LETTER TO THE EDITOR

Angular distribution and polarization of gamma radiation emitted by aligned radioactive nuclei

Recently the influence of alignment of radioactive nuclei (by means of very low temperatures) on nuclear radiations has been established experimentally [1] [2]. In this note we give some theoretical results for the angular distribution and polarization of γ-radiation of aligned nuclei, which have already been used in the discussion of an experiment [3]. These results are valid for any degree of alignment (older results [4] give only the angular distribution if the orientation of the nuclei is rather small). The calculations can be made with the use of methods similar to those for the angular correlation of γ-radiation [5].

The angular distribution of dipole radiation by which the nuclear spin changes from $j_i \to j_i - 1$ is given by

$$\tfrac{1}{2}\, W(\vartheta) = 1 + \tfrac{3}{2}\, N_2 f_2 P_2 (\cos \vartheta). \tag{1}$$

The angular distribution of quadrupole radiation by which the nuclear spin changes from $j_i \to j_i - 2$ is given by

$$\tfrac{1}{2}\, W(\vartheta) = 1 - (15/7)\, N_2 f_2 P_2 (\cos \vartheta) - 5 N_4 f_4 P_4 (\cos \vartheta). \tag{2}$$

ϑ is the angle between the direction of emission $\mathbf{k}$ of the γ-quantum and the direction of alignment $\boldsymbol{\eta}$ (which is the axis of rotational symmetry for the orientation of the nuclei). $P_2 (\cos \vartheta)$ and $P_4 (\cos \vartheta)$ are the normalized Legendre polynomials. (1) and (2) are normalized in such a way that $\int W(\vartheta)\, d\Omega = 8\pi$. The quantities N_2 and N_4 are for a nucleus with spin j

$$N_2 = j/(2j - 1), \quad N_4 = j^3/(j - 1)\,(2j - 1)\,(2j - 3). \tag{3}$$

The orientation of the nuclei is represented by the relative populations a_m (for example given by a Boltzmann-distribution) of the different m-sublevels (m with respect to $\boldsymbol{\eta}$; $\Sigma_m a_m = 1$). Another way of representing the orientation is by means of the parameters f_k *); for $k = 1, 2, 3, 4$ these are given by (summation over m from $-j$ to j)

$$\begin{aligned}
f_1 &= (1/j)\, \Sigma_m\, m\, a_m, \\
f_2 &= (1/j^2)\, [\Sigma_m\, m^2 a_m - \tfrac{1}{3} j(j + 1)], \\
f_3 &= (1/j^3)\, [\Sigma_m\, m^3 a_m - \tfrac{1}{5} (3j^2 + 3j - 1)\, \Sigma_m\, m\, a_m], \\
f_4 &= (1/j^4)\, [\Sigma_m\, m^4 a_m - \tfrac{1}{7} (6j^2 + 6j - 5)\, \Sigma_m\, m^2 a_m + (3/35) j(j-1)(j+1)(j+2)].
\end{aligned} \tag{4}$$

The polarization of the γ-radiation is represented with the aid of the Stokes parameters (cf. e.g., [6])): three perpendicular unit vectors describe the two states of linear polarization (with polarization planes rotated over 45°) and the state of left circular polarization $\boldsymbol{\chi}_c \cdot \boldsymbol{\chi}_{|}$ is the state of linear polarization for which the electrical vector lies in the plane determined by

*) These parameters f_k are the statistical tensors $\langle | (j'j)\, kq \rangle$ introduced by Fano [7]), if $j = j'$, $q = 0$ (in case the state of the system has an axis of rotational symmetry) multiplied by

$$\frac{1}{j^k} \left[\frac{(2j + k + 1)!}{(2k + 1)\,(2j - k)!}\right]^{\frac{1}{2}} \left[\sum_{\nu=0}^{k} \left(\frac{k!}{\nu!(k - \nu)!}\right)^2\right]^{-1}.$$

$\boldsymbol{\eta}$ and $\mathbf{k}$. An arbitrarily polarized beam is characterized by a unit vector $\boldsymbol{\xi}_0$ (describing the way in which it is polarized) and P ($0 \leqslant P \leqslant 1$), its degree of polarization. If $P < 1$, the beam is partially polarized and can be described as a superposition of a totally polarized beam represented by $\boldsymbol{\xi}_0$ and an unpolarized beam.

The polarization of electric dipole radiation with $j_i \to j_i - 1$ is given by ($W(\vartheta)$ has the value according to (1))

$$W(\vartheta)\, P\, \boldsymbol{\xi}_0 = -(9/2)\, N_2 f_2\, (1 - \cos^2\vartheta)\, \boldsymbol{\chi}_{\parallel} + 3N_1 f_1\, (\cos\vartheta)\, \boldsymbol{\chi}_c. \qquad (5)$$

The polarization of electric quadrupole radiation with $j_i \to j_i - 2$ is given by ($W(\vartheta)$ has the value according to (2))

$$W(\vartheta) P \boldsymbol{\xi}_0 = [(45/7)\, N_2 f_2 (\cos^2\vartheta - 1) + (25/4)\, N_4 f_4 (-7\cos^4\vartheta + 8\cos^2\vartheta - 1)]\boldsymbol{\chi}_{\parallel} + [2N_1 f_1 \cos\vartheta + 5N_3 f_3\, (-5\cos^3\vartheta + 3\cos\vartheta)]\, \boldsymbol{\chi}_c. \qquad (6)$$

The quantities N_1 and N_3 are for a nucleus with spin j

$$N_1 = 1, \; N_3 = j^2/(j - 1)\,(2j - 1). \qquad (7)$$

We now consider the case that the γ-radiation is preceded by a β-radiation, which transforms a nucleus with spin j_0 into a nucleus with spin j_i, which emits the γ-radiation. We can give a relation between the f_k before and after the β-transition

$$(N_k)_0\, (f_k)_0 = (N_k)_i\, (f_k)_i. \qquad (8)$$

This is valid if the nuclear matrix element for the β-transition has a vectorial character and if $j_i = j_0 - 1$. The same formula is valid if the orientation of the nuclei is changed by a preceding quadrupolar γ-radiation with $j_i = j_0 - 2$.

The formulae given are sufficient for a discussion of the phenomena concerning the angular distribution and polarization of the γ-quanta emitted by a source with aligned ^{60}Co nuclei [1] [2]).

A full account of the theory also containing the formulae for other cases and a discussion of possibilities for experiments will appear later in Physica. We are indebted to Prof. S. R. de Groot for his interest and for valuable discussions.

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