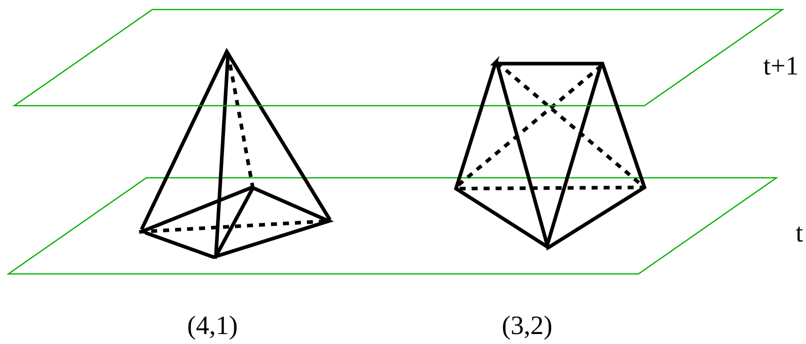

Figure 1: The two fundamental building blocks of CDT are four-simplices with flat, Minkowskian interior. They are spanned by spacelike edges, which lie entirely within spatial slices of constant time t, and timelike edges, which interpolate between adjacent slices of integer time. A building block of type (m, n) has m of its vertices in slice t, and n in slice $t + 1$.

"democratically", without distinguishing any given geometry (say, as a preferred background). Of course, these nice properties of the regularized path integral are only useful because *we are able to evaluate* $Z^{\rm CDT}$ *quantitatively*, with an essential role being played by Monte Carlo simulations. These, together with the associated finite-size scaling techniques [Newman and Barkema, 1999], have enabled us to extract information about the nonperturbative, strongly coupled quantum dynamics of the system which is currently not accessible by analytical methods, neither in this nor any other approach to quantum gravity. It is reminiscent of the role played by lattice simulations in pinning down the nonperturbative behaviour of QCD (although this is a theory we already know *much* more about than quantum gravity).

3 What CDT quantum gravity is not about

Although causal dynamical triangulation is sometimes called a discrete approach, this is potentially misleading. First, one can of course think of the simplicial building blocks as discrete objects, but they are assembled into spacetimes that are perfectly continuous and not discrete. The space of geometries *is* discretized in the sense that both four-volume and curvature contribute in discrete "bits" to the total action. However, this is only a feature of the chosen regularization, and has no physical significance as such. As explained in the previous section, the characteristic edge length a plays the role of an intermediate regulator and UV cut-off for the geometry. In the continuum limit, a is to be taken to zero strictly. In practice, what will usually suffice is to choose a significantly smaller than the scale at which one is trying to extract physical results, hence $a \ll \ell_{\rm Pl}$ if we want to establish Planck scale dynamics.

Adherents of the idea of fundamental discreteness might be tempted to identify the edge length a with a fundamental, shortest length scale, typically, the Planck length. However, this would be an ad hoc prescription which is in no way required by the construction. Besides, it has the unpleasant feature that physics at the Planck scale will then depend explicitly on the details of the chosen regularization. For example, choosing squares instead of triangles, or choosing a different discrete realization of the Einstein action will in general lead to different Planckian dynamics, thus introducing an infinite ambiguity *at that scale*. It is not good enough if all these different theories produce identical classical physics on large scales, because in quantum gravity one is of course interested in finding a (hopefully unique) description of physics at the Planck scale.

Instead of putting it in by hand, the issue of fundamental discreteness in quantum gravity needs to be addressed *dynamically*. Is such a scale generated by the dynamics of the theory? Although there are numerous claims that Planck-scale discreteness is almost "self-evident" (often, to render one's favourite calculation of black hole entropy finite), there is at this stage no concrete evidence for such a discreteness in full, four-dimensional quantum gravity[3]. We have up to now not seen any indication of it, but it is conceivable that there exist nonperturbative quantum operators in CDT quantum gravity which measure lengths (or higher-dimensional volumes) and have a discrete spectrum as $a \to 0$, thus indicating fundamental discreteness. Even if such a discreteness were found, whether or not the currently unknown "fundamental excitations of quantum gravity" are discrete or not may be yet another issue. It is not even particularly clear what one means by such a statement and whether it can be turned into an operationally well-defined question in the nonperturbative theory, and not one which is merely a feature of a particular representation of the quantum theory.

As we will see in more detail below, CDT is – as far as we are aware – the only nonperturbative approach to quantum gravity which has been able to dynamically generate its own, physically realistic background from nothing but quantum fluctuations. More than that, because of the minimalist set-up and the methodology used (quantum field theory and critical phenomena), the results obtained are robust in the sense of being largely independent of the details of the chosen regularization procedure and containing few free parameters. This is therefore also one of the perhaps rare instances of a candidate theory of quantum gravity which can potentially be falsified. In fact, the Euclidean version of the theory

[3]The derivation of discrete aspects of the spectrum of the area and volume operators in loop quantum gravity [Rovelli and Smolin, 1995, Loll, 1995] disregards dynamics (in the form of the Hamiltonian constraint), quite apart from the fact that one can argue that discreteness has been put in "by hand" by choosing a quantum representation where one-dimensional Wilson loops are well-defined operators.

extensively studied in the 1990s has already been falsified because it does not lead to the correct classical limit [Bialas et al., 1997, de Bakker, 1996]. CDT quantum gravity improves on this previous attempt by building a causal structure right into the fabric of the model. Our investigations of both the quantum properties and the classical limit of this candidate theory are at this stage not sufficiently complete to provide conclusive evidence that we have found *the* correct theory of quantum gravity, but results until now have been unprecedented and most encouraging, and have thrown up a number of nonperturbative surprises.

4 CDT key achievements I - Demonstrating the need for causality

We will confine ourselves to highlighting some of the most important results and new insights obtained in CDT quantum gravity, without entering into any of the technical details. The reader is referred to the literature cited in the text, as well as to the various overview articles available on the subject [Ambjørn et al., 2009] for more information.

The crucial lesson learned for nonperturbative gravitational path integrals from CDT quantum gravity is that the ad hoc prescription of integrating over curved Euclidean *spaces* of metric signature $(++++)$ instead of the physically correct curved Lorentzian space*times* of metric signature $(-+++)$ generally leads to inequivalent and (in $d=4$) incorrect results. "Euclidean quantum gravity" of this kind, as advocated by S. Hawking and collaborators [Gibbons and Hawking, 1993], adopts this version of doing the path integral mainly for the technical reason to be able to use real weights $\exp(-S^{\rm eu})$ instead of the complex amplitudes $\exp(iS^{\rm lor})$ in its evaluation. The same prescription is used routinely in perturbative quantum field theory on flat Minkowski space, but in that case one can rely on the existence of a well-defined Wick rotation to relate correlation functions in either signature. This is *not* available in the context of continuum gravity beyond perturbation theory on a Minkowski background, but one may still hope that by *starting out* in Euclidean signature and quantizing this (wrong) theory, an inverse Wick rotation would then "suggest itself" to translate back the final result into physical, Lorentzian signature. Alas, this has never happened, because – we would contend – no one has been able to make much sense of nonperturbative Euclidean quantum gravity in the first place, even in a reduced, cosmological context[4].

CDT quantum gravity has provided the first explicit example of a nonperturbative gravitational path integral (in a toy model of two-dimensional gravity) which is exactly soluble and leads to distinct and inequivalent results depending

[4]a discussion of the kind of problems that arise can be found in [Halliwell and Louko, 1990]

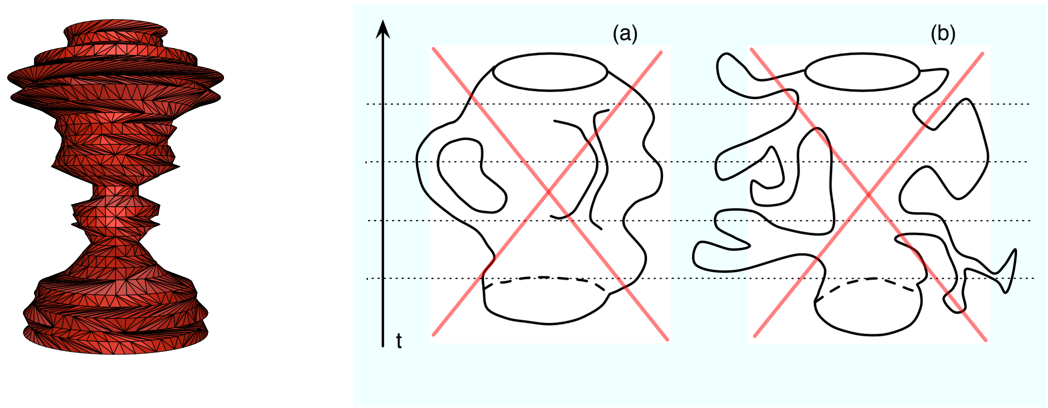

Figure 2: Typical history contributing to the loop-loop correlator in the 2d Lorentzian CDT path integral (left), time t is pointing up. The essential difference with the corresponding Euclidean amplitude is that the (one-dimensional) spatial slices, although quantum-fluctuating, are not allowed to change topology as a function of time t, thus avoiding causality-violating branching and merging points. This excludes spaces with wormholes (right picture, a) and those with 'baby universes' branching out in the time direction (right picture, b).

on whether the sum over histories is taken over Euclidean spaces or Lorentzian spacetimes (or, more precisely, Euclidean spaces which are obtained by Wick rotation – which *does* exist for the class of simplicial spacetimes under consideration – from Lorentzian spacetimes). The Lorentzian path integral was first solved in [Ambjørn and Loll, 1998], and a quantity one can compute and compare with the Euclidean version found in [Ambjørn and Makeenko, 1990] is the cylinder amplitude (Fig. 2). In the Lorentzian CDT case, only those histories are summed over which possess a global time slicing *with respect to which no spatial topology changes are allowed to occur*. After Wick rotation, this set constitutes a strict subset of all Euclidean (triangulated) spaces. In the latter there is no natural notion of 'time' or 'causality' and branching geometries are thus always present.

In the two-dimensional setting this means that one has identified a new class of anisotropic statistical mechanical models of fluctuating geometry. Several intriguing results that have been found from numerical simulations of matter-coupled versions of the model (which have so far resisted analytical solution) indicate that their geometric disorder is less severe than that of their Euclidean counterparts, and in particular that they seem to lead to critical matter exponents identical to those of the corresponding matter model on fixed, flat lattices

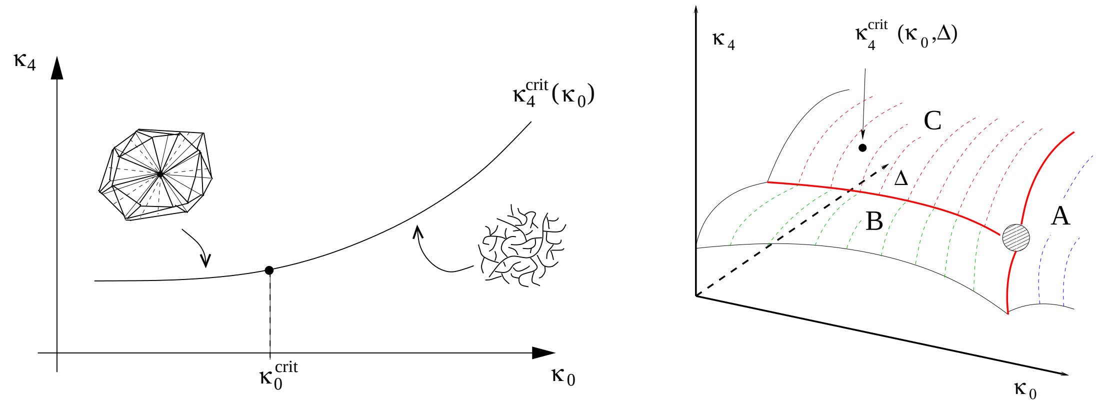

Figure 3: The phase diagrams of Euclidean (left) and Lorentzian (right) quantum gravity from dynamical triangulations, with κ_0 and κ_4 denoting the bare inverse Newton's constant and (up to an additive shift) the bare cosmological constant. After fine-tuning to the respective subspace where the cosmological constant is critical (tantamount to performing the infinite-volume limit), there are (i) two phases in EDT: the crumpled phase $\kappa_0 < \kappa_0^{\rm crit}$ with infinite Hausdorff dimension and the branched-polymer phase $\kappa_0 > \kappa_0^{\rm crit}$ with Hausdorff dimension 2, none of them with a good classical limit, (ii) three phases in CDT: A and B (the Lorentzian analogues of the branched-polymer and crumpled phases), and a *new* phase C, where an extended, four-dimensional universe emerges. The parameter Δ in CDT parametrizes a finite relative scaling between space- and time-like distances which is naturally present in the Lorentzian case.

[Ambjørn et al., 1999, Ambjørn et al., 2008a].

Another new direction in which the two-dimensional CDT model has been generalized is a controlled relaxation of the ban on branching points, while adhering to a global notion of proper time [Loll and Westra, 2003, Loll et al., 2006]. This has culminated recently in the formulation of a fully-fledged CDT string field theory in zero target space dimensions [Ambjørn et al. 2008]. The matrix model formulation of the theory makes it possible to perform the sum over two-dimensional topologies explicitly [Ambjørn et al., 2009a]. These developments are described in more detail elsewhere in this volume [Ambjørn et al., 2009b].

Returning to the implementation of strict causality on path integral histories, a key finding of CDT quantum gravity is that a result similar to that found in two dimensions also holds also in dimension four. The geometric degeneracy of the phases (in the sense of statistical systems) found in Euclidean dynamical triangulations [Bialas et al., 1996, Bialas et al., 1997], and the resulting absence of a good classical limit can in part be traced to the 'baby universes' present in the Euclidean approach also in four dimensions. As demonstrated by the results in

[Ambjørn et al., 2004a, Ambjørn et al., 2005a], the requirement of microcausality (absence of causality-violating points) of the individual path integral histories leads to a qualitatively new phase structure, containing a phase where the universe on large scales is extended and four-dimensional (Fig. 3), as required by classical general relativity. Apart from the nice result that the problems of the Euclidean approach are cured by this prescription, this reveals an intriguing relation between the microstructure of spacetime (micro-causality = suppression of baby universes in the time direction at sub-Planckian and bigger scales) and its emergent macrostructure. Referring to the questions raised at the beginning of Sec. 1, the more general lessons learned from this are that (i) "causality" is not emergent, but needs to be put in by hand on each spacetime history, and (ii) similarly, "time" is not emergent. It is put into CDT by choosing a preferred (proper-)time slicing at the regularized level, but this turns out to be only a necessary condition to have a notion of time (as part of an extended universe) present in the continuum limit, at least on large scales. It is not sufficient, because in other phases of the CDT model (Fig. 3) the spatial universe apparently does not persist at all (B) or only intermittently (A), see also reference [Ambjørn et al., 2005a].

5 CDT key achievements II - The emergence of spacetime as we know it

This brings us straight to the nature of the extended spacetime found in phase C of CDT quantum gravity. What is it, and how do we know? We cannot just 'look at' the quantum superposition of geometries, which individually of course get wilder and spikier as the continuum limit $a \to 0$ is approached, just like the nowhere differentiable paths of the path integral of the nonrelativistic particle [Reed and Simon, 1975]. We need to define and measure geometric *quantum observables*, evaluate their expectation values on the ensemble of geometries and draw conclusions about the behaviour of the "quantum geometry" generated by the computer simulations (that is, the ground state of minimal Euclidean action). Rather strikingly, inside phase C the many microscopic building blocks superposed in the nonperturbative path integral 'arrange themselves' into an extended quantum spacetime whose macroscopic shape is that of the well-known *de Sitter universe* [Ambjørn et al., 2007, Ambjørn et al., 2008b]. This amounts to a highly nontrivial test of the classical limit, which is notoriously difficult to achieve in models of nonperturbative quantum gravity. The precise dynamical mechanism by which this happens is unknown, however, it is clear that "entropy" (in other words, the measure of the path integral, or the number of times a given weight factor $\exp(-S)$ is realized) plays a crucial role in producing the outcome. This is reminiscent of phenomena in condensed matter physics, where

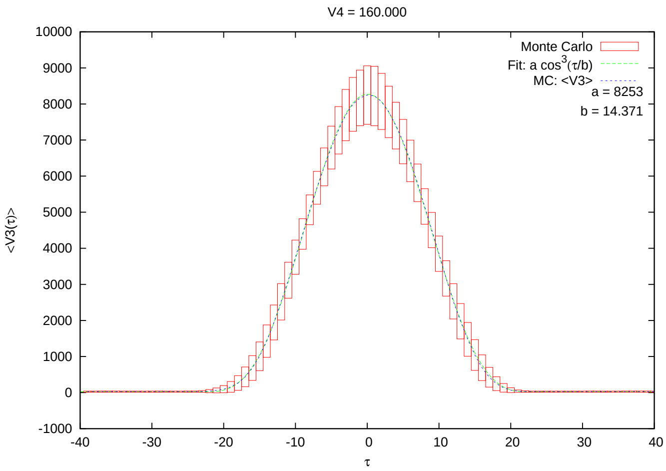

Figure 4: The shape $\langle V_3(\tau)\rangle$ of the CDT quantum universe, fitted to that of Euclidean de Sitter space (the "round four-sphere") with rescaled proper time, $\langle V_3(\tau)\rangle = a\cos^3(\tau/b)$. Measurements taken for a universe of four-volume $V_4 =$ 160.000 and time extension $T = 80$. The fit of the Monte Carlo data to the theoretical curve for the given values of a and b is impressive. The vertical boxes quantify the typical scale of quantum fluctuations around $\langle V_3(\tau)\rangle$.

systems of large numbers of microscopic, interacting constituents exhibit macroscopic, "emergent" behaviour which is difficult to derive from the microscopic laws of motion. This makes it appropriate to think of CDT's de Sitter space as a *self-organizing quantum universe* [Ambjørn et al., 2008c].

The manner in which we have identified (Euclidean) de Sitter space from the computer data is by looking at the expectation value of the volume profile $V_3(t)$, that is, the size of the spatial three-volume as function of proper time t. For a classical Lorentzian de Sitter space this is given by

$$V_3(t) = 2\pi^2 (c \cosh \frac{t}{c})^3, \quad c = const. > 0, \tag{4}$$

which for $t > 0$ gives rise to the familiar, exponentially expanding universe, thought to give an accurate description of our own universe at late times, when matter can be neglected compared with the repulsive force due to the positive cosmological constant. Because the CDT simulations for technical reasons have to be performed in the Euclidean regime, we must compare the expectation value of the shape with those of the analytically continued expression of (4), with respect to the Euclidean time $\tau := -it$. After normalizing the overall four-volume and adjusting computer proper time by a constant to match continuum proper time, the averaged volume profile is depicted in Fig. 4.

A few more things are noteworthy about this result. Firstly, despite the fact that the CDT construction deliberately breaks the isotropy between space and time, at least on large scales the full isotropy is restored by the ground state of the theory for precisely one choice of identifying proper time, that is, of fixing a relative scale between time and spatial distances in the continuum. Secondly, the computer simulations by necessity have to be performed for finite, compact spacetimes, which also means that a specific choice has to be made for the spacetime topology. For simplicity, to avoid having to specify boundary conditions, it is usually chosen to be $S^1 \times S^3$, with time compactified[5] and spatial slices which are topological three-spheres. What is reassuring is the fact that the bias this could in principle have introduced is "corrected" by the system, which clearly is driven dynamically to the topology of a four-sphere (as close to it as allowed by the kinematical constraint imposed on the three-volume, which is not allowed to vanish at any time). Lastly, we have also analyzed the quantum fluctuations around the de Sitter background - they match to good accuracy a continuum saddlepoint calculation in minisuperspace [Ambjørn et al., 2008b], which is one more indication that we are indeed on the right track.

6 CDT key achievements III - a window on Planckian dynamics

Having discussed some of the evidence for obtaining the correct classical limit in CDT quantum gravity, let us turn to the *new* physics we are after, namely, what happens to gravity and the structure of spacetime at or near the Planck scale. We will describe one way of probing the short-scale structure, by setting up a *diffusion process* on the ensemble of spacetimes, and studying associated observables. The speed by which an initially localized diffusion process spreads into an ambient space is sensitive to the dimension of the space. Conversely, given a space M of unknown properties, it can be assigned a so-called *spectral dimension* D_S by studying the leading-order behaviour of the average return probability $\mathcal{R}_V(\sigma)$ (of random diffusion paths on M starting and ending at the same point x) as a function of the external diffusion time σ,

$$\mathcal{R}_V(\sigma) := \frac{1}{V(M)} \int_M d^d x \; P(x,x;\sigma) \propto \frac{1}{\sigma^{D_S/2}}, \qquad \sigma \leq V^{2/D_S}, \tag{5}$$

where $V(M)$ is the volume of M, and $P(x_0,x;\sigma)$ the solution to the heat equation on M,

$$\partial_\sigma P(x_0,x,\sigma) = \nabla_x^2 P(x_0,x,\sigma). \tag{6}$$

[5]the period is chosen much larger than the time extension of the universe and does not influence the result

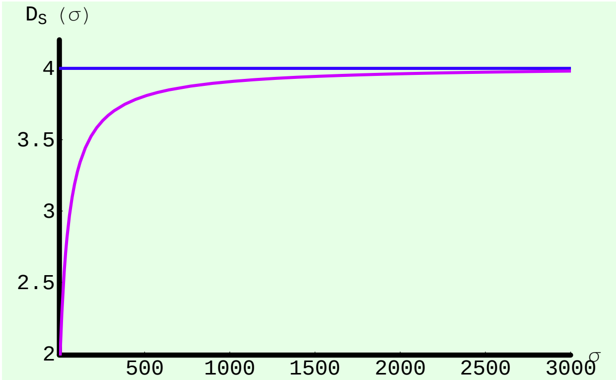

Figure 5: The spectral dimension $D_S(\sigma)$ of the CDT-generated quantum universe (lower curve, error bars not included), contrasted with the corresponding curve for a classical spacetime, simply given by the constant function $D_S(\sigma) = 4$. We assume $\sigma \ll V^{2/D_S}$, so no finite-volume effects are present.

Diffusion processes can be defined on very general spaces, for example, on fractals, which are partially characterized by their spectral dimension (usually not an integer, see [ben-Avraham and Havlin, 2000]). Relevant for the application to quantum gravity is that the expectation value $\langle \mathcal{R}_V(\sigma) \rangle$ can be measured on the ensemble of CDT geometries, giving us the spectral dimension of the dynamically generated quantum universe, with the result that $D_S(\sigma)$ depends nontrivially on the diffusion time σ [Ambjørn et al., 2005]! Since the linear scale probed in the diffusion is on the order of that of a random walker, $\sqrt{\sigma}$, short diffusion times probe the short-scale structure of geometry, and long ones its large-scale structure. The measurements from CDT quantum gravity, extrapolated to all values of σ, lead to the lower curve in Fig. 5, with asymptotic values $D_S(0) = 1.82 \pm 0.25$, signalling highly nonclassical behaviour near the Planck scale, and $D_S(\infty) = 4.02 \pm 0.1$, which is compatible with the expected classical behaviour. Previous Euclidean models never showed such a scale-dependence, reflecting their lack of an interesting geometric structure as a function of scale. For CDT in three space-time dimensions, there is evidence for an analogous scale dependence [Benedetti and Henson, 2009].

This somewhat unexpected result found in nonperturbative CDT quantum gravity has brought into focus the role of "dynamical dimensions" as diffeomorphism-invariant indicators of nonclassicality at the Planck scale[6]. Interestingly, a similar dimensional reduction from four to two near the Planck scale has since been found in disparate approaches to quantum gravity, most prominently, a nonperturbative renormalization group flow analysis [Lauscher and Reuter, 2005],

[6]Other notions of dimensionality are the Hausdorff and the fractal dimension.

and so-called Lifshitz gravity [Hořava, 2009]. The coincidence is certainly intriguing and could mean that at a more fundamental level the approaches have more in common than we currently understand (*and* capture a true aspect of nonperturbative quantum gravity). Reproducing dimensional reduction could then even become a "test" of quantum gravity, similar to how the derivation of the black hole entropy formula $S_{\mathrm{BH}} = A/4$ is often viewed, with the difference that the latter is usually associated with a semiclassical context, whereas the former is thought to characterize the behaviour of the theory in the deep UV.

Further evidence of nonclassicality on short scales in CDT comes from measurements of geometric structures in spatial slices $\tau = const$, including a measurement of their Hausdorff and spectral dimensions [Ambjørn et al., 2005a].

7 Open issues and outlook

As we have summarized above, significant strides have been made in the causal dynamical triangulations quantization program in demonstrating its compatibility with classical general relativity on large scales, and at the same time exploring its true quantum properties on small scales. A number of important issues are the subject of ongoing and future research. Firstly, as explained in more detail in [Ambjørn et al., 2008b], one would like to tune the bare parameters of the CDT simulations so as to obtain a better length resolution and get even closer to and, if possible, below the Planck scale. (Current simulations operate with quantum universes of the order of 10-20 Planck lengths across.) This would enable us to look for more direct evidence of the existence or otherwise of the nontrivial UV fixed point seen in truncated renormalization group flows [Lauscher and Reuter, 2001].

An important challenge is to reproduce further aspects of the classical limit correctly, one of which is the derivation of Newton's law "from scratch" in the nonperturbative theory. As a possibly first step towards this goal, some physical consequences of the presence of an isolated point mass in CDT's quantum de Sitter universe have been analyzed in [Khavkine et al., 2010]. Another natural area of application is the early universe, with or without the addition of a scalar field ("inflaton"), to check and discriminate between the often ambiguous and contradictory claims of quantum cosmological models in a context where *all* fluctuations of the geometry are present, not just the overall scale factor. A concrete example of what one might be able to do is nailing down factor-ordering ambiguities in the cosmological path integral [Maitra, 2009].

Coming back to some of the questions we raised at the outset of this article, the preliminary conclusion about the nature of quantum spacetime is that it is nothing like a four-dimensional classical manifold on short scales. In addition to its anomalous spectral dimension, its naïve Regge curvature diverges, indicating a singular behaviour reminiscent of (but surely worse than) that of the particle paths constituting the support of the Wiener measure. However, it apparently is *not* literally a "spacetime foam", if by that one means some bubbling, topology-changing entity: one of the main findings of dynamically triangulated models of nonperturbative quantum gravity is that allowing for local topology changes and making them part of the dynamics renders the quantum superposition inherently unstable and is incompatible with a good classical limit. Even if local topology change is not part of quantum-gravitational dynamics, we saw that global topology, as well as short-scale dimensionality are determined dynamically and do not necessarily coincide with the (somewhat arbitrary) choices made for them as part of the regularized formulation. What makes these perhaps surprising findings possible is the fact that CDT quantum gravity allows for large curvature fluctuations on short scales, and that the construction of the final theory involves a nontrivial limiting process, which the computer simulations are able to approximate.

In summary, if there is indeed a unique, interacting quantum field theory of spacetime geometry in four dimensions, which does not contain any exotic ingredients, and has general relativity as its classical limit, the CDT approach has a good chance of finding it. It relies only on a minimal set of ingredients and priors: the quantum superposition principle, locality, (micro-)causality, a notion of (proper) time and standard tools from quantum field theory otherwise[7], has few free parameters (essentially the couplings of the phase diagram of Fig. 3), and by virtue of its construction through a scaling limit can rely on a considerable degree of universality in the sense of critical system theory. Although many issues remain to be tackled and understood, the interesting new results and insights CDT has produced to date make for a pretty good start.

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[7]This puts it about on a par with the renormalization group approach of [Niedermaier and Reuter, 2006, Niedermaier, 2006], makes fewer assumptions than loop quantum gravity [Thiemann, 2007], but is not quite as minimalistic as the causal set approach [Henson, 2006].

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