In the name of God the merciful, the compassionate, and we ask for your assistance, o Generous One!

Kūshyār ibn Labbān ibn Bāshahrī al-Jīlī says: When I examined the *zīj*es composed in the art of astronomy and reflected on them, <I found that> there was incorrectness in some of them that needed rectification; some had long-windedness and difficulty that needed simplification; and some had omissions that needed completion. <Even> the *Almagest* is not free of them (i.e., the defects). All of them (i.e., the *zīj*es) <contain> careless calculations, devoid of clear exposition and unsupported by adequate demonstration. <Therefore,> I made up my mind to work out a *zīj* combining theory and practice, in which I <would> rectify the incorrectness, bring closer what was far-fetched, fill up for deficiencies, elucidate every <technical> term with a comment, and provide proofs for every calculation in it. Therefore, any difference found in anything between this <*zīj*> and the others, is <caused by my> rectification of the incorrectness or <my> bringing closer the far-fetched or <my> filling of gaps. I have discussed practice before theory in order to facilitate the beginner's access to it and to quicken his benefiting by it. I have composed this <work> in four Books: the first on elementary calculations, the second on their (i.e., of the calculations) tables, the third on commentary and astronomy, and the fourth on the demonstration of the accuracy of the elementary calculations.

When I resolved to do this and reaffirmed my intention about it, I begged God for success and guidance.

<List of the chapters of> Book I: On elementary calculations, <in> 8 sections and 85 chapters

Section 1: On eras, <in> 6 chapters
1. On the beginnings of ancient eras and <the difference> between any two of them in years and days.
2. On the three eras used in our time.
3. On converting the years of these eras into days, and the days into <corresponding> years by calculation and <by using> the table<s>.
4. On extracting <dates in> these eras from each other.
5. On the weekday of <any date of> these eras.
6. On the feasts and <other> events in these eras.

Section 2: On Sines and Chords, <in> 6 chapters
1. Introduction to the knowledge of the Sine <function>.
2. On interpolation between two lines of the Sine <table> and other tables.
3. On <finding> the Sine of a <given> arc and the arc of a <given> Sine from the table.
4. On <finding> the Sagitta of a <given> arc and the arc of a <given> Sagitta from the <special> table and <from> the Sine table.
5. On <finding> the Chord of a <given> arc and the arc of a <given> Chord from the Sine table.
6. On correcting the Sine whenever we have doubt about any <value> of it.

Section 3: On Tangents and Cotangents, <in> 3 chapters
1. On calculating the Tangent and Cotangent, their two Hypotenuses (i.e., Secant and Cosecant) and their two arcs.
2. On <finding> the Tangent of a <given> arc and the arc of a <given> Tangent from the table.
3. On converting Tangents and Cotangents to different gnomons.

Section 4: On <finding> the true longitudes of the planets, and their situations, <in> 12 chapters
1. On the epoch values and the preliminaries for <finding> the mean longitudes of the planets.
2. On deriving the mean longitudes from their tables.
3. On converting the mean longitudes from <localities having> one <geographical> longitude to another.
4. On the positions of the apogees and the nodes, and <on> their motions.
5. On the equation of time.

6. On the true longitude of the sun.
7. On the true longitude of the moon and its node<s>.
8. On the true longitudes of the five planets.
9. On the latitude of the moon.
10. On the latitudes of the five planets.
11. On the retrogradation of the planets, their direct motion, and their first and last visibility.
12. On the ascension and descension of the planets in their spheres.

Section 5: On the operations relating to the ascendants during the day and the night, <in> 22 chapters
1. On the first declination.
2. On the right ascensions of the <zodiacal> signs.
3. On the second declination.
4. On the distance of the stars from the celestial equator.
5. On the latitude of <any> locality.
6. On the ortive amplitudes of the sun and <any> star.
7. On the equation of daylight of the sun and <any> star.
8. On the ascensions for a locality (i.e., oblique ascensions).
9. On the maximum altitude of the sun and <any> star.
10. On half the day arc of the sun and <any> star.
11. On the <equinoctial> day hours of the sun and <the> stars and the degrees of their <seasonal> hours.
12. On the <ecliptical> degree of the transit of a star through the meridian.
13. On the <ecliptical> degree of the rising and setting of a star.
14. On <finding> the arc of revolution of the celestial equator since the rising of the sun or the star<s> from the altitude of the <sun or the star at a given> time.
15. On <finding> the <elapsed> hours from the arc of revolution.
16. On <finding> the ascendant from the arc of revolution.
17. On <finding> the arc of revolution from the ascendant.
18. On <finding> the altitude of the <sun at a given> time from the arc of revolution.
19. On <finding> the arc of revolution since sunset from the ascendant.
20. On <finding> the ascendant from the arc of revolution since sunset.
21. On a base <value> applying to most operations concerning day and night.
22. On the equalization of houses.

Section 6: On eclipses and what pertains to them, <in> 20 chapters
1. On the motion of the two luminaries in <one> day and <one> hour.
2. On the magnitude of the <apparent> diameter of the two luminaries and the diameter of the shadow <of the earth>.

3. On the <ecliptical> degree of a conjunction and opposition, their hours and ascendants.
4. On the absolute and adjusted magnitudes of a lunar eclipse in digits.
5. On the absolute and adjusted timing of a lunar eclipse.
6. On drawing the figure of a lunar eclipse.
7. On finding the distance of the moon from the earth.
8. On the altitude of the pole of the ecliptic which is called 'the latitude of the clime of visibility'.
9. On the altitude of any desired degree of the ecliptic.
10. On the <equatorial> distance between the meridian and the <right> ascension of a known point of the ecliptic.
11. On the parallax of the two luminaries in the altitude circle.
12. On the six angles which are needed in <the calculation of> solar eclipses.
13. On <finding> the longitudinal and latitudinal parallax of the moon from these angles.
14. On the absolute and adjusted magnitudes of a solar eclipse in digits.
15. On the absolute and adjusted times of a solar eclipse.
16. On drawing the figure of a solar eclipse.
17. On <finding> the altitude of the moon taking account of its latitude.
18. On <finding> the longitudinal and latitudinal parallax of the moon by a method which can be proved.
19. On extracting the longitudes of localities.
20. On <determining> the visibility of the <lunar> crescent and the planets from <certain> arcs defined for them.

Section 7: On the operations relating to astrology, <in> 6 chapters
1. On <finding> the distance between the <ecliptical> degree of a planet and the cardines in <terms of> hours.
2. On <finding> the projection of the ray by means of equal (i.e. ecliptical) degrees.
3. On <finding> the projection of the ray by means of ascension (i.e. equatorial) degrees.
4. On <finding> the prorogations (i.e., astrological progressions).
5. On <finding> the transfers of the years and their ascendants.
6. On converting the ascendant of the world year from one locality to another.

Section 8: On the operations which are less needed, <in> 10 chapters
1. On <finding> the latitude of a locality from the hours (i.e., the duration) of <its> longest day.
2. On the altitude with no (i.e., zero) azimuth.
3. On <finding> the azimuth for any altitude which we assume.

4. On <finding> the altitude from the azimuth.
5. On the distance between two stars of which <only> one has a <non-zero> latitude.
6. On the distance between two stars both having <non-zero> latitudes.
7. On the extraction of the meridian line.
8. On the deviation of <the directions of> localities with known longitudes and latitudes from the meridian of our locality.
9. On of the fixed stars, and the features of some of them in order to recognize them by seeing.
10. On the names of the lunar mansions, and their rising days.

These are the <titles of the> chapters of this Book. I have presented them in order of their importance, and I have devoted most attention to the ones that are most necessary. God is the One who makes <us> successful in what is correct, and to Him we shall return.

Section 1: On eras, <in> 6 chapters
Chapter 1: On the beginnings of the ancient eras and the <numbers of> years and days between any two of them.

The famous eras preserved by the ancients (i.e., those who lived up to the author's time) are: the era of the Deluge, the era of Nabonassar, the era of Philippus, the era of <Alexander> the Two-Horned, the era of Augustus, the era of Diocletianus, the era of the Hejira, and the era of Yazdigird.

The Deluge: The era of the Deluge is used by the authors of the ancient *zīj*es such as the *Sindhind zīj* and *Shāh zīj*. Its beginning was the Friday close to the occurrence of the Flood in the time of Noah – peace be upon him! On that day, at sunrise, the sun was in Aries and the moon was in conjunction with it in the beginning of Aries, and the other planets were around the beginning of Aries. Subsequent eras are related to it (i.e., the Deluge).

Nabonassar: He was Nabonassar I, among the kings of Babylon.* The first day of his era was a Wednesday. Ptolemy rendered the mean motions of the planets in the *Almagest* for this era,* and he rendered the positions of the fixed stars for the beginning of the year 886 of it, which was the first day of the reign of Antoninus. Between Friday, the first day of <the era of> the Deluge, and Wednesday, the first day of this era, there are 860,172 days, which are equal to 2,356 Persian-Egyptian years of 365 days, and 232 completed days.

Philippus: He was Philippus, known as the Mason,* father of the Two-Horned*. He was one of the kings of Athens. He <reigned> after the death of Alexander of Macedonia (Alexander III). Theon of Alexandria based his *zīj*, called the *Canon*, on this era. The first day of his era was a Sunday, between which and the era of the Deluge there were 1,014,834 days or 2,780 years and 134 days.

The Two-Horned: He was Alexander II, known as the Two-Horned.* The first day of his era was a Monday, which was the first day of the seventh year of his reign, when he left the land of Macedonia, traveled over the <whole> Earth, and reached <very remote places of> the inhabited world. Between the Monday <which was the beginning> of this era and the epoch of the Deluge there were 1,019,273 days or 2,792 <completed> years and 193 completed days.

Augustus: He was one of the Roman kings. Christ was born in some year of his <reign>. The first day of this era was a Thursday, between which and the epoch of the Deluge there were 1,122,316 days or 3,074 years and 306 days.

Diocletianus: He was one of the kings of Christendom. The first day of his era was a Wednesday, between which and the epoch of the Deluge there were 1,236,639 days or 3,388 <completed> years and 19 completed days.
The Hejira was the emigration of the Prophet—God bless him and grant him salvation!—from Mecca to Medina. He entered it (i.e., Medina) on Monday, the eighth of the month Rabīʿ al-awwal, and the era is reckoned from the beginning of that year, which was a Thursday, the first day of Muḥarram. Thus between it and that <day of emigration> there are 67 days. The year <of the Hejira calendar> is 354 days plus 1/5 plus 1/6 <of a day>. When <the accumulation of> these fractions exceeds half a day, one day is added to the days of Dhu'l-ḥijjah, so <the number of> its days becomes 30, and <the number of> the days of this year becomes 355. This happens 11 times in the computation of every 30 years, because 11 is 1/5 plus 1/6 of 30. Between this epoch and the epoch of the Deluge there are 1,359,973 days or 3,725 years and 348 days. The determination of the intercalation is such that you should cast out thirties from the <elapsed> years including the desired year, and you should multiply the remainder by 11 and cast out thirties <from the product>. If the remainder is greater than 15, then the <given> year is a leap year, and if it is less, then it is not.
Yazdigird: He was Yazdigird, son of Shahriyār, son of Kisrā, the last of the Persian kings. The first day of the year in which he acceded to the throne was a Tuesday, between which and the epoch of the Deluge there were 1,363,597 days or 3,735 years and 322 days.
If we want to know <the number of the days or years> between any two epochs, we subtract the <number of> years or days closer to the epoch of the Deluge from the <number of> years or days farther from it, and the remainder is the <number of> years or days between them.

Chapter 2: On the three calendars used in our time.

The calendars used among us and in our time are: (a) The calendar of the Two-Horned, which is the Greek and the Syrian <calendar> because there is no difference between them except in the names of the months. The first Greek month is *Kānūn al-thānī* (i.e., *Kānūn* II) with <its> Greek name, and the following <months are based> on its arrangement (i.e., the arrangement of the Syrian months regarding the number of the days in each month); (b) the calendar of the Hejira, that is the Arabian calendar; and (c) the calendar of Yazdigird, that is the Persian calendar.
As to the Syrian <calendar>, its beginning was a Monday as has been mentioned before. The Syrian names of the months and the numbers of their

days, added up and separately, are as I say: *Tishrīn* I, 31 days, **31**; *Tishrīn* II, 30 days, **61**; *Kānūn* I, 31 days, **92**; *Kānūn* II, 31 days, **123**; *Shubāṭ*, 28 days and a quarter of a day, **151**; *Ādhār*, 31 days, **182**; *Nīsān*, 30 days, **212**; *Ayyār*, 31 days, **243**; *Ḥazīrān*, 30 days, **273**; *Tammūz*, 31 days, **304**; *Āb*, 31 days, **335**; *Aylūl*, 30 days, **365**. So a year has 365 days and a quarter of a day. Whenever <the accumulation of> the quarter is greater than half a day, the number of days of *Shubāṭ* is increased by one, so <the number of> its days becomes 29. The <number of> days of this year becomes 366, and it is a leap year. To know it (i.e., the leap year), you cast out fours from the number of years including the desired year. If the remainder is 3, then this is a leap year, and if the remainder is less, it is not.

As to the Arabic <era>, its beginning was a Thursday, the first day of the year in which the Prophet <Muḥammad>—God bless him and grant him salvation!—emigrated <to Medina>. It is the 15th of *Tammūz* of the year 933 of <the era of> the Two-Horned. The names of its months and the numbers of their days, added up and separately, are as I say: *Muḥarram*, 30 <days>; *Ṣafar*, 29 <days>, **59**; *Rabī‘ I*, 30 <days>, **89**; *Rabī‘ II*, 29 <days>, **118**; *Jumādā I*, 30 <days>, **148**; *Jumādā II*, 29 <days>, **177**; *Rajab*, 30 <days>, **207**; *Sha‘bān*, 29 <days>, **236**; *Ramaḍān*, 30 <days>, **266**; *Shawwāl*, 29 <days>, **295**; *Dhu'l-qa‘da*, 30 <days>, **325**; *Dhu'l-ḥijja*, 29 <days> plus a fifth and a sixth of a day, **354; <22>**. Thus a year <has> 354 days plus a fifth and a sixth of a day. Whenever <the accumulation of> these fractions exceeds half a day, its calculation is as has already been mentioned. The <numbers of> the days of these months are found in this way: You subtract the mean daily motion of the sun from the mean daily motion of the moon, and a complete revolution (i.e., 360°) is divided by the remainder. The result is 29;31,50 days approximately. Thus the months were established <as having> 30 days and 29 days alternately, and we add the extra fractions, i.e. the excesses over half a day, at the end of the year; this adds up to a fifth and a sixth of a day.

As to the Persian <calendar>, its beginning was a Tuesday, the first day of the year in which Yazdigird, son of Shahriyār, acceded to the throne. It is the 22nd of Rabī‘ I of the year 11 of Hejira, and the 16th of Ḥazīrān of the year 943 of the <era of the> Two-Horned. The names of its months and the numbers of their days, separately and added up, are as I say: *Farwardīn-māh*, 30 <days>, **30**; *Ardībahisht-māh*, 30 <days>, **60**; *Khurdād-māh*, 30 <days>, **90**; *Tīr-māh*, 30 <days>, **120**; *Murdād-māh*, 30 <days>, **150**; *Shahrīr-māh*, 30 <days>, **180**; *Mihr-māh*, 30 <days>, **210**; *Abān-māh*, 35 <days>, **245**; *Ādhar-māh*, 30 <days>, **275**; *Day-māh*, 30 <days>, **205**; *Bahman-māh*, 30 <days>, **235**; *Isfandārmadh-māh*, 30 <days>, **265**. Thus a year <has> 365 days. The five days added at the end of Abān-māh are called the *mustaraqa* ("stolen") <days>. Since the Persian year is approximately a quarter of a day less than a solar year, this becomes one day in every four years and one month in every 120 years. During the period of their domination, the Persians observed one intercalary month every 120 years. Thus this year had 13 months. They counted the first month of this year twice: once at the beginning of the year and once more at the end of the year. They put the extra five <days> in the intercalary month (i.e., at the end of the year). <Thus,> the first month of the year was the one in which the sun entered Aries. So, the five <days> and the beginning of the year were moved from one month to the next every 120 years. In the time of Kisrā, son of Qubād, Anūshervān, the sun entered Aries in Ādhar-māh, and the five <days> were placed at the end of Abān-māh. When 120 years had passed, it was the end of the reign of the Persians, the disruption of their government, and <the beginning of> the domination of the Arabs over them. So, this tradition was neglected, and the five <days> remained at the end of Abān-māh until the year 375 Yazdigird, when the sun entered Aries on the first day of Farwardīn-māh. We have been informed that in <the province> Fārs and those areas <near it>, the five <days> were moved to the end of Isfandārmadh-māh according to the ancient tradition. But in our areas, which are Rayy, Jurjān and Ṭabaristān, they are <still observed> at the end of Abān-māh. People think that it is <something related to> the Zoroastrian religion and tradition, and should not be replaced and changed. Each day of the <Persian> months has a special name by which it is called, viz.: *Hurmazd, Bahman, Ardībahisht, Shahrīr, Isfandārmadh, Khurdād, Murdād, Day-ba-ādhar, Ādhar, Abān, Khūr, Māh, Tīr, Kūsh, Day-ba-mihr, Mihr, Surūsh, Rashan, Farwardīn, Bahrām, Rām, Bād, Day-ba-Dīn, Dīn, Ard, Ashtād, Asmān, Zāmyād, Mārasfand, Anīrān,* and the five 'stolen' days <are> *Ahunavad, Ushtavad, Isfandmad, Vahukhshatra,* <and> *Vahishtavasht.*

Chapter 3: On converting the years of these calendars into days, and the days into <the corresponding> years by calculation and by <using> table<s>.

Calculation for the Syrian <calendar>: You multiply the <number of> completed Syrian years by 21,915, you divide the product by 60, and thus the <number of> days in those years will be obtained. If the division has a remainder greater than 30, we restore it to one day. You multiply the given <number of> days by 60 and you divide the product by 21,915: The <number of> years <contained> in those days will be obtained. We divide the remainder of the division by 60: The <number of > days of the incomplete year will be obtained.

<Calculation for> the Arabian <calendar>: You multiply the <number of> completed Arabian years by 21,262 and you divide the product by 60: The <number of> days in those years will be obtained. You multiply the given <number of> days by 60 and you divide the product by 21,262: The <number of> years <contained> in those days will be obtained. We divide the remainder of the division by 60: The <number of> days of the incomplete year will be obtained.

<Calculation for> the Persian <calendar>: You multiply the <number of> completed Persian years by 365: The <number of> days in those completed years will result. You divide the given <number of> days by 365: The <number of> completed years will be obtained. The remainder is the <number of> days in the incomplete year.

<Conversion by means of> the table: If we compile tables, we record in them the multiple or single years, and months, and opposite them, the numbers of days in them in sexagesimals. Then the first <digit> of them (i.e., these numbers) is the absolute <number of> days. The second of them is a multiple of 60, i.e., once divided by 60. The third one is a multiple of 60×60, i.e., twice divided by 60. The fourth one is a multiple of $60\times60\times60$. If we want <to find> the <number of> days of given years and months, we enter with the completed years in the table of the multiple years. We take the <number of> days corresponding to the nearest number below it, and write it down (B adds: "on the <dust> board"). Then we enter with the remainder <of the years> in the table for the single years, take the <number of> days corresponding to it, and add it to what we wrote down before, any <sexagesimal> digit to its corresponding <sexagesimal> digit. Then we take the <number of> days corresponding to the completed months and add it to the sum already obtained. Then the <number of> days in the given years and months will be obtained.

If we want <to find> the <numbers of> years and months <corresponding to a certain number> of days, we enter with the days in <the column for> the multiples of days, take the <number of> years corresponding to the nearest lesser number, and write it down. Then we subtract the <number of> days found in the table from the given <number of> days, each digit from its corresponding digit. Then we enter with the remainder of the days in <the column for> the single days and take the <number of> years corresponding to the nearest lesser number. Then we add it to the <number of> years that we wrote down before. We subtract the <number of> days found in the table of single <days> from the <remaining> days that we have, any digit from its corresponding digit. We take the <number of> months corresponding to the nearest number below the <number of> remaining days. What remains from the <number of> days is the <number of> days of the incomplete month.

Chapter 4: On extracting <dates in> these calendars from each other.

If <a date in> one of these three calendars is known and we want to know <the corresponding date in> another calendar, we convert the known date into days until the present day, and keep it in mind. Then if <the era of> the known <date> precedes the <era in which the date is> unknown, we subtract the <number of> days between the two eras from the <number of> days that we kept in mind. If the <epoch of which the date is> unknown precedes the <epoch of which the date is> known, we add the <number of> days between the two eras to the <number of> days that we kept in mind. Then the remainder or the sum is the unknown <date of the desired> calendar in days. Then we convert it into years as already described. The <beginning of the> Syrian era precedes the <beginning of the> Arabian era by 340,700 days, and precedes the <beginning of the> Persian era by 344,324 days; the <beginning of the> Arabian era precedes the <beginning of the> Persian era by 3624 days. In order to check <the correctness of> the result of <converting> the calendar, we determine the weekday of the given date in the known calendar, and the weekday of the unknown date <in the desired calendar>. If they agree, then it is correct, and if they differ one or two days, we adjust the unknown <date> according to the known <date>.

Chapter 5: On the weekday of <any date of> these calendars.

The Syrian <calendar>: We convert its date into the <number of> days up to the desired day, plus this day. Then we cast out sevens and count the remainder from Monday. The <week->day at which <the number> finishes, will be the weekday <corresponding to> the given day. If we want to, we <may> cast out twenty-eights from the <number of> years including the desired year. We enter with the remainder in the weekday table, and take the weekday of <the beginning of> the desired month.
The Arabian <calendar>: We convert its date into the <number of> days as has already been discussed for the Syrian <calendar>.Then we cast out sevens and we count the remainder from Thursday. The <week->day at which the number finishes will be the weekday of the <given> day. If we want to, we <may> cast out multiples of twohundred-ten from the <number of> years including the given year. We enter with the remainder in the

weekday table and we take <the number corresponding to> the weekday <of the beginning> of the desired year. Then we add to it <the number corresponding to> the weekday of the desired month.
The Persian <calendar>: We cast out sevens from the <number of> years including the given year and we count the remainder from Tuesday. The <week->day at which <the number> finishes will be the weekday of <the beginning of> that year. For each month after Farwardīn we add two days, but we do not add anything for the weekday of Ādhar-māh because the weekday of <the first of> Abān-māh and that of Ādhar-māh are the same on account of the <five> "stolen" <days>.

Chapter 6: On the feasts and <other> events in these calendars.

Syrian <feasts>:
Mā'althā (for the literal meaning of the names of the feasts and their equivalents, see the commentary): If the 29th of Tishrīn I (October) is a Sunday, it is *Mā'althā*, otherwise, <it is> the Sunday which follows it. *Subbār*: If the 28th of Tishrīn II (November) is a Sunday, it is *Subbār*, otherwise, <the Sunday> that follows it.
Mīlād: the night which is followed by the morning of the 25th of Kānūn I (December).
Dinḥ: the 6th of Kānūn II (January).
Ṣaum al-'adhārā: It is the feast of *Ghayṭās*, the Monday which follows *Dinḥ*.
Ṣaum Naynawī: <It consists of> three days beginning on a Monday 22 days before *al-Ṣaum al-kabīr*.
'Īd al-haykal: the 2nd of Shubāṭ (February).
Al-Ṣaum al-kabīr: <For its> calculation we take the years of the Two-Horned <era> with the year we desire (i.e., the current year), and we add five to it. We cast out nineteens and we multiply the remainder by nineteen. If the product is greater than 250, we always subtract one from it; if it is less, we do not subtract anything. We cast out thirties from the result. Then we observe the remainder. If it is equal to <the number of days of> Shubāṭ <in that year> or less than that, then the <beginning of the> fast is on that day of Shubāṭ, if it is a Monday. Otherwise, the Monday after it <is the beginning of the fast>. If it (i.e., the remainder) is greater than the <number of> days of Shubāṭ <in that year>, we subtract the <number of> days of Shubāṭ from it. The remainder, <taken> as <number of the day> of Ādhār, is the beginning of the fast if it is a Monday. Otherwise, the Monday after it <is the beginning of the fast>.

We have compiled a table for it. For working with it, we take the years of <the era of> the Two-Horned with the year we desire (i.e., the current year), and we write it down in two positions. We divide one of the <numbers written in the> two positions by twenty eight and we divide the <number in the> other position by nineteen, after adding five to it. We enter along the length of the table with the remainder of the division by twenty eight, and along the width of the table with the remainder of the division by nineteen. The crossing position of the <column and the row of the> two numbers is the beginning of the fast. If it is <written> in black, it is in Shubāṭ, and if it is <written> in red, then it is in Ādhār.

Another method: It (i.e., the beginning of the fast) is on the nearest Monday to the conjunction which occurs between the 2nd of Shubāṭ (February) and the 8th of Ādhār (March). If we are in doubt about the nearest Monday, then it is <the Monday> which lies between Sha'ānīn and the *Fiṭr* that follows it.

Sha'ānīn: the Sunday, the 42nd of the days of the fast.

Fiṭr: the Sunday next to *Sha'ānīn*.

Al-Sha'ānīn al-ṣaghīra: the Friday following *Fiṭr*.

Sullāq: the Thursday 40 days after *Fiṭr*.

Finṭīqusṭī: the Sunday 10 days after *Sullāq*.

Ṣaum al-Salīḥīn: the Monday after *Finṭīqusṭī*.

Ṣaum Mārt Maryam: the first day of Āb (August).

Ẓuhūr al-Masīḥ: 6th of Āb (August).

Fiṭr Maryam: 15th of Āb (August).

'Īd al-ṣalīb: 14th of Īlūl (September); 13th of Īlūl (September) according to the Nestorians; 15th of Īlūl (September) according to the Romans and the Jacobites.

Suqūṭ al-jimār: the 7th, 14th, and 21st of Shubāṭ (February).

Ayyām al-'ajūz: Seven days starting on the 26th of Shubāṭ (February).

Nayrūz al-Mu'taḍid: 11th of Ḥazīrān (June).

Ayyām al-bāḥūr: Eight days starting on the 19th of Tammūz (July). The variation of the weather on these days indicates that during (the first to the eighth month of) the next year.

Arabian <feasts>:

'Āshūrā: It is the date of the murder of Ḥusayn b. 'Alī—May God honor him and be pleased with him!—<which occurred on> the 10th of Muḥarram.

Maulid al-Nabī - may the exalted God bless him and grant him salvation!: 12th of Rabī' I.

Yaum al-jamal: 15th of Jumādā I.

Mab'ath al-Nabī - may God bless him and grant him salvation!: 26th of Rajab.
Mi'rāj: the night of the 27th of Rajab.
Laylat al-ṣakk: the night of the 15th of Sha'bān.
Ṣaum: the days of Ramażān.
Fatḥ Makka: 20th of Ramażān.
'Īd al-Fiṭr: 1st of Shawwāl.
Al-Tarwīya: 8th of Dhu'l-ḥijja.
'Arafa: 9th of Dhu'l-ḥijja.
'Īd al-ażḥā: 10th of Dhu'l-ḥijja.
Ghadīr Khumm: 18th of Dhu'l-ḥijja.

Persian <feasts>:
Nayrūz: 1st of Farwardīn-māh (i.e., the month of Farwardīn).
Nayrūz al-khāṣṣa: 6th of Farwardīn-māh.
Mihrajān: 16th of Mihr-māh.
Mihrajān al-khāṣṣat al-ṣaghīr: 21st of Mihr-māh.
Gāgīl: 15th of Day-māh.
Bahmanjana: 2nd of Bahman-māh.
Sadaq: the night of the 10th of Bahman-māh.
Wādhīra: 22nd of Bahman-māh.
Katb al-ruqā': 5th of Isfandārmadh-māh, <based on placing> the "stolen" days at the end of Abān-māh.
The six *Jāhanbār*s: first, 26th of Ardībahisht-māh; second, 26th of Tīr-māh; third, 16th of Shahrīr-māh; fourth, 15th of Mihr-māh; fifth, 11th of Day-māh; sixth, the five "stolen" <days> of Isfandārmadh-māh.

Commentary

I.1.1 Historians from the Islamic period have confused Nabonassar, the king of Assyria whose reign began in 747 B.C. and whose era was later used in Ptolemy's *Almagest*, with Nabuchadnezzar (Nabokolassar), king of Babylonia, who reigned in the period 604-562 B.C., and who conquered Jerusalem. So, they have referred to the former by the arabicized form of the latter's name, i.e., *Bukhtanaṣṣar*.

Ptolemy lived in the time of Antoninus Pius (fl. 137 C.E.) and used the era of Nabonassar because, as he says in *Almagest* III.7, this was the era beginning from which ancient observations were preserved down to his time.

The Philippus after whom the epoch 324 B.C. is named, is a son of Alexander III (the Great) and a halfbrother of Alexander IV. His reign started in the same year as that of Alexander IV (323 B.C.), namely with the death of Alexander the Great. The title Mason (*al-bannāʾ*) is mentioned in all mss. except L. It does not occur in other sources that I have seen, save the *Muṣṭalaḥ Zīj* (MS BN arabe 2513), whose chapter on chronology seems to depend, to some extent, on Kūshyār.

In fact, it was Ptolemy's *Handy Tables* (Theon did not write a *zīj*), in which the Philippus era was adopted. This era also occurs in the *Almagest* as 'the death of Alexander' [Ptolemy 1984, 10, fn. 16].

It is generally accepted both by Muslim commentators and occidental scholars that the 'Two-Horned' (*Dhu'l-qarnayn*) mentioned in the Holy Koran, and used by Arab authors, Muslims, and Christians is to be identified with Alexander the Great (356-323 B.C.). He was Alexander III (not Alexander II, as Kūshyār calls him) of Macedonia. The era erroneously named after Alexander is actually the Seleucid era, which started with the death of Alexander IV and the accession of Seleucus, the founder of the Seleucid dynasty, to power [Ginzel 1906-1914, I, 136; Taqizadeh 1939, part 2, 124-27].

Al-Bīrūnī also mentions Diocletianus as "one of the kings of Christendom" [1879, 105], and says elsewhere that "He was the last of the pagan Emperors of Rome; after him they became Christians" [1934, 173]. In the Byzantine tradition, Diocletianus is primarily remembered as a prosecutor, for his edict of prosecution against the Christians that started in 303 C.E.

In early *zīj*es, if the remainder of a division for the determination of the intercalation of the Arabian years was 15, the resulting half of a day was usually truncated, which led to an ordinary 15th year and an intercalary 16th

year in every 30-years cycle. However, in table 2 of Book III of the *Jāmi' Zīj* for the number of days in multiples of Arabian years, Kūshyār gives the number of days in 15 Arabian years equal to 5316 =15×(354+11/30)+0.5 days. This means that, as was more common in later Persian *zīj*es, he rounded upwards the half of a day resulting from the accumulation of the fractions which led to an intercalary 15th year [cf. van Dalen 2000, 267].

Following is a summary of the numerical data given in this section:

Era	**Weekday**	**Days after the Deluge**	**Years+days**
Nabonassar (Assyrian, 26 Feb. 747 B.C.)	Wednesday	860172	2356y+232d
Philippus (Greek, 12 Nov. 324 B.C.)	Sunday	1014834	2780y+134d
Alexander (Seleucid, 1 Oct. 312 B.C.)	Monday	1019273	2792y+193d
Augustus (Roman, 30 Aug. 30 B.C.)	Thursday	1122316	3074y+306d
Diocletianus (Roman, 29 Aug. 284 C.E.)	Wednesday	1236639	3388y+19d
Hejira (Arabian, 15 July 622 C.E.)	Thursday	1359973	3725y+348d
Yazdigird (Persian, 16 June 632 C.E.)	Tuesday	1363597	3735y+22d

In this table, we see the number of days that had passed since the Deluge, at the beginning of each of the seven eras. Each number of days is also converted by Kūshyār into Persian years plus remaining days. Kūshyār's data imply that the epoch of the Deluge was taken to be Friday, 18 Feb. 3102 B.C., which was commonly used and is also implied in Kūshyār's astrological treatise [Kūshyār 1997, 140/141].

The above numbers of days for the Nabonassar, Alexander, Hejira and Yazdigird epochs are the most common ones [cf. van Dalen 2000, 266, table 2]. The correct number of days since the Deluge for the Philippus epoch is 1014932. The above number given by Kūshyār (1014834, found in the mss. C, Y, B and P) is probably an error by Kūshyār or the scribes. In the ms. L this number is given as 1014934, which is still wrong but closer to the correct number. Presumably the original digit 9 was miswritten as 8 (a possible error in the Arabic script), and the digit 2 was then changed to 4, in order to accord with the correct weekday (Sunday). For the Augustus era, the number given by Kūshyār (1122316, corresponding to 13 Nov. 30 B.C.) is one of two that are found in various other sources. It is based on the assumption that New Year in the ancient Egyptian and the Coptic calendar coincided in the time of Philippus instead of Augustus [cf. van Dalen 2000, 266]. Also the implied date for the Diocletian era, 12 Nov. 284 C.E., is one of two that were used in various early sources [cf. van Dalen 2000, 266].

I.1.2 In Arabic texts from the Islamic period, the adjective *Rūmī* (Roman) means either 'Roman' or 'Greek'. Here it refers to the Greek era. The modern names (and the numbers of days) of the 'Greek' months are for instance given by al-Bīrūnī in *al-Tafhīm* and his *Chronology*: Yanwārīūs (31), Febrārīūs (28), Mārṭīūs (31), Afrīlīūs (30), Māīūs (31), Yūnīūs (30), Yūlīūs (31), Aghusṭūs (31), Sebṭembrīūs (30), Aqṭubrīūs (31), Nuāmbrīūs (30), and Duqambrīūs (31). Kūshyār has observed the rule for determining the Syrian leap years in table 1 of Book II of the *Jāmi' Zīj* for the number of days in multiples of Syrian years.

The "conventional" Arabian lunar months have alternately 30 and 29 days. In the lunar months based on the visibility of the lunar crescent, generally used in modern time, the first day of any lunar month is the day following the first observation of the lunar crescent. In this system it is possible to have two consecutive 30-day months, or two consecutive 29-day months.

The Iranian calendar at the time of the advent of Islam was based on a vague solar year of 365 days consisting of 12 months of 30 days plus five extra days that were added at the end of the eighth month Abān. This year was originally taken from the Egyptian calendar. Some modern scholars have tried to determine the date of introduction of the Egyptian year in Iran on the basis of Kūshyār's description of the five epagomenai being at the end of Abān in the year 375 of the Yazdigird era (1006-7 C.E.), found in this chapter. For instance, Taqizadeh [1938, 12] believes that the introduction happened in the second decade of the fifth century B.C. However, none of the results have been fully satisfactory [Taqizadeh 1938, 5]. According to Kūshyār, as well as al-Bīrūnī and some other authors, Iranians intercalated one full month in each 120 years to compensate for the difference between the Egyptian year and the tropical year (about one-fourth of a day) and to keep the beginning of their year close to the vernal equinox [see e.g., Ginzel

1906-1914, 290-91]. Taqizadeh thinks that this sort of year was by no means a wholly fictitious year, as some seem to believe [1938, 57]. Recently François de Blois [1996] has tried to show that such an intercalation process was a mere "legend". However, in particular his "negative" argumentation has not convinced me.

De Blois starts his discussion with the assertion that no reference to an Iranian intercalary month is found in ancient sources and no event is reported to have happened in such a month. But from a mathematical point of view, the probability of a random event happening in an intercalary month following a 120 years period as mentioned above is $1/(120\times12+1)=1/1440$, which is less than 0.07%. He then casts doubt on the reliability of the accounts provided by Kūshyār and al-Bīrūnī for the intercalation in the Iranian calendar. Here his argument that Kūshyār prepared a manuscript of his *Jāmi' Zīj* in 393 A.H./1002-3 C.E. and hence could not have mentioned a calendar reform in 375 A.Y./1006-7 C.E. turns out to be invalid. Inspection of the Alexandria manuscript of the *zīj* shows that the date of Kūshyār's autograph was 'Sunday the 2nd of Bahman-māh of the year 393' [A.Y./8 Dhu'l-qa'da 415A.H./10 January 1025 C.E.], so Kūshyār's reference to the reform can be correct. Moreover, in the second chapter of the text presented in this article, Kūshyār says that the transfer of the five epagomenai had not yet been accepted by the inhabitants of Rayy, Jurjān and Ṭabaristān, but in the Persian translation, ms. P, prepared in 483 A.H., Rayy is omitted from the names of the cities. This indicates that Kūshyār and the translator were giving a realistic and up-to-date account of what was going on around them.

In my opinion, de Blois's arguments regarding the problem of having two anniversaries for Zoroastre's death being 8 months apart, mentioned in *Zādspram* (chapter 25), the other passage that he quotes from *Zādspram* (chapter 34), and finally, the reference he makes to *Dinkard* [de Blois 1996, 43] are consistent with Kūshyār's clear description that after each intercalation the first month of the year shifted to the next one, so that the months drifted slowly through the seasons but the epagomenai always kept trace of the vernal equinox (e.g., before 375 A.Y. the year began with Ādhar-māh, but the vernal equinox was at the beginning of Farwardīn-māh). Kūshyār's description of the arrangement of the *Jāhanbārs* also confirms that a calendar reform took place in 375 A.Y. that followed the intercalation system of the pre-Islamic Iranian calendar (see Chapter 6 and its commentary). For a recent discussion of the subject that confirms the intercalation system mentioned by al-Bīrūnī and Kūshyār, see [Ghasemlou 2003, 825-26].

Even after the advent of Islam the Persian solar calendar was used in Iran beside the Hejira lunar calendar until the 5th/11th century. In the year 471 A. H./1079 C.E., the Jalālī or Malikī calendar was constituted. In this calendar the years began with the vernal equinox based on astronomical observation or calculation.

The modern version of the Persian names of the months as mentioned by Kūshyār in this chapter has been used in the formal Iranian calendar since 1925. In this calendar, the year begins with Farvardīn; the first six months have 31 days, the next five months have 30, and the last month, Esfand has 29 days in normal years and 30 days in leap years. The leap years usually occur every four years, but sometimes they are five years apart. This is determined by the exact moment of the vernal equinox being before or after local solar noon on the 29th of Esfand. The 1st of Farvardīn is the first day whose noon is after the exact time of the vernal equinox.

I.1.3 The lengths of Syrian and Arabian years are $21915{:}60=365\frac{1}{4}$ and $21262{:}60=354\frac{11}{30}$ days, respectively.

By "completed" years and months, Kūshyār means those which have passed. An "incomplete" year or month refers to a year or month which has not yet been completed. So, when we are in the month m of the year y of any calendar, m-1 completed months and y-1 completed years have passed from the beginning of the era. The month m and the year y themselves are incomplete.

The results of Section I.1.3 are used in Section I.1.4.

I.1.4 The Syrian date is based on the Seleucid era. The following chapter gives the method of determining the weekday for any date in each of the calendars. These methods can be used for checking the correctness of a date conversion from one calendar to another.

I.1.5 The second method for finding the weekday of a date in the Syrian calendar is based on the fact that 28 times 365.25 (days) is a multiple of 7. In table 4 of Book II of the *Jāmiʿ Zīj*, the weekdays of the first day of any Syrian month for the years 1 to 28 are given directly. Then it will be easy to find the weekday of any date in a given month. The weekdays are shown in the table in the conventional *abjad* numbers from 0 to 6, corresponding to Saturday, Sunday,..., Friday, respectively. This allows us to convert the final remainder into weekdays directly, because the Arabic names for Sunday up to Thursday are derived from the Arabic words for 'one' to 'five', respectively.

The second method for finding the weekday of a date in the Arabian calendar works because 210 times $354\frac{11}{30}$ is a multiple of 7. Table 5 of Book II of the *Jāmi' Zīj* is in two parts: In one part, the weekdays of the first day of the years 1 to 210 are listed. The other part displays the weekdays of the first day of the 12 Arabian months (assuming 0 for the first month, because its beginning is the same as the beginning of the year).

The method for the Persian years is valid because 365 is a multiple of 7, plus 1. For any month we add 2 days, because $30 = 4\times7+2$. We do not add anything for Ādhar-māh, because with the five epagomenae Abān-māh has 35 days, which is a multiple of 7. Table 6 of Book II gives the number (0 to 6) corresponding to the weekday of the beginning of each Persian month for each remainder r (1 to 7) of the number of years y of the Yazdigird era, if $y = 7k+r$ for an integer k.

Examples:

The weekday of the first day of Tishrīn I of the Syrian year 1359 is found as follows:

1358 (completed years)$\times 21{,}915 \div 60 \approx 496{,}009$

$496{,}009+1 = 496010 = 7\times70858+4$

The fourth day counting from the epoch Monday is Thursday. So the desired weekday is Thursday.

If we want to use table 4 of Book II, we proceed as follows:

$1359 = 28\times48+15$

The table entry for 15 (remainder of the Syrian year) is 5, which corresponds to Thursday.

The weekday of the first day of *Ramażān* of the year 439 of the Hejira era is found as follows:

438 (entire years)$\times 21{,}262 \div 60 \approx 155{,}213$

The number of the months from the beginning of the year to the first of *Ramażān* is $4\times30+4\times29=236$, and we add one for inclusion of the desired day itself:

$155{,}213+236+1=155{,}450=22207\times7+1$

The first day counting from the epoch Thursday is Thursday itself. So, the desired weekday is Thursday.

If we want to use table 5 of Book II, we proceed as follows:

$439=210\times2+19$

The table entry for 19 (remainder of the Arabian year) is 0, and the table entry for *Ramażān* is 5. Since 5+0=5, the corresponding weekday is a Thursday.

The weekday of the first day of *Mihr-māh* of the year 416 of the Yazdigird era is found as follows:

$416=59\times7+3$

The third day counting from the epoch Tuesday is Thursday. So, the weekday of the beginning of the year is a Thursday. Now, since *Mihr-māh* is the 7th month of the Persian year, we add 12 for the six preceding months:

$3+12=15=2\times7+1$

The first day counting from the epoch Tuesday is Tuesday itself. So, the weekday of the beginning of *Mihr-māh* is Tuesday. In table 6 of Book II, the entry corresponding to r = 3 and *Mihr-māh* is 3, which corresponds to Tuesday.

I have taken these examples from the treatise *al-Lāmi' fī amthilat al-Zīj al-jāmi'* ("Explanation of the examples of the *Jāmi' Zīj*") by Abu'l-Ḥassan 'Alī b. Aḥmad al-Nasawī mentioned in the introduction of this dissertation. Al-Nasawī's calculation (fols. 51r-52r) shows some insignificant differences with what I have provided above because he made a mistake in finding the weekday of the beginning of *Tishrīn I* of the year 1359 of the Syrian era by calculation. Note that all three examples are for the years 1047-8 C.E., the time of composition of al-Nasawī's commentary.

I.1.6 The modern equivalents and the meanings of these feasts are as follows:

NAME	EQUIVALENT	MEANING
Syrian:		
Mā'althā	Presentation of Christ	
Subbār	Annunciation	
Mīlād	Christmas	Birth of Christ
Dinḥ	Epiphany	
Ṣaum al-'adhārā (Ghayṭās)		The Fast of the Virgins
Ṣaum Naynawī		The Fast of Nineveh
'Īd al-haykal	Wax Feast	The Feast of the Temple
Al-Ṣaum al-kabīr	Lent	The great Fast
Sha'ānīn	Palm Sunday	

Al-Sha'ānīn al-ṣaghīra		The lesser Sha'ānīn
Fiṭr	Easter	Fast-breaking
Sullāq	Ascension day	
Finṭīqusṭī	Pentecost, Whitsunday	
Ṣaum al-Salīḥīn		Fast of the Apostles
Ṣaum Mārt Maryam		Fasting for the illness of Mary
Ẓuhūr al-Masīḥ		Advent of Christ
Fiṭr Maryam		Fast-breaking in commemoration of Mary's death
'Īd al-ṣalīb		Feast of the Cross
Suqūṭ al-jimār		Falling of pebbles
Ayyām al-'ajūz		Days of the old woman
Nayrūz al-Mu'tażid		Mu'tażid's New Day
Ayyām al-bāḥūr	Dog days	

Arabic:

'Āshūrā'	The 10th day of Muḥarram
Maulid al-Nabī	Birth of the Prophet
Yaum al-jamal	The day of the Camel Battle
Mab'ath al-Nabī	Appointment day of the Prophet
Mi'rāj	Ascension day of the Prophet
Laylat al-ṣakk	The great Liberation night
Ṣaum	Fasting
Fatḥ Makka	Conquest of Mecca
'Īd al-Fiṭr	Feast of fast-breaking
Al-Tarwīya	Watering
'Arafa	Recognition
'Īd al-ażḥā	Feast of Immolation

Persian:

Ghadīr Khumm		Khumm pool
Nayrūz	Pers. *Nowrūz*	New Day
Al-Nayrūz al-khāṣṣa		*Nayrūz* of the nobility
Al-Mihrajān al-khāṣṣat al-ṣaghīra		The lesser specific Mihrjān
Katb al-ruqā'		Charms against scorpions
Jāhanbārs	Pers. *Gāhanbār-hā*	Seasonal feasts

The calculation of Lent by means of Kūshyār's tables is explained in [Saliba 1970, 197-98]. The explanation for *Ayyām al-bāḥūr* in parentheses in the translation is taken from al-Bīrūnī, whose account is clearer [1934, 184]. All of the feasts and fasts mentioned by Kūshyār are also described by al-Bīrūnī [1879, 199-334; 1934, 174-186; 1954-1956, I, 238-270] whose account is more complete and gives a more extensive explanation for each case. Since al-Bīrūni dedicated his *Chronology* to Qābūs in 390 A.H/999-

1000 C.E., it is highly probable that Kūshyār made use of it. In fact, he repeats the mistakes made by al-Bīrūnī (see below). In only a few cases he gives different data.

Thus Kūshyār says that first of Āb is called *Ṣaum Mārt Maryam*. But according to al-Bīrūnī [1879, 296; 1954-1956, I, 242] this is the *Ṣaum maraż Maryam* ("Fasting on account of the illness of Mary"), and he puts *Ṣaum Mārt Maryam* on the Monday that follows *Subbār* [1879, 310; 1954-1956, 245]. Kūshyār says that the *Ayyām al-bāḥūr* are eight days beginning on the 19th of Tammūz. Al-Bīrūnī's account in *al-Tafhīm* [1934, 184] is the same as Kūshyār's, but in [1879; 268; 1954-1956, I, 270] al-Bīrūnī says that they are seven days beginning on the 18th of Tammūz.

Al-Bīrūnī [1879, 329; 1954-1956, 256] puts *Yaum al-jamal* on the 3rd of Jumādā I. Only in ms. C of the *Jāmi' Zīj* it is mentioned to be on the 15th of Jumādā I. Other mss. do not mention it at all. According to Kūshyār (as found in all mss. that contain Book I), *Fatḥ Makka* ("the Conquest of Mecca") was on the 20th of Ramażān, but al-Bīrūnī [1879, 330; 1954-1956, 256] puts it on the 19th of Ramażān.

Al-Bīrūnī [1879, 214] calls the feast on the 22nd of Bahman *Bād-rūz* instead of Kūshyār's *Wādhīra*. Also instead of *Gāgīl*, we read *Kākthl* and *Kāvkīl* in al-Bīrūnī [1879, 212; 1954-1956, 260].

Each *Jāhanbār* (Persian *Gāhānbār*, lit. "The feasts of the [six] times [of creation]") consists of five days and Kūshyār defines their beginnings. Al-Bīrūnī's account of the beginnings of the six *Jāhanbārs* [1879, 204, 205, 207, 210, 212, 217; 1954-1956, 259-60] is different from Kūshyār's. The dates according to al-Bīrūnī are as follows: I) 11th of Day-māh, II) 11th of Isfandārmadh-māh, III) 26th of Ardībahisht-māh, IV) 26th of Tīr-māh, V) 16th of Shahrīwar-māh, VI) the five 'stolen days' at the end of Abān-māh. There is a shift of two in the numbers of the *Jāhanbārs* between Kūshyār and al-Bīrūnī. Kūshyār puts the 6th *Jāhanbār* at the end of Isfandārmadh-māh and al-Bīrūnī puts it at the end of Abān-māh. Zoroastrian sources are not consistent in this regard [Taqizadeh 1938, 11] and there were different accounts of the beginnings of the *Jāhanbārs*. Kūshyār's account matches with an old Pahlavi text *Āfaringān Gāhanbār* and with the calendar reform of 375 A.Y., and his system is now used by the Zoroastrians [Taqizadeh 1937, footnotes of pp. 18-10].

Most of the feasts listed by Kūshyār (and al-Bīrūnī) are still celebrated, but not always on the same dates. In the present lithurgical calendar of the Syrian Orthodox Church *Mā'althā* is celebrated on February 2nd as the presentation of Christ at the Temple of Jerusalem. Kūshyār's description for *Mā'althā* is valid for the present feast Sanctification of the Church, which corresponds to *'Īd al-haykal*. The latter falls on a Sunday in late October or early November. Kūshyār confused these two feasts with each other. The first Sunday of the Advent now falls on the 28th of November if it is a Sunday; otherwise it is the next Sunday. Kūshyār mentions this as *Subbār*. However, at present *Subbār* is celebrated on March 25th. *Ṣaum Maryam* now begins on the 10th of August, and ends at the date given by Kūshyār (the 15th of August). The fast of the Apostles is now celebrated on June 26th-29th, while the corresponding fast in Kūshyār's account, *Ṣaum al-Salīḥīn*, was on the Monday after Pentecost, so depended on Easter.

Nayrūz al-Mu'tażid was actually a Persian feast, but it was adjusted with the Syrian date 11th of *Hazīrān* (June) [cf. al-Bīrūnī 1934, 185-86]. *Ayyām al-'ajūz* and *Soqūṭ al-jimār* are Arabian occasions but defined by the solar (Syrian) dates. Al-Bīrūnī says that, according to the Greeks, *Ayyām al-bāḥūr* (Dog days) are connected with the (heliacal) rising of the Dog-star of Orion, i.e., Sirius [see al-Bīrūnī 1934, 183].

The Arabian feasts have mostly been preserved up to now, because they are actually connected to Islamic occasions and rituals. However, their importance (manifested in being a formal holiday or not) is not the same in different Islamic countries and among different sects. Also their exact dates are not always agreed unanimously. *Ramażān* (the month of fasting) and *'Īd al-Fiṭr* (the feast of fast breaking), as well as the occasions connected with the Prophet, i.e., *Maulad al-Nabī* (his birth), and *Mab'ath al-Nabī* (his appointment), and those connected with *Ḥajj* (pilgrimage to Mecca), i.e., *'Arafa* (recognition) and *'Īd al-ażḥā* (immolation), are evenly important in all the Islamic world. *'Ashūrā* and *Ghadīr Khumm* are of particular importance in Shi'ism.

In present Iran, *Nowrūz* (in Arabic *Nayrūz*) is celebrated as the most important formal national feast on 1-4 Farvardīn (usually 21-24 March). *Mihrgān* (in Arabic *Mihrajān*) now falls on the 10th (and not 16th) of *Mihr* because each of the first six Iranian months now have 31 days (not 30 days). *Sadeh* (in Arabic *Sadaq*) still falls on the 10th of *Bahman*. Its name is derived from the Persian word *sad* or *ṣad* which means "hundred", because on this day 50 days plus 50 nights remain until *Nowrūz* [Cf. Bīrūnī 1934, 182; 1954-1956, 260]. The latter two feasts are still remembered and celebrated on a limited level, but not as formal holidays. *Gāhanbār-hā* (in Arabic *Jāhanbārāt*) as well as *Mihrgān* and *Sadeh*, are regarded as important national and religious feasts among the Zoroastrians who also celebrate other old Iranian feasts.

Section 2: On Sines and Chords, <in> 6 chapters
Chapter 1: On introduction to the knowledge of the Sine <function>.

The Sine is a rule (i.e., function) referred to in finding the magnitudes of all arcs. The greatest Sine, i.e. half the diameter of a circle, can be supposed <to be divided into> any <number of > parts, but the easiest and most comprehensive <method> for calculation is <supposing it> to consist of 60 <parts>. The Cosine of an arc is the Sine of its complement to 90 degrees, as in the case of the Cosine of 36 <degrees> by which is meant the Sine of 54 <degrees>, and the Cosine of 54 <degrees> by which is meant the Sine of 36 <degrees>. We shall content ourselves with <calculating> the Sines of the degrees of a quadrant, because, for any <arc> exceeding the quadrant, the Sine is the same as <the Sine of> the degrees of the quadrant <counted> backward from 90 to 1. So, the Sine of 91 <degrees> is equal to that of 89 <degrees>, the Sine of 92 <degrees> is <equal to> the Sine of 88 <degrees>, and so on until the Sine vanishes at 180 <degrees>. Then, we begin a second time, according to the first description up to 360 <degrees>.
The Sagitta of an arc reaches 120 degrees, and this is the diameter of the circle. In calculations, whenever we say that a certain <number> is multiplied by another <number>, lowered, or a certain <number> is divided by another <number>, lowered, we mean that we lower this <second> number by one <sexagesimal> place; so, if it is in degrees, we take it as minutes, and if it is in minutes, we take it as seconds, and so on.
After this introduction, <I say that> the exact value of the Sine of 1 degree cannot be found, but its approximate value has been investigated, so that there is no difference between it and the exact value in <arithmetical> operations. I have derived it by <detailed> investigation to be <equal to> 1;2,49,38,31. We shall explain how to calculate it in the chapter on <its> proof.
The calculation of the Sine of <the arcs> greater than <one> degree is easy, and it should be preceded by an introduction on how to know the Cosine of the arcs whose Sines are known. To calculate it, you subtract the square of the known Sine from the square of the greatest Sine, and take the square root of the remainder. The result is the Cosine of the arc with known Sine. By this calculation, the Cosine of 1 degree, which equals the Sine of 89 <degrees>, is found to be 59;59,27,6,12,39.
If we want <to find> the Sines of other degrees, we multiply the Sine of the preceding degree by the Cosine of 1 degree, lowered, multiply the Sine of 1 degree by the Cosine of the preceding degree, lowered, and add the products to find the desired Sine of the degree. Example: We want <to find> the Sine of 24 <degrees>. We multiply the Sine of 23 <degrees> by the Cosine of 1 degree lowered. Then we multiply the Sine

of 1 degree by the Cosine of 23 <degrees> lowered, and add the two products to find the Sine of the desired degree, i.e. the Sine of 24 <degrees>. The Sine of 24 <degrees> is obtained not <only> through 1 and 23 <degrees>, but through any pair of numbers whose sum is 24, and the calculation is the same as for 1 and 23, like 10 and 14, 12 and 12, 18 and 6, and so on.

Chapter 2: On interpolation between two lines of the Sine <table> and other tables.

In all tables, the ratio of the difference between two <successive> values of the argument to the difference between the two <corresponding> entries of the table equals the ratio of <any> part of the difference between the two values of the argument to the <corresponding> part of the difference between the two entries of the table. Thus there are four proportional numbers: the difference A between two <successive> values of the argument; the difference B between two <corresponding> entries of the table; <any> part C of the difference between the two <successive> values of the argument; <the corresponding> part D of the difference between the two entries of the table. The desired unknown E is either the part of the difference between the two entries of the table, or the part of the difference between the two values of the argument. If the part of the difference between the two entries of the table is desired, we multiply the known part c of the difference between the two values of the argument by the difference b between the two entries of the table, and divide it by the difference a between the two values of the argument. If the part of the difference between the two values of the argument is desired, we multiply the known part d of the difference between the entries of the table by the difference a between the two values of the argument and divide it by the difference b between the two entries of the table. Thus the desired unknown will be obtained. Then, if it should be added to the entry in the table or value of the argument, we add it; and if it should be subtracted, we subtract it.

Difference A between arguments in two <successive> rows	Difference B between entries in two <successive rows>
Partial diff. C betw. arguments in two <successive> rows	Partial diff. D betw. entries in two <successive> rows

<figure>

Chapter 3: On <finding> the Sine of a <given> arc and the arc of a <given> Sine from the table.

If we want <to find> the Sine of a given arc, we enter the arc in the row of the arcs, which is the row of the arguments, and take the Sine corresponding to it, <and we correct it, if necessary> according to what has been already said about interpolation. If we want <to find> the arc of a given Sine, we enter the Sine in the table and take the arc corresponding to it, <and we correct it, if necessary> according to what has been said on interpolation.

Chapter 4: On <finding> the Sagitta of a <given> arc and the arc of a <given> Sagitta from the <special> table and from the Sine table.

For the Sagitta a table has been presented from which the Sagitta of a <given> arc and the arc of a <given> Sagitta can be taken in the same way that the Sine of a <given> arc and the arc of a <given> Sine are taken from the Sine table. If we want <to find> the Sagitta of an arc from the Sine table, we look at it: If the arc is less than 90 <degrees>, we subtract it from 90 <degrees>, take the Sine of the remainder, and subtract it (i.e., the Sine) from 60. If the arc is greater than 90 <degrees>, we subtract 90 <degrees> from it, take the Sine of the remainder, and add 60 to it (i.e. the Sine). If we want to <find> the arc of a <given> Sagitta, we look at it: If the Sagitta is less than 60, we subtract it from 60, take the arc of the remainder from the Sine table, and subtract it (i.e., the arc) from 90 <degrees>. If the Sagitta is greater than 60, we subtract 60 from it, take the arc of the remainder, and add it (i.e., the arc) to 90 <degrees>.

Chapter 5: On <finding> the Chord of a <given> arc and the arc of a <given> Chord from the Sine table.

We do not need any of these Chords in this book; we mention them <just> to complete the <list of> operations. If we want <to find> the Chord of a <given> arc, we halve the arc, take its Sine, and double it. If we want <to find> the arc of a <given> Chord, we halve the Chord, convert it to an arc <by means of the Sine table>, and double it.

Chapter 6: On correcting the Sine whenever we have some doubt about any <value> of it.

The calculation of the Sine table, and the checking of its correctness have <already been> finished; so we do not need to repeat anything about it and its calculation. However, if we doubt <the exactness of> the Sine of any degree, we look at it: If half the <number of the> degrees is an integer, we take its (i.e. this) half, multiply its Sine by its Cosine lowered, and double the result. This result will be the Sine of the degree about which there was some doubt. Example: <Let us suppose> we doubt the correctness of the Sine of 24 <degrees>. We multiply the Sine of 12 <degrees> by the Sine of its complement, 78 <degrees>, lowered, and double the result. <Thus> the Sine of 24 <degrees> will be obtained.

If there is no integer half for <the number of> these degrees, we take two arcs the sum of which equals <the number of> these degrees. Then we multiply the Sine of the smaller arc by the Cosine of the greater arc lowered, multiply the Sine of the greater arc by the Cosine of the smaller arc lowered, and add the results. It will be the Sine of the degree about which there was some doubt. Example: <Let us suppose> we doubted <the correctness of> the Sine of 25 <degrees>. Of the many pairs of arcs whose sum equals 25 <degrees>, let us take 10 <degrees> and 15 <degrees>. Then we multiply the Sine of 10 <degrees> by the Sine of 75 <degrees> lowered, multiply the Sine of 15 <degrees> by the Sine of 80 <degrees> lowered, and add the results. Then the Sine of 25 <degrees> will be obtained. If we find the Sine of 24 <degrees> also according to this computation and this example, it will be correct, but that <other> method for even numbers is easier.

Commentary

I.2.1 Kūshyār follows Ptolemy (*Almagest* I.10) in taking the radius of the circle to be equal to 60 units. Thus Kūshyār's Sine and Cosine functions are 60 times the modern sine and cosine functions, respectively. This has the consequence that Kūshyār often has to divide the product of two Sines by 60. Since he is working in the sexagesimal system, the division by 60 is easy. For dividing by 60, he uses a special term "lowered" (*munḥaṭṭan*), because division by 60 corresponds to change of one sexagesimal position. See e.g. the commentary to I.2.6.

Based on the value of the Sine of 1 degree as mentioned by Kūshyār, I have computed the Cosine of 1 degree as 59; 59,27,6,12,38.72. In the text, the sixth sexagesimal digit is rounded to the nearest integer (39).

The fifth sexagesimal digit is given as 57 (instead of 12) in the manuscripts C, L, and B (the other manuscripts do not contain this fragment). A scribe must have misread the *abjad* numeral يب (12) as نز (57).

Kūshyār gives the Sine of 1° to five sexagesimal digits, while he computes the Cosine of 1 degree up to the sixth sexagesimal digit. We can assume that Kūshyār computed Cos 1° according to $Cos 1^\circ = \sqrt{(R^2 - Sin^2 1^\circ)}$ with $R = 60$. Then the error in Cos 1° can be shown to be approximately 1/60 of the error in Sin 1°. So, Kūshyār's method can be justified. However, L and B give the sixth sexagesimal digit of the Sine of 1 degree as 0, but a sixth sexagesimal digit is not found in C.

The modern value of the sine of 1 degree is 0.017452406..., which corresponds to the Sine of 1 degree equal to 1;2,49,43,11,... Thus Kūshyār's value is correct to 3 sexagesimal digits. His value corresponds to sin 1°= 0.017452046..., which represents the correct value up to the 5th decimal digit. Starting from this smaller Sine, he actually finds a greater Cosine. His result, 59;59,27,6,12,39, corresponds to 0.999847701, whereas the correct values of Cos 1° and cos 1° are 59;59,27,6,7,45,... and 0.999847695..., respectively. The calculation method is presented in IV.1.11, where Kūshyār describes how to calculate the Chord of 1 degree. The same method is applicable for deriving the Sine of 1 degree.

At the end of this chapter, he uses the formulae

$Sin^2 (x) + Cos^2 (x) = 60^2$, and

$Sin(x+1)=Sin(x)Cos(1)/60+Sin(1)Cos(x)/60$,

to find the Sines of arcs greater than 1 degree.

Kūshyār's Sine table in II.8 is quoted in the manuscript of Yaḥyā b. Abī Manṣūr's *Zīj al-Ma'mūnī al-mumtaḥan* [1986, 101] by a scribe who lived after Kūshyār.

I.2.2 Here Kūshyār discusses two problems: 1) If n<x<n+1, find Sin(x); 2) If Sin(n)<y<Sin(n+1), find x so that Sin(x)=y. For these two purposes, he uses linear interpolation. Thus he assumes that in very small intervals, the increase in Sin(x) is proportional to the increase in x. The Arabic term for argument (independent variable) in the text is the same term for "number": *ʿadad*. For the entry (function or dependent variable) he simply uses the term *jadwal* ("table"). For the "value" of an argument or entry, he uses the word *saṭr* ("line"). Kūshyār uses the Arabic expression *taʿdīl bayn al-saṭrayn* (adjustment between two <consecutive> values) for the process of interpolation.

I.2.3 The word *saṭr* literally means "row", but Kūshyār apparently uses the term *saṭr* in the more general sense of "line". In the Sine tables, arcs are usually written in a column, as is the case in F, Y, and B (and quoted in Yaḥyā's *zīj* mentioned in the commentary of I.2.1). However, the arcs may also be given in a row at the top of the table, as in the detailed Sine tables of L and B (see also the figure in Chapter I.2.2).

I.2.4 Here the Sagitta of any given arc is defined as Sag(x)=60-Cos(x), and the arc of any given Sagitta is discussed. Table II.9 in F is for the Sagittae. We also find it in Y, L, and B. Al-Bīrūnī [1934, 5] and Ḥabash Ḥāsib [in his *zīj*, MS Berlin Ahlwardt 5750 (WE. 90), fol. 80v] call this function as *al-jayb al-maʿkūs* ("Versed Sine"). Al-Bīrūnī [1934, 4] uses the term *sahm* ("Sagitta") as "the line between the middle of a chord and the middle of the corresponding arc", and mentions that the Versed Sine of an arc is equal to the Sagitta of its double.

I.2.5 This chapter is about the Chords of arcs, which are related to Sines and modern sines by Chord(x)=2Sin(x/2)=120sin(x/2). There is no table for Chords in the *Jāmiʿ Zīj*, whereas the table of Chords is the only plane trigonometrical table given by Ptolemy (*Almagest* I.11).

I.2.6 Here Kūshyār provides a method to check the correctness of any entry in the Sine table, using the formulae

$$\mathrm{Sin}(2x)=2\mathrm{Sin}(x)\mathrm{Cos}(x)/60,$$

and

$$\mathrm{Sin}(x+y)=[\mathrm{Sin}(x)\mathrm{Cos}(y)+\mathrm{Cos}(x)\mathrm{Sin}(y)]/60.$$

At the end of *Almagest* I.10, Ptolemy describes similar ways of checking the correctness of the value of a suspected Chord with different methods.

Section 3: On Tangents and Cotangents, <in> 3 chapters

Chapter 1: On calculating the Tangent and Cotangent, their two Hypotenuses (i.e., Secant and Cosecant) and their two arcs.

The Tangent (lit. "the First Shadow") is obtained from gnomons parallel to the horizon plane; it is called the Reversed Shadow. This is the one that we have included in the table for the elementary calculations. The Cotangent (lit. "the Second Shadow") is taken from gnomons perpendicular to the horizon plane; it is called the Horizontal Shadow. This is the one which we have included it in the table for knowing <the length of shadows in> digits and feet at mid-day (see the commentary of the next chapter). It is <also> provided in the calendars.

The gnomon can be supposed <to be divided by> any <number of> parts, however, the easiest <method> in the elementary calculations is <supposing it> to consist of 60 parts. Therefore, we have taken <the values of> the Tangent based on the gnomon being <divided into> 60 parts, and the Cotangent based on the gnomon being <divided into> 12 digits or 7 feet. If the gnomons have identical divisions, the Tangent of any arc is <equal to> the Cotangent of the complement of that arc (i)[1]. If any number is multiplied by the Tangent of any arc, <lowered>, or divided by the Tangent of the complement of the arc, <lowered>, the results> are equal; the product and the quotient are the same (ii).

The Secant (lit. "the Hypotenuse of the Shadow") is the line connecting the tip of the gnomon and the end of the shadow. The arc of a Tangent is the arc of the altitude <angle>, which increases and decreases <depending> on the shadow of the gnomons.

After this introduction, <I say that> if we want the Tangent of an arc, we divide the Sine of the arc by the Cosine of the arc, lowered. The result is the Tangent, based on a gnomon <of> 60 parts (iii). If we want its Secant, we divide the Tangent by the Sine of the arc, lowered: The result is the Secant (iv). If we want <to use another method>, we add the square of the Tangent to the square of the gnomon, and we take its square root (v). If we want the arc of the Tangent <and we have no Tangent table>, we divide the Tangent by its Secant, lowered: The result is the Sine of the arc (vi). If we want the Cotangent of an arc, we divide the Cosine of the arc by the Sine of the arc, lowered: The result is the Cotangent, based on a gnomon <of> 60 parts (vii). If we want its Cosecant, we divide the Cotangent by the Cosine of the arc, lowered: The result is the Cosecant (viii). If we want <to use another method>, we add the square of the Cotangent to the square of the gnomon and we take its square root (ix). If

[1] See the commentary on this chapter, where the corresponding modern formulas are presented referring to the Roman numbers in this translation.

we want the arc of the Cotangent, we divide the Cotangent by the Cosecant, lowered: The result is the Cosine of the arc (x).

Chapter 2: On <finding> the Tangent of a <given> arc and the arc of a <given> Tangent from the table.

If we want the Tangent of an arc, we take <the entry> opposite to the arc, from the Tangent tables, as already mentioned in <the chapter on> the Sine. If we want the arc of a Tangent, we take the arc opposite to the Tangent <from the tables>.

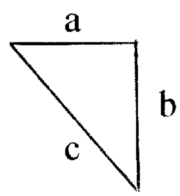
Section: We render the arc of the Tangent in the table up to 45 degrees, because, if it exceeds 45 <degrees>, the difference between <the values in> two <consecutive> lines will be great and the operation will not be correct, except potentially (i.e., theoretically). If we want to multiply any number by the Tangent of an arc, <lowered>, the arc being greater than 45 <degrees>, we divide the number by the Tangent of the complement of the arc <,lowered>. If we want to divide a number by the Tangent of an arc <,lowered>, the arc being greater than 45 <degrees>, we multiply the number by the Tangent of the complement of the arc <, lowered>. The number here is either a Sine or the Tangent of an arc less than 45 <degrees>. However, multiplication of the Tangent of an arc by the Tangent of another arc, both greater than 45 <degrees>, or division of the Tangent of an arc greater than 45 <degrees> by a number, <can> not <be carried out by this method>. In this case, <the operation> will be limited to <using> the Sine and what derives from it, without using the Tangent.

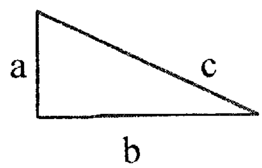
Chapter 3: On converting Tangents to different gnomons.

The ratio of the <number of the> parts <into which> a gnomon <is divided> to the parts of <another> gnomon is like the ratio of a Tangent to <another> Tangent. These are four proportional numbers. We take the <parts of the> gnomon of the known Tangent as the first <number> *A*; <the parts of> the gnomon of the unknown Tangent as the second <number> *B*; the known Tangent as the third <number> *C*, and the unknown Tangent as the fourth <number> *D*. We multiply the second <number> by the third, and divide it by the first <number>: the fourth <number> results. In case of digits and feet, if we multiply the digits by 35 minutes, they turn into feet, based on <taking> the gnomon <divided into> 7 parts (i.e., feet). If we divide the feet by 35 minutes, they turn into digits, based on <taking> the gnomon <divided into> 12 parts (i.e., digits).

Commentary

I.3.1 In modern terminology, the Tangent (Arabic *ẓill*, lit. "shadow") of an arc h is R tg h. It was also called the First or Reversed (Arabic *ma'kūs*) Shadow. The Cotangent of an arc h is R cotg h. It was also called the Second or Horizontal (Arabic *mustawī*) Shadow. For the Tangents, Kūshyār takes $R = 60$, and for the Cotangents, he takes $R = 12$ or $R = 7$. For R/cos h and R/sin h, Kūshyār uses the terms *quṭr al-ẓill al-awwal* ("The Hypotenuse of the First Shadow") and *quṭr al-ẓill al-thānī* ("The Hypotenuse of the Second Shadow"). I have used the modern terms Secant and Cosecant for them.





a = gnomon
b = Tangent (Tg)
(First Shadow, Reversed Shadow)
c = Secant (Sec)
(Hypotenuse of the First Shadow)

a = gnomon
b = Cotangent (Cotg)
(Second Shadow, Horizontal Shadow)
c = Cosecant (Cosec)
(Hypotenuse of the Second Shadow)

Kūshyār uses the term *miqyās* for gnomon. According to al-Bīrūnī [1976, I, 64], the term *miqyās* (lit. "scale, measure") is used for gnomon (*shakhṣ*), especially in calculations.

In this chapter, Kūshyār describes the relations corresponding to the following modern formulas:

(i) $\text{Tg}\, h = \text{Cotg}\,(90^\circ - h)$
(ii) $a\,\text{Tg}\, h = a / \text{Tg}\,(90^\circ - h)$
(iii) $\text{Tg}\, h = \text{Sin}\, h / (\text{Cos}\, h / R)$
(iv) $\text{Sec}\, h = \text{Tg}\, h / (\text{Sin}\, h / R)$
(v) $\text{Sec}\, h = (\text{Tg}^2\, h + R^2)^{\frac{1}{2}}$
(vi) $\text{Sin}\, h = \text{Tg}\, h / (\text{Sec}\, h / R)$
(vii) $\text{Cotg}\, h = \text{Cos}\, h / (\text{Sin}\, h / R)$
(viii) $\text{Cosec}\, h = \text{Cotg}\, h / (\text{Cos}\, h / R)$
(ix) $\text{Cosec}\, h = (\text{Cotg}^2\, h + R^2)^{\frac{1}{2}}$
(x) $\text{Cos}\, h = \text{Cotg}\, h / (\text{Cosec}\, h / R)$

Kūshyār mentions all these rules for $R = 60$. In the mss. C and Y there are additional notes for calculation of the Cotangents when R is equal to

12 digits or 7 feet: "If we want the Cotangent of an arc based on <dividing> the gnomon into 12 digits or 7 feet, we multiply the Cosine of the arc by the <number of the> parts of the gnomon and divide it by the Sine of the arc: the Cotangent will be obtained. If we want its Cosecant, we multiply the Cotangent by the <number of the> parts of the gnomon and divide it by the Cosine of the arc; or we add the square of the <number of the parts of the> gnomon to the square of the shadow, and we take its square root. If we want its arc, we multiply <the number of> the parts of the gnomon by the Cotangent and divide it by the Cosecant; the result will be the Cosine of the arc." These rules correspond to the rules for $R = 60$.

In his treatise on *Shadows*, al-Bīrūnī frequently refers to the contents of this chapter of Kūshyār's *Jāmiʿ Zīj*. For example, he says [1976, 93]: "What Kūshyār proposes for dividing the Cosine of the altitude by the Sine of the altitude, lowered, is exactly what he (al-Nayrīzī) does .... And Abu al-Wafā' proceeded like him, except that he did not lower it, for he had assumed the gnomon to be one".

I.3.2 Kūshyār provides the values of 60 tg h in table II.10 for each degree of h from 1 to 45, with the differences between any two consecutive entries. In table II.11, entitled "Cotangents or Horizontal Shadows for knowing the mid-day Shadows", he gives the values of 12 cotg h and 7cotg h for each degree of h from 1 to 90 degrees.

I don't know why in the "Section" appended to this chapter, Kūshyār requires that "the number here is a Sine or the Tangent of an arc less than 45 degrees".

Al-Bīrūnī [1976,104] says that Kūshyār gave the values of the Tangent for the arcs up to one eighth of a revolution (i.e., 360°), because the tabular differences of the Tangent values for the arguments beyond 45 degrees are so great that the Tangent calculated (by interpolation) can hardly be correct.

I.3.3 Al-Bīrūnī says [1976, 82] that Kūshyār, in his *Jāmiʿ Zīj*, converts a sexagesimal Tangent into other units, through multiplication by the number of the parts into which the gnomon is divided, and division by 60. This is what we read at the beginning of this chapter in a general form. Kūshyār's rules for converting digits and feet into each other are valid because 7/12=35/60=0; 35.

Section 4: On <finding> the true longitudes of the planets, and their situations, <in> 12 chapters

Chapter 1: On mentioning the epoch values and preliminaries for <finding> the mean longitudes of the planets.

We scrutinized the ancient <astronomical> observations as well as the new ones made in al-Ma'mūn's time and later, we examined and checked them through the conjunctions and the meridional altitudes <of the celestial bodies>, and studied each of them exhaustively, after having abandoned our passion, and avoided <personal> inclination to one side, and having abandoned racial and national partisanship, for <many> years. We found that the observations <made> by Muḥammad b. Jābir al-Ḥarrānī, better known as al-Battānī, were the most correct of all, with the least defect and inconsistency, and the closest to our time. <We also found that> its author <had developed the> most precise <point of> view, and <had> examined most thoroughly the observations which he had made. In many things that can be found by observation, he relied on Ptolemy's observations. This is <an evidence that> he (i.e., al-Battānī) <is> most inclined to veracity and fondest of truth. For these reasons, his observations are the most reliable ones, although observations are not (i.e., never) free from inconsistencies. He made observations in <various> localities in Syria, relying however on his observations in Raqqa. He composed a *zīj* in which he provided the true longitudes of the planets for the Syrian and Arabian eras. Using these two eras together with the Persian era is difficult, because they have leap <years> and fractions <for the number of days of the year>, and <there is> difference in <the number of> the days of the months. We have converted the epoch values of the mean longitudes to the Persian era. <By doing this,> we made easier the operation of finding the true longitudes, and we corrected the defect which we found in the composition and presentation of some equations <in the tables>. The description <of our procedure> will be provided in the Book on proofs. If any difference is found between the true longitude of a planet based on this *zīj* and based on the *zīj* of al-Battānī, it is due to the improvement in its equation. This mostly <occurs> in <the case of> Mars, and the difference reaches <a few> degrees. As to the other planets, it is negligible, and in <the case of> the sun and the moon, it does not occur. We subtracted from the epoch values of the mean longitudes <found> for Raqqa the motion <of the respective celestial body> in 1 hour and 7 minutes, so that they (i.e., the tabular mean longitudes) can be based on a <geographical> longitude 90 <degrees> from the Canary Islands, and are <thereby> clearer in their layout, and easier in accessibility. The mean longitudes <corresponding to> the <geographical> longitude difference are always additive <if the

other locality is> between the West (i.e., the Atlantic coast) and the longitude of 90 <degrees>. The Canary Islands are situated off the Atlantic coast. Ptolemy says that they were inhabited in ancient times. <The distance> between them and the coast is 10 degrees of the revolution of the sphere, i.e., two thirds of an hour.

Chapter 2: On deriving the mean longitudes from the tables.

If we want this, we take the Yazdigird (i.e., Persian) years including the year, month and day <for> which we want <to find the mean longitude>. Then we enter with <the number of> the years in the table of the multiple years, and take the mean longitude which corresponds to the closest number which is less than it and write it down on the board. We then take <the mean longitude> corresponding to the remainder of the years in the table of the single years. Then we take what corresponds to the month and the day. We add all that: <the result> will be the mean longitude for the noon of this day at the <geographical> longitude of 90 <degrees>. Then we adjust it by the equation of longitude, as we shall mention later. If there are entire hours left after noon, we take the corresponding <value> in the table of hours. If there are fractions with the hours, and the fractions are in minutes, we take the corresponding <value> in the table of hours, lowered once. If the fractions are in seconds, we take the corresponding <value>, lowered twice, and so on.

Chapter 3: On converting the mean longitudes from <localities having> one <geographical> longitude to another.

We have already said that these mean longitudes have been calculated for the localities whose <geographical> longitude is 90 <degrees east> of the Canary Islands in the Atlantic. We should convert it (i.e., the mean longitude) to the <geographical> longitude of the locality where we are, so that the true longitude <of the planet> comes out correctly. If we want <to do> this, we take the difference between the <geographical> longitude of our locality and the <geographical> longitude of 90 <degrees>. We take one hour for each 15 degrees of difference, and 4 minutes of an hour for each degree. The result is the hour difference between the two <geographical> longitudes. If the <geographical> longitude of our locality is less than 90 <degrees>, we add the hour difference between the two <geographical> longitudes to the given time. If the <geographical> longitude of our locality is greater than 90 <degrees>, we subtract the hour difference between the two <geographical> longitudes from the given time. The sum or the remainder is the time adjusted for the <geographical> longitude difference. On this basis, we derive the mean longitudes for our locality. We find the <geographical> longitudes of the localities, by taking them from the table compiled for them, or by deriving them by calculation, as we shall mention in Chapter 19 of Section 6.

Chapter 4: On the positions of the apogees and the nodes, and <on> their motions.

The positions of the apogees for the beginning of the Yazdigird era are: the sun: Gemini 18; 31°; Saturn: Sagittarius 0;45°; Jupiter: Virgo 10;45°; Mars: Leo 3;15°; Venus: Gemini 18;31°; Mercury: Libra 17;44°. Their motion is one entire cycle in 24,000 solar years, that is 54 seconds in each year. If we want to adjust <the apogee values>, we take the Yazdigird years elapsed after the (i.e., any) known adjusted apogee, and subtract from it one tenth of it. What remains is the motion of the apogee in minutes. If we wish so, we <may> obtain their motions from the tables composed for them and we add them to their previously adjusted positions.

As for nodes, we do not need anything about them in this book, save their positions at the beginning of the Yazdigird era <i.e.,> Saturn: Cancer 10:45°; Jupiter: Cancer 0;45°; Mars: Taurus 3;15°; Venus: Pisces 18;31°; Mercury: Capricorn 17;44°. Their motions follow those of the apogees. For deriving their positions: we subtract 50 degrees from the apogee of Saturn, then we subtract 90 degrees from the remainder; we add 20 degrees to the apogee of Jupiter and subtract 90 degrees from the sum; we subtract 90 degrees from the apogees of Mars and Venus and 90 degrees from the <point> opposite to the apogee of Mercury. The result is the position of the nodes for this <given> time.

Chapter 5: On the equation of time.

There is a special adjustment for the time for which the true longitudes of the two luminaries are found, which is known as the "equation of time". If we want this, we subtract 10 signs and 16 degrees from the mean longitude of the sun at the <given> time. The remainder is <called> "the result of the mean longitude". We <also> subtract 10 signs, 22 degrees and 4 minutes from the right ascension of the true longitude of the sun. The remainder is <called> "the result of the right ascension". Then we take the excess of the result of the mean longitude over the result of the right ascension, and multiply it by 4. Then we take the degrees as minutes and the minutes as seconds. <The result> is the equation of time in minutes of an hour. We subtract it from the time adjusted for the difference between the two <geographical> longitudes. <The result> becomes the time adjusted for the equation of time. Another method: We add 6 degrees and 4 minutes to the mean longitude of the sun, and take the difference between it and the right ascension of its true longitude; we multiply <the result> by 4 and make it "lowered", i.e., we lower the place of degrees into minutes, minutes into seconds, and seconds into thirds. The result will be the equation of time in minutes of an hour and parts of the minutes of an hour. We always subtract it from the time adjusted for the difference between the two <geographical> longitudes. The result will be the time adjusted for the equation of time. <Based> on this calculation, we have compiled a table in which we have written the mean longitudes of the sun and, opposite to them, the equation of time in minutes and seconds of an hour, so that we do not need to find the true longitude of the sun twice. <The table is> based on <taking> the apogee in Gemini 24°. The motion of the apogee does not affect this equation sensibly, except in long time intervals. There is no need at all to apply this equation for finding the true longitudes of the five planets.

Chapter 6: On the true longitude of the sun.

We write the mean longitude of the sun in two positions and we subtract the adjusted apogee for the <given> time from one of the <numbers put in the two> positions. The result is the adjusted mean anomaly. We take the equation opposite to it <from the table>, and we interpolate it. Then we always add it (i.e., the equation) to the mean longitude, and the result is the true longitude <of the sun>.

Chapter 7: On the true longitude of the moon and its node<s>.

We write the mean longitude, the mean anomaly, and the double elongation. Then we take the first equation corresponding to the double elongation, and always add it to the mean anomaly. The result is the true anomaly (i.e., the adjusted position of the moon on the epicycle). Then we take the second equation corresponding to it, and keep it in mind. Then we take the difference <in epicyclic equation> at the lesser distance <of the epicycle center> corresponding to the double elongation and the sixtieths corresponding to the true anomaly. We multiply them one by another, and divide the product by 60. The result is the adjusted difference <in epicyclic equation>. If the true anomaly appears in the upper <part> of the table for the sixtieths, we add the adjusted difference to the second equation; and if the true anomaly appears in the lower <part> of the table for the sixtieths, we subtract the adjusted difference from the second equation. Then we always add this sum or remainder of the <second> equation to the mean longitude. The result is the true longitude <of the moon>. For all planets <and the moon>, the true anomaly is the same as the adjusted mean anomaly.

<For finding the true longitude of> **the** <ascending> **node**, we subtract its "mean longitude" from a <complete> rotation (i.e., 360°), and what remains is the true longitude of the ascending node. The descending node is always in opposition to the position of the ascending node.

Chapter 8: On the true longitude of the five planets.

We write the mean longitude and the mean anomaly, and <then> we subtract from the mean longitude the adjusted apogee for the <given> time. The remainder is the mean centrum. We take the first equation corresponding to it, and we always add it to the mean centrum and subtract it from the mean anomaly. The result of adding it to the mean centrum is the adjusted centrum. The remainder from the mean anomaly is the true anomaly (i.e., the adjusted position of the planet on the epicycle), and we take the second equation corresponding to it, and bear it in mind. Then we take the difference <in epicyclic equation> at greater or lesser distance <of the epicycle center from the earth> -whichever we <may> find- corresponding to the adjusted centrum, and the sixtieths corresponding to the true anomaly. We multiply them one by another, and divide <the product> by 60. The result is the adjusted difference. If the true anomaly appears in the upper <part> of the table for the sixtieths, we add the adjusted difference to the second equation. If the true anomaly appears in the lower <part> of the table for the sixtieths, we subtract the adjusted difference from the second equation. We always add this sum or remainder of the <second> equation to the adjusted centrum. We add to this sum the apogee; the result is the <desired> true longitude <of the planet>.

Section. This is not the true adjusted centrum, because it is according to the displacement of the equations in this *zīj*. If we want <to obtain> the true <adjusted centrum> in order to use it for the latitudes and the determination of the stations for the retrogradations and direct motions, we add to it 7 degrees for Saturn, 12 degrees for Jupiter, 47 degrees for Mars, 48 degrees for Venus, and 26 degrees for Mercury.

Chapter 9: On the latitude of the moon.

We subtract the true longitude of the node from the true longitude of the moon, or we add the "mean longitude" of the node to the true longitude of the moon. The remainder or the sum is the 'argument of the latitude'. Then we take the latitude corresponding to it. If the argument is less than 3 <zodiacal> signs, then the latitude is northerly, ascending, and

increasing. If it is greater than 3 and less than 6 <zodiacal signs>, then the latitude is northerly, decreasing, and descending. If it is greater than 6 and less than 9 <zodiacal signs>, then the latitude is southerly, descending, and increasing. If it is greater than 9 <zodiacal signs> up to an entire rotation, then the latitude is southerly, ascending, and decreasing.
Its calculation: We subtract the true longitude of the node from the true longitude of the moon. The remainder is the argument of the latitude. We multiply its Sine by the Shadow of the maximum latitude, lowered. The result is the Shadow of the latitude. The maximum latitude is 5 degrees.

Another method: We multiply the Sine of argument of the latitude by the Sine of the maximum latitude, lowered. The result is the Sine of the latitude of the argument. Then we multiply the Cosine of the argument by the Sine of the maximum latitude, lowered. The result is the Sine of the latitude of the complement of the argument. We find its arc, and obtain its Cosine, and divide by it the Sine of the latitude of the argument, lowered. The result is the Sine of the latitude. All people in the art <of astronomy> shorten this calculation, multiplying the Sine of the argument of the latitude by the Sine of the maximum latitude, lowered. They believe that the result is the Sine of the latitude. However, this is, not the Sine of the latitude of the moon, but the Sine of an arc close to the latitude <of the moon>.

Chapter 10: On the latitudes of the five planets.

The superior planets: We take the true adjusted centrum mentioned at the end of Chapter 8 of this section. For Saturn, we add 50 degrees to it; for Jupiter, we subtract 20 degrees from it; For Mars, we leave it as it is. Then we enter with it in the two rows of numbers (i.e., arguments), and take the corresponding <number of the> 'proportional minutes for the latitude', and write it down. If the centrum appears in the upper half of the two rows of numbers, we take the northern latitude of the planet corresponding to the true anomaly. If the centrum appears in the lower half, we take the southern latitude of the planet corresponding to the true anomaly. We multiply the resulting quantity by the 'proportional minutes for the latitude'. The result is the latitude of the planet in the direction which was found.

Venus and Mercury: We take the inclination and slant corresponding to the true anomaly, and we write down each of them separately. If for Mercury, specifically, the adjusted centrum appears in the upper half of the two rows of numbers, we subtract from its slant one-tenth of it. If it appears in the lower half, we add to its slant one-tenth of it. The result is the slant to be used, different from the initial one, and we keep it in mind. Then we add to the true adjusted centrum of Venus 3 <zodiacal> signs, and to that of Mercury 9 <zodiacal> signs. We use this for taking the proportional minutes for the latitude. We multiply it (i.e., the number of the proportional minutes for the latitude) by the inclination. The result is the first latitude <component>, which is the inclination of the epicycle. If the augmented centrum and the true anomaly both appear in the same half of the two rows of numbers, the first latitude <component> is southerly. If their positions are <in> different <halves>, the first latitude <component> is northerly. Then we take the true adjusted centrum for Venus as it is, and for Mercury <the true adjusted centrum> plus 6 <zodiacal> signs. We take the <number of the> proportional minutes for the latitude corresponding to it, and write it in two positions. We multiply <the quantity in> one <of these> positions by the slant. The result is the second latitude <component>, which is <called> the 'deflected latitude'. If this centrum from which we took the proportional minutes falls in the upper half and the true anomaly is less than 6 <zodiacal> signs, then the second latitude <component> is northerly, <but> if the true anomaly is greater than that, then it (i.e., the second latitude component) is southerly. If the centrum falls in the lower half, and the true anomaly is less than 6 <zodiacal> signs, then the second latitude <component> is southerly, <but> if the true anomaly is greater than that, the second latitude <component> is northerly. Then we take the proportional minutes written in the other position. We multiply them by 10 minutes for Venus, and by 45 minutes for Mercury. The result is the third latitude <component>, which is the inclination of the eccentric orb. For Venus, it is always northerly, and for Mercury, always southerly. Among these three latitude components, we add those in the same direction. When they have different <signs>, we subtract the smaller from the greater one, and we find the direction of the result (i.e., the direction of the greater one). That is the latitude of the planet in the direction which resulted.

Ascension and descension: We calculate the latitude for 10 days later. If it is northerly at the first <given date> and increases in the second <date>, it is ascending; if it decreases at the second <date>, it is descending. If it is southerly at the first <given date> and increases at the second <date>, it is descending; if it decreases at the second <date>, it is ascending. If it is northerly at the first <given date>, and southerly at the second <date>, it is 'descending in the north'. If it is southerly at the first <given date>, and northerly at the second <date>, it is 'ascending in the south'. The maximum latitude is <as follows>: for Saturn 3;2° northern, 3;5° southern; for Jupiter 2;5° northern, 2;8° southern; for Mars 4;21° northern, 7;50° southern; for Venus 6;22° in both directions; for Mercury 4;5° in both directions.

Chapter 11: On the retrogradation of the planets, their direct motion, and first and last visibility.

We take the first equation corresponding to the centrum, and keep it in mind. We add the mean <motion of the planet in> longitude for one day to the centrum, and take its <first> equation again. We subtract the smaller equation from the greater one. If the equation is additive, we add the difference to the mean <motion in> longitude for the day; if it is subtractive, we subtract <the difference from the mean motion in longitude for the day>. The remainder or the sum is the adjusted mean <motion in> longitude for the day. Then we take the second equation corresponding to the true anomaly, and bear it in mind. We add the mean <motion in> anomaly relating to one day to the true anomaly and take its <second> equation again. We subtract the smaller equation from the greater one. The remainder is the difference of the equation of the day. If the difference is less than the adjusted mean <motion in> longitude for the day, the planet is <in> direct <motion>. If it is greater, then the planet is retrograde. If it is equal to it (i.e. the adjusted mean motion in longitude for the day), the planet is stationary <before> retrogradation or direct motion.

Another method: We enter with the adjusted centrum in the table for the first station <column of the> table, and take the corresponding entry. We subtract the first station from a complete rotation (i.e., 360°). The remainder is the second station. Then we look at the true anomaly: If it is less than the first station and greater than the second station, then the planet is <in> direct <motion>. If it is greater than the first station and less than the second station, then the planet is retrograde. If it is equal to the first station, then it is stationary <before> retrogradation. If it is equal to the second station, it is stationary <before> direct motion. If <the difference> between them is a few degrees, we divide it by the daily <motion in the> anomaly of the planet. The result is the period of time until the planet retrogrades or since its retrogradation, or until it moves directly, or since it has moved directly. The daily <motions in> anomaly of the planets <are as follows>: For Saturn 0; 57°; for Jupiter 0;54°; for Mars 0; 28°; for Venus 0; 37°; for Mercury 3; 6°. We have written the retrogradation, direct motion, first and last visibility in their approximate positions in the table for the second equation. We take any of these situations (i.e., being in direct motion, stationary, or retrograde) corresponding to the true anomaly <from the table for the second equation>. If <the difference> between the true anomaly and one of these situations is a few degrees, we divide it by the daily <motion in> anomaly of the planet as we have mentioned before. The result is the period of time until it retrogrades, or since its retrogradation, or until it moves directly, or since it has moved directly, or until its apparition, or since its apparition, or until its last visibility, or since its last visibility. If the planet is seen rising before sunrise, it is 'eastward', and if it is seen setting after sunset, it is 'westward'. The limit of orientality (being eastward) and occidentality (being westward) for the superior planets is 60 degrees, for Venus 47 degrees, and for Mercury 26 degrees. This is their (i.e., the inferior planets') maximum elongation. The combustion of the superior planets <occurs> approximately in the middle of the days of their direct motion. Their opposition to the sun <occurs> approximately in the middle of the days of their retrogradation. The combustion of Venus and Mercury is approximately in the middle of the days of direct motion and the middle of the days of retrogradation.

Chapter 12: On the ascension and descension of the planets in their spheres.

The ascension and descension are meant to be <ascension and descension> in the <relevant> zones in the spheres of the apogee (i.e., the eccentric orb) and of the epicycle. On the sphere of apogee <is> the center of the epicycle, and on the epicycle <is> the body of the planet. The zones in the sphere of the apogee corresponding to <the positions of> the center are written in the table of the first equation. The zones in the epicycle corresponding to the true anomaly <are written down> in the table of the second equation. If the center and the true anomaly are found between the maximum and mean distances in the order of the <zodiacal> signs, then the center of the epicycle or the body of the planet on the epicycle is descending from maximum to mean distance. <If they are> between the mean and minimum distances, <they are> descending from mean to minimum distance. <If they are> between the minimum and the second mean distance, <they are> ascending from minimum to mean distance. <If they are> between the mean and the maximum distance, <they are> ascending from the mean to the maximum distance. As to the ascension of the planet and its descension, i.e., the ascension of the planet itself in the sphere of the apogee and its descension in it, they are clear if the position of the apogee is known.

Commentary

The term *aḥwāl* ("situations") in the title of this section refers to the apparent motion of the planets that may be direct, stationary, or retrograde (see Chapter 11).

I.4.1 According to Kūshyār, the geographical longitudes of Jurjān and Raqqa with respect to the Canary Islands are 90;0° and 73;15°, respectively (table II.54). By dividing their longitude difference, 16;45° by 15 degrees/hour, one obtains 1 hour and 7 minutes, in accordance with the time interval which Kūshyār uses to convert the positions of the celestial bodies from the local time at Raqqa into that at Jurjān.

Al-Battānī mentioned in his *Zīj al-Ṣābī* [1899-1907, III, 7] that he found Ptolemy's *Almagest* most reliable and followed it in his work.

I.4.2 In table II.13, the mean longitude of the sun is given to the nearest second for the years 1, 21, 41, ..., 581 of the Yazdigird era, and its motion is given for 1, 2, 3,..., 20 Persian years, 40, 60, 80, 100, 200, 300, 400, 500 Persian years, the 12 Persian months from Farwardīn-māh to Esfandārmadh-māh, 1, 2, 3,..., 30 days, and for 1, 2, 3,..., 60 hours. In table II.17 the same functions are given for the moon with the same precision. The same functions, to the nearest minutes are also given for the five planets: Saturn in II.22; Jupiter in II.25; Mars in II.28; Venus in II.31; and Mercury in II.34. For minutes and seconds of an hour, the relevant entries for the hours are lowered once or twice.

I.4.3 The planetary mean longitudes in this *zīj* are given for the geographical longitude 90°. In his table II.54 for the geographical coordinates of the localities, only Jurjān has this geographical longitude, and in I.1.2, he mentions Jurjān as a locality in which he lived. So, the tables must have been composed for Jurjān. Kūshyār says that for the localities west or east of the geographical longitude 90°, we add or subtract 1 hour for each 15 degrees, and 4 minutes for each degree of longitude difference. Thus the tables for the mean longitudes of the celestial bodies can also be used in other localities. For each celestial body, in tables II.13, II.17, II.18, II.19, II.21, II.22, II.23, II.25, II.26, II.28, II.29, II.31, II.32, II.34 and II.35, he provides in a final column the positive or negative corrections for the geographical longitudes 71, 72, 73,..., 100°.

I.4.4 In the margin of table II.12 (preliminaries for mean longitudes), Kūshyār gives these positions of the apogees for the beginning of the Yazdigird era. There he says that they are taken from the *zīj* of al-Battānī.

Kūshyār gives their motion, which is the same as the precession of the equinoxes, as one complete rotation per 24,000 years, or 54 seconds per year. The modern values are 25,770 years and 50.29 seconds respectively. Hipparchus, who described this motion first, evaluated it as "not less than 1° per 100 years", and Ptolemy as 1° per 100 years [Ptolemy 1984, 328]. The Mumtaḥan astronomers and al-Battānī had already improved the Ptolemaic value to 1° per 66 years [van Dalen 2006,]. Kūshyār computes this motion by subtracting one-tenth of the elapsed years in the Yazdigird era, and he takes the result as the motion of the apogee in minutes of arc. This computation is valid, because 0;0,54° = 0;1°.(1- 0.1). The motions may also be taken directly from table II.14.

I.4.5 Here Kūshyār provides two equivalent methods for finding the equation of time. In modern notation they are as follows:

$$E_t(\bar{\lambda}_d) = 0;4[(\bar{\lambda}_d - 10^s16°) - (\alpha(\lambda) - 10^s22;4°)], \text{ and}$$

$$E_t(\bar{\lambda}_d) = 0;4[(\bar{\lambda}_d + 6;4° - \alpha(\lambda)].$$

Here $E_t(\bar{\lambda}_d)$ is the equation of time, $\bar{\lambda}_d$ the displaced mean longitude (see below) of the sun, λ the true longitude of the sun, and $\alpha(\lambda)$ the right ascension of the sun, for the given time. In both methods, Kūshyār applies "lowering" or "division by 60", however, he describes it in two different ways. In table II.15, Kūshyār gives the values of equation of time for each six degrees of mean longitude of the sun, with a column of increments for each degree that may be used for interpolations. This is the original form of the table, which is found in mss. F and Y; in mss. B and L, the entries are given for each degree of the argument. This chapter (in a rather abridged form) and the table are appended to the manuscript of Yaḥyā b. Abī Manṣūr's *Al-zīj al-Ma'mūnī al-mumtaḥan* [1986, 121-22], where Kūshyār's table is similar to the version found in B and L. Of course, since Kūshyār lived after Yaḥyā, this chapter had been added by someone who prepared the manuscript based on Yaḥyā's work, some time after Kūshyār composed his *Jāmiʿ Zīj.*

Prof. E. S. Kennedy recomputed Kūshyār's table for equation of time by computer [1988, 4]. In Y and L, there is an extra phrase to the effect that the 16° implied in the first method was originally 18°, and that Kūshyār subtracted 2° from it. Kūshyār used "displaced" mean solar longitudes, which were 2° less than mean longitudes, to avoid negative values for the solar equation [Van Dalen 1993, 138-139; 1996, 236-238]. As Dr. Benno van Dalen remarked [1993, 140], the 10^s 16° in the formula is close to the value of the displaced mean solar position for which the equation of time assumes its minimum value, and the 10^s 22°4' is approximately equal to the right ascension of the former solar position added to the displaced solar equation 3;38,21° to result the true solar position in the given time. See also the commentary on IV.4.1.

As Kūshyār says at the end of this chapter, since the argument of the table is mean longitude, there is no need to find the true longitude twice. If we have a table for the equation of time as a function of the true longitude, we in principle need to carry out an iteration, since the true longitude for the time adjusted for the equation of time is a little different from the true longitude for the original time. Then we would have to carry out the whole calculation again for the newly found time. See also the commentary to IV.4.1.

I.4.6 In table II.16 in four pages, the values of the solar equation are precise to the nearest second for each degree of anomaly. This table also includes columns of tabular differences to be used in interpolation.

I.4.7 In the relevant theory in the *Almagest*, the first equation is for converting mean anomaly into true anomaly. The second equation is the epicyclic equation at apogee. The difference in epicyclic equation accounts for the increment of maximum epicyclic equation (for any elongation). The sixtieths determine the portion of this increment to be applied for arbitrary true anomaly. Kūshyār's method of finding the true longitude of the moon is ultimately based on *Almagest* V.9 [1984, 237-39]. However, in the *Almagest* the increment in epicyclic equation is found as a function of the true anomaly and the sixtieths are found as a function of the double elongation, whereas we find the inverse in Kūshyār's *zīj*. This is due to an interesting innovation of Kūshyār who applies a different interpolation process for adjustment of the second equation. As demonstrated by Glen Van Brummelen [1998] for the planets, Kūshyār's attempt was conscious. His method is simpler and less accurate, but shows that Kūshyār "was no mere copyist".

Since the revolution of the nodes is opposite to the order of the zodiacal signs, "the mean longitudes" (or merely "the longitudes", because the motion is uniform) given in table II.21 are subtracted from an entire cycle. Among different methods of tabulating the longitude of the nodes (with positive or negative motions), Kūshyār chose to tabulate the supplement of the longitude. So, "the mean [longitude]" here actually means "the supplement of the longitude".

I.4.8 This method is also found in *Almagest* XI.1 [1984, 554]. But in Kūshyār's *zīj*, the first equation is always added to the mean centrum and subtracted from the mean anomaly, whereas in the *Almagest*, this is only for arguments from 180 to 360 degrees. For arguments less than 180 degrees, the equation is subtracted from the mean centrum and added to the mean anomaly. A similar difference is encountered in the calculation of the second equation. This is due to the displacement method used by Kūshyār in order to avoid negative values for the equation [Van Brummelen 1998, 268-270]. The first equation is the equation of centrum. The second equation is the equation of anomaly for the mean distance [Pedersen 1974, 279-294]. Here again, contrary to Ptolemy's procedure, Kūshyār uses the adjusted centrum and the true anomaly as arguments for obtaining the difference relating to greater or lesser distance (depending on the planet's position being between the mean distance and the apogee or the perigee), and the sixtieths, respectively. The reason is the same as in the case of the moon, i.e., application of a different interpolation process by Kūshyār [Van Brummelen 1998].

The tabular values of the mean centrum in the table for the first equation are not really the mean centrum, because Kūshyār shifts the mean centrum in order to compensate the above-mentioned displacement of the first and the second equation. For example, in the case of Mars, since he adds 12° to all tabular values of the first equation and 47° to all values of the second equation, he subtracts 59° from each tabular value of the mean centrum (to which the two equations are to be added). By adding the first equation, the result is

(mean centrum - 59°) + (first equation + 12°) =

(mean centrum + first equation) - (59° - 12°) = adjusted centrum - 47°

This is why Kūshyār adds certain amounts to the resulting centrum of each planet in order to obtain its real value. Of course, in the process of finding the true longitude of the planet, e.g., Mars, the remaining 47° is implicitly added later, in the calculation of the second equation [Van Brummelen 1998, 270].

I.4.9 The two methods for finding the argument of latitude are equivalent (see I.4.7). Note that the "mean longitude" of a node means a complete rotation minus its true longitude. Kūshyār provides a compact table for it in II.37. There is a similar table for the latitude of the moon in al-Battānī's *zīj* [1899-1907, II, 78-83, column 7]. Kūshyār's first method for calculating the latitude of the moon is:

$$tg\beta = \sin a_{\beta} tg\beta_m$$

where β is the latitude of the moon, α_β is the argument of latitude, and β_m is the maximum latitude of the moon. He also provides another method, i.e.,

$$\sin\beta' = \sin a_\beta \sin\beta_m \ , \ \sin\beta'' = \cos a_\beta \sin\beta_m \ , \ \sin\beta = \sin\beta' / \cos\beta''$$

where the auxiliary parameters β' and β'' are called 'the Sine of the latitude of the argument' and 'the Sine of the latitude of the complement of the argument', respectively. The second method can be derived from the first in the following way (not mentioned by Kūshyār):

$$\sin^2\beta=(1+\cot g^2\beta)^{-1}=(1+1/\sin^2\alpha_\beta tg^2\beta_m)^{-1}=\sin^2\alpha_\beta tg^2\beta_m/(1+\sin^2\alpha_\beta tg^2\beta_m)=$$
$$\sin^2\alpha_\beta\sin^2\beta_m/(\cos^2\beta_m+\sin^2\alpha_\beta\sin^2\beta_m)=\sin^2\alpha_\beta\sin^2\beta_m/[1-\sin^2\beta_m(1-\sin^2\alpha_\beta)]$$
$$=\sin^2\alpha_\beta\sin^2\beta_m/(1-\sin^2\beta_m\cos^2\alpha_\beta)=\sin^2\beta'/\cos^2\beta''$$

Kūshyār mentions the fact that some astronomers take β' as the latitude of the moon, whereas it is not equal to the latitude of the moon, but an approximation to it (cf. IV.4.8). Maybe Kūshyār is criticizing al-Battānī who did this [1899, III, 113]. Al-Battānī provides a method equivalent to the modern formula $\sin\beta=\sin\alpha_\beta\sin\beta_m$, which is the correct approach and produces Kūshyār's β'. It seems that Kūshyār misunderstood the geometrical concept of "argument of latitude" as meant by al-Battānī (see the commentary to IV.4.8). In Kūshyār's method the final result is divided by $\cos\beta''$. So he overestimates $\sin\beta$ by a factor whose maximum value is 1/cos 5°≈ 1.00382. Thus the maximal error in the calculation of $\mathrm{Sin}\,\beta$ is 0.382 percent and the maximum absolute error in Kūshyār's computation for β is about 1 minute and 9 seconds. Table II.37 gives the latitude of the moon, in degrees and minutes, for each degree of the argument of latitude.

I.4.10 The methods for finding the latitudes of the superior and inferior planets are taken from *Almagest* XIII.6 [1984, 635-36]. As remarked by Toomer in a footnote to his translation of the *Almagest* [1984, 635], the amounts to be applied to the true adjusted centrum of the superior planets represent the (rounded) distance between the apogee and the northpoint of the inclined orb. The tabular entries of *daqā'iq ḥiṣaṣ al-'arż* are interpolation coefficients used for calculating the latitudes of all planets. They should not be confused with the quantity *ḥiṣṣat al-'arż* (argument of latitude) used for the moon.

In the case of Mercury, the addition and subtraction of one tenth of the tabular value of the slant, relates to the fact that the maximum slant is taken as 2;30° in the table. However, Ptolemy found that the maximum slant actually differs from 2;30° by 13' in the negative direction at the apogee and by 16' in the positive direction at the perigee. He takes a middle value 15' or 1/4° for both these differences. Since 15' is one-tenth of 2;30°, he adds or subtracts one-tenth of each tabular value of the slant of Mercury [Ptolemy 1984, 630].

An explanation of the procedure for finding the third latitude component of the inferior planets is given by O. Neugebauer [1975, I, 224]. See also the commentary to IV.4.9.

I.4.11 The first method is a direct approach in which Kūshyār compares the motion in longitude and the epicyclic equation; so, in fact he computes the difference in true longitude. The second method is found in al-Battānī's *zīj*, chap. 46 [1899-1907, III, 173]. Ptolemy discusses it in detail in *Almagest* XII 7 & 8 [1984, 583-88]. Kūshyār gives the position of the first stations of each planet in degrees and minutes for each six degree of the argument in the same table for its latitude (tables II.38 to II.42). The different situations of each planet (direct motion, stagnation, retrogradation, occultation, and emergence west or east of the sun) depending on the value of the true anomaly are mentioned in the table for its second equation (tables II.24, II.27, II.30, II.33, II.36).

Kūshyār's definition of maximum orientality or occidentality for the superior planets (being 60° distant from the sun) is conventional, because they can have arbitrary distances from the sun. In this case, al-Bīrūnī assumes the maximum orientality (or occidentality) to be 30°. Then he calls the planet in the interval from 30° to 90° from the sun to be 'weakly oriental' (or 'weakly occidental'). Al-Bīrūnī also mentions that the lower limits of orientality (and occidentality) are conventional and the planets can be invisible after passing this limit [1934, 296-296a].

I.4.12 In this chapter, Kūshyār defines the meaning of the ascension and descension of the planets in their spheres, possibly because these may be interpreted in different ways [al-Bīrūnī 1934, 110-111]. Kūshyār's discussion of these terms is related to the variation of the distance of the planet from the earth between its maximum and minimum value.

Section 5: On the operations relating to the ascendants of the day and the night, <in> 22 chapters
Chapter 1: On the first declination.

We multiply the Sine of the degrees <of a point on the ecliptic> whose declination we want <to find> by the total declination, lowered: The result is the Sine of the first declination. Through successive observations, we found the total declination 23; 35°. Based on this calculation, a table for it (i.e., the first declination) has been compiled <in this *zīj*>.

Chapter 2: On the right ascensions of the <zodiacal> signs.

We divide the Cosine of the degrees <of a point on the ecliptic> of which we want <to find> the right ascension, by the Cosine of the <first> declination of the degrees, lowered: The result is the Cosine of the right ascension. We find the corresponding arc <from the Sine table> and subtract it from 90°.

Another method: We divide the Tangent of the degrees <of a point on the ecliptic> by the Tangent of the total declination: The result is the Sine of the right ascension of those degrees.

Another method: If <the values of> the second declination are known <by means of the relevant table>, then we find the arc corresponding to the first declination of those degrees in the table for the second declination. The result is the right ascension for those degrees. A table has been compiled for it.

Chapter 3: On the second declination.

We divide the Sine of the <first> declination of those degrees <of a point on the ecliptic> by the Cosine of the <first> declination of the complement of the degrees, lowered: The result is the Sine of the second declination.

Another method: We multiply the Sine of those degrees by the Tangent of the total declination, lowered. The result is the Tangent of the second declination, and its maximum <value> is <equal to> the maximum <value> of the first declination.

Another method: If <the values> of the right ascension are known <through the relevant table>, then we find the arc corresponding to the degrees in <the table for> the right ascensions: The result is the inverse of the right ascension. We take its first declination: <The result> is the second declination for those degrees. A table has been compiled for it.

Chapter 4: On the distance of the stars from the celestial equator.

If the latitude of the star and the second declination of its degree <on the ecliptic> are in the same direction, we add them; if they are in different <directions>, we subtract the lesser from the greater, and <on this basis> we know the direction of the remainder. Then we multiply its Sine by the Cosine of the total declination and divide it by the Cosine of the second declination obtained for the degree of the star (i.e., for its ecliptical longitude): The result is the Sine of the distance of the star from the celestial equator. Its direction is the <same> direction that we have found. This distance of the star is similar to the first declination of the sun.

Chapter 5: On the latitude of <any> locality.

We obtain the maximum <value> of the altitude of the sun on any day by one of the altitude <measurement> instruments. We know the <first> declination of the degree of the sun (i.e., of its ecliptical longitude). If the <first> declination is northern, we subtract it from the maximum <value> of the altitude. If it is southern, we add it to the maximum <value> of the altitude: The result is the complement of the latitude of the locality. Should <the result> become more than 90°, we subtract it from 180°. The remainder is the complement of the latitude of the locality.

Chapter 6: On the ortive amplitude of the sun and the star<s>.

We divide the Sine of the <first> declination of the degree of the sun (i.e., its ecliptical longitude), or the Sine of the distance of the star from the celestial equator, by the Cosine of the latitude of the locality, lowered: The result is the Sine of the ortive amplitude.

Another method: If half the day arc of the degree <on the ecliptic> or of the star is known, then we multiply the Cosine of the <first> declination of the degree <on the ecliptic> or the Cosine of the distance of the star from the celestial equator by the Sine of half the day arc of the degree <on the ecliptic> or of the star, lowered: The result is the Cosine of the ortive amplitude. We find the corresponding arc <from the Sine table> and we subtract it from 90°. Half the day arc is <discussed> in Chapter 10 of this section.

Chapter 7: On the equation of daylight of the sun and the star<s>.

We divide the Cosine of the ortive amplitude of the sun or the star by the Cosine of the <first> declination of the sun or the Cosine of the distance of the star from the celestial equator, lowered: The result is the Cosine of the equation of daylight.

Another method: We multiply the Sine of the <first> declination of the sun, or the Sine of the distance of the star from the celestial equator, by the Sine of the latitude of the locality, and divide it by the Cosine of the <first> declination or the distance. The result is <called> the 'base'. Then we divide the base by the Cosine of the latitude of the locality, lowered: The result is the Sine of the equation of daylight.

Another method: We multiply the Tangent of the <first> declination of the sun, or the Tangent of the distance of the star from the celestial equator, by the Tangent of the latitude of the locality, lowered: The result is the Sine of the equation of daylight. A table for the Tangent of the <first> declination has been compiled <in this *zīj*>.

Another method: For the degrees of the ecliptic if the equation of daylight for the first of Cancer or of Capricorn, i.e., the maximal equation of daylight is known: We multiply the Sine of the maximal equation of daylight by the Sine of the right ascension of the degree <of the ecliptic>, lowered: The result is the Sine of the equation of daylight of the degree <of the ecliptic>. A table for the equation of daylight for the latitude of 36° has been compiled <in this *zīj*>.

Chapter 8: On the ascensions for a locality (i.e., oblique ascensions).

<For> the northern degrees <of the ecliptic>, i.e., from the first of Aries until the end of Virgo, we subtract the equation of daylight <of the degree> from their right ascensions. <For> the southern degrees <on the ecliptic>, i.e., from the first of Libra until the end of Pisces, we add the equation of daylight <of the degree> to their right ascensions: The result is the oblique ascension of that degree for that locality. A table for the oblique ascensions of <the zodiacal signs for> the latitude of 36° has been compiled <in this *zīj*>.

Chapter 9: On the maximum altitude of the sun and the star<s>.

If the declination of the sun or the distance of the star from the celestial equator is northern, we add it to the complement of the latitude of the locality. If the declination or the distance is southern, we subtract it from the complement of the latitude of the locality. The sum or the remainder is the maximum altitude of the sun or the star. If the sum is over 90°, we subtract it from 180°. The remainder is the maximum altitude in the northern direction.

Chapter 10: On half the day arc of the sun and <any> star.

If the declination of the sun or the distance of the star from the celestial equator is northern, we add its equation of daylight to 90°. If the declination or the distance is southern, we subtract its equation of daylight from 90°. The sum or the remainder is half the day arc of the sun or the star.

Another method: We subtract the oblique ascension of the degree <of the ecliptic> from the oblique ascension of its opposite <degree>. The remainder is the day arc. If we subtract the day arc of the sun or the star from 360°, the remainder is the night arc.

Chapter 11: On the <equinoctial> day hours of the sun and the star<s> and the degrees of their <seasonal> hours.

We multiply the equation of daylight of the sun or the star by 8 minutes. Then, if the declination of the degree of the sun <on the ecliptic> or the distance of the star from the celestial equator is northern, we add it (i.e., the product) to 12 <hours>. If the declination or the distance is southern, we subtract it from 12 <hours>. The sum or the remainder is the <number of the equinoctial> hours of the daylight of the sun or the star.

We multiply the equation of daylight by 10 minutes. Then, if the declination or the distance is northern, we add it to 15 <degrees>. If the declination or the distance is southern, we subtract it from 15 <degrees>. The sum or the remainder is <the number of> degrees in one <seasonal> hour of the sun or the star.

Another method: We divide the day arc of the sun or the star by 15 <degrees>: The result is the <number of the> equinoctial hours of the day. We also divide it by 12 <hours>: The result is the <number of the> degrees in one seasonal hour of the day. If we subtract the <number of the equinoctial> hours of the day from 24, the remainder is <the number of> the hours of the night. If we subtract <the number of> the degrees in one <seasonal> hour of the day from 30, the remainder is <the number of> the degrees in one <seasonal> hour of the night.

If we add to <the number of> the equinoctial hours of the day one-fourth of it, the sum is <the number of> the degrees in one seasonal hour of the day. If we subtract from <the number of> degrees in one seasonal hour of the day one-fifth of it, the remainder is <the number of> the equinoctial hours of the day.

Chapter 12: On the <ecliptical> degree of the transit of a star through the meridian.

If the star has no (i.e., zero) latitude, the <ecliptical> degree of its transit is the same as its longitude. If it has <a non-zero> latitude, we multiply the Cosine of the latitude of the star by the Sine of the distance to the solstice nearest to it, either before it or after it, and we divide it by the Cosine of the distance of the star from the celestial equator: The result is the Sine of the adjusted distance from the solstice. We find the corresponding arc, and add it to the beginning of the <zodiacal sign of the> solstice if the star lies after it in the sequence of the <zodiacal> signs, and we subtract it from it (i.e., from the beginning of the zodiacal sign of the solstice) if the distance <of the star> from it is in the order opposite <to that of the zodiacal signs>: The result is the right ascension of the degree of transit <counted> from the beginning of Aries. We find the <ecliptical> arc corresponding to the right ascension: The result is that <ecliptical> degree which passes the meridian <simultaneously> with the star.

Chapter 13: On the <ecliptical> degree relating to the rising and setting of a star.

If the distance of the star from the celestial equator is northern, we subtract its equation of daylight from the right ascension of the degree of its transit. If the distance is southern, we add its equation of daylight to the right ascension of the degree of its transit: The result is the oblique ascension of the <ecliptical> degree that rises <simultaneously> with the star. We add the day arc of the star to the <right> ascension of the <ecliptical> degree of rising <simultaneously with the star>. We find the arc corresponding to the sum in <the table for> the oblique ascension. Then we take its opposite, and this is the <ecliptical> degree setting <simultaneously> with the star.

Chapter 14: On <finding> the arc of revolution of the celestial equator since the rising of the sun or the star<s> from the altitude of the <sun or the star at a given> time.

We multiply the Sine of the altitude of the <given> time by the Sagitta of half the day arc and divide it by the Sine of the maximum altitude: The result is <called> the "arrangement Sine" of the arc of revolution. We subtract it from the Sagitta of half the day arc. The remainder is the Sagitta of the excess of the arc of revolution. We find the corresponding arc which is the excess of the arc of revolution. If the altitude of the

<given> time is eastern, we subtract the excess of the arc of revolution from half the day arc. If the altitude is western, we add the excess of the arc of revolution to half the day arc: The result is the arc of revolution of the celestial equator.

Chapter 15: On <finding> the <elapsed> hours from the arc of revolution.

We divide the arc of revolution of the celestial equator by 15: The result is <the number of> the equinoctial hours <elapsed> since the rising of the sun or the star. We divide the arc of revolution by <the number of> degrees in the <seasonal> hours corresponding to the degree of the sun or the star: The result is <the number of> the seasonal hours since the rising of the sun or the star.

Chapter 16: On <finding> the ascendant from the arc of revolution during the day and at night.

We add the arc of revolution from the rising of the sun or the star to the oblique ascension of the sun or the oblique ascension of the <ecliptical> degree which rises <simultaneously> with the star: The sum is the oblique ascension of the ascendant. We find the corresponding arc in the table for the oblique ascensions, and thus the ascendant will be obtained.

Chapter 17: On <finding> the arc of revolution from the ascendant.

We subtract the oblique ascension of the sun or the oblique ascension of the degree which rises <simultaneously> with the star from the oblique ascension of the ascendant: The remainder is the arc of revolution of the celestial equator since the rising of the sun or the star.

Chapter 18: On <finding> the altitude of the <sun at a given> time from the arc of revolution.

We obtain the difference between the arc of revolution and half the day arc: The result is the excess of revolution. We subtract its Sagitta from the Sagitta of half the day arc: The result is the Sine of the altitude of the sun or the star at the <given> time corresponding to the given arc of revolution. We find the corresponding arc: It is the altitude.

Chapter 19: On <finding> the arc of revolution since sunset from the ascendant.

We subtract the oblique ascension of the degree opposite to the sun from the oblique ascension of the ascendant at the time of measurement: The remainder is the arc of revolution of the celestial equator since sunset.

Chapter 20: On <finding> the ascendant from the arc of revolution since sunset.

We add the arc of revolution of the celestial equator since sunset to the oblique ascension of the degree opposite to the sun: The sum is the oblique ascension of the ascendant. We find its <corresponding> arc in the table for the oblique ascensions: It is the ascendant.

Chapter 21: On a base <value> applying to most operations concerning day and night.

We multiply the Cosine of the declination of the degree of the sun by the Cosine of the latitude of the locality, lowered twice: The result is the base <value>.

<Finding> the arrangement Sine from the altitude of the <given> time: We divide the Sine of the altitude of the <sun at the given> time by the base value: The result is the arrangement Sine (i).

<Finding> the altitude from the arrangement Sine: We multiply the base value by the arrangement Sine of the arc of revolution: The result is the Sine of the altitude (ii).

<Finding> the Sagitta of half the day arc which is called the "day Sine": We divide the Sine of the maximum altitude by the base value: The result is the day Sine (iii).

<Finding> the meridian altitude from the day Sine: We multiply the base value by the day Sine: The result is the Sine of the meridian altitude (iv).

<Finding> the excess of the arc of revolution: We divide the difference between the Sine of the altitude of the <sun at the given> time and the Sine of the meridian altitude by the base value: The result is the Sagitta of the excess of the arc of revolution (v).

<Finding> the altitude of the <sun at the given> time from the Sagitta of the excess of the arc of revolution: We multiply the Sagitta of the excess of the arc of revolution by the base value. We subtract the remainder from the Sine of the meridian altitude: The remainder is the Sine of the altitude <of the sun> (vi).

<**Finding**> **the equation of daylight**: Half of the day arc is known from its Sagitta. The difference between half the day arc and 90° is the equation of daylight.

<**Finding**> **the arc of revolution of the celestial equator**: The excess of the arc of revolution is known from its Sagitta. Half of the day arc is <also> known from its Sagitta. If the altitude is eastern, we subtract the excess from half the day arc. If the altitude is western, we add the excess to half the day arc. The sum or the remainder is the arc of revolution of the celestial equator.

Chapter 22: On the equalization of houses.

We obtain the <number of the> degrees in the <seasonal> hours of the ascendant, double it, and keep it. We subtract this double from 60: The remainder is twice the <number of the> degrees in the <seasonal> hours of the degree opposite to the ascendant (i.e., the descendant). We keep it. Then we subtract 90° from the right ascension of the ascendant: The remainder is the right ascension of the tenth <house>. Then we write the right ascension of the ascendant in two positions. We subtract from one of the positions twice the <number of the> degrees in the <seasonal> hours of the ascendant. We add to the other <position> the <number of the> degrees in the <seasonal> hours of the degree opposite <to the ascendant>. The remainder is the right ascension of the twelfth <house> and the sum is the right ascension of the second <house>. We subtract the subtrahend from the remainder <again> and add the addend to the sum <again>: The result of the subtraction is the right ascension of the eleventh <house>, and that of the addition is the right ascension of the third <house>. We find the arcs corresponding to each of these right ascensions. The results are the <ecliptical> degrees of <the cusps of> the houses. Then, the fourth <house> is <diametrically> opposite to the tenth <house>; the fifth <house> is opposite to the eleventh <house>; the sixth <house> is opposite to the twelfth <house>; the seventh <house> is opposite to the ascendant; the eighth <house> is opposite to the second <house>; and the ninth <house> is opposite to the third <house>. If we want to check the operation to know if we have worked correctly or wrongly, we subtract from the right ascension of the eleventh <house> twice the <number of the> degrees in the <seasonal> hours of the ascendant, that we had subtracted, and we add to the right ascension of the third <house> twice the <number of the> degrees in the <seasonal> hours of the opposite <of the ascendant>. If the remainder is equal to the right ascension of the tenth

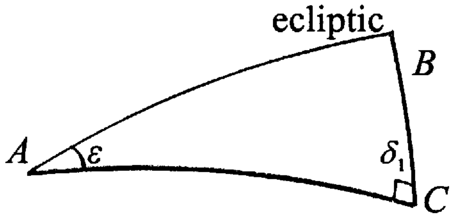
<house> and the sum is equal to its opposite, then we have worked correctly. If not, we have worked wrongly, and we repeat the operation.

Another method: We write the right ascension of the tenth <house> in two positions. We add to one position twice the <number of the> degrees in the <seasonal> hours of the ascendant. We subtract from the other <position> twice the <number of the> degrees in the hours of the opposite <of the ascendant>. The sum <is> the right ascension of the eleventh <house> and the remainder <is> the right ascension of the ninth <house>. We add the addend to the sum <again> and we subtract the subtrahend from the remainder <again>: The sum is the right ascension of the twelfth <house>, and the remainder is the right ascension of the eighth <house>. The arcs of these right ascensions are the <ecliptical> degrees of <the cusps of> the houses, and their opposites are <found> as mentioned before.

Commentary

I.5.1 In modern notation: $Sin\delta_1 = Sin\lambda Sin\varepsilon / R$, where δ_1 is the absolute value of the first declination, λ the true longitude, ε the total declination, and *R* the radius of the trigonometric circle (usually taken equal to 60 in medieval Islamic trigonometry). Ptolemy [1984, 69-70] explains this method by solving the problem for special cases. Al-Battānī gives the same formula as Kūshyār but in terms of the Chord function [1899-1907, III, 18]. A proof of the validity of the formula for the first declination is given in IV.5.1. A table for the first declination is given in II.43.

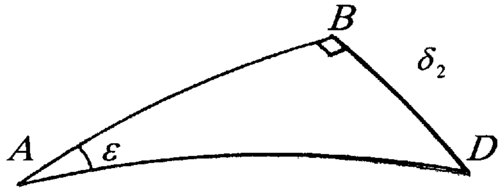
I.5.2 For a discussion of the right ascension see [Pedersen 1974, 99-101]. Kūshyār's methods for finding the right ascensions are equivalent to $CosA_0(\lambda) = Cos\lambda /(Cos\delta_1 / R)_1$ and $SinA_0(\lambda) = Tg\delta_1 /(Tg\varepsilon / R)$, where A_0 is the right ascension and the other symbols are as in I.5.1. The third method, in which we use the table for the second declination, is based on the symmetrical relation between the first and the second declinations. I will now explain Kūshyār's rather confusing description of the third method. Apparently, he provides the method for finding the right ascension by means of a table for the second declination, if the value of the first declination is known.



In the figure, *AB* is the true longitude of the ecliptical degree, *AC* the corresponding right ascension, δ_1 the first declination, and ε the total declination or the angle between the ecliptic and the celestial equator. Now, if we take *AC* as an arc on the ecliptic and *AB* as the celestial equator, then δ_1 will be the second declination of *AC*. So, if we find the argument of the tabular entry equal to δ_1 (which Kūshyār implicitly assumes to be known), in the table for the second declination, the argument will be equal to the required right ascension.

Ptolemy solves the problem for the special cases λ=30° and λ=60°, using a method equivalent to the second formula [1984, 71- 73]. Al-Battānī gives the second formula but instead of Sines and Tangents he uses the Chord function [1899-1907, III, 20]. Kūshyār gives a table for the right ascensions in II. 45 and proofs of the formulas in IV.5.2.

I.5.3 The formulas are equivalent to $Sin\delta_2 = Sin\delta_1 /[Cos\delta_1(90° - \lambda)/R]$ and $Tg\delta_2 = Sin\lambda Tg\varepsilon / R$, where δ_2 is the second declination, and δ_1(90°-λ) is the first declination of the complement of the true longitude (other symbols are as defined formerly). The third method in which we use the table for the rising times on the equator, is based on the symmetry between the first and the second declinations. Kūshyār's description in the third method is again confusing (cf. the third method in I.5.2). Apparently, he supposes that we have the table for the right ascensions and we can find the first declination (by computation or from the corresponding table).



In the figure, *AB* is the arc on the ecliptic, and δ_2 is the corresponding second declination. So, *BD* is perpendicular to *AB*. Now, if we take *AD* as an arc on the ecliptic, and *AB* as the celestial equator, then δ_2 may be regarded as the first declination of *AD*. Kūshyār calls *AD ʿaks al-maṭāliʿ* ("the inverse of the right ascension", meaning: the ecliptical longitude where the right ascension is *AD*). Proofs of the three methods are given in IV.5.3. A table for the second declination is given in II.43.

I.5.4 In modern notation: $Sin(d) = Sin(\beta \pm \delta_2)Cos(\varepsilon)/Cos(\delta_2)$, where *d* is the distance from the celestial equator, and β the ecliptical latitude of the star (other symbols as defined above). In al-Battānī's *zīj*, *d* is found through a method equivalent to the first formula given below in I.5.6. Kūshyār gives a proof of his formula in IV.5.4.

I.5.5 In modern notation: $\varphi = 90° - (h_{max} \pm \delta_1)$, where φ is the geographical latitude of the locality, and h_{max} is the maximum altitude of the sun on the day of measurement, and δ_1 is the declination of the sun on that day. A proof of this formula is given in IV.5.5. If h_{max} is measured in the south, the + sign is for a southern declination and the – sign is for northern declination. If h_{max} is measured in the north, the + sign is for a northern declination and the – sign is for a southern declination. The above formula can be written as $90° - \varphi = h_{max} \pm \delta_1$. Now if $h_{max} + \delta_1$ exceeds 90° (In the northern hemisphere, this happens when northern δ_1 exceeds φ), then $90° - \varphi = 180° - (h_{max} + \delta_1)$. This is possible in those localities where φ<ε. For example in Mecca, where φ=21.4°, this happens near the summer solstice, when $\delta_1 > 21.4°$.

I.5.6 The ortive amplitude (θ) is the arc on horizon between the East point and the rising point of the sun or the star [Kennedy & Sharkas 1962]. In modern notations, the first method is $Sin\theta = Sin\delta_1 /(Cos\varphi / R)$ for the sun, and $Sin\theta = Sind /(Cos\varphi / R)$ for the stars, where *d* is the distance of the star from the celestial equator. The second method may be expressed in modern notation as $Cos\theta = Cos\delta_1 Sin\frac{D}{2} / R$ for the sun and $Cos\theta = CosdSin\frac{D}{2} / R$ for the stars, where *D* is the day arc (see Chapter 10 of this section). Proofs of the validity of these formulas are given in IV.5.6. In the *Almagest*, the second method is used for finding the maximum ortive amplitude at Rhodes where φ=36° [1984, 76-77]. In al-Battānī's *zīj*, both methods for finding the ortive amplitudes are described [1989-1907, III, 29-30].

I.5.7 The equation of daylight (Δ*D*) is defined as Δ*D* = |*D*/2 - 90°|. For the sun, the four methods are equivalent to the following formulas:
$Cos\Delta D = Cos\theta /(Cos\delta_1 / R)$,
$Sin\Delta D = Sin\delta_1 Sin\varphi /(Cos\delta_1 Cos\varphi / R) = Tg\delta_1 Tg\varphi / R = SinMSinA_0(\lambda) / R$.
In the first three formulas, we may substitute *d* for δ_1 to obtain the formulas for the stars (see the last sentence in I.5.4). In the last formula, *M* is the maximum value of Δ*D*. Proofs of the formulas are given in IV.5.7, where he also provides the proof for another method that is not mentioned here. A table of the function Δ*D* for the latitude 36° and a table of the function *M* for latitudes of 16°, 17°, 18°, ...,45° are given in II.48 and II.47 in F, respectively. However, the table in II.47 is left blank in F and it is also missing in the other manuscripts. In Y, L, and B, the table II.48 is given for the latitude of 35;30°. At the end of Y, there is a table for Δ*D* calculated for the latitude of 30;5°. In the *Almagest* we find an application of a method equivalent to Kūshyār's second method, for the latitude of 36° relating to Rhodes [1984, 78-79], without referring to the 'base'. Al-Battānī also uses the second method for finding the equation of the daylight arc [1989-1907, III, 48-49] in order to find the daylight arc itself (see I.5.10).

I.5.8 For a discussion of the oblique ascensions see [Pedersen 1974, 99-101]. A proof of the validity of Kūshyār's method for finding the oblique ascensions is found in IV.5.8. A table of the oblique ascensions for the latitude of 36° (possibly a rounded value for 36;50°, the latitude of Jurjān) is given in II.46 in F. The corresponding tables in Y, L, and B are given for the latitude of 35;30°. At the end of Y, corresponding tables for latitudes of 38° (possibly for Daylam in Gīlān) and 32; 23° (Isfahan) are given. In P, the relevant tables for latitudes of 38° and 36; 30° are given among the seven tables added in the sequel of Book I.

I.5.9 The method described in this chapter is equivalent to the modern formula $h_{\max} = (90° - \varphi) \pm \delta_1$ where the positive sign is used for northern declinations, and the negative sign for southern declinations of the sun. For the planets and stars, we substitute the distance from the celestial equator for δ_1. A proof of the validity of this method is given in IV.5.9.

I.5.10 The method for finding half the day arc is equivalent to the formula $D/2 = 90° \pm \Delta D$. The positive and negative cases are for the northern and the southern declination of the sun or the distance of the planet from the celestial equator, respectively. A proof of this method is given in IV.5.10.

In the second method, if the oblique ascension of a certain ecliptical degree λ is $A(\lambda)$, then we have $A(\lambda+180°) = A(\lambda)+180° \pm 2\Delta D$ (this relation can be deduced from the rule for finding the oblique ascension given in I.5.8 above and the symmetries of the oblique ascension described in [Ptolemy 1984, 90-92]). So the difference is $180° \pm 2\Delta D$, which is the day arc. The plus sign is used for the first and fourth quadrants, and the minus sign is used for the second and third quadrants.

I.5.11 Since each degree of rotation of the celestial equator corresponds to 4 minutes of time, the length of a day is

$$4(180° \pm 2\Delta D) \text{ minutes} = 12 \text{ hours} \pm 8\Delta D \text{ minutes}$$

The number of degrees in a seasonal hour is

$$D/12 = (180° \pm 2\Delta D)/12 = 15° \pm (10/60)\,\Delta D$$

Dividing $2\Delta D$ by 15 or 12 is equivalent to multiplying $2\Delta D$ by 4 minutes (i.e., 4/60) or multiplying ΔD by 10 minutes (i.e., 10/60).

So the alternative methods are equivalent to those formulated above. The number of degrees in a seasonal hour of the night is

$$(360° - D)/12 = 30° - D/12, \text{ i.e.,}$$

30° minus the number of degrees in a seasonal hour of the day.

It is obvious that $\frac{D}{15}(1+\frac{1}{4}) = \frac{D}{12}$ and $\frac{D}{12}(1-\frac{1}{5}) = \frac{D}{15}$. There is no section corresponding to I.5.11 in Book IV.

I.5.12 The Sine of the 'adjusted distance' (d_a) of the star from the solstice (arc TE in the figure of IV.5.11), or the difference between the right ascensions of the ecliptical degree relating to the star, and the nearest solstice is found by a method equivalent to the formula $Sind_a = Cos\beta Sind_s / Cosd$, where β is the latitude of the star, d_s is its distance to the nearest solstice, and d is the distance of the star from the celestial equator. By adding or subtracting this auxiliary magnitude d_a to or from the longitude of the relevant solstice, we find the right ascension of the star, and hence, by the table of right ascensions, its ecliptical degree of transit. A proof of the validity of this method is given in IV.5.11. Kūshyār's method is different from and simpler than the method given by Ptolemy [1984, 411-12] and followed by al-Battānī [1899-1907, III, 48].

I.5.13 Proofs of these methods are given in IV.5.12. Kūshyār's methods are simpler than those provided by al-Battānī [1899-1907, III, 49-50] for this subject.

I.5.14 The arc of revolution is the distance covered by a point on the celestial equator due to the apparent revolution of the celestial sphere (1° per 4 minutes of time) between two moments of time. Usually these two moments are sunrise or sunset and the time of observation. In this chapter, Kūshyār wants to compute the arc of revolution from the altitude which he has found by observation (for a definition of the Sagitta function see I.2.4 and its commentary). In modern notation, first we obtain the so called 'arrangement sine' of the arc of revolution; then we obtain the excess of the arc of revolution, and then the arc of revolution, as follows:

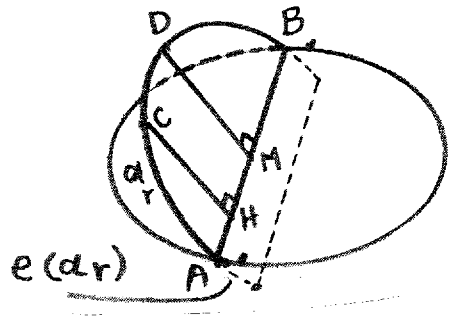
Let $$As(a_r) = Sinh_t(R - Cos\frac{D}{2}) / Sinh_{max},$$

then $$R - Cose(a_r) = R - Cos\frac{D}{2} - As(a_r),$$

and $$a_r = \frac{D}{2} \pm e(a_r).$$

$As(a_r)$ is called the 'arrangement Sine' of the arc of revolution, h_t is the altitude of the sun or the star at the given time, h_{max} is the maximum altitude, and $e(a_r)$ is the excess of the arc of revolution.

A proof of the validity of this calculation is given in IV.5.13. Al-Battānī gives a similar method for this calculation [1899-1907, III, 45], but he does not use the term 'arrangement Sine'. Al-Kāshī used the term in the Book V of his *Khāqānī zīj* [Kennedy 1985, 45; 1998, 38]. In the introduction of Book V, he defines the 'arrangement Sine' as "the perpendicular from one end of an arc to a chord that passes through the other end of the arc". He adds that "if a perpendicular is drawn on the base of a circular segment and reaches the circumference and divides the arc into two [unequal] arcs, the perpendicular is regarded as the 'arrangement Sine' of either of the two arcs."



In the chapter entitled "A compendium of astronomy" in Book III, Kūshyār defines the 'arrangement Sine' in a similar way. In his definition $ACDB$ is the day arc and CH, the 'arrangement Sine' of the 'arc of revolution AC, is parallel to the Sagitta of half the day arc (DM).

Example:

Suppose the altitude of the sun is 53° eastern, the maximum altitude is 72;58°, and half the day arc is 104;24°.

Using the above formulas, we have:

$As(a_r)=Sin53°(60-Cos104;24°)=62.5807$,

$60-Cos\,e(a_r)=60-Cos104;24°-62.5807=12.3407$, so $Cos\,e(a_r)=47.6593$, $e(a_r)=37.24°$. Now the arc of revolution can be found by the last formula above, taking the minus sign (because the sun is in the eastern half of the sky): $a_r=D/2-37;24°=67;0°$.

The example is taken from al-Nasawī's *al-Lāmi' fī amthilat al-Zīj al-jāmi'* (see the commentary on I.1.5), fol. 61v. Al-Nasawī finds a_r equal to 67;1°. The difference with the above result is only 1 minute that can be due to the rounding errors.

With the data of the example, the geographical latitude of the locality of observation can be found equal to 35;57.5°. This accords with the historical information that al-Nasawī worked in or around Rayy (with geographical latitude 35;40° N).

Having the arc of revolution, we can easily find the time of the day, as the equinoctial hours passed since sunrise (see I.5.15):

67;0/15= 4 hours + 28 minutes.

We can also find the local time:

12- 37;24°/15= 9;30.4 AM

I.5.15 Since a complete revolution of the equator occurs in 24 hours, by dividing the arc of revolution into 15=360:24, we obtain the number of the hours elapsed since the rising of the sun or the star. The method for finding the number of the seasonal hours elapsed since sunrise is obvious.

I.5.16 A proof of this method is given in IV.5.14. This method is also given by al-Battānī [1899-1907-, III, 45].

I.5.17 A proof of this method is also given in IV.5.14.

I.5.18 This is the inverse of the method given in I.5.14. Al-Battānī provides a similar method [1899-1907, III, 46].

I.5.19 This method is valid because the ecliptic is a great circle, and a semicircle of it is always above the horizon. Then, the setting of any point on the ecliptic is simultaneous with the rising of the diametrically opposite point on the ecliptic.

I.5.20 This is simply the inverse of the method mentioned in I.5.19.

I.5.21 In modern notation, the "base" B (not the same B mentioned in I.5.7) as well as the first six rules in the chapter are as follows:

$$B = Cos\delta_1 Cos\varphi / R^2$$

i) $As(a_r) = Sinh_t / B$

ii) $Sinh_t = BAs(a_r)$

iii) $R - Cos\frac{D}{2} = Sinh_{max} / B$

iv) $Sinh_{max} = B(R - Cos\frac{D}{2})$

v) $R - Cose(a_r) = (Sinh_{max} - Sinh_t)/B$

vi) $Sinh_t = Sinh_{max} - B[R - Cose(a_r)]$

The validity of formula i is proved in IV.5.15. The formulas ii, iii, iv, v, and vi can be derived from i, by applying the definition of the base and by the formulas given in I.5.14.

I.5.22 'Equalization of the Houses' is a method of division the ecliptic into 12 parts, different from the zodiacal signs, for astrological purposes. The beginnings of the first, fourth, seventh and tenth house were usually regarded to be the ascendant, the lower midheaven, the descendant, and the upper midheaven, respectively. These points are called the 'cardines' (Arabic: *autād*). Then the arcs between these four points were divided into three parts for finding the beginnings of the other houses. There were different methods for this division. Kūshyār's method was called "the well- known method" by al-Bīrūnī [1954-1956, III, 1357-59; 1985, 276] and "the Standard Method" in modern literature [North 1986, 4; Kennedy 1996, 538, 548]. It was actually the most popular one among the medieval Islamic authors. The origin of this method is pre-Islamic [North 1986, 6], however, some modern authors ascribe it to Alchabitius (Latinized form of al-Qabīṣī), the court astrologer of the Buyid Sayf al-Dawla [Kennedy 1996, 539-40].

In this method the cardines are projected onto the celestial equator along great circle arcs through the equatorial poles. Then the four resulting segments are trisected. The trisection points are projected back onto the ecliptic along great circle arcs through the equatorial poles to obtain the corresponding cusps (i.e., the beginnings of the houses). Kūshyār first finds $\Delta\alpha_d$, the equatorial arc corresponding to 2 diurnal unequal hours (for the sun at the ascendant). He also finds $\Delta\alpha_n = 60^\circ - \Delta\alpha_d$, the equatorial arc corresponding to 2 nocturnal unequal hours. By subtracting $\Delta\alpha_d$ from the right ascension of the ascendant and adding $\Delta\alpha_n$ to this right ascension, he finds the right ascensions of the cusps of the twelfth and the second houses. By repeating this process, he finds the right ascensions of the cusps of the eleventh and the third houses. Now he finds the ecliptical degrees corresponding to these right ascensions. He also finds the cusps of the other houses, which are opposite to the former houses correspondingly. In the second method, he starts from the tenth house and follows a process similar to the first one and differing in the order of the houses to be found.

Al-Battānī mentions the same method, and he follows the order given in Kūshyār's second method [al-Battānī 1899-1907, III, 110-11].

Section 6: On eclipses and what pertains to them, <in> 20 chapters
Chapter 1: On the motion of the two luminaries in <one> day and <one> hour.

The daily motion is the difference between the true longitude of <any> one of the two luminaries for any day and the true longitude for the next or the previous day. It is called 'daily rate'. The hourly motion is the result of dividing the daily motion by twenty-four. It is called 'hourly rate'. Or <using another method,> we find the true longitude of <any> one of the two luminaries for the given time; then <we find it> for six hours later or earlier. We take the difference between the two true longitudes and multiply it by 10 minutes (i.e., 10/60). A table is compiled for it (i.e., for the hourly rate) <in this *zīj*>. If the hourly rate of the sun is subtracted from the hourly rate of the moon, the remainder is the adjusted rate. It is called 'the lunar gain'.

Chapter 2: On the magnitude of the <apparent> diameter of the two luminaries and the diameter of the shadow <of the earth>.

<For finding> the diameter of the sun, we multiply its daily motion by 33 minutes (i.e. 33/60), or we multiply its hourly motion by $13\frac{1}{5}$: The result is its diameter according to its distance from the earth. As for the diameter of the moon, we multiply its daily motion by 2 minutes and 26 seconds (i.e., 2/60+26/60×60=146/3600), or we multiply its hourly motion by 58 minutes and 25 seconds (i.e., 58/60+25/60×60=3505/3600): The result is its diameter according to its distance from the earth. For the diameter of the shadow <of the earth>, we multiply the diameter of the moon by $2\frac{3}{5}$: The result is the diameter of the shadow according to the distance of the moon from the earth, the sun being at its maximum distance <from the earth>. If we want extreme precision, we take the excess of the hourly motion of the sun over 2 minutes and 23 seconds, multiply <the result> by 10, and subtract <this product> from the diameter of the shadow which was found: The result is the adjusted diameter of the shadow according to the distance of the sun also from the earth. A table is compiled for these diameters, with the hourly motions of the two luminaries <in this *zīj*>.

Chapter 3: On the <ecliptical> degree of a conjunction and opposition, their hours and ascendants.

We find the true longitudes of the two luminaries for noon of the day nearest to the conjunction or the opposition, and we take the distance between the two true longitudes from those true longitudes in the case of conjunction. But in the case of opposition, <we take the distance> after we have added 6 <zodiacal> signs to the position of the moon. We note which of the two <luminaries> precedes the other <in the ecliptic>. Then we multiply the distance by 5 minutes and we call the result 'the part of the distance', and we keep it; <then> we add it to the distance: The result is the distance plus 'the part of the distance'. Then we look: if the sun precedes <the moon>, we add the distance plus its part to the <true longitude of the> moon and we add the 'part of the distance' to the <true longitude of the> sun. If the moon precedes <the sun>, we subtract the distance plus its part from the <true longitude of the> moon, and we subtract 'the part of the distance' from the <true longitude of the> sun. <The sun and the moon> will be in conjunction or opposition in the same second <of a degree>. **The hours**: Then we find the hourly motions of the two luminaries. We subtract the hourly motion of the sun from the hourly motion of the moon. The remainder is <called> 'the lunar gain'. We divide the distance by 'the lunar gain'. The result is the <number of> hours corresponding to the distance. If the sun precedes <the moon>, we add the hours corresponding to the distance to the hours of the noon. If the sum is less than the hours of the entire day, then it is the <number of the> hours of the day elapsed <before conjunction>. If it is greater than the <number of the> hours of the day, we subtract the <number of the> hours of the day from it. The remainder is the <number of the> hours of the next night elapsed <before conjunction>. If the moon precedes <the sun>, and the <number of the> hours corresponding to the distance is less than the hours of the noon, we subtract the <number of the> hours corresponding to the distance from the hours of the noon. The remainder is the <number of the> hours of the day elapsed <before the conjunction>. If it is greater than <the number of> the hours of the noon, we subtract the <number of the> hours corresponding to the distance from the sum of the hours of the noon and the hours of the night. The remainder is the <number of the> hours of the previous night elapsed <before the conjunction>. Then we find the true longitudes of the two luminaries for the resulting hours (i.e., for the resulting time). If they coincide with the <ecliptical> degree that had been computed before, the <number of the> the hours is correct. If their positions are different, we take the difference between them, and we operate with them in the same way that we operated with the true longitude<s> at noon and the distance between the two luminaries. The result of this second time is the <ecliptical> degree of the conjunction and opposition and the time <of conjunction and opposition> with <more> precision. We find the ascendant for the resulting time. It is the ascendant of the conjunction and the opposition.

Chapter 4: On the absolute and adjusted magnitudes of a lunar eclipse in digits.

We consider the latitude of the moon at the <time of> opposition: If it is more than 63 minutes northern or southern, the moon will not be eclipsed, if it is less than this <limit>, it can be eclipsed. Then we find the diameter of the moon and the diameter of the shadow, we add them, and we halve the result. It is <called> half the <sum of the> diameters (i.e., the sum of the two radii). If the latitude of the moon is greater than the <sum of the> two radii or equal to it, the moon will not be eclipsed. If the latitude is less <than that, the moon> will be eclipsed. The excess of the <sum of the> two radii over the latitude is the <magnitude> of the eclipse in minutes. If it is greater than the diameter of the moon, <then the moon> will be totally eclipsed and will remain in it (i.e., in the eclipsed situation) for some time. If it is equal to the diameter of the moon, <then the moon> will be totally eclipsed, but it will not remain in the <total> eclipse. If it is less than the diameter of the moon, <then the moon> will be partially eclipsed. We multiply the <magnitude> of the eclipse in minutes by 12 and divide it by the diameter of the moon. The result is the absolute <magnitude> of the eclipse in digits, in which we take the diameter <of the moon equal to> 12 digits. For <finding> its adjusted <value>, we subtract the <magnitude> of the eclipse in minutes from the diameter of the moon, and <also> from the diameter of the shadow, and we add the remainders. Then we multiply the remainder of the diameter of the moon by the <magnitude> of the eclipse in minutes, and divide it by the sum of the two remainders. The result is the Sagitta of the shadow. We subtract it from the <magnitude> of the eclipse in minutes. The remainder is the Sagitta of the moon. Then we subtract the Sagitta of the moon from the diameter of the moon, multiply the remainder by the Sagitta of the moon, and take the square root of the result. The <final> result is the absolute Sine. We keep it. Then we multiply the absolute Sine by 60 and divide it by the radius of the moon. The result is the adjusted Sine. We find the corresponding arc. If the Sagitta of the moon is less than its radius, then this arc is <called> 'the arc of the moon<'s disk>'. If the Sagitta is greater than the radius <of the moon>, we subtract the arc from 180 <degrees>. The remainder is 'the arc of the moon<'s disk>'. Then we multiply the diameter of the moon by 22 and divide it by 7. The result is the circumference of the moon's disk. Then we multiply half of it by the radius of the moon. The result is the area of the moon's disk. Then we multiply the circumference of the disk by the arc <of the moon's disk>, and divide <the product> by 360 <degrees>. The result is half the 'arc of the sector'. We multiply it by the radius of the moon. The result is the <area of the> sector of the moon. Then we obtain the difference between the Sagitta and the radius, and multiply <the remainder> by the absolute Sine. The result is <the area of > the triangular <portion> of the moon. If the Sagitta is less than the radius <of the moon>, we subtract the <area of the> triangular <portion> from the <area of the> sector. If the Sagitta is greater <than the radius>, we add it <to the area of the sector>. The sum or the remainder is <the area of> 'the segment of the moon'. Then we repeat the operation for the shadow <instead of the moon> from the absolute Sine on. However, the Sagitta of the shadow does not reach the value of its radius. When we find the segment of the shadow, we add it to the segment of the moon. The result is the adjusted <magnitude> of the eclipse in minutes. We multiply it by 12 and we divide the result by the area of the moon's disk. The result is the adjusted <magnitude> of the eclipse in digits, based on taking the area of the <moon's> disk <equal to> 12 digits. <The procedure given in> this chapter is also sufficient for finding the adjusted magnitude of solar eclipses in digits, if we let the disk of the sun take the place of the disk of the moon in this <procedure>, and let the moon's disk take the place of the disk of the shadow. We follow the conditions that we laid down in the case of the moon, its Sagitta, its arc, and its triangular <portion>. A table for finding the approximate adjusted magnitudes of the two (i.e., solar and lunar) eclipses is compiled <in this *zīj*>.

Chapter 5: On the absolute and adjusted times of a lunar eclipse.

The time of the opposition is that of the middle of the lunar eclipse. The other times are <as follows:> the beginning of the lunar eclipse, the beginning of the duration <of totality>, the beginning of the emersion <of the eclipse>, and the end of the emersion. If there is no duration <of totality>, <then the times are:> the beginning of the lunar eclipse, and the end of the emersion. We subtract the square of the latitude of the moon at the middle of the lunar eclipse from the square of the <sum of the> two radii and obtain its square root. <The result> is the <magnitude> of immersion from the beginning of the lunar eclipse to its middle in minutes, whether or not it has a duration <of totality>. We divide it by the lunar gain. The result is the <number of the> hours of the immersion from the beginning to the middle <of the lunar eclipse>. We subtract it from the time of the middle of the lunar eclipse, and we <separately> add it <to the time of the middle>. The remainder is the time of the beginning <of the lunar eclipse>, and the sum is the time of the end of emersion. If <the moon> has a duration <of totality>, we subtract the radius of the moon from the radius of the shadow. Then we subtract from the square of the remainder the square of the latitude <of the moon> at the middle of the lunar eclipse, and obtain the square root of the <last> remainder. It is the <magnitude> of the immersion from the beginning of the duration <of totality> to its middle in minutes. We divide it by the lunar gain. The result is the <number of the> hours of the immersion from the beginning of the duration <of totality> to its middle. We subtract it from the time of the middle <of the lunar eclipse>, and we <separately> add it <to the time of the middle>. The remainder is the time of the beginning of the duration <of totality>, and the sum is the time of the beginning of emersion. For finding its adjusted value, we subtract the square of the latitude of the moon at the beginning of the lunar eclipse from the square of <the sum of> the two radii. We add what remains from the square of <the sum of> the two radii to the square of the difference between the latitude of the moon at the beginning of the lunar eclipse and its latitude at the middle of the lunar eclipse. We obtain the square root of the result. It is the adjusted <magnitude> of immersion from the beginning to the end <of the eclipse> in minutes. We divide it by the lunar gain. The result is the adjusted <duration of> the immersion in hours. We subtract it from the time of the middle of the lunar eclipse. The remainder is the adjusted time of the beginning <of the eclipse>. Then we also subtract the square of the latitude of the moon at the end of the emersion from the square of <the sum of> the two radii. We add the remainder to the square of the difference between the latitude of the moon at the end of the emersion and its latitude at the middle of the lunar eclipse. We obtain the square root of the result. It is the second adjusted <magnitude> of the immersion in minutes; it is <the duration> from the middle <of the lunar eclipse> to the end of emersion. We divide it by the lunar gain. The result is the second adjusted <duration of> the immersion in hours. We add it to the <time of the> middle of the lunar eclipse. The result is the adjusted time of the end of the emersion. Finding the adjusted <values> of the other times <involved in a lunar eclipse> is of no use.

Chapter 6: On drawing the figure of a lunar eclipse.

We draw a straight line <segment> of arbitrary length. We divide it by the number of the minutes of <the sum of> the two radii. Then we draw a circle the radius of which is equal to this line <segment>. It is the circle of <the sum of> the two radii. We take from the line <segment a part of it> equal to the radius of the shadow and we draw a circle with this segment as its radius and centered on the center of the first circle. It is the circle of the shadow. We draw the two diameters of the two circles, which intersect at the center at right angles. We write on their directions the four <geographical> orientations: east opposite to west, and north opposite to south. Then we take from the line <segment a part of it> equal to the latitude of the moon in the middle of the lunar eclipse. We put one leg of the compasses at the center of the two circles, and the other where it falls on the south-north line depending on the direction of the latitude, and we make a mark there. It is the center of the moon at the middle of the lunar eclipse. Then we take from the line <segment a part of it> equal to the radius of the moon and we draw a circle with this segment as its radius and centered on the center of the moon. It is the circle of the moon in the middle of the lunar eclipse. The portion of it inside the circle of the shadow is the eclipsed part of the moon.

Chapter 7: On <finding> the distance of the moon from the earth.

First we consider the double elongation <of the moon>. If it is <equal to> zero, then the distance of the center of the epicycle from the center of the earth is <put equal to> 60 parts. If the double <elongation> is exactly 6 <zodiacal> signs, then the distance of the center <of the epicycle from the center of the earth is> $39\frac{1}{3}$ parts. If the double <elongation> is exactly 3 or 9 signs, we subtract the square of $10\frac{1}{3}$ parts from the square of $49\frac{2}{3}$ parts, and obtain the square root of the remainder. The distance of the center <of the epicycle from the center of the earth> is <found to be> approximately $48+\frac{1}{3}+\frac{1}{4}$ parts. If the double <elongation> is between these <values>, we multiply both its Sine and Cosine by $10\frac{1}{3}$ minutes (i.e., $31/3\times60$), and we subtract the square of the product of the Sine <of the double elongation by $10\frac{1}{3}$ minutes> from the square of $49\frac{2}{3}$ parts. Then we obtain the square root of the remainder. <Now,> if the double <elongation> is less than 3 signs or greater than 9 <signs>, we add to the square root the product of the Cosine of the double <elongation by $10\frac{1}{3}$ minutes >. If the double <elongation> is greater than 3 signs and less than 9 signs, we subtract from the square root the product of the Cosine of the double <elongation by $10\frac{1}{3}$ minutes>. The <final> result is the distance of the center of the epicycle from the center of the earth. **The body of the moon**: Then we obtain from the equation tables the difference <in epicyclic equation> at lesser distance <of the moon from

the earth> corresponding to the double <elongation>, and the sixtieths corresponding to the true anomaly. We multiply them by each other. We add <the result> to 5 parts and 1 minute (i.e., 5+1/60 parts). We obtain the Sine of the result. The <last> result is the adjusted radius of the epicycle. Then if the true anomaly is <equal to> zero, we add the adjusted radius of the epicycle to the distance of the center of the epicycle. <The result> is the distance of the moon from the center of the earth. If the true anomaly is exactly 6 signs, we subtract the radius of the epicycle from the distance of the center of the epicycle. The remainder is the distance of the moon from the center of the earth. If the true anomaly is exactly 3 or 9 signs, we add the square of the adjusted radius of the epicycle to the square of the distance of the center <of the epicycle>. We obtain its square root, and <the result> is the distance of the moon <from the center of the earth>. If the true anomaly is between these <values>, we multiply both the Sine and the Cosine of the true anomaly by the adjusted radius of the epicycle, lowered. Then, if the true anomaly is less than 3 signs or greater than 9, we add the product of the Cosine <by the adjusted radius of the epicycle, lowered> to the distance of the center <of the epicycle from the center of the earth>. If the true anomaly is greater than 3 signs and less than 9, we subtract the product of the Cosine <by the adjusted radius of the epicycle, lowered> from the distance of the center <of the epicycle from the center of the earth>. We add to the square of the sum or the remainder the square of the product of the Sine <by the adjusted radius of the epicycle, lowered>. We obtain the square root <of the final result>. It is the distance of the moon from the center of the earth. A table is compiled <in this *zīj*> for the distance of the moon <from the earth> sufficient for what we need in <calculating> solar eclipses and the <lunar> crescent visibility. The distance of the moon is found in the table where the double elongation <is shown> in the <first row along the> width and the true anomaly, in the <first column along the> length <of the table>. This is sufficient for the calculation. Calculating the distance of the sun <from the earth> is not very necessary for us. Its calculation is like that of the moon, except that we use its (i.e., the sun's) mean anomaly instead of the true anomaly <used in the case of the moon>: <We use> 2 degrees and 1 minute instead of the adjusted radius of the epicycle, and <we use> 60 instead of the distance of the epicycle. Then we multiply what we find for the distance by $18\frac{4}{5}$. <The result> is the distance <of the sun> from the center of the earth. Its maximum distance is approximately 1,255 parts; its mean distance is approximately 1,208 parts; its minimum distance is approximately 1,161 parts.

Chapter 8: On the altitude of the pole of the ecliptic, which is called 'the latitude of the clime of visibility'.

We divide the Sine of the altitude of <the sun at a given> time by the Sine of the arc between the tenth <house> (i.e., mid-heaven) of <the given> time and its ascendant on the ecliptic, lowered. The result is the Cosine of the altitude of the pole. We find the <corresponding> arc <in the table of Sines> and subtract it from 90 <degrees>. The remainder is the altitude of the pole.

Chapter 9: On the altitude of any desired degree of the ecliptic.

We multiply the Sine of the arc between the <given> degree and the ascendant or the descendant by the Sine of the altitude of the tenth <house>. We divide <the product> by the Sine of the arc between the tenth <house> and the ascendant or descendant. The result is the Sine of the altitude of the <given> degree and <also> of the altitude of any planet of zero latitude.

Chapter 10: On the <equatorial> distance between the meridian and the <right> ascension of a known point of the ecliptic.

If the known point is between the tenth <house> and the ascendant, we subtract the right ascension of the tenth <house> from the right ascension of the <known> point. The remainder is the distance of <the ascension of> the point from the meridian. If the known point is between the seventh and tenth <house>, we subtract the right ascension of the <known> point from the right ascension of the tenth <house>. The remainder is the distance of the <ascension of the known> point from the meridian. If the remainder is greater than 90 <degrees>, we subtract it from 180 <degrees>. The remainder is the <desired> distance.

Chapter 11: On the parallax of the two luminaries in the altitude circle.

We obtain both the Sine of the altitude of the <ecliptical> degree of the moon and the Sine of the complement (i.e., the Cosine) of the altitude, lowered. We subtract the result of <lowering> the Sine of the altitude from the distance of the moon from the earth (as in Chapter I.5.7, the maximal distance is 60). We add to the square of the remainder the square of the result of <lowering> the Cosine of the altitude. We obtain the square root of the <final> result. Then we divide the result of

<lowering> the Cosine of the altitude by this square root, lowered. The result is the Sine of the parallax of the moon in the altitude circle. If the moon is on the horizon, we add to the square of the distance of the moon from the earth the square of the radius of the earth, which is <taken as> one part. We obtain the square root <of the sum>. Then we divide the radius of the earth by this square root, lowered. The result is the Sine of the parallax <of the moon>. If we subtract the parallax from the calculated altitude of the <ecliptical> degree of the moon, the remainder is the apparent altitude <of the moon, seen> from the surface of the earth.

Section: The parallax of the sun is calculated in a similar way, by using its mean distance from the earth. However, its parallax at different distances does not differ in a <noticeable> magnitude. Its maximum parallax is about 3 minutes. We need this to subtract it from the parallax of the moon. The remainder is the adjusted parallax of the moon in the altitude circle. This <is used> for <reaching> extra precision in <calculating> solar eclipses. A table is compiled <in this *zīj*> for obtaining it from the complement of the altitude of the sun.

Chapter 12: On the six angles which are needed in <the calculation of> solar eclipses.

The first angle: It is <in the case> when the position of the moon is the first <degree> of Aries or Libra, and <also> the <ecliptical> degree of the ascendant of a <given> time. It (i.e., the angle) is equal to the complement of the altitude of the beginning of Cancer or of Capricorn, whichever is on the meridian circle <above the horizon>. It is <called> ‘the latitude angle,’ and its complement <is called> ‘the longitude angle.’ **The second angle**: It is <in the case> when the position of the moon is the first <degree> of Aries or Libra, and <also> the <ecliptical> degree of the tenth <house> of the <given> time. It (i.e., the angle) is equal to the complement of maximum declination <of the sun>. It is <called> ‘the latitude angle,’ and its complement <is called> ‘the longitude angle.’ **The third angle**: It is <in the case> when the position of the moon is other than the first <degree> of Aries or Libra, and <also> the <ecliptical> degree of the ascendant of the <given> time. It (i.e., the angle) is equal to the altitude of the pole of the ecliptic at the <given> time. It is <called> ‘the latitude angle,’ and its complement is <called> ‘the longitude angle.’ **The fourth angle**: It is <in the case> when the position of the moon is the first <degree> of Cancer or Capricorn, and <also> the <ecliptical> degree of the tenth <house> of the <given> time. It (i.e., the angle) is a right angle. In this case, there is no longitude angle. **The fifth angle**: It is <in the case> when the position of the moon is other than the equinoxes or solstices, and <also> the <ecliptical> degree of the tenth <house> of the <given> time. Then we consider the declination of the <ecliptical> degree of the tenth <house> and the <geographical> latitude of the locality. If the declination is northern, we subtract the smaller <one of these two quantities> from the greater <one> (if $0<\varphi<\delta_1<\varepsilon$, then the tenth house is northern with respect to the zenith). If the declination is southern, we add it to the <geographical> latitude of the locality. The sum or the remainder is <equal to> the distance of the ecliptic from the zenith <along the meridian>. Then we divide the Sine of the altitude of the pole of the ecliptic by the Sine of the distance of the ecliptic from the zenith <along the meridian>, lowered. The result is the Sine of the latitude angle. We find the <corresponding> arc: it is the latitude angle; its complement is the longitude angle. **Another method**: We divide the Sine of the <right> ascension of the distance of the equinoctial point above the horizon from the meridian <,which is found> according to what <is described> in the relevant chapter (i.e., I.5.2), by the Sine of <the arc> between the tenth <house> and the equinoctial point of the ecliptic, lowered. The result is the Sine of the latitude angle. We find the <corresponding> arc: it is the latitude angle; its complement is the longitude angle. **The sixth angle**: It is <in the case> when the position of the moon is in an arbitrary <ecliptical> degree, between the ascendant and the descendant. Then we divide the Sine of the altitude of the pole of the ecliptic by the Cosine of the altitude of the <ecliptical> degree of the moon, lowered. The result is the Sine of the latitude angle. We find the <corresponding> arc: it is the latitude angle; its complement is the longitude angle.

Chapter 13: On <finding> the longitudinal and latitudinal parallax of the moon from these angles.

We multiply both the Sine of the latitude angle and the Sine of the longitude angle by the parallax in the altitude circle, lowered. The result from the latitude angle is the latitudinal parallax. The result from the longitude angle is the longitudinal parallax. If the distance of the moon from the zenith, when it reaches the meridian, is towards the south and the latitude of the moon is southern, or the distance is towards the north and the latitude of the moon is northern, we add the latitudinal parallax to the latitude. If they differ <in direction>, we subtract the smaller from the greater one. The result is the apparent latitude. Its direction is that of the sum of the latitude and the parallax, or the direction of the greater of the two. The place of the moon in most northern localities is southern with respect to the zenith.

Chapter 14: On <measuring> the absolute and adjusted magnitudes of a solar eclipse in digits.

If the latitude of the moon at the <time of> conjunction is southern, and more than 35 minutes, or northern and more than 95 minutes, the sun will not be eclipsed. If the latitude is less than that, it can be eclipsed. If it can be eclipsed, we find the time of the conjunction, its ascendant, the longitudinal parallax of the moon, and its apparent latitude <at the time>. Then we divide the longitudinal parallax by the lunar gain. The result is the <longitude> difference in hours. If the distance of the <ecliptical> degree of the <new> conjunction from the ascendant is less than 90 degrees, we subtract the <longitude> difference in hours from the time of the conjunction, and the longitude difference in minutes from the <ecliptical> degree of conjunction, and from the argument of the latitude, in order to learn the latitude from it. If the distance of the <ecliptical> degree of the <new> conjunction from the ascendant is greater than 90 <degrees>, we add the <longitude> difference in hours to the time of the conjunction, and the <longitude> difference in minutes to the <ecliptical> degree of the conjunction, and to the argument of the latitude. The sum or the remainder of the time of the conjunction is the time of apparent conjunction. The sum or the remainder of the <ecliptical> degree of the conjunction is the <ecliptical> degree of the apparent conjunction. We compute the ascendant for the time of the apparent conjunction. We obtain from this ascendant and from the <ecliptical> degree of the apparent conjunction, the apparent latitude of the moon and its longitudinal parallax. Then we divide <this> parallax by the lunar gain. The result is the second <longitude> difference in hours. If the distance of the first <ecliptical> degree from the ascendant of the apparent conjunction is less than 90 <degrees>, we subtract the <longitude> difference in hours from the first time of the conjunction, and the <longitude> difference in minutes <of arc> from the first <ecliptical> degree of the conjunction. If the distance of the first <ecliptical> degree of the conjunction from the ascendant of the apparent conjunction is greater than 90 degrees, we add the <longitude> difference in hours to the first time of the conjunction, and the <longitude> difference in minutes <of arc> to the first <ecliptical> degree of the conjunction. The sum or the remainder in hours is the time of the adjusted apparent conjunction and the time of the middle of the solar eclipse. The sum or the remainder in <ecliptical> degrees of the conjunction is the position of the moon in the middle of the solar eclipse (i.e., no further iteration is necessary). That is because if we derive this time of the conjunction from the ascendant and from the position of the moon in which the longitudinal parallax <is considered>, it (i.e., the observed time of the middle of the conjunction) will be equal to the second result or so close that <the difference> cannot be observed. When the time of the middle of the solar eclipse and its ascendant are found, we add the radii of the sun and the moon, which we call 'the <sum of the> two radii'. If the apparent latitude <of the moon> is equal to or greater than the <sum of the> two radii, the sun will not be eclipsed. If it is less <than the sum of the two radii>, the sun will be eclipsed. The excess of the <sum of the> two radii over the apparent latitude is the <magnitude of the> solar eclipse in minutes. We multiply the <magnitude of the> eclipse in minutes by 12 and divide it by the diameter of the sun. The result is the <magnitude of the> solar eclipse in digits. It is the eclipsed part of its diameter, based on <taking> the diameter <to be equal to> 12 digits. If the conjunction happens to be before sunrise, then the sun rises <in an> eclipsed <situation>. We <then> use the lower mid-heaven instead of the upper mid-heaven and we change the ascendant to the descendant in all the operations relating to the solar eclipse. Also, when we finish finding the <longitude> difference in hours and the <longitude> difference in minutes <of arc>, we always subtract the <longitude> difference in hours from the time of the conjunction and the <longitude> difference in minutes <of arc> from the <ecliptical> degree of the conjunction. Finding the adjusted magnitude of a solar eclipse is like finding the adjusted magnitude of a lunar eclipse, both in calculation and <in using> tables; the disk of the moon in this case plays the role of the disk of the shadow in that case, and the disk of the sun in this case plays the role of the disk of the moon in that case.

Chapter 15: On the absolute and adjusted times of a solar eclipse.

We subtract the square of the apparent latitude <of the moon> in the middle of the solar eclipse from the square of <the sum of> the two radii, and we obtain the square root of the remainder. The result is the <arc of> immersion in minutes. Then we divide it by the lunar gain. The result is the <duration of> immersion in hours. We subtract it from and <also> add it to the time of the middle of the solar eclipse. The remainder is the time of the beginning of the solar eclipse, and the sum is the time of the end of the emersion. Finding the adjusted magnitude of these two times is like finding the adjusted magnitudes of the times relating to the lunar eclipse, where we substitute the apparent latitude <of the moon> here for the absolute latitude <of the moon> there. The solar eclipse has no (i.e., zero) duration <of totality>.

Chapter 16: On drawing the figure of a solar eclipse.

We draw a straight line <segment> of arbitrary length. We divide it by the number of the minutes of <the sum of> the two radii. We draw a circle with a radius equal to that <line segment>, so that its radius will be equal to this line segment. It is <called> 'the circle of <the sum of> the two radii'. We draw two of its diameters which intersect in the center at right angles. We write around it the four directions: east opposite to west, and north opposite to south. Then we take <a part> from the line <segment> equal to the radius of the sun, and we draw a circle with a radius equal to that <part> and centered at the center of the circle of <the sum of> the two radii. It is <called> the circle of the sun. Then we take <a part> from the line <segment> equal to the apparent latitude. We put one arm of the compasses on the center of the two circles, and the other <arm> where it occurs on the north or south line, depending on the direction of its apparent latitude. We make a mark there to stand for the center of the moon in the middle of the solar eclipse. Then we take <a part> from the line <segment> equal to the radius of the moon. We take the mark as the center and draw the circle of the moon around it. The portion of the circle of the sun which falls in the circle of the moon is the amount of its (i.e., the sun's) eclipsed part.

Chapter 17: On <finding> the altitude of the moon taking account of its latitude.

Ptolemy and the experts in the art <of astronomy> who followed him, all calculated the magnitude of the parallax of the moon in the altitude circle and the measures of the six angles, which we have <already> mentioned, assuming the moon to have no latitude at all. They found the longitudinal and latitudinal parallax <of the moon> by substituting straight lines for the arcs of small circles. There is no noticeable disadvantage in what they did to <find> the latitude, except that precision has superiority to approximation, and exactness is more accepted <by people> than approximation. It was possible for us <to find> a method with proof which is not much different from the first <method> in difficulty and length, by which the altitude of the moon, its apparent latitude, and its longitudinal parallax is determined taking account of its latitude. It is <as follows:> We multiply the Cosine of the latitude by the Cosine of the distance of its <ecliptical> degree from the ascendant or descendant of the <given> time, whichever being less than 90 <degrees>, lowered. The result is a Sine. We find the <corresponding> arc, and subtract <the arc> from 90 <degrees>. The remainder is <called> 'the first arc'. Then we divide the Sine of the latitude by the Sine of 'the first arc', lowered. The result is <called> 'the Sine of the second arc'. We find the <corresponding> arc. If the latitude is northern, we add this arc to the complement of the altitude of the pole of the ecliptic. If the latitude is southern, we subtract it (i.e., the arc) from it. The sum or the remainder is <called> 'the result from the complement of the altitude of the pole'. Then we multiply the Sine of 'the first arc' by the Sine of 'the result from the complement of the altitude of the pole', lowered. The result is the Sine of the altitude <of the moon or the planet or star> taking account of the latitude of the moon or the other planet or star which has a <non-zero> latitude. The parallax in the altitude circle is obtained from this altitude.

Chapter 18: On <finding> the longitudinal and latitudinal parallax of the moon by a method <the validity of> which can be proved.

We have said in chapter 11 of this section that the altitude obtained by calculation is the true altitude that we would find if we observed from the center of the altitude circle. <If> the parallax is subtracted from it, <the remainder> is the apparent altitude <as observed> from the surface of the earth. Following what we have mentioned, <we add that the subject of> this chapter may occur in 5 cases. **First**: <The case in> which the altitude of the tenth <house> of the <given> time is 90 degrees, and the moon has no <non-zero> latitude. <Then> the parallax in the altitude circle is <the same> longitudinal parallax alone. It (i.e., the moon) has no latitudinal parallax. **Second**: <The case in> which the distance of the <ecliptical> degree of the moon from the ascendant of the <given> time is 90 degrees, the moon either having or not having a <non-zero> latitude. Then the parallax in the altitude circle affects the apparent latitude alone. It has no longitudinal parallax. **Third**: <The case in> which the altitude of the tenth <house> of the <given> time is 90 degrees, and the moon has a <non-zero> latitude. <For finding> the apparent latitude, we multiply the Sine of the latitude of the moon by the Cosine of the apparent altitude, and we divide <the product> by the Cosine of the true altitude. The result is the Sine of the apparent latitude. Its direction is the same as that of the latitude of the moon. As for the longitudinal parallax, we divide the Sine of the apparent altitude by the Cosine of the apparent latitude, lowered. The result is <considered> a Sine. We find the <corresponding> arc, and subtract it (i.e., the arc) from the distance of the <ecliptical> degree of the moon from the ascendant or descendant (whichever is closer). The remainder is the longitudinal parallax. **Fourth**: <The case in> which the altitude of the tenth <house> of the <given> time is less than 90

<degree>, and the moon has no <non-zero> latitude. <For finding> the apparent latitude, we multiply the Sine of the parallax in the altitude circle by the Sine of the altitude of the pole of the ecliptic, and divide <the product> by the Cosine of the true altitude. The result is the Sine of the apparent southern latitude. <For finding> the longitudinal parallax we divide the Cosine of the parallax in the altitude circle by the Cosine of the apparent latitude, lowered. The result is the Cosine of the longitudinal parallax. **Fifth**: <The case in> which the altitude of the tenth <house> is less than 90 <degrees>, and the moon has a <non-zero> latitude. <For finding> the apparent latitude, we multiply the Cosine of the latitude of the moon by the Cosine of the arc between its <ecliptical> degree and the ascendant or descendant of the <given> time, whichever is less than 90 <degrees>, lowered. The result is <called> 'the Sine of the first arc'. We multiply it by the Cosine of the apparent altitude. We divide <the product> by the Cosine of the true altitude. The result is <called> 'the Sine of the second arc'. We find the <corresponding> arc. Then we divide the Cosine of the apparent altitude by 'the Sine of the second arc', lowered. The result is <called> 'the Sine of the third arc'. We find the <corresponding> arc. We obtain the difference between it and the complement of the altitude of the pole of the ecliptic. The result is <called> 'the fourth arc'. We multiply the Sine of 'the fourth arc' by the Cosine of 'the second arc', lowered. The result is the Sine of the apparent latitude. If 'the third arc' is greater than the complement of the altitude of the pole of the ecliptic, the direction of the latitude is northern. If 'the third arc' is less <than that>, the direction of the latitude is southern. <For finding> the latitudinal parallax, we divide the Sine of the second arc by the Cosine of the apparent latitude, lowered. The result is <called> 'the Sine of the first arc'. We find the <corresponding> arc and keep it. Then we subtract <the distance> between the <ecliptical> degree of the moon and the ascendant or descendant, whichever is less than 90 <degrees>, from 90 <degrees>. We subtract the remainder from 'the first arc' which we kept. The <final> remainder is the longitudinal parallax.

Chapter 19: On extracting the longitudes of the localities.

We calculate a solar eclipse for the <geographical> longitude 90 <degrees>, and we find the time of its beginning or the time of the end of its emersion. Then we observe one of these two times in our locality, as accurately as possible. We obtain the altitude <of the sun> for this time. We find the <local> time from it. If the observed time is greater than the calculated <one>, then our locality is eastern with respect to <the meridian of> longitude 90 <degrees>. If the observed time is less <than

that>, then our locality is western with respect to <the meridian of> longitude 90 <degrees>. The difference between the calculated and the observed times is the <difference in> hours between the two longitudes. We multiply it by 15. The result is the longitude <difference> between the two localities. If our locality is eastern, we add it to the latitude 90 <degrees>. If it is western, we subtract it from the latitude 90 <degrees>. The sum or the remainder is the longitude of our locality. In a lunar eclipse, the altitude of the moon is not faultless because of its parallax, and it is difficult to obtain the precise altitude of the fixed stars, and we cannot rely on their true position. So, we may obtain the altitude of a planet whose true position we know. Then this planet will be <useful> like the sun in what we require. **Another method** is <as follows>. We find the true longitude of the sun for noon of the <given> day, relating to the <geographical> longitude 90 <degrees>. Then we observe its altitude at noon of that day <in our locality> with one of the reliable, precise devices for <measuring> the altitude. If the sun is in <one of> the northern <zodiacal> signs, we subtract the complement of the latitude of our locality from the altitude which was found. If the sun is in <one of> the southern signs, we subtract the altitude which was found from the complement of the altitude of the locality. The remainder is the declination of the sun. We find its <argument> arc in the table of declinations relating to the quadrant in which the sun <occurs>. The result is the position of the sun in our locality. We obtain the difference between it and the first true longitude. We enter it (i.e., the difference) in <the table for> the mean motion of the sun in <any number of> hours, and we obtain the corresponding <number of> hours. The result is the <difference> between the two longitudes in hours. We multiply it by 15. The result is the <difference> between the two <geographical> longitudes in degrees. If the position of the sun in our locality is <relating to a> smaller <zodiacal longitude> compared with its first position, our locality is eastern with respect to <the meridian of> longitude 90. Then we add <the difference> between the two <geographical> longitudes to 90 <degrees>. If its position in our locality is <relating to a> greater <zodiacal longitude>, then our locality is western with respect to <the meridian of> longitude 90 <degrees>. Then we subtract <the difference> between the two <geographical> longitudes from 90 <degrees>. The sum or the remainder is the <geographical> longitude of our locality. Whenever the sun is closer to the equinoctial points, <the result> is more correct because the declination is more distinct and its increments are greater here. **Another method**, used by the ancients by approximation, and from which are derived the <geographical> longitudes of most localities in the books and tables, is

<as follows>. We consider the <distance> between our locality and a locality of known <geographical> longitude and latitude in parasangs, and the <number of the> days <to travel> the road <between them>. Then we take one degree for each two days of <travelling> the road or for each 20 parasangs. We multiply <the number of the parts> by itself, bisect the result, and keep it. If the latitudes of the two localities are equal, we obtain the approximate square root of the bisected result. The <final> result is the longitude <difference> between the two localities. If the latitudes of the two localities are different, we subtract the less from the greater. We multiply the remainder by itself and subtract it from the bisected result. We obtain the square root of the remainder. The result is the longitude <difference> between the two localities. This is something obtained by approximation, not based on proof. A table is compiled <in this *zīj*> for the longitudes and latitudes of some localities. We have registered the famous localities in it, so that they (i.e., their coordinates) may be known approximately.

Chapter 20: On <determining> the visibility of the <lunar> crescent and the planets from <certain> arcs defined for them.

As for the <lunar> crescent visibility, none of the ancients spoke about it, because they knew (i.e., defined) the beginnings of the lunar months from the conjunctions. Each one of the moderns, when they needed the <anticipation of the lunar> crescent visibility for Islamic religious observances, worked out a chapter and a calculation <method> in this <matter> according to his own belief. For most inhabited <regions>, there is nothing general in it (i.e., their speculation about lunar crescent visibility) that can be relied upon. Their calculation thereof is not based on any valid rule and principle, and is not immune to the mistakes relating to the <degree of> clearness of the atmosphere and sharpness of the eyes <of the observer>. <Our method for> it is <as follows:> We obtain the distance between the two luminaries taking account of the latitude <of the moon>. This is <described> in chapter 5 of section 8 <of Book I of this *zīj*>. <We also obtain> the distance between the sun and the degree <on the ecliptic> which sets at the same time as the moon, <measured> in terms of descension degrees. The limit of the first arc is 10 degrees and of the second arc 8 degrees. Then we find the altitude of the moon at sunset or sunrise taking account of its latitude. We subtract from the result the parallax <of the moon> in the altitude circle. The remainder is <called> the 'visibility arc'. Its limit is 6 degrees. If the three arcs are <equal to> the above mentioned limits or greater than them, then the <lunar> crescent is visible. If they are less <than the

limits, then the lunar crescent> is not visible. If two of them witness to the visibility, then the judgment should be <based> on them. The difficult visibility will be related to the <third> deficient arc. If <the deficient arc> is the visibility arc, then <difficulty in visibility> is because of <small> altitude. If <the deficient arc> is the distance between the two luminaries, then it (i.e., the difficult visibility) is because of light insufficiency. If <the deficient arc> is the distance between them in terms of descension, then it (i.e., the difficult visibility) is because of the too small time it (i.e., the moon) remains above the horizon <after sunset>, and of the speed of its setting. **Another method**: We multiply the Sine of <the arc> between the <ecliptical> degree of the sun and the degree on the ecliptic which sets at the same time as the moon, by the Cosine of the pole of the ecliptic, lowered. The result is the Sine of the visibility arc under the earth (i.e., below the horizon). If <the sum of> the arc of visibility and the distance between the two luminaries taking account of the latitude <of the moon is> 18 <degrees> or more than that, then the <lunar> crescent will be seen. If it is less <than 18 degrees, then the lunar crescent> will not be seen.

The calculation of the visibility of the planets is similar to this. But in <the calculation of> their visibility we do not need <to consider their> parallax, and the distance between them and the sun. Their altitude is known (i.e., found) taking account of their latitudes. If that (i.e., the altitude) is <at least> equal to 'the visibility arc', <then the planet> is visible. If it is less, <then the planet> is not visible. The visibility arcs <for the planets> according to what was found in ancient <times is as follows:> For Saturn, 11°; for Jupiter, 10°; for Mars, 11; 30°; for Venus 5° and Mercury, 10°.

Commentary

I.6.1 The second method is equivalent to the first one, because multiplication by 10 minutes is equivalent to division by 6. The Arabic term *buht* for the daily motion of a planet is derived from the Sanskrit *bhukti* [al-Bīrūnī, 1910, II, 346; id., 1934, 105-106]. Table II.49 gives the hourly rate of the two luminaries as a function of their mean anomaly.

I.6.2 Multiplication of the daily motion by 0;33° is equivalent to multiplication of the hourly motion by $13\frac{1}{5}$, because 24 (0; 33°) = $13\frac{1}{5}$. The two methods are equivalent for the moon too, because 24(0; 2, 26°) = 0; 58, 25°. According to Kūshyār, the parameters 0; 33° and 0; 2, 26° are the constant ratios of the apparent diameter of the sun and the moon to their daily motion. Kūshyār's assumption of a constant ratio between diameter and daily motion is not accurate. According to Kepler's model, the ratio of $(\text{diameter})^2$ to daily motion is constant. Kūshyār's assumption of a constant ratio between the apparent diameter of the moon and the diameter of the earth's shadow is approximately true. The rules given in this chapter are found in al-Khwārizmī's *zīj* and may be traced beck to the Indian *zīj* Khandakhādyaka [al-Khwārizmī 1962, 57-59]. Table II.49 gives the diameter of the sun, the moon, and the shadow as functions of their mean anomaly.

I.6.3 Multiplication by 5 minutes is equivalent to division by 12. Here, for a first approximation of the ecliptical degree where the conjunction or opposition occurs, the hourly motion of the moon is taken 12 times that of the sun. Then if the distance of the sun and the moon on the ecliptic is *d*, we first suppose that they will be in conjunction after the sun has covered *d*/12 degrees and the moon has covered *d*+ *d*/12 degrees. This method is described by Ptolemy [1984, 281-82], and by al-Battānī [1899-1907, III, 142]. However, they divide *d*+ *d*/12 by the hourly motion of the moon, whereas Kūshyār divides *d* by the 'lunar gain'. The requirement that the sun and the moon should be in the same second is an exaggerated accuracy. By the expression "hours of the noon", Kūshyār means the number of the equinoctial hours elapsed since sunrise, at noon.
Worked example: On the day of conjunction, the sun is in Aries 24;59° and the moon is in Aries 26;39°. The distance between the two luminaries is: 26;39° – 24;59° = 1;40°.
The 'part of the distance' is: 1;40°×0;5=0;8,20°. The distance plus 'the part of the distance' is: 1;40°+0;8,20°=1;48,20°. Now we can find the ecliptical degree of the conjunction both by subtracting the 'part of the

distance' from the longitude of the sun and by subtracting the distance plus 'the part of the distance' from the longitude of the moon:

24;59° - 0;8,20° = 24;50,40°,

26;39° - 1;48,20° = 24;50,40°.

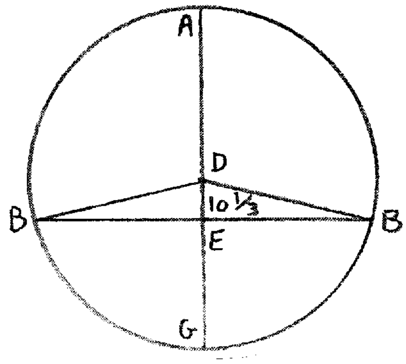
Then the ecliptical degree of the conjunction is Aries 24;50,40°.
Since in 30 days, the sun describes 30°, and the moon describes 30°+360° approximately, the lunar gain is about 12°/day or 30′/hour. We divide the distance by the lunar gain: 1;40°: 30′=3; 20. So the conjunction occurs 3 hours and 20 minutes before the noon, approximately. This example is based on al-Nasawī's example for this chapter (fol. 69 v). Al-Nasawī takes the lunar gain equal to 0;28,2°, so he finds the time of the conjunction 3 hours and 34 minutes before the noon (see the commentary on I.1.5). As can be seen in this example, the is no need to repeat the process of finding the time of the conjunction (as Kūshyār says at the end of this chapter).

I.6.4 Kūshyār expresses the magnitude of a lunar eclipse in two different ways. The 'absolute' magnitude of the eclipse is the length of the part of the line segment between the center of the moon and the center of the shadow which is inside both circles. The 'absolute' magnitude is expressed in (linear) digits, where 12 digits correspond to the diameter of the lunar disk. The 'adjusted' magnitude of the eclipse is the area of the common part of the two circles. The 'adjusted' magnitude is expressed in (area) digits, where 12 digits correspond to the area of the full moon [Kennedy 1956, 143]. The difference between 'the arc of the moon<'s disk>' and 'the arc of the sector' is only in the units of measurement. However, for finding 'half the arc of the sector', we should multiply "half" the circumference of the moon's disk (and not the whole circumference, as Kūshyār says) by the arc of the moon's disk, and divide the product by 360 degrees. Kūshyār proves his method in IV.6.1. Similar linear and area digits may be defined for a solar eclipse. Ptolemy used a similar method of finding the area digits for a special case [1984, 302-305]. Al-Battānī also used a similar method [1899-1907, III, 149-50]. Table II.52 of Kūshyār's *zīj* gives the adjusted digits as a function of absolute digits, from 1 to 12, for solar and lunar eclipses. It is a reproduction of the relevant table given by Ptolemy [1984, 308]. The same table is provided by al-Battānī [1899-1907, II, 890].

I.6.5 In IV.6.2, Kūshyār proves the validity of the calculation of the immersion time from the beginning of the partial eclipse and from the beginning of the total eclipse to the middle of the eclipse. In this chapter, 'adjusted value' means 'more precise value' for these times, in which the variation of the latitude of the moon is also considered. A proof of this more precise method is given in IV.6.3. The first method is provided by Ptolemy [1984, 300-301]. Both methods are presented by al-Battānī [1899-1907, III, 147-48].

I.6.6 A sample of such a drawing is provided in IV.6.4. A similar drawing is provided by al-Battānī [1899-1907, III, 154].

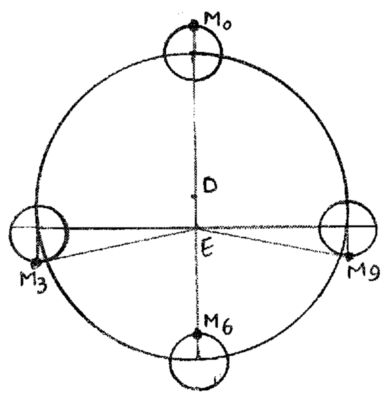
I.6.7 Kūshyār first finds the distance of the center of the moon's epicycle from the earth, where the maximal distance is assumed to be 60 "parts". Kūshyār's model for lunar motion is the same as that of Ptolemy; see [Pedersen 1974, 159-202] and [Ptolemy 1984, 173-254]. For the cases in which the double elongation of the moon is 0°, 6 zodiacal signs, and 3 or 9 zodiacal signs, the figure below justifies Kūshyār's calculation. The parameter 39⅓ is a rounding of the Ptolemaic one 39;22 parts [Ptolemy 1984, 251]. The parameter 5;1° is the maximum value of the epicyclic equation.



If the double elongation is between these values, in the figure drawn in IV.6.5, *ZD* is 'the product of the Sine' and *ZE* is 'the product of the Cosine'. Then it can be deduced from the figure that,

$$(BD^2 - ZD^2)^{\frac{1}{2}} = BZ \text{ , } EB = BZ + ZE.$$

Again, if the true anomaly of the moon is 0°, 6 zodiacal signs, and 3 or 9 zodiacal signs, the figure below justifies Kūshyār's calculation of the distance of the body of the moon from the earth.



If the true anomaly is between these values, then in the figure of IV.6.5, *IH* is 'the product of the Sine' and *IB* is 'the product of the Cosine'.

The computation of an "adjusted radius of the epicycle" in I.6.7 seems to be superfluous; instead of this one can simply take the radius of the epicycle, as in the proof in IV.6.5. We may also use table II.50 instead of calculation. A simplified version of this method may be used for the sun; however, it is rarely necessary, according to Kūshyār. As Kūshyār mentions in Chapter 11 of this section, 'part' means the unit of measurement, which is equal to the radius of the earth. The coefficient $18\frac{4}{5}$ is the ratio of the mean distance of the sun from the earth to the maximum distance of the moon from the earth. Kūshyār mentions this value, as well as the maximum, mean, and minimum distance of the sun from the earth, in the chapter 'On the distances and sizes of the celestial bodies', in Book III of this *zīj*. Ptolemy [1984, 250-51] and al-Battānī [1899-1907, III, 81-82] provide similar methods but for concrete numerical values. Kūshyār tacitly assumes that the distance of the apogee of the center of the lunar epicycle to the earth is 60 earth radii (Ptolemy's value is 59 earth radii).

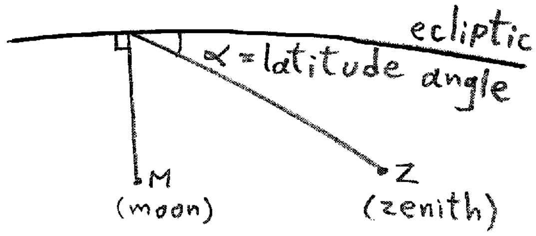
I.6.8 A proof for this method is given in IV.6.6. The "latitude of the clime of visibility" or "the latitude of visible climate" (*'arż iqlīm al-ru'ya*) was a standard term in Islamic spherical astronomy and probably taken over from the Hindus. It is the shortest distance from the zenith to the ecliptic, or the complement of the angle between the ecliptic and the horizon [Kennedy 1983, 168, 290].

I.6.9 A proof for this method is given in IV.6.7.

I.6.10 The case when the "remainder" in the last sentence is greater than 90° cannot occur. Kūshyār omits the cases where the point is between the ascendant and the fourth house, or between the fourth house and the descendant. Then the distance is between the right ascension of the point and the lower half of the meridian. There is no proof for this method in IV.6. The term 'ascension' in the title must refer to right ascension, because the latitude of the locality is not involved in this method. This chapter is not found in L. It is not mentioned in the table of contents at the beginning of B; however, the chapter belongs to the missing part of B. It is not mentioned in the table of contents of P, but the chapter itself is found in P. The term 'ascension' is missing in the title of the chapter in P.

I.6.11 A proof for this method is given in IV.6.8. Similar methods are provided by Ptolemy [1984, 256-64] and al-Battānī [1899-1907, III, 118]. The values for the solar parallax are given in table II.51.

I.6.12 This chapter provides the computation of the smaller angle between the ecliptic and the arc of the great circle passing through the zenith and the ecliptical position (perpendicular projection) of the moon (see the figure below). This angle is called the 'latitude angle' and its complement is called the 'longitude' angle. Kūshyār discusses six cases. He first treats the five special cases where the moon is situated on one of the equinoxes or solstices, and is in rising or setting position or at its culmination point, respectively. His sixth case is the general case. In 'the other method' provided in the fifth case, he first finds the complement of the right ascension of the equinoctial point (*KM* in the figure of IV.6.9) using the method provided in I.5.2. L and Y give only 'the other method' for the fifth case, but P gives both methods. Ptolemy [1984, 123-29] provides a special table for this angle, which is entitled "Table of zenith distances and ecliptic angles". Al-Battānī [1899-1907, III, 115] discusses different cases of this angle. See also [Pedersen 1974, 118-121, Neugebauer 1975, 48-50].



I.6.13 A geometrical proof of this method is given in IV.6.10. Al-Battānī presents a calculation method for this subject [1899-1907, III, 115], which is also given by Ptolemy [1984, 266]. This method presupposes that the relevant spherical triangles are considered as plane triangles. Kūshyār's rule for addition or subtraction of the latitudinal parallax is valid when the moon reaches the meridian. But when the moon is close to the horizon, especially near the equinoxes, so that a southern latitude may lead to an increase in the altitude, the rule is not valid.

I.6.14 In the description of the conditions in which the sun may be eclipsed, Kūshyār assumes that the case in which the distance of the moon from the zenith is northern (mentioned in Chapter I.6.13) will not occur. For finding the precise time of the middle of a solar eclipse, the time of conjunction is adjusted in two stages for the longitudinal parallax of the moon. The procedure for finding the magnitude of a solar eclipse in linear digits and area digits is like the procedure for lunar eclipses. Similar methods are provided by Ptolemy [1984, 310-13] and al-Battānī [1899-1907, III, 157-58, 164-65].

I.6.15 This method is similar to the method for lunar eclipses. See al-Battānī [1899-1907, III, 162].

I.6.16 An example of such a drawing is found in IV.6.11. Al-Battānī [1899-1907, III, 171] presents a similar drawing.

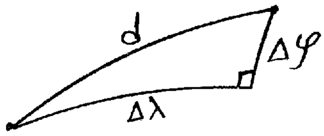
I.6.17 In IV.6.12, Kūshyār proves his precise method of finding the altitude of the moon. In the figure for Chapter IV.6.12, *IK* is the latitude of the moon, *IB* is 'the first arc', *HZ* is 'the second arc', and *ZA* is 'the result from the complement of the altitude of the pole <of the ecliptic>'. There is "another method" in this chapter which is not correct and authentic, so it has been omitted here. See the commentary to IV.6.12. The maximum difference in the parallax of the moon due to considering the latitude of the moon is equal to 5 minutes which is not negligible.

I.6.18 This is the continuation of the preceding chapter and contains Kūshyār's method for finding the precise values of the latitudinal and longitudinal parallax of the moon. This method that 'can be proved' is more precise than the method in I.6.13 which is approximate. By 'true altitude' he means the altitude of the moon if it is observed from the center of the earth. The first two cases are self-evident. In the third and fourth cases, F and C also give alternative methods for finding the longitudinal parallax, but the alternative methods are not correct and authentic. Y, B, and P give only the first method for finding this quantity, while L only gives the alternative method. So we have only quoted the first method here. F, C, V, Y, and even L provide the proof for the first method in the third and fourth cases. F, C and L give the proofs for the third, fourth, and fifth cases in IV.6.13.

I.6.19 Kūshyār presents three methods for finding the longitude difference between a given locality and another locality whose longitude and latitude are known. None of them is acceptable. The first method does not take into account the non-simultaneity of the solar eclipse phases as seen from different localities. Of course, finding the altitude of a planet at some moment (e.g., the beginning of emersion in lunar eclipses) in two localities is a practical way to calculate the geographical longitude difference. The second method does not consider that the change in the declination of the sun during a few hours is negligible. However, it is in principle correct, though not practical. As I will show below, the third method could be acceptable if we divided the distance in parasangs by 10 instead of 20, at the beginning. The length of an arc of one degree of a great circle on the earth is:

$$(40'000'000:6000)/360 \approx 18 \text{ parasangs}$$

For two localities with the same geographical latitude, we take the arc on the parallel circle for this latitude. Kūshyār's method in this case is equivalent to division by $10\sqrt{2}$. The length of an arc of one degree on the parallel circle is 18cosφ parasangs, where φ is the geographical latitude of the two localities. Then $10\sqrt{2}=18\cos\varphi$, and φ=38°13'. This is approximately for the geographical latitude for Kūshyār's position. For the case in which the geographical latitudes are different, Kūshyār simply applies the Pythagorean rule for the sides of a right triangle whose hypotenuse is the arc between the two localities and whose other sides are the arcs along the latitude parallel circle and the terrestrial meridian.



I.6.20 In this chapter, two methods for determining lunar crescent visibility are presented. The first method is based on the minimal values of three parameters: the distance between the two luminaries (10°), the distance between the sun and the ecliptical degree which sets simultaneously with the moon (8°), and the altitude of the moon at sunset in terms of its latitude (visibility arc) (6°). Kūshyār's second method is based on two parameters: the visibility arc below the horizon, and the distance between the two luminaries. The second method requires that the sum of these two parameters should not be less than 18°. Kūshyār presents here a rule for finding the visibility arc in the second method. A proof for this rule is given in IV.6.14. There Kūshyār says that the minimum values of the visibility arc have been found "from 6; 30° to 7°". B gives only the first method, and L mentions only the second method. Ptolemy dose not discuss lunar crescent visibility. The minimal values of the visibility arc for the planets in Kūshyār's *zīj* are the same as those given by Ptolemy [1984, 639-40]. Al-Battānī [1899-1907, III, 133] provides a method similar to Kūshyār's first method, but based on the first two parameters only. Al-Battānī does not use the visibility arc. He gives a greater minimal value for the two parameters: 13; 40° instead of 10° for the first parameter and 10; 50° instead of 8° for the second one. In his *Zīj-i Sanjarī* (no. 27 in Kennedy's *Survey*), Abū al-Fatḥ ʿAbd al-Raḥmān al-Khāzinī who lived about a century after Kūshyār, follows Kūshyār's methods for lunar crescent visibility. But al-Khāzinī provides the minimal values of the parameters as intervals rather than single values, considering the position of the moon in its orbit around the earth: 10°-12° (instead of 10°) for the first arc; 8°-12° (instead of 8°) for the second arc; and 6°-8° (instead of 6°) for the third arc. For the visibility arc below horizon, he mentions the interval 8°-10° instead of Kūshyār's 6;30°-7° (MS 682, ex-Sepahsālār library, Tehran, fol. 18r).

Section 7: On the operations relating to astrology, <in> 6 chapters

Chapter 1: On <finding> the distance between the <ecliptical> degree of a planet and the cardines in <terms of> hours.

If the planet is above the earth, we obtain its distance from the <cusp of the> tenth <house>, <whether it is situated> before or after <the tenth house>, <measured> on the <basis of the> right ascensions. If it is under the earth, we obtain its distance from the fourth <house>, <whether it is situated> before or after <the fourth house>, <measured> on the <basis of the> right ascensions. Then, if the planet is above the earth, we divide the distance by the <number of the> degrees in each hour for the <ecliptical> degree of the planet. If the planet is under the earth, we divide the distance by the <number of the> degrees in each hour for the <degree> opposite the <ecliptical> degree of the planet. The result is the distance of the planet from the tenth <house> or the fourth <house> cardine, <whether it is situated> before or after <the cardine>, in seasonal hours. If these <number of> hours are subtracted from 6, the remainder is the distance from the ascendant or the descendant.

Chapter 2: On <finding> the projection of the ray by means of equal (i.e., ecliptical) degrees.

Projections of the rays <measured> in equal degrees are the arcs of the ecliptic whose magnitudes are 60°, 90°, 120°, and 180°. If the planet has <non-zero> latitude, these arcs are obtained from a circle passing through the planet. Then they are transferred to the circle of the ecliptic. This is <found as follows:> We divide the Cosine of 60°, i.e. the Sine of 30°, by the Cosine of the latitude of the planet, lowered. Then we find the arc <corresponding to the quotient>. The result is the Sine of the difference between 90° and the arc of the sextile <ray> or <that of> the trine <ray>. We subtract it from 90°; the arc of the sextile <ray> remains. We add it (i.e., the difference) to 90°; the sum is the arc of the trine <ray>. As for <the arc of> the quartile <ray>, it is always 90°, and the <arc of> opposition is always 180°.

Chapter 3: On <finding> the projection of the ray by means of ascension (i.e., equatorial) degrees.

This is <similar> to the calculation of the equalization of the houses, except that it is found for ascensions of the horizon of the planet (i.e., the great circle through the planet and the North and South points of the horizon), in the same way that the equalization of the houses is

<calculated> for the ascensions of the horizon of the locality. The astrologers unanimously accept this <method of> equalization. If that is correct, it is suitable to correctly compute the projections of the rays by this calculation <method>. For this <purpose>, we need to know the <number of> degrees in each hour for the <ecliptical> degree of the planet, based on its position <with respect to the horizon>. To do this, we check the <ecliptical> degree of the planet. If it is <the same as> the degree of the <cusp of the> tenth <house> or that of the <cusp of the> fourth <house>, then the <number of the> degrees in each hour for them is 15. If it (i.e., the ecliptical degree of the planet) is <the same as> the degree of the ascendant or descendant, then the <number of the> degrees in each hour for them is <equal to the number of the> degrees in each hour for the ascendant or the descendant (see I.5.11). If it is between the two cardines, we take the difference between <the number of> the degrees in each hour of its (i.e., the planet's) <ecliptical> degree and 15, multiply it by the distance of the <ecliptical> degree from the cardine of the <cusp of the> tenth <house> or the <cusp of the> fourth <house> (as described in I.7.1), and divide it by 6. The result is the equation. If the <ecliptical> degree is between the <cusp of the> tenth <house> and the ascendant, or in the quadrant opposite to this one, and if 15 is the greater value, we subtract the equation from it; otherwise, we add the equation to it (i.e., to 15). If the <ecliptical> degree is between the ascendant and the <cusp of the> fourth <house> or in the quadrant opposite to this one, and if the <number of> degrees in each hour for the <ecliptical> degree is the greater value, we subtract the equation from it; otherwise, we add the equation to it. The result is the <number of> degrees in each hour for the <ecliptical> degree of the planet based on its position. Then we obtain the right ascension of the <ecliptical> degree of the planet, and we subtract from it the <number of> degrees in each hour for it, multiplied by 4. We find the arc corresponding to the remainder in the <table for the> right ascensions. The result is the position of the left sextile. The right trine is opposite to it. We also subtract from the right ascension of the <ecliptical> degree of the planet the <number of> degrees in each hour for it, multiplied by 6. We find the arc corresponding to the remainder in the <table for the> right ascensions. The result is the position of the left quartile. The right quartile is opposite to it. Then, we subtract the <number of the> degrees in each hour for the <ecliptical> degree from 30. We multiply the remainder by 4 and add it to the right ascension of the planet. We find the arc corresponding to the result in <the table for> these ascensions. The <final> result is the position of the right sextile. The left trine is opposite to it. This expression, i.e. the projection of the ray, has no correct meaning other than one of <the meanings given in> these two chapters.

Chapter 4: On <finding> the prorogations (i.e., astrological progressions).

The prorogations are of four types. One of them is 13 <ecliptical> signs in a solar year. It is <called> the minor prorogation, because it has the highest speed. The second <type> is one <ecliptical> sign in a solar year. It is <called> the medium prorogation. The third <type> is one ascensional degree in a solar year. It is <called> the major prorogation, because it has the lowest speed. The fourth <type> is the prorogation of the transfer indicators, like the mean motion of the sun. It is <called> the transfer prorogation. We have compiled two tables for the minor and medium prorogations, from which the degrees in the table may be obtained for any given <number of> months and days; or the <number of> months and days <may be obtained> for any given <number of> degrees.
The transfer prorogation is known from the table for the mean longitudes of the sun. For the major prorogation, we need <to carry out some> operation. Its calculation is <as follows:> We check if the <ecliptical> degree to be moved by prorogation is the degree of the <cusp of the> tenth or fourth <house>; or, if the <ecliptical> degree of the planet is in them, we subtract the right ascension of the <cusp of the> tenth or fourth <house> from the <right> ascension of the <ecliptical> degree in which the prorogation ends. <We take> one year for each degree, and 6 days for each minute of the remainder. During such <number of> years and days, the <ecliptical> degree moved by prorogation will reach the <degree> in which the prorogation ends. If the <ecliptical> degree moved by prorogation is the <ecliptical> degree of the ascendant, or if the <ecliptical> degree of the planet is in it, we subtract the oblique ascension of the ascendant from the <oblique> ascension of the <ecliptical> degree in which the prorogation ends. <We take> one year for each degree, and 6 days for each minute of the remainder. If the <ecliptical> degree <moved by prorogation> is the <ecliptical> degree of the descendant, or if the <ecliptical> degree of the planet is in it, we subtract the oblique ascension of the ascendant from the <oblique> ascension of the opposite of the <ecliptical> degree in which the prorogation ends. <We take> one year for each degree, and 6 days for each minute of the remainder.
If the <ecliptical> degree moved by prorogation is between two cardines, we obtain the right ascension and the oblique ascension of that <ecliptical> degree. We multiply the difference between the two ascensions by the distance of the <ecliptical> degree of the planet from the cardine rising prior <to it> in hours (as described in I.7.1). We divide <the product> by 6. The result is the equation. If the <ecliptical> degree is between the <cusp of the> tenth <house> and the <cusp of the> fourth <house>, or in the quadrant opposite to this, and the right ascension has

the greater value, we subtract the equation from it; otherwise, we add the equation to it. If the <ecliptical> degree is between the ascendant and the <cusp of the> fourth <house>, or in the quadrant opposite to this, and the oblique ascension has the greater value, we subtract the equation from it; otherwise, we add the equation to it. The result is the ascension of the <ecliptical> degree, based on its position.
Then we find the ascension of the <ecliptical> degree in which the prorogation ends, through a similar operation. However, we use in it the distance of the first <ecliptical> degree moved by prorogation (instead of the ecliptical degree of the planet) from the cardine which was used formerly, in hours. We use the ascension <here> as we used it in that <operation>. Then we subtract the ascension of the <ecliptical> degree moved by prorogation from the ascension of the <ecliptical> degree in which the prorogation ends. <We take> one year for each degree, and 6 days for each minute of the remainder.
If the <period of> time is known, and we want to know where the terminal point reaches from a given <ecliptical> degree during this <period of> time, <we carry out> **its calculation** <as follows>. If the given <ecliptical> degree is the <ecliptical> degree of the <cusp of the> tenth or fourth <house>, or if the <ecliptical> degree of the planet is in them, we add to its right ascension one degree for each year, and one minute for each 6 days, <considered> from a known time. We find the arc corresponding to the result in <the table for> right ascension. It will be the terminal point from that <ecliptical> degree. If the given <ecliptical> degree is the <ecliptical> degree of the ascendant or the descendant, or if the <ecliptical> degree of the planet is in them, we add to the <oblique> ascension of the ascendant one degree for each year and one minute for each 6 days, <counted> from a known time. We find the arc corresponding to the result in <the table for> oblique ascension. It will be the terminal point from the <ecliptical> degree of the ascendant. The opposite of this terminal point is the terminal point from the <ecliptical> degree of the descendant. If the given <ecliptical> degree is between two cardines, we add to the right ascension and the oblique ascension of the <ecliptical> degree one degree for each year, and one minute for each 6 days, <counted> from a known time. We find the arc corresponding to each one in <the table for> its ascension. Then we obtain the difference between the two arcs. We multiply it by the distance of the <ecliptical> degree from the cardine rising prior <to it> in hours, and divide it by 6. The result is the equation. If the <ecliptical> degree is between the <cusp of the> tenth and fourth <house>, or in the opposite <quadrant>, and the arc of the right ascension has the greater value, we subtract the equation from it; otherwise, we add the equation to it. If the <ecliptical> degree is between the ascendant and the <cusp of the> fourth <house> or in the

<opposite> quadrant, and the arc of the oblique ascension has the greater value, we subtract the equation from it; otherwise, we add the equation to it. The result is the terminal point from that degree. An example for this <operation:> The ascendant <is in> 4° of Pisces; the <cusp of the> tenth <house> <is in> 15° of Sagittarius; Venus <is> in 24° of Capricorn; and Mars <is> in 20° of Aquarius. We move Venus to the <ecliptical> degree of Mars by prorogation. It (i.e., Venus) reaches it in 23 years and 150 days. We want to know where the terminal point from Venus reaches at the completion of this <period of> time. It is <found> 20; 23° of Aquarius.

Chapter 5: On <finding> the transfers of the years and their ascendants.

In this chapter we need <to know> the mean longitude of the sun for the transfer and the time of the true longitude. It is the time for which we should find the true longitude of the planets for the transfer. <We also need> the time of the transfer and its ascendant. It should be known that when we subtract the <number of the> base year of the beginning, from the <number of the> year in which the transfer occurs, in the Yazdigird era, the remainder is the <number of the> entire years which followed this beginning. The transfer is the entering into the next year upon the sun's return to its original position. An example of this <follows:> The beginning occurred in the year 332. We want <to know> the transfer of the year in the year <3>89. We subtract 32 from 89. The remainder is 57. It is <the number of> the entire years which followed the beginning. The transfer is entering into the 58th year upon the sun's return to its original position. <Finding the> **transfer mean longitude**: We write down the true longitude of the sun for the base <year> somewhere on the dustboard to be known <during the operation>. Then we write it down in three positions, and subtract the adjusted apogee for the time of the transfer from the first position. The remainder is the <true> anomaly. We obtain the equation for it, and subtract it from the anomaly and from the second and third positions. Then we obtain the equation for this anomaly and add it to the second position. We check if it exceeds the true longitude of the epoch <year>. <If so,> we subtract the excess from the anomaly and from the third position. If it is less than the true longitude of the epoch <year>, we add the deficit to the anomaly and to the third position. We make the second <position> like the third <one>. Then we obtain again the equation for this anomaly and add it to the second position. We check if it exceeds the true longitude of the epoch <year>; <if it does,> we subtract the excess from the anomaly and from the third position. If it is less than the true longitude of the epoch <year>, we add the deficit to the anomaly and to the third position. We make the second <position> like the third

<position>. Then we obtain again the equation for this anomaly and add it to the second position. We check if it exceeds the true longitude of the epoch <year>; <if it does,> we subtract the excess from the anomaly and from the third position. If it is less than the true longitude of the epoch <year>, we add the deficit to the anomaly and to the third position. What results for the anomaly in this iteration, is the anomaly of the transfer. What results in the third position is the transfer mean longitude. <Finding> **the time of the true longitude** <is as follows>: For <finding> the time of the true longitude, we find, from the <relevant> table, the mean longitude of the sun for the beginning of the year in which the transfer occurs. Then <we find the mean longitudes> for the months and the days of the year, and the hours and their fractions, so that it will be equal to the mean longitude corresponding to the transfer. What results from the months, days and hours is the arc of revolution after midday in hours. <Finding the> **time of the true longitude from the arc of revolution in hours** <is as follows: If the <geographical> longitude of the locality is less than 90°, we obtain the difference between the longitude difference in hours and the equation of time in hours. If the longitude difference <in hours> is the greater value, we add it (i.e., the final difference) to the arc of revolution in hours. If the equation of time is the greater value, we subtract it from the arc of revolution in hours. The sum or the remainder is the time of the true longitude from the day or the night. Then we know its distance from the midday. If the <geographical> longitude of the locality is greater than 90°, we add the longitude difference in hours and the equation of time in hours, and subtract the sum from the arc of revolution in hours. The remainder is the time of the true longitude from the day or the night. Then we know its distance from the midday. **Section on the transfer time**: For <finding> the transfer time, if the <geographical> longitude of the locality is less than 90°, we subtract the <geographical> longitude difference in hours from the time of the true longitude. If the <geographical> longitude of the locality is greater than 90°, we add the <geographical> longitude difference in hours to the time of the true longitude. We add to the sum or the remainder the equation of time in hours. The result is the <time of> the transfer in hours after the midday. If it is less than the half-day hours, we add it to the time of the midday in hours. The result is the <number of the> hours elapsed <from the beginning of> the present day. If it is more than half-day hours, we subtract the half-day hours from it. The remainder is the <number of the> hours elapsed <from the beginning of> the next night. If it is greater than the sum of the half-day hours and the <number of the> hours of the next night, we subtract the <number of the> half-day hours and <the number of> the hours of the next night. The remainder is the <number of the> hours elapsed <from the beginning of> the next day. **The ascendant:**

When this (i.e., the transfer time) is found, we multiply it by 15. <The product> is the arc of revolution of the equator from the rising of the sun or from its setting until the time of the transfer. If it is <in> the day, we add it to the <oblique> ascension of the <ecliptical> degree of the sun. If it is <in> the night, we add it to the oblique ascension of the opposite of the <ecliptical> degree of the sun. We find the arc corresponding to the sum in the table for ascensions. The result is the ascendant. This operation <may be used> for finding the transfer of the sun into any ecliptical degree. The return of the sun to its <original> position <occurs> after <describing> a single cycle. The excess of the arc over a cycle of the equator is 86; 36°.

Chapter 6: On converting the ascendant of the world year from one locality to another.

We obtain the <geographical> longitude difference of the two localities in degrees. It is <equal to> the arc of revolution. If the second locality has a greater <geographical> longitude, we add the arc of revolution to the <right> ascension of the ascendant relating to the first locality. If the second <locality> has a smaller <geographical> longitude, we subtract the arc of revolution from the <right> ascension of the ascendant relating to the first locality. We find the arc corresponding to the sum or the remainder in the <table for> the <oblique> ascensions of the second locality. The result is the ascendant in the second locality.

Commentary

I.7.1 This is a method for finding the time interval that a point of the ecliptic needs for moving, by the daily motion of the universe, from its present position to one of the cardinal positions (in the horizon or meridian planes), or vice versa. The unit of measurement is the "seasonal hour"; that is one sixth of the time which the point needs to move from the last cardinal position before its present position to the first cardinal position after its present position. Kūshyār uses this computation in I.7.3 and I.7.4.

I.7.2 Ancient astrologers believed that each planet P casts seven visual rays to other points of the ecliptic whose positions are defined by the vertices of a regular hexagon, a square and an equilateral triangle with *P* as their common vertex. For a discussion of this theory see [Kennedy & Krikorian-Preisler 1972, 3-7; Hogendijk 1989, 170-72]. Kūshyār did not discuss this subject in his *Introduction to Astrology.* According to the *Jāmiʿ Zīj*, when the planet has a non-zero latitude, it casts its rays to the points on the ecliptic whose distances from the planet are 60°, 90°, etc. The ecliptical longitude of these points can be found by the method provided in this chapter, based on the Cosine theorem for spherical right triangles. For the sextile rays, we determine an arc $P(r)$ by:

$$\text{Cos } P(r) = R\text{Cos } 60° / \text{Cos } \beta,$$

where β is the latitude of the planet, and R = 60. If the ecliptical longitude of the planet is λ, it casts its two sextile rays to points with longitude $\lambda \pm P(r)$. A proof of this method is presented in IV.7.1. Al-Battānī provides another method for finding the projections of the rays [1899, III, 196-197], but his method is lengthy and complicated, as Kūshyār remarks in IV.7.1. Al-Bīrūnī provides two methods for finding the projection of the rays, taking the planet's latitude into account. His first method is similar to the method given by Kūshyār in this chapter [Kennedy & Krikorian-Preisler 1972, 6].

I.7.3 This alternative method can be shown to have the following geometrical rationale: first the position of the planet is projected onto the equator along arcs of great circles from the north point to the south point of the horizon. Kūshyār approximates this geometric determination by a computation which produces a very imperfect result, but which was nevertheless standard in his time. The method is described in terms of "hours" determined by the position of the planet in the ecliptic (compare I.7.1). When the planet is in its culmination point or the point opposite to it, each hour corresponds to 15 degrees of the celestial equator (i.e., the seasonal hour for geographical latitude 0°). When the planet is rising or setting, each hour is equal to the seasonal hour corresponding to the ecliptical degree of the planet (for the latitude of the locality; day hour [at rising] and night hour [at setting]). For other positions of the planet, the number of degrees on the celestial equator corresponding to each unequal hour may be found by linear interpolation. The results may be used for finding the whole series of the projections of the rays [Hogendijk 1989, 178-80].

I.7.4 The concept of 'prorogation' (or progression, in Arabic *Tasyīr*) was connected with an astrological method for anticipating important events in the life of a person, based on the positions of the celestial bodies at the moment of his or her birth [Yano and Viladrich 1991, 1-3]. Kūshyār discussed this subject in Chapter 20 and 21 of the third Book of his *Introduction to Astrology* [1997, 216-35]. M. Yano and M. Viladrich have discussed the content of Chapter 21 [Yano & Viladrich 1991]. Table II.53 of Kūshyār's *zīj* provides the minor and medium prorogations. The ms. F quotes a fragment "from another manuscript" under the misleading title "another method" after referring to the tables for minor and medium prorogations. But this is actually an explanation and example for the application of these tables, which does not sound authentic. The transfer prorogation is not mentioned in manuscript L. For the major prorogation, we should find the adjusted ascensions of the given point on the ecliptic. When the given point on the ecliptic to be moved by prorogation is in the tenth or fourth house, the adjusted ascensions are the same as the right ascensions. When the point is rising or setting, oblique ascensions are used for this purpose. If the given point is situated between the cardines, a combination of the right and oblique ascensions is applied, using linear interpolation. This is called the ascension of the ecliptical degree based on its position. The same process is carried out for the 'terminal point'. The required period of time is found by counting one year for each degree and 10 days for each minute of the difference between the initial and final adjusted ascensions. Inversely, for finding the terminal point relating to the major prorogation for a given period of time, a similar method is used. I checked the calculation of the example given for this case, using the right ascensions in table II.45 and the oblique ascensions for the latitude of 36° provided in table II.46: The result was in accordance with the terminal point given in the text with negligible deviation.

I.7.5 In this chapter Kūshyār wants to find the moment in a given year t, at which the true longitude of the sun has a given value c (equal to its true longitude at a definite moment a number of years ago). This problem is solved by an iterative procedure. Kūshyār first finds the position $A(t)$ of the apogee in year t. He uses $x_1 = c - A(t)$ as a first approximation of the mean centrum at *t*, and computes the equation $q(x_1)$. Then $x_2 = c \pm q(x_1)$ is a second approximation of the mean longitude, and so on. This method was also used by other Islamic period astronomers; see [Kennedy 1969, 249-50]. Al-Battānī deals with this matter in two places in his *zīj* [1899-1907, III,192-93, 223], but his approach is different from Kūshyār's. The mean longitude is then used for finding the hours relating to the arc of revolution after midday for the transfer. Then the number of the hours past midday is corrected for the geographical longitude difference and equation of time for finding the number of the hours relating to the true longitude in a locality with geographical longitude 90°. Then again the longitude difference and the equation of time are applied for finding the transfer times in different localities. The number of the hours past sunrise or sunset is used for finding the ascendant of the transfer time, using the ascension tables. The excess of revolution after a solar cycle is mentioned to be equal to 86; 36°. This magnitude divided by 360° gives the excess of a solar year over an integer number (365) of days, equal to 0.2405 of a day. The modern value for this magnitude is 0.2422 of a day.

I.7.6 The ascendant of the world year found for a locality can be used for finding the same ascendant for another locality with a different geographical longitude, using the oblique ascension tables, as described in this chapter. For a description of different systems of world years see [Kennedy 1962] and [Kennedy and van der Waerden 1963]. By the ascendant of the world year, Kūshyār apparently means the ascendant of the position of the prorogation (*Tasyir*) indicator on the celestial equator (see I.7.4 above) which makes a complete revolution on the celestial equator during one world year.