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## An alternative BRST operator for topological Landau-Ginzburg models

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### Abstract

We propose a new BRST operator for the B-twist of  $N = 2$  Landau-Ginzburg (LG) models. It solves the problem of the fractional ghost numbers of Vafa's old BRST operator and shows how the model is obtained by gauge fixing a zero action. An essential role is played by the anti-BRST operator, which is given by one of the supersymmetries of the  $N = 2$  algebra. Its presence is needed in proving that the model is indeed a topological field theory. The space of physical observables, defined by taking the anti-BRST cohomology in the BRST cohomology groups, is unchanged.

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# 1 Introduction

Topological field theories (TFT) [1, 2] are field theories with a BRST symmetry, and whose energy-momentum tensor is BRST exact. Formally this implies, via the Ward identity, that the partition function of the theory is independent of the metric on the manifold on which the theory is defined. A large class of TFT's can be constructed by gauge fixing a topological invariant [3] or by the so-called *twisting* of theories with  $N = 2$  [1, 5] or  $N = 4$  [4] supersymmetry. This twisting, in turn, can be done in two different ways, the so-called A- and B-twist [6, 7]. Let us consider the twisting of two dimensional  $N = 2$  Landau-Ginzburg models (LG). These twists both involve changes in the spins of the fermionic fields, and the choice of a BRST operator, with the help of the susy charges of one of the two the  $N = 2$  algebras [8]. The relevant physical operators (observables) are representatives of the BRST cohomology classes at some definite ghost number. The assignment of these ghost numbers for the A-twist is straightforward, but for the B-twist it is problematical. The most obvious assignments lead to an action that has (in part) a ghost number different from zero. A more elaborate assignment, involving ghost numbers that are generically non-integer for most of the fields, allows for a consistent ghost number zero action, but makes the subdivision of fields in classical fields, ghosts and antighosts unclear (to say the least).

In this letter we intend to show that, by re-interpreting the customary BRST charge for the B-twisted model as the sum of a BRST and an anti-BRST charge, all ghost number assignments fall into place. We propose to take for the BRST operator *one* of the  $N=2$  supersymmetry charges used by Vafa, and the other as the anti-BRST operator. The corresponding ghost number assignments make a conventional separation in classical fields, ghosts and antighosts straightforward, but the usual symmetry between BRST and anti-BRST transformations is not present yet. The interpretation of the anti-BRST transformation takes an entirely standard form, if one changes to a different basis of fields, which is related in a (mildly) nonlocal way with the customary basis.

As a consequence of our procedure, the  $(++)$  component of the energy momentum tensor is anti-BRST exact while the  $(--)$  component is BRST exact. This implies that we also need the Ward identity for the anti-BRST operator in order to prove that the theory is metric independent. Moreover, it also implies that observables are subjected to two conditions, namely they should be BRST invariant and their anti-BRST transformation should be BRST exact. This leads us to define the physical spectrum as being the elements of the anti-BRST cohomology defined in the BRST cohomology. We have computed the spectrum in this way and it leads to the same topological observables as in Vafa's approach.

## 2 Anti-BRST and the topological twist

We first remind the reader of the basic ingredients of the LG models, and the topological models obtained from them by twisting, mainly to fix the notation. Although a description in an  $N = 2$  superfield formalism in two dimensions is possible, the component notation seems most fit for the present purposes. The Lagrangian of the model is

$$S = \int d^2x \left[ -\partial_+ X^{i*} \partial_- X^j \eta_{i*j} + 2i\psi^j \partial_- \psi^{i*} \eta_{i*j} + 2i\xi^j \partial_+ \xi^{i*} \eta_{i*j} \right. \\ \left. + 4\kappa\psi^i \xi^j \partial_i \partial_j W - 4\kappa\psi^{i*} \xi^{j*} \partial_{i*} \partial_{j*} W^* - F^i F^{j*} \eta_{ij*} - 2\kappa F^{i*} \partial_{i*} W^* - 2\kappa F^j \partial_j W \right]. \quad (1)$$

The bosonic fields  $X, X^*, F$  and  $F^*$  all are spin zero fields, while the fermionic fields  $\psi, \psi^*$  have spin (helicity)  $-\frac{1}{2}$  and the fermionic fields  $\xi, \xi^*$  have spin  $\frac{1}{2}$ . The potential  $W(X)$  that determines the interaction, is a quasi-homogeneous potential of degree  $d$  with scaling weights  $\omega_i$ , which means that for all complex  $\lambda$ ,

$$W(e^{\omega_i \lambda} X^i) = e^{d\lambda} W(X^i). \quad (2)$$

This action possesses an  $N = 2$  global symmetry algebra. The supersymmetry transformation rules are given by

$$\begin{aligned} \delta X^i &= \psi^i \epsilon^- + \xi^i \tilde{\epsilon}^- & \delta X^{i*} &= -\psi^{i*} \epsilon^+ - \xi^{i*} \tilde{\epsilon}^+ \\ \delta \psi^i &= -\frac{i}{2} \partial_+ X^i \epsilon^+ - \frac{1}{2} F^i \tilde{\epsilon}^- & \delta \psi^{i*} &= \frac{i}{2} \partial_+ X^{i*} \epsilon^- - \frac{1}{2} F^{i*} \tilde{\epsilon}^+ \\ \delta \xi^i &= -\frac{i}{2} \partial_- X^i \tilde{\epsilon}^+ + \frac{1}{2} F^i \epsilon^- & \delta \xi^{i*} &= \frac{i}{2} \partial_- X^{i*} \tilde{\epsilon}^- + \frac{1}{2} F^{i*} \epsilon^+ \\ \delta F^i &= -i \partial_+ \xi^i \epsilon^+ + i \partial_- \psi^i \tilde{\epsilon}^+ & \delta F^{i*} &= -i \partial_+ \xi^{i*} \epsilon^- + i \partial_- \psi^{i*} \tilde{\epsilon}^- . \end{aligned} \quad (3)$$

There are four different global parameters. When the supersymmetries are made local, the transformations close using the Lorentz transformations, for which the  $\psi$  and  $\xi$  fields have spin  $\frac{1}{2}$  as mentioned. Also there are two additional global  $U(1)$  symmetries. The charges of all the fields are given in the table below (the index  $i$  is suppressed but understood in the table, and we abbreviated  $h = \omega/d$ ). The symmetry for the  $q_+$  charges is due to the quasihomogeneity of the potential.

	$X$	$X^*$	$F$	$F^*$	$\psi$	$\psi^*$	$\xi$	$\xi^*$
$q_+$	$-2h$	$2h$	$2 - 2h$	$-2 + 2h$	$1 - 2h$	$-1 + 2h$	$1 - 2h$	$-1 + 2h$
$q_-$	$0$	$0$	$0$	$0$	$1$	$-1$	$-1$	$1$

In two dimensions, the Lorentz symmetry is a  $U(1)$  symmetry. The basic idea behind the topological twist is that one can redefine the Lorentz symmetry by combining the original one with the other (global)  $U(1)$  symmetries present in the model. The two standard ways [6, 7] are

$$\begin{aligned} s_A &= s + \frac{1}{2} q_+ & \text{A - twist} \\ s_B &= s - \frac{1}{2} q_- & \text{B - twist} . \end{aligned} \quad (4)$$

Here,  $s$  denotes the spin of the field before the twist, i.e. for the N=2 LG model, while  $s_A$  and  $s_B$  denote the spins in the A-twisted and the B-twisted model respectively. Note that the spins take on generically non-integer values for the A-twist. In the B-twist (4) all fields have spin  $s_B = 0$ , except the fermions  $\psi^i$ , which have spin  $s_B = -1$ , and  $\xi^i$ , which have spin  $s_B = 1$ . We will focus on the B-twist from now on.

This redefinition of spin has two main consequences. The first is that the coupling to gravity changes, i.e. there is a change in the energy-momentum tensor. The second is that two of the four supersymmetries now have zero  $s_B$  spin, namely those parametrised by  $\epsilon^+$  and by  $\tilde{\epsilon}^+$ . Following [8], these two supersymmetries can be used to construct a

spinless fermionic operator  $\delta$  (acting from the left):

$$\begin{aligned}
\delta X^{i*} &= \psi^{i*} + \xi^{i*} & \delta F^i &= i(\partial_+ \xi^i - \partial_- \psi^i) \\
\delta \xi^{i*} &= \frac{1}{2} F^{i*} & \delta \xi^i &= -\frac{i}{2} \partial_- X^i \\
\delta \psi^{i*} &= -\frac{1}{2} F^{i*} & \delta X^i &= 0 \\
\delta F^{i*} &= 0 & \delta \psi^i &= -\frac{i}{2} \partial_+ X^i .
\end{aligned} \tag{5}$$

From these expressions it is obvious that  $\delta^2 = 0$ . It is proposed in [8] to interpret  $\delta$  as a BRST operator of a so far unspecified gauge symmetry. The action of the LG model can be written as

$$\begin{aligned}
S &= 4\kappa \psi^i \xi^j \partial_i \partial_j W - 2\kappa F^i \partial_i W \\
&\quad + \delta[F^i(\psi^{i*} - \xi^{i*}) + 4\kappa \partial_{i*} W^* \psi^{i*} + iX^{i*}(\partial_- \psi^i + \partial_+ \xi^i)] \\
&\equiv S^0 + \delta\Psi .
\end{aligned} \tag{6}$$

This is of the same form as a classical action, supplemented by a gauge fixing action which is the BRST variation of a gauge fermion. It is still not specified what the gauge symmetry of this action would be. One can show that the energy-momentum tensor is BRST exact, and therefore equivalent to zero, which makes the theory into a metric-independent one, i.e. a topological field theory.

If one tries to supply the missing ingredients to make contact with the usual gauge fixing approach to TFT, where the operator  $\delta$  clearly is to be interpreted as a BRST operator of a closed symmetry algebra, one encounters a problem. This can most easily be seen by trying to figure out the ghost numbers of all the fields. They have to be chosen such that the action has ghost number zero and the BRST operator raises ghost number by one. There is one solution to these requirements, namely to assign ghost numbers equal to minus the  $q_+$ -charge [9]. These assignments are unsatisfactory however. The fact that all are non-integer is uncommon, although not necessarily wrong. One could imagine that, due to an anomaly in the ghost number current of the quantum theory, shifts of the ghost number are induced. We are working on the classical level however, where such a feature seems impossible in the conventional approach. There the ghost number assignments follow from a definite procedure, and necessarily come out to be integer (positive or negative). Thus an interpretation as a gauge fixed theory is impossible. If one tries to rescue this, by assigning integer ghost numbers to all the fields, such that the BRST operator has ghost number one, then the action contains terms of ghost number minus 2 [7].

Another approach is to try and find the gauge symmetry directly. Looking at  $S^0$  in (6) one of the gauge symmetries is a certainly a shift in  $X^{i*}$ , and one would therefore introduce a ghost field for this symmetry. Looking at the transformation rule eq.(5) for  $X^{i*}$ , one sees that in fact we have introduced *two* ghost fields for only one symmetry. The theory is then reducible, and  $F^{i*}$  are the ghosts for ghosts. This seems to be an unnecessary complication. The BRST algebra for this symmetry is the left column of (5). For the right column, the interpretation is not so clear. There seems not to be a gauge symmetry and a corresponding action, for which the right column is the BRST algebra.

To remedy this situation, *we propose to change the BRST operator*. The previous BRST operator was obtained from the supersymmetries with as BRST parameter  $\Lambda = \epsilon^+ = \tilde{\epsilon}^+$ . Instead, we propose to use simply the first of these supersymmetries, and interpret it as a BRST operator by itself. The second supersymmetry we propose to identify with the anti-BRST operator<sup>1</sup>. We will call these operators  $\mathbf{s}$  and  $\bar{\mathbf{s}}$  respectively. The transformation rules are:

$$\begin{aligned} \bar{\mathbf{s}}X^{i*} &= \psi^{i*} & \mathbf{s}X^{i*} &= \xi^{i*} \\ \bar{\mathbf{s}}\psi^i &= -\frac{i}{2}\partial_+ X^i & \mathbf{s}\psi^{i*} &= -\frac{1}{2}F^{i*} \\ \bar{\mathbf{s}}\xi^* &= \frac{1}{2}F^{i*} & \mathbf{s}\xi^i &= -\frac{i}{2}\partial_- X^i \\ \bar{\mathbf{s}}F^i &= i\partial_+ \xi^i & \mathbf{s}F^i &= -i\partial_- \psi^i, \end{aligned} \tag{7}$$

with all the other (anti)BRST transformations vanishing. One easily verifies the important nilpotency relations  $\mathbf{s}^2 = \bar{\mathbf{s}}^2 = \mathbf{s}\bar{\mathbf{s}} + \bar{\mathbf{s}}\mathbf{s} = 0$ . Comparing with (5) we see that the BRST operator introduced by Vafa is the sum,  $\delta = \mathbf{s} + \bar{\mathbf{s}}$ . The invariance of the action under  $\mathbf{s}$  and  $\bar{\mathbf{s}}$  follows of course from the original supersymmetries.

The condition that fixes the ghost number assignments is now that  $\mathbf{s}$  raises the ghost number by one unit,  $\bar{\mathbf{s}}$  lowers it by one unit, and the action has ghost number zero. All ghost numbers are integers. In fact, the ghost number turns out to be nothing but the  $q_-$  charge (see table).

With this new interpretation, the action of the LG model can still be written as the sum of a classical action and a gauge fixing part. One easily computes

$$\begin{aligned} S &= 4\kappa\psi^i\xi^j\partial_i\partial_j W - 2\kappa F^i\partial_i W \\ &\quad + \mathbf{s}\bar{\mathbf{s}} \left[ 4\kappa W^* + 2X^{i*}F^i \right]. \end{aligned} \tag{8}$$

The classical part does not depend on  $X^{i*}$ , and therefore one has a gauge (shift-)symmetry  $\delta X^{i*} = \epsilon^{i*}$ , and the corresponding ghosts  $\xi^{i*}$ . In accordance with the spirit of the BRST-anti-BRST scheme [11], one introduces also an antighost  $\psi^{i*}$ , and its BRST variation  $F^{i*}$ . Apart from this quartet, there is a second set of fields transforming into each other, viz.  $F^i, \psi^i, \xi^i$  and  $X^i$ . The reason for the presence of the latter fields, and for their transformations, eq.(7), is not obvious at this stage, but we will come back to their interpretation. It is now clear that the gauge fixing part  $\mathbf{s}\bar{\mathbf{s}}K$  fixes the shift symmetries, as one would do starting from a zero action to construct a TFT [3].

The identifications above do not yet exhibit the usual structure of BRST-anti-BRST. A first signal is that the first term in (8) should not be present in the underlying gauge invariant classical action, since  $\psi^i$  and  $\xi^i$  have non zero ghost number, and the classical action supposedly depends only on classical fields. This term should rather be a part of a gauge fixing term instead. A second point is that there should be more symmetry between ghosts and antighosts. The anti-BRST transformation of the classical fields are identical to their BRST transformation, when replacing ghosts with antighosts. This is not the case for the second set, since we then also have to interchange  $\partial_+$  and  $\partial_-$ .

In the  $N = 2$  LG model, the starred and unstarred fields occur symmetrically. The twist has lifted this symmetry: the former are all spinless, but  $\psi^i$  and  $\xi^i$  have helicities 1 and -1 respectively. One can construct  $\psi^i dx^+$  and  $\xi^i dx^-$ , which behave as one forms under holomorphic coordinate transformations. The asymmetry is mirrored in the derivatives

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<sup>1</sup>For a review of the use of BRST-anti-BRST symmetry of gauge theories, we refer to [11].

in the transformation laws for the second set, which is in accordance with the helicity-assignment. At the same time, one can also consider  $F^i$  to be a two-form, which can not be distinguished from a scalar in the treatment with a flat metric. The BRST–anti-BRST symmetry can be redressed by the following non-local change of field variables:

$$\begin{aligned}\psi^i &= \partial_+ \chi^i \\ \xi^i &= \partial_- \rho^i \\ F^i &= \partial_- \partial_+ H^i .\end{aligned}\tag{9}$$

All the fields on the right hand side are scalars. Remark that the Jacobian of this transformation is one, at least formally, since the contributions from the fermions cancel against the bosons. For the new variables we can take the transformation rules

$$\begin{aligned}\bar{s}\chi^i &= -\frac{i}{2}X^i & s\rho^i &= -\frac{i}{2}X^i \\ \bar{s}H^i &= i\rho^i & sH^i &= -i\chi^i ,\end{aligned}\tag{10}$$

to reproduce the so far unexplained rules in (7). They now correspond to a shift symmetry for the field  $H^i$ , introducing the ghost field  $\chi^i$ . The antighost is  $\rho^i$ , and  $X^i$  completes the quartet. It is clear that we have uncovered a manifest BRST anti-BRST symmetry.

The action, when written in terms of the new fields, is BRST exact:

$$\begin{aligned}S &= s[4\kappa i\partial_+ H^i \partial_- \rho^j \partial_i \partial_j W] + s\bar{s}[4\kappa W^* + 2X^{i*}F^i] \\ &= s[\xi^i(-2i\partial_+ X^{i*} + 4\kappa i\partial_+ H^j \partial_i \partial_j W) + \psi^{i*}(2F^i + 4\kappa \partial_{i*} W^*)] .\end{aligned}\tag{11}$$

This allows the following interpretation. One starts from two classical fields,  $X^{i*}$  and  $H^i$ . The classical action is zero, and the symmetries are shift symmetries, with ghosts  $\xi^{i*}$  and  $\chi^i$ . Then one introduces antighosts  $\psi^{i*}$  and  $\rho^i$ , and Lagrange multipliers  $X^i$  and  $F^{i*}$ . This completes the field content of the theory. The gauge fixed action is the BRST exact functional, given in eq.(11). Note that the actual content of the resulting TFT depends heavily on the gauge fixing procedure, as usual: there are no physical local fluctuations, but global variables may remain. We have nothing to add on this point, so we refer to the existing literature[8].

We conclude that our proposal for the BRST operator has led, via the transformation of (10), to a re-interpretation of the B-twisted Landau-Ginzburg model which makes contact with the alternative view on topological field theories, as arising from gauge fixing a zero action.

Having changed the BRST operator, we now discuss the implications of this change. First of all, we investigate whether we still have a topological theory in the sense that the energy-momentum is BRST exact for the new BRST operator. Afterwards, we will investigate whether the physical content (observables) of the theory has changed.

### 3 The energy-momentum tensor

To prove the topological nature of the theory we have to show that the energy momentum tensor is trivial. To compute it, it is not necessary to covariantize the (flat space) LG action (2) completely. The transformation properties of the various fields can be seen by

considering *holomorphic* changes of coordinates only. One finds that  $\psi^i dx^+$  and  $\xi^i dx^-$  are  $(1, 0)$  and  $(0, 1)$  forms respectively, which we assemble in  $A^i = \psi^i dx^+ + \xi^i dx^-$  purely for notational convenience. Likewise,  $F^i$  is a  $(1, 1)$  form component (we use the bold symbol for the form itself). The BRST and anti-BRST transformation respect the form degrees. The covariant action is

$$\begin{aligned}
S = & \int \frac{1}{2} dX^i \wedge [dX^{i*}]_* + iA^i \wedge d(\psi^{i*} - \xi^{i*}) - iA^i \wedge [d(\psi^{i*} + \xi^{i*})]_* \\
& + \int 2\kappa A^i \wedge A^j \partial_i \partial_j W - 4\kappa \psi^{i*} \xi^{j*} \partial_{i*} \partial_{j*} W^* \sqrt{g} dx^+ \wedge dx^- \\
& - \int F^{i*} \mathbf{F}^i + 2\kappa F^{i*} \partial_{i*} W^* \sqrt{g} dx^+ \wedge dx^- + 2\kappa \mathbf{F}^i \partial_i W,
\end{aligned} \tag{12}$$

where the lower star is used to denote the Hodge dual,  $[\omega]_* = \sqrt{g} g^{\mu\nu} \epsilon_{\nu\rho} \omega_\mu dx^\rho$ . The metric dependence is in the volume element and in the definition of the Hodge dual. The derivative of this action w.r.t. the metric gives the energy momentum tensor :

$$\begin{aligned}
T_{++}^B = & -\partial_+ X^{i*} \partial_+ X^i + 2i\psi^i \partial_+ \psi^{i*} = \bar{s} [2i\psi^i \partial_+ X^{i*}] \\
T_{--}^B = & -\partial_- X^{i*} \partial_- X^i + 2i\xi^i \partial_- \xi^{i*} = s [2i\xi^i \partial_- X^{i*}] \\
T_{+-}^B = & 4\kappa \psi^{i*} \xi^{j*} \partial_{i*} \partial_{j*} W^* + 2\kappa F^{i*} \partial_{i*} W^* = s\bar{s} [4\kappa W^*].
\end{aligned} \tag{13}$$

After the derivation, we have taken the metric to be flat. These are therefore the relevant operators for variations of correlation functions around a flat metric.

The  $(++)$  component is anti-BRST exact, but not BRST exact in spite of (11). The reason is that the BRST operator depends on the metric and one cannot commute the BRST variation and the derivative w.r.t. the metric [10].

To prove metric independence of correlation functions, one needs not only BRST invariance, but also the Ward identity for the anti-BRST operator. What is needed is that the physical operators are BRST invariant, *and* that their anti-BRST variation is BRST exact. In that case one can argue as follows. Denoting by  $\mathcal{O}_i, i = 1, \dots, N$  a set of (metric independent) physical operators that satisfy  $s\mathcal{O}_i = 0, \bar{s}\mathcal{O}_i = sV_i$  and  $T_{++} = \bar{s}X_{++}$  :

$$\begin{aligned}
\frac{\delta}{\delta g^{++}} \langle \mathcal{O}_1 \dots \mathcal{O}_N \rangle &= \langle T_{++} \mathcal{O}_1 \dots \mathcal{O}_N \rangle \\
&= \langle \bar{s}X_{++} \mathcal{O}_1 \dots \mathcal{O}_N \rangle \\
&= \langle X_{++} \sum_{i=1}^N \mathcal{O}_1 \dots sV_i \dots \mathcal{O}_N \rangle \\
&= \langle (sX_{++}) \sum_{i=1}^N \mathcal{O}_1 \dots V_i \dots \mathcal{O}_N \rangle \\
&= \langle \psi^i y_{\xi^i} \sum_{i=1}^N \mathcal{O}_1 \dots V_i \dots \mathcal{O}_N \rangle,
\end{aligned} \tag{14}$$

where  $y_{\xi^i} \equiv \frac{\overleftarrow{\delta} S}{\delta \xi^i}$ . We have assumed that there are no BRST nor anti-BRST anomalies. In order to have a topological field theory the result should vanish. Classically, one may use the field equations  $y = 0$ . In fact, also in the formulation of [8], field equations were used implicitly. The  $(++)$  component of his energy momentum tensor is only BRST exact

modulo the *same* field equations, i.e.  $T_{++} = \delta(-2i\partial_+ X^{i*}\psi^i) + \psi^i y_{\xi^i}$ . On the other hand, in the quantum theory, these field equations might contribute to order  $\hbar$ . This contribution can in principle be computed using the Schwinger–Dyson equations. This formally leads to products of operators at the same space-time point. To make a proper computation one needs a regularisation scheme. That computation is beyond the scope of this paper, but is currently under study [12].

The proper cohomological formulation is, that one first determines the  $\mathbf{s}$  cohomology, a space of equivalence classes. The operator  $\bar{\mathbf{s}}$  is well defined and nilpotent in that space, so that *its* cohomology can be used as our characterisation of physical states<sup>2</sup>. This characterisation is not arbitrary, but more or less forced upon us by the requirement that the energy momentum tensor is trivial. We now investigate this cohomology.

## 4 The spectrum

The observables in Vafa’s picture are the solutions of the  $\delta$ -cohomology, while in our interpretation they are the solutions of the  $\bar{\mathbf{s}}$ -cohomology in the  $\mathbf{s}$ -cohomology. Let us for example consider observables which are integrals of functions  $\int \Phi^{(2)}$ , over the Riemann surface. For simplicity we restrict ourselves here to integrals over  $(1, 1)$ -forms with respect to holomorphic coordinate transformations, although the two spectra coincide even if one imposes no restriction at all. For the  $\delta$  operator, we have to solve the descent equations :

$$\begin{aligned}\delta\Phi^{(2)} &= -\mathbf{d}\Phi^{(1)} , \\ \delta\Phi^{(1)} &= -\mathbf{d}\Phi^{(0)} , \\ \delta\Phi^{(0)} &= 0 ,\end{aligned}\tag{15}$$

where  $\mathbf{d}$  is the graded exterior derivative, defined as  $(-1)^{gh}\mathbf{d}$ , and  $\Phi^{(k)}$  is a  $k$ -form. The equalities are always taken modulo field equations, i.e. we compute the weak cohomology. We denote by  $M$  the space of formal sums  $\Phi = \Phi^{(2)} + \Phi^{(1)} + \Phi^{(0)}$ . The descent equations take the form:

$$(\delta + \mathbf{d})\Phi = 0.\tag{16}$$

and the relevant cohomology is translated into the  $\delta + \mathbf{d}$  cohomology. The solution is given by the polynomial ring of the LG potential [7] :

$$\begin{aligned}\Phi^{(0)} &= P(X) , \\ \Phi^{(1)} &= -2i\partial_i P(\psi^i dx^+ + \xi^i dx^-) , \\ \Phi^{(2)} &= [-4\partial_i\partial_j P\psi^i\xi^j + 2\partial_i P F^i]dx^+ \wedge dx^- ,\end{aligned}\tag{17}$$

where  $P(X)$  is a polynomial corresponding to some non trivial element of the ring determined by the potential  $W(X)$ . Indeed, the vanishing relations  $\partial_i W = 0$  follow from the field equation of  $F^i$ , which imply that  $\kappa\partial_i W$  is weakly equal to  $\delta\psi^{i*}$ .

Now we turn to the  $\mathbf{s}$  cohomology. In a first step,  $H(\mathbf{s}, M) = \frac{\ker(\mathbf{s}+\mathbf{d})}{\text{im}(\mathbf{s}+\mathbf{d})}$ . Since  $\mathbf{d}$  is the graded exterior derivative,  $\bar{\mathbf{s}} + \mathbf{d}$  is well defined on  $H(\mathbf{s}, M)$ , and we can calculate its cohomology in  $H(\mathbf{s}, M)$ . The physical observables are then given by  $H(\bar{\mathbf{s}}, H(\mathbf{s}, M)) =$

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<sup>2</sup>In the usual gauge theories, the more common procedure of choosing an anti-BRST invariant representative in each equivalence class amounts to the same thing.



$\frac{\ker(\bar{\mathbf{s}}+\mathbf{d})}{\text{im}(\bar{\mathbf{s}}+\mathbf{d})}$ . The classes of  $H(\bar{\mathbf{s}}, H(\mathbf{s}, M))$  are represented by those forms  $\Phi$  for which there exists a  $Y = Y^{(2)} + Y^{(1)} + Y^{(0)}$  such that

$$(\mathbf{s} + \mathbf{d})\Phi = 0 \quad (\bar{\mathbf{s}} + \mathbf{d})\Phi = (\mathbf{s} + \mathbf{d})Y , \quad (18)$$

and which itself is not  $\bar{\mathbf{s}} + \mathbf{d}$ -trivial. The solution is given by:

$$\begin{aligned} \Phi^{(2)} &= (-4\partial_i\partial_j P\psi^i\xi^j + 2\partial_i P F^i)dx^- \wedge dx^+ & Y^{(2)} &= -\Phi^{(2)} , \\ \Phi^{(1)} &= 2i\partial_i P\psi^i dx^+ & Y^{(1)} &= -2i\partial_i P\xi^i dx^- , \\ \Phi^{(0)} &= 0 & Y^{(0)} &= P . \end{aligned} \quad (19)$$

This shows that the spectra coincide. Needless to say, the same result is true if we interchange the order in which the cohomologies are computed.

## 5 Conclusions

The main observation of the present paper is that in the topologically (B-)twisted 2d Landau-Ginzburg model, the topological symmetry is most naturally interpreted in terms of a BRST–anti-BRST symmetry. This interpretation allows one to reconcile the requirements of integer ghost number, and ghost number zero for the action.

The anti-BRST symmetry has been used in TFT to study topological Yang-Mills theory [13], and also recently in a more general context [14]. In these cases, it seems more a matter of choice whether one mentions the anti-BRST symmetry or not. In contrast, for our proposal the anti-BRST is forced upon us in order to make sense of the twisted model as the gauge fixing of a zero action, and also from the requirement that the model is topological, so that its energy momentum tensor must be “trivial”. Finally we want to remark that this procedure can be extended to the interpretation of the B-twist of any model 2d  $N = 2$  model. Indeed, the interpretation of the  $N = 2$  SUSY transformations in terms of (anti)BRST transformations is much more general than the precise model that we used, since it only relies on the off-shell formulation of the  $N = 2$  algebra (using auxiliary fields). This formulation of the algebra is the same in e.g.  $\sigma$ -models, such that there too a more natural interpretation of the twisted model may be based on the BRST–anti-BRST symmetry.

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