

PHASE SHIFTS IN BALANCED MIXING

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A very simple argument to derive the properties of an optical balanced mixer system is given and the effect of eventual asymmetry is indicated.

In superheterodyne receivers the use of a balanced mixer has several advantages, especially in the suppression of local oscillator noise. The phase relationships required to produce the suppression are obtained for microwaves in a magic T, as shown by Pound [1] by detailed analysis of the fields and of currents and voltages in the various arms. He also mentioned examples of other circuits which are equivalent to the magic T in their behaviour. Van de Stadt [2] has recently developed an optical balanced mixer with similar advantages.

It can be shown that the phase relationships which are responsible for the major advantage of a balanced mixer are not specific to certain devices but are implied by simple energy considerations. For purposes of illustration we consider a 45° thin film, splitting a light beam into two equal portions as shown in figs. 1a and 1b, but the arguments apply equally well to any

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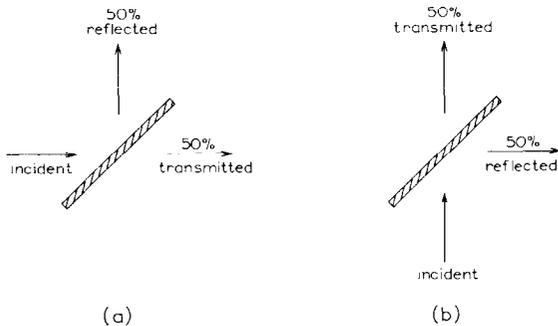


Fig. 1. Symmetric properties of the nonabsorbing 50% beam splitter.

four-terminal network in which energy coming from two of the directions is split equally into the other two.

Since the phase relationships must be independent of amplitudes we can choose, with no loss of generality, unit amplitudes for the incident waves. Then, using the notation illustrated in fig. 2, A_r , A_t , B_r and B_t must each have an amplitude $1/\sqrt{2}$, since power varies as the square of the amplitude. Resultant amplitudes can easily be seen from the construction of fig. 3 to be given by

$$R_{AB} = (1 + \cos \varphi_{AB})^{1/2}; \quad R_{BA} = (1 + \cos \varphi_{BA})^{1/2}.$$

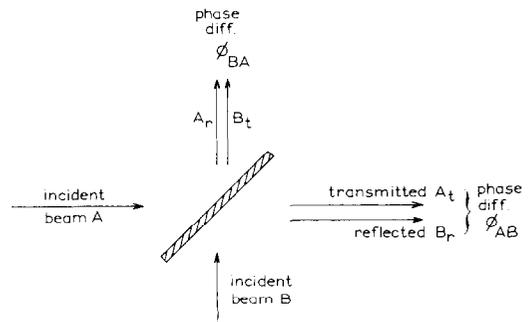


Fig. 2. Combination of two waves in a beam splitter.

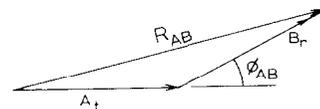


Fig. 3. Addition of wave amplitudes to generate a resultant amplitude.

Conservation of energy requires that $R_{AB}^2 + R_{BA}^2 = 2$, leading to

$$\cos \varphi_{BA} + \cos \varphi_{AB} = 0$$

satisfied by

$$\varphi_{BA} = \varphi_{AB} \pm \pi.$$

This is the result which makes possible a cancellation of local oscillator noise. If the detected signals are combined so that variations due to changes in local oscillator amplitude, say A, cancel, then those fluctuations due to beats between A and B will add.

This conclusion concerning phase shifts is readily generalized to other than 50-50% beam splitters by the assumption of transmission, reflection, and absorption coefficients which are the same for both directions of incidence. (It should be noted that fig. 1a implies fig. 1b.) In the general case where the splitter shows absorption it is possible for the absorption to be different for the two directions of incidence, but we consider first here only the case in which the film is symmetric. Consider again fig. 2. Let the transmission coefficient for both directions be α and the absorption coefficient κ ; thus the reflection coefficient ρ amounts to $1 - \alpha - \kappa$. We then have

$$A_r^2 = (1 - \alpha - \kappa)A^2, \quad B_r^2 = (1 - \alpha - \kappa)B^2,$$

$$A_t^2 = \alpha A^2, \quad B_t^2 = \alpha B^2,$$

and

$$A_r^2 + A_t^2 = (1 - \kappa)A^2, \quad B_r^2 + B_t^2 = (1 - \kappa)B^2.$$

Hence

$$R_{AB}^2 = \alpha A^2 + (1 - \alpha - \kappa)B^2 + 2AB\alpha(1 - \alpha - \kappa) \cos \varphi_{AB},$$

$$R_{BA}^2 = (1 - \alpha - \kappa)A^2 + \alpha B^2 + 2AB\alpha(1 - \alpha - \kappa) \cos \varphi_{BA}.$$

Adding these two expressions gives

$$R_{AB}^2 + R_{BA}^2 = (1 - \kappa)(A^2 + B^2) + 2AB\alpha(1 - \alpha - \kappa)(\cos \varphi_{AB} + \cos \varphi_{BA}).$$

But we also have

$$R_{AB}^2 + R_{BA}^2 = A^2 + B^2 - \kappa A^2 - \kappa B^2 = (1 - \kappa)(A^2 + B^2).$$

Hence we obtain again

$$\cos \varphi_{AB} + \cos \varphi_{BA} = 0.$$

Finally, it is easily shown in the same way that if the beam splitting film has asymmetric proportion, that is, if the transmission and reflection coefficients for beam A are α_A and ρ_A , and those for beam B are α_B and ρ_B , respectively, then

$$\cos \varphi_{AB} = \frac{\rho_A \alpha_B}{\rho_B \alpha_A} \cos \varphi_{BA}$$

and the phase difference now indeed differs from π by an angle δ which for small values of the quantity

$$\frac{\rho_A \alpha_B}{\rho_B \alpha_A} - 1$$

is given (in first order) by

$$\text{tg } \delta = \frac{(\rho_A \alpha_B / \rho_B \alpha_A) - 1}{\text{tg } \varphi_{AB}}.$$

REFERENCES

- [1] R. V. Pound, Microwave mixers (McGraw-Hill, New York, 1948) ch. 6.
- [2] H. van de Stadt, Appl. Opt., submitted for publication.