Spin-Drag Hall Effect in a Rotating Bose Mixture

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We show that in a rotating two-component Bose mixture, the spin drag between the two different spin species shows a Hall effect. This spin-drag Hall effect can be observed experimentally by studying the out-of-phase dipole mode of the mixture. We determine the damping of this mode due to spin drag as a function of temperature. We find that due to Bose stimulation there is a strong enhancement of the damping for temperatures close to the critical temperature for Bose-Einstein condensation.

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Introduction.—Electronic transport is one of the main topics of interest in condensed-matter physics, and an especially important phenomenon in electronic transport is the Hall effect, discovered in the late 19th century by Hall. He observed that if a magnetic field **B** is applied perpendicular to the current density **j** through a conductor, the Lorentz force leads to a voltage drop in the direction perpendicular to both the current density and the magnetic field. This voltage is proportional to $\mathbf{j} \times \mathbf{B}$, with a proportionality constant that depends only on the density of electrons and not on any other material parameters [1]. While the discovery of the Hall effect predates that of the electron, note that electronic transport is in fact fermionic transport since it is mediated by the movement of electrons. By now many variations of the Hall effect have been found: the (integer and fractional) quantum Hall effects in which the Hall voltage is quantized [2–4], the spin Hall effect [5,6], and the quantum spin Hall effect [7]. Recently, a spin Hall drag effect has also been proposed [8]. The latter is, as all spin Hall effects are, due to spin-orbit interactions that play no role in atomic Bose gases if they are not externally introduced by applying an appropriate laser field configuration [9]. The proposal of Ref. [8] is therefore physically very different from what we discuss below.

An important field of physics which connects few-body atomic physics with many-body and condensed-matter physics is that of cold atoms. The Bose-Einstein condensation of bosons at very low temperatures was already predicted by Einstein in 1924, but only observed directly in 1995 [10]. Over the past 15 years, techniques have been getting steadily more refined, and it is now possible to make all sorts of degenerate atomic mixtures consisting of several spin states or of several different atomic species of either fermions or bosons. These mixtures are always trapped in optical and/or magnetic potentials, in which the atoms can be set into rotation, for example, by stirring with a so-called laser spoon [11].

It is tempting to combine the above two fields to also get more insight into the physics of bosonic transport. This seems less than straightforward, since it is not possible to simply attach leads to a cloud of cold atoms to set up a steady-state transport current of atoms through the mixture. Moreover, in the cold-atom situation there are no obvious mechanisms that relax the particle current and give nonzero resistivities. However, we can use the phenomenon of spin drag as a bridge between these two worlds [12–14]. Spin drag was first proposed by D'Amico and Vignale by making an analogy with Coulomb drag between two electron layers [15]. It was later observed by Weber *et al.* [16]. Whereas in the classic Coulomb drag experiment electrons are differentiated by the layer they occupy, in spin drag the spin of the electron is the relevant degree of freedom, i.e., electrons of one spin species drag along electrons of the other spin species. The resistivity created by this spin drag, which is a resistivity to spin but not to charge currents, typically goes as $\rho_D \propto T^2$ in electronic systems, with T the temperature. This is the trademark of a Fermi-liquid like behavior.

In our earlier work [13], we investigated the situation in which the particles involved in the spin drag are bosons instead of fermions. We proposed an idealized setup in which spin-1 bosons in the state $|m_F = +1\rangle$ are accelerated along a torroidal trap by a time-dependent magnetic-field texture that creates a fictitious electric field. The atoms in state $|m_F = 0\rangle$, which are also present, do not feel this force but experience spin drag due to collisions with the other species. We found that due to the Bose enhancement of interatomic scattering, the drag resistivity increases at lower temperatures. For the one-dimensional setup considered, it in fact behaves as $\rho_D \propto T^{-5/2}$ for low temperatures, in strong contrast with the usual quadratic Fermi-liquid result.

In this Letter, we discuss spin drag in a rotating Bose mixture. We consider the realistic situation of a three-dimensional Bose mixture with two spin components, present in equal numbers, just above the temperature for Bose-Einstein condensation. We first consider the homogeneous case and look in linear response for steady-state solutions of the appropriate Boltzmann equation with a nonzero spin current. We find that the drag resistivity now becomes a 3×3 matrix with nonzero off-diagonal elements that are proportional to the rotation speed, which represents a Hall effect. Indeed, these off-diagonal elements are analogous to those found in the electronic Hall effect and, in particular, do not depend on the specific collisional details of the mixture that determine the diagonal resistivities, but only on the atomic density and external rotation frequency.

Such steady-state solutions no longer exist in the realistic situation that the atomic mixture is trapped in an external harmonic potential. The spin-drag Hall effect can nevertheless be observed in that case by considering the dipole mode in which the two spin components oscillate out of phase with each other, because this mode obtains an orthogonal, i.e., a transverse component due to the spin-drag Hall effect. Moreover, the longitudinal spin drag leads to damping of this mode, which makes it interesting to find out how the relaxation rate of these modes depends on temperature. To obtain this, we again solve the Boltzmann equation for this specific case in linear response, and find that the relaxation rate shows a substantial increase as the temperature gets closer to the critical temperature. In three dimensions the relaxation rate does, however, remain finite at the transition temperature.

Spin-drag Hall effect.—To illustrate the spin-drag Hall effect we consider first a homogeneous three-dimensional Bose mixture of two spin states, which we label $|0\rangle$ and $|1\rangle$, in the normal state, where no Bose-Einstein condensate is present. We assume that the bosons in spin state $|1\rangle$ couple to an external force $\mathbf{F}$, and that the other spin state does not couple to this external force. This would, for example, be the case if the external force is due to the Zeeman effect in a magnetic field, and if the two spin states correspond to the $m_F = 0$ and $m_F = 1$ projections of an $F = 1$ hyperfine state. We assume that the system is rotating, which gives rise to a Coriolis force that is the equivalent of the Lorentz force from the electronic Hall effect.

The appropriate Boltzmann equation is

$$\frac{\partial f_1}{\partial t} + \left[\frac{\mathbf{F}}{\hbar} + \mathbf{\Omega} \times \mathbf{k}\right] \cdot \frac{\partial f_1}{\partial \mathbf{k}} = \Gamma_{\text{coll}}[f_0, f_1]. \qquad (1)$$

Here, $f_1(\mathbf{k}, t)$ is the distribution function for the bosons in state $|1\rangle$. The Boltzmann equation for $f_0(\mathbf{k}, t)$ is found by replacing $f_1 \leftrightarrow f_0$ and setting $\mathbf{F}$ to zero. Furthermore, $\mathbf{\Omega} = \Omega \mathbf{z}$ is the rotation vector, and $\mathbf{\Omega} \times \mathbf{k}$ gives the Coriolis force. The collision term, $\Gamma_{\text{coll}}[f_0, f_1]$, describes collisions of atoms with different spin. We will give its precise definition later on. There are, of course also collisions between atoms with an identical spin but they do not play a role for the spin drag.

We solve the Boltzmann equation by using the ansatz $f_1(\mathbf{k}, t) = N_B[\epsilon_{\mathbf{k}-m\mathbf{v}_1(t)/\hbar}]$, with a similar expression for $f_0(\mathbf{k}, t)$ in terms of $\mathbf{v}_0(t)$. Here, $N_B[\epsilon] = [e^{\beta(\epsilon-\mu)} - 1]^{-1}$ is the Bose-Einstein distribution function with $\beta = 1/k_B T$ the inverse thermal energy, k_B Boltzmann's constant, and T the temperature. The single-particle dispersion is $\epsilon_{\mathbf{k}} = \hbar^2 \mathbf{k}^2/2m$ with m the particle mass. The chemical potential μ is determined by the condition that the density of atoms is constant. The Boltzmann equation leads to the following equations of motion for the drift velocities $\mathbf{v}_0(t)$ and $\mathbf{v}_1(t)$:

$$nm\frac{d\mathbf{v}_0}{dt} = 2nm\mathbf{\Omega} \times \mathbf{v}_0 - \mathbf{\Gamma}(\mathbf{v}_0 - \mathbf{v}_1); \qquad (2)$$

$$nm\frac{d\mathbf{v}_1}{dt} = n\mathbf{F} + 2nm\mathbf{\Omega} \times \mathbf{v}_1 + \mathbf{\Gamma}(\mathbf{v}_0 - \mathbf{v}_1). \qquad (3)$$

Here, n is the particle density per spin. We assume this density to be equal for the two spin species. Note that generalizations of the above to spin and mass imbalanced systems are straightforward.

In general the frictional spin drag is determined by the full nonlinear (vector-valued) function $\mathbf{\Gamma}(\mathbf{v}_0 - \mathbf{v}_1) = \int d\mathbf{k}\hbar\mathbf{k}\Gamma_{\text{coll}}(N_B[\epsilon_{\mathbf{k}-m\mathbf{v}_0/\hbar}], N_B[\epsilon_{\mathbf{k}-m\mathbf{v}_1/\hbar}])/(2\pi)^3$. Because the collision integral is isotropic we have in linear response that $\mathbf{\Gamma}(\mathbf{v}_0 - \mathbf{v}_1) \simeq \Gamma'(0)(\mathbf{v}_0 - \mathbf{v}_1)$. We now introduce the relative particle current $\mathbf{j} = n(\mathbf{v}_1 - \mathbf{v}_0)$, which up to dimensionful prefactors is equal to the spin current, and solve the above equations of motion for the steady state, i.e., $d\mathbf{j}/dt = 0$. We then find that $\mathbf{j} = \boldsymbol{\sigma} \cdot \mathbf{F} = \boldsymbol{\rho}^{-1} \cdot \mathbf{F}$, which defines the conductivity and resistivity tensors $\boldsymbol{\sigma}$ and $\boldsymbol{\rho}$, respectively. Note that these are 3×3 matrices since the force and current are three-dimensional vectors. We find that the longitudinal resistivities $\rho_{xx} = \rho_{yy} = \rho_{zz} = 2\Gamma'(0)/n^2$, which are related to the spin-drag relaxation time τ via a Drude-like formula as $\rho_{xx} \equiv m/n\tau$. This relaxation time is the time scale on which the spin current decays due to collisions of atoms in different spin states. The Hall resistivities are given by $\rho_{xy} = -\rho_{yx} = 2m\Omega/n$. All other components of the resistivity and conductivity tensors are zero. Like the Hall resistivity in electronic systems, the transverse components of the resistivity do not depend on the specifics of the processes that lead to a nonzero longitudinal resistivity, but only on the density and strength of the Coriolis force. The longitudinal resistivity, however, that determines the dissipation of the relative momentum current via frictional spin drag, depends on the inter-spin-species collisions. We dub this Hall effect the spin-drag Hall effect, as spin drag is needed to get a finite steady-state spin Hall current. This is further demonstrated by the fact that the spin Hall conductivity for small rotations is proportional to the square of the spin-drag relaxation time, i.e., $\sigma_{xy} \propto \tau^2$.

Collective modes.—To implement the spin-drag Hall effect in a realistic cold-atom experiment, we have to take into account the effects of the trapping potential. In this situation the collective-mode spectrum of the mixture provides an experimental method to determine the spin-drag resistivities. We consider a harmonic trapping potential $V_{\text{trap}}(\mathbf{x}) = m\omega_0^2(x^2 + y^2)/2 + m\omega_z^2 z^2/2$ with radial trapping frequency ω_0 and axial frequency ω_z. We now have for the Boltzmann equation

$$\frac{\partial f_1}{\partial t} + \left[\mathbf{\Omega} \times \mathbf{k} - \frac{1}{\hbar}\nabla V\right] \cdot \frac{\partial f_1}{\partial \mathbf{k}} + \frac{\hbar\mathbf{k}}{m} \cdot \frac{\partial f_1}{\partial \mathbf{x}} = \Gamma_{\text{coll}}[f_0, f_1], \qquad (4)$$

where $V(\mathbf{x}) = V_{\rm trap}(\mathbf{x}) - m\Omega^2(x^2+y^2)/2$ includes the centrifugal force. The equation for f_0 is again found by replacing $f_1 \leftrightarrow f_0$.

We solve this inhomogeneous Boltzmann equation by making the ansatz $f_1(\mathbf{x},\mathbf{k},t) = N_B[\epsilon_{\mathbf{k}-m\mathbf{v}_1(t)/\hbar} + V(\mathbf{x}-\mathbf{x}_1(t))]$, with a similar expression for $f_0(\mathbf{x},\mathbf{k},t)$. This ansatz is now parameterized by the center-of-mass velocities $\mathbf{v}_{0,1}(t)$ and positions $\mathbf{x}_{0,1}(t)$ of the two atomic clouds. From this, we get the equations of motion.

$$Nm\frac{d\mathbf{v}_1}{dt} = 2Nm\mathbf{\Omega}\times\mathbf{v}_1 - N\frac{dV(\mathbf{x}_1)}{d\mathbf{x}_1} + \mathbf{\Gamma}(\mathbf{v}_0-\mathbf{v}_1,\mathbf{x}_0-\mathbf{x}_1); \tag{5}$$

$$Nm\frac{d\mathbf{v}_0}{dt} = 2Nm\mathbf{\Omega}\times\mathbf{v}_0 - N\frac{dV(\mathbf{x}_0)}{d\mathbf{x}_0} - \mathbf{\Gamma}(\mathbf{v}_0-\mathbf{v}_1,\mathbf{x}_0-\mathbf{x}_1), \tag{6}$$

with N the particle number per spin state. Note that due to the centrifugal force, we need to have that $|\Omega| < \omega_0$.

We again linearize the above equations using that $\mathbf{\Gamma}(\mathbf{v},\mathbf{x}) \simeq \Gamma'\mathbf{v}$ due to the isotropy of the collision integral. We next observe that all the motion in the z direction decouples. We therefore only consider the motion of the clouds in the $x-y$ plane, since this contains the spin-drag Hall effect. The linearized equations then yield a collective-mode spectrum with eight modes in total. There are four modes in which the two clouds of particles move in phase, and in which there is, as a result, no drag effect. The modes correspond physically to in-phase harmonic oscillations of the two clouds with frequencies $\omega_0 \pm \Omega$. There are two different frequencies because the degeneracy due to the two equivalent directions of oscillation in the effective two-dimensional system, is split by the external rotation. The four out-of-phase modes correspond physically to the two atomic clouds moving relative to each other. This results in transfer of momentum between the two clouds, leading to spin drag and damping of these modes. These modes have the frequencies $\omega = -i\gamma \pm \Omega + \sqrt{\omega_0^2 + (i\gamma\pm\Omega)^2}$. The imaginary part of the above frequencies gives the damping rate of the modes, and is for $\Omega \ll \omega_0$ given by $\gamma \equiv 1/2\tau = \Gamma'/Nm$. This relaxation time gives the longitudinal spin-drag resistivity as $\rho_{xx} = \rho_{yy} = m/n\tau$, and is estimated next. From the eigenvectors of the modes we find that the plane of oscillation of the out-of-phase dipole mode is not fixed in the corotating frame, which implies a transverse spin current. This is the trap equivalent of the spin-drag Hall effect discussed in the previous section.

Spin-drag relaxation time.—In the inhomogeneous case, we find that

$$\mathbf{\Gamma}(\mathbf{v}_0-\mathbf{v}_1,\mathbf{x}_0-\mathbf{x}_1) = \int d\mathbf{x}\int\frac{d\mathbf{k}}{(2\pi)^3}\hbar k\Gamma_{\rm coll}\{N_B[\epsilon_{\mathbf{k}-m\mathbf{v}_0(t)/\hbar} + V(\mathbf{x}-\mathbf{x}_0(t))], N_B[\epsilon_{\mathbf{k}-m\mathbf{v}_1(t)/\hbar} + V(\mathbf{x}-\mathbf{x}_1(t))]\}, \tag{7}$$

where

$$\Gamma_{\rm coll}[f_0,f_1] = \frac{(2\pi)^4}{\hbar}(T_{01}^{2B})^2\int\frac{d\mathbf{k}_2}{(2\pi)^3}\int\frac{d\mathbf{k}_3}{(2\pi)^3}\int\frac{d\mathbf{k}_4}{(2\pi)^3}\times\delta(\mathbf{k}+\mathbf{k}_2-\mathbf{k}_3-\mathbf{k}_4)\delta(\epsilon_{\mathbf{k}}+\epsilon_{\mathbf{k}_2}-\epsilon_{\mathbf{k}_3}-\epsilon_{\mathbf{k}_4}) \times\{[1+f_1(\mathbf{x},\mathbf{k},t)][1+f_0(\mathbf{x},\mathbf{k}_2,t)]f_1(\mathbf{x},\mathbf{k}_3,t)f_0(\mathbf{x},\mathbf{k}_4,t) - f_1(\mathbf{x},\mathbf{k},t)f_0(\mathbf{x},\mathbf{k}_2,t) \times[1+f_1(\mathbf{x},\mathbf{k}_3,t)][1+f_0(\mathbf{x},\mathbf{k}_4,t)]\}.$$

Here, T_{01}^{2B} is the two-body T matrix, which equals $4\pi a\hbar^2/m$, with a the scattering length for inter-spin-species collisions. Introducing the response function

$$\chi(\mathbf{x};\mathbf{q},\omega) = \int\frac{d\mathbf{k}}{(2\pi)^3} \times\frac{N_B[\epsilon_{\mathbf{k}}+V(\mathbf{x})] - N_B[\epsilon_{\mathbf{k}+\mathbf{q}}+V(\mathbf{x})]}{\epsilon_{\mathbf{k}+\mathbf{q}}-\epsilon_{\mathbf{k}}+\hbar\omega+i0}, \tag{8}$$

we find

$$\Gamma' = \frac{\hbar^2}{12\pi\beta}(T_{01}^{2B})^2\int d\mathbf{x}\int\frac{d\mathbf{q}}{(2\pi)^3}\int_{-\infty}^{\infty}d\omega q^2 \times\frac{[\mathrm{Im}(\chi(\mathbf{x};\mathbf{q},\omega))]^2}{\sinh^2(\beta\hbar\omega/2)}, \tag{9}$$

from which we can determine the spin-drag relaxation time. The imaginary part of the response function is worked out explicitly to yield

$$\mathrm{Im}[\chi(\mathbf{x};\mathbf{q},\omega)] = \frac{m}{2\hbar^2\Lambda q}\log\left(\frac{e^{q^2\Lambda^2/16\pi-\beta\mu(\mathbf{x})-\hbar\beta\omega/2+\pi(\hbar\beta\omega)^2/q^2\Lambda^2} - e^{-\hbar\beta\omega}}{e^{q^2\Lambda^2/16\pi-\beta\mu(\mathbf{x})-\hbar\beta\omega/2+\pi(\hbar\beta\omega)^2/q^2\Lambda^2} - 1}\right), \tag{10}$$

with Λ the thermal de Broglie wavelength and $\mu(\mathbf{x}) = \mu - V(\mathbf{x})$.

We estimate the above expression for a three-dimensional homogeneous system with density n for which we, in first approximation, have take the central density in the trap to make connection with the inhomogeneous case. The result for τ is evaluated numerically, and is shown in Fig. 1. We see that Bose enhancement is indeed at

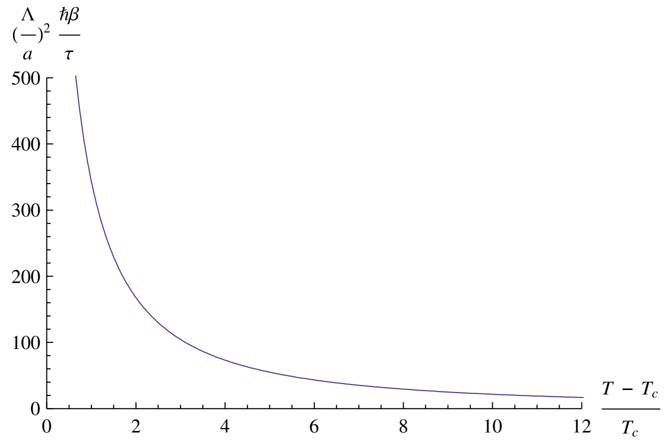

FIG. 1 (color online). Bose enhancement of the spin-drag relaxation time τ upon approaching the critical temperature T_c for Bose-Einstein condensation from above.

play: $1/\tau$ gets dramatically bigger as the temperature approaches the critical temperature. We do find, however, that it remains finite as $T \downarrow T_c$. Furthermore, from our numerical results, we find that $1/\tau(T) - 1/\tau(T_c) \propto -\sqrt{-\beta\mu} \propto T_c - T$.

Discussion and conclusion.—We have introduced the spin-drag Hall effect, i.e., the emergence of a transverse spin current, in rotating Bose mixtures. To determine whether the spin-drag relaxation rate can be measured in principle, we make estimates of the relaxation time for some realistic values of the parameters. Taking for instance ^{87}Rb at temperatures between 10 and 100 nK, with an interspecies scattering length of about 100 Bohr radii, we find values of the order of 1–100 ms for densities $n = 10^{11}$–10^{12} cm^{-3}. Considering that the trapping potential usually has $\omega_0/2\pi \simeq 0.01$–1 kHz, this means the damping should indeed happen on an observable time scale. To compare with electronic systems we note that the Drude formula $m/ne^2\tau$ (here e is the electronic charge used to convert to units of electrical resistivity) with our result for τ yields resistivities of the order of 10^{-6}–10^{-2} Ω m, many orders of magnitude larger than the spin-drag resistivity in an electronic system [17].

One interesting aspect of the Bose enhancement of the $1/\tau$ is the behavior close to the critical temperature. Here, we numerically found that $1/\tau(T) - 1/\tau(T_c) \propto -(T - T_c)^{\kappa}$, with $\kappa = 1$. In future work we intend to investigate the value of this exponent in various dimensions with renormalization-group methods, taking into account critical fluctuations that are not captured by the Boltzmann approach presented here. In previous work concerning one spatial dimension, we found $\kappa = -5/2$ [13], also with Boltzmann methods. An interesting aspect of a two-component Bose mixture is that it also may become ferromagnetic above the critical temperature of Bose-Einstein condensation. We intend to study also the effects of this transition on the spin drag.

The collective-mode spectrum determined theoretically in this Letter can be observed experimentally by setting the two spin states in relative motion. This can, for example, be achieved by shortly applying small magnetic-field gradient to excite the spin-dipole mode. Although the spin-dipole mode has not been studied experimentally for bosons yet, it has been for fermions [18], and it is feasible with current experimental techniques for bosons as well. Another possibility to study spin drag is to use a state-selective laser in a manner that is similar to Ref. [19].

We hope that the close collaboration between theory and experiments in this area, will lead to more insight into bosonic transport and, on the long run, may eventually lead to the development of useful atomtronics devices [20], where atoms rather than electrons are the main carriers of transport.

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