# General Introduction

## 1. Introduction

Between the eighth and fifteenth centuries A.D., the main centers for the study of the exact sciences were in the Islamic world. Astronomy was one of the most important sciences studied in medieval Islamic civilization, not only because of its relevance to Islamic timekeeping and prayers, but also for its own sake and because of its relationship to astrology. By the eighth century, the mathematical and astronomical traditions of pre-Islamic Iran and India had been assimilated. From the ninth century onwards, the Islamic astronomers studied the Greek tradition, and several Arabic translations were produced of the most important astronomical work of Greek antiquity, the *Almagest* of Ptolemy. The Ptolemaic models of planetary motion are physically 'wrong', because the earth is considered as the center of the universe. Important basic features of the Ptolemaic models can be mathematically transformed to heliocentric ones by a simple change of coordinates. This explains to a modern reader why these Ptolemaic models, which were used by Islamic astronomers, can be fitted very well to observed positions of the planets. The resulting predictions are so accurate that the errors can hardly be noticed with naked eye.

From the ninth century onwards, the Islamic astronomers made new observations in order to check and correct some of the parameter values that were used in Ptolemy's astronomical models, such as the value of the solar eccentricity. In the tenth century and later, the Ptolemaic models were themselves modified. For example, it was discovered that the apogee (point of furthest distance) of the sun had a very slow motion with respect to the signs of the zodiac and also with respect to fixed stars. We shall not be concerned with the modifications of the Ptolemaic models made for purely philosophical reasons in the Eastern Islamic world in the twelfth century and later, in order to make these models completely consistent with the Aristotelian dogma of uniform circular motion.

Libraries in and outside the present Islamic world contain thousands of medieval Arabic astronomical manuscripts. Only few of these have been studied to date.[1] An important genre in the Arabic astronomical literature is a group of treatises called *zīj*es (plural of *zīj*). From the ninth century onwards, the Islamic tradition produced constantly astronomical handbooks with instructions and tables for the computation of solar, lunar and planetary positions. These handbooks were often also provided with auxiliary trigonometrical tables, tables for lunar visibility and prayer times, geographical tables, tables of stellar positions, and astrological tables. The total number of pages with tables in a single handbook may be 150-200. These handbooks were called *zīj*, from Old Persian *zīg*, meaning "thread" or "chord". By extension the word came to mean "the set of parallel threads making up the warp of a fabric". Then, since the closely drawn vertical lines of a numerical table are similar to the parallel strings of a textile, the meaning was further extended to include the former. And finally, the word came to denote whole sets of astronomical tables with instructions.

More than 200 *zīj*es are known to have been written, of which more than 100 are extant, sometimes in many different manuscripts. Surveys of the entire *zīj* literature can be found in [Kennedy 1956]. A new survey of Islamic *zīj*es is currently under preparation by Dr. Benno van Dalen. For more information about *zīj*es see [King and Samsó 2001]. Only a few *zīj*es have been published to date. The *zīj* of al-Battānī appeared between 1899 and 1907 in a critical Arabic edition with a Latin translation in three volumes [al-Battānī 1899-1907]. Al-Bīrūnī's *al-Qānūn al-Mas'ūdī* or *Canon Masudicus* in 13 Books can also be considered a *zīj*. This was published in India between 1954 and 1956 in an uncritical edition [al-Bīrūnī 1954-1956]. The text was not analyzed mathematically and astronomically, and consequently many errors in the manuscripts which were used remained uncorrected. (These errors were not corrected in the Russian translation published in 1973 and 1976 either [al-Bīrūnī 1973-1976]). The Persian introductions to the 15th century *zīj* of Ulugh Beg (but only with the chronological tables) appeared in an edition with French translation in [Sédillot 1847 and 1853]. Some medieval Latin reworkings of Arabic *zīj*es have also been published (e.g., [Al-Khwārimī 1962]). Modern researchers have studied many other Arabic *zij*es, and have often published smaller sections thereof, but no edition with

[1] The most up-to-date lists of mathematical and astronomical manuscripts can be found in [Sezgin 1974 and 1976; Rosenfeld & Ihsanoğlu 2003].

translation of whole *zījes* has appeared, and no Arabic *zīj* has ever been translated into English hitherto. The *zījes* contain a mass of material which, once properly evaluated, will lead to a detailed knowledge of the development and transmission of astronomical and mathematical knowledge in Islamic civilization, and thus shed light on an aspect of Islamic culture which is severely underrepresented and underestimated in modern historical accounts and political discussions. My dissertation is a small contribution to the enormous amount of research work that still remains to be done.

The present dissertation contains an edited Arabic version with an English translation of a large part of the *Jāmiʿ Zīj* ("Comprehensive astronomical handbook with tables") by the Iranian mathematician and astronomer Kūshyār ibn Labbān (ca. 1025 A.D.). Kūshyār's *Jāmiʿ Zīj* has drawn the attention of modern historians of science, and several sections of it have been published on the basis of a few Arabic manuscripts. These publications will be discussed in Part 3 below. The whole *Jāmiʿ Zīj* has never been critically edited. The part which I am presenting in this dissertation consists of Books I and IV, containing the astronomical instructions (Book I) and the corresponding "Proofs" (Book IV). I am currently preparing editions and translations of the remaining Books II (numerical tables) and III (cosmology). The *Jāmiʿ Zīj* will be the first one to be translated into English in its entirety.

The reader may wonder why I have chosen Kūshyār's *zīj* from the 100 or so extant unpublished *zījes*. The fact that Kūshyār was a native my homeland Gīlān (north Iran) constitutes an emotional reason. In addition, there are good scientific and historical reasons. Kūshyār's *zīj* dates back to a relatively early period, and unlike most other early *zījes*, Kūshyār presents not only astronomical instructions (in Book I), but also the corresponding "Proofs" (in Book IV). Thus Kūshyār's work, unlike many other similar works, allows us to have a more intimate knowledge of the eleventh-century astronomical view and thought. As we shall see in Part 4 below, Kūshyār was a competent mathematician, who had studied the works of his predecessors critically, and who seems to be the author of some new methods in his *zīj* not to be found elsewhere. Also, Section I.1 of the *Jāmiʿ Zīj* is one of the earliest extant Arabic treatments of calendars, and provides important information especially on the old Persian calendar that was still in use in his time. The earliest documented change of one of the equations for Mars by a Muslim astronomer is that of the equation of centrum by Kūshyār [Van Dalen 2004a, 22] (cf. [Van Brummelen 1988, 268]). Finally, Kūshyār's work is nearly contemporary with al-Bīrūnī's colossal *Canon Masudicus*, which has already been mentioned, and which is probably the major work of Islamic astronomy (comparable to Ptolemy's *Almagest*). I hope that my edition of Kūshyār's work can serve as an incentive for future editions and translations of other *zīj*es, including the *Canon Masudicus* of al-Bīrūnī.

In Part 2 of this preface, I discuss the available information about Kūshyār's life and his other works. In Part 3, I report the studies of Kūshyār's *Jāmi' Zīj* by modern historians and a summary of its contents. In Part 4, I discuss the characteristics of Kūshyār's *zīj*, and some medieval astronomical terminology with which the reader of the *Jāmi' Zīj* should be familiar. Part 5 deals with the extant Arabic manuscripts of Kūshyār's *Jāmi' Zīj*, and the editorial procedure which I have followed in establishing the Arabic text and the translation.

The reader interested in more discussions about different aspects of the Islamic period astronomy may be referred to *Studies in Islamic exact sciences* by E. S. Kennedy, his colleagues and former students [Kennedy 1983], and the different works by B. Van Dalen, J. P. Hogendijk, D. A. King, F. J. Ragep [Al-Ṭūsī 1993] and G. Saliba, mentioned in the bibliography below (Part 6).

Publications will be referred to by author's name and year of publication, followed by page number(s) if necessary, thus: [Kennedy 1956, 125]. The full reference can be found in the bibliography in Part 6. Dates will be given in Christian chronology (A.D.), Islamic lunar chronology (A.H.), and the Persian Yazdigird chronology (A.Y.). The Islamic chronology began on July 15, 622 A.D., and its years are lunar years of 12 months (354 or 355 days). The Yazdigird chronology began on June 16, 632 A.D., and its years are solar years of 365 days.

I shall use the standard sexagesimal notation for all values, in which, e.g., 3,34;0,15 means $3\times60+34+0/60+15/60^2$. Kūshyār uses the technical term *inḥiṭāṭ* ("lowering") for division by 60 which actually leads to a shift of sexagesimal position.

## 2. Life and work of Kūshyār ibn Labbān

Kūshyār's complete name was *Kīā* Abu'l-Ḥasan Kūshyār ibn Labbān Bāshahrī al-Jīlī. The word *kīā* meant "king" or "ruler" in classical Persian. It was also prefixed to the names of some authorities and scholars in the Caspian province of Gīlān. His *kunya* Abu'l-Ḥasan, literally "Ḥasan's father", shows that he was a Muslim. However, his given name –Kūshyār– is a pure Persian name connected with the Zoroastrian religion. Its original form was *Gūshyār*, consisting of the name *Gūsh* and the suffix *-yār*. In the pre-Islamic Iranian calendar, each month of a year had 30 days, and each day of the month had a special name. The 14th day of each month was called *Gūsh-rūz* (the day of *Gūsh*; see Chapter 2 of the second section in Book I of the *Jāmi' Zīj* ), after Gūsh, the guardian angel of useful quadrupeds in the Zoroastrian religion, which still had some followers in Gīlān in Kūshyār's time. *Gūshyār* literally means "a gift of Gūsh" or "aided by Gūsh", generally taken to mean "fortunate". Maybe the Arabic title *Sa'īd* ("fortunate, auspicious") for Kūshyār, found in some manuscripts of his works, was merely a translation of the word *Gūshyār* [Mu'īn 1952, 202-04]. His *nisba* al-Jīlī (arabicized form of *Gīlī*) attributes him to Gīlān. The arabicized form Jīlī is also used by European authors. He is now referred to in Iran as Kūshyār-e Gīlānī (or Gīlī).

We know little about Kūshyār's biography. He was an eminent Iranian mathematician and astronomer who lived in the second half of the 10th and the early 11th century A.D. [Saidan 1973; Qurbani 1996, 414-420; Yano 1997; Jaouiche 1986; Pingree 2002; Bagheri 2006a]. He was from the Gīlān province situated in the northern part of Iran, on the southern coast of the Caspian Sea. Since he finished writing a copy of his *Jāmi' Zīj* in 393 A.Y./1025 A.D., and that, according to al-Nasawī, he was dead in 416 A.Y./1048 A.D. (see below), he must have died between 1025 and 1048 A.D. In Book I of the *Jāmi' Zīj* (Chapter 5, Section 7), Kūshyār presents an example of a nativity in 332 A.Y./963-4 A.D. that may refer to his own date of birth. He then finds the years that had elapsed from that year up to 389 A.Y./1020-21 A.D., which may be taken as the year in which he wrote Book I of the *Jāmi' Zīj*. A detailed account of the social conditions of Gīlān in Kūshyār's time is provided in the introduction to the French translation of his treatise on arithmetic [Mazahéri 1975].

Kūshyār spent part of his life in Rayy[2], as he explicitly mentions it in I.1.3. We know from al-Bīrūnī [1985, 101,139,143] that he met Kūshyār (evidently in Rayy). Kūshyār told al-Bīrūnī that he had abridged al-Khujandī's *qānūn al-hay'a* (lit., "Rule of cosmology"; i.e., the sine theorem in right spherical triangles) and renamed it *al-Mughnī* (lit., "making [one] able to dispense [with Menelaus' Theorem]"). See IV.3.1 and its commentary.

Kūshyār was probably an astronomer at the court of Voshmgīr (d. 357 A.H./967-8 A.D.), the Iranian local ruler in Māzandarān province, on the southern coast of the Caspian Sea, immediately east of Gīlān. In *Tārīkh-i Māzandarān* ("A history of Māzandarān") composed in the 17th century A.D., we read: "One day in the month of Muḥarram 357 A.H., in the city of Jurjān[3], Kūshyār advised the ruler of Māzandarān, Voshmgīr, not to ride horses throughout that day lest he should be killed. All the saddles were taken off the horses, and the ruler did not ride all day long. However, in the evening he heard the grunt of a wild boar, and he could not help riding. He mounted a horse and followed the wild boar; the boar rushed towards the horse, Voshmgīr fell and died" [Gīlānī 1973, 78]. This account is not consistent with the above assumption for Kūshyār's date of birth. However, older sources such as [Ibn Isfandiyār 1941, part 2, 3-4], composed in the early 14th century A.D., which mentions Kūshyār among the astronomers of Ṭabaristān (an older name for Māzandarān) [ibid., part 1, 137], and [Mar'ashī 1954, 131], composed in the late 15th century A.D., give similar accounts of the same event without naming Kūshyār. Therefore it seems that the astronomer in this story was someone else, and Kūshyār was in fact at the court of Voshmgīr's son, Qābūs (reigned 367-403 A.H./977/8-1012/3 A.D.), to whom al-Bīrūnī presented his *Āthār al-bāqiya* (*Chronology of ancient nations*) in 390 A.H/999-1000 A.D. The following account confirms this conjecture.

In the medical treatise *Dhakhīra-yi Khwārazmshāhī* ("Khwārazmshāh's treasure"), written in Persian by Sayyed Ismā'īl Jurjānī in 504 A.H./1110-11 A.D., the author says that Kūshyār was a learned astronomer from Gīlān who lived in Gurgān in the service of Qābūs (Voshmgīr's son). Then Jurjānī narrates his encounter with some descendants of Kūshyār in Qum. They showed him treatises written by Kūshyār in a very neat and nice form. They told Jurjānī that "Kūshyār wrote only when he was calm and relaxed, and his books are written very

[2] An old city of Iran, now adjacent to the south-eastern part of Tehran.

[3] This is the arabicized name of Gurgān, an old city in Māzandarān province. The ruins of Gurgān are near Gunbad-i Qābūs in present-day Iran, about 100 kilometers north-east of modern Gurgān.

neatly in a nice hand; when Kūshyār was told that his writing style required too much time to complete a single book, he replied, 'yes, it takes much time, but once I am gone, people won't be concerned with how long I took to write them, but rather with the quality and contents of the books'" [Jurjānī 1976, 644].

In his article on Kūshyār, the historian Beyhaqī quotes the following dictum from him: "If two persons are interested in a single thing, the one ignoring the defects of that thing is really unfair to himself." [Beyhaqī 1935, 84].

Sa'dī, the famous Iranian poet of the 13th century, in the following poem on humbleness, names Kūshyār as the symbol of a wise scholar [Sa'dī 1879, 245-246]:

"Some one was a little knowledgeable about stars;
But he was drunk with arrogance.
From afar he went to Kūshyār,
With a heart full of devotion [to him and] a head full of conceit.
The wise man (i.e., Kūshyār) utterly ignored him,
[And] did not teach him anything.
Thus frustrated, when he decided to travel [back home],
The learned glorious man said to him:
'You have imagined yourself full of wisdom;
[Well,] how can a brim-full vessel contain more?
You are full of pretensions; therefore, you go empty-handed from me;
Come [to me] empty; in order to be filled with knowledge'.
......................................................................"[4]

In March 1988, Kūshyār's millennium was celebrated at Gīlān University during the 19th Annual Iranian Mathematics Conference. Kūshyār's works have attracted the attention of modern scholars since the early 19th century A.D. All his works are written in Arabic, the lingua franca of his time. For a detailed list of his works and their manuscripts see [Sezgin 1974, 343-345; 1978, 246-249; 1979, 182-183; Rosenfeld-Ihsanoğlu 2003, # 308, 118-119]. For a list of my remarks and additional data on the entry on Kūshyār ibn Labbān in the latter see [Bagheri 2006b, 2].

In Part 3 of this General Introduction, I will list the contents of

[4] English translation by Dr. Hushang A'lam; the last two lines are not translated, because they are not related to Kūshyār.

Kūshyār's *Jāmi' Zīj*, and I review the publications by modern authors related to the *Jāmi' Zīj*. In Part 4, I present some basic astronomical concepts which are necessary to understand the *Jāmi' Zīj*, and I also discuss the innovations made by Kūshyār.

Kūshyār's only known mathematical work is entitled *Uṣūl ḥisāb al-Hind* ("Principles of Hindu Reckoning"). It was translated into Hebrew by Shālôm ben Joseph 'Anābī in the 15th century A.D. (see [Cecotti 2004]). An edition of the Arabic text of this treatise was published by Saidan [Saidan 1967]. In recent decades, it has been translated into English, French, Persian, and Russian [Kūshyār 1965; Mazahéri 1975, 73-133; Kūshyār 1988a; Abdullazade 1990, 233-250]. For a comparative survey of the different versions of this work extant in four mss. in Istanbul, Tehran, Bombay and Cairo, see [Bagheri 2004].

Kūshyār's astrological treatise is titled *al-Madkhal fī ṣinā'at aḥkām al-nujūm* ("Introduction to the art of astrology"). An edition of the original Arabic text has been published by Prof. Michio Yano with an English translation and with an edition of the medieval Chinese translation prepared in 1383 A.D. [Kūshyār 1997]. There are also medieval Persian and Turkish translations of this treatise which have not yet been published [Sezgin 1979, 183; Pingree 2002, 408].

Kūshyār's treatise on the astrolabe is extant in several manuscripts. Mr. Taro Mimura has prepared an edition of the Arabic text under the supervision of M. Yano at Kyoto Sangyo University, and plans to publish it with an English translation. There is an old Persian translation of this work in Tashkent (MS 3894/1). Abdullazade has provided a table of the contents of this treatise [Abdullazade 1990, 194-212] and I have published an edition of the old Persian translation with an introduction [Kūshyār 2004].

### 3. Kūshyār's *Jāmiʿ Zīj* and its contents

Kūshyār's most important astronomical work is the *Jāmiʿ Zīj* (*al-Zīj al-Jāmiʿ* (lit., "Comprehensive astronomical handbook with tables"). In the Iranian literary tradition, Kūshyār's *zīj* was reputed as involving very elaborated and complicated subjects. An Iranian poet of the 13th century A.D., Muḥammad ibn al-Badīʿ al-Nasawī, writes in a poem quoted by [ʿAwfī 1906, 241]:

چو حل شدست مرا زیج گوشیار سخن

کجا به طیره شوم من ز ریش خند و زنخ

"Since [the problems of] Kūshyār's *zīj* of literature/poetry have been solved for me,
"I may not by any means be angered by [people's] derision and idle talk."

Here "Kūshyār's *zīj*" is used as a metaphor for "complicated and abstruse subjects". Kennedy gives a summary account of Kūshyār's *Jāmiʿ Zīj* in [Kennedy 1956, 125, 156-57]. He maintains that the elements of this *zīj* were taken from al-Battānī's *Ṣābī Zīj*, and that it is improbable that new observational data were incorporated into it. The *Jāmiʿ Zīj* was well known and influential in the Islamic astronomical tradition. Although it is influenced by Ptolemy's *Almagest* and al-Battānī's *zīj*, the *Jāmiʿ Zīj* has a special value because Kūshyār systematically presents geometrical proofs of the underlying theorems and algorithms. This feature is found only in a few other extant *zīj*es, e.g., Abu'l-Wafā's *Almagest*, al-Bīrūnī's *Canon Masudicus*, and al-Kāshī's *Khāqānī Zīj*.

The *Jāmiʿ Zīj* consists of four *maqāla*s ("Books"): Book I on elementary trigonometrical and astronomical calculations; Book II contains numerical trigonometrical and astronomical tables; Book III is on cosmology (*hay'a*); Book IV on "proofs" of the computations in Book I. Muḥammad ibn ʿUmar ibn Abī Ṭālib Tabrīzī translated the first book of the *Jāmiʿ Zīj* into Persian in 483 A.H./1090 A.D. [cf. Bagheri 1998]. Versions in Hebrew characters of different parts of the *zīj* are kept in four manuscripts that cover the whole work altogether [Langermann 1996, 151]. ʿAlī ibn Aḥmad al-Nasawī, probably a disciple of Kūshyār, wrote an Arabic commentary on the first book of the *Jāmiʿ Zīj* entitled *al-Lāmiʿ fī amthilat al-Zīj al-jāmiʿ* ("Explanation of the examples in the *Jāmiʿ Zīj*")

(MS Or. 45/7, Columbia University, New York, fols. 49r-75v)[5]. He presented numerical examples for each of the 85 chapters in Book I of the *Jāmi' Zīj* except for six chapters[6] that, according to him, did not need any example and two chapters which he simply skipped[7]. The folios of this ms. are not in their correct order[8] and there is a lacuna from the middle of chapter 6.14 to the middle of chapter 6.20. It is particularly interesting that on folios 50r and 51v al-Nasawī mentions the year 416 of Yazdigird era (1047-8 A.D.) as "the present year". So he flourished around 1050, and since at the beginning of the treatise he names Kūshyār with the prayer "may God have mercy on him!", this confirms that Kūshyār had died at that date.

A manuscript kept in the National Library of Tunis is said to be a commentary on Kūshyār's astronomical treatise (*Sharḥ kitāb Kūshyār ibn Labbān fi'l-falak*) by 'Abd al-Karim Dakālī [*Fihris* 1977-81, vol. 1, 106]. This may be another commentary on the *Jāmi' Zīj* [cf. Pingree 2002].

Partial editions, translations and studies of the *Jāmi' Zīj* have appeared during the last two and half centuries. Muḥammad A'lā al-T$^{h}$ānawī in his *Kashshāf iṣṭilāḥāt al-funūn* (A dictionary of the technical terms [used in the sciences of the Musulmans]), composed in 1158 A.H./1745-46 A.D., quoted from Kūshyār's *Jāmi' zīj* about the similarities of the Greek and the Syrian calendars, in his entry on chronology (*al-ta'rikh*) [al-T$^{h}$ānawī 1862, I, 57].

Ludwig Ideler published an edition of some fragments of the chapter on calendars with German translation [Ideler 1825-1826, II, 623-633]. Joachim Lelewel cited some data from the table of geographical coordinates given in the *Jāmi' Zīj*, and compared them with those of al-Bīrūnī and Ibn Yūnus [Lelewel 1852, xlvi-xlix]. E. Wiedemann translated the preface of the *zīj* into German [Wiedemann 1920, 132]. An edition of Chapter II.32, "On the distances and sizes [of celestial bodies]" appeared in India [Kūshyār 1948], and a Persian translation of it was published in Iran [Kūshyār 1988b]. Prof. J. L. Berggren published a translation with a commentary of Section IV.3 of the *Jāmi' Zīj* on

[5]. The late Prof. A. S. Saidan erroneously attributed this work to Kūshyār and gave wrong manuscript data for it [Saidan 1973, 531, 533]. He seems to have followed Salih Zeki who in his *Āthār al-bāqiya* (lit., "The existing remnants") provides similar information in the entry on Kūshyār [Zeki 1911, 166].

[6] These are Chapters 2.1 (commentary to Chapter 1 of Section 2), 4.1, 6.6, 8.7, 8.9, and 8.10.

[7] These are Chapters 4.7 and 4.8.

[8] A fragment from the middle of 5.21 to the middle of 6.3 is misplaced into the middle of 7.1; one folio from 7.4 is misplaced into the middle of 5.21; and one folio of a Persian treatise on arithmetic is misplaced into the middle of 7.4.

spherical trigonometry. He concluded that, while Kūshyār's account of the trigonometry of his day was not particularly original, it did contain the latest results and showed Kūshyār's taste for systematic exposition based on simple argumentation [Berggren 1987]. Prof. E. S. Kennedy studied Kūshyār's method for the calculation of the equation of time [Kennedy 1988, 2-4]. Khurshid F. Abdullazade extensively discussed the spherical trigonometry, mathematical astronomy and geographical material in the *Jāmi' Zīj* [Abdullazade 1990, 61-193, 213-230]. Dr. Benno van Dalen analyzed the table for the equation of time in Book II of the *Jāmi' Zīj* and was able to explain its method of computation, by taking into account that the tabular values are influenced by the displacement of the solar mean motion, as explained in Part 4 of this General Introduction. Van Dalen used statistical methods in order to determine the parameter values which Kūshyār used [Van Dalen 1993, 134-41]. He also analyzed a table for the true solar longitudes found in the sequel of the Berlin ms. of the *zīj* and showed that it probably derives from Yaḥyā ibn Abī Manṣūr [Van Dalen 1994b].

Glen Van Brummelen described Kūshyār's ingenious innovative interpolation scheme for composing double argument tables for the planetary equations of anomaly on the basis of the tables in Book II of the *Jāmi' Zīj*. The process significantly simplified the determination of a planet's longitude at a given time, although at the cost of some inaccuracy in the result. Van Brummelen took only the tables in Book II into account, but his mathematical reconstructions are confirmed by Kūshyār's text which we are now publishing; compare Sections I.4 and IV.4. Van Brummelen concludes that "Kūshyār was no mere copyist" [Van Brummelen 1998, 279].

Toshiaki Kashino discussed the planetary longitude theory in the *Jāmi' Zīj* and provided an edition of the Arabic text and English translation of Chapters I.4.1 to I.4.8, tables II.12 to II.14, II.16 to II.36, II.56, Chapters III.13, III.16 to III.19 and IV.4.1 to IV.4.7 of Kūshyār's *Jāmi' Zīj*, in his unpublished thesis [Kashino 1998].

In his *Introduction to astrology*, Kūshyār mentions his other *zīj* entitled *al-Zīj al-Bāligh* ("The extensive astronomical handbook with tables") [Kūshyār 1997, 6/7, 216/217]. No manuscript of the integral text of this work has been reported up to now. However, a short chapter entitled *Fī isti'māl adwār al-kawākib 'alā madhhab al-Hind min Zīj al-Bāligh li-Kūshyār* ("On the application of the cycles of the planets according to the Indian method from Kūshyār's *Zīj al-Bāligh*") kept in Bombay (MS R. I 86, Mulla Firuz collection, Cama Oriental Institute) is reported by F. Sezgin [1974, 248]. I have discussed the content of this chapter in an unpublished paper presented at the 17th Annual Conference for the History of Arabic Science, Suweida (Syria), 1993.

Abu'l-Fażl 'Allāmī mentions in his *Ā'īn-i Akbarī* ['Allāmī 1983, vol. 1, 185], besides the *Jāmi' Zīj* and the *Bāligh Zīj*, another work by Kūshyār entitled the *'Ażudī Zīj*. But the existence of such a work has never been confirmed by another reference in the works of Kūshyār or other authors. We know only one work named *al-Zīj al-'Ażudī*, composed by Ibn A'lam (ca. 960 A.D.) which has not come down to us [Kennedy 1956, 134].

Now I present very briefly a list of the contents of the *Jāmi' Zīj*. Books I and IV each consist of 8 sections. I explain the subjects of Book I and Book IV jointly, because they are directly related to each other. This list will be followed in Part 4 by an explanation of some basic concepts which Kūshyār uses.

In Section I.1, Kūshyār discusses different types of calendars used in ancient times and in his own time. He describes the methods for converting a date between any of the three calendars used in his time (Greek, Arabic and Persian). He also presents a method for finding the weekday corresponding to any date in any of these three calendars, and he lists the feasts in the three calendars.

In Section I.2, he discusses the trigonometric functions sine, cosine, and versed sine. Since he takes the radius of the base circle equal to 60 parts, his trigonometric functions are always 60 times the functions we use. He also discusses the chord function that was used by Ptolemy. He gives the values of the chords of 1/3, 1/4, 1/5, and 1/10 of the circle, and the chords of the sum and the difference of two arcs. He also provides the values of the sine and cosine of 1 degree and their application in setting up a sine table. The definitions of the trigonometric functions are not presented in Book I. We find them in Section IV.1, where Kūshyār also proves the validity of the results for the above-mentioned trigonometric functions which he presents in I.2.

In Section I.3 Kūshyār discusses the trigonometric functions tangent and cotangent and methods to compute them from sine tables. Again his tangent and cotangent are 60 times the functions we use. He also mentions other definitions in which the coefficient of the cotangent is 7 or 12 instead of 60. If a vertical gnomon is divided into 12 units, then the length of the cotangent or, as Kūshyār calls it, the horizontal shadow, is measured in the same units, called digits. If the gnomon is divided into 7 units, the cotangent is measured in units called feet. The validity of the methods of this section is proved in Section IV.2.

In Section I.4, Kūshyār provides his methods for computing the position of the sun, the moon, its nodes, and the five planets. He also discusses the equation of time and the latitudes of the moon and the five planets. He ends the section with a discussion on the retrogradation of the planets. The geometrical background of some of these methods is discussed in Section IV.4. The previous Section IV.3 provides some preliminary theorems in spherical trigonometry.

Section I.5 is devoted to the calculations of different quantities used in mathematical astronomy, such as the first and the second declinations, right and oblique ascensions, ortive amplitudes, the day arc for the sun, the altitude of the sun and the ascendant (We will explain some of these concepts below). Kūshyār also discusses the astrological concept of houses, which involves mathematical computation. Geometrical proofs of the mathematical calculations in this section are given in Section IV.5.

Section I.6 is about lunar and solar eclipses and their magnitudes and durations. Kūshyār also discusses the parallax of the moon and the sun and lunar crescent visibility. The corresponding "proofs" are given in Section IV.6.

Section I.7 is on operations relating to astrology. Here Kūshyār deals with astrological concepts as "projection of rays", prorogation etc. The geometrical background of the method for projection of rays is given in Section IV.7.

Section I.8 is entitled "On operations which are less needed". In this section he provides methods for finding the following quantities, among others: the geographical latitude of a locality from the duration of its longest day, the altitude of the sun or a planet when it is due East or West, the apparent distance between two stars from their ecliptical coordinates, the terrestrial meridian line, the direction to Mecca, etc.
At the end of this section Kūshyār presents the names of the fixed stars, some features for recognizing them, and the lunar mansions. The

"proofs" of the mathematical methods of this section are presented in Section IV.8.

Book II includes 55 tables. The first 7 tables are related to calendrical calculations. The next 4 tables are tables of the sine, versed sine, tangent and cotangent. Then follow 5 tables for finding the true longitude of the sun and 5 tables for finding the true longitude of the moon. For each of the planets Saturn, Jupiter, Mars, Venus and Mercury, 3 tables give the values of mean motion, anomaly and equations. The next 6 tables are for the latitudes of the moon and the planets. Then 6 tables give the first and second declination, right and oblique ascensions, and the equation of day. The next 4 tables are for calculations relating to eclipses. One table is for the astrological function called prorogation. Finally 2 tables provide the names and coordinates of the cities and the fixed stars.

Book III on cosmology (*hay'a*) contains 32 chapters on different astronomical subjects such as the climates, the size of the earth, the ascensions, equinoctial and temporal hours, the orbs of the celestial bodies, retrogradations, sizes and distances of the celestial bodies, lunar phases, and eclipses. Two chapters of the third book entitled *al-Ab'ād wa'l-ajrām* ("<On> the distances and sizes <of the celestial bodies>"), and *Jawāmi' 'ilm al-hay'a* ("A compendium of the science of cosmology" containing definitions of around 130 astronomical terms) were also copied, translated, and circulated as independent treatises.

## 4. Kūshyār's *Jāmi' Zīj* between tradition and innovation

Kūshyār does not inform us what books his reader should have read in order to understand this *zīj*. But it is evident that he assumed a thorough knowledge of Books I through VI of the *Elements* of Euclid (ca. 300 B.C.) on plane geometry of straight lines and line segments, triangles, circles, ratios and proportions. The reader of Kūshyār's Book IV, on "Proofs", should have a good command of Euclid's *Data* as well. This work contains theorems of the type that if certain elements are known in a figure, other elements can also be determined. Euclid's *Elements* and *Data* were available in good Arabic translations. Unlike Euclid, Kūshyār and his contemporaries routinely used numerical approximations of irrational ratios. Thus the reader should also be familiar with, e.g., square root extraction. Since much of Kūshyār's work concerns spherical trigonometry, his reader needs to know some materials on the geometry of the sphere, which is explained, for example, in the *Spherics* of Theodosius (ca. 100 B.C.). This work was also available in Arabic translation.

We now introduce the reader to some basic concepts and terminology of later Greek (Ptolemaic) and medieval Islamic mathematical astronomy, which were traditional concepts and terminology in Kūshyār's time. This introduction may facilitate the reading of the translation of Kūshyār's work. Kūshyār himself explains some of these concepts in Book III of his *zīj*, which is not published here. I do not intend to give a complete exposition of the Ptolemaic system or of medieval astronomy. Of course, the reader may pursue the translation, and refer to this exposition only when necessary.

The most important concept in later Greek and Islamic spherical geometry is that of a great circle, which is the intersection of a sphere by a plane through the center of the sphere. In Kūshyār's work, an "arc" on a sphere almost always means an arc of a great circle. Using arcs of great cicles, Menelaus of Alexandria (ca. 75 A.D.) defined spherical triangles. Each pair of points on a sphere, which are not on the same diameter, can be joined by precisely one great circle arc less than 180 degrees, and thus three points on a sphere, such that no two of them are on the same diameter, determine a spherical triangle. Spherical trigonometry was further developed by Islamic geometers in the tenth century A.D., and Kūshyār makes some contributions to this field in Book IV of the *Jāmi' Zīj*.

Another important concept, often used by Kūshyār in Book IV, is the "pole" of a circle on a sphere (not necessarily a great circle). The two poles of any circle on a sphere are the two points of intersection of the sphere with the straight line passing through the center of the sphere perpendicular to the plane of the circle. Every point on a sphere is the pole of precisely one great circle. (The reader may recall the familiar example of the terrestrial equator with its two "poles", namely the North and South poles.) Kūshyār sometimes assumes the following property of poles of great circles: If $P$ and $Q$ are the poles of two different circles $p$ and $q$, the points of intersection of $p$ and $q$ are the poles of the great circle through $P$ and $Q$.

Kūshyār provides most of the proofs in Book IV with a figure. In the case of theorems on spherical trigonometry, the figures in the works of Kūshyār and his contemporaries are not perspective drawings of the sphere; the theory of perspective was unknown in medieval Islamic mathematics. Kūshyār's figures are symbolic representations in which one side of the sphere (for example, the part above the horizon) is represented on the paper inside one boundary circle (for example, the representation of the horizon circle). Arcs on the sphere are represented as circular arcs on the paper, in such a way that their relative positions on the sphere are conserved. Arcs on the other side of the sphere may extend outside the boundary circle. There is no evidence that Kūshyār (or any of his contemporaries) used a consistent method of projection in drawing figures for geometrical theorems and proofs. Of course, such systems of projections were known at that time, and used in making metal astrolabes and maps.

Kūshyār used the sine as his basic trigonometric function. In Part 3 of this General Introduction, I have referred to Kūshyār's very detailed explanation of the use and computation of Sines in Section 2 of Books I and IV. Here, I only call attention to the capital initial letter in my translation of Sine, which indicates that Kūshyār's Sine is 60 times the modern sine. Kūshyār defines the Sine in a circle whose radius he divides into 60 "parts", and he expresses the Sine sexagesimally in parts, minutes and seconds. Therefore his Sine of 45 degrees is 42 parts, 35 minutes and 25 seconds ($30\sqrt{2} \approx 42+35/60+25/3600+ \ldots$). The term "total Sine", which often occurs in Kūshyār's work and in my translation, means the maximal Sine (of 90 degrees), that is the radius of the circle.

Now we turn from mathematics to mathematical astronomy. Kūshyār, his contemporaries and predecessors used the celestial sphere for many astronomical computations. This is a very large imaginary sphere, which may coincide with the outermost sphere of the universe. The center of the celestial sphere coincides with that of the earth and the sphere is so large that the radius of the earth can be neglected in all computations. On the celestial sphere, different points and circles are defined. The celestial equator and the celestial North and South poles are the intersections of the celestial sphere with the plane of the terrestrial equator, and the line through the terrestrial North and South poles. Almost all ancient and medieval astronomers, including Kūshyār, assumed that the earth is at rest and that the universe performs a daily rotation around the axis through the celestial North and South poles.

The second fundamental circle on the celestial sphere is the ecliptic, which is defined by the motion of the sun. Ancient and medieval astronomers believed that the sun performs a yearly motion around the earth. This motion (more precisely, the motion of the center of the sun) takes place in a plane passing through the center of the earth, and the ecliptic is the intersection of that plane with the celestial sphere. The ecliptic and the equator intersect at two points, which are called the vernal point (or vernal equinox), and the autumnal point (or autumnal equinox). The moments when the sun is at the vernal and autumnal point, define the beginning of the spring and the fall on the northern hemisphere of the earth; then the day and night have equal length. The two points on the ecliptic at maximal distance of the celestial equator are called the two solstitial points, or solstices. When the sun is at the summer solstice, which is in the northern celestial hemisphere, summer begins on the northern hemisphere on the earth, and the day is longest in the temperate regions north of the equator (which include Iran). Similarly, winter begins in these temperate regions when the sun is at the winter solstice; then the day is shortest. In ancient and medieval astronomy, the ecliptic together with the equinoxes and solstices partake in the daily rotation of the universe around the earth.

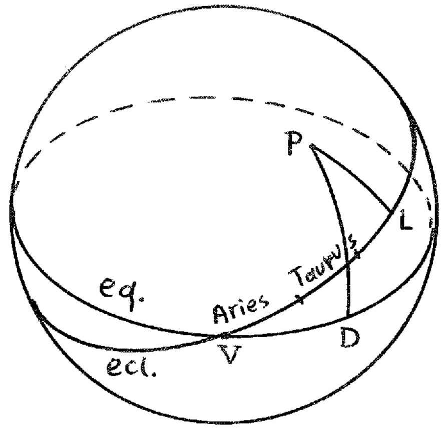
The Babylonians were the first to define the ecliptic. They divided it into 12 "signs" of equal length. These signs were also used in Greek, Islamic and European astronomy and astrology. In the order of the yearly motion of the sun, and beginning with the beginning of spring, the Latin names of the signs are as follows: Aries, Taurus, Gemini, Cancer, Leo, Virgo, Libra, Scorpio, Sagittarius, Capricorn, Aquarius and Pisces. These arbitrary names were assigned from stellar constellations. In Greek, Islamic and European astronomy, the spring equinox is the beginning of Aries, the summer solstice the beginning of Cancer, the autumnal equinox the beginning of Libra, and the winter solstice the beginning of

Capricorn. The Babylonians divided each sign of the ecliptic into 30 degrees; so the whole ecliptic is divided into 360 equal degrees. The Greek astronomers adopted from the Babylonians this division into signs, and the Greek astronomer Hipparchus (ca. 150 B.C.) divided all other circles into 360 degrees as well.

Hipparchus was the first to realize that the position of the sun at the beginning of spring (when the day and night are equal in length) changes very slowly with regard to the fixed stars. He and his followers, including Kūshyār, supposed that this phenomenon is caused by a very slow motion of the "fixed stars" with respect to the signs of the ecliptic. This motion was supposed to be a uniform rotation around an axis perpendicular to the plane of the ecliptic. The axis intersects the celestial sphere in the two "poles" of the ecliptic. When Kūshyār refers to the "pole" of the ecliptic, he means the North pole of the ecliptic, which is close to the celestial North pole and always above the horizon in Iran. Kūshyār believed that one complete rotation of the fixed stars with respect to the ecliptic takes place in 24,000 years. In modern astronomy, the phenomenon is called "precession of the equinoxes" and described as a motion of the equinoxes, i.e., the plane of the equator, with respect to the fixed stars, rather than the other way around. The precession is explained as the result of a slow motion of the earth's axis. As a result of precession, the signs slowly move away from the stellar constellations from which the names of the signs were originally derived in the first centuries B.C. Thus most part of the constellation "Pisces" is now in the sign "Aries", and so on.

The three basic coordinate systems on the celestial sphere can now be described. The first system uses ecliptical longitude and latitude, and was used by Kūshyār and his contemporaries in the computation of planetary, lunar and solar positions, and in most astrological applications. This system is hardly used in modern astronomy.

Figure 1



To find the ecliptic coordinates of a point *P*, draw a great circle arc *PL* through it, perpendicular to the ecliptic and less than 90 degrees, meeting the ecliptic in *L*. (Precisely one such arc can be drawn if *P* is not one of the ecliptical poles.) The ecliptical latitude is the length of this arc in degrees, and the latitude is called "northern" if *P* is between the ecliptic and the North pole of the ecliptic, and "southern" if *P* is between the ecliptic and the south pole of the ecliptic. Thus, Kūshyār does not work with negative latitudes. In his words, a point on the ecliptic has "no latitude"; we would say that the point has "zero latitude". The arc between the vernal point *V* and *L*, measured along the direction of the yearly motion of the sun, is the ecliptical "longitude" of the point *L*. The ecliptic longitude is always between 0 and 360 degrees. If *L* is in Aries, the longitude is between 0 and 30 degrees, etc. Figure 1 displays the celestial sphere from the outside. *V* is the vernal point, *P* a point on the northern half of the celestial hemisphere. *L* is approximately in Gemini.

The second coordinate system is defined similarly, but with respect to the celestial equator. We draw a great circle arc *PD* through *P*, perpendicular to the equator (see Figure 1). The arc *VD* on the equator, measured in the direction nearly parallel to the yearly motion of the sun, is called the "right ascension" of point *P*, and the arc *PD* is simply called "distance to the equator".

Modern astronomers use the general concept "declination" instead, but Kūshyār uses the term "declination" (*mayl*) only if *P* is on the ecliptic. If Kūshyār refers to "the declination of (a certain) ecliptical degree", he means the declination of a point on the ecliptic which is the endpoint of an arc beginning at the vernal point and ending at the degree in question. For Kūshyār the declination is "northern" or "southern", but never negative. Another curious term is "total declination", meaning: the declination of one of the two solstitial points. Nowadays this "total declination" is called "obliquity of the ecliptic", and it is equal to the angle between the equator and the ecliptic at the vernal equinox. Kūshyār and most of his Islamic predecessors used the value 23 degrees and 35 minutes.

The third coordinate system which we have to mention is defined with respect to the horizon. The plane tangent to the earth at the locality of the observer intersects the celestial sphere in a circle, which can be considered as a great circle because the radius of the earth is ignored. This great circle is called the "(local) horizon". The line joining the center of the earth to the observer intersects the celestial sphere above the horizon at the zenith (Arabic: *samt al-ra's*) of the locality, and below the horizon at the nadir (Arabic: *naẓīr al-samt*) of the locality. The meridian is the great circle through the zenith and the celestial north and south poles. The meridian intersects the horizon in two points: the North point (closer to the celestial north pole) and the South point. The arcs between the North and South points are bisected by the East and West points. Different localities on earth have different zeniths on the celestial sphere. (In fact, one can map the earth on the celestial sphere by mapping every locality on its zenith. In this way, the celestial sphere was often used for terrestrial computations in medieval Islamic geometry. An example is Kūshyār's determination of the direction of Mecca in IV.8.)

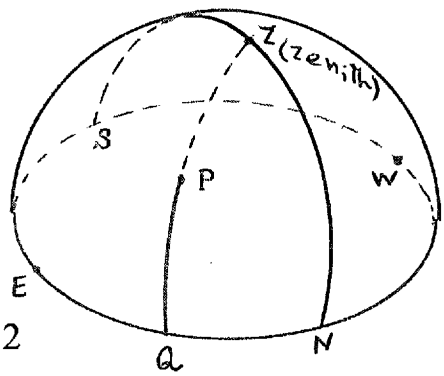



Figure 2

For any point $P$ on the celestial sphere, not the zenith and nadir, we can draw a unique great circle arc $PQ$ less than 90 degrees perpendicular to the horizon, and meeting the horizon at $Q$. This arc, or its length in degrees, is called the "altitude" (or "depression" if the point is below the horizon). The arc between $Q$ and the East or West point, whichever is closer, is called the azimuth (Arabic: *al-samt*). See Figure 2.

Points on the prime vertical (the great circle through the East and West points and the zenith) are said to have "no azimuth". These conventions are contrary to the modern ones, which prescribe that the zero point of the azimuth is in the North point and that the azimuth ranges between 0 and 180 degrees East and West.

Now we continue with an introduction to some of the traditional concepts and terminology of Ptolemaic astronomy which were used by Kūshyār. This introduction is meant to give the reader some idea about what he may encounter in the translation of Kūshyār's *zīj*. For further details, I refer him to the standard expositions in [Pedersen 1974] and [Neugebauer

1975], and to the translation of *Almagest* by Toomer [Ptolemy 1984]. I often refer to these works in my own commentary. Some of the parameter values of Kūshyār in the following introduction are taken from Books II and III of his *zīj*, which I am currently preparing for publication.

We begin with the motion of the sun. As early as the fifth century B.C., the Babylonians had already observed that the sun does not move uniformly on the ecliptic. In the spring and summer the motion is slightly slower than in the fall and winter. Hipparchus and Ptolemy, who believed in the Aristotelian dogma of uniform motion, explained this "anomaly" by means of the following model (Figure 3).

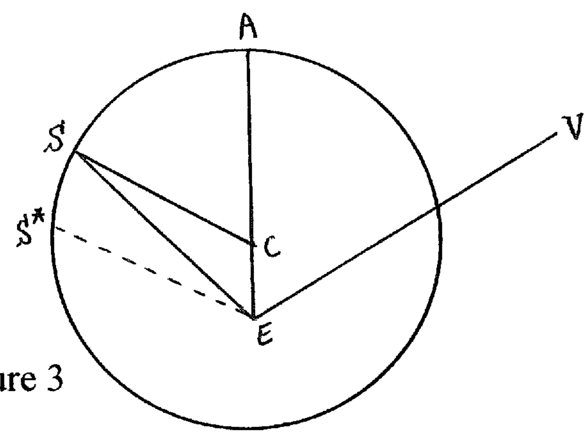



Figure 3

They assumed that the sun *S* moves uniformly on a circle, the *deferent*, whose center *C* does not coincide with the center *E* of the earth. Ptolemy put the radius of the circle equal to 60 "parts". In Book III of the *Almagest* he explains in detail how he (allegedly) derived the parameter values in this model from observations of solar altitude above the horizon. His conclusions are that the solar eccentricity *CE* is 2;29,30 "parts" (which he often rounded to 2;30 "parts"); that the apogee *A* is in 5;30 Gemini; and that the sun performs one complete rotation on the deferent in 365+(1/4)–(1/300) days, so the daily motion on the deferent is 0;59,8,17,13,12,31 degrees.[9]

The Ptolemaic model of the solar motion agrees reasonably well with the model based on Newtonian mechanics. That is to say the two models predict approximately the same solar position in the ecliptic. Ancient and

[9] For a description of the sexagesimal notation which has become standard since the work of Neugebauer, and which I will use, see Part 1 of this General Introduction.

medieval astronomers were unable to measure the variations in the distance between the sun and the earth. According to Newtonian mechanics, the sun is at rest and the earth moves around it in a Keplerian ellipse with the sun at one of the foci (we ignore the gravitational effects of bodies other than the sun and the earth). To an observer on earth, the sun seems to move in a Keplerian ellipse with the earth at one of its foci. Uniform circular motion on a deferent with center *C* and the earth *E* close to it, is a reasonable approximation of Keplerian motion on an ellipse with center *C* and earth *E* at one of the foci. Thus the concept of "eccentricity" could be transformed from the length of *CE* (in "parts") in the Ptolemaic solar (and planetary) models to the elliptic models of Kepler, and hence to the geometrical description of the ellipse in general. In modern geometry, the eccentricity of an ellipse means the distance of the center to any focus of the ellipse, divided by half the major axis.

I now explain some additional traditional technical terminology. Figure 3 displays the deferent in the plane of the ecliptic, which is the plane in which the solar motion takes place. *V* is the (direction of the) vernal point, *A* is the apogee, and angle *VEA* is its ecliptical longitude.

Kūshyār calls angle *ACS* the "mean argument"[10] of the sun and the angular sum *VEA*+*ACS* the "mean longitude". These two quantities are linear functions of time. Modern authors often introduce the "mean sun", that is, a point *S** in the ecliptic so that *ES** is parallel to *CS*. Then the "mean longitude" of the sun *S* is the ecliptical longitude of *S**.

In order to compute the position of the sun at a given time, we need to know the position of the apogee *A*, and the position of *CS* at a conveniently chosen zero point in time. For this zero point, Kūshyār chose noon on the first day of the Yazdigird era at a locality with a terrestrial longitude of 90 degrees east of the Canary Islands (at that time the westernmost part of the known inhabited world). The Yazdigird era (A.Y.) is the most common Iranian calendar of which the first day was June 16, 632 A.D. The Yazdigird era is very convenient for astronomical calculations because every year in this calendar has a constant length of 365 days.[11] From the *zīj* of his predecessor al-Battānī, Kūshyār derived that at this moment of time, the apogee *A* was in 18;31 degrees Gemini, and the "mean longitude" of the sun was 26;24,36 degrees Gemini (i.e., *VEA*+*ACS* = 86;24,36 degrees). The geographical longitude 90 degrees

[10] I have translated the Arabic term *khāṣṣa* by "mean anomaly" for the sake of consistency with the theory of the planets, although this translation may be somewhat misleading in the case of the sun.
[11] Kūshyār discusses the Yazdigird era and other calendars that were used in his time in Section 1 of book I.

was very convenient for Kūshyār because it was assumed to be the longitude of the city of Jurjān where he lived. The reader will notice that Kūshyār's apogee in 18;31 Gemini is different from Ptolemy's apogee in 5;30 Gemini (in particular Ptolemy's solar parameters were quite bad). Kūshyār supposes that the apogee $A$ has the same slow motion as the fixed stars, namely 0;0,54 degrees per year.

The extreme precision in the daily motion of the sun is due to the fact that it is based on observations spanning an interval of more than 1000 years. Of course the precision in the position of the sun and also in the position of the apogee is illusory.

The position of the sun as seen from the earth is defined by the angle *VES*, where $V$ is the vernal point. To compute this angle at a given moment of time in the Yazdigird era, we first need to find the "adjusted apogee", that is the position of $A$ for the given moment, by adding 0;0,54 times the year number to 18;31 Gemini. Then we find the "mean longitude" as the sum of 26;24,36 degrees Gemini plus the number of elapsed days times the daily mean solar motion of 0;59,8,20,46,56,14 degrees. In Book II, Kūshyār provides tables for facilitating this computation. We subtract the "adjusted apogee" (angle *VEA*) from the "mean longitude" (angle *VEA* + angle *ACS*), and thus we obtain the "mean argument", i.e., the angle *ACS*.

From the "mean argument" *ACS*, Ptolemy and Kūshyār compute the correction angle *ESC* by trigonometrical calculations, which will be discussed below. This correction angle *ESC* is called "equation". This term is misleading for a modern reader, because no mathematical equation is involved here. The confusion can be explained by the fact that the word "equation" in the astronomical sense was derived, via the Latin *equatio*, from the Arabic *ta'dīl*. This word has the same root as the Arabic word *mu'ādala*, which means "algebraic equation", and which was also translated into Latin as *equatio*.

Call $c$ the "mean argument", angle *ACS*, and $q(c)$ the corresponding equation angle *ESC*. Ptolemy's computation of $q(c)$ is equivalent to the following formula:

$$\sin q(c) = e \sin c / \sqrt{(d + e\cos c)^2 + (e \sin c)^2}\ ,$$

where $d = CS = 60$, and $e = CE = 2;30$.

Ptolemy and most Islamic astronomers computed the true solar longitude *VES* by adding or subtracting the equation *ESC* to or from the mean longitude *VEA*+*ACS*. In figure 3, the equation has to be subtracted. Apparently Kūshyār wanted the equations to be always additive in order to avoid the possible confusion that quantities from tables had to be added or subtracted depending on sometimes obscure conditions. So he made the following formal change in his computations. I give a general description of his idea below (cf. [van Dalen 2004b, 840-43] and [Van Brummelen 1998]). Choose an integer $n$ greater than the maximum value of $q(c)$ for all $c$ (for the sun, Kūshyār chooses $n = 2$). Kūshyār defines two new quantities: (1) the "displaced" mean argument $c' = c - n$; and (2) the "shifted" equation $q'(c') = n \pm q(c'+n)$. The minus sign is used if $c'+n$ is less than 180 degrees (so the equation has to be subtracted), and the plus sign if $c'+n$ is between 180 and 360 degrees (so the equation has to be added). The terms "displaced" and "shifted" are modern. In Book II, Kūshyār tabulates the shifted equation $q'(c')$ whose values are always positive. Kūshyār then computes the angle *VES* as $c' + q(c')$.

For some further technical terminology and another innovation of Kūshyār we turn to the motion of the moon. For the sake of simplicity we define in Figure 3 the 'mean sun' as an imaginary body $S^*$ in the ecliptic so that *CS* is parallel to $ES^*$. Thus the longitude of $S^*$ is equal to the "mean longitude" of the sun.

We shall now describe Ptolemy's complicated lunar model, which was used by Kūshyār, without further motivation. I realize that the description may be a little bewildering for the reader. For the reason why Ptolemy adopted precisely this model, the reader may refer to [Pedersen 1974, 159-202] and [Neugebauer 1975, 68-99]. The moon moves in a plane which makes a small angle (5 degrees) with the plane of the ecliptic, and the points of intersection with the ecliptic are called the lunar nodes. (The name "ecliptic" is derived from the fact that solar and lunar eclipses take place when the moon is close to the ecliptic.) In the rest of my description, I will ignore the ecliptical latitude of the moon, and identify the moon with its perpendicular projection on the ecliptic. Consequently, Figure 4 displays the ecliptic with earth *E*, the (direction of the) vernal point *V*, and the moon *M*. The body of the moon is represented as a point. Point $S^*$ denotes the (direction of the) mean sun. We assume that the motion of $S^*$ in Figure 4 is counter-clockwise.

The lunar motion is composed of three components: The moon *M* moves clockwise on an epicycle (a small circle) with center *C*. Point *C* moves with a fast counter-clockwise motion on a greater circle (the deferent) with center *D*, and point *D* moves clockwise on a small circle (not drawn in the figure) with the earth at its center. We will not discuss the question as to what extent this model is a faithful representation of the lunar motion according to modern theories.

Before describing these motions in detail, it is probably a good idea to introduce the "mean longitude" of the moon, namely angle *VEC*. The mean longitude is a linear function of time. According to Kūshyār, it increases by 13;10,35 degrees per day, and the position of *C* at noon of the first day of the Yazdigird era at Jurjān is 4;10,28 degrees Aries. Angle *CES** is called the "elongation"; it is also a linear function of time, and increases by 13;10,35 – 0;59,8 = 12;11,27 degrees per day.

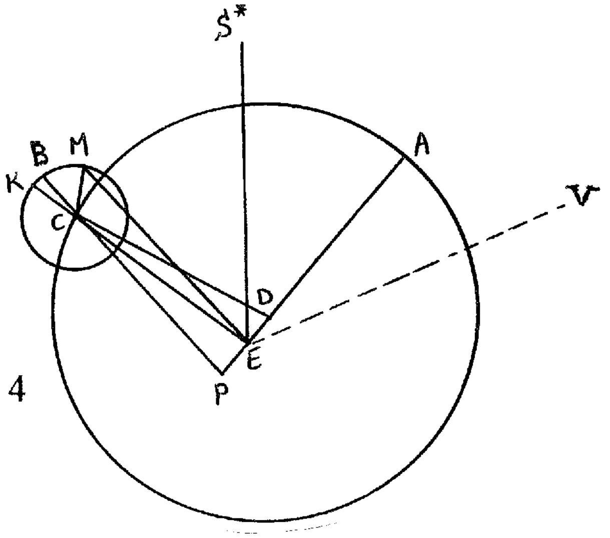


Figure 4

Here are the precise definitions of the three components of the lunar motion. Kūshyār puts the radius *DA* of the deferent equal to 60 "parts".

(1) Point *D* moves uniformly on a circle around the earth *E*, with radius 12;30 parts, in the direction contrary to the solar motion. Line *ED* extended intersects the deferent in its apogee *A*. The daily motion of *D* (and *A*) in the ecliptic is the daily increase of the elongation minus the mean solar daily motion, namely 11;12,19 degrees.

(2) Point *C* moves on the deferent *D* in such a way that the mean lunar longitude *VEC* increases by the above mentioned value 13;10,35 degrees per day. This means that angle *CEA* increases by 24;22,54 degrees per day, that is twice the daily increase in the elongation 12;11,27. Angle *CEA* is accordingly called the "double elongation". The position of *C* is defined by the requirement that line *ES** always bisects angle *AEC*. Thus, if the elongation *AES** is 0 or 180 degrees, point *C* coincides with *A*, and the epicycle center is at maximum distance of 60 "parts". If the elongation *AES** is 90 degrees, *C* is on *EDA* such that *CE*=*CD*–*DE*= 35 "parts".

(3) The moon $M$ moves on this epicycle in the following way: Choose $P$ on $DE$ extended such that $EP = DE$. Draw $PC$ and extend it to meet the epicycle at $B$. Then the "mean anomaly" angle $BCM$ increases as a linear function of time.[12] The daily increase is according to Kūshyār 13;3,54 degrees, and the epoch value (at noon of the first day of the Yazdigird era at locality 90 degrees East of Canary Islands) is 307;4,26 degrees.

Extend $EC$ to meet the epicycle at $K$. Then $B$ coincides with $K$ if the elongation is a multiple of 90 degrees, and the absolute value of arc $BK$ is maximal for an elongation equal to 45, 135, 225 or 315 degrees. Ptolemy put $EA = 60$ and found the radius of the epicycle to be equal to 5;15 "parts". Kūshyār does not specify what value he used, but it must have been the product of the scaling factor 1;12,30 times the Ptolemaic value. Later, Kūshyār assumes that one "part" is equal to one earth's radius; his maximum distance 60 "parts" of the epicycle center $C$ to the earth is almost equivalent to the Ptolemaic value of 59 earth radii.

Now the problem is how to compute the true position of the moon, which is the angle $VEM$, from these data.

First, the mean anomaly (angle $BCM$) is changed to the "true anomaly" (angle $KCM$) by adding or subtracting the "first equation" (angle $BCK$). Kūshyār computes this angle essentially in the Ptolemaic way, but subjected to a cosmetic change to avoid negative values, in a similar way as in the computation of the sun. The details are not to be mentioned here.

Then using the "true anomaly" $KCM$ as an argument, Kūshyār computes the "second equation" $MEC$. The lunar longitude $VEM$ is computed by adding or subtracting angle $MEC$ to the mean longitude $VEC$. The computation of the second equation is of interest here, because it involves a change compared to Ptolemy's computation. Kūshyār also applies a cosmetic change in order to avoid subtraction, but we will describe his procedure and compare it to Ptolemy's procedure as if the cosmetic change had not taken place. Call $a$ the true anomaly (angle $KCM$), and $c$ the double elongation, angle $AEC$. The distance $EC$ is a function of $DE$, $DA$ and $c$ and will be denoted as $d(c)$. So $d(c) = EC$.

We have $d(c) = \sqrt{(60 + e\cos c)^2 + (e\sin c)^2}$, with $e = 12;30$.

Call $r = BC$ the radius of the epicycle. Call angle $MEC = q(a,c)$.

We have in modern terms

$$\sin q(a,c) = r\sin a / \sqrt{[d(c) + r\cos a]^2 + (r\sin a)^2}\text{, with } r = 6;20.$$

[12] The features (2) and (3) imply a contradiction with the principle of uniform circular motion.

A table of this function for all $a$ and $c$ would contain tens of thousands of values. Ptolemy and Kūshyār both use an approximation in order to simplify the computation. Ptolemy computes tables for the two functions $q\,(a,0)$ and $q\,(a,180)-q\,(a,0)$. For a fixed $c$ the maximal value $m(c)$ of $q\,(a,c)$ can easily be found from $\sin m(c) = r/d(c)$. He then defines an interpolation coefficient $s\,(c) = 60\frac{m(c)-m(0)}{m(180)-m(0)}$, rounded to integers. The number $s\,(c)$ is always between 0 and 60 and is called the "sixtieth". His computation of $q\,(a,c)$ boils down to

$$q\,(a,c) \approx q\,(a,0)+[s\,(c)/60].[q\,(a,180)-q\,(a,0)].$$

In the same notation, Kūshyār computes only $q\,(a,0)$ and the "difference for lesser distance"[13] $m(c)-m(0)$. He also computes a "sixtieth" $S\,(a) = 60\ q(a,0)/m(0)$, rounded to integers.[14]

He then puts

$$q\,(a,c) \approx q\,(a,0)+[S\,(a)/60].[m\,(c)-m\,(0)].$$

This approximation is an interesting variation on the method of Ptolemy. Kūshyār's approximation is somewhat less accurate but saves some computational work.

The reader has now got the flavor of the traditional Ptolemaic astronomical models and computations and their modifications by Kūshyār. The latter makes similar simplifications in the computation of the planetary longitudes, described in [Van Brummelen 1998].

The best description of Ptolemy's theory of planetary latitudes is to be found in [Pedersen 1974, 355-86]. The way in which Kūshyār handles Ptolemy's theory of latitudes has not yet been investigated by modern historians of science. Kūshyār made some modifications in his description of the calculations; see my commentary on the relevant chapters. It seems to me that Kūshyār (and for example his predecessor al-Battānī) understood the geometric rationale of Ptolemy's theory of latitude of planets, but not the fine points of the corresponding computations.

The parameter values in Kūshyār's models (eccentricities and radii of the epicycles) are, apart from scaling factors, almost always the same as in the *Almagest* of Ptolemy or the *zīj* of al-Battānī. In the case of Mercury, Kūshyār says in Book III that he takes the eccentricity $e = 3;10$, while Ptolemy and al-Battānī had taken eccentricity $e = 3;0$ parts. However, according to Kashino [1998, 17] and [Van Brummelen, 268],

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[13] Kashino's translation "difference for the nearest distance" [1998, 26, 45, 98,99] is misleading.

[14] In Kashino's formula (2.34), $2\eta$ should be equal to zero [1998, 13].

Kūshyār's tabular values are actually based on taking $e$ equal to 3;0 parts, which was used by Ptolemy and al-Battānī. In Section I.4.4, Kūshyār mentions a change he made in the parameter values involving the equation of Mars. He does not specify his new parameter values, and refers only vaguely to observations of meridian altitudes and of conjunctions, without reference to specific observations. According to the mathematical analysis by Van Brummelen [1998, 268] based on Book II of the *Jāmi' Zīj*, Kūshyār changed a parameter value $e = 6$ which Ptolemy and al-Battānī used in the Mars model to a value in the neighbourhood of 6;2,35.

Although Book IV of the *Jāmi' Zīj* is said to contain the "Proofs" on Book I, Book IV does not contain anything like the determination of parameter values of the planetary models from observations, as explained by Ptolemy in the *Almagest*, and by Kūshyār's contemporary al-Bīrūnī in the *Canon Masudicus*. The reader may well ask what the word "proofs" in the title of Book IV really means.

It seems to me that Kūshyār referred to classical Greek geometrical proofs. If a very complicated quantity is computed in Book I, Kūshyār presents in the corresponding section of Book IV a geometrical figure with an abstract proof, in the style of the *Data* of Euclid. In the proof, Kūshyār demonstrates that a certain line segment or arc is "known". The reader is supposed to work out for himself that this line segment or arc in the figure corresponds to the quantity to be computed. Kūshyār often leaves it to his reader to identify in the figure in Book IV the given line segments or arcs, which correspond to the quantities that he supposes to be known in the computation in Book I.

It turns out, not surprisingly, that most innovations in the *Jāmi' Zīj* are mathematical in character. We have already seen the cosmetic changes in order to avoid subtractions, and the simplified interpolation procedure for the "second equation" of the moon and planets.

Kūshyār also presents what may be his own theoretical proofs and procedures in parallax and eclipse computations in Section IV.6. In these calculations, Ptolemy assumes that a number of (small) circular arcs can be approximated by straight line segments. Kūshyār describes Ptolemy's approximate methods (in Sections I.6.13, IV.6.10 and I.6.18, IV.6.13), but he then presents exact methods (in Sections I.6.8, IV.6.13 and I.6.17, IV.6.12), which are probably his own. The difference in the result of computation is often negligible and irrelevant for all practical purposes. Thus Kūshyār belongs to a tradition of Islamic mathematicians who were interested in theoretical proofs and methods, regardless of practical applicability. This tradition included famous mathematicians such as Ibn al-Haytham (ca. 965-1041 A.D.) who wrote a work of over 100 pages on the fact that the moon and planets may not culminate exactly in the meridian if their proper motion is not parallel to the celestial equator[15].

[15] This is the *Kitāb hay'at ḥarakāt al-kawākib al-sab'*, mentioned in [Sezgin 1974, 260, no. 27].

## 5. Manuscripts and editorial procedures

The manuscripts of the *Jāmi' Zīj* which I have used in my work and the abbreviations used for them are as follows (an asterisk * refers to the mss. used as bases for the Arabic edition).

A Alexandria, Baladiya Library, MS 4285 *jim* [Zaydān 1926-1929, I, 216-17]; Books III and IV, copied in 566 A.H./1170-71 A.D. from an autograph dated 393 A.Y./415 A.H./ 1025 A.D., fols. 1v-73v.

B Berlin, Staatsbibliothek, MS Mq. 101 [Ahlwardt 1887-1899, V, 203-206, no. 5751]; Books I and II, copied in 806[1] A.H./1403-04 A.D., pp. 2-221.

C* Cairo, Dār al-kutub, MS Muṣṭafā Fāżil Mīqāt 213/1 [King 1981-1986, I, 414; II, 104]; Book I, copied in 1169 AH/1755-56 A.D., fols. 1v-26r.

F* Istanbul, Fatih, MS 3418/1 [Krause 1936, 472] (Cat., p. 196); Books I-IV, copied in 545 A.H./1150-51 A.D., fols. 1v-175v.

L Leiden Universiteitsbibliotheek, MS Or. 8 [Voorhoeve 1957, 405; De Jong & De Goeje 1865-1866, III, 84-86, no. 1054]; Books I-IV, copied in 634 A.H./1236-37 A.D., fols. 1v-124r.

M Moscow, Russian state Library, MS 154/3 [Matvievskaya & Rosenfeld 1983, II, 217]; Books III and IV, copied in 525 A.H./1130-31 A.D., fols. 36v-111r.

P Leiden Universiteitsbibliotheek, MS Or. 523/1 [De Jong & De Goeje 1865-1866, III, 87-88, no. 1056]; Persian translation of Book I, copied in 689 A.H./1290-91, 31 fols.

V* Istanbul, Vehbi Efendi, MS 893 [Krause 1936, 472]; Book IV, copied in 427 A.H./1035-36 from an autograph, fols. 1r-75r.

Y Istanbul, Yeni Cami, MS 784/3 [Krause 1936, 472], (Cat. Ahmed III, p. 64); Books I-IV, copied in the 6th century A.H./12th century A.D., fols. 230r-362r.

The following mss. extant in Cairo [King 1986, 45] were not accessible to me:
Dār al-kutub Mīqāt: no. 400 (Books I and II, ca. 650 A.H./1250 A.D., [King 1981-1986, I, 62]); no. 691 (Books I and II, ca. 700 A.H./1300 A.D., [King 1981-1986, I, 120]); no. 188/2 (Book II, ca. 1200 A.H./1785 A.D., [King 1981-1986, I, 53]); Ṭal'at Riyāża 102/3 (Book IV, 1128 A.H./1716 A.D., [King 1981-1986, I, 533]). On these manuscripts see also [King 1981-1986, II, 104-05].

[1]. The date 832 A.H. (=1428-1429 A.D.) is also written on the ms. by a later hand.

A manuscript kept in Hyderabad (Āṣaf. I, 798, no. 305) is mentioned in [Sezgin 1978, 248] as a Persian translation of the *Jāmi' Zīj* (cf. [Rosenfeld & Ihsanoğlu 2003, 118] that mentions it as *Zīj* of Kūshyār al-Jīlī). At my request, my colleagues in the Encyclopaedia Islamica Foundation (Tehran), Mr. Hasan Taromi Rad and Dr. Mohsen Ma'sumi, inspected this manuscript in their trip to Hyderabad in January 2006. This manuscript is not really by Kūshyār. It is a copy of Ulugh Beg's *zīj.*

In order to justify the choice of manuscripts for my edition, it is now necessary to provide some further information on the manuscripts and their relationship.

F is the oldest extant manuscript containing all four books of the *Jāmi' Zīj*. It is written in a clear hand and has very few scribal errors and omissions. F has a lacuna from the middle of the table of contents in the beginning of Book I until the middle of Chapter I.2.2.

The Arabic text of book I is also contained in manuscripts Y, L, B, and C. In B there is a lacuna from the middle of Chapter I.8 (corresponding to I.2.2 in F) to the middle of Chapter I.69 (corresponding to I.6.16 in F).

The Arabic text of Book IV is also contained in manuscripts V, M, A, Y, and L. Manuscript Y has a lacuna from the beginning of Book IV until the end of Chapter IV.9 (corresponding to IV.1.9 in F), and from Chapter IV.33 (corresponding to IV.5.6 in F) until the middle of Chapter IV.43 (corresponding to IV.5.16 in F).

The manuscripts can be divided into three groups according to the way in which Books I and IV are subdivided. (Subdivision of Books II and III are similar in all extant manuscripts).

Group 1: In manuscripts F, C, V and M Books I and IV are subdivided into different sections (*fuṣūl* ), and each section is further subdivided into chapters (*abwāb*). Thus, in F, Books I and IV are divided into 8 sections, and Sections 1 through 8 of Book I are divided into 6, 6, 3, 12, 22, 20, 6, and 10 chapters respectively. Thus the total number of the chapters of Book I is 85. Inspection of the manuscript P has shown to me that it was translated from a manuscript of the same group as F.

Group 2: In manuscripts B, and A Books I and IV are directly divided into chapters. Thus, book I is divided into Chapters 1 through 84 in manuscript B. According to [King 1981-1986, II, 104], Book I was also subdivided into 84 chapters in the manuscript Dār al-kutub Mīqāt 400, and Book IV was subdivided into 63 chapters in the manuscript Ṭal'at Riyāża 102/3.

Group 3: Finally, the manuscript L subdivides Book I into consecutive chapters (as in manuscripts B and Y), and Book IV into 8 sections (as in manuscripts F, V and M. See the description in [De Jong and De Goeje 1865-1866, III, 84-86]. We note that Book I is divided into 80 chapters in L. On the other hand, manuscript Y subdivides Book I into sections and Book IV into chapters, which is different from the division of L in both Books I and IV.

The differences between the groups 1 and 2 concern not only the division of the Books, and additions or omissions made to the text (compare IV.3.1 and IV.3.4), but also to some extent the mathematical content. In IV.6.9, for example, the manuscripts F, V, L and M have only one figure for the first four cases of the proof, but manuscripts A and Y have four figures, one for each case. On the whole, the mathematical differences between the two groups are minor.

Since manuscript V (in group 1) and A (in group 2) both contain a statement to the effect that they are a copy of an autograph, I tentatively conclude that Kūshyār compiled more than one version of the *Jāmiʿ Zīj*, and that the groups 1 and 2 descend from different autograph versions. Since the text in A is to some extent mathematically superior to F, it is likely that group 2 represents a later version than group 1. In other words, Kūshyār originally started with the division of Books I and IV into 8 sections, subdivided into chapters, and he later decided to remove the sections and adopt a subsequent numbering of chapters in Books I and IV. He also made some minor mathematical changes to Book IV in the process. The many similarities between manuscripts F, V and C support to my mind the assumption that they descend from the first version of the *Jāmiʿ Zīj* which Kūshyār compiled. Manuscript L and Y in group 3 represent mixed versions.

Manuscript F was written in a classical Arabic language with few deviations from classical grammatical rules. Only very occasionally, the text violates the rules about agreement between genders (masculine and feminine) and between number (single or plural). Such violations may well be due to Kūshyār. At the end of manuscript A the scribe wrote a note to the effect that he found some grammatical flaws, such as confusion between genders and between singular and plural, in the autograph of Kūshyār's text from which A was copied. The scribe adds that he copied the text as it was without any change. One can try to interpret such deviations from the classical norms in Kūshyār's writings as traces of Middle Arabic. However, the deviations may also be due to the fact that Kūshyār's native language was not Arabic but Persian. The *Jāmiʿ Zīj* was a very technical text, written for a specialized audience with a long and thorough training in mathematics and astronomy. In such a text, one expects language of a formal nature. I have recorded all variant readings in manuscripts C and V in my apparatus, and very few of these variants may be traces of Middle Arabic. For example, on p. 25 note 34 of my edition, I have noted the variant readings ست عشر درجة in the manuscript C, which may be a Middle Arabic form for the classical form ست عشرة درجة in F (compare [Blau 1966-1967, I, 239]. I tentatively interpret the variant reading اي درجة (Arabic text p. 77, footnote 27) in V as a Middle Arabic form of اية درجة in F (and in Kūshyār's original), but the deviant form may have been in Kūshyār's autograph. It is impossible to decide such matters.

After much deliberation, I have opted for the following procedure for establishing the Arabic text.

I have chosen manuscript F as the base for my Arabic edition. As the main alternative manuscript for restoring illegible or missing words in F, I have used C for the edition of Book I and V for the edition of Book IV. In reconstructing the original text, I have used the other manuscripts in cases where V and C also have ambiguities or lacunae. I have not corrected minor grammatical flaws in the text of F, because such flaws may be due to the fact that Arabic was not Kūshyār's native language. I have included all variant readings of F, C and V in my critical apparatus. I have only recorded variants in other manuscripts A, B, L, M and Y if they seemed to me in relation to the meaning of the text.[2]

Since it is my aim to reconstruct the first original version of Kūshyār's *Jāmiʿ Zīj*, and due to the formal scientific character of the text, I have not embarked on a systematic investigation of other manuscripts from a linguistic point of view. For the same reason, I have not hesitated to adapt the orthography to modern standards in some cases, although in most cases I have followed the spelling of F. I have attempted to make the text accessible to modern scientists and historians of science in the middle East and elsewhere, i.e., the modern equivalent of the audience for which Kūshyār wrote his *Jāmiʿ Zīj*.

I have also used a copy of the only known manuscript of the Arabic treatise *al-Lāmiʿ fī amthilat al-Zīj al-Jāmiʿ* ("Explanation of the examples of the *Jāmiʿ Zīj* ") by Abu'l-Ḥasan ʿAlī b. Aḥmad Nasawī (MS Or. 45/7,

[2] Only in exceptional cases, such as Section IV.6.9, I have adopted some of Kūshyār's adaptations in the later version of the *Jāmiʿ Zīj* in my text and translation, since they contribute to the clarity of the contents. See the commentary to the relevant passages.

Columbia University, New York, fols. 49r-75v), for providing some worked examples.

I have followed F for the spellings of the words. Whenever the Arabic letters are used to denote *Abjad* (sexagesimal alphabetic) numbers, I have printed them in boldface in my Arabic edition. I have not added punctuation marks. The chapters and sections are written continuously in the manuscript F, but I have started each chapter from a new line and each section on a new page. Whenever applicable, I have also divided each chapter into paragraphs for sake of clarity.

I have used the following abbreviations in the apparatus:

om. for omitted word or phrase
add. for added word or phrase

In the body of the Arabic I have used angular brackets < > for restoring the omissions of the text. I have put superfluous words in rectangular brackets [ ]. Significant marginal notes from all other manuscripts are also mentioned in the critical apparatus.

In the English translation, I have tried to maintain the structure of the sentences as much as possible. When this was not possible, then I have added a word or an expression in angular brackets < > to make the translation understandable. My explanatory additions to the translation are provided in parentheses ( ). Kūshyār usually writes the numbers in words, but I have used numbers in digits. For the technical terminology, I have tried to use the most recognized equivalents for Arabic astronomical terms. However, since there are usually different variant for the English equivalents of the Arabic astronomical terms, I have essentially followed Prof. E. S. Kennedy in his different publications. For some concepts that were not translated into English in previous works, I have used the equivalents that Prof. Kennedy wrote in a personal communication to me.

For the sexagesimal numbers, I have used the standard notation in which 21,33,8;24,17 stands for $21\times60^2+33\times60+8+24/60+17/60^2$.

The diacritical marks used for the pronunciation of Arabic terms or proper names are as follows:

ā for the long vowel *alef* ا

ū for the long vowel wāw و

ī   for the long vowel *yā'* ي

ʿ = ع   ' = ء (*hamza*)

th=ث   dh=ذ   gh=غ   q=ق

ḥ=ح   ṣ=ص   ẓ=ض   ṭ=ط   ẓ=ظ

I have used roman numbers for the four books of the *Jāmiʿ Zīj*, and Hindu-Arabic numbers for the sections and chapters. In referring to the sections and chapters, I have used the abbreviated form such as I.5 for Section 5 of Book I, and IV.6.8 for Chapter 8 of Section 6 in Book IV.

The commentaries to the chapters of each section are provided at the end of the relevant section, using the above abbreviations to denote the chapters. Since most of the astronomical theories in Kūshyār's *Jāmiʿ Zīj* are essentially Ptolemaic, I have not gone into complete details. I have referred the reader to the standard works such as Toomer's translation of the *Almagest* [Ptolemy 1984], and Pedersen's *Survey of the Almagest* [Pedersen 1974]. On the other hand, Kūshyār is influenced by al-Battāni's *al-Zīj al-Ṣābī*. So I have referred to the corresponding discussion in al-Battānī's work (Nallino's publication of the *Zīj* [al-Battānī 1899-1907]), whenever applicable. For the calculation methods provided by Kūshyār, I have given the formula in modern notation. In some cases, I have provided a worked example to make the method more comprehensible. Prof. J. L. Berggren has published a translation of IV.3 [Berggren 1987] and Mr. T. Kashino has provided an edition and English translation of the chapters and tables relating to planetary longitudes in all four Books of the *Jāmiʿ Zīj*, as mentioned in Part 3 of this preface [Kashino 1998]. I have noticed significant differences between these publications and my edition, in my commentaries to the chapters and in my critical apparatus to the Arabic text. References are given in brackets and include author name, year of publication and page number(s).

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