ON THE THEORY OF ANGULAR CORRELATION OF TWO SUCCESSIVE GAMMA QUANTA EMITTED BY ORIENTATED NUCLEI

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We have calculated the angular correlation function of two γ-quanta in the directions $\mathbf{k}_1$ and $\mathbf{k}_2$ in the case that the quanta are emitted by nuclei orientated along an axis $\boldsymbol{\eta}$. The formula for this problem is complicated on account of the dependence on three directions, viz. $\mathbf{k}_1$, $\mathbf{k}_2$ and $\boldsymbol{\eta}$.

The orientation is described by the parameters f_k [1]) which are related to the relative populations a_m of the levels m by

$$f_k = \Sigma_m(-1)^{j-m} \langle jmj - m \mid jjk\,0 \rangle a_m.$$

The directions $\mathbf{k}_1$ and $\mathbf{k}_2$ are characterized by a rotation in space. S is the rotation which transforms $\mathbf{k}_1$ into $\boldsymbol{\eta}$ and T does the same with $\mathbf{k}_2$.

With the aid of the algebra of tensor operators developed by Racah [2]) [3]), we obtain a formula for arbitrary multipole radiations of the two quanta and arbitrary transitions $j_i \rightarrow j_e \rightarrow j_f$.

$$W(\mathbf{k}_1\mathbf{k}_2\boldsymbol{\eta}) = \Sigma_k f_k \Sigma_{\sigma k'} C_\sigma^{kk'}(j_i j_e j_f L_1 L_2)\, D_{0\sigma}^{(k)}(S) . D_{\sigma 0}^{(k')}(S^{-1}T).$$

$D^{(k)}$ is the $(2k+1)$ dimensional irreducible representation of the rotation group. $C_\sigma^{kk'}(j_i j_e j_f L_1 L_2)$ depends on the multipole orders of the radiations and the spin quantum numbers of the three nuclear states. Though this formula is rather complicated, it shows us the type of functional dependence on the directions $\mathbf{k}_1$ and $\mathbf{k}_2$. It can also be seen that the general formula for the angular correlation in the case of randomly orientated nuclei only contains Legendre polynomials, since in this case only the term with $k = 0$ is left. Legendre polynomials also occur if we integrate over the direction of the second quantum; then only the term with $k' = 0$ is left. In this case we find the formula for the directional distribution of γ-radiation emitted by orientated nuclei [4]).

In order to make an estimate of the order of the effect, we have derived formula which are suitable for numerical calculations in

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the cases of two successive dipole radiations and two successive quadrupole radiations [5]). In this transition we have assumed the spin quantum number to change as $j_i \rightarrow j_i - L \rightarrow j_i - 2L$ $(L = 1, L = 2)$. In these cases the correlation function is symmetric in $\mathbf{k}_1$ and $\mathbf{k}_2$. The numerical results give an effect of the same order as in the case of the directional distribution. Although in the correlation function more parameters f_k are involved, it gives practically no more information on the nuclear orientation since the f_k with higher values of k are negligible compared with the other terms. Nevertheless, the measurements on the angular correlation can give a valuable check of the theory. We add some remarks on a possible effect which is characteristic for the orientation.

We apply the formula to two successive quadrupole radiations with the nuclear transitions $j_i \rightarrow j_i - 2 \rightarrow j_i - 4$. This is the case for ^{60}Co which, after a β-desintegration, gives two successive γ-quanta of the above mentioned type. There are two directions of orientation $\boldsymbol{\eta}_1$ and $\boldsymbol{\eta}_2$. We now take $\mathbf{k}_1$ perpendicular to the plane of $\boldsymbol{\eta}_1$ and $\boldsymbol{\eta}_2$ and $\mathbf{k}_2$ in the plane of $\boldsymbol{\eta}_1$ and $\boldsymbol{\eta}_2$. The angles are defined by $(\boldsymbol{\eta}_1 \, \boldsymbol{\eta}_2) = \cos 68°$, $(\mathbf{k}_2 \, \boldsymbol{\eta}_1) = \cos 20°$ and $(\mathbf{k}_2 \boldsymbol{\eta}_2) = \cos 88°$.

For the relative population of the level m we have $a_m = \cosh \beta m$. a_m gives the orientation of the nuclei before the beta transition as the disorientating effect of the beta emission has been taken into account. Now the correlation probability $W_s(\beta)$ for this special directions only depends on β. The following result is then obtained. $W_s(\beta = 0) = W_s(\beta = \infty)$; this means the correlation probabilities for randomly orientated nuclei $(\beta = 0)$ and for totally orientated nuclei $(\beta = \infty)$ are equal. For $\beta = 0.8$ there is a maximum of about 12%. Thus, if a sufficiently high degree of orientation could be obtained, then the correlation probability would first increase and then decrease as the apparatus is warmed up during the experiment.

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