

## NOISE AND HOT CARRIER EFFECTS IN A SINGLE INJECTION SOLID STATE DIODE

A. GISOLF and R. J. J. ZIJLSTRA

Fysisch Laboratorium, Rijksuniversiteit, Utrecht, The Netherlands

(Received 3 December 1973; in revised form 21 January 1974)

**Abstract**—In an earlier paper, analytical expressions for the scattering noise in single injection diodes operating far in the hot carrier regime were derived. In this paper the scattering noise is numerically calculated for the whole range of applied voltages of interest and, in addition, an extrinsic semiconductor is considered as starting material. The results are presented in two figures.

In an earlier paper [1] the noise due to scattering of charge carriers was calculated for a single injection solid state diode, operating in the hot carrier regime. The following restrictions were made: the treatment was one dimensional; only scattering by long wavelength acoustical phonons was considered; diffusion was neglected in the electric current; and carriers were assumed to be injected into a pure insulator.

An analytical expression for the spectral density of the a.c. open circuit voltage fluctuations was derived, valid for  $(\mu_0 V_0/L)/u \gg 1$ . Here  $\mu_0$  = low field mobility,  $V_0$  = bias voltage,  $L$  = contact spacing and  $u$  = sound velocity.

In this note numerical results will be presented for the noise at frequencies small with respect to the reciprocal carrier transit time, but without restrictions on the applied voltage. In addition we extended the calculations to the case of an extrinsic semiconductor, with carriers of the same type as those injected. In the usual notation we have the current equation for electrons, and Poisson's equation

$$\frac{I}{A} = -q\mu(E)n(x)E(x) + H(x, t) \quad (1)$$

$$\frac{\partial E}{\partial x} = -\frac{q}{\epsilon}[n(x) - n_d]$$

with  $A$  = area,  $I$  = current in the outer circuit, where  $H(x, t)$  is the Langevin source, describing the scattering fluctuations, and where  $n_d$  = density of fully ionized donors. Considering fluctuations around a steady state and making a Fourier analysis gives

$$\begin{aligned} \frac{\tilde{I}}{A} = \epsilon E_0 \mu(E_0) \frac{\partial \tilde{E}}{\partial x} + \tilde{E} \frac{d}{dE_0} [E_0 \mu(E_0)] \\ \times \left[ \epsilon \frac{dE_0}{dx} - qn_d \right] - \tilde{H} \end{aligned} \quad (2)$$

and

$$\frac{dE_0}{dx} = \frac{I_0}{\epsilon A} \frac{1 + (qAn_d/I_0)E_0\mu(E_0)}{E_0\mu(E_0)} \quad (3)$$

where Fourier transforms are indicated by tilde and steady state values are denoted by subscript 0. For the applied voltage we find with the help of equation (3)

$$V_0 = - \int_0^L E_0 dx = - \frac{\epsilon A}{I_0} \int_0^{E_L} \frac{E_0^2 \mu(E_0) dE_0}{1 + (qAn_d/I_0)E_0\mu(E_0)}. \quad (5)$$

Here the lower limit of the integral is determined by the boundary condition  $E(0) = 0$  [6, 1], whereas in a numerical calculation  $E_L$  can be found from the equality

$$L = \int_0^{E_L} \frac{dx}{dE_0} dE_0 = \frac{\epsilon A}{I_0} \int_0^{E_L} \frac{E_0 \mu(E_0) dE_0}{1 + (qAn_d/I_0)E_0\mu(E_0)}. \quad (6)$$

Note that  $I_0, V_0 \geq 0$  and  $E_0, E_L \leq 0$ .

For an a.c. open circuit ( $\tilde{I} = 0$ ) a formal solution of equation (2) gives  $\tilde{E}$  and  $\tilde{V} = - \int_0^L \tilde{E} dx$  in terms of  $\tilde{H}$ . We find, by changing variables and using partial integration

$$\tilde{V} = \frac{\epsilon A^2}{I_0^2} \int_0^{E_L} \tilde{H} \frac{(E_L - E_0)E_0\mu(E_0) dE_0}{[1 + (qAn/I_0)E_0\mu(E_0)]^2}.$$

Hence the spectral density of the voltage fluctuations is given by

$$S_V(f) = \frac{\epsilon^2 A^4}{I_0^4} \int_0^{E_L} \int_0^{E_L} S_H(x_1, x_2, f) \times \frac{(E_L - E_1)(E_L - E_2)E_1 E_2 \mu(E_1) \mu(E_2) dE_1 dE_2}{[1 + (qAn_d/I_0)E_1 \mu(E_1)]^2 [1 + (qAn_d/I_0)E_2 \mu(E_2)]^2}.$$

According to van der Ziel[2] and van Vliet[3] we have

$$S_H(x_1, x_2, f) = \frac{4q^2 Dn}{A} \delta(x_1 - x_2) = -\frac{4qI_0^2 D(E_1)[1 + (qAn_d/I_0)E_1 \mu(E_1)]}{\epsilon A^3 E_1^2 \mu^2(E_1)} \times \delta(E_1 - E_2)$$

where  $E_1 = E_0(x_1)$  and  $E_2 = E_0(x_2)$ , and  $D$  is the diffusion constant.

Hence

$$S_V = -\frac{4\epsilon q A}{I_0^2} \int_0^{E_L} \frac{D(E_0)(E_L - E_0)^2 dE_0}{[1 + (qAn_d/I_0)E_0 \mu(E_0)]^3}. \quad (7)$$

Note that  $n_d$  still may depend on  $x$ . In the following, however,  $n_d$  will be assumed space independent. Equations (5)–(7) are amenable to numerical calculations provided that the functional dependence of the carrier mobility and diffusion constant on electric field strength is known.

Earlier[1] we derived the following expressions for  $\mu$  and  $D$ :

$$\mu = \frac{2}{3} \frac{1_a q}{m} \left\langle \frac{1}{v} \right\rangle, \quad D = \frac{1}{3} 1_a \langle v \rangle \quad (8)$$

where  $1_a$  = electron mean free path for acoustical phonon scattering,  $m$  = carrier effective mass, and where brackets denote averages over the local velocity distribution:

$$f_0 = C \left( 1 + \frac{\frac{1}{2} m v^2}{p k T} \right)^p \exp \left( -\frac{m v^2}{2 k T} \right) \quad (9)$$

as derived by Davydov[4] and by Yamashita and Watanabe[5] for a homogeneous semiconductor. Here  $C$  = normalizing constant,  $p = q^2 E_0^2 1_a^2 / 6 m u^2 k T$  and  $T$  = lattice temperature. Using equations (8) and (9) numerical expressions for  $\mu$  and  $D$  as a function of electric field strength can be found.

In Figs. 1 and 2 results of numerical calculations are plotted for the current-voltage characteristic and the noise temperature vs bias voltage respectively. This was done by introducing the following dimensionless variables:  $I_0/I_c$ ,  $V_0/V_c$  and  $n_d/n_c$ , where  $V_c = E_c L$ ,  $n_c = (\epsilon/q) E_c / L$ ,  $I_c = q \mu_0 n_c A E_c$ ,

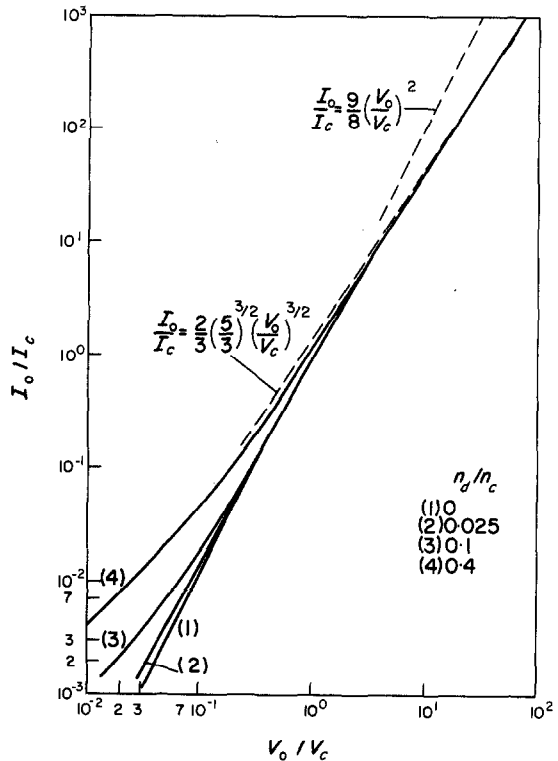


Fig. 1. d.c. current  $I_0$  plotted vs d.c. voltage  $V_0$  with the donor concentration  $n_d$  as a parameter. The quantities are presented in terms of the dimensionless variables  $V_0/V_c$ ,  $I_0/I_c$  and  $n_d/n_c$  where  $V_c = E_c L$ ,  $n_c = (\epsilon/q) E_c / L$ ,  $I_c = q \mu_0 n_c A E_c$  and  $E_c$  is the critical field-strength for the onset of hot carrier effects. The dashed lines represent analytical results derived before.

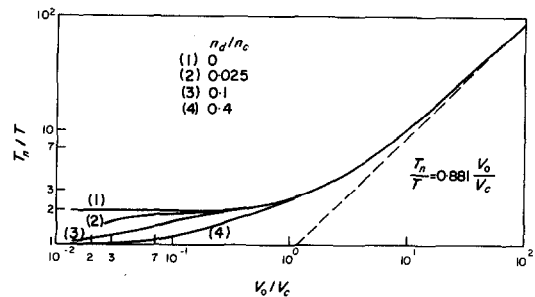


Fig. 2. The spectral density of the a.c. open circuit voltage fluctuations  $S_V$  plotted vs d.c. voltage  $V_0$  with  $n_d$  as a parameter. The noise is represented in terms of the equivalent noise temperature  $T_n = S_V / (4k \partial V_0 / \partial I_0)$ .  $V_0$  is represented in terms of  $V_0/V_c$  with  $V_c = E_c L$ . The dashed curve represents the analytical result derived before.

while  $E_c = 1.51 u/\mu_0$  as defined in ref. [1]. In Fig. 1,  $I_0/I_c$  is plotted vs  $V_0/V_c$  with  $n_d/n_c$  as a parameter.

Note that for  $V_0 \ll V_c$  and  $n_d \gg n_c$  a linear dependence is shown corresponding with Ohm's law.

For  $V_0 \ll V_c$  and  $n_d/n_c \rightarrow 0$  a square voltage dependence is shown corresponding with the well known Mott and Gurney law for carrier injection into pure insulators [6]. For  $V_0 \gg V_c$ , however,  $I_0$  is proportional to  $V_0^{3/2}$ , in agreement with the analytical expression derived in ref. [1].

In Fig. 2  $T_n/T$  is plotted vs  $V_0/V_c$  with  $n_d/n_c$  as a parameter where the noise temperature  $T_n$  is defined by  $S_V = 4kT_n (dV_0/dI_0)$ . Note that in the hot carrier regime ( $V_0 \gg V_c$ ) the results converge towards  $T_n/T = 0.881 V_0/V_c$  as derived before [1].

For  $V_0 \ll V_c$  we have  $T_n/T = 2$  for  $n_d/n_c = 0$  as expected for space charge limited flow in a pure insulation [7], and  $T_n/T \rightarrow 1$  for larger values of  $n_d/n_c$  as expected for an ohmic conductor.

#### REFERENCES

1. A. Gisolf and R. J. J. Zijlstra, *Solid-St. Electron.* **16**, 571 (1973).
2. A. van der Ziel and K. M. van Vliet, *Solid-St. Electron.* **11**, 508 (1968).
3. K. M. van Vliet, *Solid-St. Electron.* **13**, 649 (1970).
4. B. Davydov, *Physik. Z. Sowjetunion* **9**, 433 (1936).
5. J. Yamashita and M. Watanabe, *Prog. Theor. Phys.* **12**, 443 (1954).
6. M. A. Lampert and P. Mark, *Current Injection in Solids*, Academic Press, New York (1970).
7. A. van der Ziel, *Solid-St. Electron.* **9**, 1139 (1966).