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Terrain Correction for Gravity Measurements, Using a Digital Terrain Model (DTM)

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ABSTRACT

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A single-term expression is given to calculate the gravitational effect for any square vertical prism with a sloping surface. A moderate measure of approximation is involved. The expression is well suited to automatic calculation of the terrain correction when a digital terrain model is available. The subjective or intuitive averaging of the elevation of a large sector is thereby diminished. For the square area immediately surrounding the gravity station, three geometrical models are presented, of which the one may be selected which best suits the local morphology as well as the desired accuracy.

INTRODUCTION

A digital terrain model (DTM), in which the topography is described by the elevation numbers on a square grid, offers the possibility of automatically calculating the terrain correction for gravity surveys. The DTM is stored in a rectangular array. For a sample area, two equivalent representations of the terrain are shown in Figs. 1 and 2. These are: a simplified contour map and a stacked-profile map, respectively. The SW corner is also given in Table I, as a part of the DTM array. The sampling grid is 100 m$\times$100 m.

The elevation number is assumed to represent the elevation of a square terrain block with sides equal to the grid unit. This first approximation may or may not be permissible, depending upon the grid unit size, the morphology and the required accuracy. The square area in the SW corner of Fig. 2 is shown in Fig. 3, using this first approximation. Details of the construction are shown separately in Fig. 4.

For any grid point 0, it is possible to calculate the gravitational effect of any other terrain block. If the height of the block is taken equal to the difference

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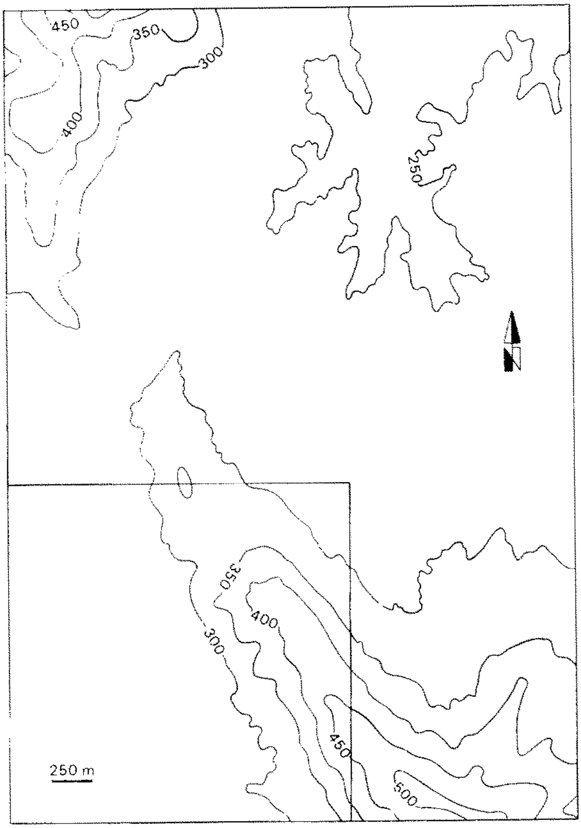

Fig. 1. Contour map of sample area. Interval = 50 m.

in elevation between the block and the point 0, the calculated gravitational effect is identical to the terrain correction. The gravitational effect of a rectangular prism with horizontal upper and lower surfaces and vertical sides, has been known since 1830 (Everest, 1830). A more accessible paper by Nagy (1966) gives equivalent expressions. They are lengthy, occupying 24 terms and a multitude of factors.

If the upper and lower surfaces of such a prism slope centrally towards point 0, the expression reduces to 8 terms (Ketelaar, 1976). The expression is a function of the distance from 0 to the block in question: $K=f(i,j)$, where i and j are grid coordinates. It must be evaluated eight times, twice for each of the four edges of the prism. As with the first approximation, this second approximation of the terrain surface may or may not be permissible, again depending upon the grid unit, the morphology and the required accuracy. Fig. 5 shows the same area as before, but using this second approximation. Details are shown in Fig. 6.

TABLE I

SW corner of the sample area, given as elevation numbers

j \ i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	245	247	250	262	260	267	250	252	262	258	253	248	253	262	267	265	275
2	255	247	262	267	258	272	250	255	258	267	258	267	253	259	258	277	267
3	268	248	272	265	268	272	255	257	255	262	267	276	260	257	257	262	262
4	268	262	267	267	268	267	262	255	257	257	265	275	275	244	263	262	264
5	275	280	270	282	268	263	252	262	265	257	267	263	263	267	267	267	270
6	283	281	272	280	285	275	262	267	258	263	262	265	269	265	270	267	272
7	284	284	290	290	296	280	262	272	270	264	265	264	269	270	270	267	268
8	295	292	287	282	275	263	272	272	275	263	268	277	272	272	272	272	272
9	295	287	280	285	270	271	275	275	280	272	269	278	273	276	277	277	277
10	272	286	282	275	270	275	275	275	280	282	272	272	277	283	283	279	282
11	275	290	280	280	275	277	287	292	285	282	272	275	277	284	283	281	287
12	272	273	287	287	274	280	280	283	280	277	272	278	282	290	290	287	297
13	275	282	285	283	277	285	288	287	278	282	283	285	295	310	310	307	302
14	282	286	283	283	282	287	287	287	287	290	294	302	312	339	337	326	314
15	290	295	294	287	284	287	292	295	296	297	305	325	345	365	365	350	337
16	287	293	294	285	292	297	300	302	307	320	340	340	368	392	392	385	365
17	283	292	296	295	297	307	310	317	325	352	375	357	380	417	417	400	377
18	292	296	302	305	307	317	326	335	360	360	385	387	400	425	425	400	370
19	302	305	312	318	330	345	350	370	390	395	407	426	430	402	407	375	345
20	312	317	325	345	365	385	398	405	425	430	433	437	430	370	375	343	320
21	327	335	342	360	390	423	428	435	452	447	433	415	397	340	345	323	315
22	342	356	380	390	425	455	465	460	452	432	405	382	360	325	325	310	302
23	372	393	415	430	452	480	478	455	427	400	372	352	332	307	307	302	297
24	397	425	455	470	477	480	460	457	400	372	350	330	317	304	305	297	287
25	425	457	487	505	490	460	430	405	387	354	332	317	307	297	297	293	287
26	455	482	510	510	475	440	410	385	372	350	330	317	307	297	297	293	283

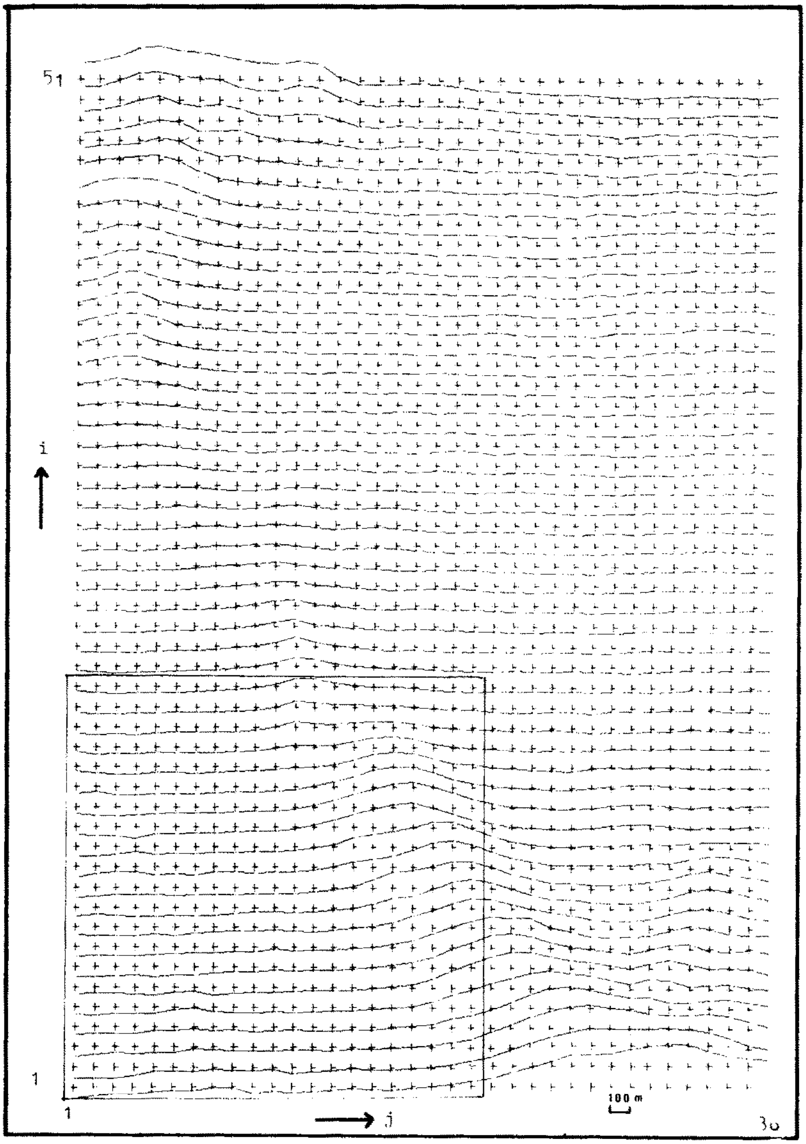

Fig. 2. The sample area represented as stacked elevation profiles. The Z scale is the same as the X and Y scale. The square in the SW corner indicates the area shown in Figs. 3 and 5.

Subjectively it is noted that this second approximation is more valid than the first in the vicinity of the point 0.

The expression for the prism with horizontal upper and lower surfaces can be simplified by a number of approximations, after which it becomes equivalent to the expression for the prism with sloping upper surface and horizontal

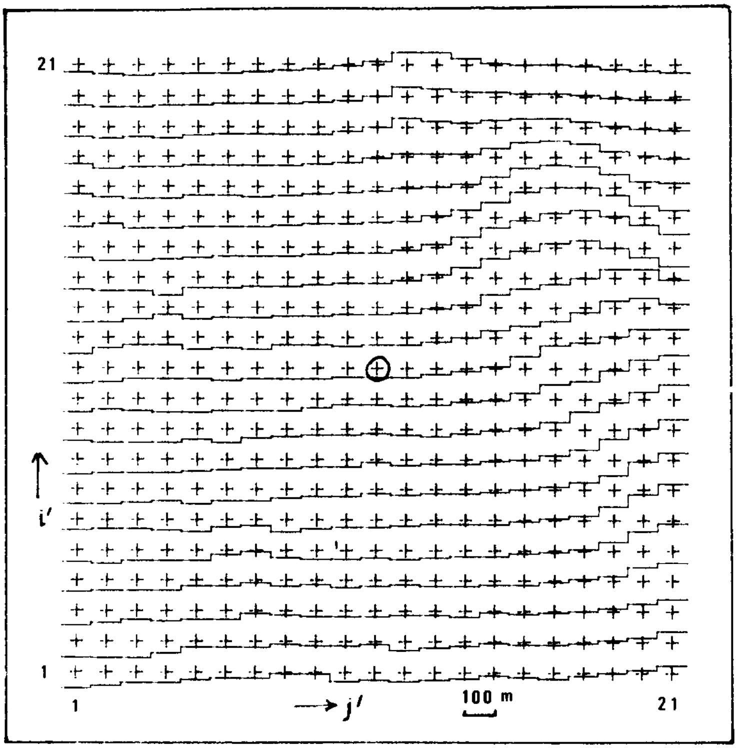

Fig. 3. SW corner of the sample area as stacked profiles with horizontal upper surface in each prism. Circle = centre.

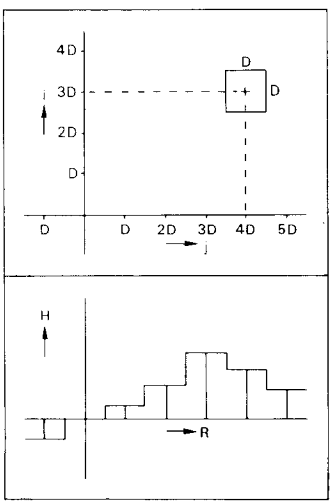

Fig. 4. Ground plan of the terrain prisms with horizontal upper surfaces.

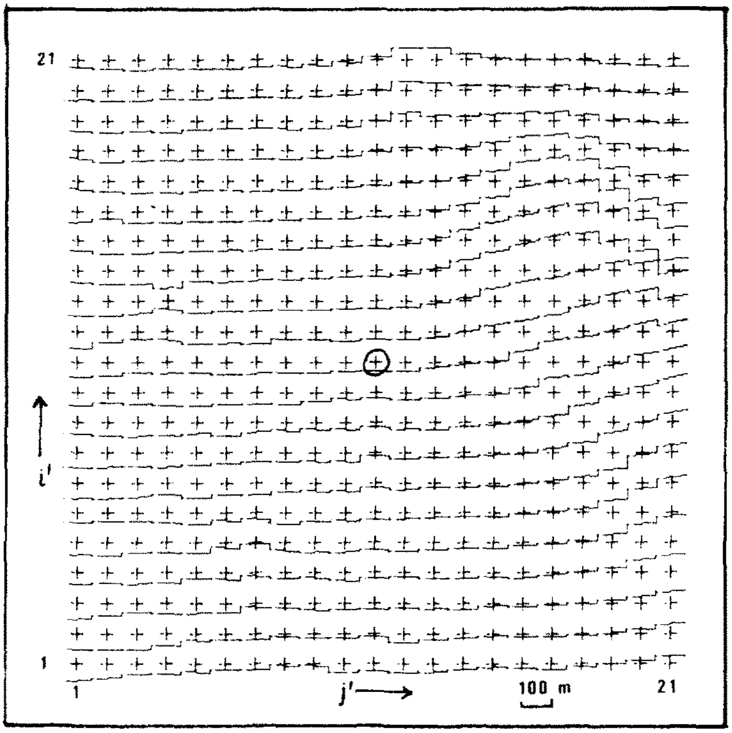

Fig. 5. SW corner of the sample area as stacked profiles with sloping upper surface in each prism. Circle = centre.

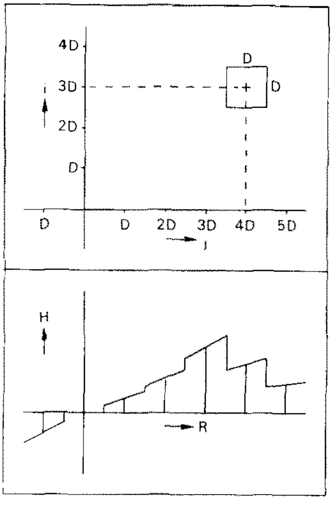

Fig. 6. Ground plan of the terrain prisms with sloping upper surfaces.

TABLE II

Relative error of $[K-(i^2+j^2)^{-0.5}]/K$, in percent

j	i: 0	1	2	3	4	5	6
1	3.666	2.428					
2	1.008	0.859	0.542				
3	0.457	0.416	0.327	0.237			
4	0.259	0.244	0.211	0.170	0.130		
5	0.165	0.160	0.140	0.122	0.102	0.085	
6	0.116	0.110	0.107	0.094	0.079	0.062	0.059

lower surface (or vice versa). This implies that the shape of the surface is not particularly important except in the immediate vicinity of the station 0.

Figs. 3 and 5 show an area with a size of 2 km×2 km, sampled on a grid of 100 m×100 m. To calculate the terrain correction in the centre 0, the contribution of each of the 440 prisms surrounding it, is evaluated and summed.

The function $K(i,j)$ can be calculated once for every (i,j) and stored in an array, after which the appropriate element can be called and used for each prism in turn. It would also be convenient to overlay the DTM array with a third array representing the rock density map of the area.

SIMPLIFICATION OF $K=f(i,j)$

The expression for $K=f(i,j)$ is given in an appendix by Ketelaar (1976), and its numerical value as well, for i and j up to 7 (inclusive).

It appears that the function K is almost equal to $(i^2+j^2)^{-0.5}$. Table II gives the relative error involved in this approximation. It is suggested that, even for the prism closest to 0, this error is so small that it is acceptable to use the approximation. The expression for the terrain correction then becomes very simple indeed:

$$g=G\sigma\ (1-\cos\alpha)D(i^2+j^2)^{-0.5} \quad (1)$$

where G is the gravitational constant, D is the grid unit and α is the slope from 0 to the prism's upper surface (or lower surface, as the case may be): σ is the density.

The angle α can be written explicitly as (see Fig. 7):

$$\alpha=ATN[\{h(i,j)-h(0,0)\}/\{D(i^2+j^2)^{0.5}\}]$$

Taking $R=D(i^2+j^2)^{0.5}$, eq. 1 can be written as:

$$g=G\sigma(1-\cos\alpha)D^2/R \quad (2)$$

This expression shows that, for a constant terrain slope α, the terrain correc-

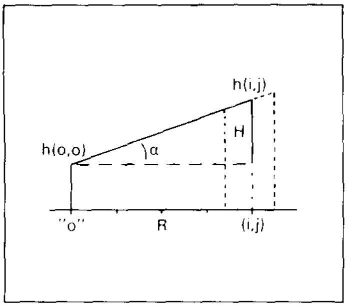

Fig. 7. Central angle α.

tion decreases linearly with the distance: this in spite of the linearly increasing height and therefore linearly increasing mass. Eq. 2 is used in Table III, which can be used to estimate the largest distance at which the terrain must be taken into account, when using prisms of 100 m × 100 m. Alternatively, when taking a constant height for all prisms outside the observation station in 0, eq. 2 gives the values shown in Table IV.

EXAMPLE

The area in Figs. 1 and 2 has the size of 3.5 km × 5 km. It was sampled on a square grid of 100 m × 100 m, so that the DTM array consists of 1836 elements.

TABLE III

Terrain correction in mgal for a prism of 100 m × 100 m and a height such that α = constant; density = 2670 kg m^{-3}

α	R = 300	600	900	1200	1500 (m)
10°	0.0090	0.0045	0.0030	0.0023	0.0018
20°	0.0358	0.0179	0.0119	0.0090	0.0072
30°	0.0795	0.0398	0.0265	0.0199	0.0159

TABLE IV

Terrain correction in mgal for prisms of 100 m × 100 m with a given centre height at different distances; density = 2670 kg m^{-3}

H	R = 300	600	900	1200	1500 (m)
100	0.0305	0.0040	0.0012	0.0005	0.0003
200	0.0970	0.0152	0.0047	0.0020	0.0010
300	0.1739	0.0313	0.0102	0.0044	0.0023
400	0.2375	0.0498	0.0171	0.0076	0.0040

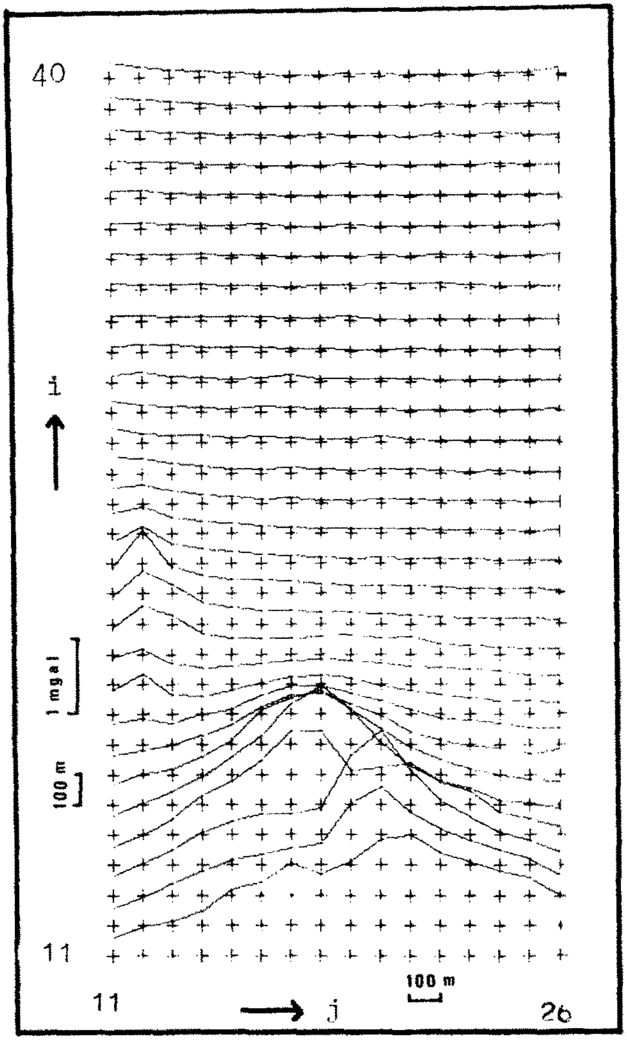

Fig. 8. Stacked profiles of the terrain corrections calculated for features up to 1 km in the cardinal directions.

For a gravity survey in this area, the terrain correction was required for features within a distance of 1 km. Using eq. 1, the terrain correction was obtained for 496 stations in a rectangle of 1.5 km$\times$3 km. Table V gives the values in milligals, using a density of 2670 kg/m^3. The table covers only the 11 southernmost lines. The corrections for the entire area are shown in the form of a stacked profiles map in Fig. 8. The correction accounts for all elements inside a square of 2 km$\times$2 km and not merely a circle with a diameter of 2 km.

The gravity stations in this survey were spaced with an interval of 25 m. The

TABLE V

S part of the sample area: terrain corrections in mgals, accounting for features up to 1 km in the cardinal directions; density 2670 kg m^{-3}

j \ i	11	12	13	14	15	16	17	18	19	20	21	22
11	0.251	0.252	0.277	0.258	0.267	0.294	0.277	0.295	0.335	0.361	0.372	0.378
12	0.395	0.386	0.430	0.419	0.422	0.439	0.332	0.345	0.544	0.488	0.643	0.712
13	0.467	0.571	0.611	0.626	0.581	0.496	0.409	0.296	0.265	0.292	0.529	0.520
14	0.613	0.725	0.852	0.922	0.796	0.649	0.532	0.408	0.255	0.228	0.284	0.294
15	0.885	0.864	0.998	1.135	1.057	0.869	0.673	0.473	0.322	0.244	0.238	0.224
16	0.988	0.946	1.095	1.416	1.329	1.268	0.910	0.666	0.429	0.299	0.234	0.198
17	1.222	1.022	1.098	1.797	1.752	1.428	1.067	0.783	0.506	0.338	0.239	0.187
18	1.094	1.098	1.180	1.781	1.988	1.539	1.088	0.786	0.529	0.356	0.248	0.173
19	1.244	1.640	1.871	1.266	1.672	1.270	0.947	0.694	0.495	0.340	0.277	0.164
20	1.548	1.851	2.200	1.312	1.236	1.009	0.798	0.581	0.399	0.289	0.238	0.150
21	1.610	1.510	1.672	1.366	0.923	0.779	0.574	0.478	0.325	0.250	0.244	0.138
22	1.378	1.291	1.266	1.108	0.739	0.618	0.477	0.355	0.276	0.233	0.157	0.118
23	1.246	1.139	1.047	0.999	0.634	0.487	0.386	0.337	0.231	0.215	0.128	0.097
24	1.124	1.017	0.847	0.705	0.479	0.406	0.374	0.293	0.192	0.182	0.110	0.092
25	1.044	0.896	0.736	0.629	0.436	0.353	0.300	0.222	0.183	0.138	0.099	0.082
26	0.830	0.694	0.590	0.525	0.374	0.323	0.342	0.231	0.164	0.129	0.095	0.075

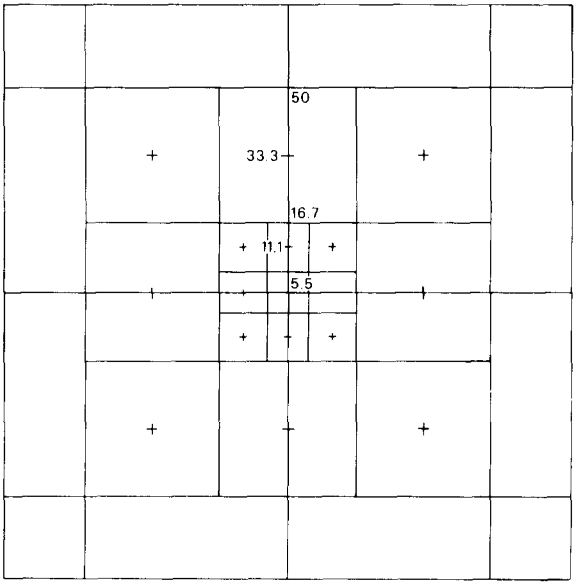

Fig. 9. Scheme for the inner square.

tabulated numbers were interpolated to obtain the correction for each station. Around each gravity station there remains an area of 4 squares of 50 m × 50 m. The effect of this area must be accounted for "by hand". This inner zone is discussed in the next section.

THE INNER SQUARE OF 100 m × 100 m

Two alternatives are open. Firstly eq. 1 can be used. It would be very costly to undertake a dense survey of the elevation of the area immediately around the gravity station. By successively reducing the grid unit D by a factor of 1/3 one can come as close as desired to the station, each reduction giving 8 smaller squares with the central, ninth, square remaining unprocessed immediately around the gravity station. The scheme is shown in Fig. 9, in which the numbers are based on the original value of D ($=100$). In the illustration D takes the values of 33.3 and 11.1. The central remaining zone consists of four squares of 5.5 m × 5.5 m and could be reduced by a further step, with $D=3.7$ and a remaining area of in all 3.7 m × 3.7 m.

The accuracy required for the elevations in the various squares can be estimated as follows. From eq. 1 we obtain:

$$\mathrm{d}g/\mathrm{d}H = G\,\sigma\,D^2\,H/(H^2+R^2)^{1.5}$$

from which:

TABLE VI

Levelling accuracy required for an accuracy of 0.001 mgal per prism (in m); density = 2670 kg m^{-3}

α(°)	Prism on axis	Prism in corner
2	1.61	3.22
5	0.65	1.30
10	0.33	0.67
15	0.23	0.47
20	0.19	0.37
25	0.16	0.32
30	0.15	0.30

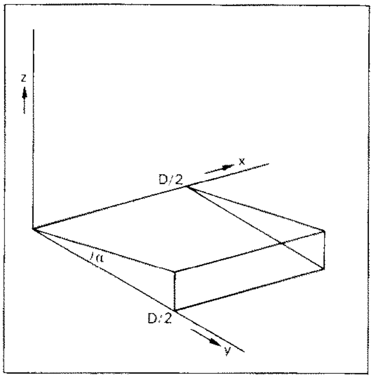

Fig. 10. Wedge-shaped terrain prism with the strike along an axis.

$$dH = dg\ (H^2 + R^2)^{1.5} / (G\ \sigma\ D^2\ H)$$

Under the condition that $i = 0$ and $j = 1$, valid for a prism on an axis, $D = R$; when $i = j = 1$, valid for a corner prism: $D = R/\sqrt{2}$.

These two conditions give the following two relations:

$$dH = dg/(G\ \sigma\ \sin\alpha\ \cos^2\alpha)$$

$$dH = 2dg/(G\ \sigma\ \sin\alpha\ \cos^2\alpha)$$

Setting the required accuracy dg at 0.001 mgal these two equations determine the accuracy with which the elevations must be known. Table VI gives the required accuracy for a few values of α.

In a number of cases a satisfactory accuracy can be obtained by using a combination of compass, rangefinder and clinometer from the gravity station's

TABLE VII

Terrain correction (mgal) for a wedge of 50 m×50 m with a surface slope α, having a density of 2670 kg m^{-3}

α (°):	2	5	10	15	20	25	30
Terrain corr. (mgal):	0	0.003	0.012	0.027	0.049	0.076	0.111
α (°):	80	85	86	87	88	89	89.5
Terrain corr. (mgal):	1.04	1.25	1.30	1.35	1.41	1.48	1.52

position to estimate the elevations in the eight squares. Note that the required accuracy is independent of the grid unit D.

The second method to approach the question is by considering a complete 50 m×50 m prism. The prism in question may show a plane upper surface which has its strike along the x-axis and a maximum slope α in the direction of the y-axis. The situation is shown in Fig. 10. The gravitational effect of this wedge is expressed as:

$$g = \int_{\text{vol.}} \frac{G \sigma z \, dx \, dy \, dz}{(x^2 + y^2 + z^2)^{1.5}} \tag{3}$$

with the following limits of integration: $x = 0$ to $D/2$: $y = 0$ to $D/2$ and $z = 0$ to $y \tan \alpha$. Integrating and using the boundary condition that for $\alpha = 0$, the gravitational effect equals zero:

$$g = G \sigma (D/2) \; [2 \log(1 + \sqrt{2}) + \cos \alpha \log(\cos \alpha) - \cos \alpha \log \{1 + (\cos^2\alpha + 1)^{1/2}\} - \log \{\cos \alpha + (\cos^2\alpha + 1)^{1/2}\}] \tag{4}$$

It is noted that an infinitely high scarp next to the gravity station is equivalent to an angle $\alpha = 90°$. This situation leads to: $g = 1.176 \, \sigma \, (D/2) \, 10^{-5}$ mgal for a prism of $D/2 \times D/2$; taking a density of 2670 and a prism of 50 m×50 m: $g = 1.57$ mgal. This situation is also approached for smaller angles.

Table VII shows the effect for some smaller slopes. The density was taken at 2670 and the prism sides were 50 m×50 m. The accuracy with which the slope α must be known in order to estimate the terrain correction with an accuracy dg, can be obtained by differentiating eq. 4, after which an evaluation of dα gives the results tabulated in Table VIII, which were calculated using dg = 0.001 mgal and a wedge of 50 m×50 m. Conversely, an estimate of the accuracy with which the wedge's slope is determined, leads to an estimate of the accuracy with which the terrain correction for that terrain block has been calculated.

TABLE VIII

Accuracy required for measuring the terrain slope α to obtain the terrain correction accurate within 0.001 mgal, valid for a wedge of 50 m×50 m and a density of 2670 kg m^{-3}

α (°):	2	5	10	15	20	25	30
dα (°):	2.1	0.8	0.4	0.3	0.2	0.2	0.1

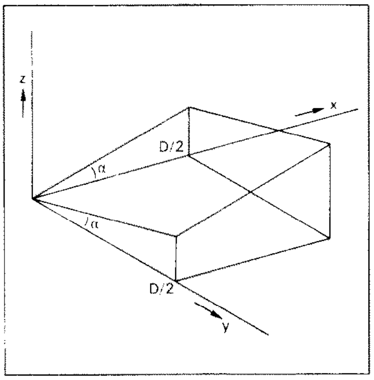

Fig. 11. Truncated prism with a central slope α towards the station.

A variant of the whole prism with sloping surface is shown in Fig. 11, where the strike is at 45° to the x and y axes.

The volume integration of eq. 3 is done within the following limits of integration: $x=0$ to $D/2$: $y=0$ to $D/2$: $z=0$ to $(x^2+y^2)^{\frac{1}{2}}\tan\alpha$. Actually this implies that the surface is not plane but conical: for all but the most extreme terrain slopes the distinction will hardly matter – the more so because we are dealing with calculations whereby approximations are present in all stages of the terrain representation.

Integrating eq. 3 and determining the constants of integration by evaluating the gravitational attraction when $\alpha=0$, gives a simple expression:

$$g=G\sigma\,(1-\cos\alpha)D\log(1+\sqrt{2}) \quad (5)$$

As is to be expected, the situation of an infinitely high vertical scarp adjacent to the gravity station provides the same value as in the previous case. Table IX gives the terrain correction for the same set of slopes as used in Table VII, and calculated for a prism of 50 m×50 m and a density of 2670 kg m^{-3}.

The accuracy with which the slope α must be measured for a given accuracy dg, may be obtained by differentiating eq. 5 and evaluating the resulting expression for dα. Table X provides some insight into the required and attainable accuracy.

TABLE IX

Terrain correction (mgal) for a truncated prism of 50 m×50 m with a slope α towards the station (density 2670 kg m^{-3})

α (°):	2	5	10	15	20	25	30
Terrain corr. (mgal):	0	0.006	0.024	0.053	0.095	0.147	0.210
α (°):	80	85	86	87	88	89	89.5
Terrain corr. (mgal):	1.30	1.43	1.46	1.49	1.51	1.54	1.56

TABLE X

Accuracy of the terrain correction d*g* and the slope error dα as a function of the terrain slope α for a truncated prism of 50 m×50 m with a density of 2670 kg m^{-3}

d*g* (mgal)	α (°): 5	10	15	20	25	30
0.001	0.4	0.2	0.1	0.1	0.1	0.1
0.005	2	1	0.7	0.5	0.4	0.3
0.01	4	2	1.4	1	0.9	0.7

CONCLUSION

The usual practice of lumping together hills and depressions inside one of the outer zones, in order to obtain an average height, leads to terrain corrections which are uncertain at best and subject to errors at all times. There is some evidence that the practice involves an "intuitive" averaging which therefore introduces noise in the final result. Calculating, let alone checking the terrain corrections is an expensive and troublesome business and every attempt at automating this process is welcome. Manual calculation on a widely spaced grid and interpolating for intermediate gravity stations, as is often done, may also lead to introducing noise into the completed, corrected profiles if, as may be the case, the terrain correction does not change linearly between the widely spaced points.

In most systems for calculating the terrain correction, the size of the outer zones increases. The correction for hills and valleys in a large outer zone should be calculated separately: when their elevations are averaged the calculated terrain correction will be too low.

The single-line expression given in this paper could be used for an automated terrain correction which avoids the abovementioned disadvantages. The three models presented for the inner zone, offer a choice which may be based on the actual shape of the terrain immediately surrounding the gravity station: the procedures outlined are essentially "manual", but do not require charts or tables.

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