

Some exact results in four-dimensional non-perturbative string theory

Daniel Robles-Llana,^{1,*} Martin Roček,^{2,†} Frank Saueressig,^{1,‡} Ulrich Theis,^{3,§} and Stefan Vandoren^{1,¶}

¹*Institute for Theoretical Physics and Spinoza Institute,
Utrecht University, 3508 TD Utrecht, The Netherlands*

²*C.N. Yang Institute for Theoretical Physics, Stony Brook University, NY 11794-3840, USA*

³*Institute for Theoretical Physics, Friedrich-Schiller-University Jena, D-07743 Jena, Germany*

We find the D(−1) and D1-brane instanton contributions to the hypermultiplet moduli space of type IIB string compactifications on Calabi–Yau threefolds. These combine with known perturbative and worldsheet instanton corrections into a single modular invariant function that determines the hypermultiplet low-energy effective action.

PACS numbers: 11.25.-w, 04.65.+e

The absence of a complete non-perturbative formulation of string theory is its main shortcoming as a full-fledged theory of quantum gravity. In special cases, however, the symmetries and dualities of the theory are powerful enough to fix the exact couplings in the low-energy effective action.

Important examples are provided by type II string compactifications on Calabi–Yau threefolds (CY), where the four-dimensional effective actions are constrained by $\mathcal{N} = 2$ supersymmetry. The massless fields are components of a supergravity multiplet, vector multiplets, or hypermultiplets. Their scalar fields parametrize moduli spaces \mathcal{M}_{VM} and \mathcal{M}_{HM} , respectively, which locally form a direct product [1]. The special geometry of \mathcal{M}_{VM} is determined by a holomorphic function $F(X)$ [2]. The exact expression for this function includes perturbative and worldsheet instanton corrections in the inverse string tension α' , which can in principle be computed by mirror symmetry, see *e.g.* [3]. (More details are given in Fig. 1.)

On the other hand, the string coupling constant g_s is set by the vacuum expectation value of the dilaton, whose four-dimensional reduction belongs to a hypermultiplet. Thus \mathcal{M}_{HM} receives both perturbative and non-perturbative g_s corrections. Building on earlier work [4, 5], the perturbative corrections have recently been understood in [6]. The non-perturbative corrections arise in the IIA case from Euclidean D2 or NS5-branes wrapping around supersymmetric three-cycles or the entire CY, respectively, and in the IIB case from D(−1)-instantons as well as D1, D3, D5, and NS5-branes wrapping holomorphic cycles in the CY [7]. Little is known about summing up such corrections – see however [8, 9] for some results in the limit where gravity decouples.

In this letter, we use the constraints from supersymmetry and the $\text{SL}(2, \mathbb{Z})$ duality symmetry of IIB string theory to determine the full D(−1) and D1-brane instanton corrected low-energy effective action for hypermultiplets in type IIB compactifications on CY. This provides a large class of four-dimensional $\mathcal{N} = 2$ supergravity theories where exact results are obtained to all orders in both α' and g_s .

Similar ideas were applied in [10, 11, 12, 13] to ob-

tain instanton corrections to higher dimensional effective actions: the D(−1) contributions were understood from the modular invariant completion of the R^4 terms in the ten-dimensional effective action, where they are linked to perturbative α' corrections. The D1-branes belong to the $\text{SL}(2, \mathbb{Z})$ multiplet formed by the (p, q) -strings [14]. Therefore their instanton contributions can be found by applying $\text{SL}(2, \mathbb{Z})$ transformations to the worldsheet instanton contributions, which arise from fundamental strings wrapping two-cycles of the compact space.

Hitherto, implementing these dualities on the four-dimensional hypermultiplet moduli space has been hampered by the complicated quaternion-Kähler geometry [15] of \mathcal{M}_{HM} . In recent years, however, it has become clear that, as for the vector multiplet case, hypermultiplet couplings to $\mathcal{N} = 2$ supergravity are determined by a single function $\chi(\phi)$ [16, 17, 18]. Moreover, when only D(−1) and D1-branes are present, enough Peccei–Quinn shift symmetries in the RR sector remain unbroken so that the hypermultiplet sector can be described in terms of tensor multiplets. Projective superspace methods [19] and the superconformal tensor calculus [18] can then be used to encode the off-shell tensor multiplet couplings in terms of a potential χ^t . After dualizing tensors into scalars, χ^t coincides with $\chi(\phi)$. This allows us to find the non-perturbative D(−1) and D1-instanton corrected low-energy effective action by imposing the constraints coming from supersymmetry and $\text{SL}(2, \mathbb{Z})$ invariance on χ^t . In contrast to the vector multiplet prepotential, the potential χ^t is real and therefore receives contributions from both instantons and anti-instantons.

At string tree-level, the c-map [20] provides a simple relation between χ^t and the vector prepotential of the T-dual compactification on the same CY [21]. The IIB hypermultiplet sector is thus determined by the holomorphic prepotential $F(X)$ of IIA vector multiplets. This prepotential describes the complexified Kähler deformations of the CY and consists of three parts. The first one encodes the classical geometry and is given by

$$F_{\text{cl}}(X) = \frac{1}{4!} \kappa_{abc} \frac{X^a X^b X^c}{X^1}, \quad (1)$$

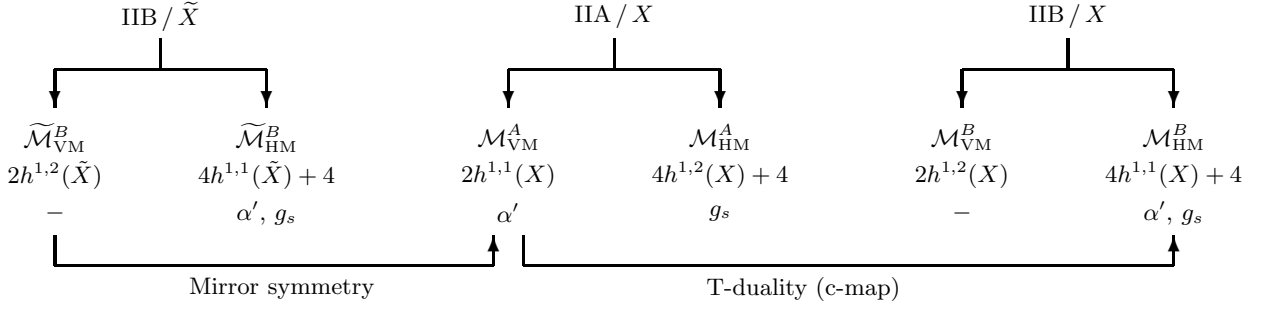


FIG. 1: Dualities relating vector and hypermultiplet moduli spaces arising in CY compactifications of type II strings. The moduli spaces for vector and hypermultiplets are denoted by \mathcal{M}_{VM} and \mathcal{M}_{HM} , respectively, with their dimensions below in terms of the Hodge numbers $h^{1,2}$ and $h^{1,1}$ of the CY X or its mirror \tilde{X} with $h^{1,1}(\tilde{X}) = h^{1,2}(X)$. The lines below indicate the kind of quantum corrections as well as some of the dualities that relate the various moduli spaces. Mirror symmetry is the statement that $\widetilde{\mathcal{M}}_{\text{VM/HM}}^B = \mathcal{M}_{\text{VM/HM}}^A$, while the c-map determines $\mathcal{M}_{\text{HM}}^{B/A}$ at string tree-level in terms of $\mathcal{M}_{\text{VM}}^{A/B}$.

where $X^\Lambda = \{X^1, X^a\}$ (with a running over $h^{1,1}$ values) are the complexified Kähler moduli and κ_{abc} are the (classical) triple intersection numbers. This prepotential receives two kinds of quantum corrections: a perturbative α' correction proportional to the Euler number $\chi_E = 2(h^{1,1} - h^{1,2})$ of the CY,

$$F_{\text{pt}}(X) = \frac{i}{8} \zeta(3) \chi_E (X^1)^2, \quad (2)$$

and non-perturbative worldsheet instanton contributions

$$F_{\text{ws}}(X) = -\frac{i}{4} (X^1)^2 \sum_{\{k_a\}} n_{k_a} \text{Li}_3(e^{2\pi i k_a X^a / X^1}), \quad (3)$$

where the n_{k_a} are genus zero Gopakumar–Vafa invariants that enumerate the rational curves of degree k_a in the CY (see, *e.g.*, [3]). The polylogarithm appearing here is defined as $\text{Li}_s(x) = \sum_{n>0} n^{-s} x^n$.

The c-map relates $F(\tilde{X})$ to the T-dual tensor multiplet Lagrangian density \mathcal{L} in superspace [21]:

$$\mathcal{L}(v^I, \bar{v}^I, x^I) = \text{Im} \oint_C \frac{d\zeta}{2\pi i \zeta} \frac{F(\eta^\Lambda)}{\eta^0}. \quad (4)$$

Here the $\eta^I \equiv v^I / \zeta + x^I - \bar{v}^I \zeta$, $I = (0, \Lambda)$, are $\mathcal{N} = 2$ tensor supermultiplets, ζ is a local coordinate on the Riemann sphere, and the integration contour C encloses one of the roots ζ_+ of $\zeta \eta^0$. The Lagrangian (4) belongs to the general class of superconformal tensor multiplet Lagrangians, which are based on an arbitrary function of the η^I homogeneous of degree one [17]. The corresponding tensor potential (which depends on the tensor multiplet scalars v^I, \bar{v}^I, x^I only) can be computed via [18]

$$\chi^t(v^I, \bar{v}^I, x^I) \equiv -\mathcal{L} + x^I \partial_{x^I} \mathcal{L}. \quad (5)$$

In this formulation the physical scalar fields of the Poincaré theory are given by $\text{SU}(2)_R$ and dilatation invariant combinations of the tensor multiplet scalars. The latter can be grouped into $\text{SU}(2)_R$ vectors $\vec{r}^I =$

$(2v^I, 2\bar{v}^I, x^I)$ with $\vec{r}^I \cdot \vec{r}^J = 4v^{(I} \bar{v}^{J)} + x^I x^J$. Taking the superconformal quotient, one obtains the following identifications [22]: the dilaton-axion system $\tau = a + i e^{-\phi_{10}} = \tau_1 + i \tau_2$ can be expressed as

$$\tau = \frac{1}{(r^0)^2} \left(\vec{r}^0 \cdot \vec{r}^1 + i |\vec{r}^0 \times \vec{r}^1| \right), \quad (6)$$

while the RR scalars are given by $A^\Lambda = \vec{r}^0 \cdot \vec{r}^\Lambda / (r^0)^2$. Finally, $z^a = \eta^a(\zeta_+) / \eta^1(\zeta_+)$ correspond to the complexified Kähler moduli of the CY. Substituting (4) into (5) and expressing the result in terms of the physical fields, the tensor potential can be computed (after a numerical rescaling of $r^0 = |\vec{r}^0|$)

$$\sqrt{2} \chi^t = -i r^0 \tau_2^2 (F_1(z) + \bar{z}^a F_a(z) - \text{c.c.}), \quad (7)$$

where $F_\Lambda = \partial F / \partial X^\Lambda$. Note that the r^0 dependence of χ^t is completely fixed by its superconformal weight.

The tensor potential

$$\chi^t = \chi_{\text{cl}}^t + \chi_{\text{pt}}^t + \chi_{\text{ws}}^t \quad (8)$$

describing the tensor multiplet geometry at string tree-level is then obtained by evaluating (7) for $F = F_{\text{cl}} + F_{\text{pt}} + F_{\text{ws}}$. The classical prepotential (1) gives rise to

$$\chi_{\text{cl}}^t = \sqrt{2} r^0 \sqrt{\tau_2} V(\sqrt{\tau_2} \text{Im } z) \quad (9)$$

with $V(v) = \frac{1}{3!} \kappa_{abc} v^a v^b v^c$, the perturbative α' corrections yield

$$2\sqrt{2} \chi_{\text{pt}}^t = r^0 \chi_E \zeta(3) \tau_2^2, \quad (10)$$

and the worldsheet instanton corrections give

$$\sqrt{2} \chi_{\text{ws}}^t = -r^0 \tau_2^2 \sum_{\{k_a\}} n_{k_a} \text{Re}(\text{Li}_3(e^{iw}) + w_2 \text{Li}_2(e^{iw})) \quad (11)$$

with $w \equiv w_1 + i w_2 = 2\pi k_a z^a$.

We now proceed to implement the $\text{SL}(2, \mathbb{Z})$ duality, and then verify that the resulting non-perturbative completions of (9) – (11) indeed satisfy the supersymmetry

constraints. The action of the modular group on the physical scalar and tensor fields has been worked out in [23]. On the scalar fields, the S generator acts as

$$\begin{aligned} \tau &\mapsto -1/\tau, & A^a &\mapsto -|\tau|^{-2}(\tau_1 A^a + \tau_2^2 \operatorname{Re} z^a), \\ \operatorname{Re} z^a &\mapsto A^a - \tau_1 \operatorname{Re} z^a, & \operatorname{Im} z^a &\mapsto |\tau| \operatorname{Im} z^a, \end{aligned} \quad (12)$$

while the T transformation just shifts

$$\tau_1 \mapsto \tau_1 + 1, \quad (13)$$

and leaves the other scalars invariant. This action is induced by linear transformations of the tensor multiplets [24]

$$\eta^0 \mapsto d\eta^0 + c\eta^1, \quad \eta^1 \mapsto b\eta^0 + a\eta^1, \quad \eta^a \mapsto \eta^a, \quad (14)$$

where the integers a, b, c, d are the entries of the $\operatorname{SL}(2, \mathbb{Z})$ transformation matrix and obey $ad - bc = 1$. In particular, (14) implies that under S transformations $r^0 \mapsto |\tau|r^0$.

The IIB hypermultiplet effective action should be invariant under the $\operatorname{SL}(2, \mathbb{Z})$ transformation of the four-dimensional fields. In other words, the $\operatorname{SL}(2, \mathbb{Z})$ transformations (12) and (13) should act as discrete isometries on the quaternion-Kähler metric underlying the hypermultiplet sector. As such they should lift to invariances of the (hyperkähler) potential $\chi(\phi)$ [16] and χ^t . Note that this condition is more restrictive than in the Kähler case, where invariance of the metric requires invariance of the Kähler potential only up to a Kähler transformation.

Let us examine the modular properties of the tensor potential (8). From the transformations (12), (13) and (14) it follows that the classical part (9) is indeed $\operatorname{SL}(2, \mathbb{Z})$ invariant, which is consistent with the invariance of the classical tensor multiplet action.

The perturbative tensor potential (10), however, is not modular invariant by itself. The form of its modular invariant completion becomes apparent after including the perturbative string loop correction [5, 6, 18]

$$\chi_{1\text{-loop}}^t = \frac{\zeta(2)}{\sqrt{2}} \chi_E r^0. \quad (15)$$

Adding this to (10) gives the perturbatively corrected tensor potential

$$2\sqrt{2}\chi_{\text{pt}}^{t'} = \chi_E r^0 \sqrt{\tau_2} \left(\zeta(3)\tau_2^{3/2} + 2\zeta(2)\tau_2^{-1/2} \right). \quad (16)$$

Inside the brackets one recognizes the first two terms in the expansion of the non-holomorphic Eisenstein series

$$Z_{3/2}(\tau, \bar{\tau}) = \sum'_{m,n} \frac{\tau_2^{3/2}}{|m\tau + n|^3}, \quad (17)$$

where the primed sum is taken over all integers $(m, n) \in \mathbb{Z}^2 \setminus (0, 0)$. This series is naturally generated by applying the method of images to (16). Because the prefactor

$r^0 \sqrt{\tau_2}$ is $\operatorname{SL}(2, \mathbb{Z})$ invariant, the modular invariant completion of (16) is

$$2\sqrt{2}\chi_{(-1)}^t = \frac{\chi_E}{2} r^0 \sqrt{\tau_2} Z_{3/2}(\tau, \bar{\tau}). \quad (18)$$

This expression encodes the perturbative and the full non-perturbative D(-1)-brane instanton corrections to the four-dimensional hypermultiplet effective action. After Poisson resummation (with K_1 the modified Bessel function of first order)

$$\begin{aligned} Z_{3/2} &= 2\zeta(3)\tau_2^{3/2} + \frac{2\pi^2}{3}\tau_2^{-1/2} + 8\pi\tau_2^{1/2} \times \\ &\times \sum_{m \neq 0, n > 0} \left| \frac{m}{n} \right| e^{2\pi i m n \tau_1} K_1(2\pi |m n| \tau_2), \end{aligned} \quad (19)$$

we find terms that are exponentially suppressed at large τ_2 ; these have a clear interpretation as D(-1)-brane instanton contributions. This is analogous to the ten-dimensional IIB case, where the perturbative and D(-1)-brane corrections combine into $\sqrt{\tau_2} Z_{3/2}$ to give the modular invariant and supersymmetry preserving prefactor of the R^4 terms (in string frame) [10, 11].

As for the perturbative correction in (10) discussed above, the non-perturbative term χ_{ws}^t is not $\operatorname{SL}(2, \mathbb{Z})$ invariant. This was to be expected, since χ_{ws}^t contains the contributions of the fundamental string wrapping supersymmetric cycles of the CY, whereas the other contributions from (p, q) -strings in the same $\operatorname{SL}(2, \mathbb{Z})$ multiplet are missing. These can be taken into account by summing χ_{ws}^t over all its $\operatorname{SL}(2, \mathbb{Z})$ transforms, modulo the stabilizer group Γ_∞ generated by T . The result is

$$\begin{aligned} 2\sqrt{2}\chi_{(1)}^t &= -r^0 \sqrt{\tau_2} \sum_{\{k_a\}} n_{k_a} \sum'_{m,n} \frac{\tau_2^{3/2}}{|m\tau + n|^3} \times \\ &\times (1 + |m\tau + n| w_2) e^{-S_{m,n}} \end{aligned} \quad (20)$$

with instanton action

$$S_{m,n} = |m\tau + n| w_2 + i m(\sigma - \tau_1 w_1) - i n w_1 \quad (21)$$

and $\sigma \equiv 2\pi k_a A^a$. The potential (20) encompasses all instanton corrections due to wrapped F- and D-strings as well as their bound states. Indeed, (12) and (13) imply

$$T : S_{m,n} \mapsto S_{m,n+m}, \quad S : S_{m,n} \mapsto S_{-n,m}, \quad (22)$$

which are precisely the $\operatorname{SL}(2, \mathbb{Z})$ transformations of the (p, q) -string action [14]. Note that the instanton corrections (20) are completely fixed by the topology of the underlying CY through its Gopakumar-Vafa invariants [25].

Finally, we must verify that the modular invariant tensor potentials constructed above satisfy the constraints arising from supersymmetry [18]. The key observation is that the supersymmetry constraints, when formulated at the level of χ^t , transform covariantly under

global general linear transformations of the tensor multiplets $\eta^I \mapsto \Lambda^I{}_J \eta^J$, with $\Lambda \in \text{GL}(n, \mathbb{R})$. In particular, the $\text{SL}(2, \mathbb{Z})$ transformations (14) are embedded in this group, and hence commute with supersymmetry. Thus, if we start with the supersymmetric potentials $\chi_{\text{pt}}^{\text{t}}$ and $\chi_{\text{ws}}^{\text{t}}$ and add their $\text{SL}(2, \mathbb{Z})$ transforms, the sum is also supersymmetric. Consequently, the modular invariant tensor potential

$$\chi^{\text{t}} = \chi_{\text{cl}}^{\text{t}} + \chi_{(-1)}^{\text{t}} + \chi_{(1)}^{\text{t}} \quad (23)$$

leads to a supersymmetric tensor multiplet Lagrangian.

We conclude with some remarks. A crucial ingredient in our derivation has been the off-shell tensor multiplet formulation of the low-energy effective supergravity action. After dualizing the tensors into scalars, one obtains the non-perturbative corrections to the hypermultiplet moduli space, together with additional isometries acting as continuous shifts on the dual scalars. These shift symmetries will however be broken by three-brane and five-brane instantons, which have not been included in our work. The main reason is that this sector transforms into itself under $\text{SL}(2, \mathbb{Z})$: D3-branes are selfdual, and D5-branes are mapped into NS5-branes and vice versa. Hence, they cannot be generated from the D(-1), D1-brane and worldsheet instantons with the method used in this letter. To determine these corrections as well, a better understanding of the full duality group beyond $\text{SL}(2, \mathbb{Z})$ is needed. This U-duality group acts non-trivially on the hypermultiplet moduli space and will transform all instanton corrections into each other. Implementing the full symmetry group will require new methods beyond the ones used in this paper (see [26] for a speculation on how some such corrections might be described). We leave this as an open problem, whose solution will undoubtedly contribute to a better understanding of non-perturbative string theory.

This work was initiated at the 4th Simons Workshop in Physics and Mathematics. DRL, FS, UT and SV thank the YITP and the Department of Mathematics at Stony Brook University for hospitality and partial support. DRL is supported by the European Union RTN network MRTN-CT-2004-005104. MR is supported in part by NSF grant no. PHY-0354776. FS is supported by the European Commission Marie Curie Fellowship no. MEIF-CT-2005-023966. UT was supported by the DFG within the priority program SPP 1096 on string theory.

* Electronic address: D.RoblesLlana@phys.uu.nl

† Electronic address: rocek@max2.physics.sunysb.edu

‡ Electronic address: F.S.Saueressig@phys.uu.nl

§ Electronic address: Ulrich.Theis@uni-jena.de

¶ Electronic address: S.Vandoren@phys.uu.nl

- [1] B. de Wit, P. G. Lauwers and A. Van Proeyen, *Lagrangians of N=2 supergravity - matter systems*. Nucl. Phys. **B255** (1985) 569.
- [2] B. de Wit and A. Van Proeyen, *Potentials and symmetries of general gauged N=2 supergravity - Yang-Mills models*. Nucl. Phys. **B245** (1984) 89.
- [3] K. Hori, S. Katz, A. Klemm, R. Pandharipande, R. Thomas, C. Vafa, R. Vakil and E. Zaslow, *Mirror symmetry*. Clay mathematics monographs, AMS, Providence USA, 2003.
- [4] I. Antoniadis, S. Ferrara, R. Minasian and K. S. Narain, *R**4 couplings in M- and type II theories on Calabi-Yau spaces*. Nucl. Phys. **B507** (1997) 571 [arXiv:hep-th/9707013].
- [5] I. Antoniadis, R. Minasian, S. Theisen and P. Vanhove, *String loop corrections to the universal hypermultiplet*. Class. Quant. Grav. **20** (2003) 5079 [arXiv:hep-th/0307268].
- [6] D. Robles-Llana, F. Saueressig and S. Vandoren, *String loop corrected hypermultiplet moduli spaces*. JHEP **0603** (2006) 081 [arXiv:hep-th/0602164].
- [7] K. Becker, M. Becker and A. Strominger, *Five-branes, membranes and nonperturbative string theory*. Nucl. Phys. **B456** (1995) 130 [arXiv:hep-th/9507158].
- [8] H. Ooguri and C. Vafa, *Summing up D-instantons*. Phys. Rev. Lett. **77** (1996) 3296 [arXiv:hep-th/9608079].
- [9] E. Witten, *Heterotic string conformal field theory and A-D-E singularities*. JHEP **0002** (2000) 025 [arXiv:hep-th/9909229].
- [10] M.B. Green and M. Gutperle, *Effects of D-instantons*. Nucl. Phys. **B498** (1997) 195 [arXiv:hep-th/9701093].
- [11] M.B. Green and P. Vanhove, *D-instantons, strings and M-theory*. Phys. Lett. **B408** (1997) 122 [arXiv:hep-th/9704145].
- [12] E. Kiritsis and B. Pioline, *On R**4 threshold corrections in type IIB string theory and (p,q) string instantons*. Nucl. Phys. **B508** (1997) 509 [arXiv:hep-th/9707018].
- [13] B. Pioline and E. Kiritsis, *U-duality and D-brane combinatorics*. Phys. Lett. **B418** (1998) 61 [arXiv:hep-th/9710078].
- [14] E. Witten, *Bound states of strings and p-branes*. Nucl. Phys. **B460** (1996) 335 [arXiv:hep-th/9510135].
- [15] J. Bagger and E. Witten, *Matter couplings in N=2 supergravity*. Nucl. Phys. **B222** (1983) 1.
- [16] B. de Wit, B. Kleijn and S. Vandoren, *Superconformal hypermultiplets*. Nucl. Phys. **B568** (2000) 475 [arXiv:hep-th/9909228].
- [17] B. de Wit, M. Roček and S. Vandoren, *Hypermultiplets, hyperkähler cones and quaternion-Kähler geometry*. JHEP **0102** (2001) 039 [arXiv:hep-th/0101161].
- [18] B. de Wit and F. Saueressig, *Off-shell N = 2 tensor supermultiplets*. JHEP **0609** (2006) 062 [arXiv:hep-th/0606148].
- [19] S. J. Gates, Jr., C. Hull and M. Roček, *Twisted multiplets and new supersymmetric nonlinear sigma models*. Nucl. Phys. **B248** (1984) 157; A. Karlhede, U. Lindström and M. Roček, *Self-interacting tensor multiplets in N=2 superspace*. Phys. Lett. **B147** (1984) 297.
- [20] S. Ferrara and S. Sabharwal, *Quaternionic manifolds for type II superstring vacua of Calabi-Yau spaces*. Nucl. Phys. **B332** (1990) 317.
- [21] M. Roček, C. Vafa and S. Vandoren, *Hypermultiplets*

- and topological strings*. JHEP **0602** (2006) 062
[arXiv:hep-th/0512206].
- [22] B. Pioline and S. Vandoren, private communication.
- [23] R. Böhm, H. Günther, C. Herrmann and J. Louis, *Compactification of type IIB string theory on Calabi-Yau threefolds*. Nucl. Phys. **B569** (2000) 229
[arXiv:hep-th/9908007].
- [24] N. Berkovits, *Conformal compensators and manifest type IIB S duality*. Phys. Lett. **B423** (1998) 265
[arXiv:hep-th/9801009].
- [25] R. Gopakumar and C. Vafa, *M-theory and topological strings. I*. [arXiv:hep-th/9809187].
- [26] L. Anguelova, M. Roček and S. Vandoren, *Quantum corrections to the universal hypermultiplet and superspace*. Phys. Rev. **D70**, 066001 (2004)
[arXiv:hep-th/0402132].