Orientifolding in $N = 2$ Superspace

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Abstract

We discuss orientifold projections on superspace effective actions for hypermultiplets. We present a simple and new mechanism that allows one to find the Kähler potential and complex structure for the $N = 1$ theory directly in terms of the parent $N = 2$ theory. As an application, we demonstrate our method for Calabi-Yau orientifold compactifications of type IIB superstrings.

1 Introduction

Compactifications of type II strings on Calabi-Yau threefolds with branes/fluxes lead to interesting semi-realistic models of four-dimensional physics with $N = 1$ supersymmetry (see [1, 2] for recent reviews). For compact Calabi-Yau's, cancellation of tadpoles requires the introduction of orientifold planes. These are objects with negative tension, and, in contrast to D-branes, have no dynamical degrees of freedom.

The orientifolds we consider can be understood as modding out the combined operation of worldsheet parity and an involution on the Calabi-Yau space. In the cases we study, the supersymmetry is reduced from $N = 2$ to $N = 1$. Here we concentrate on the effects of the projection only, freezing brane moduli and switching off fluxes (superpotentials) and quantum corrections. In that case the kinetic term in the resulting $N = 1$ supersymmetric low-energy effective action has a very particular structure. It is locally the product of two Kähler manifolds. These can be seen as submanifolds of the special Kähler and quaternion-Kähler moduli spaces of the $N = 2$ vector and hypermultiplets respectively.

Whereas it is straightforward to exhibit the Kähler structure of the $N = 1$ subsector arising from truncating $N = 2$ vector multiplets, the projection is more subtle for the hypermultiplet sector [3]: the quaternion-Kähler geometry of the hypermultiplet moduli space is generically not Kähler. In this note we demonstrate that this problem is easily overcome by working off-shell in $N = 2$ superspace. The off-shell formulation of matter coupled $N = 2$ supergravity uses a compensator that restores the superconformal symmetry. The orbifold projection is then easily implemented on $N = 2$ projective superfields. As a result, the $N = 2$ superconformal quotient yielding the original $N = 2$ theory reduces to an $N = 1$ Kähler quotient which gives the Kähler potential of the orientifolded theory. We illustrate our findings by considering $\mathcal{O}3/\mathcal{O}7$ and $\mathcal{O}5/\mathcal{O}9$ orientifolds of type IIB string theory on Calabi Yau threefolds. A similar analysis can be performed for type IIA strings.

2 Orientifolds and projective superspace

In this section we outline the orientifold projection of $N = 2$ supergravity theories coupled to tensor multiplets or hypermultiplets. As mentioned in the introduction, rather than working with the Poincaré theory, we use the superconformal tensor calculus to give an off-shell description in projective superspace. In such a formalism the general couplings are based on the $N = 2$ superspace Lagrangian density [4]

$$\mathcal{L}(v^I, \bar{v}^I, x^I) = \mathrm{Im} \oint_C \frac{\mathrm{d}\zeta}{2\pi\mathrm{i}\,\zeta} H(\eta^I) \ , \tag{1}$$

where C is an appropriately chosen contour, and ζ is a local coordinate on the Riemann sphere. The function $H(\eta)$ must be homogeneous of degree one for superconformal invariance of the action [5]. Projective superfields are defined by

$$\eta^I(\zeta) \equiv \frac{v^I}{\zeta} + x^I - \bar{v}^I \zeta \ , \tag{2}$$

and describe $N = 2$ tensor supermultiplets. In terms of $N = 1$ superfields, the components v^I are chiral $N = 1$ multiplets whereas the x^I are $N = 1$ tensor multiplets containing one real scalar and a tensor. They both have scaling weight two under dilatations, and the three scalars form a triplet under $SU(2)_R$, which is part of the superconformal symmetry group.

To dualize to hypermultiplets, one first defines the tensor potential [6]

$$\chi(v^I, \bar{v}^I, x^I) \equiv -\mathcal{L} + x^I \partial_{x^I} \mathcal{L} \ , \tag{3}$$

and performs a Legendre transformation by defining dual $N = 1$ chiral superfields

$$w_I + \bar{w}_I = \frac{\partial \mathcal{L}}{\partial x^I} \ , \tag{4}$$

which have scaling weight zero. After eliminating the x^I in terms of the chiral superfields, this potential becomes the Kähler potential for the superconformal hypermultiplets,

$$K(v, \bar{v}, w + \bar{w}) = \chi(v^I, \bar{v}^I, x^I(v, \bar{v}, w + \bar{w})) \ . \tag{5}$$

In fact, the scalars of the hypermultiplets parametrize a hyperkähler manifold.

Finally, to descend to the Poincaré theory, one couples to the Weyl multiplet and performs a superconformal quotient. In this formulation the physical scalar fields of the Poincaré theory are given by $SU(2)_R$ and dilatation invariant combinations of the tensor multiplet scalars.

The orientifold projections we consider break half of the supersymmetries and truncate the theory to $N = 1$. As we explicitly demonstrate in the next section, the orientifold projection can be implemented in projective superspace by defining a parity operator on the complex coordinate ζ, and requiring projective superfields to be either parity-even or parity-odd

$$\begin{aligned} \eta(-\zeta) &= \eta(\zeta) \quad \Longleftrightarrow v = \bar{v} = 0 \ , \\ \eta(-\zeta) &= -\eta(\zeta) \Longleftrightarrow x = 0 \ . \end{aligned} \tag{6}$$

We see that this projects out either the $N = 1$ chiral superfield v or the $N = 1$ tensor multiplet x, and hence there is only $N = 1$ supersymmetry left over, as required.

Since we are working in an off-shell formulation, this projection can be carried out on the superspace Lagrangian density $\mathcal{L}$ or on the Kähler potential K. The different $N = 2$ tensor multiplets $\eta(\zeta)$ can be subject to either of the two conditions in (6). The resulting $N = 1$ theory is then based on the Kähler potential $K(v, \bar{v}, w + \bar{w})$ with a number of chiral multiplets v^M and a number of $N = 1$ tensor multiplets x^u. One then integrates this function over the standard $N = 1$ superspace measure, and the theory still has $N = 1$ superconformal invariance with dilatation and $U(1)_R \subset SU(2)_R$ symmetry inherited from the $N = 2$ theory. To descend to the Poincaré theory, one couples to the $N = 1$ Weyl multiplet and performs an $N = 1$ superconformal quotient. This can be done by first defining the scale and $U(1)_R$ invariant coordinates (with $M = \{0, A\}$)

$$\tau^M \equiv \frac{v^M}{v^0} = \{1, \tau^A\} \ . \tag{7}$$

In other words, v^0 is the compensator. Gauge-fixing dilatations and $U(1)_R$ gives old minimal $N = 1$ supergravity coupled to chiral multiplets [1]. Because of the homogeneity properties, v^0 can be scaled out of the Kähler potential K and after the $N = 1$ superconformal quotient, the resulting Kähler potential for the chiral multiplets is

$$\mathcal{K}(\tau^A, \bar{\tau}^A, w^u + \bar{w}^u) = -\log K(\tau^A, \bar{\tau}^A, w^u + \bar{w}^u) \ . \tag{8}$$

In more geometric terms, this quotient is just a standard $U(1)$ Kähler quotient [7].

3 Calabi-Yau orientifolds of IIB superstrings

We consider the compactification of type IIB strings on Calabi-Yau orientifolds leading to $N = 1$ supersymmetric actions in four dimensions. After briefly summarizing the results of [8], we rederive them starting from the superspace formulation of the $N = 2$ supersymmetric theory and apply the orientifold projection directly in superspace.

3.1 Orientifold projections and complex structures

The four-dimensional effective actions have been worked out in [8]. Before orientifolding the massless fields of the $N = 2$ supergravity theory are summarized in Table 1.

gravity multiplet	1	$(g_{\mu\nu}, V^0)$
vector multiplets	$h^{(1,2)}$	(V^K, X^K)
tensor multiplets	$h^{(1,1)}$	(t^a, b^a, c^a, E_2^a)
double-tensor multiplet	1	(B_2, C_2, ϕ, l)

Table 1: Massless spectrum of type IIB strings compactified on a Calabi-Yau threefold.

There are two ways of performing the orientifold projection, which lead to CY orientifolds with either $\mathcal{O}3/\mathcal{O}7$ or $\mathcal{O}5/\mathcal{O}9$ planes. From the point of view of the $N = 2$ effective theory the difference between these theories arises from the double tensor multiplet sector. In $\mathcal{O}3/\mathcal{O}7$ orientifolds the B_2, C_2 tensors are projected out and one is left with a chiral multiplet containing ϕ, l. For the $\mathcal{O}5/\mathcal{O}9$ case the l, B_2 are projected out and ϕ, C_2 form a linear multiplet.

To find the spectrum of the other fields, one decomposes the cohomology classes $H^{(1,1)}$ and $H^{(1,2)}$ into subspaces which are even and odd under the orientifold action.

$$H^{1,1} = H_+^{(1,1)} \oplus H_-^{(1,1)} \ , \qquad H^{(1,2)} = H_+^{(1,2)} \oplus H_-^{(1,2)} \ , \tag{9}$$

with dimensions $h_+^{(1,1)}, h_-^{(1,1)}, h_+^{(1,2)}$ and $h_-^{(1,2)}$, respectively. From this one readily works out the $N = 1$ spectrum of the orientifolded theory. The fields appearing in the $\mathcal{O}3/\mathcal{O}7$ and $\mathcal{O}5/\mathcal{O}9$ are then summarized in Table 2. To bring the $N = 1$ effective action into standard form one has to

[1] There is an alternative way of performing the superconformal quotient by using an $N = 1$ tensor multiplet as a compensator. This yields new minimal $N = 1$ supergravity, but we do not discuss this option in this paper.

	$\mathcal{O}3/\mathcal{O}7$		$\mathcal{O}5/\mathcal{O}9$	
gravity multiplet	1	$g_{\mu\nu}$	1	$g_{\mu\nu}$
vector multiplets	$h_{+}^{(1,2)}$	V^{k}	$h_{-}^{(1,2)}$	V^{κ}
chiral multiplets	$h_{-}^{(1,2)}$	X^{κ}	$h_{+}^{(1,2)}$	X^{k}
chiral multiplets	1 $h_{-}^{(1,1)}$	(ϕ, l) $(b^{\lambda}, c^{\lambda})$	– $h_{+}^{(1,1)}$	– (t^{σ}, c^{σ})
linear multiplets	– $h_{+}^{(1,1)}$	– $(t^{\sigma}, E_2^{\sigma})$	1 $h_{-}^{(1,1)}$	(ϕ, C_2) $(b^{\lambda}, E_2^{\lambda})$

Table 2: Bosonic fields in the spectrum of Calabi-Yau orientifold compactifications with $\mathcal{O}3/\mathcal{O}7$ and $\mathcal{O}5/\mathcal{O}9$ planes.

find a complex structure for the chiral fields spanning a Kähler space. In the vector multiplet sector this is simply the complex structure provided by the $N = 2$ vector multiplet scalars. For the remaining chiral multiplets, these holomorphic coordinates are provided by

$$\tau \equiv l + \mathrm{i}e^{-\phi}\,, \qquad G^{\lambda} \equiv c^{\lambda} - \tau b^{\lambda}\,, \qquad \text{and} \qquad \tau^{\sigma} \equiv \mathrm{e}^{-\phi}\, t^{\sigma} + \mathrm{i}\, c^{\sigma}\ , \tag{10}$$

for the $\mathcal{O}3/\mathcal{O}7$ and $\mathcal{O}5/\mathcal{O}9$-orientifold, respectively. We collectively denote these complex coordinates by τ^A and will show that they coincide with (7). For the clarity of notation, we have $\tau^A = \{\tau, G^{\lambda}\}$ ($\mathcal{O}3/\mathcal{O}7$) and $\tau^A = \{\tau^{\sigma}\}$ ($\mathcal{O}5/\mathcal{O}9$). One can then determine the Kähler potential in terms of τ^A, following [8]. We give explicit formulae in later subsections.

3.2 Superspace description

The fields introduced above can be expressed through $SU(2)_R$ and dilatation invariant combinations of the scalar fields of the superconformal theory. The resulting expressions for the $N = 2$ theory are given by [9, 10]

$$l + \mathrm{i}\,\mathrm{e}^{-\phi} = \frac{1}{2\sqrt{2}(r^0)^2}\left[\vec{r}^{\,0}\cdot\vec{r}^{\,1} + \mathrm{i}\,|\vec{r}^{\,0}\times\vec{r}^{\,1}|\right]\,, \quad b^a + \mathrm{i}\,t^a = \frac{\eta_+^a}{\eta_+^1}\,, \quad \sqrt{2}(lb^a - c^a) = \frac{\vec{r}^{\,0}\cdot\vec{r}^{\,a}}{2(r^0)^2}\,. \tag{11}$$

Here $\vec{r}^{\,I} = \left[-i(v^I - \bar{v}^I)\,,\, v^I + \bar{v}^I\,,\, x^I\right]$ and, with $I = \{0, \Lambda\}$ and $\Lambda = \{1, a\}$

$$\eta_+^{\Lambda} = x^{\Lambda} + \frac{x^0}{2}\left(\frac{v^{\Lambda}}{v^0} + \frac{\bar{v}^{\Lambda}}{\bar{v}^0}\right) + \frac{r^0}{2}\left(-\frac{v^{\Lambda}}{v^0} + \frac{\bar{v}^{\Lambda}}{\bar{v}^0}\right)\ . \tag{12}$$

Finally, the (tree-level) tensor potential is given by [10]

$$\chi = \sqrt{2}\,|\vec{r}^{\,0}|\mathrm{e}^{-2\phi}\ V(t)\ , \tag{13}$$

with $V(t) = \frac{1}{3!}\kappa_{abc}t^a t^b t^c$, where κ_{abc} are the triple intersection numbers of the Calabi-Yau manifold. Similarly, an explicit expression for the superspace Lagrangian density was given in [9].

We can now apply the orientifolding on the level of the off-shell theory. We first consider the $\mathcal{O}3/\mathcal{O}7$ orientifold before turning to the $\mathcal{O}5/\mathcal{O}9$ case.

3.2.1 Orientifolding to $\mathcal{O}3/\mathcal{O}7$

This truncation can be effected by setting the $N = 1$ linear superfields

$$\underbrace{x^0 = 0\,, \quad x^1 = 0}_{\text{universal sector}}, \qquad \underbrace{x^\lambda = 0}_{h_-^{(1,1)}\text{ parity odd}}\,, \qquad \underbrace{v^\sigma = 0}_{h_+^{(1,1)}\text{ parity even}}\,. \tag{14}$$

Substituting this truncation into (11) gives

$$\tau \equiv l + \mathrm{i}e^{-\phi} = \frac{1}{2\sqrt{2}}\frac{v^1}{v^0}\,, \qquad G^\lambda \equiv c^\lambda - \tau b^\lambda = -\frac{1}{2\sqrt{2}}\frac{v^\lambda}{v^0}\,, \quad t^\sigma = \mathrm{i}|v^0|\frac{x^\sigma}{v^1\,\bar{v}^0 - \bar{v}^1\,v^0}\,, \tag{15}$$

and projects out $B_2, C_2, t^\lambda, E_2^\lambda, b^\sigma, c^\sigma$. Comparing to Table 2 we see that (14) reproduces the spectrum of the $\mathcal{O}3/\mathcal{O}7$ orientifold projection together with the complex structure (7) for the chiral fields (recall that in our notation $\tau^A = \{\tau, G^\lambda\}$).

From (8) and (13) it is easy to determine the Kähler potential. After scaling out v^0, we find, up to an irrelevant additive constant,

$$\mathcal{K} = -2\log[-i(\tau - \bar{\tau})] - \log[V(t)]\,. \tag{16}$$

Taking into account the proper rescaling of t^σ this agrees with [8]. We have written the result in terms of τ and the real linear superfields t^σ. The latter still need to be replaced in terms of the chiral superfields through the Legendre transform. We refrain from giving explicit formulae, as this procedure is identical as in [8].

3.3 Orientifolding to $\mathcal{O}5/\mathcal{O}9$

As a second example we discuss orientifold compactifications with $\mathcal{O}5/\mathcal{O}9$-planes. In this case we set

$$\underbrace{x^0 = 0\,, \quad v^1 = 0}_{\text{universal sector}}, \qquad \underbrace{v^\lambda = 0}_{h_-^{(1,1)}\text{ parity odd}}\,, \qquad \underbrace{x^\sigma = 0}_{h_+^{(1,1)}\text{ parity even}}\,. \tag{17}$$

Substituting (17) into the expression for the $N = 2$ Poincaré fields gives

$$l = 0\,, \qquad e^{-\phi} = \pm\frac{1}{4\sqrt{2}}\frac{x^1}{|v^0|}\,, \qquad b^\lambda = \frac{x^\lambda}{x^1}\,, \qquad \tau^\sigma = -\frac{i}{2\sqrt{2}}\frac{v^\sigma}{v^0}\,, \tag{18}$$

while $l, b^\sigma, E_2^\sigma, c^\lambda, t^\lambda$ and one of the universal tensors are projected out. Comparing to Table 2 we see that (17) reproduces the spectrum of the $\mathcal{O}5/\mathcal{O}9$ orientifold projection together with the complex structure (7) for the chiral fields. The Kähler potential can again be obtained from (8) and (13) after performing the appropriate dualization of the linear multiplets.

The general technique outlined in Section 2 should also apply to Calabi-Yau orientifolds of the type IIA string [11].

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