Redefining B-twisted topological sigma models[1]

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ABSTRACT

The recently proposed procedure [1] to perform the topological B-twist in rigid $N = 2$ models is applied to the case of the σ model on a Kähler manifold. This leads to an alternative description of Witten's topological σ model, which allows for a proper BRST interpretation and ghost number assignment. We also show that the auxiliary fields, which are responsible for the off shell closure of the $N = 2$ algebra, play an important role in our construction.

1. Introduction

From the physical point of view, topological field theories (TFT) [2, 3] are interesting because they describe certain apects of $N = 2$ or $N = 4$ models. They can be solved exactly since the semi-classical approximation for these theories is exact. For more on motivation and introduction, see the contribution of R. Dijkgraaf to this volume.

Topological field theories are field theories whose energy-momentum tensor is BRST exact. Formally this implies, via the Ward identity, that the partition function of the theory is independent of the metric on the manifold on which the theory is defined. A large class of TFT's can be constructed by gauge fixing a topological invariant [7] or by the so-called *twisting* $N = 2$ theories with [2, 3, 8]. This twisting, in turn, can be done in two different ways, the so-called A- and B-twist [9, 10].

In this paper we consider the twisting of two dimensional $N = 2$ σ models. These twists both involve changes in the spins of the fermionic fields, and the choice of a BRST operator, with the help of the susy charges of one of the two the $N = 2$ algebras [9, 3]. The relevant physical operators (observables) are representatives of the BRST cohomology classes at some definite ghost number.

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[1]Talk given by S. Vandoren

The assignment of these ghost numbers and the BRST interpretation for the A-twist is straightforward, but for the B-twist it is unclear. If we do not introduce auxiliary fields in the $N = 2$ algebra, the action after the B-twist does not have the structure of a gauge fixed action of an underlying gauge theory. If we do introduce them, it is the interpretation of the BRST charge itself which is doubtful.

In this contribution we intend to show that, by rewriting the customary BRST charge for the B-twisted model as the sum of a new BRST charge and an anti-BRST charge, the ghost number assignments and the BRST interpretation fall into place. This procedure can be applied to any (rigid) $N = 2$ theory. For topological Landau-Ginzburg models, see [1].

We propose to take for the BRST operator *one* of the N=2 supersymmetry charges used by Witten, and the other as the anti-BRST operator. The corresponding ghost number assignments make a conventional separation in classical fields, ghosts and antighosts straightforward, but the usual symmetry between BRST and anti-BRST transformations is not present yet. The interpretation of the anti-BRST transformation takes an entirely standard form, if one changes to a different basis of fields, which is related in a (mildly) nonlocal way with the customary basis.

As a consequence of our procedure, the $(++)$ component of the energy momentum tensor is anti-BRST exact while the $(--)$ component is BRST exact. This implies that we also need the Ward identity for the anti-BRST operator in order to prove that the theory is metric independent. Moreover, it also implies that observables are subjected to two conditions, namely they should be BRST invariant and their anti-BRST transformation should be BRST exact. This leads us to define the physical spectrum as being the elements of the anti-BRST cohomology defined in the BRST cohomology. We argue that this cohomology problem leads to the same observables as in the old approach.

2. N=2 σ-models

One can formulate $N = 2$ models with or without auxiliary fields. Including these fields, one realises the algebra off shell. We will treat here both cases and comment on the difference between the two B-twists.

2.1. On shell formulation
The N=2 σ-model action on the target Kähler manifold $\mathcal{M}_K$ is

$$
\begin{aligned}
S \quad = \quad & -\frac{1}{2}g_{ij^*}(\partial_+ X^i \partial_- X^{j^*} + \partial_- X^i \partial_+ X^{j^*}) \\
& +ig_{ij^*}(\psi^i \nabla_- \psi^{j^*} + \psi^{j^*}\nabla_- \psi^i) \\
& +ig_{ij^*}(\xi^i \nabla_+ \xi^{j^*} + \xi^{j^*}\nabla_+ \xi^i) \\
& +4R_{ij^*kl^*}\psi^i \psi^{j^*}\xi^k \xi^{l^*} \ ,
\end{aligned}
\tag{1}
$$

where e.g. $\nabla_- \psi^i = \partial_- \psi^i - \Gamma^i_{jk}\partial_- X^j \psi^k$, and an integral over a Riemann surface Σ is

also understood. The supersymmetry rules are

$$\delta X^i = \psi^i \epsilon^- + \xi^i \tilde{\epsilon}^- \qquad\qquad \delta X^{i^*} = -\psi^{i^*} \epsilon^+ - \xi^{i^*} \tilde{\epsilon}^+$$
$$\delta \psi^i = -\tfrac{i}{2}\partial_+ X^i \epsilon^+ - \tilde{\epsilon}^- \Gamma^i_{jk} \xi^j \psi^k \quad \delta \psi^{i^*} = \tfrac{i}{2}\partial_+ X^{i^*} \epsilon^- + \tilde{\epsilon}^+ \Gamma^{i^*}_{j^*k^*} \xi^{j^*} \psi^{k^*} \qquad (2)$$
$$\delta \xi^i = -\tfrac{i}{2}\partial_- X^i \tilde{\epsilon}^+ - \epsilon^- \Gamma^i_{jk} \psi^j \xi^k \quad \delta \xi^{i^*} = \tfrac{i}{2}\partial_- X^{i^*} \tilde{\epsilon}^- + \epsilon^+ \Gamma^{i^*}_{j^*k^*} \psi^{j^*} \xi^{k^*} \ .$$

The supersymmetry algebra only closes on shell.

2.2. Off shell formulation

To find an off shell formulation, we introduce auxiliary fields F^i, F^{i^*}. The action given above is then obtained by integrating out the auxiliary fields in the following action :

$$
\begin{aligned}
S \ = \ &-\frac{1}{2}g_{ij^*}(\partial_+ X^i \partial_- X^{j^*} + \partial_- X^i \partial_+ X^{j^*}) \\
&+ ig_{ij^*}(\psi^i \nabla_- \psi^{j^*} + \psi^{j^*} \nabla_- \psi^i) \\
&+ ig_{ij^*}(\xi^i \nabla_+ \xi^{j^*} + \xi^{j^*} \nabla_+ \xi^i) \\
&- F^i F^{j^*} g_{ij^*} + 2\Gamma^i_{jk} \xi^j \psi^k g_{ij^*} F^{j^*} + 2F^i g_{ij^*} \Gamma^{j^*}_{k^*l^*} \psi^{l^*} \xi^{k^*} \\
&- 4\psi^i \psi^{j^*} \xi^k \xi^{l^*} \partial_i \partial_{j^*} \partial_k \partial_{l^*} K \ ,
\end{aligned}
\qquad (3)
$$

where K is the Kähler potential with $g_{ij^*} = \partial_i \partial_{j^*} K$. The $N = 2$ susy transformation rules now become

$$\delta X^i = \psi^i \epsilon^- + \xi^i \tilde{\epsilon}^- \qquad\qquad \delta X^{i^*} = -\psi^{i^*} \epsilon^+ - \xi^{i^*} \tilde{\epsilon}^+$$
$$\delta \psi^i = -\tfrac{i}{2}\partial_+ X^i \epsilon^+ - \tfrac{1}{2}F^i \tilde{\epsilon}^- \qquad \delta \psi^{i^*} = \tfrac{i}{2}\partial_+ X^{i^*} \epsilon^- - \tfrac{1}{2}F^{i^*} \tilde{\epsilon}^+$$
$$\delta \xi^i = -\tfrac{i}{2}\partial_- X^i \tilde{\epsilon}^+ + \tfrac{1}{2}F^i \epsilon^- \qquad \delta \xi^{i^*} = \tfrac{i}{2}\partial_- X^{i^*} \tilde{\epsilon}^- + \tfrac{1}{2}F^{i^*} \epsilon^+ \qquad (4)$$
$$\delta F^i = -i\partial_+ \xi^i \epsilon^+ + i\partial_- \psi^i \tilde{\epsilon}^+ \quad \delta F^{i^*} = -i\partial_+ \xi^{i^*} \epsilon^- + i\partial_- \psi^{i^*} \tilde{\epsilon}^- \ .$$

The twist we will perform is based on these transformation rules. They are the same for any (rigid) $N = 2$ theory in two dimensions. It is therefore not surprising that the results of [1] can also be applied to the case of sigma models. The field equations of the auxiliary fields are

$$
\begin{aligned}
F^i \ &= \ 2\Gamma^i_{jk} \xi^j \psi^k \\
F^{i^*} \ &= \ 2\Gamma^{i^*}_{j^*k^*} \psi^{j^*} \xi^{k^*} \ .
\end{aligned}
\qquad (5)
$$

Using these field equations in the action and in the transformation rules, one recovers the on shell formulation of the previous subsection.

3. B-Twisting: the old approach

Following Witten [9, 3], one can perform the B twist of the $N = 2$ model by setting $\epsilon^- = \tilde{\epsilon}^- = 0$. The other two supercharges build up a spinless BRST operator $Q = G^+ + \tilde{G}^+$. All the fields have zero spin, except ψ^i and ξ^i, which have resp. spin -1 and 1.

3.1. On shell twisting

For the on shell formulation, we get the BRST rules (acting from the left) :

$$\delta X^{i^*} = \psi^{i^*} + \xi^{i^*} \qquad\qquad \delta X^i = 0$$

$$\delta \xi^{i^*} = \Gamma^{i^*}_{j^*k^*}\psi^{j^*}\xi^{k^*} \qquad\qquad \delta \xi^i = -\frac{i}{2}\partial_- X^i$$

$$\delta \psi^{i^*} = -\Gamma^{i^*}_{j^*k^*}\psi^{j^*}\xi^{k^*} \qquad\qquad \delta \psi^i = -\frac{i}{2}\partial_+ X^i \ . \tag{6}$$

This BRST operator is nilpotent, and the action can be rewritten as

$$\begin{aligned}
S &= 4R_{ij^*kl^*}\psi^i\psi^{j^*}\xi^k\xi^{l^*} + ig_{ij^*}(\xi^{j^*} - \psi^{j^*})(\nabla_+\xi^j - \nabla_-\psi^j) \\
&\quad + \delta[-ig_{ij^*}(\psi^i\partial_- X^{i^*} + \xi^i\partial_+ X^{j^*})] \\
&\equiv S^0 + \delta\Psi \ . \tag{7}
\end{aligned}$$

First we assign ghost numbers to all the fields. Imposing that the action has ghost number zero, and the BRST operator has ghost number one, it follows that X^i and X^{i^*} have ghost number zero, ξ^{i^*} and ψ^{i^*} have ghost number one and ψ^i and ξ^i have ghost number minus one. With these assignements, the part S^0 of the action still contains ghosts and antighosts, i.e. it is not the classical action, which only depends on the classical fields and not on the ghosts. Secondly, the fields X^i are merely lagrangian multipliers since they do not transform under BRST. So the only classical field would be X^{i^*}, but then the classical action should only depend on X^{i^*}. It is clear that in this formulation the BRST interpretation is obscure. We will now see that some of these problems disappear when including the auxiliary sector.

3.2. Off shell twisting

When the auxiliary fields are included, the BRST transformation rules are :

$$\delta X^{i^*} = \psi^{i^*} + \xi^{i^*} \qquad\qquad \delta F^i = i(\partial_+\xi^i - \partial_-\psi^i)$$

$$\delta \xi^{i^*} = \frac{1}{2}F^{i^*} \qquad\qquad \delta \xi^i = -\frac{i}{2}\partial_- X^i$$

$$\delta \psi^{i^*} = -\frac{1}{2}F^{i^*} \qquad\qquad \delta \psi^i = -\frac{i}{2}\partial_+ X^i$$

$$\delta F^{i^*} = 0 \qquad\qquad \delta X^i = 0 \ . \tag{8}$$

From these expressions it is obvious that $\delta^2 = 0$. It is proposed in [9] to interpret δ as a BRST operator of a so far unspecified gauge symmetry. The action of the σ model can be written as

$$\begin{aligned}
S &= \delta[(F^i - 2\Gamma^i_{jk}\xi^j\psi^k)(\psi^{j^*} - \xi^{j^*})g_{ij^*} - ig_{ij^*}(\psi^i\partial_- X^{j^*} + \xi^i\partial_+ X^{j^*})] \\
&\equiv S^0 + \delta\Psi \ . \tag{9}
\end{aligned}$$

This is of the same form as a classical action S^0, supplemented by a gauge fixing action which is the BRST variation of a gauge fermion. The classical action in this case is

simply zero. There are two gauge fixing conditions. One that restricts the X's to be constant maps from the Riemann surface Σ to the target manifold $\mathcal{M}_K$. The other puts the auxiliary fields on shell. The ghost numbers are the same as in the previous subsection, and for the auxiliary fields one has $gh(F^{i^*}) = 2 = -gh(F^i)$. This means that they are ghosts for ghosts and the gauge algebra is reducible. Indeed, there is a gauge symmetry corresponding to arbitrary shifts in X^{i^*}. For this we have introduced two ghosts instead of one, namely ξ^{i^*} and ψ^{i^*}. Whereas this redundancy somewhat complicates matters, it is adequately handled by the ghosts for ghosts F^{i^*}. When looking for an interpretation of the column on the right in eqs.8, there does arise a problem. It seems that it can also be understood as a reducible multiplet with F^i as a classical field. The form of its transformation rule leads to extra transformation on the ghosts ξ^i and ψ^i, and X^i would be a multiplier. However, this contradicts the ghost number assignments: we can not interpret F^i as a classical field, since it has ghost number minus two. Analogous statements hold for the would-be ghosts ξ^i and ψ^i.

As we will see in the next section, all these problems can be solved by defining a new BRST operator.

4. A new formulation

To remedy this situation, *we propose to change the BRST operator*. The previous BRST operator was obtained from the supersymmetries with as BRST parameter $\Lambda = \epsilon^+ = \tilde{\epsilon}^+$. Instead, we propose to use simply the first of these supersymmetries, and interpret it as a BRST operator by itself. The second supersymmetry we propose to identify with the anti-BRST operator[2]. We will call these operators $\mathbf{s}$ and $\bar{\mathbf{s}}$ respectively. The transformation rules are:

$$
\begin{array}{ll}
\bar{\mathbf{s}}X^{i*} = \psi^{i*} & \mathbf{s}X^{i*} = \xi^{i*} \\
\bar{\mathbf{s}}\psi^i = -\frac{i}{2}\partial_+ X^i & \mathbf{s}\psi^{i*} = -\frac{1}{2}F^{i*} \\
\bar{\mathbf{s}}\xi^{i*} = \frac{1}{2}F^{i*} & \mathbf{s}\xi^i = -\frac{i}{2}\partial_- X^i \\
\bar{\mathbf{s}}F^i = i\partial_+ \xi^i & \mathbf{s}F^i = -i\partial_- \psi^i \ ,
\end{array}
\tag{10}
$$

with all the other (anti)BRST transformations vanishing. One easily verifies the important nilpotency relations $\mathbf{s}^2 = \bar{\mathbf{s}}^2 = \mathbf{s}\bar{\mathbf{s}} + \bar{\mathbf{s}}\mathbf{s} = 0$. Comparing with eq.(8) we see that the previous BRST operator is the sum, $\delta = \mathbf{s} + \bar{\mathbf{s}}$. The invariance of the action under s and $\bar{s}$ follows of course from the original supersymmetries. The condition that fixes the ghost number assignments is now that s raises the ghost number by one unit, $\bar{s}$ lowers it by one unit, and the action has ghost number zero. All the bosons have ghost number zero, and the fermions $\psi^i, \psi^{i^*}, \xi^i, \xi^{i^*}$ have ghost numbers resp. 1,-1,-1,1.

With this new interpretation, the action of the σ model can still be written as the

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[2]For a review of the use of BRST–anti-BRST symmetry of gauge theories, we refer to [5].

sum of a classical action and a gauge fixing part. One easily computes

$$S = \mathbf{s}[-2ig_{ij^*}\partial_+ X^{j^*}\xi^i + 2g_{ij^*}\psi^{j^*}(F^i - 2\Gamma^i_{jk}\xi^j\psi^k)] \ . \tag{11}$$

The classical part does not depend on X^{i*}, and therefore one has a gauge (shift-)symmetry $\delta X^{i*} = \varepsilon^{i*}$, and the corresponding ghosts ξ^{i*}. In accordance with the spirit of the BRST–anti-BRST scheme [5], one introduces also an antighost ψ^{i*}, and its BRST variation F^{i*}. Apart from this quartet, there is a second set of fields transforming into each other, viz. F^i, ψ^i, ξ^i and X^i. They ensure that one restricts the X^i to be constant, which was also the case in the δ picture. Indeed, the BRST gauge fixing condition $\mathbf{s}\xi^i = -\frac{i}{2}\partial_- X^i$ forces these maps to be holomorphic. It is then the anti-BRST operator that kills the anti-holomorphic part of X^i, since it is anti-BRST exact.

There are still two things that are unsatisfactory. First, if F^i is interpreted as a classical field, and the classical action is zero, then the gauge symmetry on F^i would be an arbitrary shift. Looking at its transformation rule we did not include this shift symmetry. Secondly the identifications above do not yet exhibit the customary structure of BRST-anti-BRST, which exhibits more symmetry between ghosts and antighosts: the anti-BRST transformation of the classical fields are usually identical to their BRST transformation, when replacing ghosts with antighosts. This is not the case for the second set of fields above, since we then also have to interchange ∂_+ and ∂_-.

In the $N = 2$ σ-model, the starred and unstarred fields occur symmetrically. The twist has lifted this symmetry: the former are all spinless, but ψ^i and ξ^i have helicities 1 and -1 respectively. One can construct $\psi^i dx^+$ and $\xi^i dx^-$, which behave as one forms under holomorphic coordinate transformations. The asymmetry is mirrored in the derivatives in the transformation laws for the second set, which is in accordance with the helicity-assignment. At the same time, one can also consider F^i to be a two-form, which can not be distinguished from a scalar in the treatment with a flat metric. The BRST–anti-BRST symmetry can be redressed by the following non-local change of field variables:

$$
\begin{aligned}
\psi^i &= \partial_+ \chi^i \\
\xi^i &= \partial_- \rho^i \\
F^i &= \partial_- \partial_+ H^i \ .
\end{aligned}
\tag{12}
$$

All the fields on the right hand side are scalars. Remark that the Jacobian of this transformation is equal to unity, at least formally, since the contributions from the fermions cancel against the bosons. For the new variables we can take the transformation rules

$$
\begin{aligned}
\bar{\mathbf{s}}\chi^i &= -\frac{i}{2}X^i & \mathbf{s}\rho^i &= -\frac{i}{2}X^i \\
\bar{\mathbf{s}}H^i &= i\rho^i & \mathbf{s}H^i &= -i\chi^i \ ,
\end{aligned}
\tag{13}
$$

6

to reproduce the so far "unexplained" rules in (10). They now correspond to a shift symmetry for the field H^i, introducing the ghost field χ^i. The antighost is ρ^i, and X^i completes the quartet. It is clear that we have uncovered a manifest BRST anti-BRST symmetry. The action, when written in terms of the new fields, is of course still BRST exact: one simply writes the exact term in eq.(11) in terms of the new variables.

This allows the following interpretation. One starts from two classical fields, X^{i^*} and H^i. The classical action is zero, and the symmetries are shift symmetries, with ghosts ξ^{i^*} and χ^i. Then one introduces antighosts ψ^{i^*} and ρ^i, and Lagrange multipliers X^i and F^{i^*}. This completes the field content of the theory. Note that the actual content of the resulting TFT depends heavily on the gauge fixing procedure, as usual: there are no physical local fluctuations, but global variables may remain.

Having changed the BRST operator, we now discuss the implications of this change. First of all, we investigate whether we still have a topological theory in the sense that the energy-momentum is BRST exact for the new BRST operator. Afterwards, we will investigate whether the physical content (observables) of the theory has changed.

5. The energy momentum tensor

There are two metrics in the model : the world sheet metric $h_{\alpha\beta}$, which was taken to be flat, and the space time Kähler metric g_{ij^*}. The world sheet metric is external. The space time metric is a function of X^i and X^{i^*}, which are integration variables in the path integral. We can thus only study the dependence of the path integral on the $h_{\alpha\beta}$ metric, by computing the energy momentum tensor. The computation is analogous to the Landau-Ginzburg model [1] and we find

$$
\begin{aligned}
T^B_{++} &= -g_{ij^*}\partial_+ X^{j^*}\partial_+ X^i + 2ig_{ij^*}\psi^i\nabla_+\psi^{j*} = \bar{\mathbf{s}}\left[2ig_{ij^*}\psi^i\partial_+ X^{j^*}\right] \\
T^B_{--} &= -g_{ij^*}\partial_- X^{j^*}\partial_- X^i + 2ig_{ij^*}\xi^i\nabla_-\xi^{j*} = \mathbf{s}\left[2ig_{ij^*}\xi^i\partial_- X^{j^*}\right] \\
T^B_{+-} &= 0 \ .
\end{aligned}
\tag{14}
$$

After the derivation, we have taken the metric to be flat. These are therefore the relevant operators for variations of correlation functions around a flat metric. We see that, although the action is BRST exact, the (++) component of the energy momentum tensor is only anti-BRST exact. This is because the BRST operator depends on the metric and one cannot commute the BRST variation and the derivative w.r.t. the metric [6].

To prove metric independence of correlation functions, one needs not only BRST invariance, but also the Ward identity for the anti-BRST operator. What is needed is that the physical operators are BRST invariant, *and* that their anti-BRST variation is BRST exact. For a more complete argument, see [1].

The proper cohomological formulation is, that one first determines the **s** cohomology, a space of equivalence classes. The operator **s̄** is well defined and nilpotent

in that space, so that *its* cohomology can be used as our characterisation of physical states[3]. This characterisation is not arbitrary, but more or less forced upon us by the requirement that the energy momentum tensor is trivial.We now investigate this cohomology.

6. The spectrum

The observables of the B twisted σ model were first computed in [9]. This was done in the on shell formulation, i.e. without using the auxiliary fields. In order to compare with the off shell formulation, one has to compute the weak cohomology of the BRST operator. In this way one eleminates the auxiliary fields again. For topological LG models, this leads to dividing out the vanishing relations $\kappa \partial_i W = 0$, where W is the LG potential, since it is weakly (using the field equations of the auxiiary fields) equal to the BRST variation of ψ^{i^*}. The local (zero forms) observables are then the elements of the chiral ring. We will see below to what it will lead in the case of the σ model.

6.1. The δ cohomology

Let us start with the δ cohomology. One can make the field redefinitions $c^{i^*} = \psi^{i^*} + \xi^{i^*}$ and $\bar{c}^{i^*} = \xi^{i^*} - \psi^{i^*}$. The BRST algebra on the new fields is then

$$\delta X^{i^*} = c^{i^*} \qquad \delta \bar{c}^{i^*} = F^{i^*}$$
$$\delta c^{i^*} = 0 \qquad \delta F^{i^*} = 0 \ , \tag{15}$$

together with the unstarred sector in the left column of eq.(8).

At first sight, one might think that there is no cohomology in the starred sector, since $\{X^{i^*}, c^{i^*}\}$ and $\{\bar{c}^{i^*}, F^{i^*}\}$ form trivial pairs that drop out of the cohomology. For two reasons this is not true. A first reason is that we are interested in the weak cohomology. The field equations imply relations between the different fields, so that the usual reasoning for eliminating trivial pairs does not apply. Going to the basis with [9]

$$\bar{c}_i = g_{ij^*} \bar{c}^{j^*} \ , \tag{16}$$

one finds for the second pair $\{\bar{c}^{i^*}, F^{i^*}\}$

$$\delta \bar{c}_i = -y_{F^i} = g_{ik^*}(F^{k^*} - \partial_{j^*} g^{k^*l} c^{j^*} \bar{c}_l) \ , \tag{17}$$

where y_{F^i} stands for the field equation of F^i. In this way, the auxiliary fields are eleminated from the spectrum. Therefore, $\bar{c}_i$ is weakly BRST invariant. Since it is not exact, it remains in the weak cohomology.

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[3]This procedure is more familiar than it sounds. When one considers a BRST cohomology class, it is quite common that it contains only one anti-BRST invariant member (up to a factor) — in which case that member is considered to be the physical one. See for example [11].

Consider now a section $V_{(q,0)}^{(0,p)} \in \Omega^{(0,p)} \times \wedge^q T^{(1,0)} \mathcal{M}_K$, i.e. $V_{(q,0)}^{(0,p)}$ takes the form

$$V_{(q,0)}^{(0,p)} = dX^{i_1^*} \wedge ... \wedge dX^{i_p^*} V_{i_1^* ... i_p^*}^{j_1 ... j_q} \frac{\partial}{\partial X^{j_1}} \wedge ... \wedge \frac{\partial}{\partial X^{j_q}} \; . \tag{18}$$

We can associate a zero form to any such section

$$\mathcal{O}[V_{(q,0)}^{(0,p)}] = c^{i_1^*} ... c^{i_p^*} V_{i_1^* ... i_p^*}^{j_1 ... j_q} \bar{c}_{j_1} ... \bar{c}_{j_q} \; . \tag{19}$$

From the identity

$$\delta \mathcal{O}[V_{(q,0)}^{(0,p)}] \approx \mathcal{O}[\bar{\partial} V_{(q,0)}^{(0,p)}] \tag{20}$$

it follows that the BRST cohomology and the twisted Dolbeault cohomology on the target manifold are isomorphic (see [9, 4] for more details): we have that $\mathcal{O}[V_{(q,0)}^{(0,p)}]$ is BRST invariant if $\bar{\partial} V_{(q,0)}^{(0,p)} = 0$ and BRST exact if $V_{(q,0)}^{(0,p)} = \bar{\partial} S_{(q,0)}^{(0,p-1)}$. This isomorphism between BRST cohomology and twisted Dolbeault cohomology only holds for the local observables, i.e the zero forms. To know the global observables, one must solve the descent equations, see [9].

From this isomorphism one can see the second reason why the δ cohomology may not be trivial, namely because of global properties of the target manifold. While one can locally write any closed form as an exact differential, one cannot do this in a global way. For the BRST cohomology this means that functions of $\{X^{i^*}, c^{i^*}\}$ may not drop out of the spectrum.

6.2. The $\bar{\mathbf{s}}$ in $\mathbf{s}$ cohomology
In [1] we, have shown the equivalence of the δ cohomology with the $\bar{\mathbf{s}}$ in $\mathbf{s}$ cohomology in topological Landau-Ginzburg models. Here, we want to argue that this equivalence also holds for B–twisted topological σ models. We will only indicate where the proof differs from the one given in [1]. In the unstarred sector, everything goes through as in [1]. In the starred sector, we have a non zero δ cohomology. The $\{c^{i^*}, \bar{c}_i\}$ basis of the previous subsection is not convenient anymore. This is because of the chiral split we have made when defining the BRST anti-BRST complex. Instead we will define

$$\bar{\psi}_i = g_{ij^*} \psi^{j^*} \; . \tag{21}$$

This leads to

$$\mathbf{s} X^{i^*} = \xi^{i^*} \qquad \bar{\mathbf{s}} X^{i^*} = g^{i^* j} \bar{\psi}_j$$
$$\mathbf{s} \xi^{i^*} = 0 \qquad \bar{\mathbf{s}} \xi^{i^*} \approx \partial_{k^*} g^{i^* j} \bar{\psi}_j \xi^{k^*}$$
$$\mathbf{s} \bar{\psi}_i = \frac{1}{2} y_{F^i} = -1/2 g_{il^*} (F^{l^*} + \partial_{k^*} g^{l^* j} \bar{\psi}_j \xi^{k^*}) \approx 0 \qquad \bar{\mathbf{s}} \bar{\psi}_i = 0, \tag{22}$$

together with the unstarred sector, see eq.(10).

To compute the $\mathbf{s}$ cohomology, we take again a section $V_{(q,0)}^{(0,p)} \in \Omega^{(0,p)} \times \wedge^q T^{(1,0)} \mathcal{M}_K$. Now, we associate with it the zero form operator

$$\mathcal{O}[V_{(q,0)}^{(0,p)}] = \xi^{i_1^*} ... \xi^{i_p^*} V_{i_1^* ... i_p^*}^{j_1 ... j_q} \bar{\psi}_{j_1} ... \bar{\psi}_{j_q} \; . \tag{23}$$

One finds again that

$$\mathbf{s}\mathcal{O}[V_{(q,0)}^{(0,p)}] \approx \mathcal{O}[\bar{\partial}V_{(q,0)}^{(0,p)}] \ . \tag{24}$$

This means that the **s** cohomology is also isomorphic to the twisted Dolbeault cohomology, and thus to the δ cohomology, at least for the zero forms (for higher forms, the reasoning may be extended using descent equations). This indicates that the second step in the computation of the $\bar{\mathbf{s}}$ in the **s** cohomology does not give further restrictions. To check this, we take the anti-BRST variation of an element of the **s** cohomology and find

$$\bar{\mathbf{s}}\mathcal{O}[V_{(q,0)}^{(0,p)}] \approx \mathbf{s}\mathcal{O}[V_{(q+1,0)}^{(0,p-1)}] \ , \tag{25}$$

where $\mathcal{O}[V_{(q+1,0)}^{(0,p-1)}]$ can be read off from eq.(23) raising an index using the metric, more explicitly:

$$\mathcal{O} = (-)^{q-1}\, p\, [\xi^{i_1^*}...\xi^{i_{p-1}^*} V_{i_1^*...i_{p-1}^* i_p^*}^{j_1...j_q} g^{i_p^* j_{q+1}} \bar{\psi}_{j_1}...\bar{\psi}_{j_{q+1}}]. \tag{26}$$

This means that, for zero forms, the $\bar{\mathbf{s}}$ operator in the **s** cohomology is equal to zero, and the $\bar{\mathbf{s}}$ cohomology in the **s** cohomology is equivalent to the δ cohomology.

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